Rigorously Speaking, What are We?

Bringsjord v. Granger

Rigorously Speaking, What are We?

Bringsjord v. Granger

Questions re S5 Problems?

(Gotta read book if new to modal logic.)

Toward Metalogical Problem 2 — option 1

(Review Speedup!)

Toward Metalogical Problem 2 — option 1

(Review Speedup!)

```
(forall (n) (= (func n 1) (inc 1)))
```

Toward Metalogical Problem 2 — option 1

(Review Speedup!)

```
(forall (n) (= (func n 1) (inc 1)))
```

```
(forall (x) (= (func 1 (inc x)) (inc (inc (func 0 x)))))
```

Toward Metalogical Problem 2 — option 1

(Review Speedup!)

```
(forall (n) (= (func n 1) (inc 1)))
```

```
(forall (x) (= (func 1 (inc x)) (inc (inc (func 0 x)))))
```

```
(forall (n x) (= (func (inc n) (inc x) (func n (func (inc n) x)))))
```

Toward Metalogical Problem 2 — option 1

(Review Speedup!)

```
(forall (n) (= (func n 1) (inc 1)))
```

```
(forall (x) (= (func 1 (inc x)) (inc (inc (func 0 x)))))
```

```
(forall (n x) (= (func (inc n) (inc x) (func n (func (inc n) x)))))
```

```
(NatNum 1)
```

Toward Metalogical Problem 2 — option 1

(Review Speedup!)

```
(forall (n) (= (func n 1) (inc 1)))
```

```
(forall (x) (= (func 1 (inc x)) (inc (inc (func 0 x)))))
```

```
(forall (n x) (= (func (inc n) (inc x) (func n (func (inc n) x)))))
```

```
(NatNum 1)
```

```
(forall (n) (if (NatNum n) (NatNum (inc n))))
```

Toward Metalogical Problem 2 — option 1

(Review Speedup!)

```
(forall (n) (= (func n 1) (inc 1)))
```

```
(forall (x) (= (func 1 (inc x)) (inc (inc (func 0 x)))))
```

```
(forall (n x) (= (func (inc n) (inc x) (func n (func (inc n) x)))))
```

```
(NatNum 1)
```

```
(forall (n) (if (NatNum n) (NatNum (inc n))))
```

```
(NatNum (func 5 5))?????
```

Toward Metalogical Problem 2 — option 2

(Review k-order ladder)

Toward Metalogical Problem 2 — option 2

(Review k-order ladder)

Does $\mathscr{L}_3$ = TOL work in HyperSlate? Partially?
Not at all? What's possible and what's not?
What exactly is needed inference-rule-wise for a full natural-deduction system for TOL. Can a chatbot like GPT-4 or Bard etc. handle TOL reasoning challenges expressed in English?
What specimens do you have for your answer?

Some Roots of the Debate

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The grammar of mammalian brain capacity

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ABSTRACT

Uniquely human abilities may arise from special-purpose brain circuitry, or from concerted general capacity increases due to our outsized brains. We forward a novel hypothesis of the relation between computational capacity and brain size, linking mathematical formalisms of grammars with the allometric increases in cortical–subcortical ratios that arise in large brains. In sum, i) thalamocortical loops compute formal grammars; ii) successive cortical regions describe grammar rewrite rules of increasing size; iii) cortical–subcortical ratios determine the quantity of stacks in single-stack pushdown grammars; iv) quantitative increase of stacks yields grammars with qualitatively increased computational power. We arrive at the specific conjecture that human brain capacity is equivalent to that of indexed grammars – far short of full Turing-computable (recursively enumerable) systems. The work provides a candidate explanatory account of a range of existing human and animal data, addressing longstanding questions of how repeated similar brain algorithms can be successfully applied to apparently dissimilar computational tasks (e.g., perceptual versus cognitive, phonological versus syntactic); and how quantitative increases to brains can confer qualitative changes to their computational repertoire.

1. Brain growth shows surprisingly few signs of evolutionary pressure

Different animals exhibit different mental and behavioral abilities, but it is not known which abilities arise from specializations in the brain, i.e., circuitry to specifically support or enable particular capacities. Evolutionary constraints on brain construction severely narrow the search for candidate specializations. Although mammalian brain sizes span four orders of magnitude [1], the range of structural variation differentiating those brains is extraordinarily limited.

An animal's brain size can be roughly calculated from its body size [2], but much more telling is the relationship between the sizes of brains and of their constituent parts: the size of almost every component brain circuit can be computed with remarkable accuracy just from the overall size of that brain [1,3–5], and thus the ratios among brain parts (e.g. cortical to subcortical size ratios) increase in a strictly predictable allometric fashion as overall brain size increases [6,7] (Fig. 1).

These allometric regularities obtain even at the level of individual brain structures (e.g., hippocampus, basal ganglia, cortical areas). There are a few specific exceptions to the well-documented allometric rule (such as the primate olfactory system [8]), clearly demonstrating that at least some brain structure sizes *can* be differentially regulated in evolution, yet despite this capability, it is extremely rare for telencephalic structures ever to diverge from the allometric rule [4,6,7,9]. Area 10, the frontal pole, is the most disproportionately expanded structure in the human brain, and has sometimes been argued to be *selected* for differential expansion, yet the evidence has strongly indicated that area 10 (and the rest of anterior cortex) are nonetheless precisely the size that is predicted allometrically [6,7,10,11].

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The modal argument for hypercomputing minds

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Abstract

We now know both that hypercomputation (or super-recursive computation) is mathematically well-understood, and that it provides a theory that according to some accounts for some real-life computation (e.g., operating systems that, unlike Turing machines, never simply output an answer and halt) better than the standard theory of computation at and below the "Turing Limit." But one of the things we do not know is whether the human mind hypercomputes, or merely computes—this despite informal arguments from Gödel, Lucas, Penrose and others for the view that, in light of incompleteness theorems, the human mind has powers exceeding those of TMs and their equivalents. All these arguments fail; their fatal flaws have been repeatedly exposed in the literature. However, we give herein a novel, formal *modal* argument showing that since it's mathematically *possible* that human minds are hypercomputers, such minds *are* in fact hypercomputers. We take considerable pains to anticipate and rebut objections to this argument.

Keywords: Computationalism; Hypercomputation; Incompleteness theorems

1. Introduction

Four decades ago, Lucas [50] expressed supreme confidence that Gödel's first incompleteness theorem (= Gödel I) entails the falsity of computationalism, the view that human persons are computing machines (e.g., Turing machines). Put barbarically, Lucas' basic idea is that minds are more powerful than Turing machines. Today, given our understanding of hypercomputation in theoretical computer science, and given the absolute consensus reigning in cognitive science that the human mind is, at least in large part, *some* sort of information-processing device, we know enough to infer that if Lucas is right, the mind is a hypercomputer. However, Lucas' arguments have

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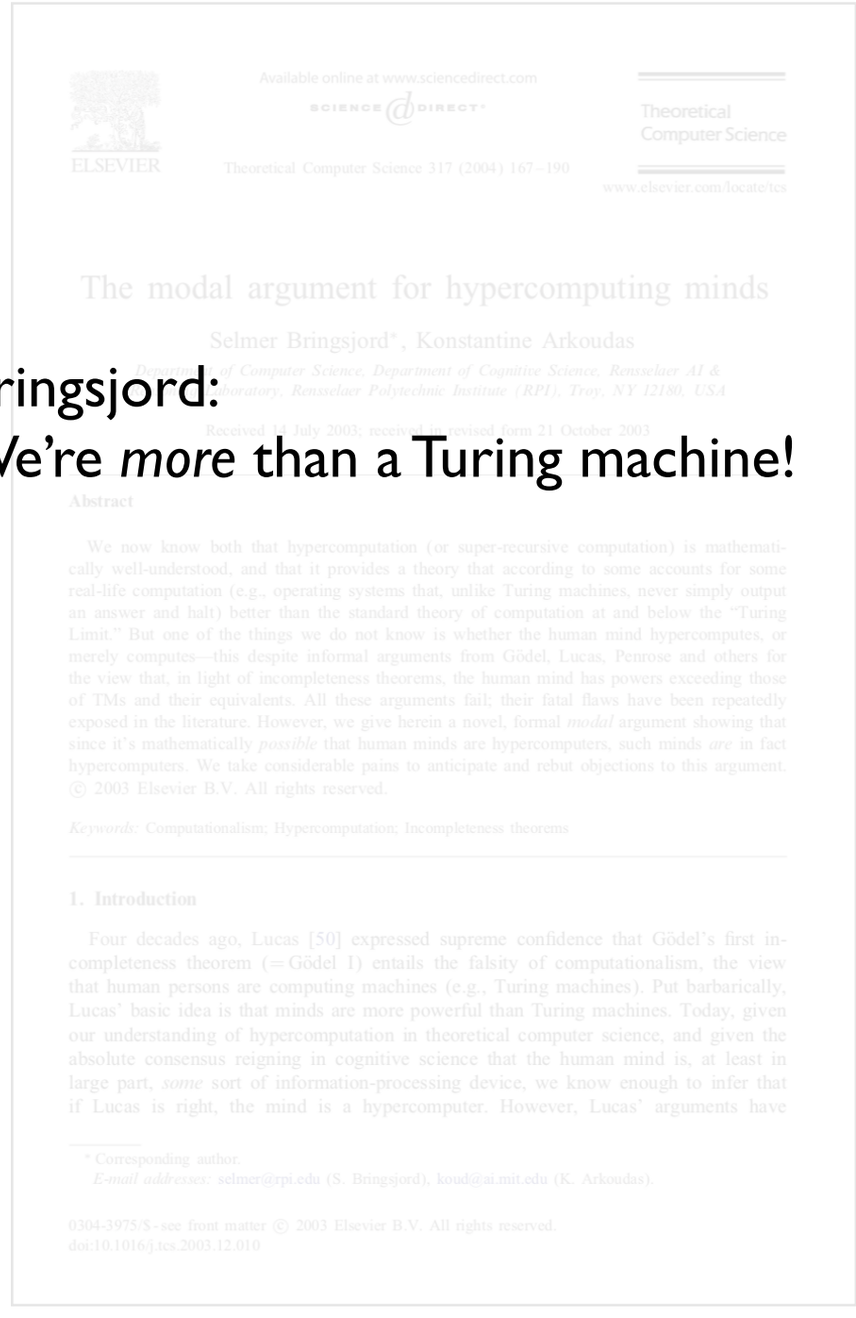
Some Roots of the Debate

Granger:
We're *less* than a Turing machine!



Bringsjord:
We're *more* than a Turing machine!

"I don't yet know how to handle 'non-linearity' in all of this, precisely. Maybe you can help. Here are some pointers, thoughts, initial constraints/structures …"

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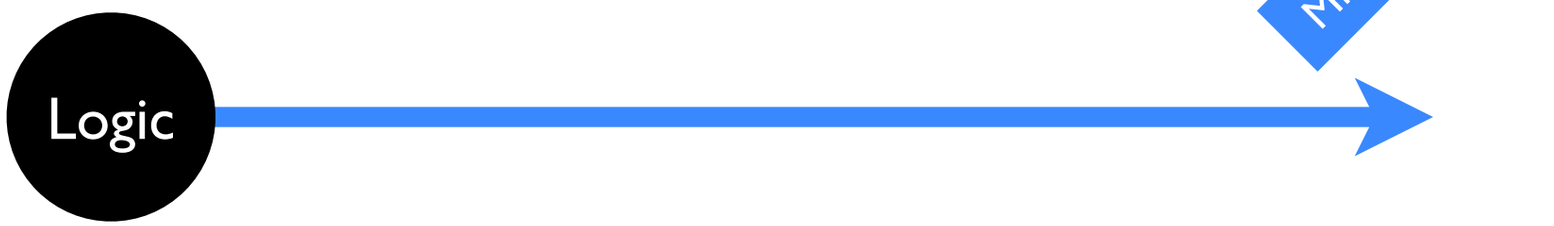
Logic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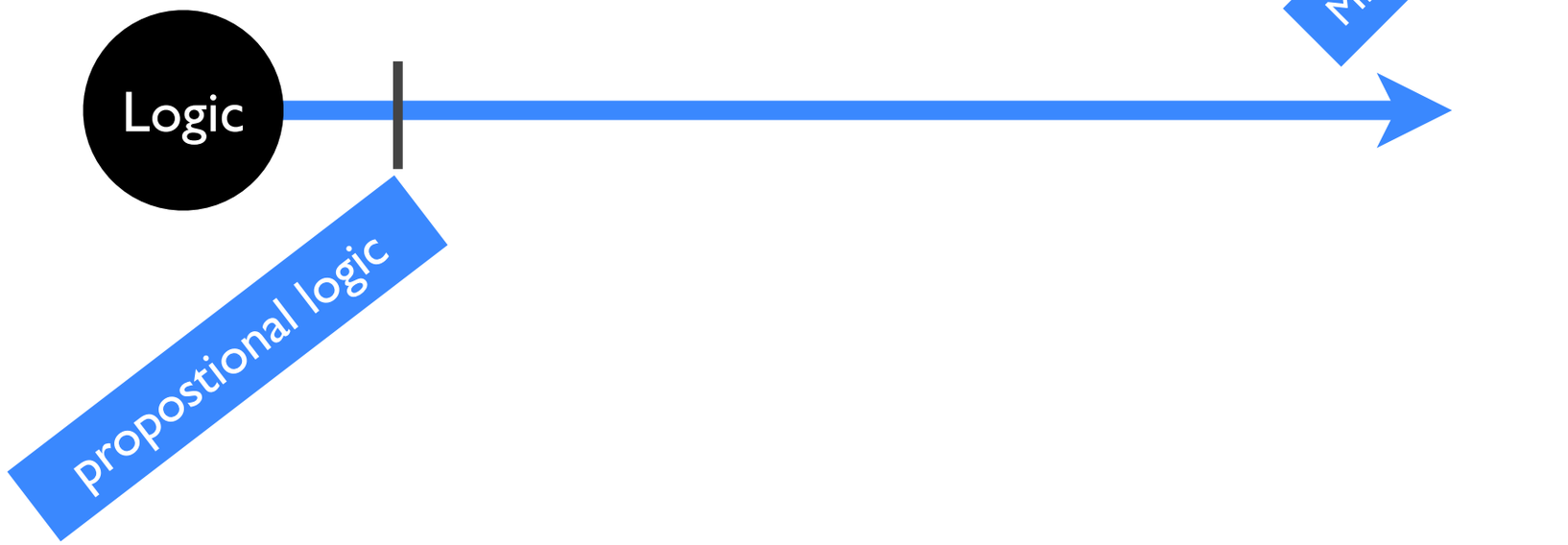
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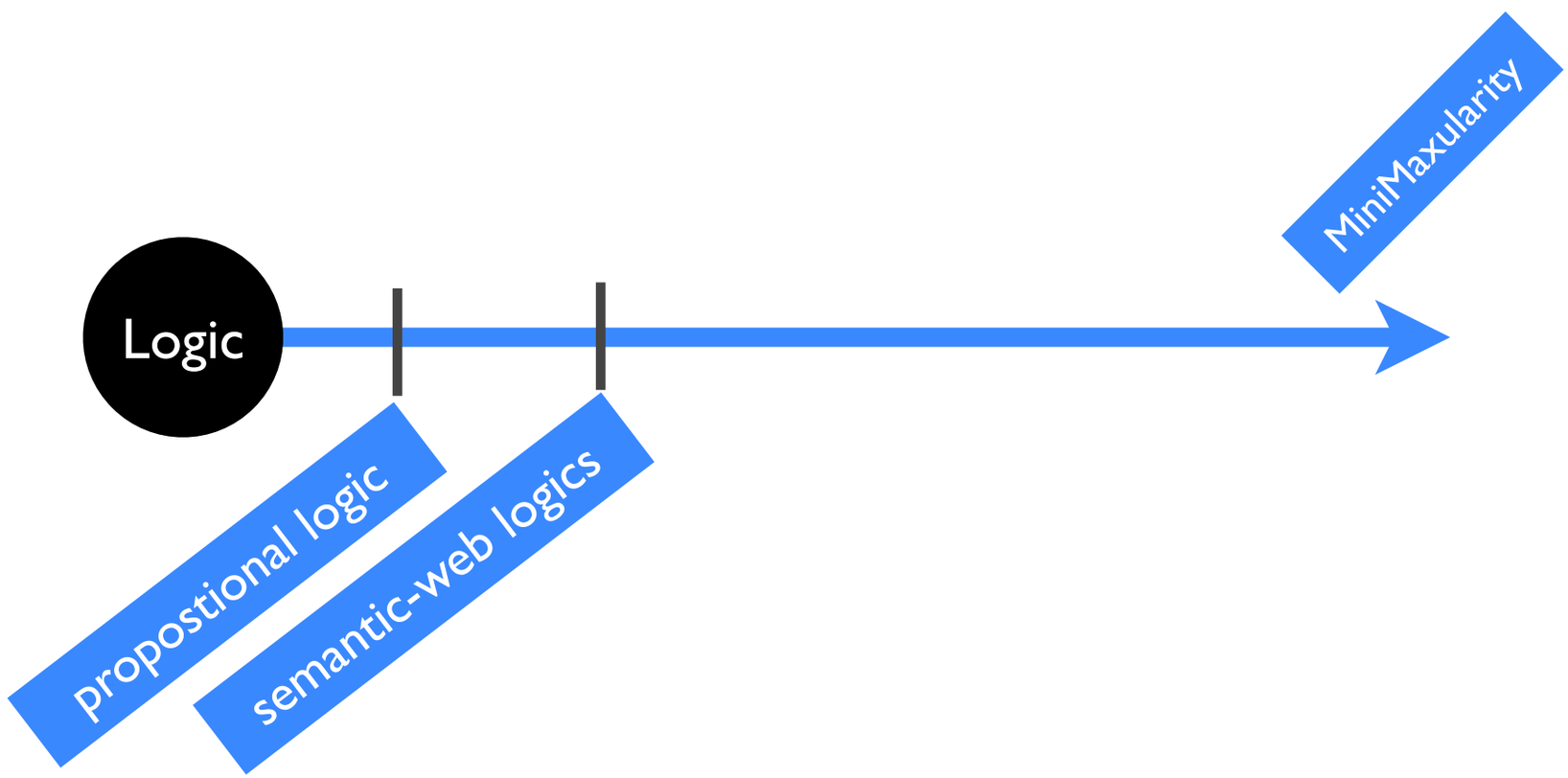
propostional logic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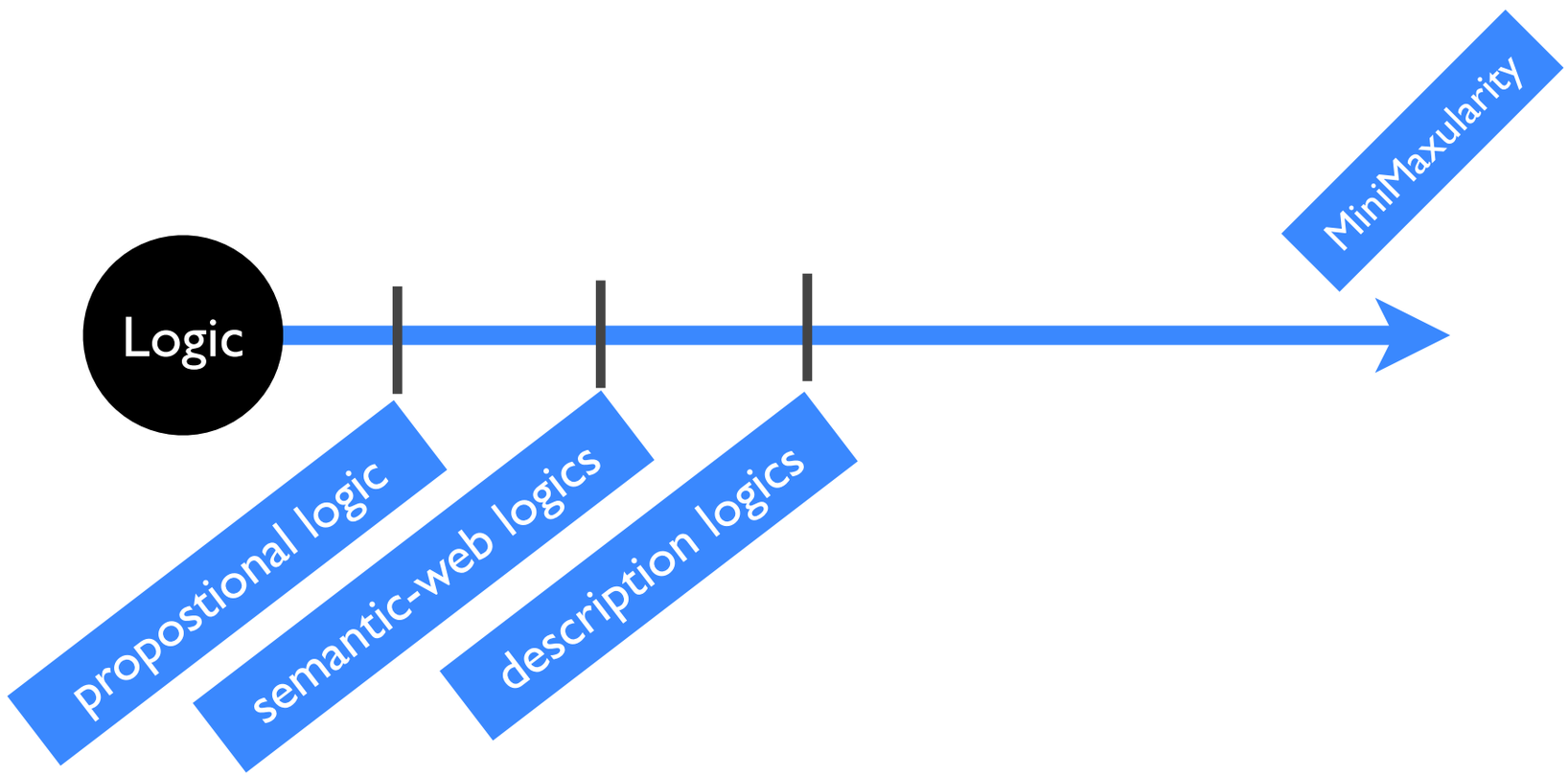
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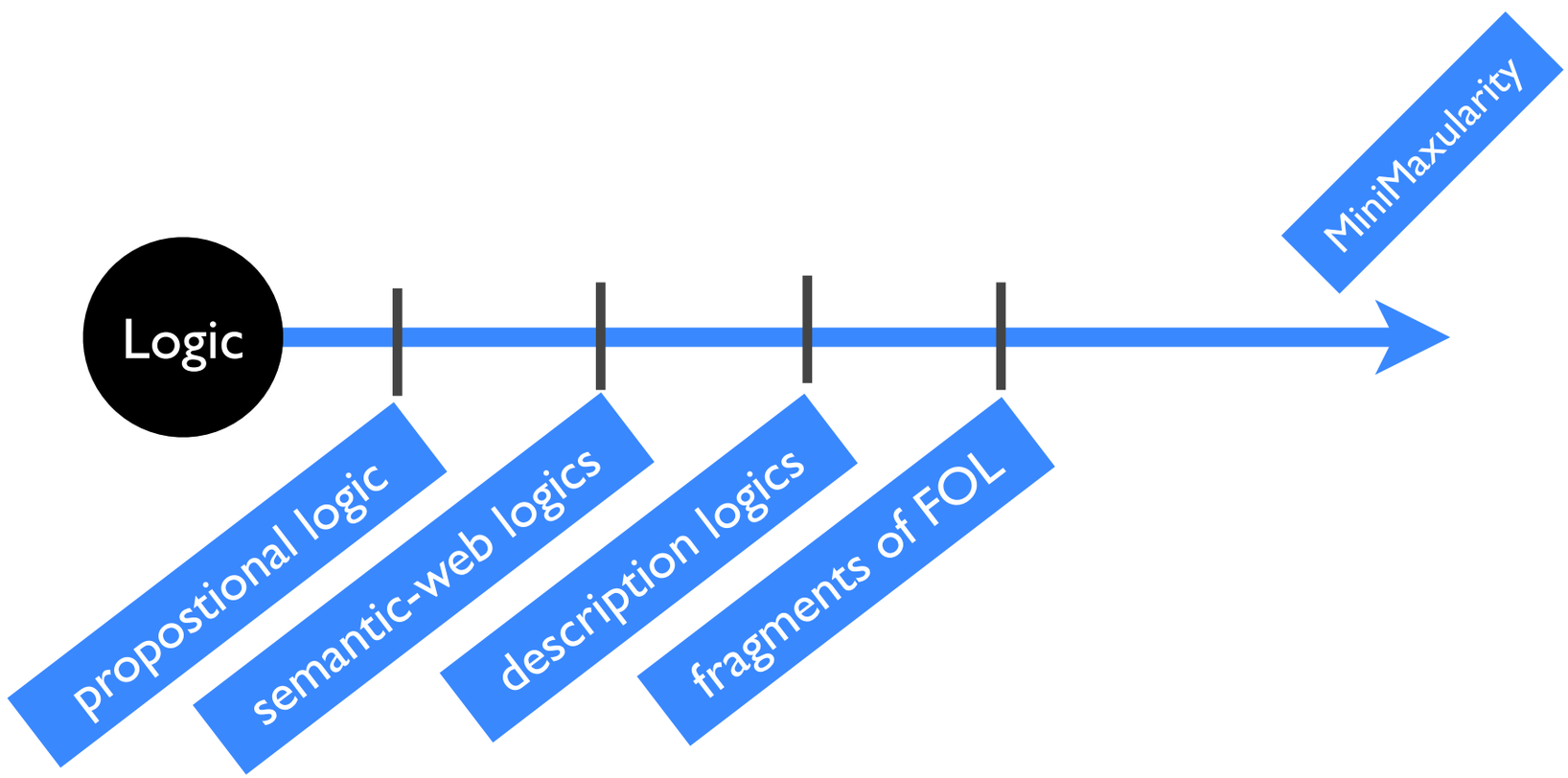
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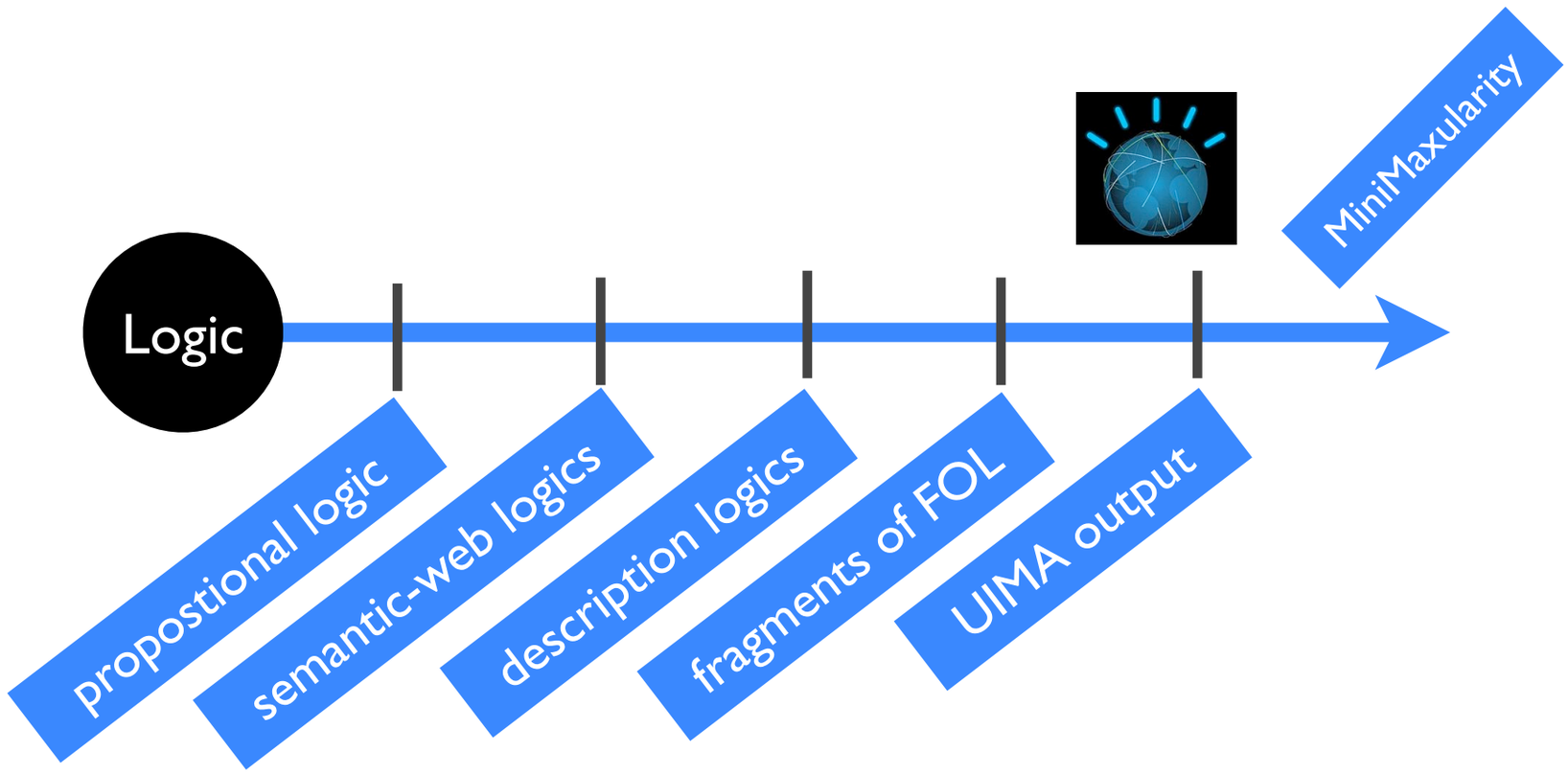
semantic-web logics
description logics
fragments of FOL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fragments of FOL
UIMA output
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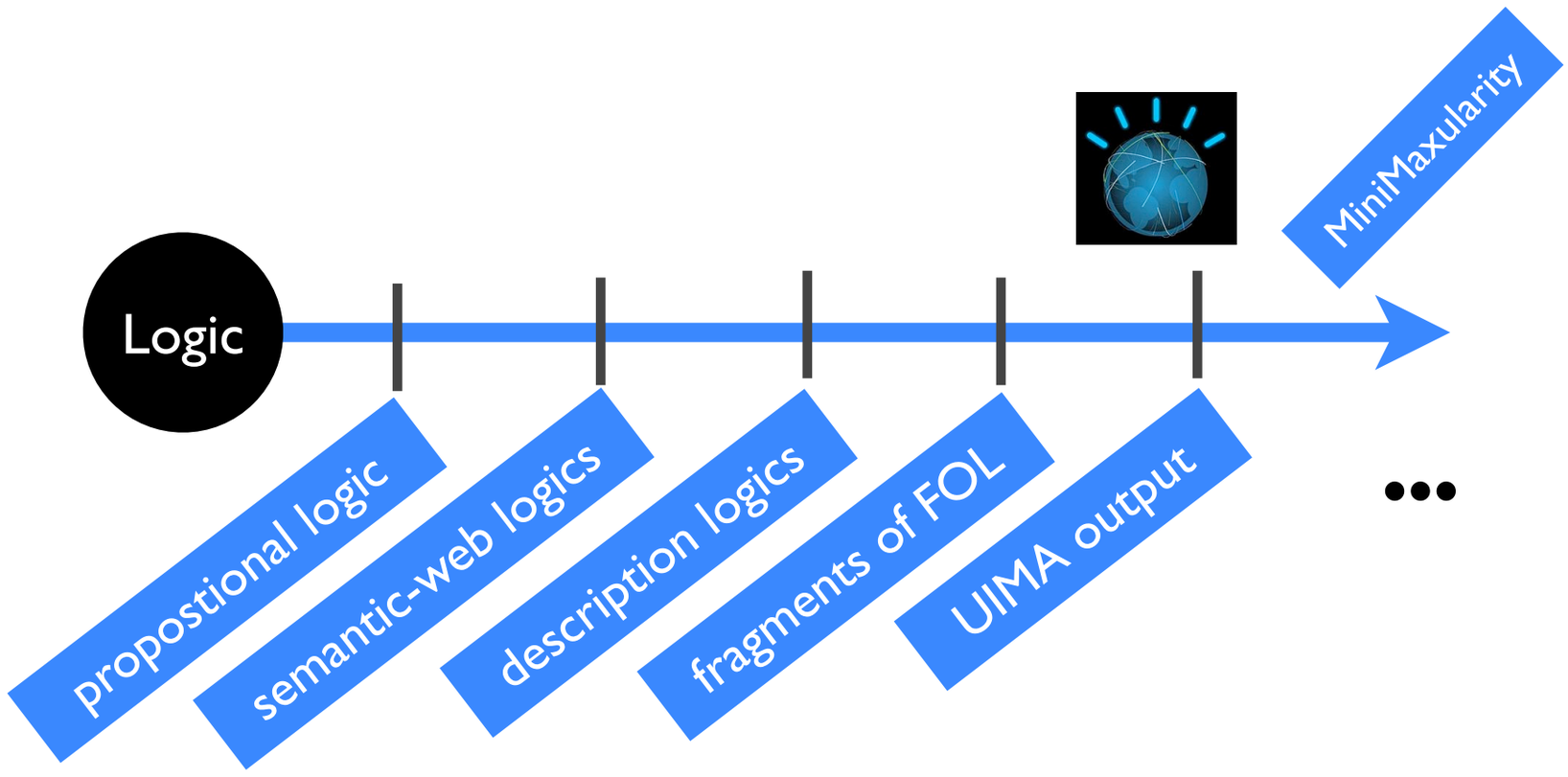
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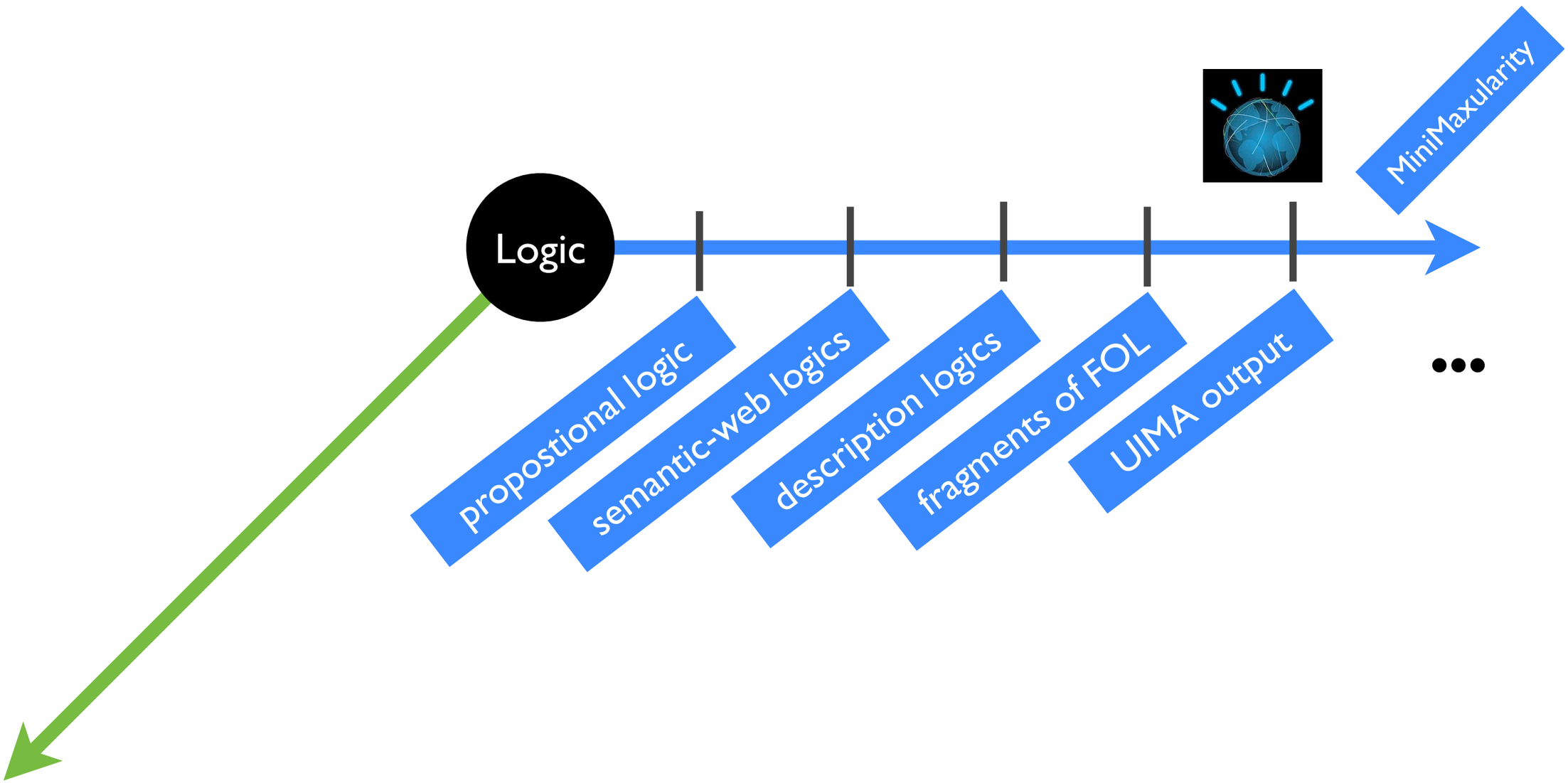
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fragments of FOL
UIMA output
MiniMaxularity
...

Art of Infallibility I

Logic

propostional logic

semantic-web logics

description logics

fragments of FOL

UIMA output

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MiniMaxularity

Art of Infallibility I

FOL

Logic

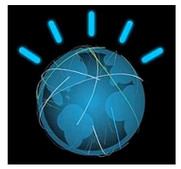
propostional logic

semantic-web logics

description logics

fragments of FOL

UIMA output



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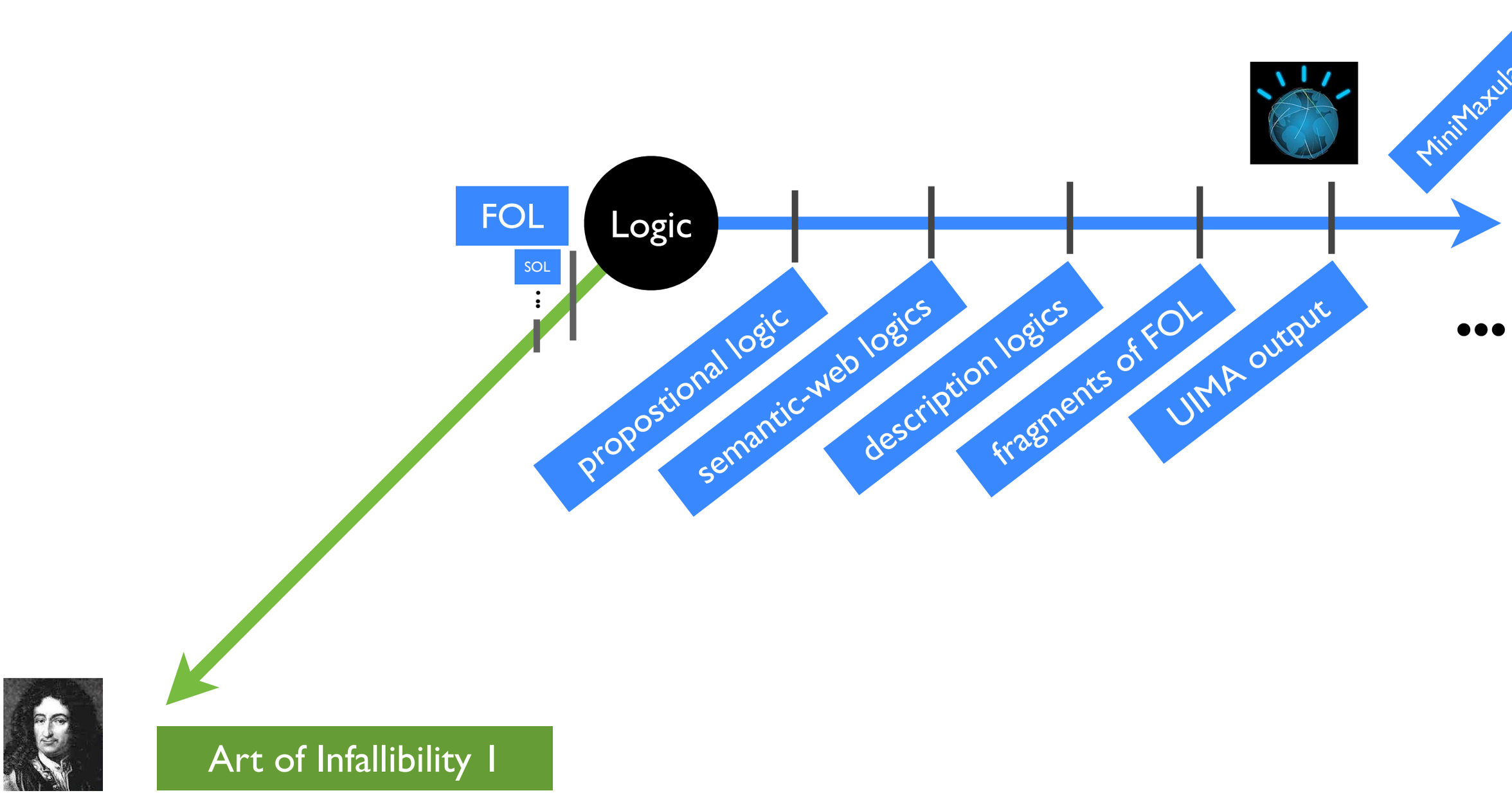
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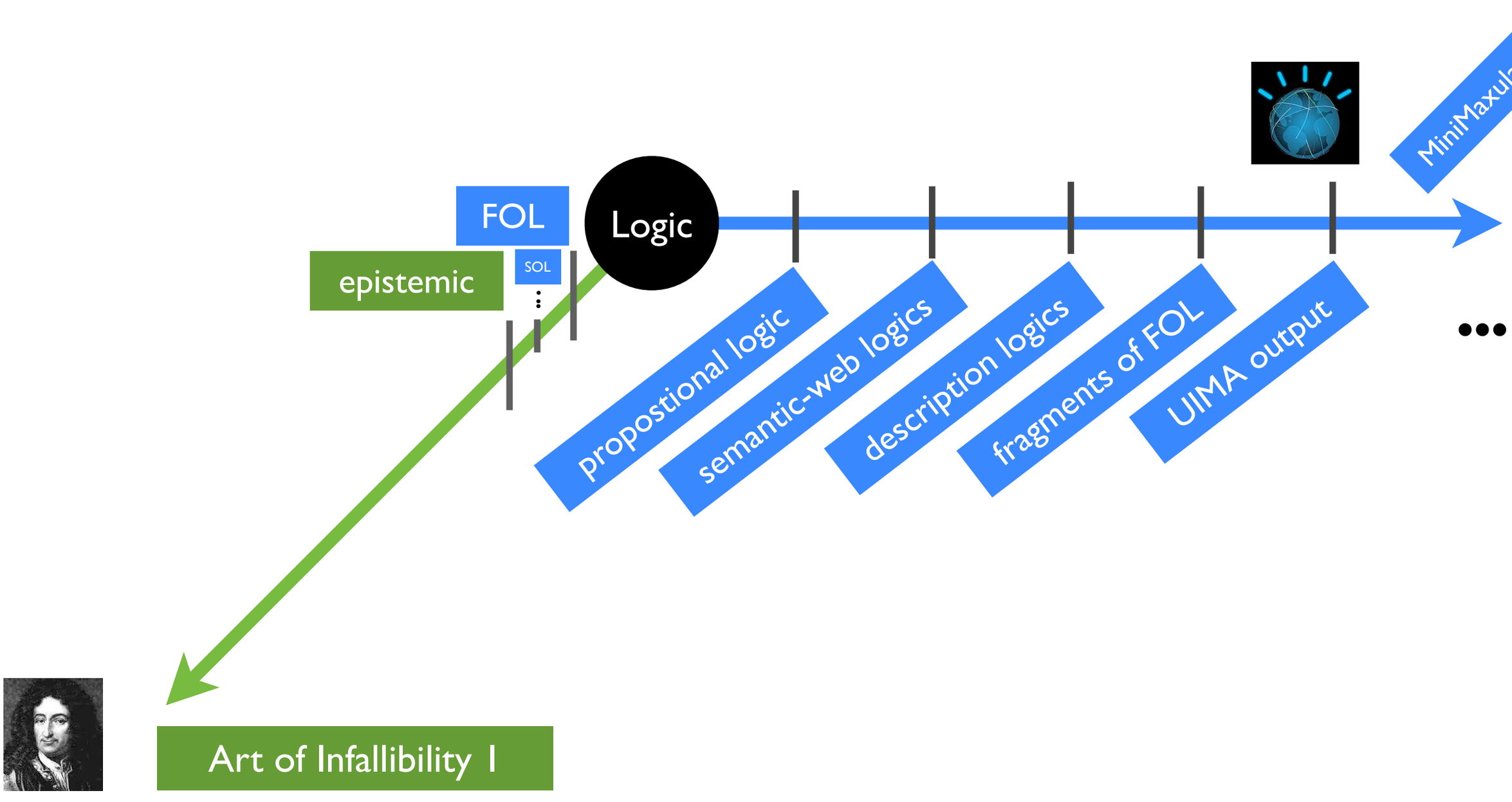
UIMA output
Art of Infallibility I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UIMA output
Art of Infallibility I

Art of Infallibility I

Logic

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UIMA output

MiniMaxularity

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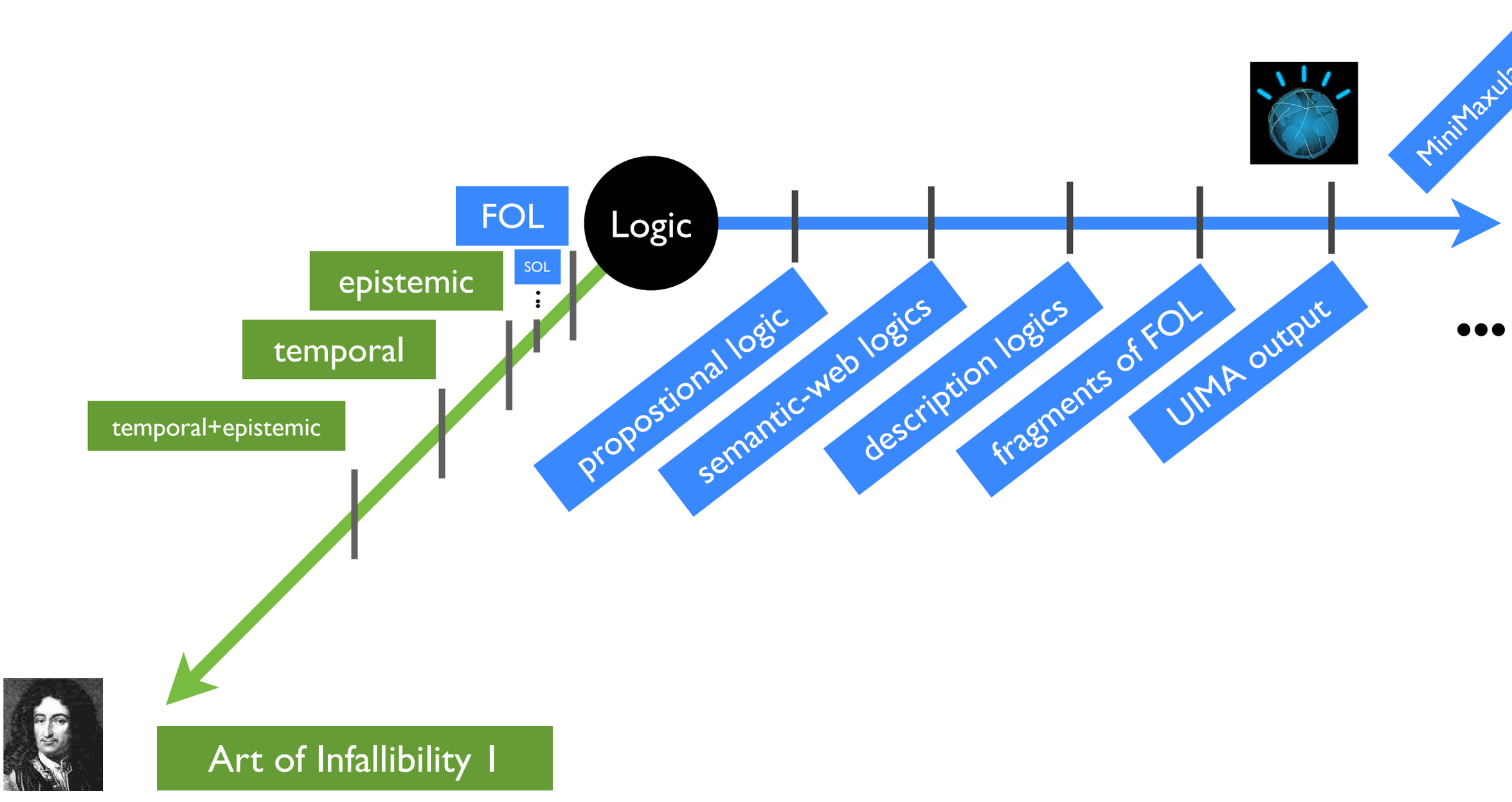
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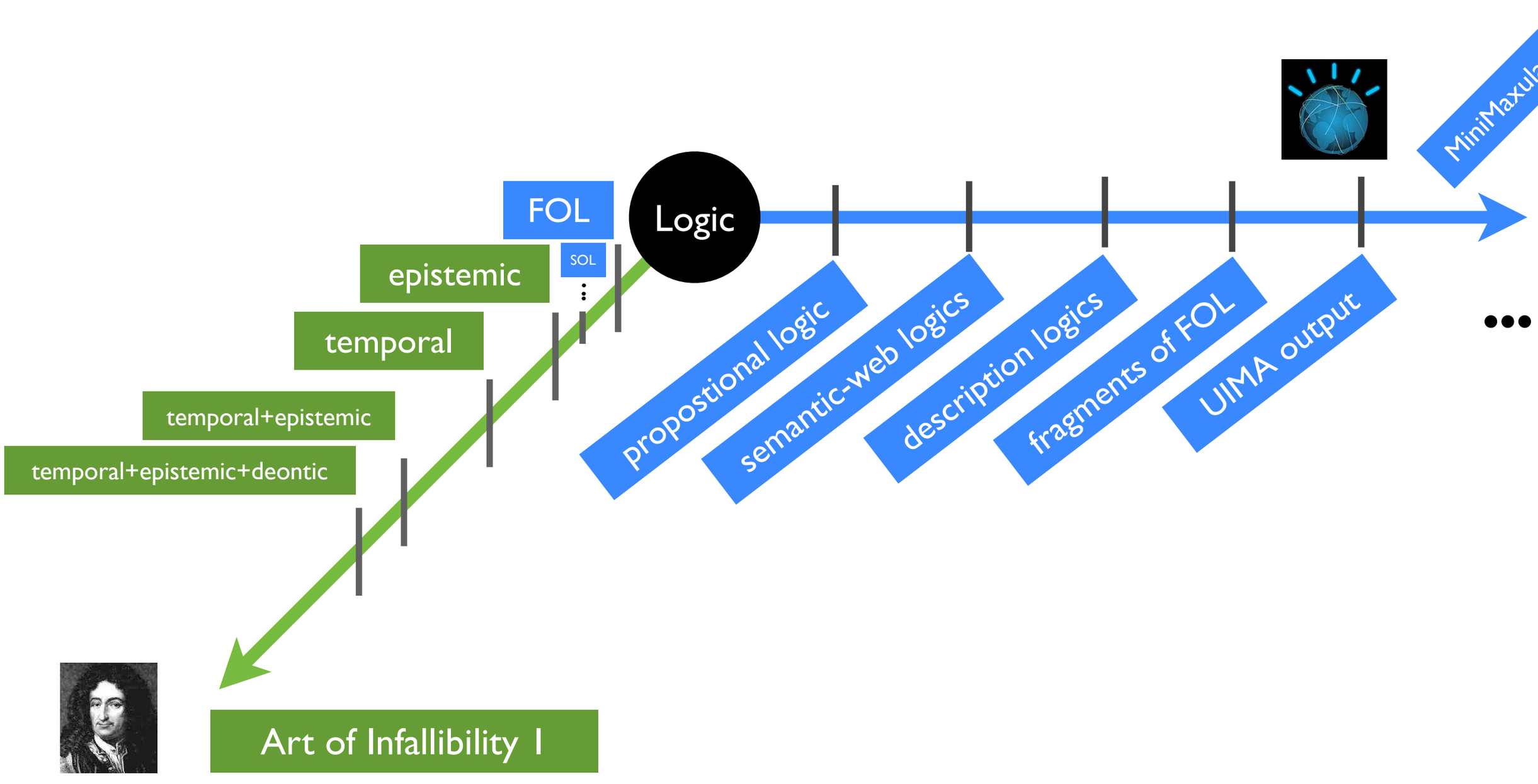
Art of Infallibility I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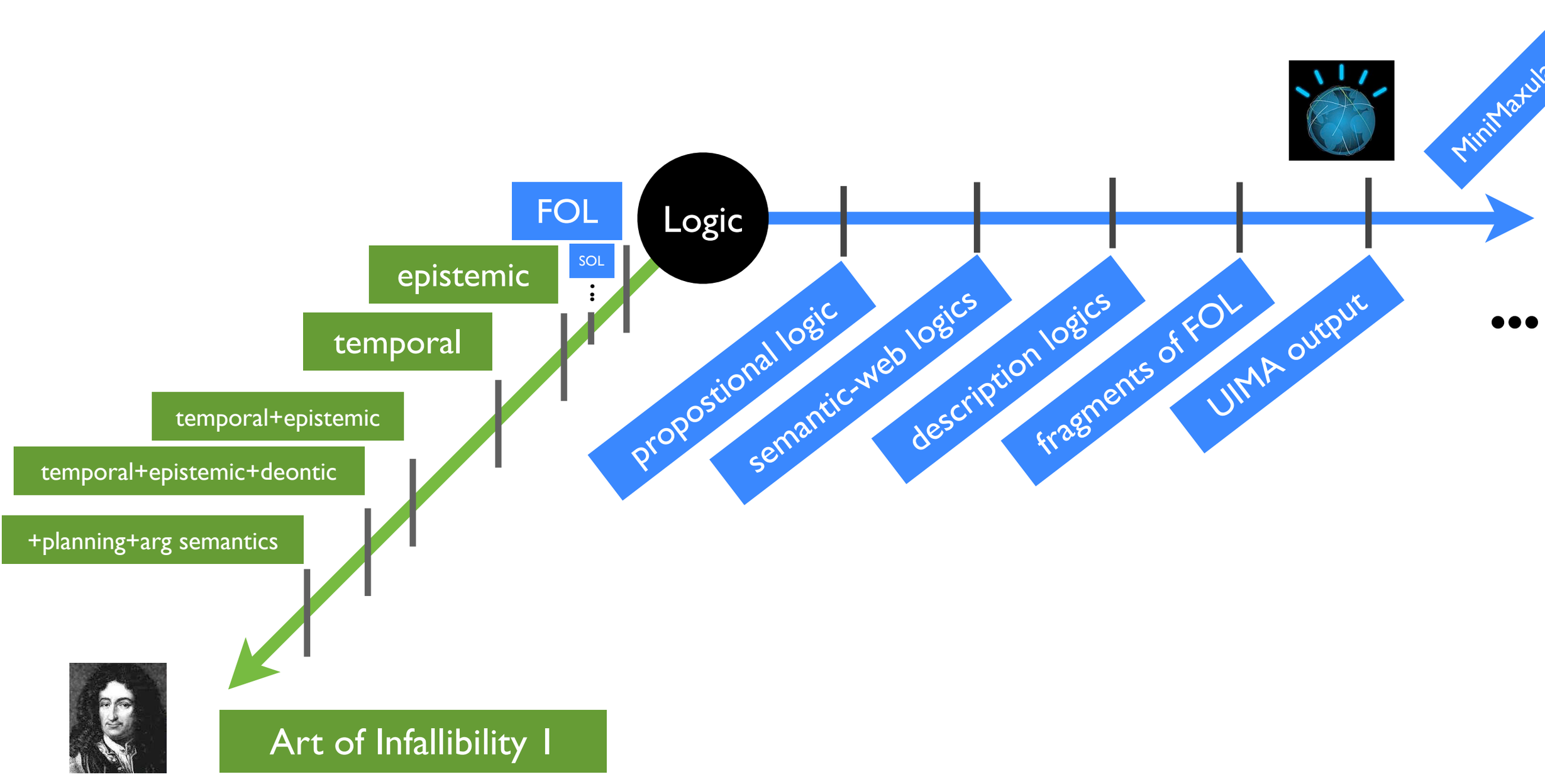
Art of Infallibility I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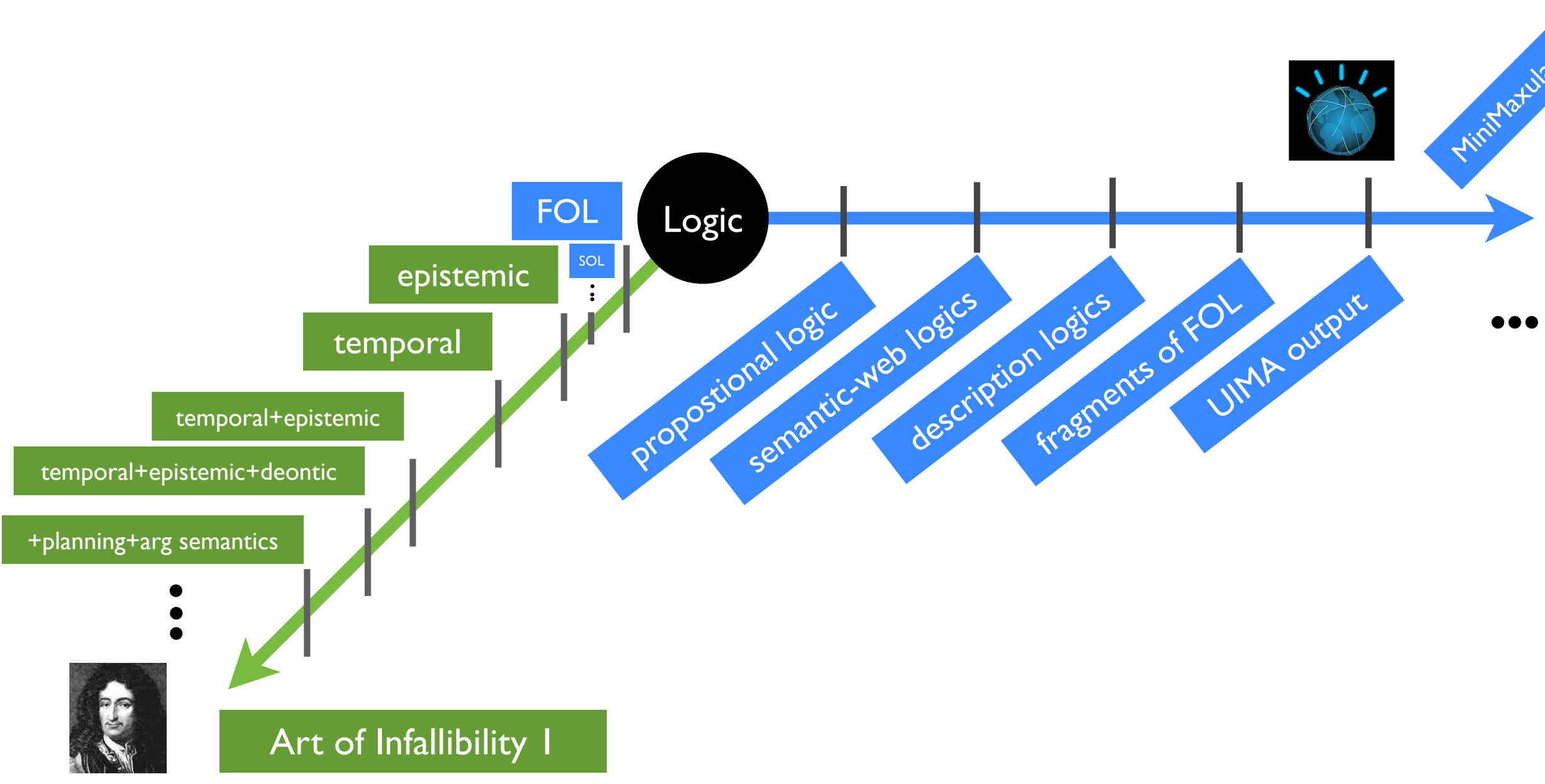
fragments of FOL
UIMA output
Art of Infallibility I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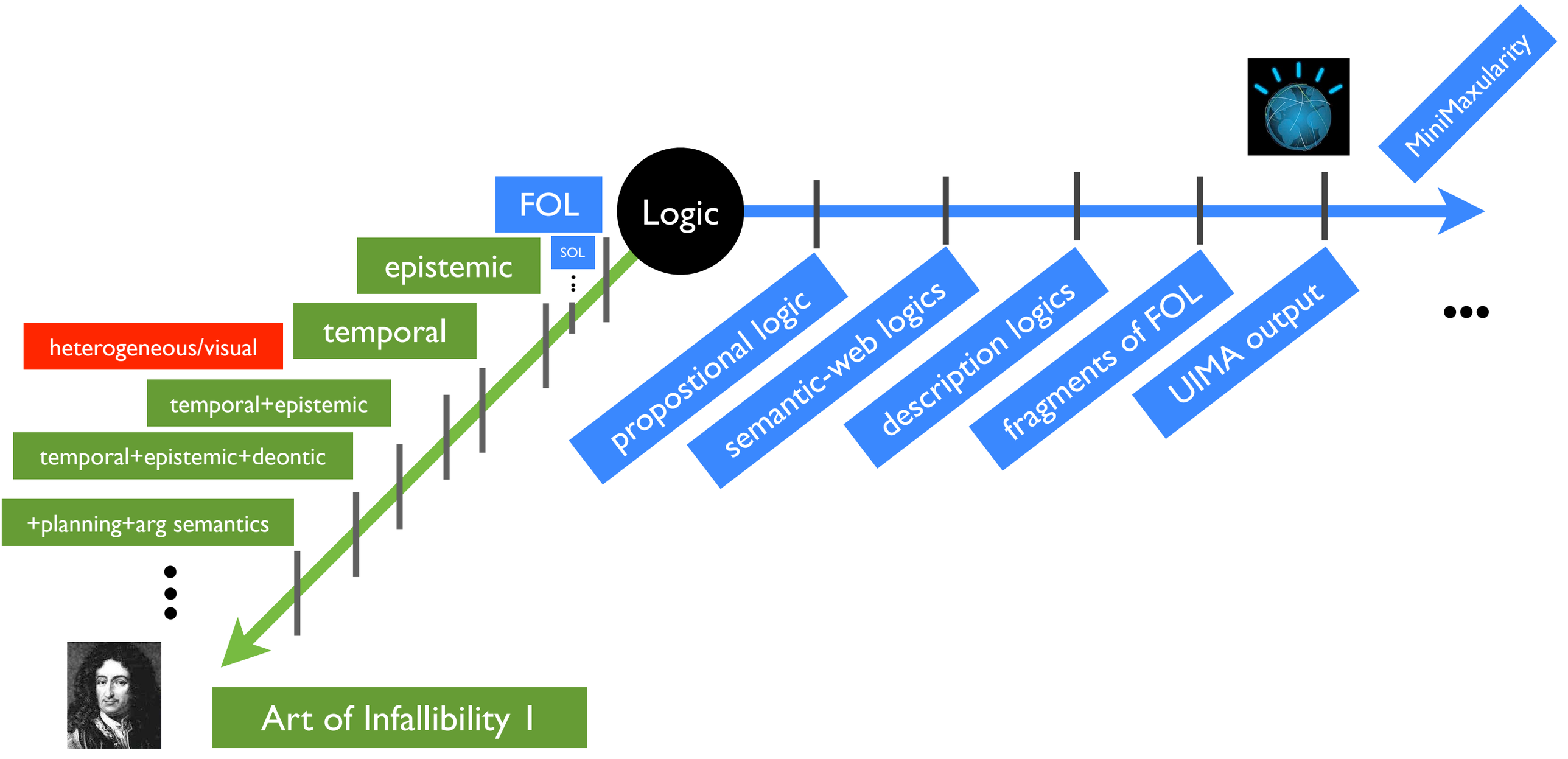
Art of Infallibility I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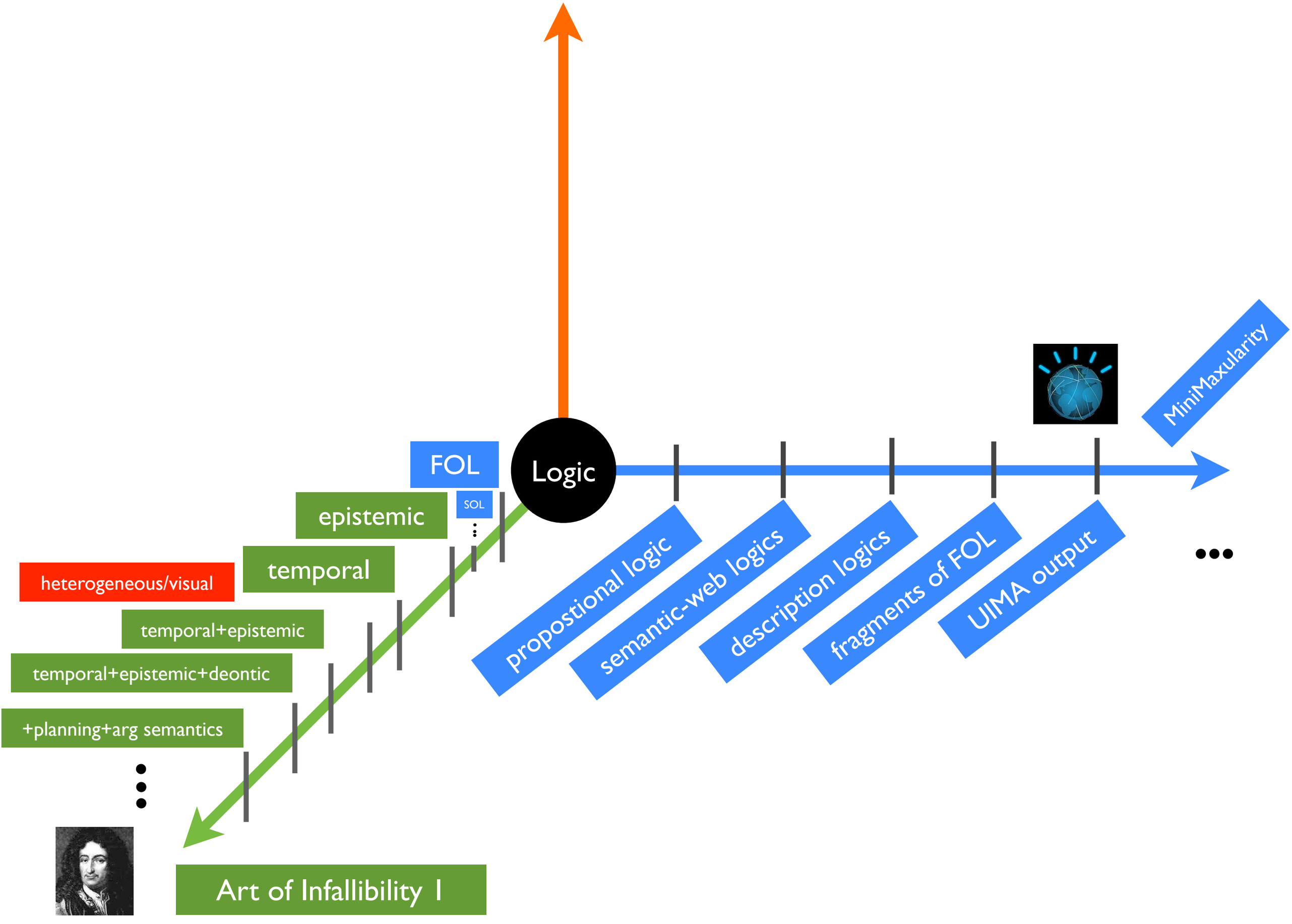
Art of Infallibility I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temporal+epistemic+deontic
+planning+arg semantics
propostional logic
semantic-web logics
description logics
fragments of FOL
UIMA output
Art of Infallibility I

Infinitary (AoI 2)

Logic

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heterogeneous/visual

temporal+epistemic

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semantic-web logics

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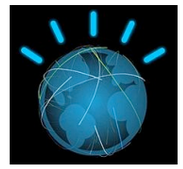
fragments of FOL

UIMA output

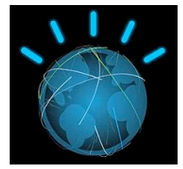
MiniMaxularity



Art of Infallibility I

Infinitary (AoI 2)

$L_{\omega 1,\omega}$



MiniMaxularity

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temporal+epistemic

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propostional logic

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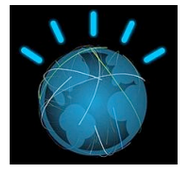
UIMA output

Art of Infallibility I

Infinitary (AoI 2)

$L_{\omega 1,\omega}$



MiniMaxularity

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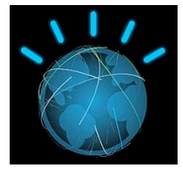
UIMA output

Art of Infallibility I

Infinitary (AoI 2)

$L_{\omega 1,\omega}$



MiniMaxularity

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temporal+epistemic+deontic

+planning+arg semantics

propostional logic

semantic-web logics

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fragments of FOL

UIMA output

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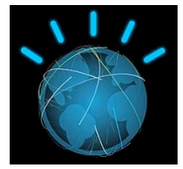
Art of Infallibility I

Infinitary (AoI 2)

$L_{\omega 1,\omega}$



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fragments of FOL

UIMA output

Art of Infallibility I

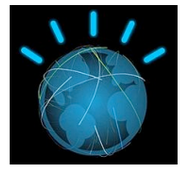
$\mathcal{DCEC}^*$

Deontic Cognitive Event Calculus

(with Castañeda's *)

Infinitary (AoI 2)

$L_{\omega 1,\omega}$



MiniMaxularity

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temporal+epistemic+deontic

+planning+arg semantics

propostional logic

semantic-web logics

description logics

fragments of FOL

UIMA output

Art of Infallibility I

$\mathcal{DCEC}^*$

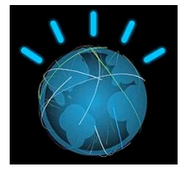
Deontic Cognitive Event Calculus
(with Castañeda's *)

1. natural language semantics (non-Montagovian)
2. higher-cognition tests (for Psychometric AI)
   (false-belief test, deliberative mind-reading
   mirror test for self-consciousness ...)
3. ethically correct robots
4. biz & econ simulation

Art of Infallibility I

Infinitary (AoI 2)

$L_{\omega 1,\omega}$



MiniMaxularity

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UIMA output

$\mathcal{DCEC}^*$

Deontic Cognitive Event Calculus

(with Castañeda's *)

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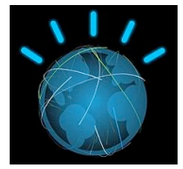
$L_{\omega 1,\omega}$



MiniMaxularity

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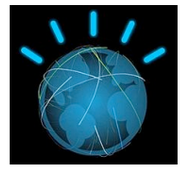
fragments of FOL

UIMA output

Art of Infallibility I

Infinitary (AoI 2)

$L_{\omega 1,\omega}$



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temporal+epistemic

temporal+epistemic+deontic

+planning+arg semantics

propostional logic

semantic-web logics

description logics

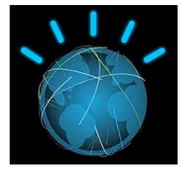
fragments of FOL

UIMA output

Art of Infallibility I

Infinitary (AoI 2)

$L_{\omega 1,\omega}$



MiniMaxularity

FOL

Logic

SOL

epistemic

temporal

heterogeneous/visual

temporal+epistemic

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UIMA output

...

Gödel's "God Theorem"

Art of Infallibility I

Infinitary (AoI 2)

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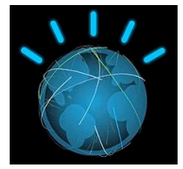

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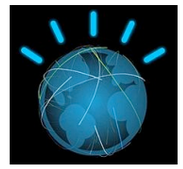

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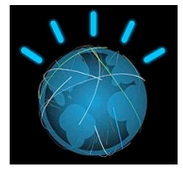
ITS (Culture, Language, Math)

Art of Infallibility I

Infinitary (AoI 2)

$L_{\omega 1,\omega}$



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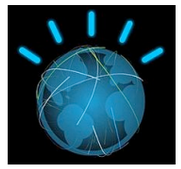

MiniMaxularity

AI-ified Axiomatic Physics!
(*Synthese*)

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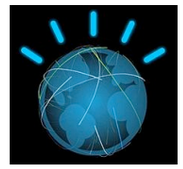

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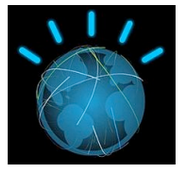
Infinitary (AoI 2)

Goodstein's Theorem!

$L_{\omega 1,\omega}$



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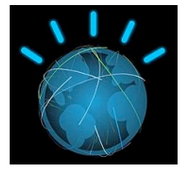

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Art of Infallibility I

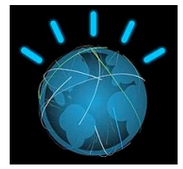
$\mathcal{DCEC}^*$

Deontic Cognitive Event Calculus

(with Castañeda's *)

Infinitary (AoI 2)

$L_{\omega 1,\omega}$



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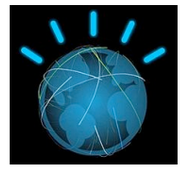
$\mathcal{DCEC}^*$

Deontic Cognitive Event Calculus
(with Castañeda's *)

1. natural language semantics (non-Montagovian)
2. higher-cognition tests (for Psychometric AI)
   (false-belief test, deliberative mind-reading
   mirror test for self-consciousness ...)
3. ethically correct robots
4. Basis for RL: **Learning Ex Nihilo**

Infinitary (AoI 2)

$L_{\omega 1,\omega}$



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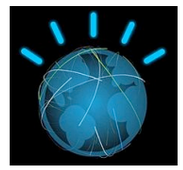

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Art of Infallibility I

Animal-Level AI

Super-Serious Human Cognitive Power

Serious Human Cognitive Power

Entscheidungsproblem

Mere Calculative Cognitive Power

Animal-Level AI

Analytical Hierarchy

Serious Human Cognitive Power

Entscheidungsproblem

Mere Calculative Cognitive Power

Animal-Level AI

Analytical Hierarchy

Arithmetical Hierarchy

Entscheidungsproblem

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Animal-Level AI

Analytical Hierarchy

Arithmetical Hierarchy

Entscheidungsproblem

Polynomial Hierarchy

Animal-Level AI

Analytical Hierarchy

Arithmetical Hierarchy

Entscheidungsproblem

Polynomial Hierarchy

$$\mathbf{P} \subseteq \mathbf{NP} \subseteq \mathbf{PSPACE} = \mathbf{NPSPACE} \subseteq \mathbf{EXPTIME} \subseteq \mathbf{NEXPTIME} \subseteq \mathbf{EXPSPACE}$$

Animal-Level AI

Analytical Hierarchy

Arithmetical Hierarchy

$\vdots$

Π_2

Σ_2

Π_1

Σ_1

Σ_0

Entscheidungsproblem

Polynomial Hierarchy

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Animal-Level AI

Analytical Hierarchy

Arithmetical Hierarchy

$\vdots$

Π_2

Σ_2

Go: AlphaGo

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Animal-Level AI

Analytical Hierarchy

Arithmetical Hierarchy

$\vdots$

Π_2

Σ_2

Jeopardy!: **Watson**

Go: AlphaGo

Π_1

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Entscheidungsproblem

Polynomial Hierarchy

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Animal-Level AI

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Chess: Deep Blue

Jeopardy!: **Watson**

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Entscheidungsproblem

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Animal-Level AI

Analytical Hierarchy

Arithmetical Hierarchy

$\vdots$

Π_2

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Checkers: Chinook

Chess: Deep Blue

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Entscheidungsproblem

Polynomial Hierarchy

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Animal-Level AI

Analytical Hierarchy

Arithmetical Hierarchy

$\vdots$

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Entscheidungsproblem

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Chess: Deep Blue

Checkers: Chinook

Go: AlphaGo

Jeopardy!: **Watson**

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Animal-Level AI

Analytical Hierarchy

Arithmetical Hierarchy

$\vdots$

Π_2

Σ_2

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Entscheidungsproblem

Polynomial Hierarchy

Chess: Deep Blue

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Animal-Level AI

Analytical Hierarchy

Arithmetical Hierarchy

Church

$\vdots$

Π_2

Σ_2

Π_1

Σ_1

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Entscheidungsproblem

Polynomial Hierarchy

Chess: Deep Blue

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Jeopardy!: **Watson**

Go: AlphaGo

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Animal-Level AI

Analytical Hierarchy

Arithmetical Hierarchy

Church

Turing

$\vdots$

Π_2

Σ_2

Π_1

Σ_1

Σ_0

Entscheidungsproblem

Polynomial Hierarchy

Chess: Deep Blue

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Jeopardy!: **Watson**

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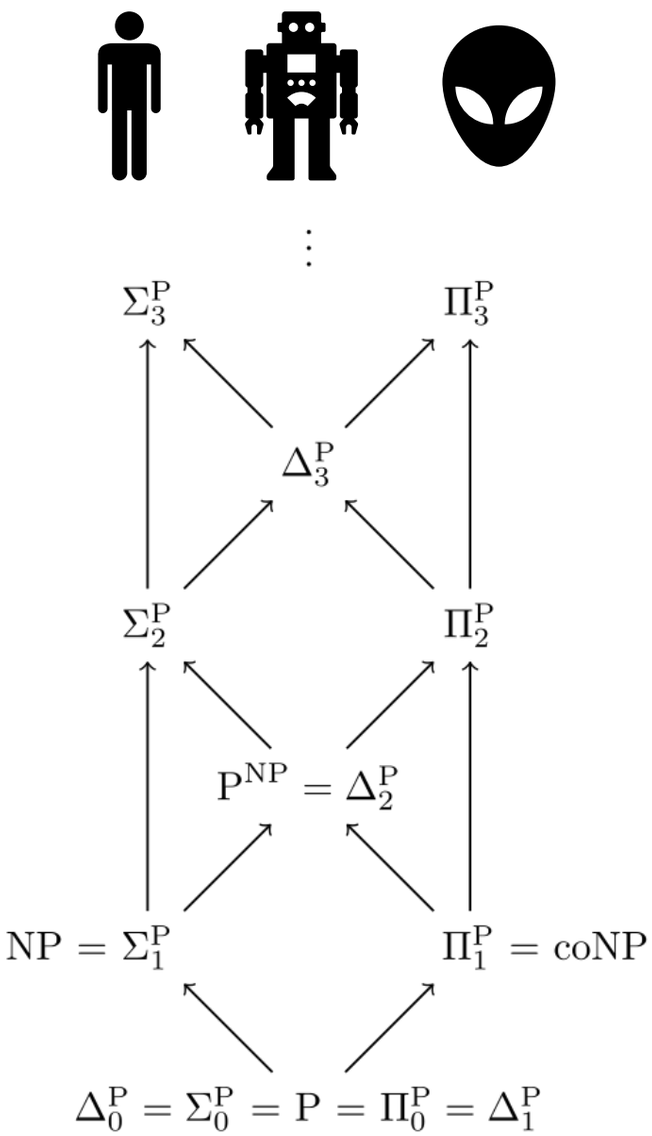
Polynomial Hierarchy, Part I

(via formal logic, directly; a start)



Polynomial Hierarchy, Part I

(via formal logic, directly; a start)

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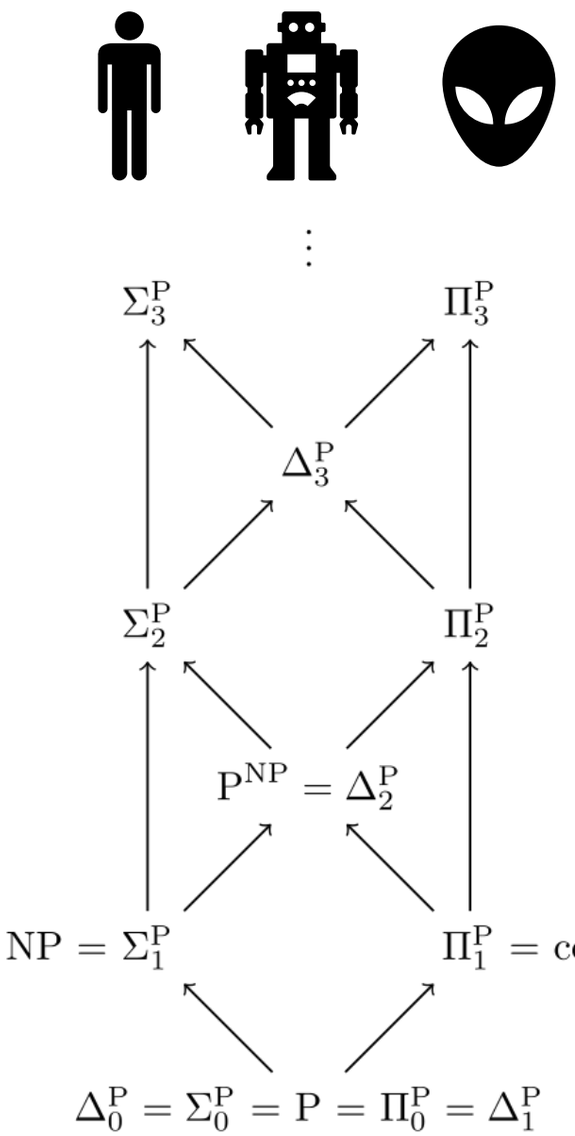

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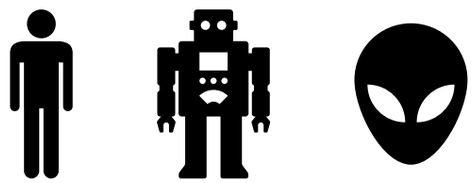

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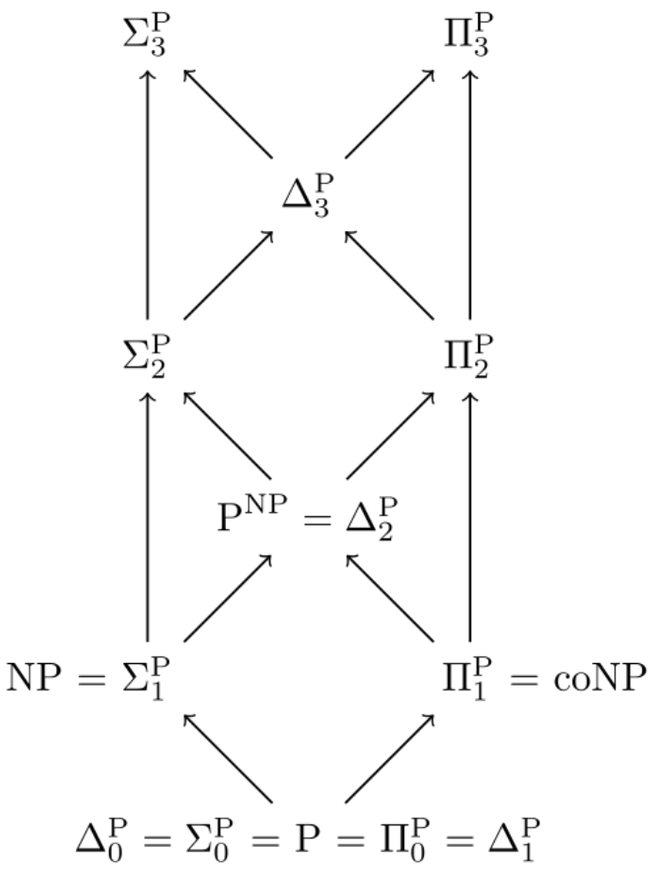

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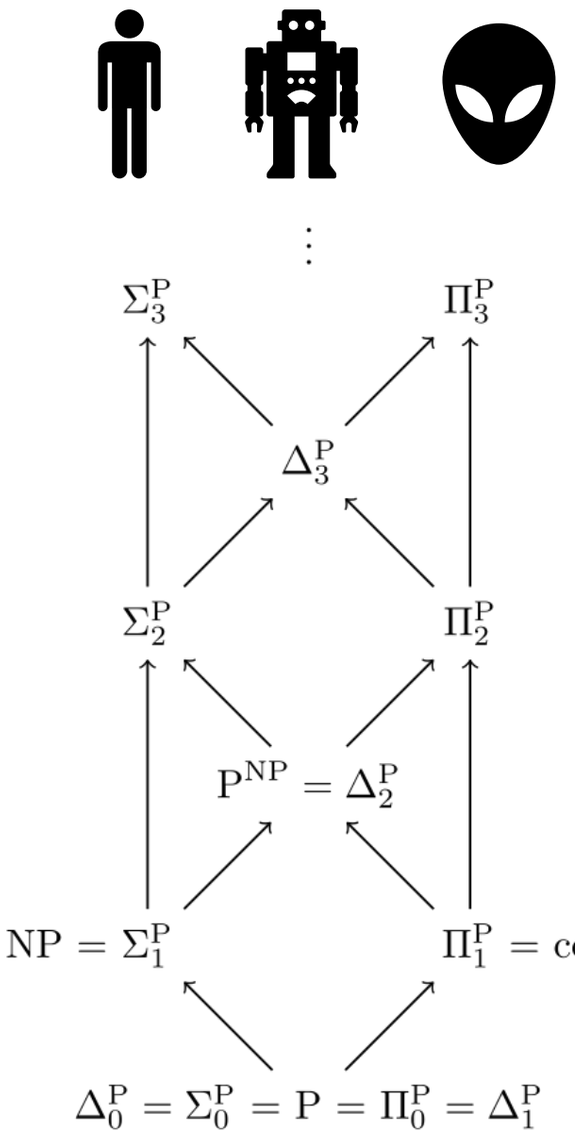

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Polynomial Hierarchy, Part I

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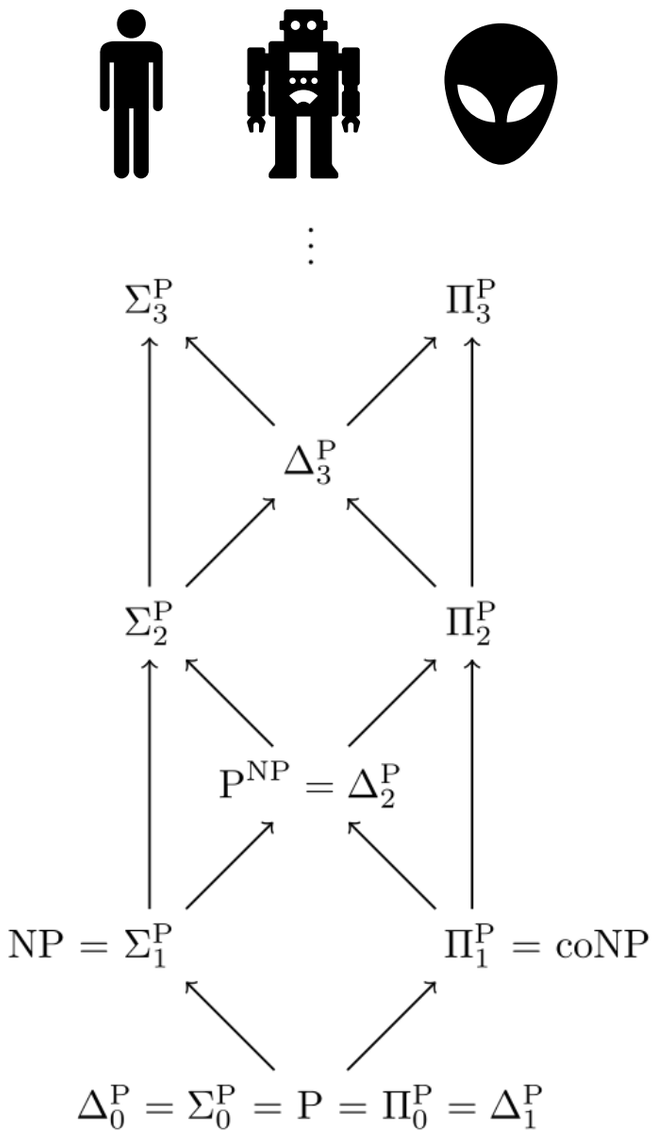

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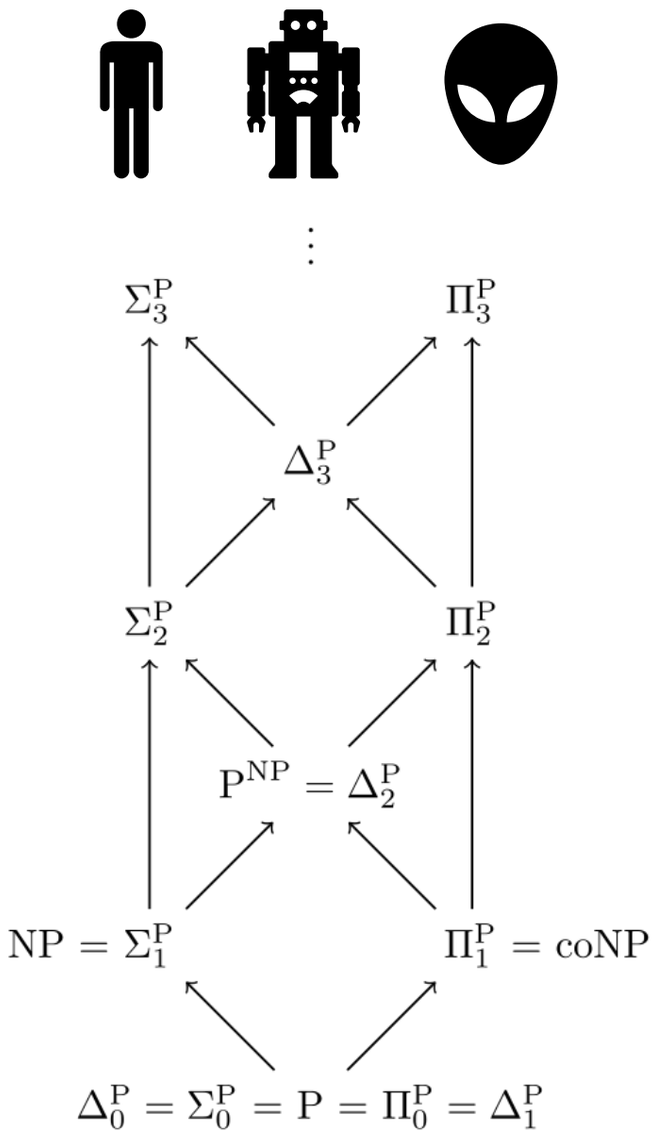

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Polynomial Hierarchy, Part I

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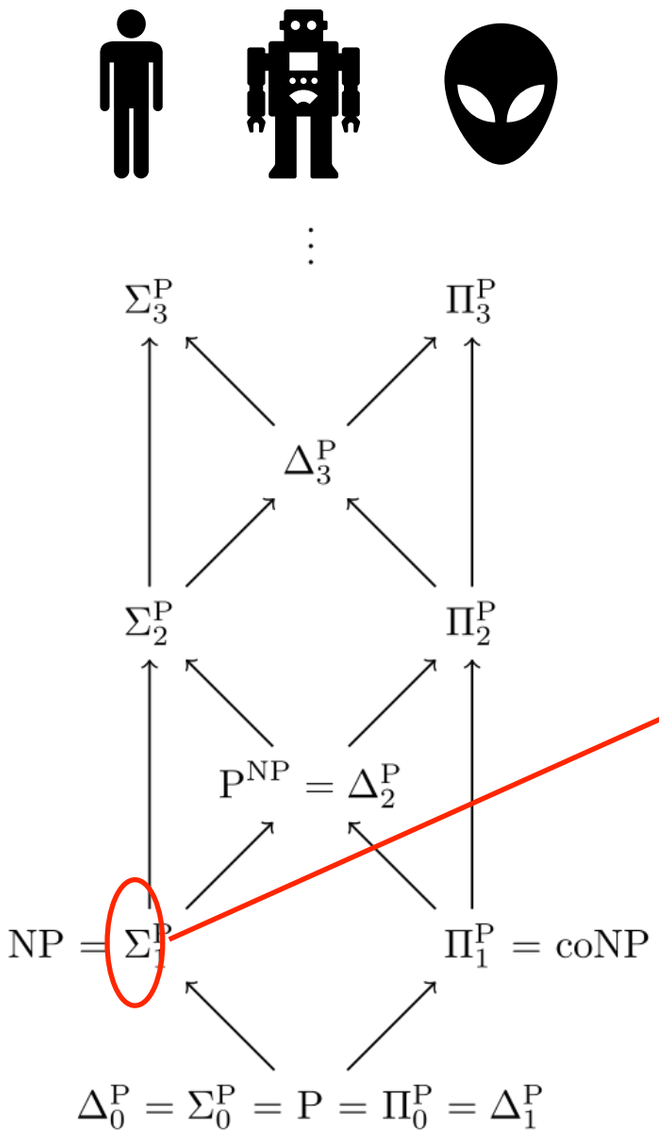

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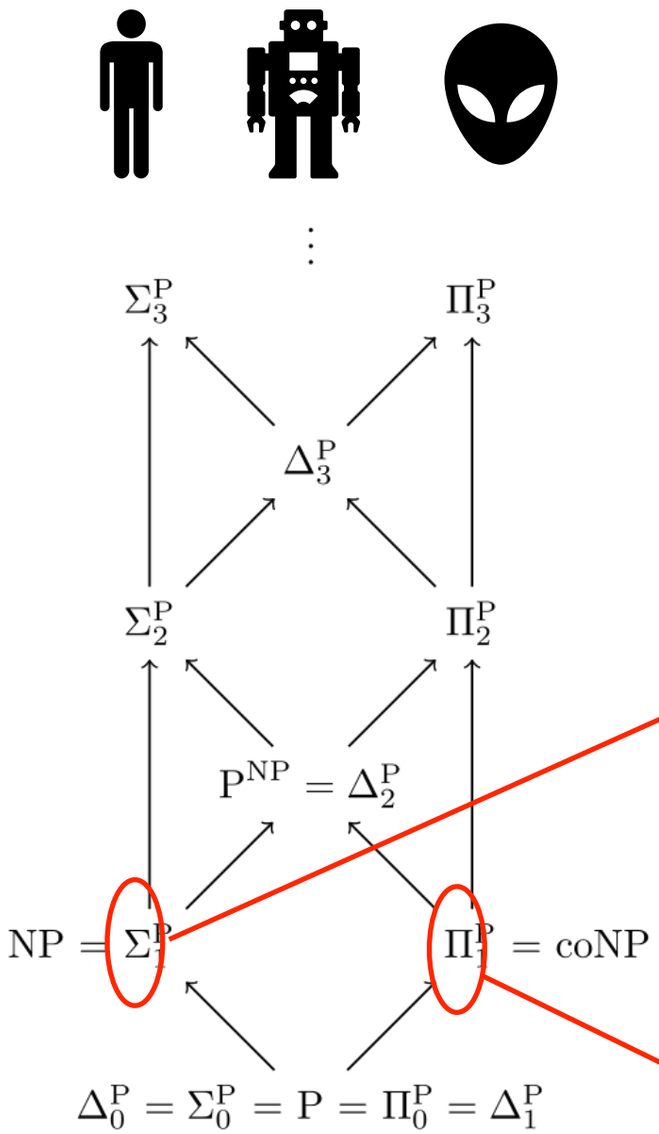

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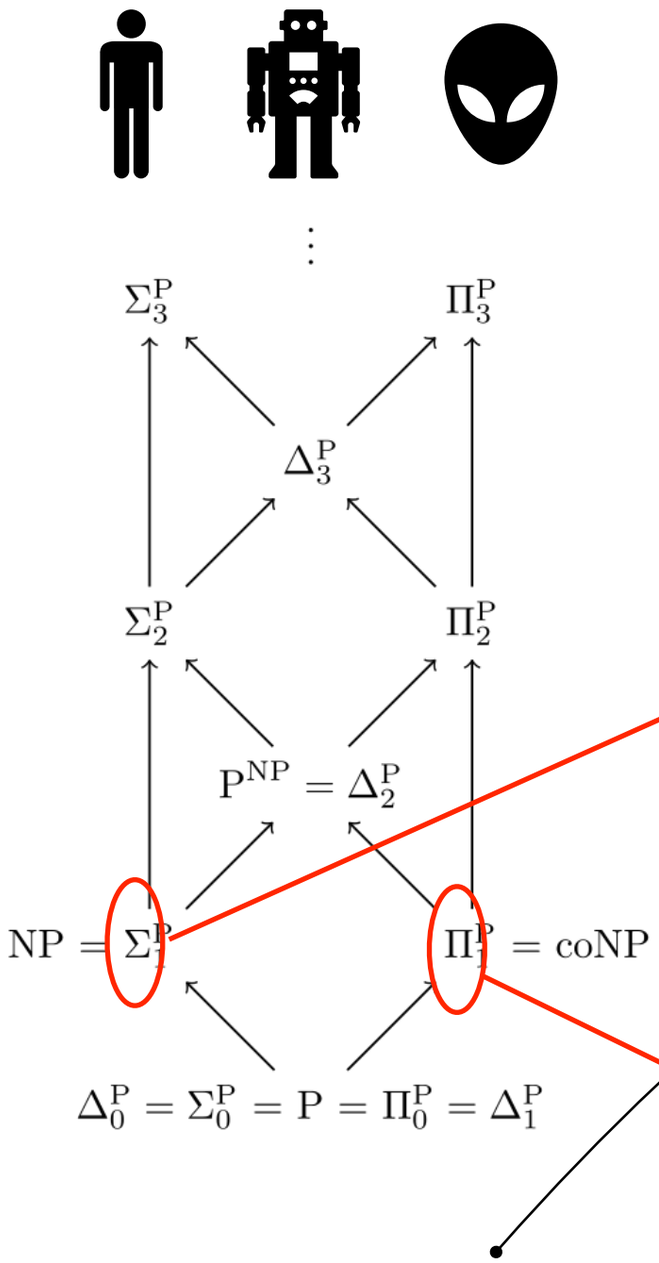

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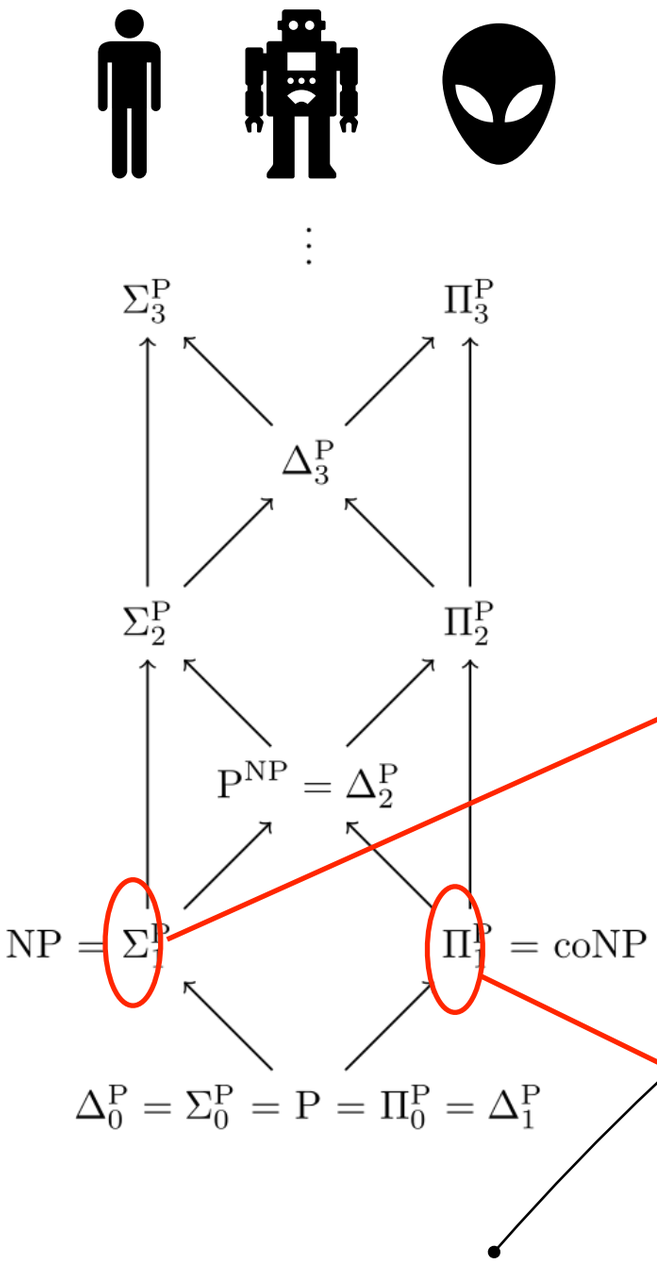

(Or a truth-graph *y* in HS® with at least one open branch.)

We say that a relation $R(u, y_1, \ldots, y_n)$ is polytime iff there is a deterministic Turing Machine $\mathfrak{m}$ and a polynomial p s.t. $\mathfrak{m}$ decides this relation in $p(|u|)$.

$L \in \mathbf{NP}$ iff: there's a polytime relation R s.t. $u \in L$ iff $\exists y R(u, y)$.

E.g.: We can prove $\mathbf{SAT} \in \mathbf{NP}$ because we have a polytime relation R s.t. $\phi \in \mathbf{SAT}$ iff $\exists y R(\phi \in \mathscr{L}_{pc}, \langle$assignments to Boolean vars$\rangle)$, where these assignments produce truth.

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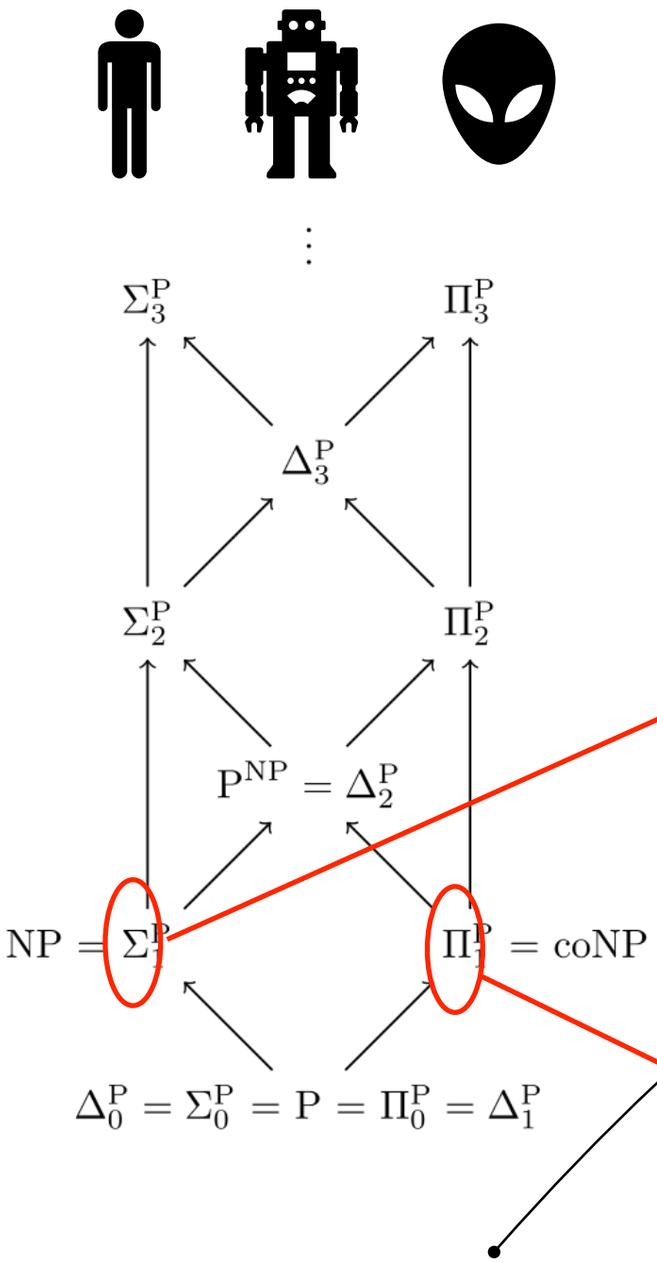
To prove $\mathbf{coSAT} \in \mathbf{coNP}$, we note that we have a polytime relation R s.t. $\phi \in \mathbf{coSAT}$ iff $\forall y R(\phi \in \mathscr{L}_{pc}, \langle$assignments to Boolean vars$\rangle)$, where the assignments produce *falsity*.

Polynomial Hierarchy, Part I

(via formal logic, directly; a start)



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(Or a truth-graph y in HS® with at least one open branch.)

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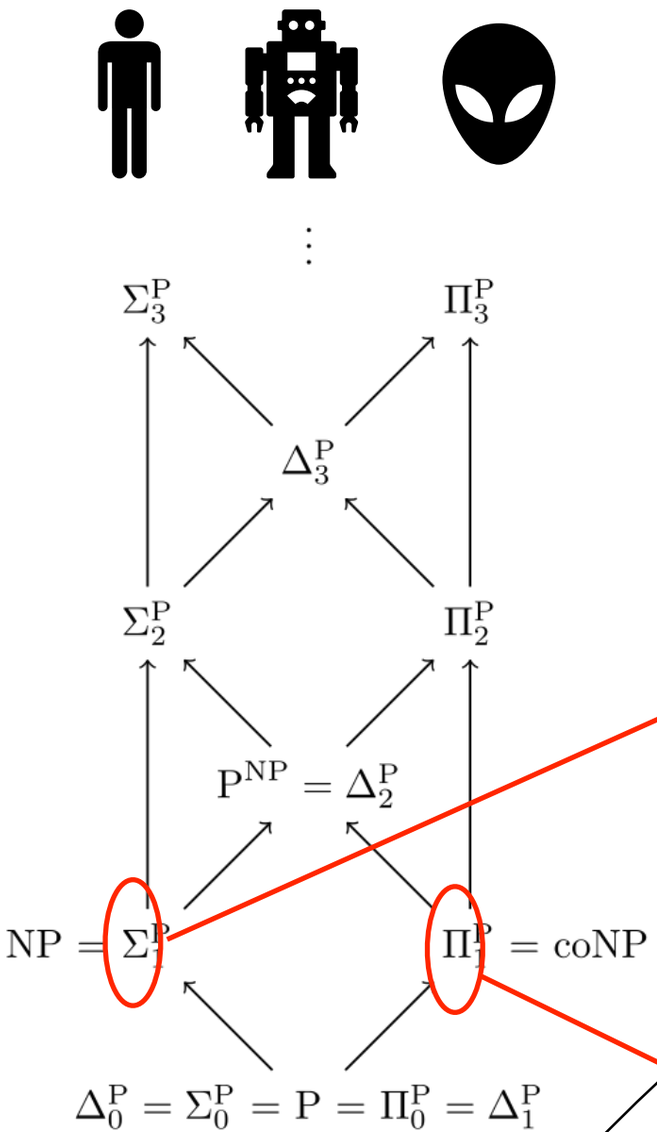

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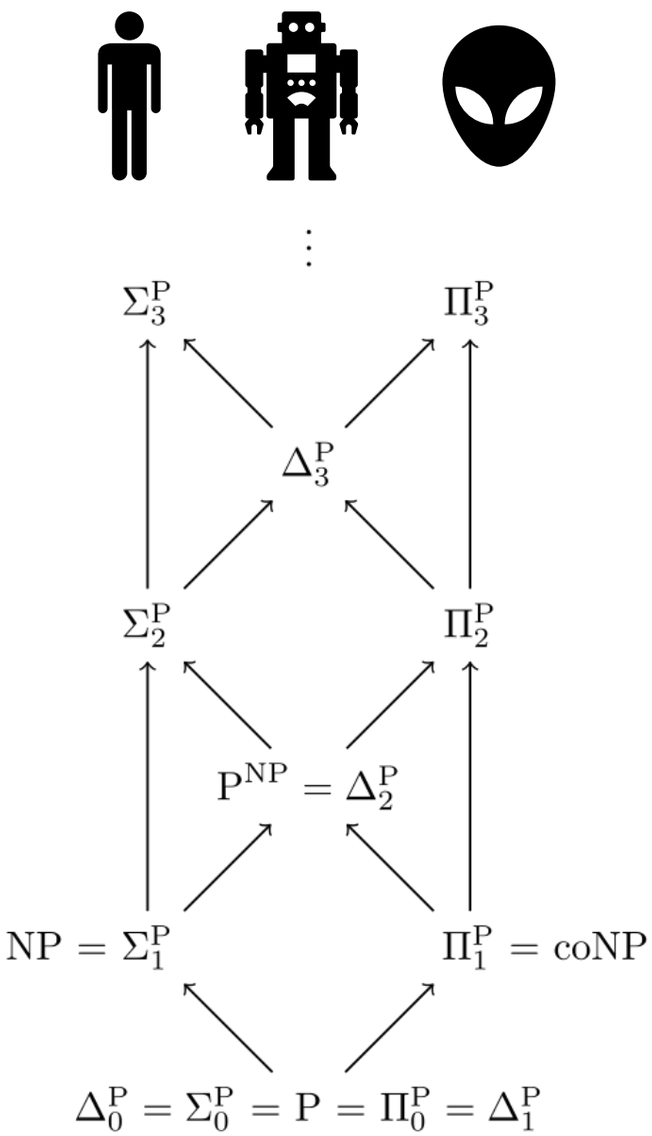
(Or a truth-graph *y* in HS® with at least one open branch.)

(Or every truth-graph *y* in HS® has no open branch.)

Polynomial Hierarchy, Part I

(via formal logic, directly; a start)



Polynomial Hierarchy, Part I

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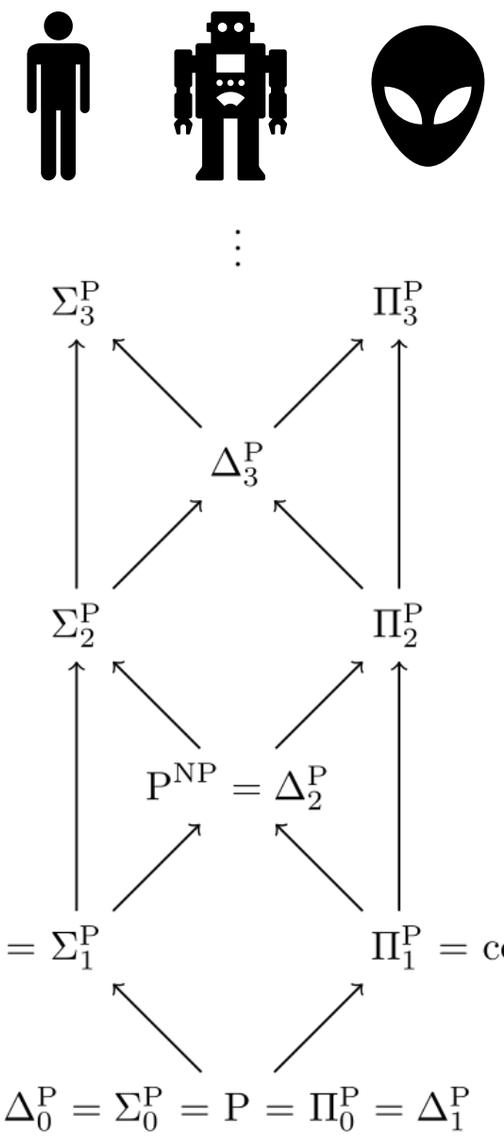
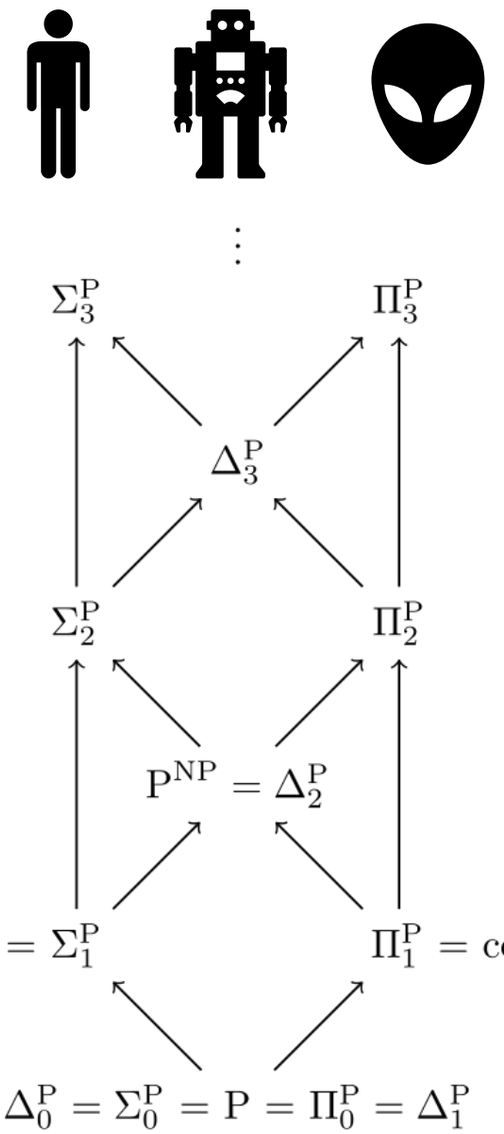


"What's that Δ??"

Polynomial Hierarchy, Part I

(via formal logic, directly; a start)



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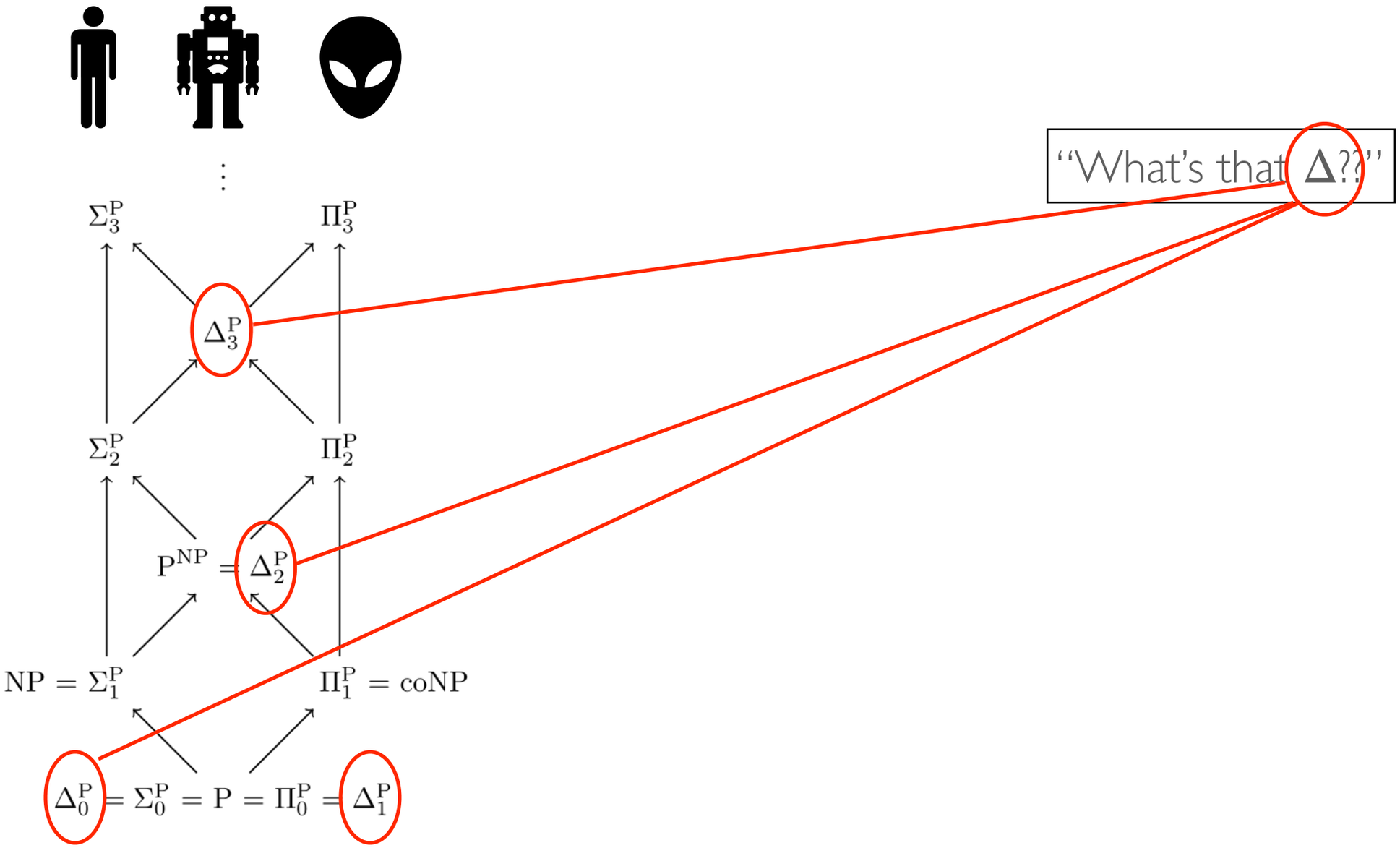

Polynomial Hierarchy, Part I

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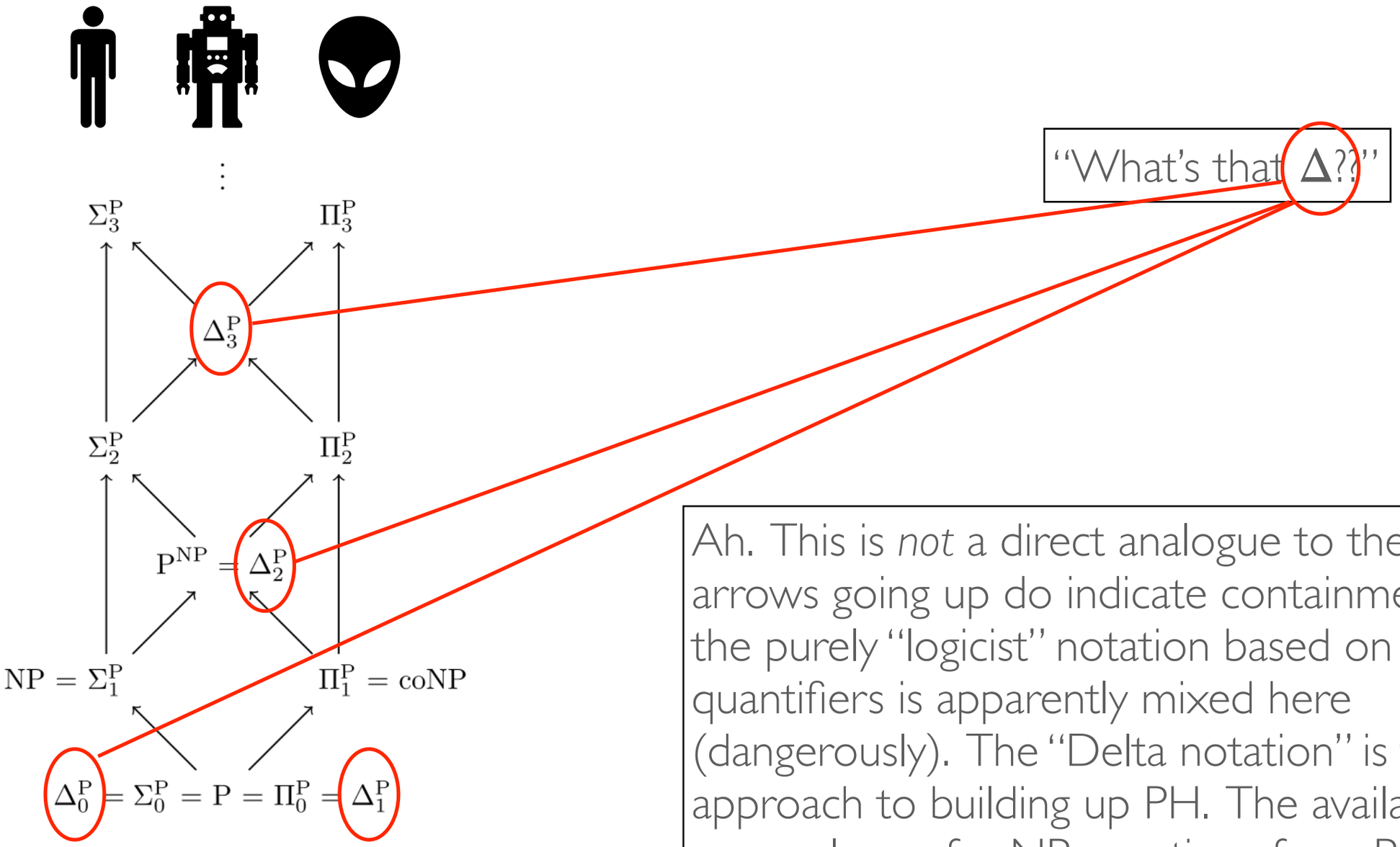
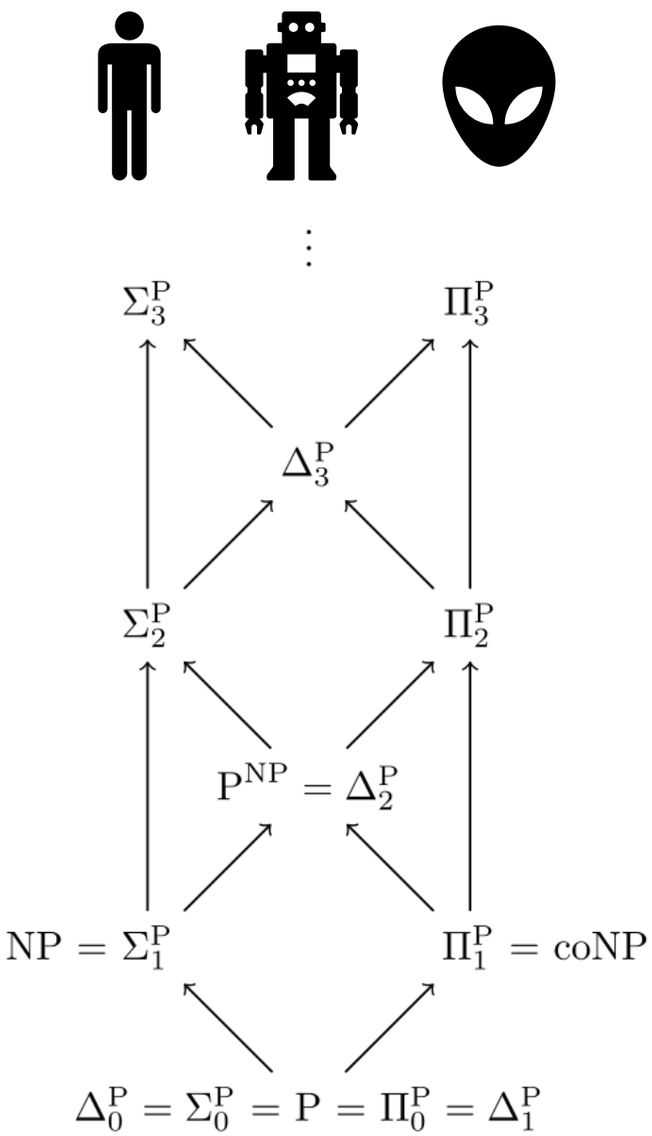


Ah. This is *not* a direct analogue to the AH. The arrows going up do indicate containment, but the purely "logicist" notation based on quantifiers is apparently mixed here (dangerously). The "Delta notation" is the oracle approach to building up PH. The availability of an oracle e.g. for NP questions from P-solving machine would subsume both NP and coNP, etc.

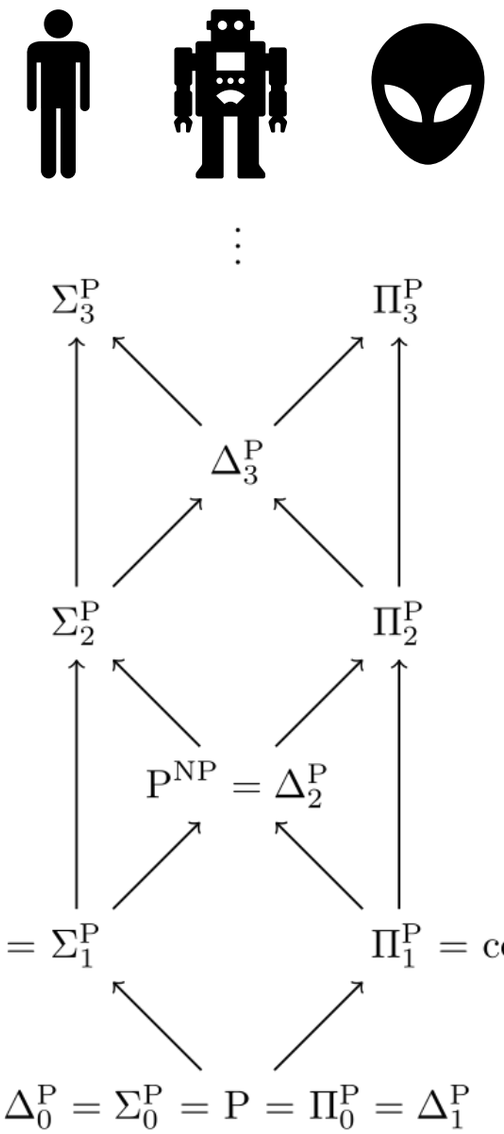
Polynomial Hierarchy, Part II

(via formal logic, directly)



Polynomial Hierarchy, Part II

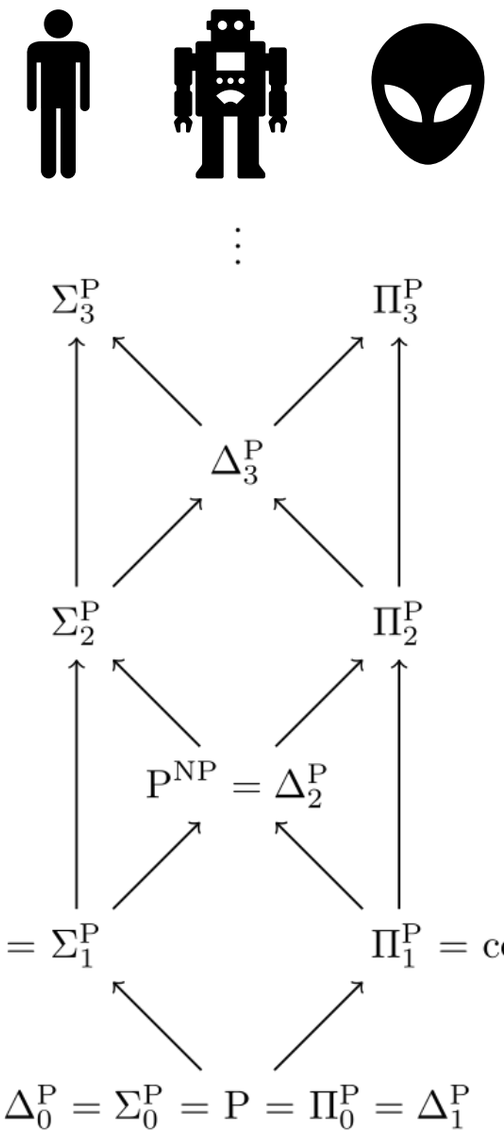
(via formal logic, directly)



Eg:

Polynomial Hierarchy, Part II

(via formal logic, directly)

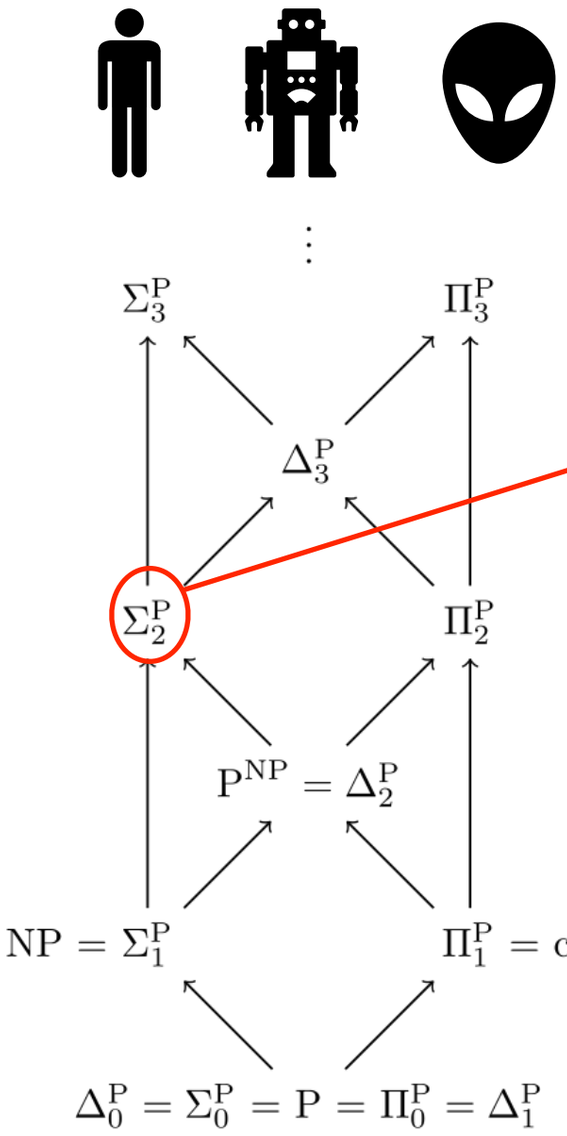


Eg:

$$\langle \phi_1, k \rangle \in L \text{ iff } \exists \phi_2 \forall \alpha KLogEquiv(\phi_1, \phi_2, |\phi_2| \leq k, \alpha(\phi_1) = \alpha(\phi_2))$$

Polynomial Hierarchy, Part II

(via formal logic, directly)



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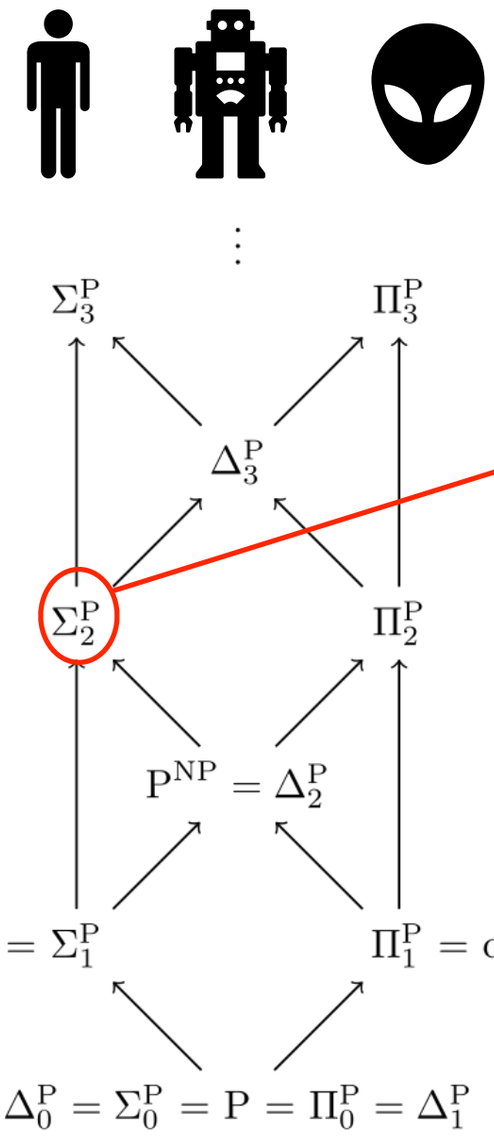

free variables

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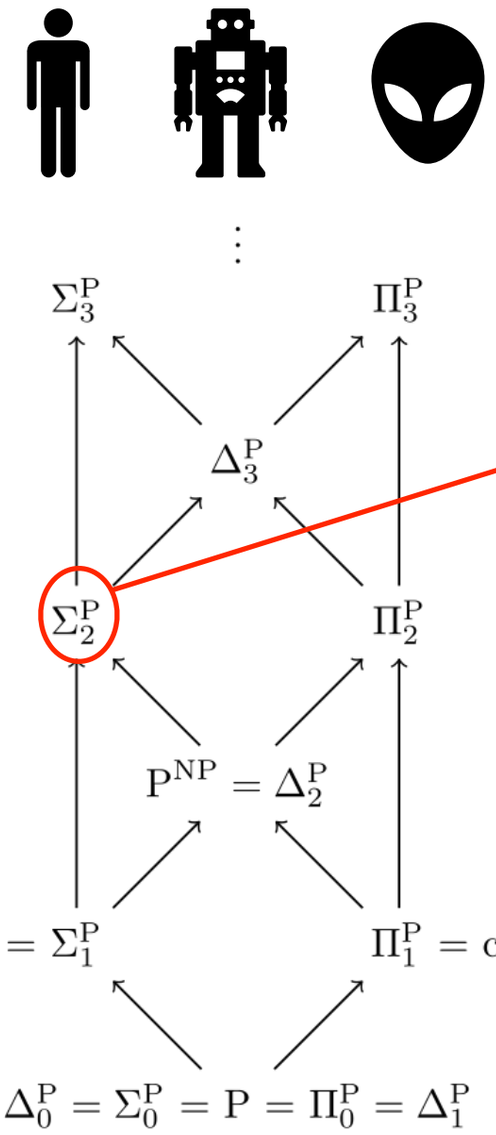

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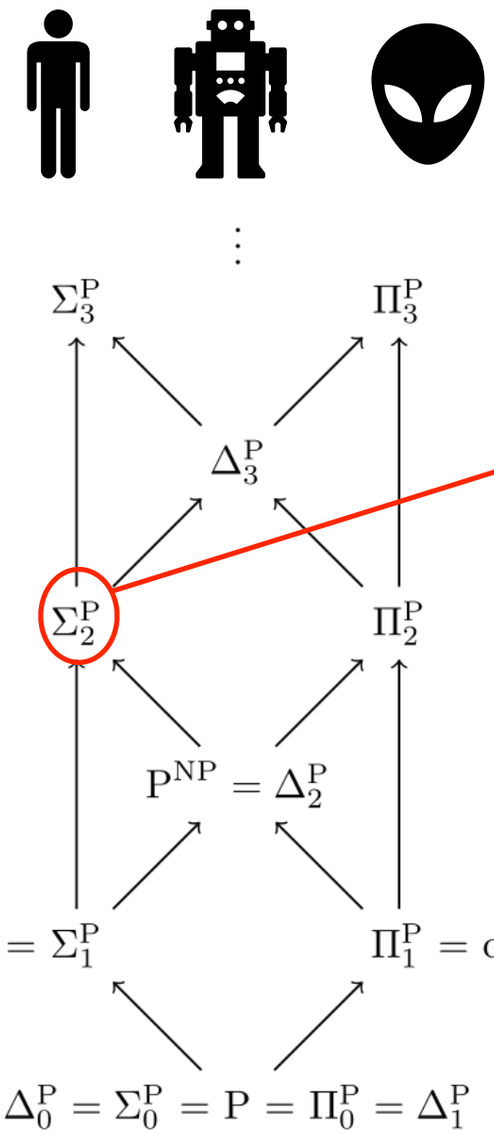
Polynomial Hierarchy, Part II

(via formal logic, directly)



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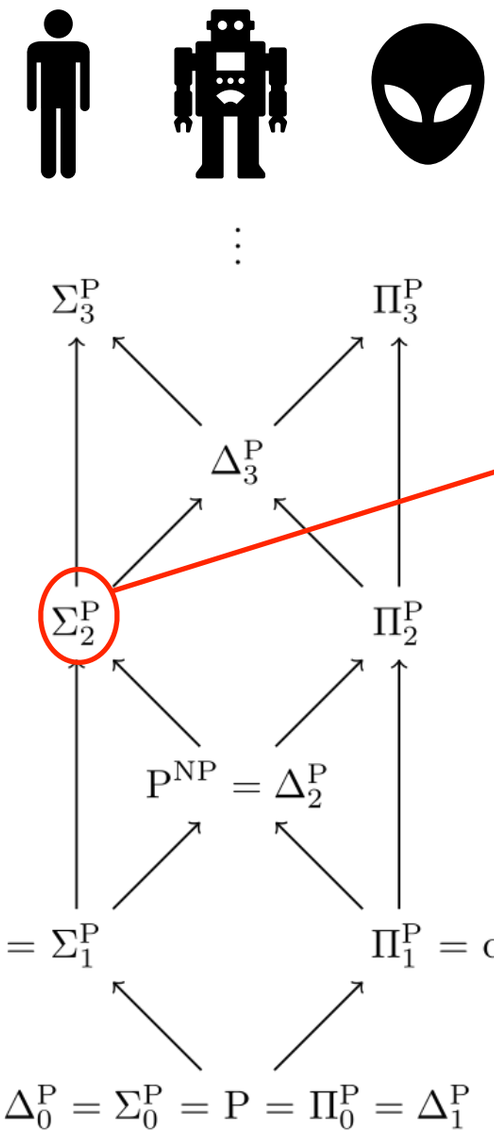

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Polynomial Hierarchy, Part II

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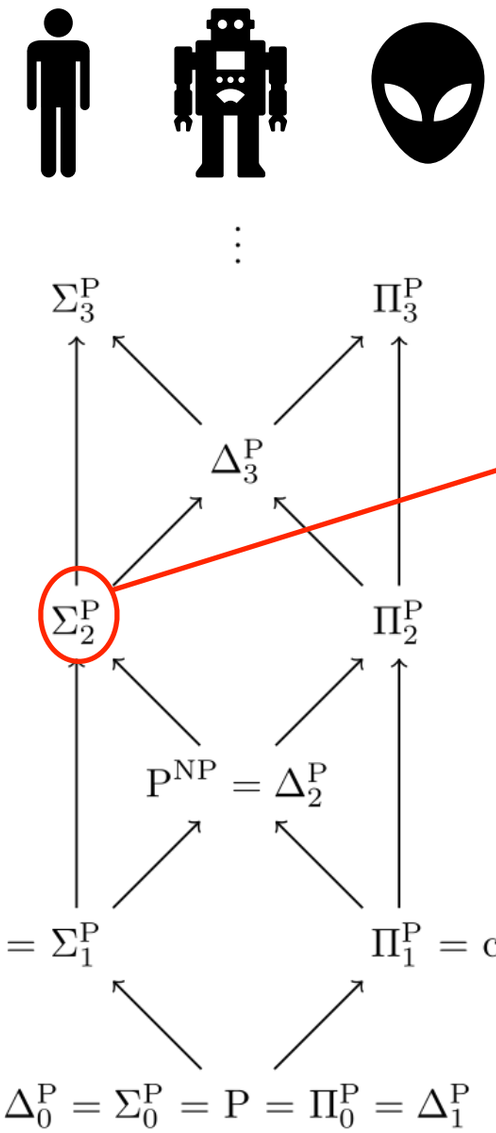

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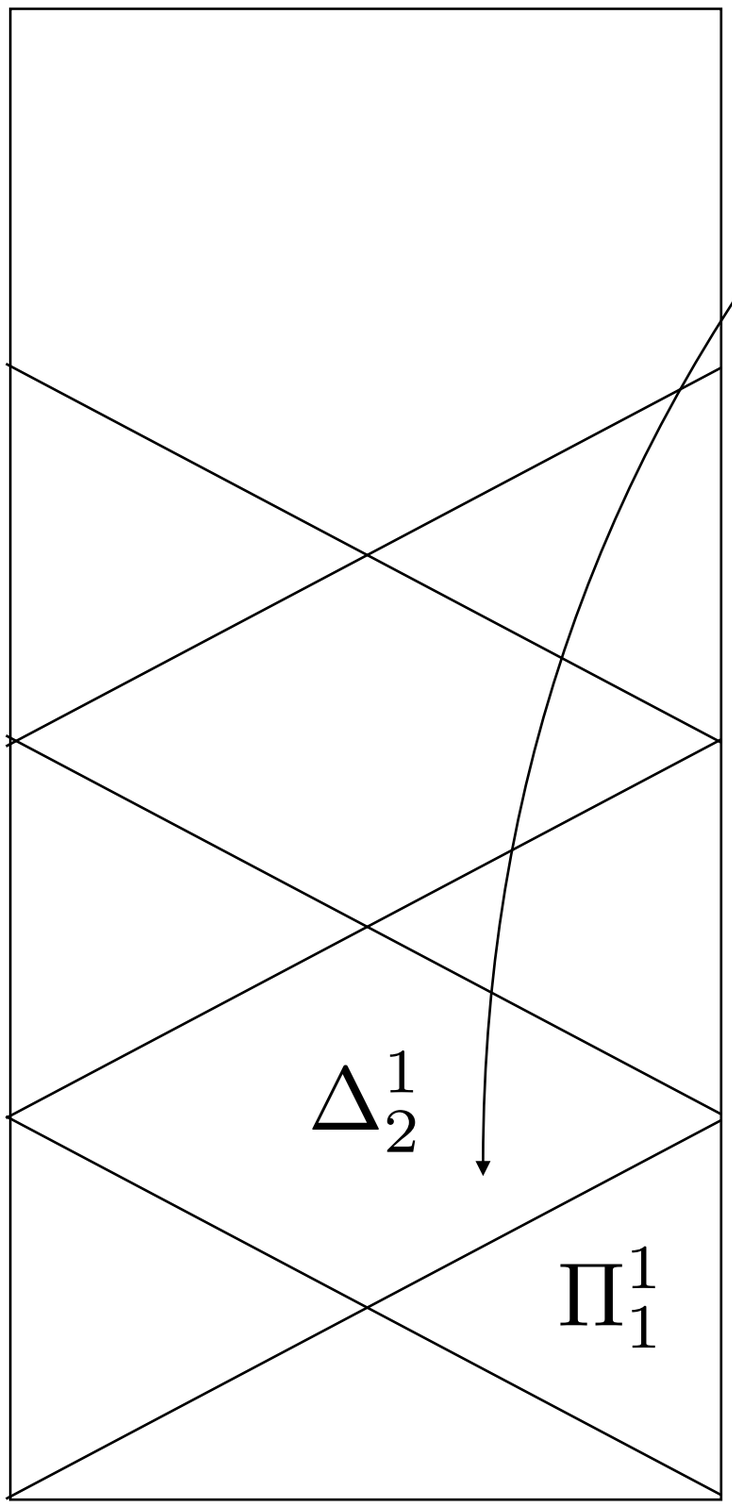
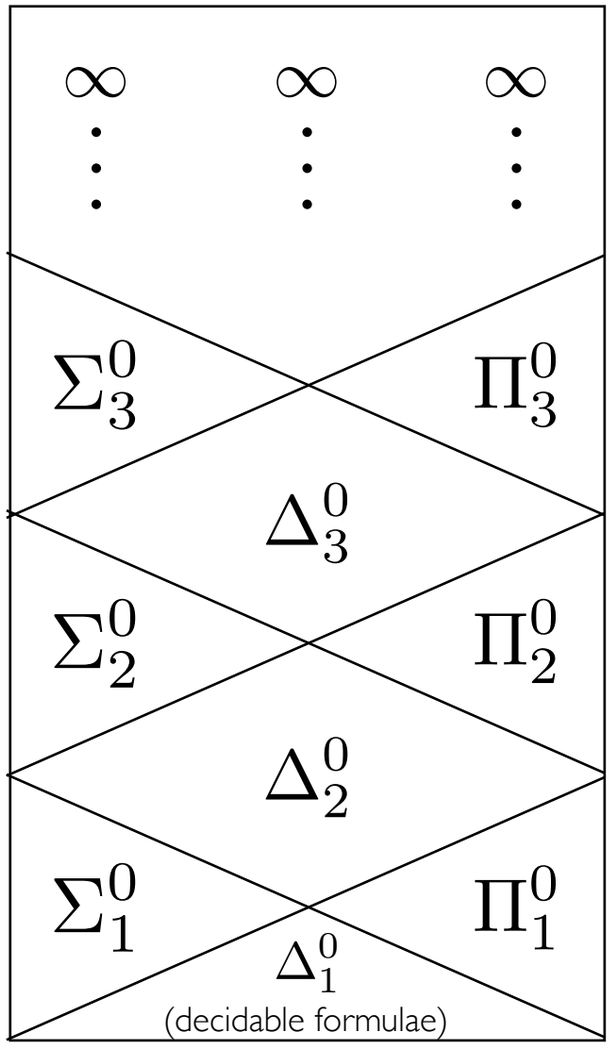
CogSci and AI need to say more about where AI falls/can fall in the landscape.



Infinite Time Turing Machines (ITTMs)



Human Persons
(according to Bringsjord)

Human Brains
(according to Granger)

$\mathcal{CH}$ (Chomsky Hierarchy)

Turing Machines (TMs)
Linear Bounded Automata (LBAs)
Push Down Automata (PDAs)
Finite State Automata (FSAs)

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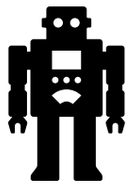

$\mathfrak{LM}$

$\mathcal{A}^n\mathcal{H}$ (Analytic Hierarchy)

Δ^1_2

Π^1_1

Infinite Time Turing Machines (ITTMs)

Human Persons
(according to Bringsjord)

$\mathcal{A}^r\mathcal{H}$ (Arithmetic Hierarchy)

∞ ∞ ∞

Σ^0_3 Π^0_3

Δ^0_3

Σ^0_2 Π^0_2

Δ^0_2

Σ^0_1 Π^0_1

Δ^0_1
(decidable formulae)

Human Brains
(according to Granger)

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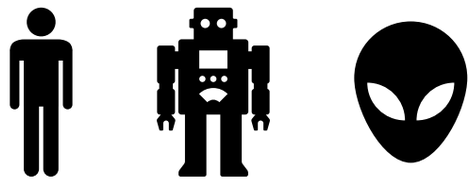
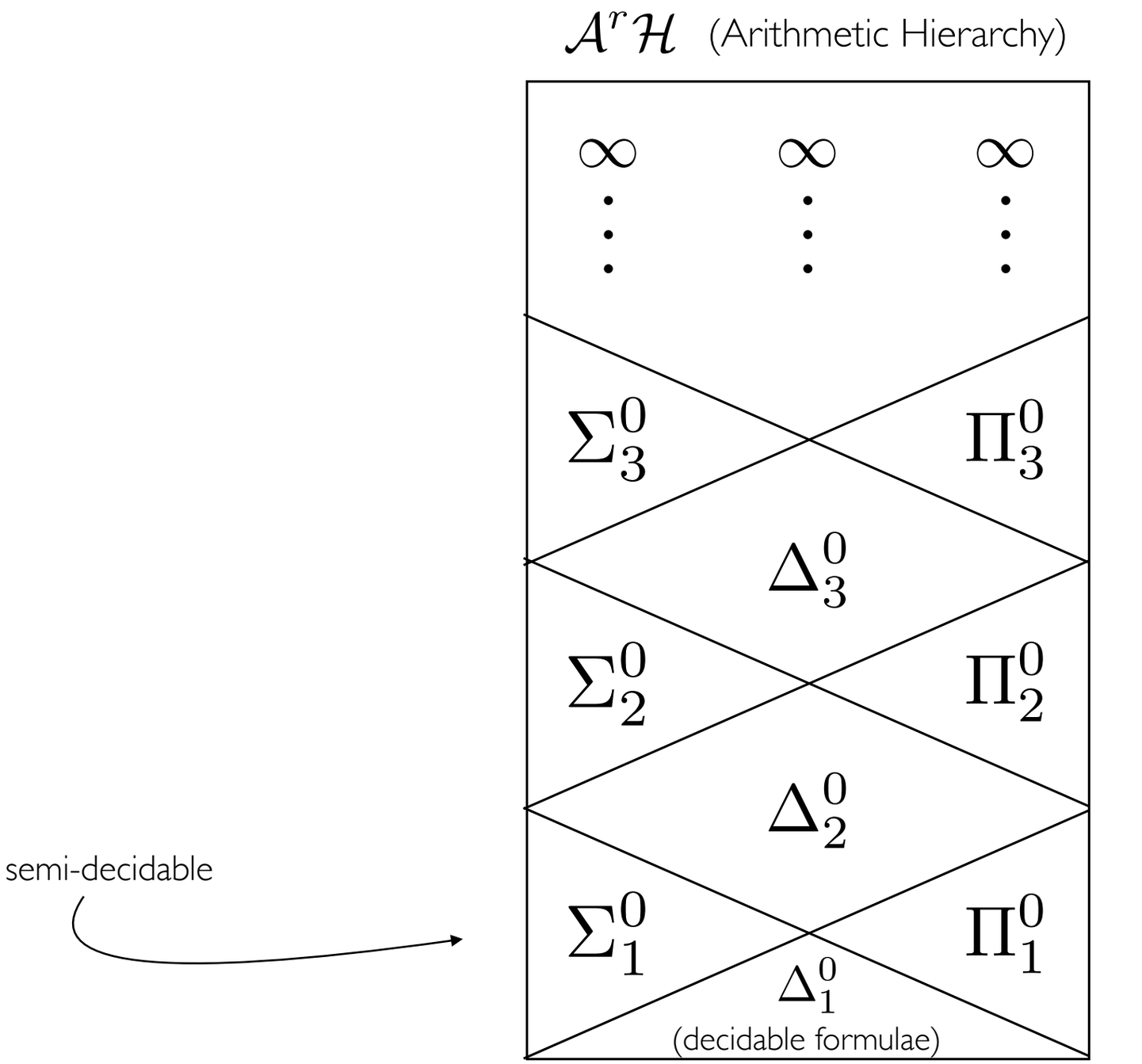


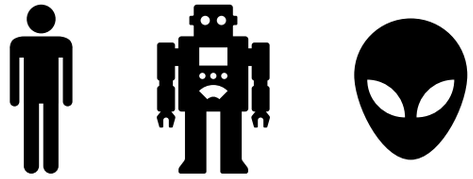
$$\mathbf{2SAMEFUNC} \coloneqq \{\mathfrak{m}_1, \mathfrak{m}_2 : \forall u \forall v [\exists k(\langle \mathfrak{m}_1, u \rangle : v, k \leftrightarrow \exists k'(\langle \mathfrak{m}_2, u \rangle : v, k')]$$



$$\Delta_1^0 = \Sigma_1^0 \cap \Pi_1^0$$

Arithmetic Hierarchy, Part I

Can you see the carryover from PH?



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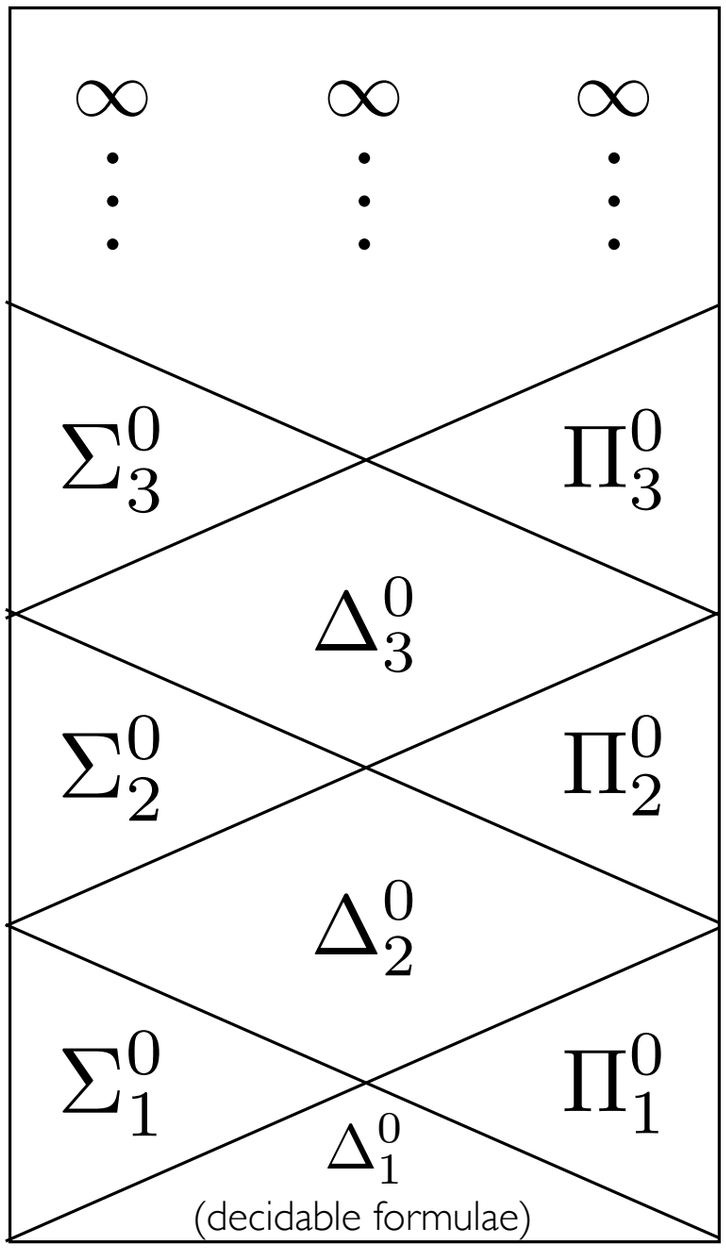
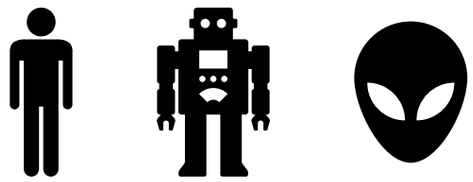
$\mathcal{A}^r\mathcal{H}$ (Arithmetic Hierarchy)



semi-decidable

$$\Delta_1^0 = \Sigma_1^0 \cap \Pi_1^0$$

Arithmetic Hierarchy, Part I



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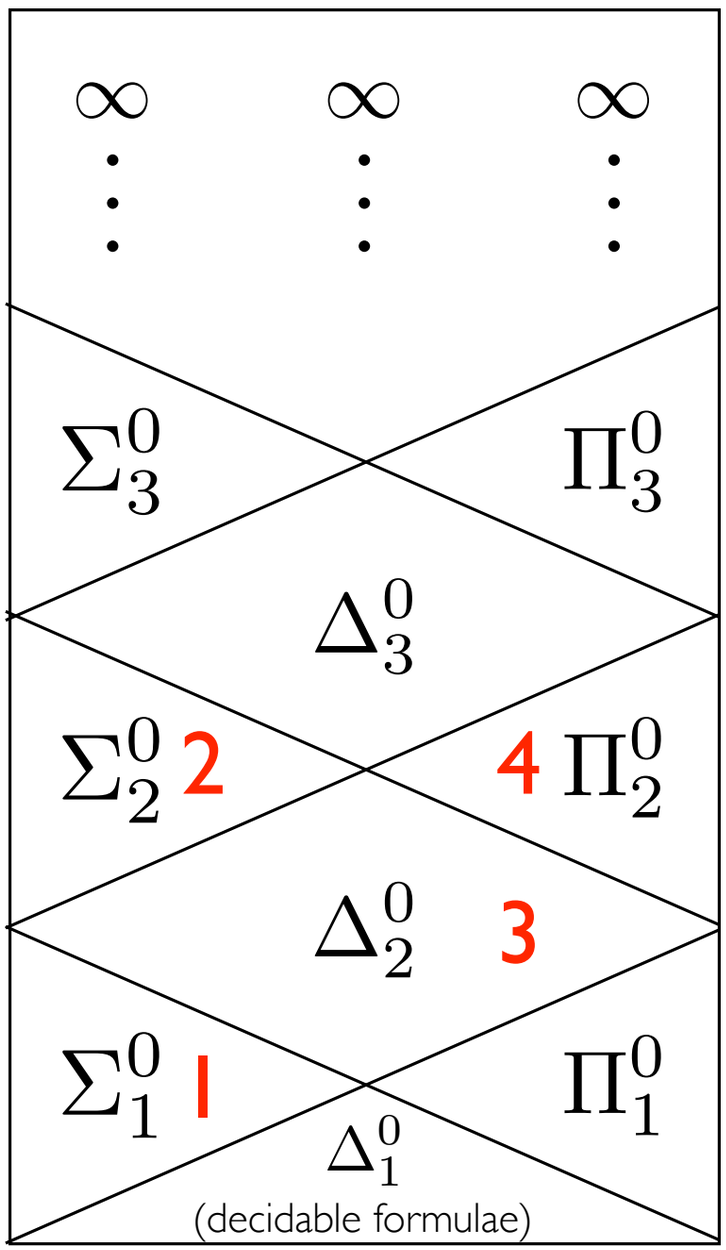

semi-decidable

Let R be a Turing-decidable (= decidable, *simpliciter*) dyadic relation. Where is the set:

$$\{x : \exists y R(x, y)\},$$

1 2 3 or 4?

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Arithmetic Hierarchy, Part I

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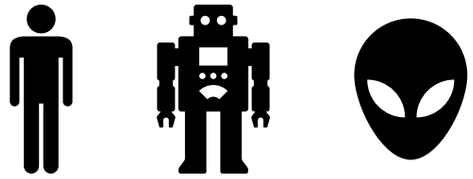

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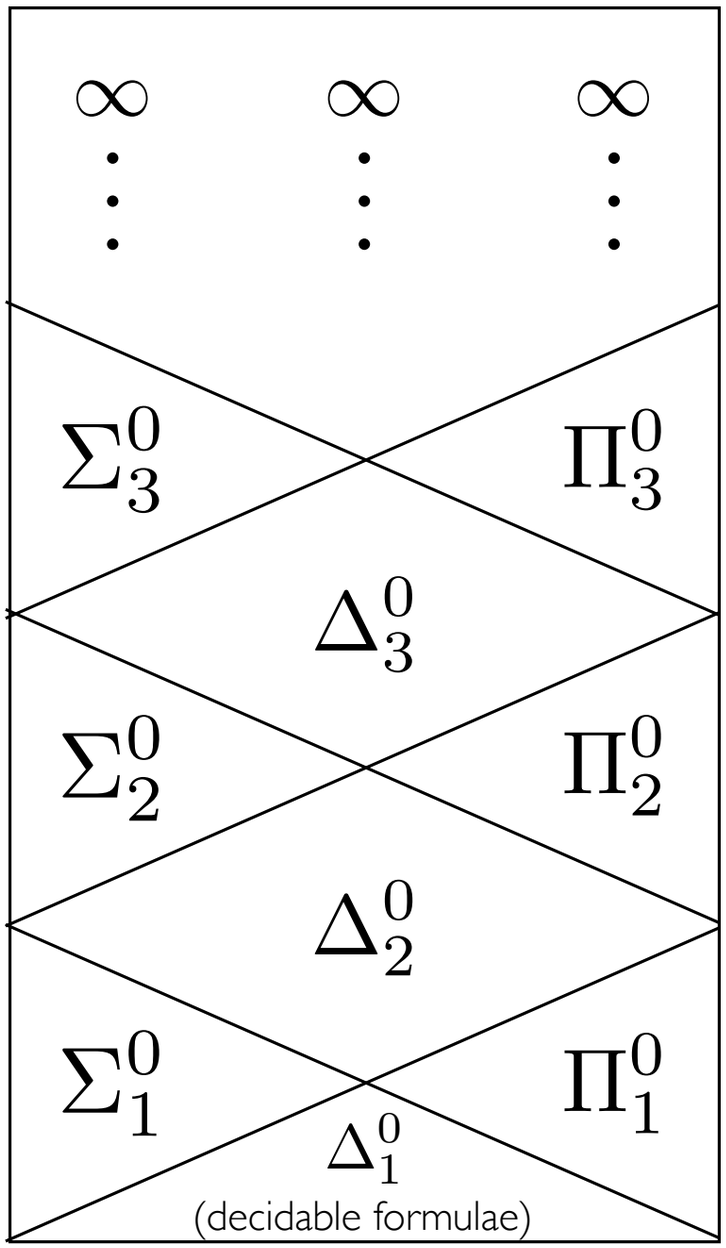

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Arithmetic Hierarchy, Part I

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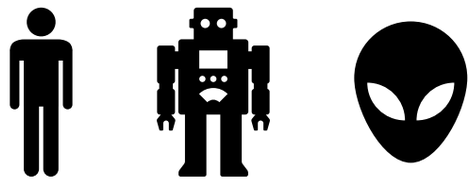

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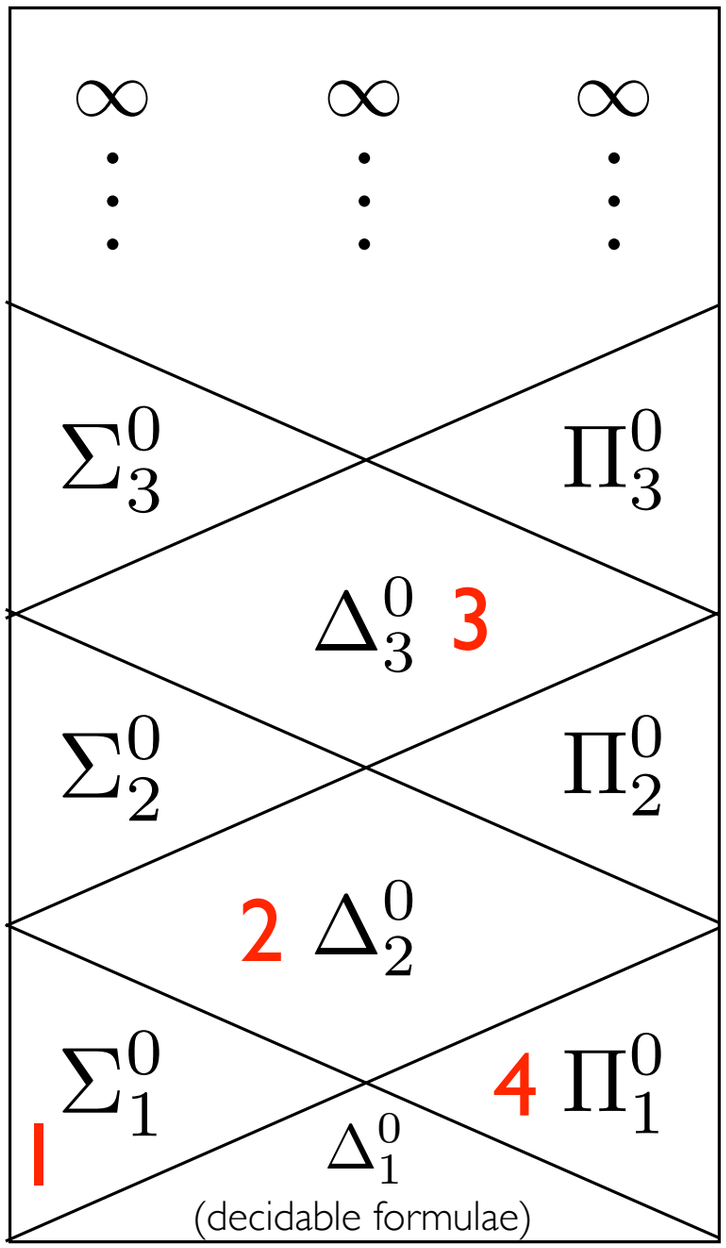

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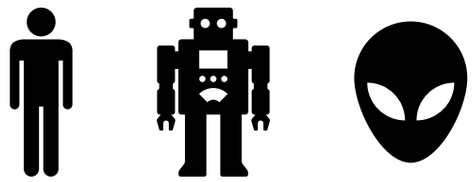

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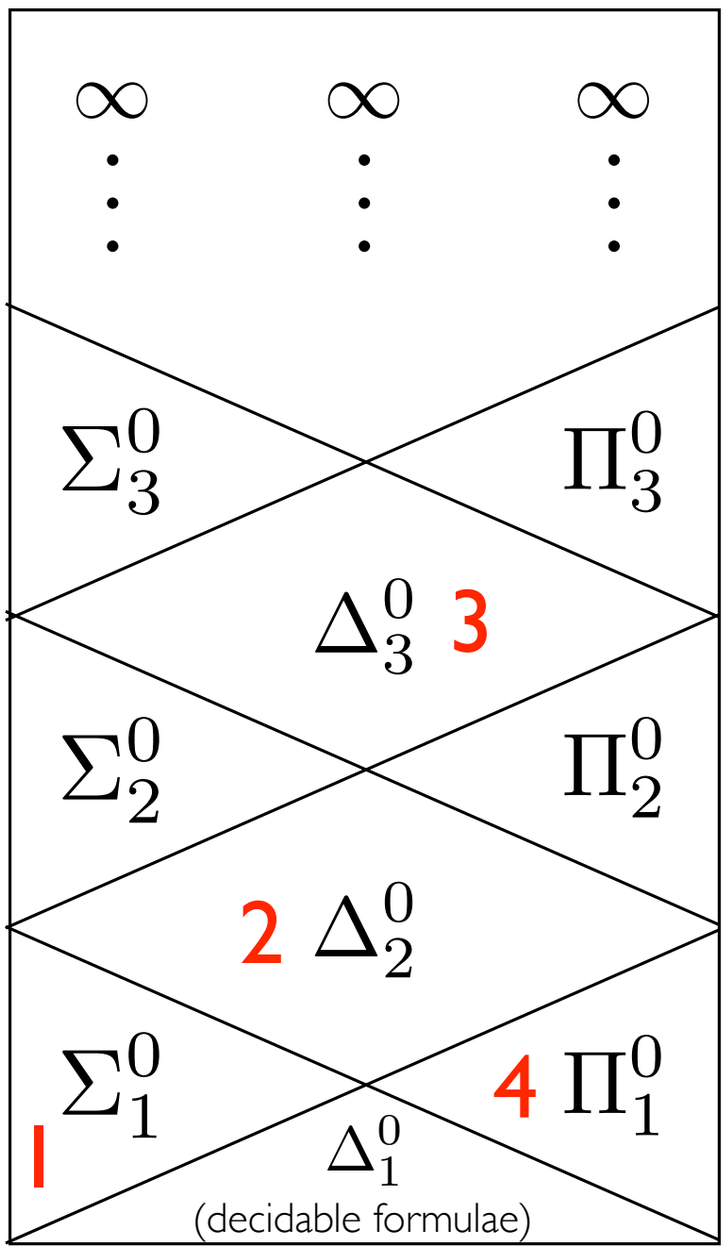

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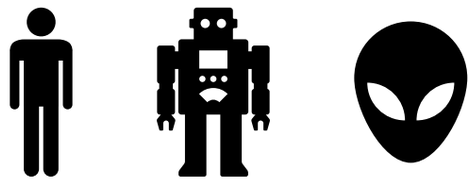
Arithmetic Hierarchy, Part I



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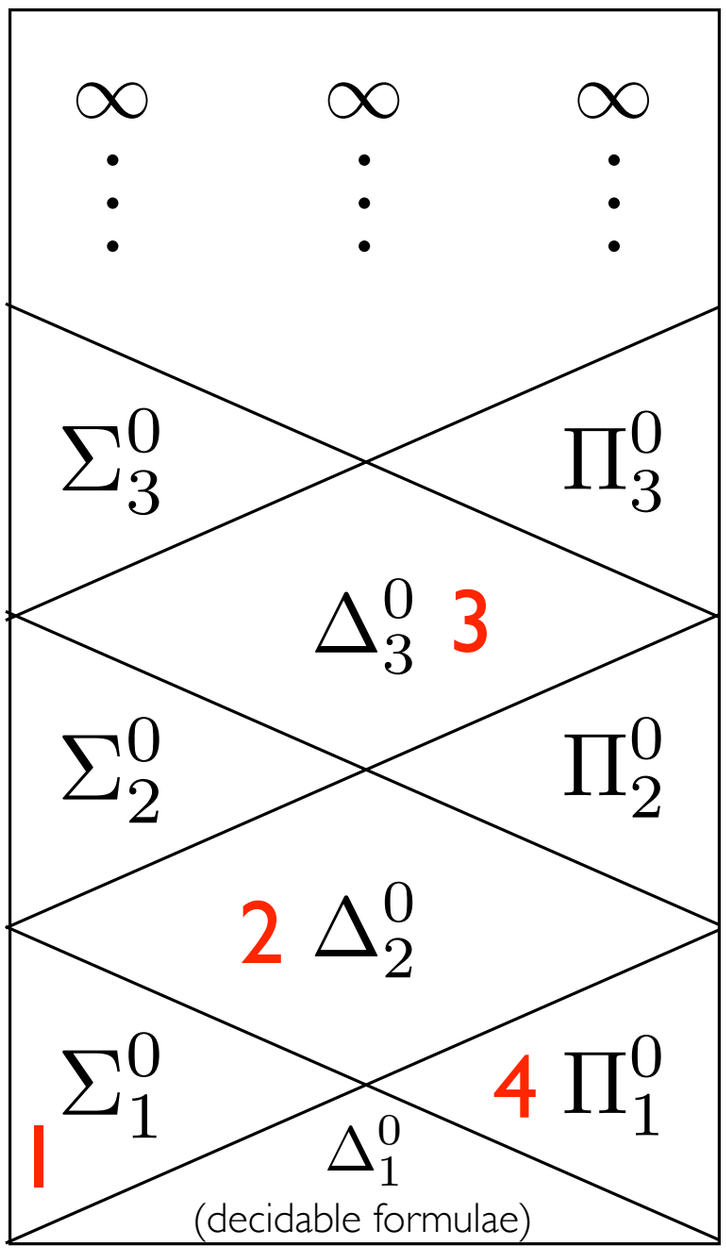

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1 2 3 or 4?

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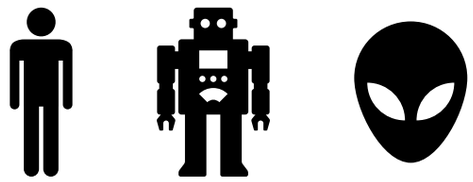
Arithmetic Hierarchy, Part I



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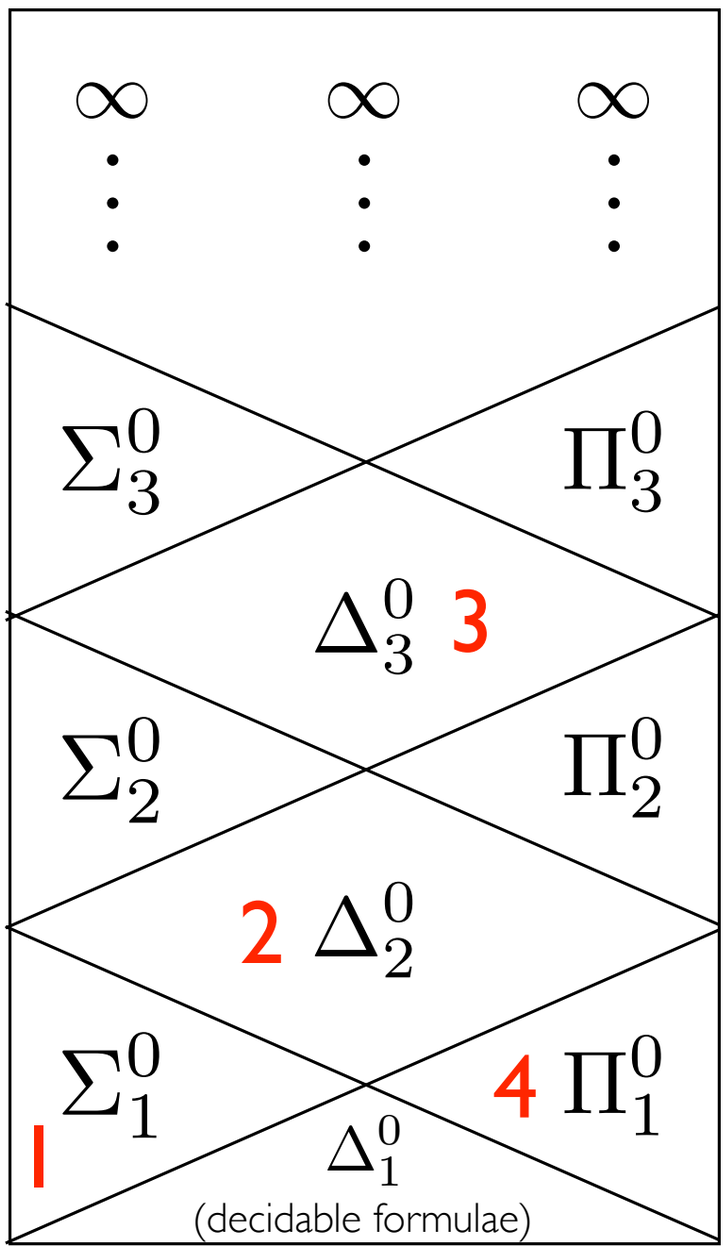

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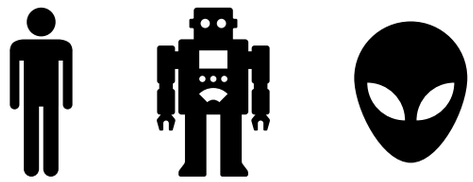

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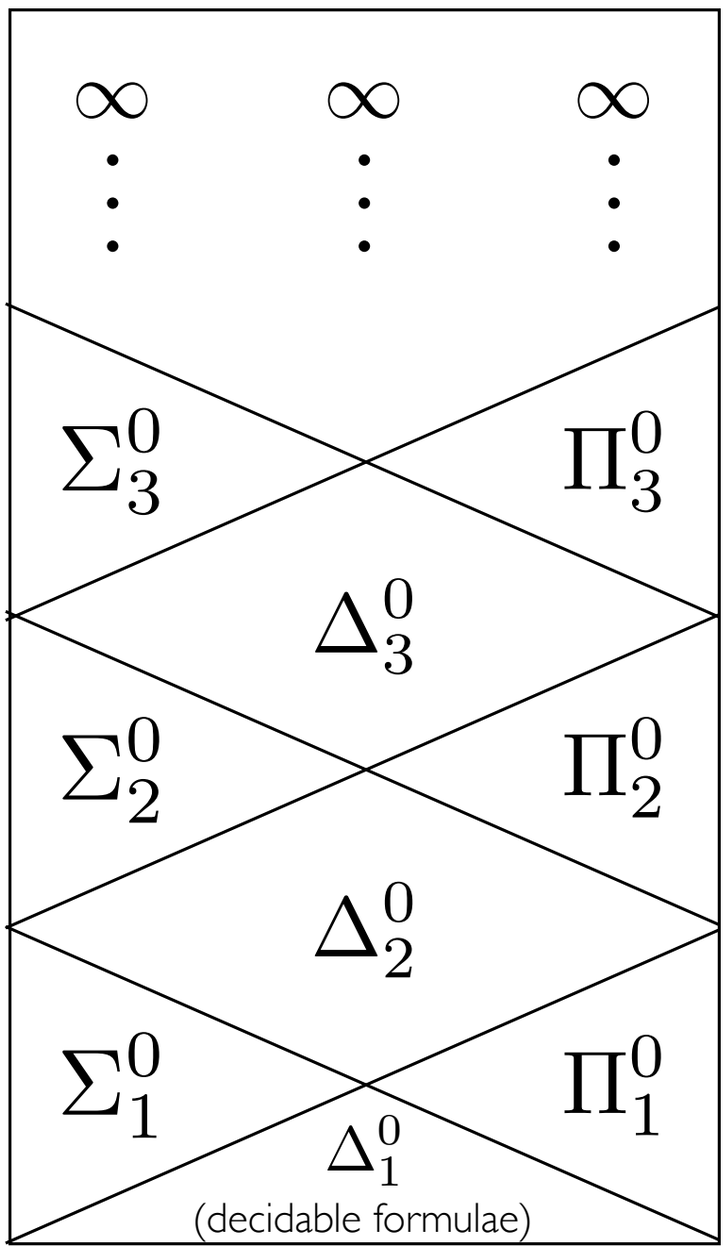

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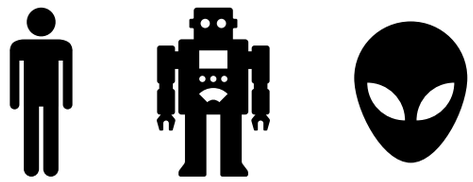

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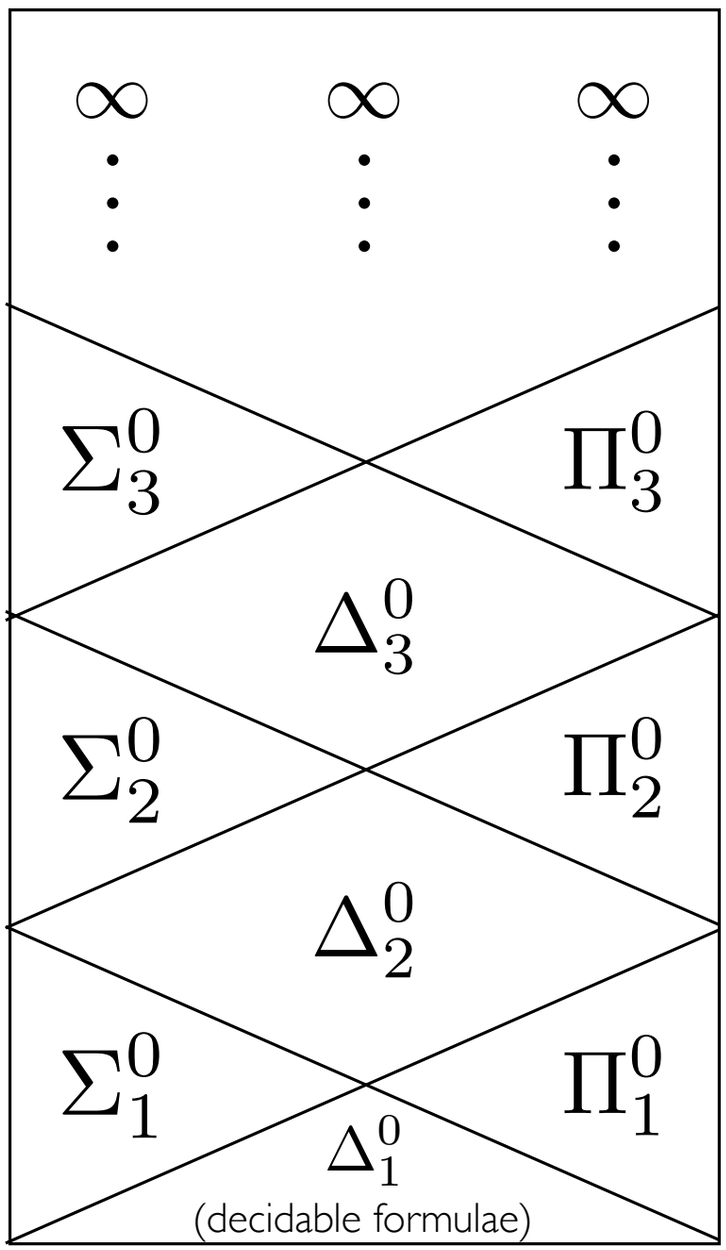

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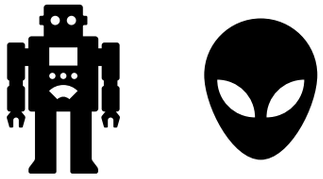

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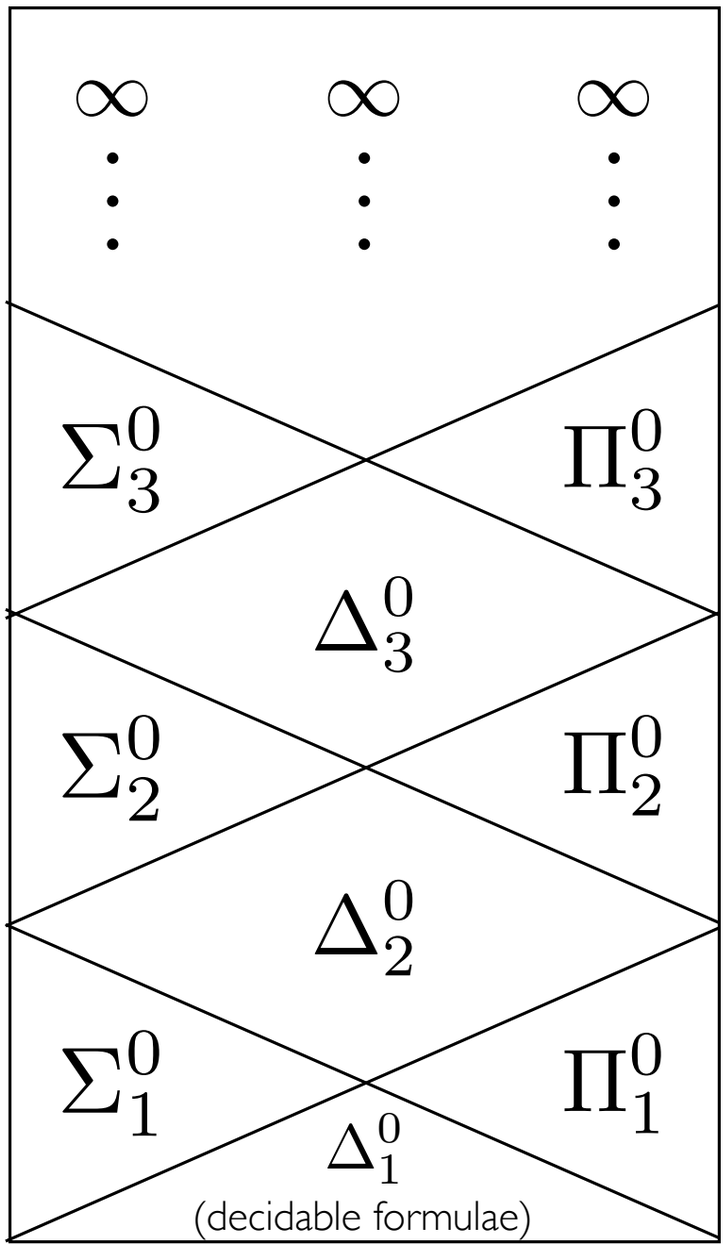

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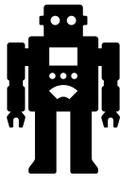

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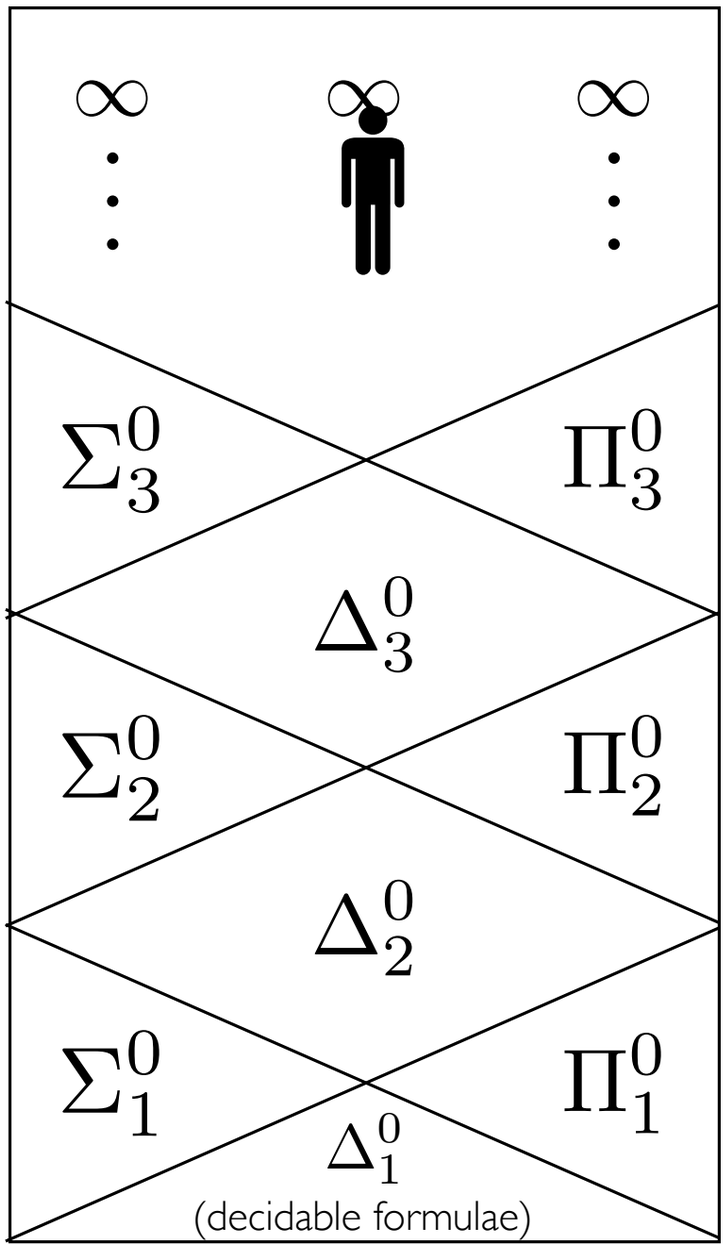

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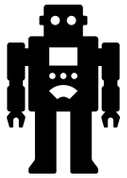

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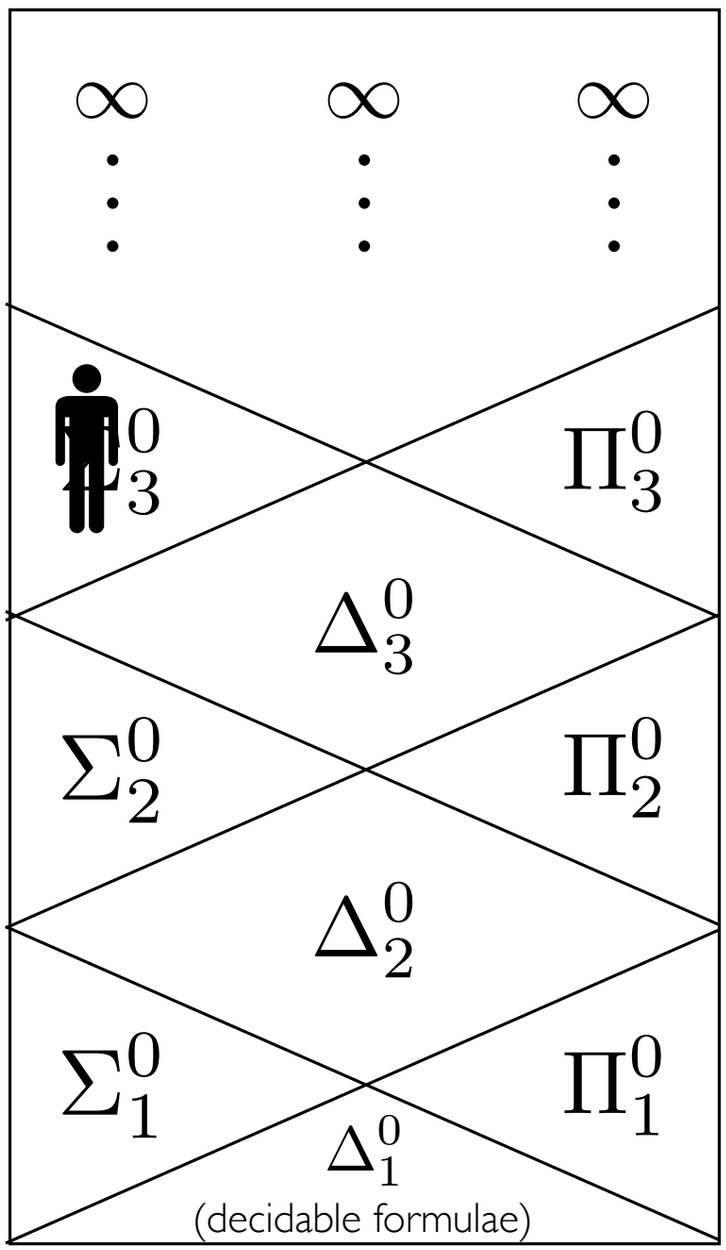

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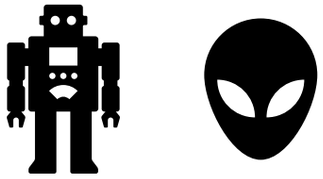

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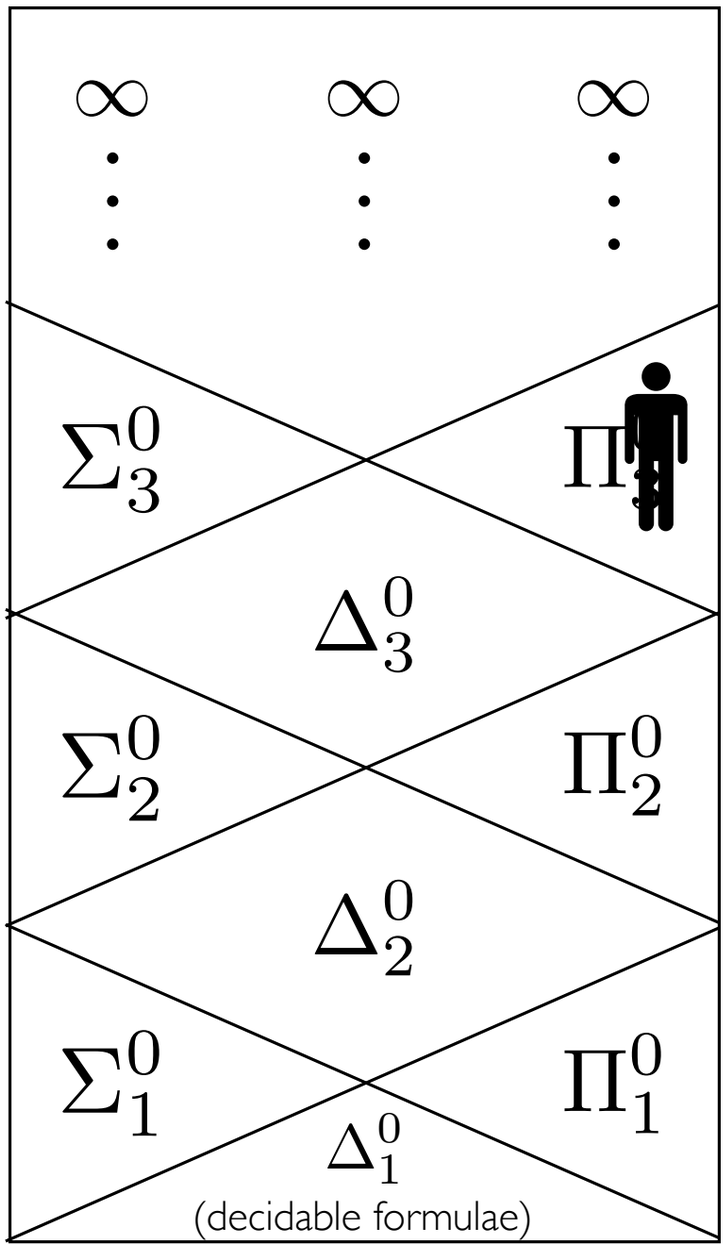

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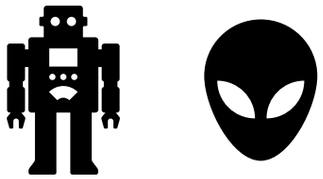

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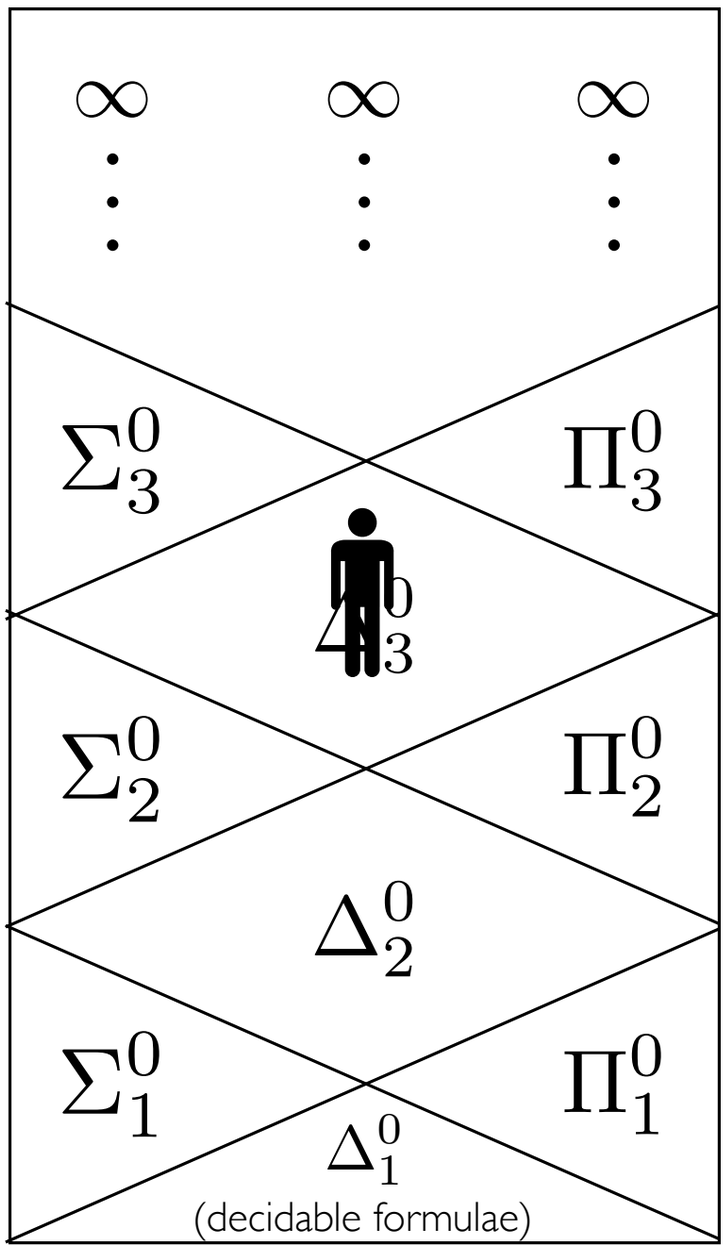

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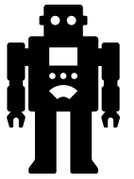

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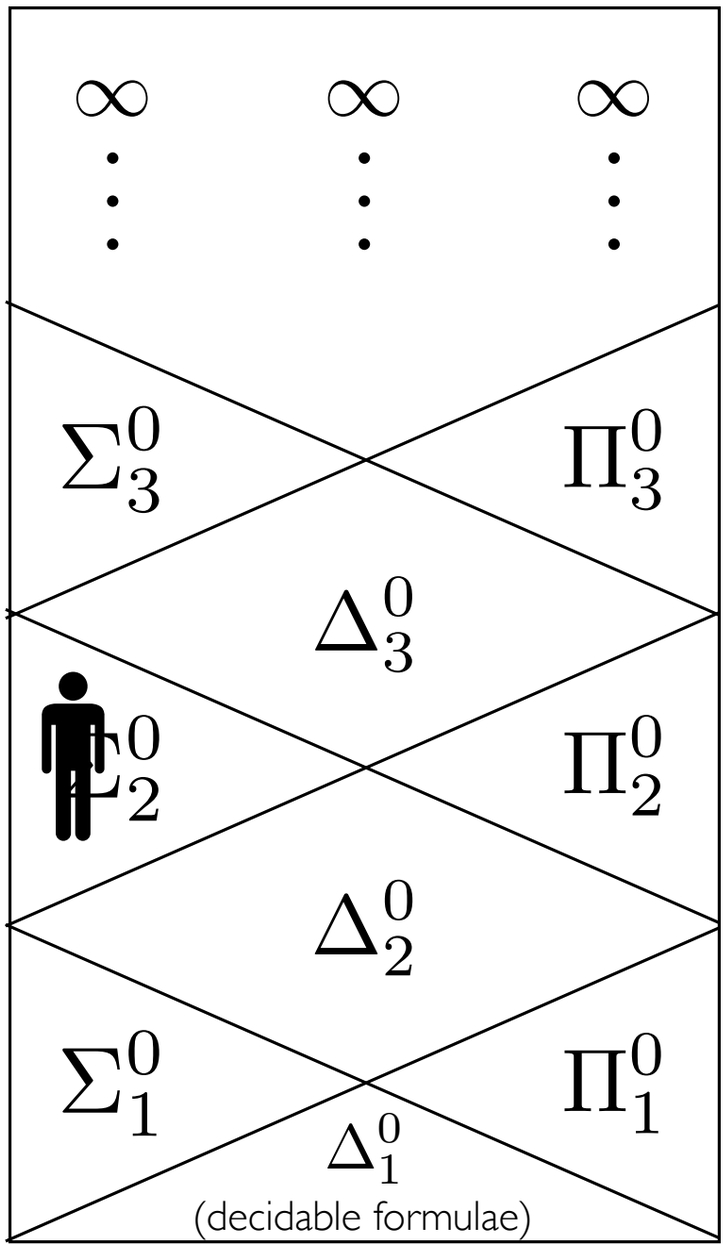

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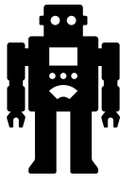

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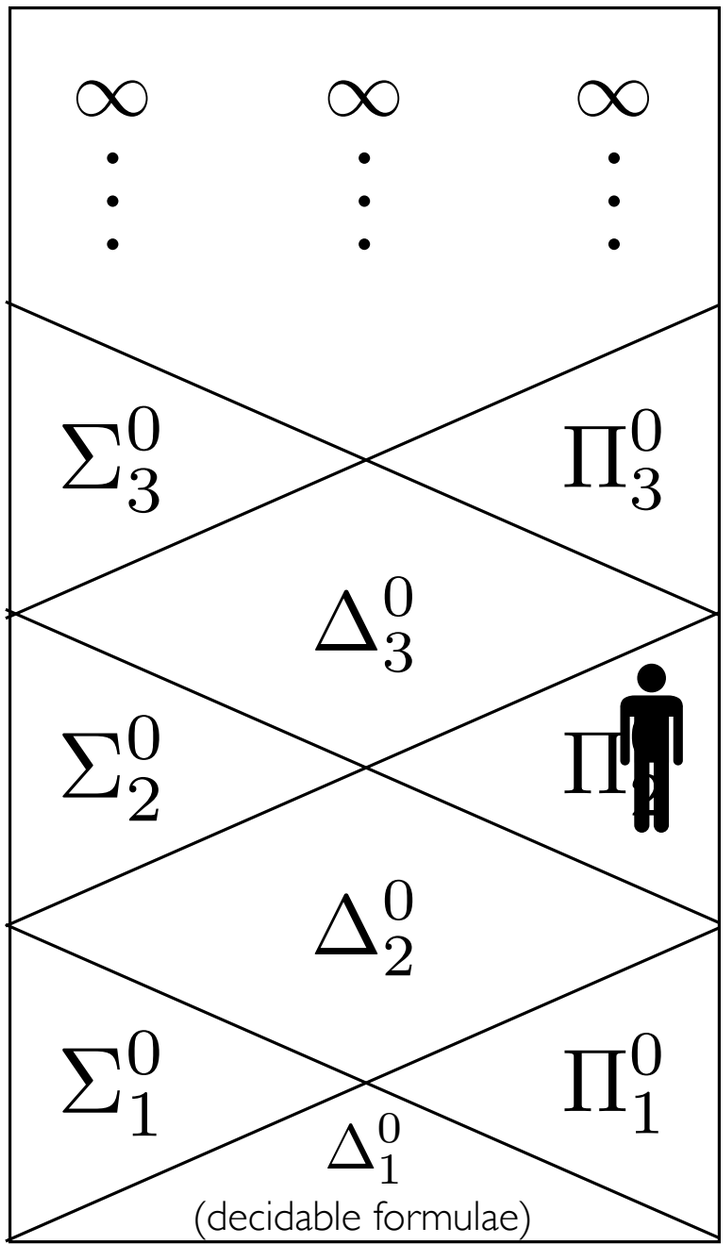

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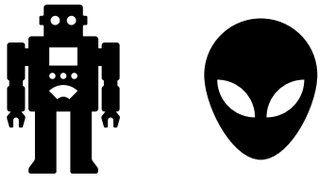

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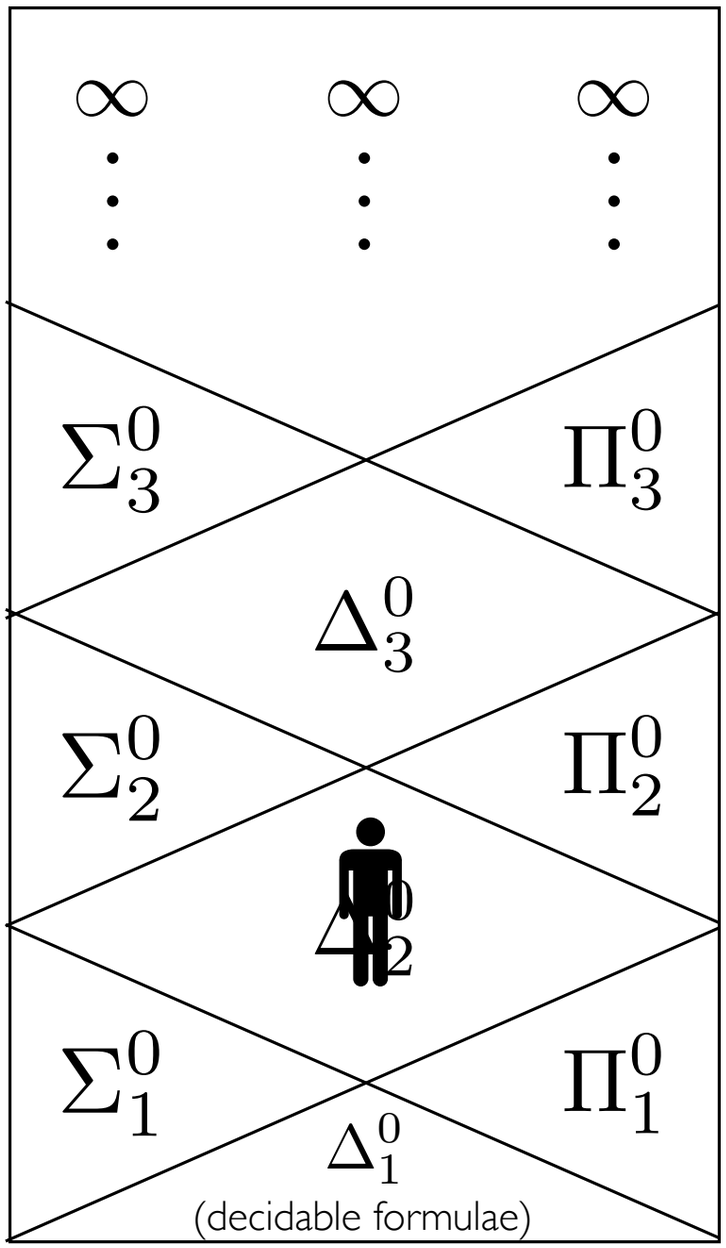

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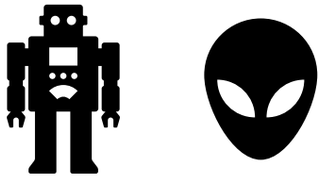

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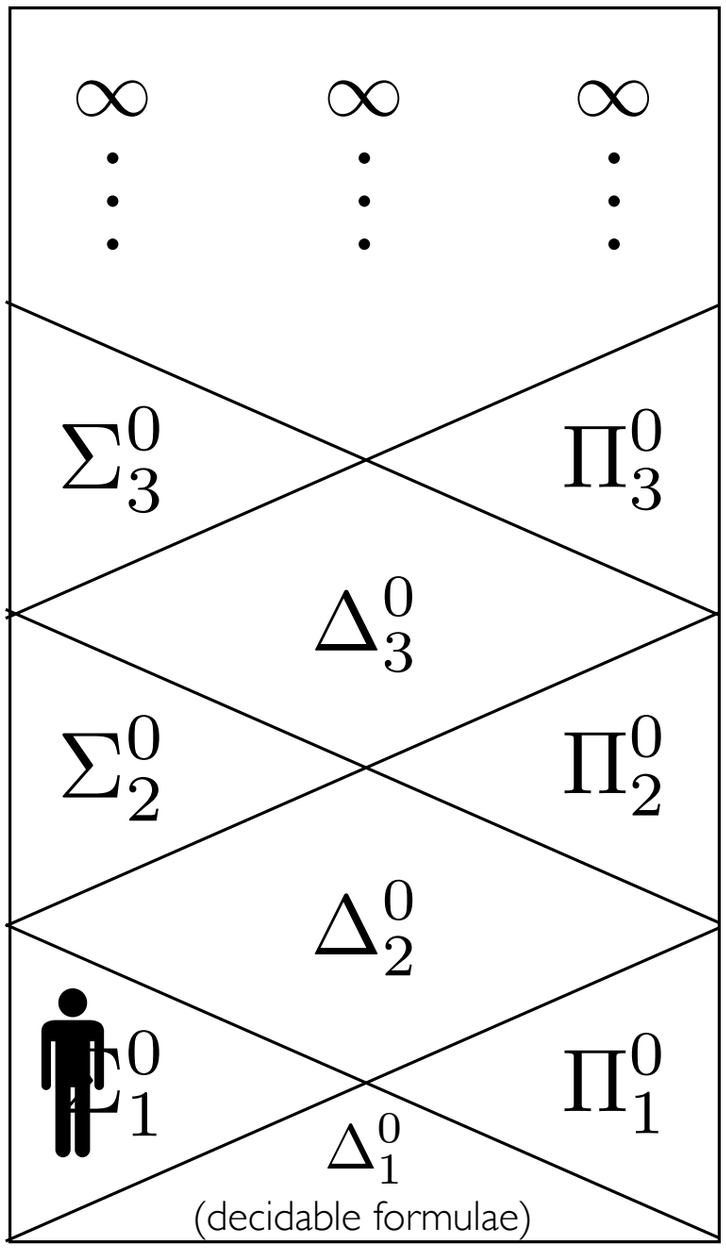

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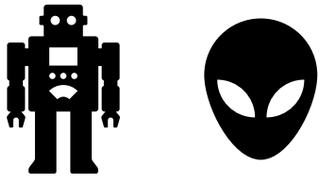

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$$\mathbf{2SAMEFUNC} := \{\mathfrak{m}_1, \mathfrak{m}_2 : \forall u \forall v [\exists k(\langle \mathfrak{m}_1, u\rangle : v, k \leftrightarrow \exists k'(\langle \mathfrak{m}_2, u\rangle : v, k')]$$

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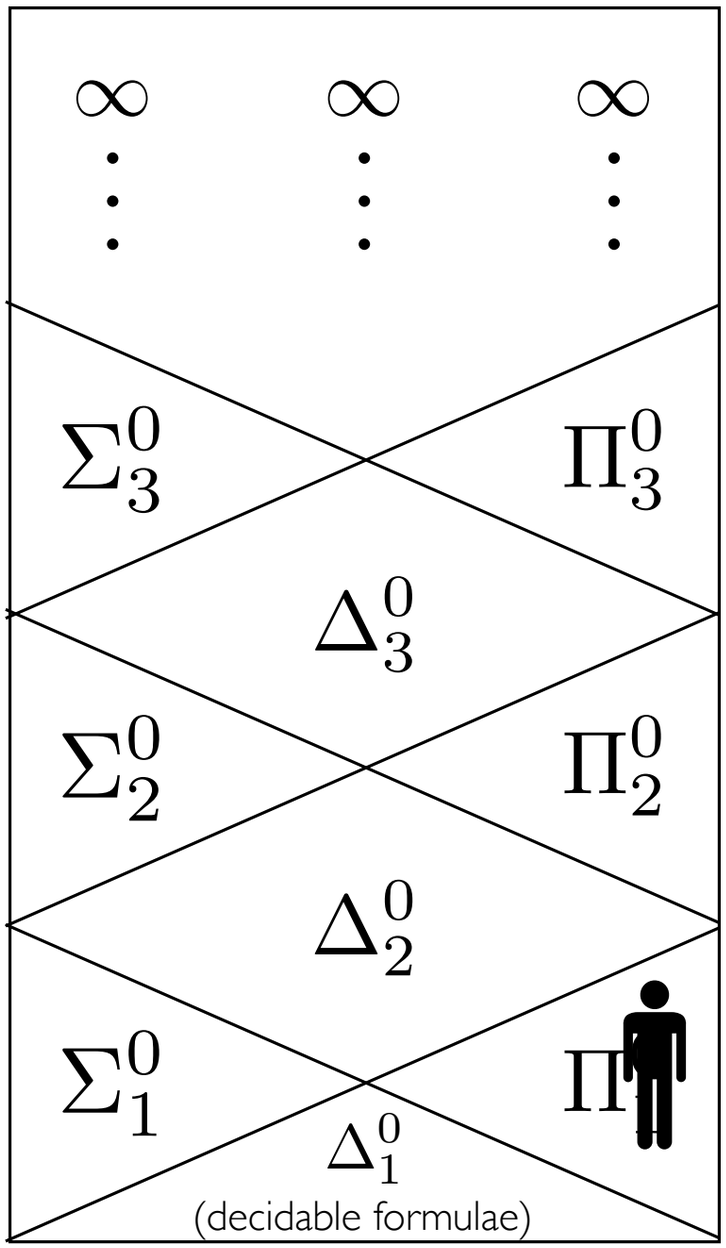

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$x \in \Sigma_i$ iff $\exists R\ \exists y_1 \forall y_2 \cdots Q_i y_i R(x, y_1, y_2, \ldots, y_i)$
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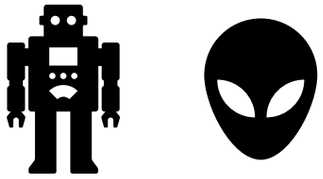
Try your hand at classifying! …

From Kleene: The set to be classified, $\mathscr{K}$, consists of all those inputs to a given Turing machine $\mathfrak{m}$ that results in this machine halting after some number of steps.

$$\Delta_1^0 = \Sigma_1^0 \cap \Pi_1^0$$

Arithmetic Hierarchy, Part I



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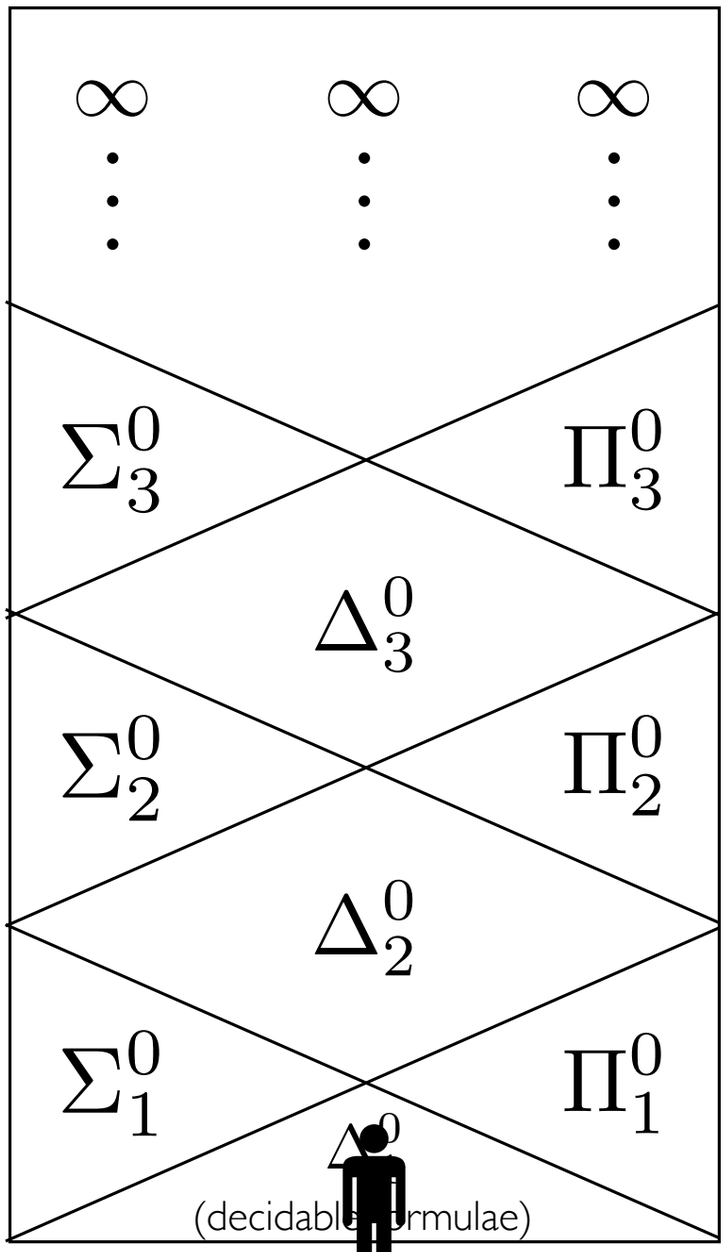

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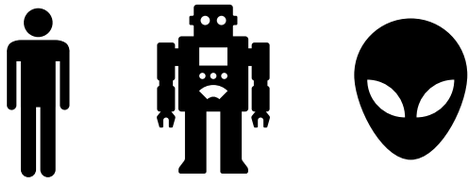

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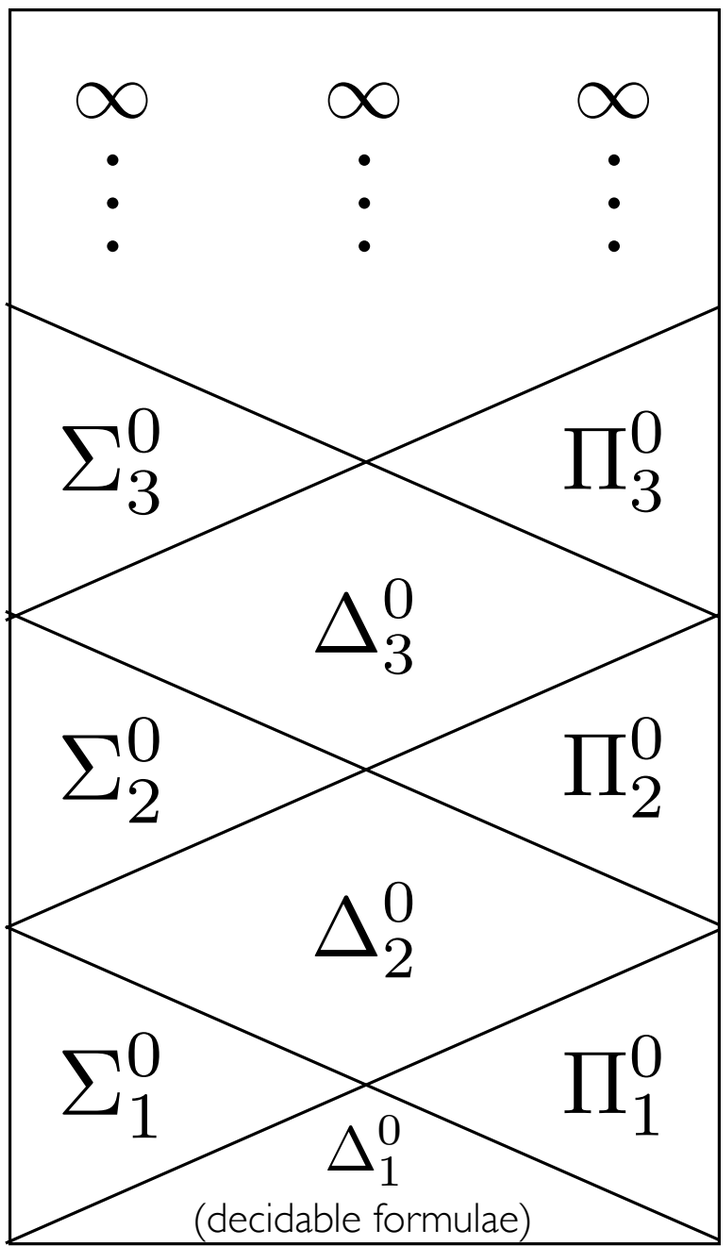

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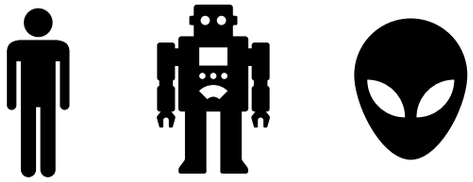

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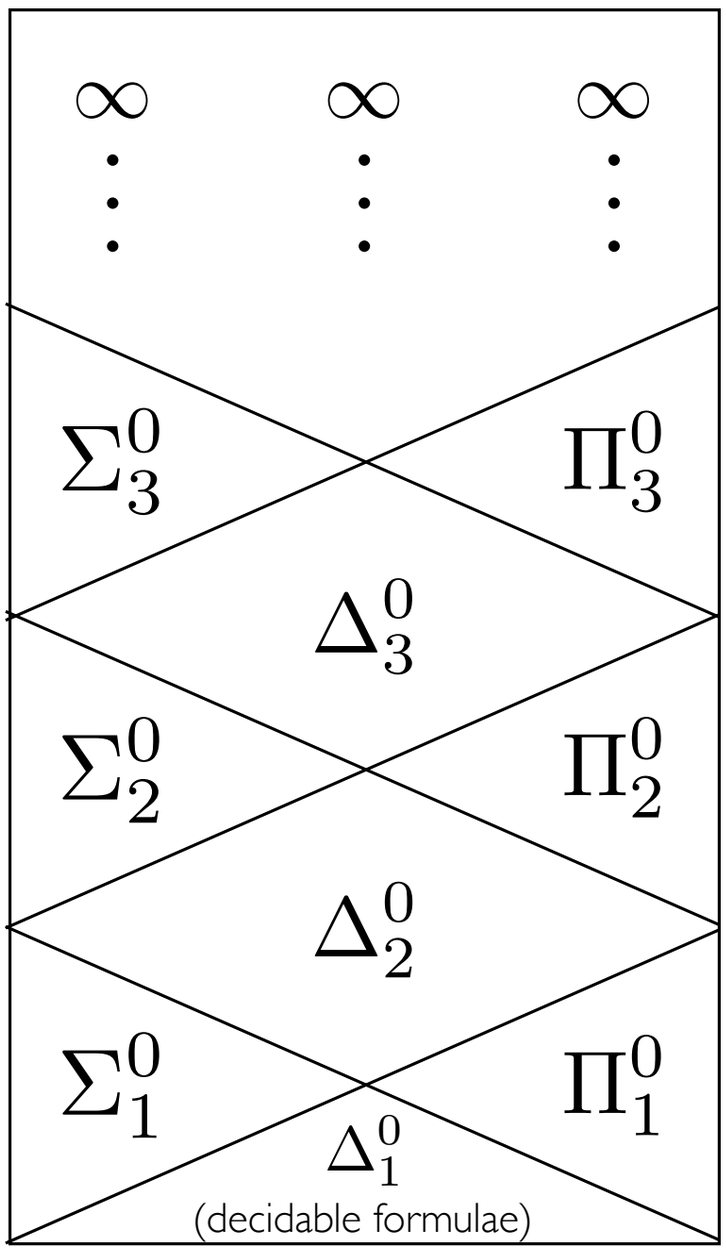

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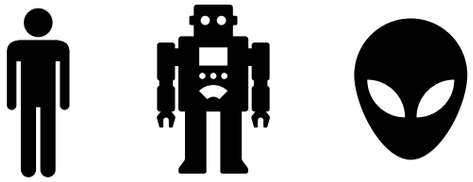

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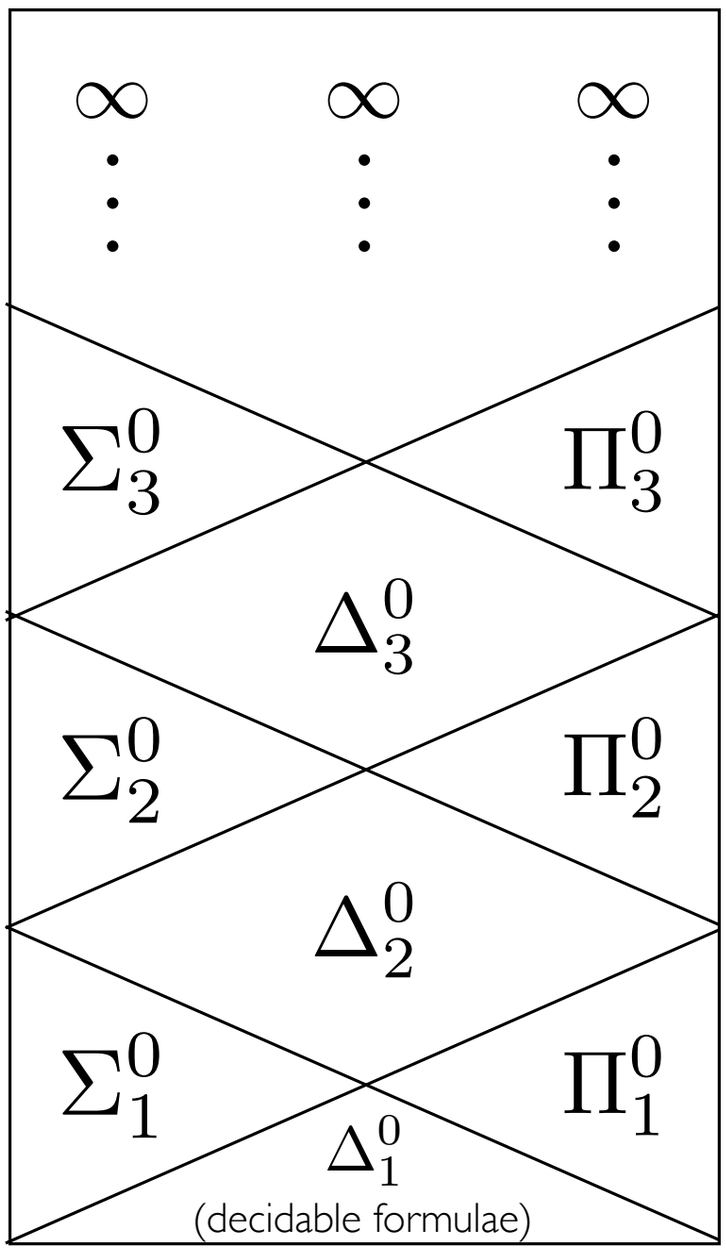

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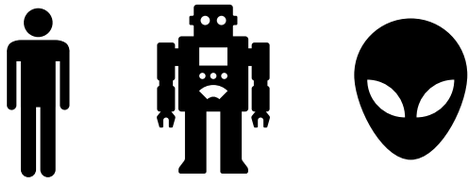

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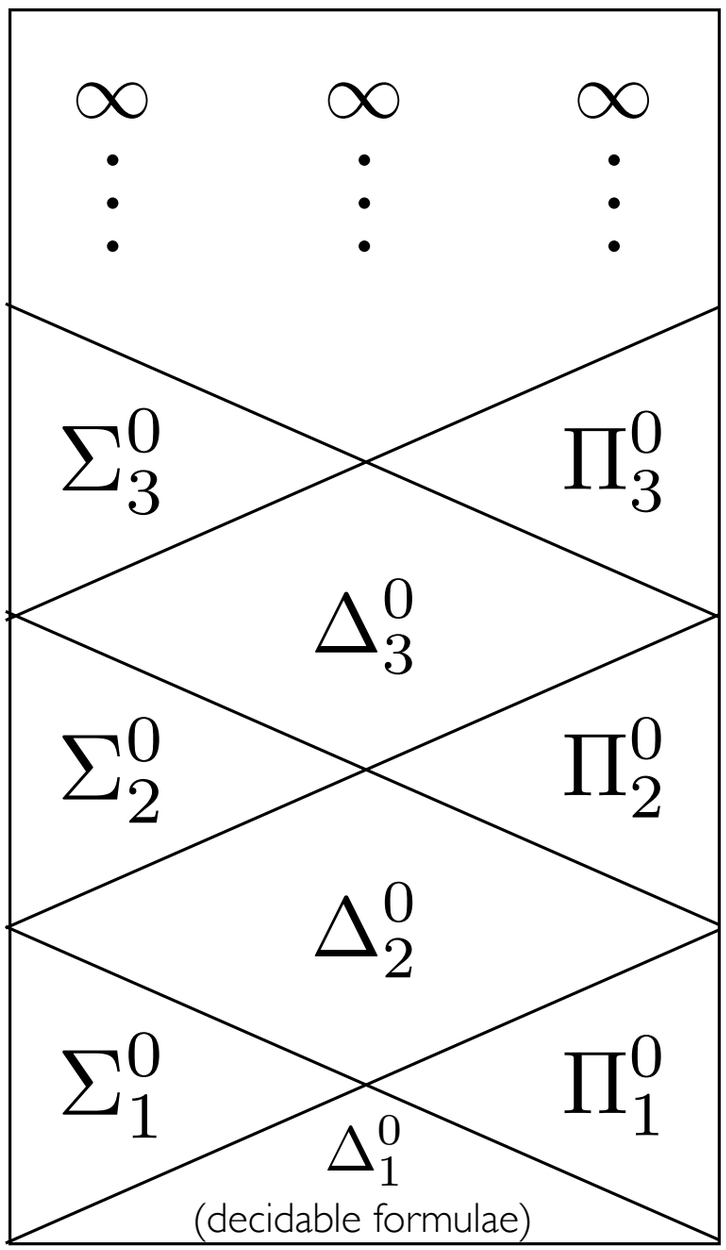

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The set to be classified is the set of all pairs of programs P_1 and P_2 s.t. both compute exactly the same functions.

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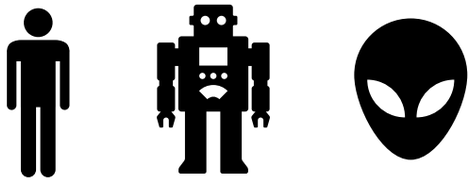

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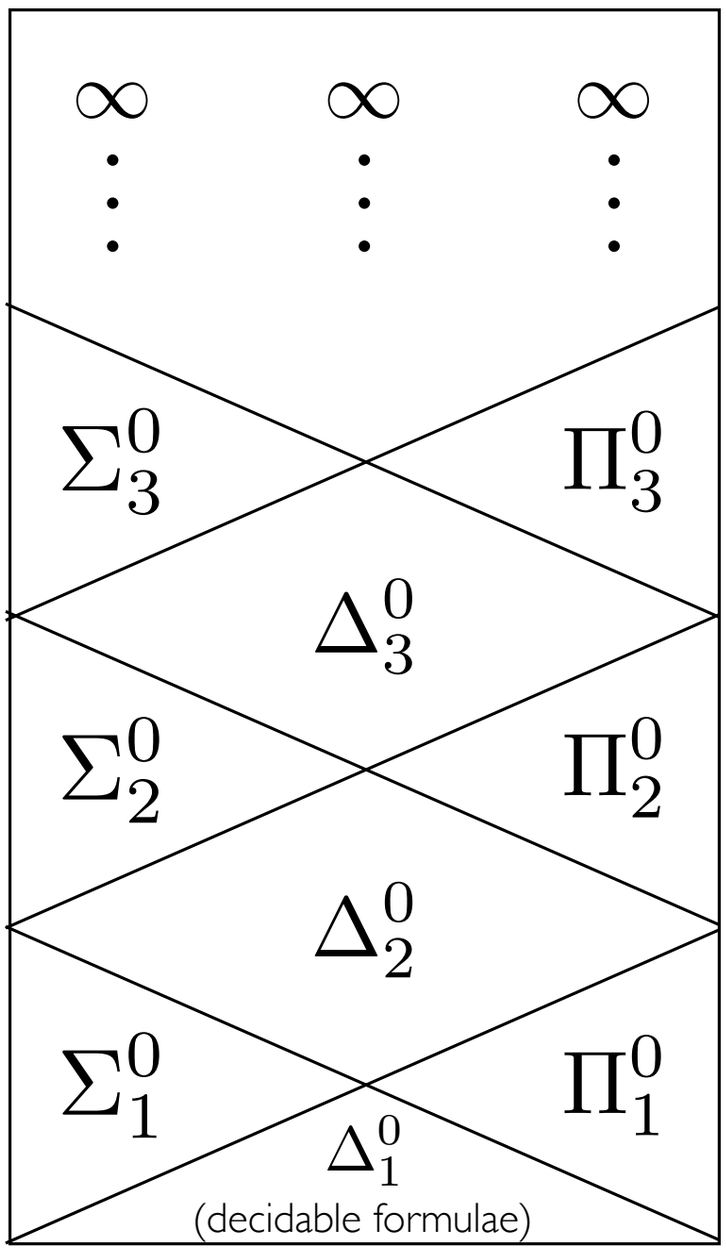

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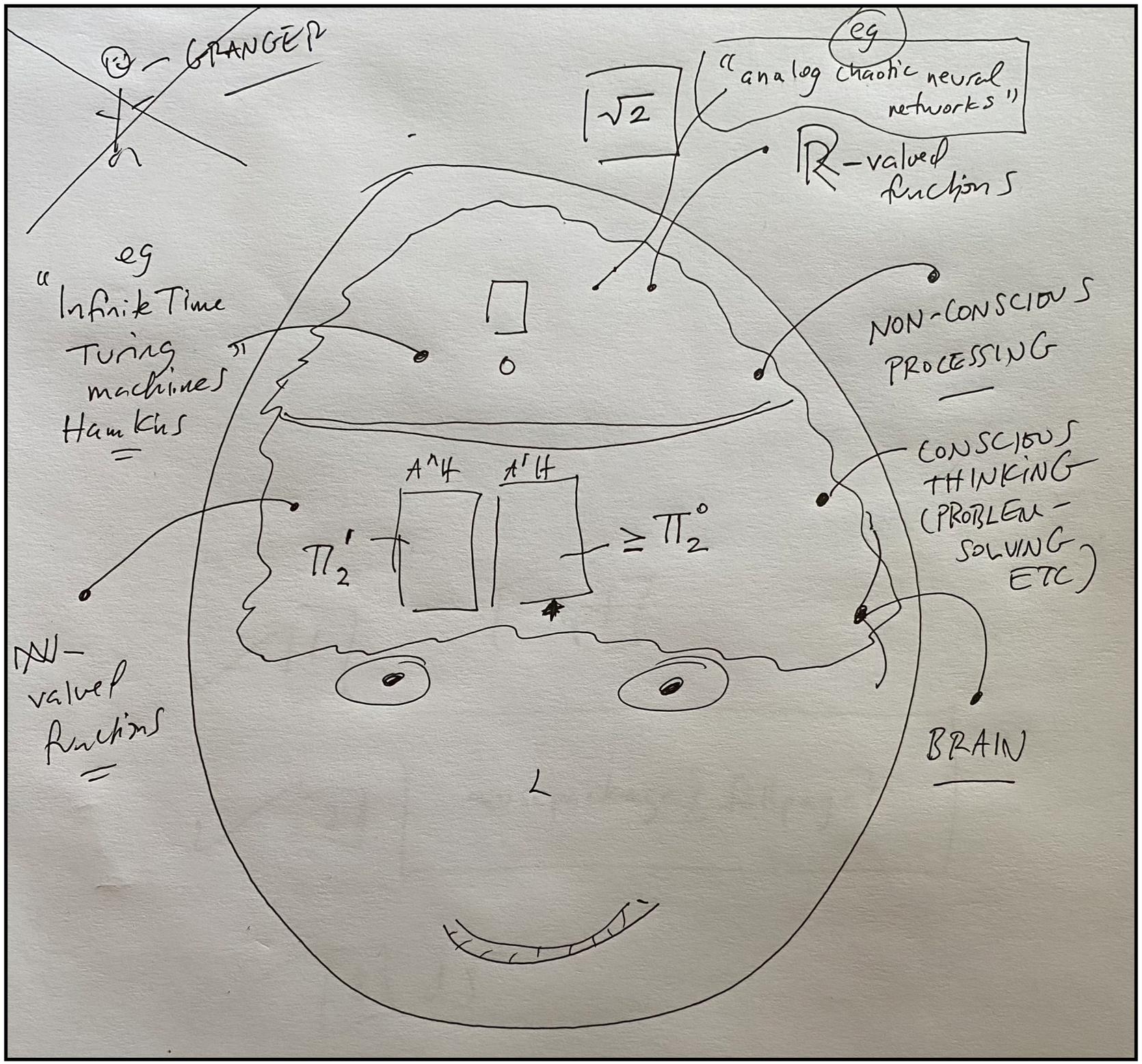
```
(forall (u v) (exists (k1 k2)
                      (iff (comp m1 u v k1)
                           (comp m2 u v k2))))
```

Arithmetic Hierarchy, Part I


eg
"analog chaotic neural networks"
$\sqrt{2}$
$\mathbb{R}$-valued functions
eg
"Infinite Time Turing machines" Hamkins
NON-CONSCIOUS PROCESSING
CONSCIOUS THINKING (PROBLEM-SOLVING ETC)
$A \wedge H$
$A' \wedge H$
Π_2^1
$\geq \Pi_2^0$
$\mathbb{N}$-valued functions
BRAIN

What about (oft vaunted) quantum computers?

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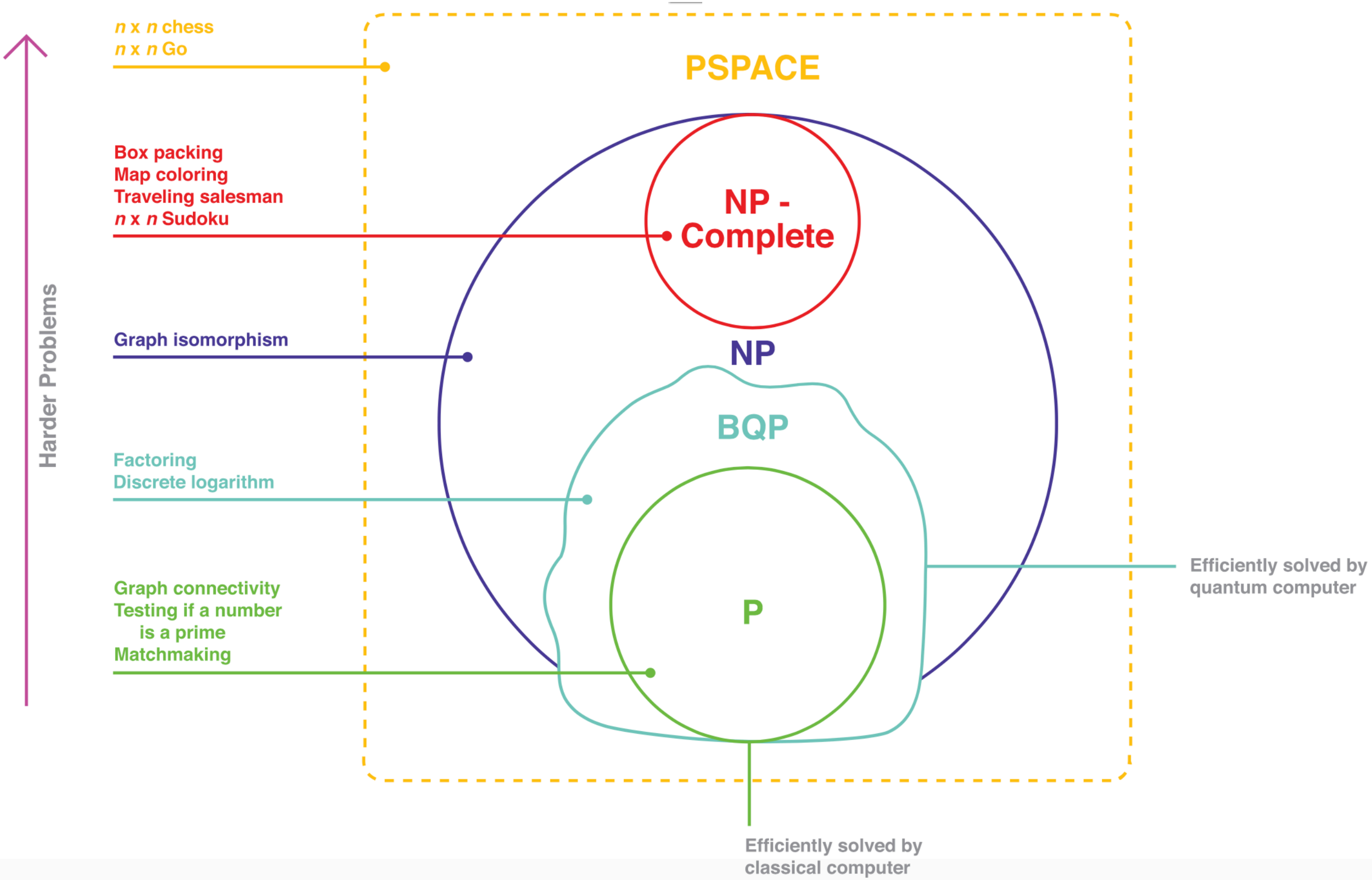

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