Gödel’s God Theorem, & Selmer’s Mental Family

Selmer Bringsjord

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RPI
Troy NY USA
version 120423Y

RAIR
Rensselaer AI and Reasoning Lab

Misc Pts re Grades/Grading

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- For those who need it, I will again today happily open any Required problem(s) on an individual "secret" basis for anyone until last class. Just come to the front in Part 2 of our class mtg today so I can input your email address to obtain such access for you, along with (a) problem(s) you want access to.

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- Make sure you submit one of the options for Question 2 in the Metalogical category for an A …

Fun Times @ Penn

"Proving that God exists is no harder than proving that 2+2=4 from **PA**."

Context …

Gödel's Great Theorems (OUP)

by Selmer Bringsjord

- Introduction ("The Wager")
- Brief Preliminaries (e.g. the propositional calculus & FOL)
- The Completeness Theorem
- The First Incompleteness Theorem
- The Second Incompleteness Theorem
- The Speedup Theorem
- The Continuum-Hypothesis Theorem
- The Time-Travel Theorem
- Gödel's "God Theorem"
- Could a Finite Machine Match Gödel's Greatness?

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Theorems …

Benzmüller-Scott-Gödel

A1 Either a property or its negation is positive, but not both:
$$\forall\phi[P(\neg\phi) \equiv \neg P(\phi)]$$

A2 A property necessarily implied by a positive property is positive: $\forall\phi\forall\psi[(P(\phi) \wedge \Box\forall x[\phi(x) \supset \psi(x)]) \supset P(\psi)]$

T1 Positive properties are possibly exemplified:
$$\forall\phi[P(\phi) \supset \Diamond\exists x\phi(x)]$$

D1 A *God-like* being possesses all positive properties:
$$G(x) \equiv \forall\phi[P(\phi) \supset \phi(x)]$$

A3 The property of being God-like is positive: $P(G)$

C Possibly, God exists: $\Diamond\exists x G(x)$

A4 Positive properties are necessarily positive:
$$\forall\phi[P(\phi) \supset \Box\, P(\phi)]$$

D2 An *essence* of an individual is a property possessed by it and necessarily implying any of its properties:
$$\phi\ ess.\ x \equiv \phi(x) \wedge \forall\psi(\psi(x) \supset \Box\forall y(\phi(y) \supset \psi(y)))$$

T2 Being God-like is an essence of any God-like being:
$$\forall x[G(x) \supset G\ ess.\ x]$$

D3 *Necessary existence* of an individ. is the necessary exemplification of all its essences: $NE(x) \equiv \forall\phi[\phi\ ess.\ x \supset \Box\exists y\phi(y)]$

A5 Necessary existence is a positive property: $P(NE)$

T3 Necessarily, God exists: $\Box\exists x G(x)$

Benzmüller-Scott-Gödel

X

A1	Either a property or its negation is positive, but not both:	$\forall\phi[P(\neg\phi) \equiv \neg P(\phi)]$
A2	A property necessarily implied by a positive property is positive:	$\forall\phi\forall\psi[(P(\phi) \wedge \Box\forall x[\phi(x) \supset \psi(x)]) \supset P(\psi)]$
T1	Positive properties are possibly exemplified:	$\forall\phi[P(\phi) \supset \Diamond\exists x\phi(x)]$
D1	A *God-like* being possesses all positive properties:	$G(x) \equiv \forall\phi[P(\phi) \supset \phi(x)]$
A3	The property of being God-like is positive:	$P(G)$
C	Possibly, God exists:	$\Diamond\exists x G(x)$
A4	Positive properties are necessarily positive:	$\forall\phi[P(\phi) \supset \Box P(\phi)]$
D2	An *essence* of an individual is a property possessed by it and necessarily implying any of its properties:	$\phi\ ess.\ x \equiv \phi(x) \wedge \forall\psi(\psi(x) \supset \Box\forall y(\phi(y) \supset \psi(y)))$
T2	Being God-like is an essence of any God-like being:	$\forall x[G(x) \supset G\ ess.\ x]$
D3	*Necessary existence* of an individ. is the necessary exemplification of all its essences:	$NE(x) \equiv \forall\phi[\phi\ ess.\ x \supset \Box\exists y\phi(y)]$
A5	Necessary existence is a positive property:	$P(NE)$
T3	Necessarily, God exists:	$\Box\exists x G(x)$

Benzmüller-Scott-Gödel

X

A Victorious Gödelian Variant?

Intelligently extracted from Gödel/Benzmüller's A1; F1a from Oppy.

$$\forall R(Pos(R) \rightarrow \neg Pos(\bar{R}))$$

Gödel/Benzmüller's A2; F2 from Oppy.

$$\forall R, R'[Pos(R) \wedge \Box \forall x(R(x) \rightarrow R'(x))] \rightarrow Pos(R')]$$

$$Pos(NE)$$

$$\forall R[Pos!(R) \leftrightarrow Pos(ER)]$$

Theorem 5 — welcome-weak? — from Oppy:

$$\forall R[Pos!(R) \rightarrow \Box \exists xER(x)]$$

Variant: *Positive* to *Great-making*

Intelligently extracted from Gödel/Benzmüller's A1; F1a from Oppy.

$$\forall R(GM(R) \rightarrow \neg GM(\bar{R}))$$

Gödel/Benzmüller's A2; F2 from Oppy.

$$\forall R, R'[GM(R) \wedge \Box \forall x(R(x) \rightarrow R'(x))] \rightarrow GM(R')]$$

$$GM(NE)$$

$$\forall R[GM!(R) \leftrightarrow GM(ER)]$$

Theorem 5 from Oppy:

$$\forall R[GM!(R) \rightarrow \Box \exists x GM(x)]$$

New Family of Mental Arguments; Email if Interested

New Family of Mental Arguments; Email if Interested

A New Family of Mental Arguments*

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New Family of Mental Arguments; Email if Interested

A New Family of Mental Arguments*

[4] I refer, note, to the *original* series; but I do so without loss of generality, since nothing fundamentally changes in subsequent spinoffs.

[5] For a full definition of personhood, see (Bringsjord 1997, Bringsjord, Noel & Caporale 2000) (or any other credible account; e.g., see Dennett 1978, Chisholm 1978). Here, without the surrounding discussion from that book, is the definition, amended slightly for the present paper: x is a person if and only if x has the *capacity*

1. to "will," to make choices and decisions, set plans and projects — autonomously;
2. for consciousness,[6] for experiencing pain and sorrow and happiness, and a thousand other emotions — love, passion, gratitude, and so on;
3. for *self*-consciousness, for being aware of his/her states of mind, inclinations, preferences, etc., and for grasping the concept of him/herself;
4. to communicate through a language;
   - Note: The language here should at minimum be at the level of one determined by a mildly Type-0 grammar. For now (I return below to the issue), I leave this formal constraint aside, and mention only that one of the extraordinary things about human persons is that the natural languages over which they have command are at least at this level, when viewed through the lens of formal logic. From the point of view of the present paper, the greatness of us, on the linguistic side, can be viewed as at least partially revealed in the rather famous (Chomsky 1956). However, many philosophers and logicians will know that so-called "Type 0" grammars in Chomsky's hierarchy were being specified, probed, and understood by Post (himself, of course, a human person) in the 1920's. Post didn't publish these grammars till much later, in (Post 1943).
5. to know and believe propositions of great complexity,
   - Note: I leave at this sport the concept of complexity informal. It would be easy enough to pin things down via both extensional (e.g. quantificational complexity regimented by the standard $\Delta_i, \Sigma_j, \Pi_k$ categorization) and intensional (e.g. layers of epistemic and other modal operators) complexity measures for formulae that capture propositions. I return to this below.

   and to believe things about what others believe (second-order beliefs), and to believe things about what others believe about one's beliefs (third-order beliefs), and so on;
6. to desire not only particular objects and events, but also changes in his or her character, and in the character of others;
7. to reason (for example, in the fashion exhibited in the writing and reading/studying of this very paper).

New Family of Mental Arguments; Email if Interested

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	(1)	The greatest things are persons, and the non-divine variety are here.
	(2′)	If the greatest things are persons, and the non-divine variety are here, then either God exists and is the ground of this state-of-affairs or E_2 is or E_3 is or ... or E_n is.
	(2a)	It is not the case that E_2 is, and it is not the case that E_3 is and ... and it is not the case that E_n is.
∴	(3)	God exists.