

# On to Intensional Logics

(In this edition of IFLAI2, S5 & DCEC only.)

**Selmer Bringsjord**

Rensselaer AI & Reasoning (RAIR) Lab  
Department of Cognitive Science  
Department of Computer Science  
Lally School of Management & Technology  
Rensselaer Polytechnic Institute (RPI)  
Troy, New York 12180 USA

*Intermediate Formal Logic & AI (IFLAI2)*  
9/7/2023



**In The Logic-and-AI News**

...



MARKETS

BUSINESS

INVESTING

TECH

POLITICS

CNBC TV

INVESTING CLUB



PRO

WATCH NOW



THE BOTTOM LINE

SHARE

# Why AI may end labor unions and become your new employer: Robert Reich

Sep 3 2023

**Mechanics ...**

# Mechanics ...

# Mechanics ...

Access codes available if not registered!

# Mechanics ...

Access codes available if not registered!

Might find helpful the “Getting Off The Ground ...” tutorial, available by link in “Tutorials” section of our course web page.

# Mechanics ...

Access codes available if not registered!

Might find helpful the “Getting Off The Ground ...” tutorial, available by link in “Tutorials” section of our course web page.

**Important:** Your “University ID” as a student @ RPI is you RIN #.



# Mechanics ...

Access codes available if not registered!

Might find helpful the “Getting Off The Ground ...” tutorial, available by link in “Tutorials” section of our course web page.

**Important:** Your “University ID” as a student @ RPI is you RIN #.

**Important:** 1 browser, 1 window w/ tabs, & immaculate housekeeping (versioning up-to-date eg).

# Mechanics ...

Access codes available if not registered!

Might find helpful the “Getting Off The Ground ...” tutorial, available by link in “Tutorials” section of our course web page.

**Important:** Your “University ID” as a student @ RPI is you RIN #.

**Important:** 1 browser, 1 window w/ tabs, & immaculate housekeeping (versioning up-to-date eg).

As part of review, a personalized problem should now be in your HG account & due by COC today. Try it! In fact let's look @ mine now, together and have a tutorial ...

On the esemplastic  
extensional-logic ladder ...  
questions?

# Climbing the $k$ -order Ladder

# Climbing the $k$ -order Ladder

$a$  is a llama, as is  $b$ ,  $a$  likes  $b$ , and the father of  $a$  is a llama as well.

# Climbing the $k$ -order Ladder

$Llama(a) \wedge Llama(b) \wedge Likes(a, b) \wedge Llama(fatherOf(a))$

$a$  is a llama, as is  $b$ ,  $a$  likes  $b$ , and the father of  $a$  is a llama as well.

# Climbing the $k$ -order Ladder

**ZOL**  $Llama(a) \wedge Llama(b) \wedge Likes(a, b) \wedge Llama(fatherOf(a))$

$a$  is a llama, as is  $b$ ,  $a$  likes  $b$ , and the father of  $a$  is a llama as well.

# Climbing the $k$ -order Ladder

There's some thing which is a llama and likes  $b$  (which is also a llama), and whose father is a llama too.

**ZOL**  $Llama(a) \wedge Llama(b) \wedge Likes(a, b) \wedge Llama(fatherOf(a))$

$a$  is a llama, as is  $b$ ,  $a$  likes  $b$ , and the father of  $a$  is a llama as well.



# Climbing the $k$ -order Ladder

$\exists x[Llama(x) \wedge Llama(b) \wedge Likes(x, b) \wedge Llama(fatherOf(x))]$

There's some thing which is a llama and likes  $b$  (which is also a llama), and whose father is a llama too.

**ZOL**  $Llama(a) \wedge Llama(b) \wedge Likes(a, b) \wedge Llama(fatherOf(a))$

$a$  is a llama, as is  $b$ ,  $a$  likes  $b$ , and the father of  $a$  is a llama as well.

# Climbing the $k$ -order Ladder

**FOL**  $\exists x[Llama(x) \wedge Llama(b) \wedge Likes(x, b) \wedge Llama(fatherOf(x))]$

There's some thing which is a llama and likes  $b$  (which is also a llama), and whose father is a llama too.

**ZOL**  $Llama(a) \wedge Llama(b) \wedge Likes(a, b) \wedge Llama(fatherOf(a))$

$a$  is a llama, as is  $b$ ,  $a$  likes  $b$ , and the father of  $a$  is a llama as well.

# Climbing the $k$ -order Ladder

Things  $x$  and  $y$ , along with the father of  $x$ , share a certain property (and  $x$  likes  $y$ ).

**FOL**  $\exists x[Llama(x) \wedge Llama(b) \wedge Likes(x, b) \wedge Llama(fatherOf(x))]$

There's some thing which is a llama and likes  $b$  (which is also a llama), and whose father is a llama too.

**ZOL**  $Llama(a) \wedge Llama(b) \wedge Likes(a, b) \wedge Llama(fatherOf(a))$

$a$  is a llama, as is  $b$ ,  $a$  likes  $b$ , and the father of  $a$  is a llama as well.

# Climbing the $k$ -order Ladder

$$\exists x \exists y \exists R [R(x) \wedge R(y) \wedge Likes(x, y) \wedge R(fatherOf(x))]$$

Things  $x$  and  $y$ , along with the father of  $x$ , share a certain property (and  $x$  likes  $y$ ).

**FOL**  $\exists x [Llama(x) \wedge Llama(b) \wedge Likes(x, b) \wedge Llama(fatherOf(x))]$

There's some thing which is a llama and likes  $b$  (which is also a llama), and whose father is a llama too.

**ZOL**  $Llama(a) \wedge Llama(b) \wedge Likes(a, b) \wedge Llama(fatherOf(a))$

$a$  is a llama, as is  $b$ ,  $a$  likes  $b$ , and the father of  $a$  is a llama as well.

# Climbing the $k$ -order Ladder

**SOL**  $\exists x \exists y \exists R [R(x) \wedge R(y) \wedge Likes(x, y) \wedge R(fatherOf(x))]$

Things  $x$  and  $y$ , along with the father of  $x$ , share a certain property (and  $x$  likes  $y$ ).

**FOL**  $\exists x [Llama(x) \wedge Llama(b) \wedge Likes(x, b) \wedge Llama(fatherOf(x))]$

There's some thing which is a llama and likes  $b$  (which is also a llama), and whose father is a llama too.

**ZOL**  $Llama(a) \wedge Llama(b) \wedge Likes(a, b) \wedge Llama(fatherOf(a))$

$a$  is a llama, as is  $b$ ,  $a$  likes  $b$ , and the father of  $a$  is a llama as well.

# Climbing the $k$ -order Ladder

Things  $x$  and  $y$ , along with the father of  $x$ , share a certain property; and,  $x R^2s y$ , where  $R^2$  is a positive property.

**SOL**  $\exists x \exists y \exists R [R(x) \wedge R(y) \wedge Likes(x, y) \wedge R(fatherOf(x))]$

Things  $x$  and  $y$ , along with the father of  $x$ , share a certain property (and  $x$  likes  $y$ ).

**FOL**  $\exists x [Llama(x) \wedge Llama(b) \wedge Likes(x, b) \wedge Llama(fatherOf(x))]$

There's some thing which is a llama and likes  $b$  (which is also a llama), and whose father is a llama too.

**ZOL**  $Llama(a) \wedge Llama(b) \wedge Likes(a, b) \wedge Llama(fatherOf(a))$

$a$  is a llama, as is  $b$ ,  $a$  likes  $b$ , and the father of  $a$  is a llama as well.

# Climbing the $k$ -order Ladder

$\exists x, y \exists R, R^2 [R(x) \wedge R(y) \wedge R^2(x, y) \wedge \text{Positive}(R^2) \wedge R(\text{fatherOf}(x))]$

Things  $x$  and  $y$ , along with the father of  $x$ , share a certain property; and,  $x$   $R^2$ s  $y$ , where  $R^2$  is a positive property.

**SOL**  $\exists x \exists y \exists R [R(x) \wedge R(y) \wedge \text{Likes}(x, y) \wedge R(\text{fatherOf}(x))]$

Things  $x$  and  $y$ , along with the father of  $x$ , share a certain property (and  $x$  likes  $y$ ).

**FOL**  $\exists x [Llama(x) \wedge Llama(b) \wedge \text{Likes}(x, b) \wedge Llama(\text{fatherOf}(x))]$

There's some thing which is a llama and likes  $b$  (which is also a llama), and whose father is a llama too.

**ZOL**  $Llama(a) \wedge Llama(b) \wedge \text{Likes}(a, b) \wedge Llama(\text{fatherOf}(a))$

$a$  is a llama, as is  $b$ ,  $a$  likes  $b$ , and the father of  $a$  is a llama as well.

# Climbing the $k$ -order Ladder

**TOL**  $\exists x, y \exists R, R^2 [R(x) \wedge R(y) \wedge R^2(x, y) \wedge \text{Positive}(R^2) \wedge R(\text{fatherOf}(x))]$

Things  $x$  and  $y$ , along with the father of  $x$ , share a certain property; and,  $x$   $R^2$ s  $y$ , where  $R^2$  is a positive property.

**SOL**  $\exists x \exists y \exists R [R(x) \wedge R(y) \wedge \text{Likes}(x, y) \wedge R(\text{fatherOf}(x))]$

Things  $x$  and  $y$ , along with the father of  $x$ , share a certain property (and  $x$  likes  $y$ ).

**FOL**  $\exists x [Llama(x) \wedge Llama(b) \wedge \text{Likes}(x, b) \wedge Llama(\text{fatherOf}(x))]$

There's some thing which is a llama and likes  $b$  (which is also a llama), and whose father is a llama too.

**ZOL**  $Llama(a) \wedge Llama(b) \wedge \text{Likes}(a, b) \wedge Llama(\text{fatherOf}(a))$

$a$  is a llama, as is  $b$ ,  $a$  likes  $b$ , and the father of  $a$  is a llama as well.



# Climbing the $k$ -order Ladder

**TOL**  $\exists x, y \exists R, R^2 [R(x) \wedge R(y) \wedge R^2(x, y) \wedge \textit{Positive}(R^2) \wedge R(\textit{fatherOf}(x))]$

$\mathcal{L}_3$  Things  $x$  and  $y$ , along with the father of  $x$ , share a certain property; and,  $x$   $R^2$ s  $y$ , where  $R^2$  is a positive property.

**SOL**  $\exists x \exists y \exists R [R(x) \wedge R(y) \wedge \textit{Likes}(x, y) \wedge R(\textit{fatherOf}(x))]$

$\mathcal{L}_2$  Things  $x$  and  $y$ , along with the father of  $x$ , share a certain property (and  $x$  likes  $y$ ).

**FOL**  $\exists x [Llama(x) \wedge Llama(b) \wedge \textit{Likes}(x, b) \wedge Llama(\textit{fatherOf}(x))]$

$\mathcal{L}_1$  There's some thing which is a llama and likes  $b$  (which is also a llama), and whose father is a llama too.

**ZOL**  $Llama(a) \wedge Llama(b) \wedge \textit{Likes}(a, b) \wedge Llama(\textit{fatherOf}(a))$

$\mathcal{L}_0$   $a$  is a llama, as is  $b$ ,  $a$  likes  $b$ , and the father of  $a$  is a llama as well.

# Climbing the $k$ -order Ladder

⋮

**TOL**  $\exists x, y \exists R, R^2 [R(x) \wedge R(y) \wedge R^2(x, y) \wedge \textit{Positive}(R^2) \wedge R(\textit{fatherOf}(x))]$

$\mathcal{L}_3$  Things  $x$  and  $y$ , along with the father of  $x$ , share a certain property; and,  $x$   $R^2$ s  $y$ , where  $R^2$  is a positive property.

**SOL**  $\exists x \exists y \exists R [R(x) \wedge R(y) \wedge \textit{Likes}(x, y) \wedge R(\textit{fatherOf}(x))]$

$\mathcal{L}_2$  Things  $x$  and  $y$ , along with the father of  $x$ , share a certain property (and  $x$  likes  $y$ ).

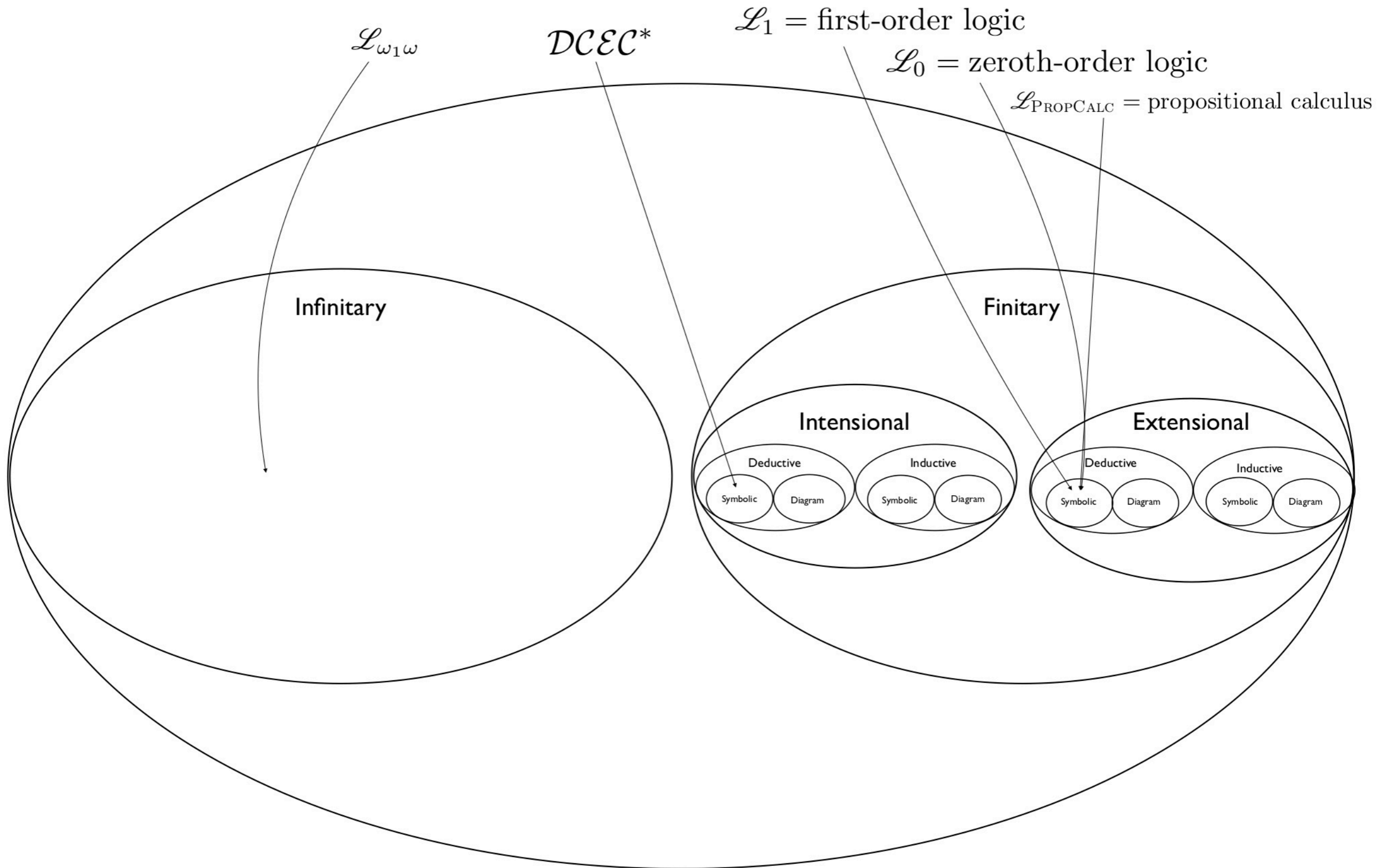
**FOL**  $\exists x [Llama(x) \wedge Llama(b) \wedge \textit{Likes}(x, b) \wedge Llama(\textit{fatherOf}(x))]$

$\mathcal{L}_1$  There's some thing which is a llama and likes  $b$  (which is also a llama), and whose father is a llama too.

**ZOL**  $Llama(a) \wedge Llama(b) \wedge \textit{Likes}(a, b) \wedge Llama(\textit{fatherOf}(a))$

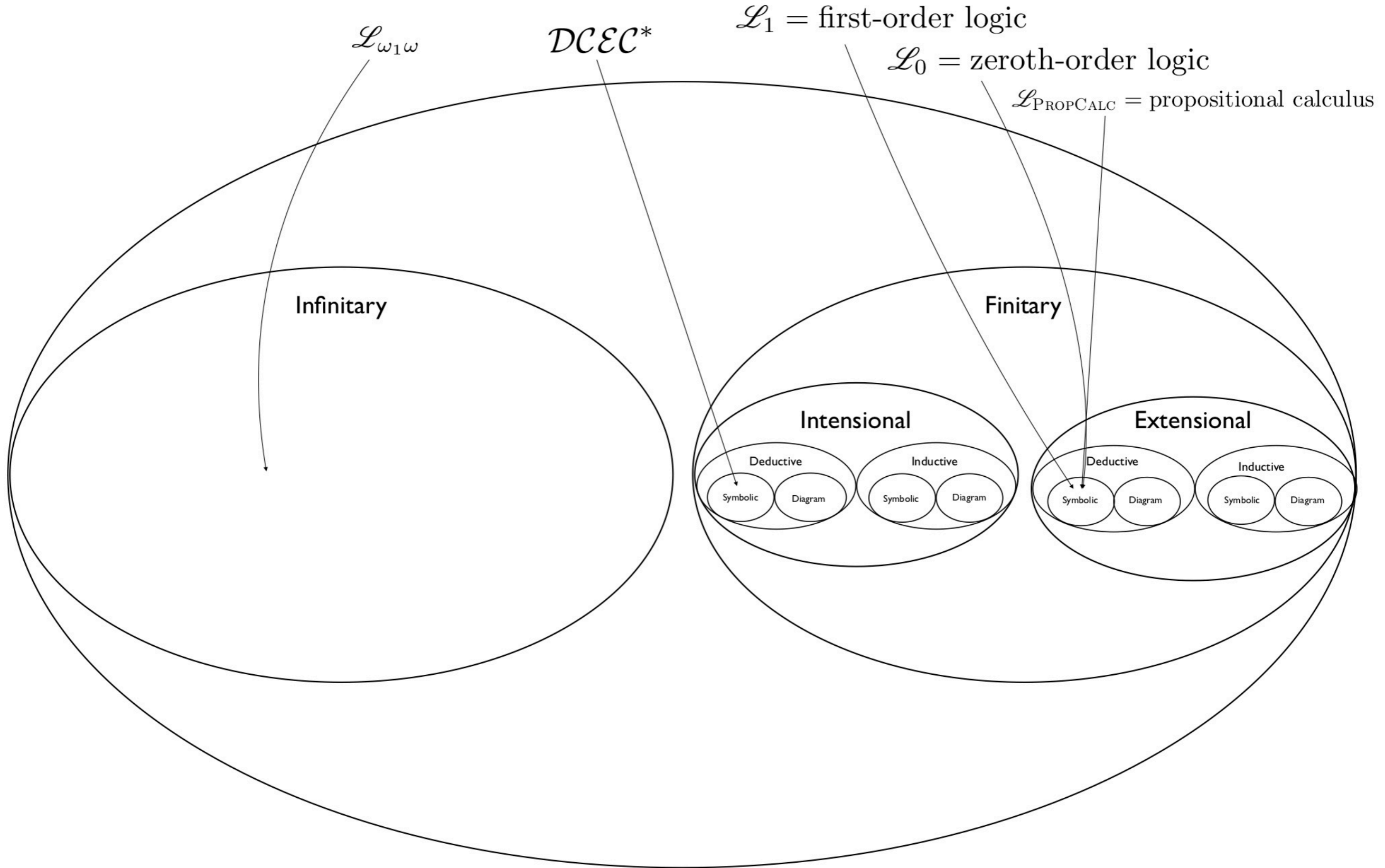
$\mathcal{L}_0$   $a$  is a llama, as is  $b$ ,  $a$  likes  $b$ , and the father of  $a$  is a llama as well.

# The Universe of Logics



$\mathcal{L}_3$   
 $\mathcal{L}_2$

# The Universe of Logics



# The Universe of Logics

$\mathcal{L}_3$

$\mathcal{L}_2$

$\mathcal{L}_{\omega_1\omega}$

$\mathcal{DCEC}^*$

$\mathcal{L}_1 =$  first-order logic

$\mathcal{L}_0 =$  zeroth-order logic

$\mathcal{L}_{\text{PROPCALC}} =$  propositional calculus

Infinitary

Finitary

Intensional

Extensional

Deductive

Inductive

Deductive

Inductive

Symbolic

Diagram

Symbolic

Diagram

Symbolic

Diagram

Symbolic

Diagram

# Climbing the $k$ -order Ladder

⋮

**TOL**  $\exists x, y \exists R, R^2 [R(x) \wedge R(y) \wedge R^2(x, y) \wedge \textit{Positive}(R^2) \wedge R(\textit{fatherOf}(x))]$

$\mathcal{L}_3$  Things  $x$  and  $y$ , along with the father of  $x$ , share a certain property; and,  $x$   $R^2$ s  $y$ , where  $R^2$  is a positive property.

**SOL**  $\exists x \exists y \exists R [R(x) \wedge R(y) \wedge \textit{Likes}(x, y) \wedge R(\textit{fatherOf}(x))]$

$\mathcal{L}_2$  Things  $x$  and  $y$ , along with the father of  $x$ , share a certain property (and  $x$  likes  $y$ ).

**FOL**  $\exists x [Llama(x) \wedge Llama(b) \wedge \textit{Likes}(x, b) \wedge Llama(\textit{fatherOf}(x))]$

$\mathcal{L}_1$  There's some thing which is a llama and likes  $b$  (which is also a llama), and whose father is a llama too.

**ZOL**  $Llama(a) \wedge Llama(b) \wedge \textit{Likes}(a, b) \wedge Llama(\textit{fatherOf}(a))$

$\mathcal{L}_0$   $a$  is a llama, as is  $b$ ,  $a$  likes  $b$ , and the father of  $a$  is a llama as well.

# Climbing the $k$ -order Ladder

⋮

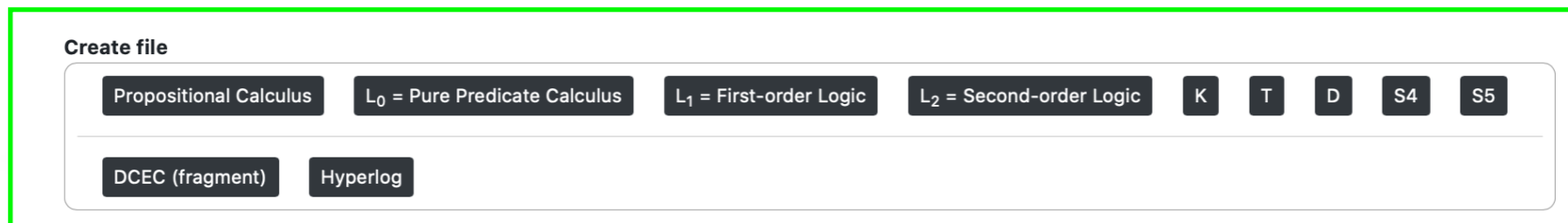
**TOL**  $\exists x, y \exists R, R^2 [R(x) \wedge R(y) \wedge R^2(x, y) \wedge \text{Positive}(R^2) \wedge R(\text{fatherOf}(x))]$

$\mathcal{L}_3$

Things  $x$  and  $y$ , along with the father of  $x$ , share a certain property; and,  $x$   $R^2$ s  $y$ , where  $R^2$  is a positive property.

**SOL**  $\exists x, y \exists R [R(x) \wedge R(y) \wedge R(x, y) \wedge \text{Positive}(R) \wedge R(\text{fatherOf}(x))]$

$\mathcal{L}_2$



Things  $x$  and  $y$ , along with the father of  $x$ , share a certain property (and  $x$  likes  $y$ ).

**FOL**  $\exists x [Llama(x) \wedge Llama(b) \wedge Likes(x, b) \wedge Llama(\text{fatherOf}(x))]$

$\mathcal{L}_1$

There's some thing which is a llama and likes  $b$  (which is also a llama), and whose father is a llama too.

**ZOL**  $Llama(a) \wedge Llama(b) \wedge Likes(a, b) \wedge Llama(\text{fatherOf}(a))$

$\mathcal{L}_0$

$a$  is a llama, as is  $b$ ,  $a$  likes  $b$ , and the father of  $a$  is a llama as well.

# Climbing the $k$ -order Ladder

⋮

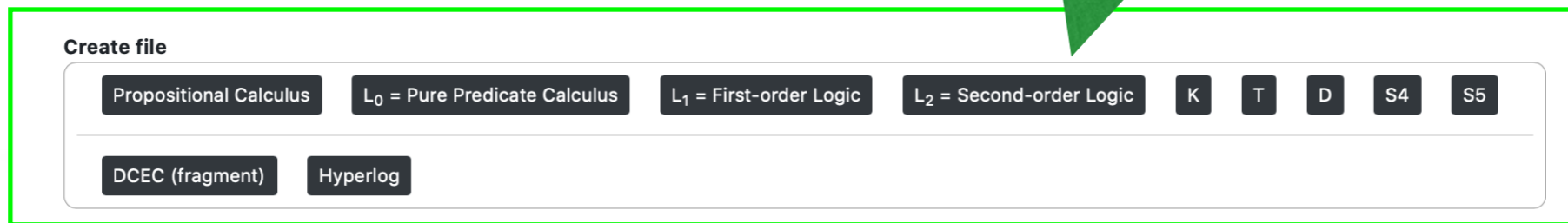
**TOL**

$\exists x, y \exists R, R^2 [R(x) \wedge R(y) \wedge R^2(x, y) \wedge \text{Positive}(R^2) \wedge R(\text{fatherOf}(x))]$

$\mathcal{L}_3$

Things  $x$  and  $y$ , along with the father of  $x$ , share a certain property; and,  $x$   $R^2$ s  $y$ , where  $R^2$  is a positive property.

**SOL**



$\mathcal{L}_2$

Things  $x$  and  $y$ , along with the father of  $x$ , share a certain property (and  $x$  likes  $y$ ).

**FOL**

$\exists x [Llama(x) \wedge Llama(b) \wedge Likes(x, b) \wedge Llama(\text{fatherOf}(x))]$

$\mathcal{L}_1$

There's some thing which is a llama and likes  $b$  (which is also a llama), and whose father is a llama too.

**ZOL**

$Llama(a) \wedge Llama(b) \wedge Likes(a, b) \wedge Llama(\text{fatherOf}(a))$

$\mathcal{L}_0$

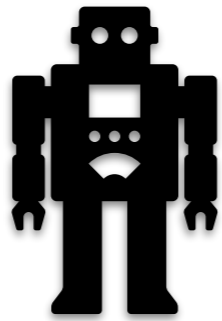
$a$  is a llama, as is  $b$ ,  $a$  likes  $b$ , and the father of  $a$  is a llama as well.



**Blinky as portal to  
intensional logics ...**

(Believes blinky (Loc ball 1))

Blinky



1



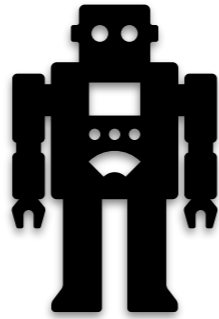
2



3

(Believes blinky (Loc ball 1))

Blinky



1



2

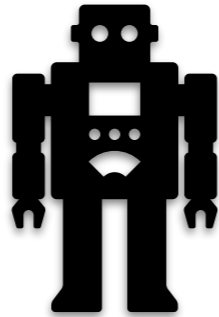


3



(Believes blinky (Loc ball 1))

Blinky



1



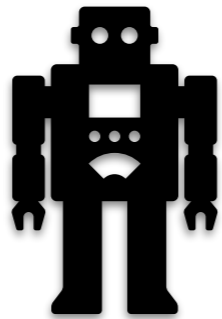
2



3

(Believes blinky (Loc ball 1))

Blinky



1



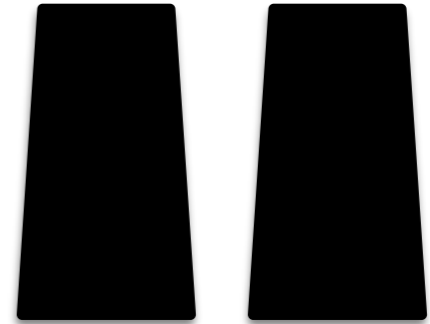
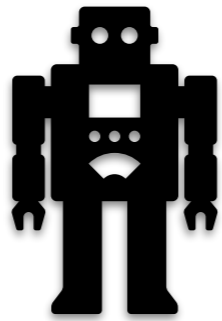
2



3

(Believes blinky (Loc ball 1))

Blinky

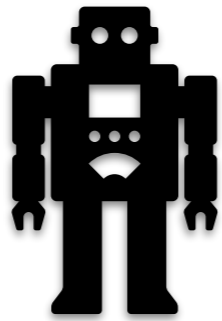


2

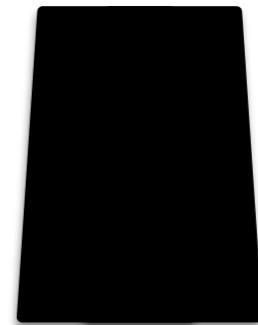
3

(Believes blinky (Loc ball 1))

Blinky



1

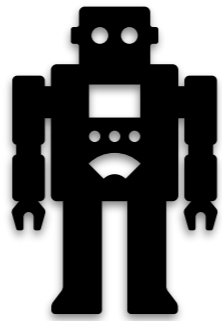


2

3

(Believes blinky (Loc ball 1))

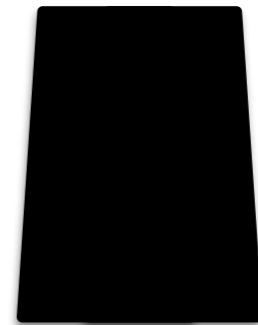
Blinky



1

2

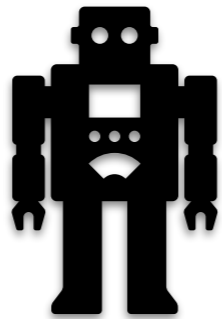
3





(Believes blinky (Loc ball 1))

Blinky



1



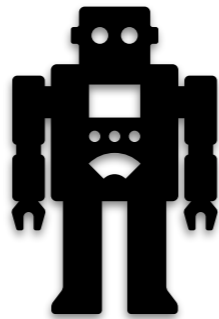
2



3

(Believes blinky (Loc ball 1))

Blinky



1



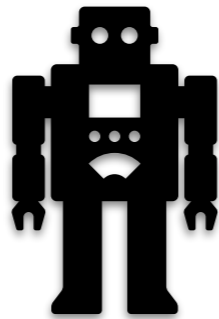
2



3

(Believes blinky (Loc ball 1))

Blinky



1



2

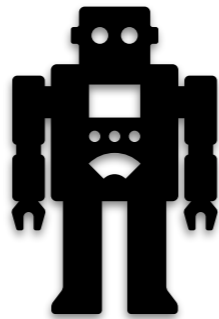


3

The ball is in the cup at location #1.

(Believes blinky (Loc ball 1))

Blinky



1



2



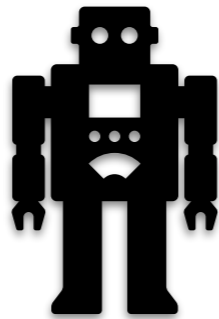
3

The ball is in the cup at location #1.

Loc(ball,1)

(Believes blinky (Loc ball 1))

Blinky



1



2



3

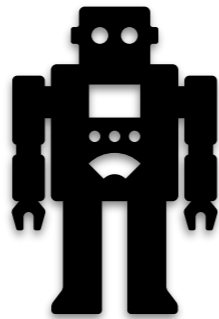
The ball is in the cup at location #1.

Loc(ball,1)

(Loc ball 1)

(Believes blinky (Loc ball 1))

Blinky



1



2



3

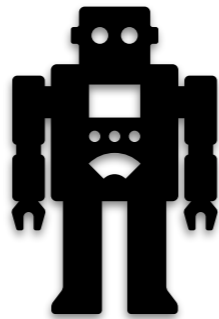
The ball is in the cup at location #1.

**FALSE** Loc(ball,1)

(Loc ball 1)

(Believes blinky (Loc ball 1))

Blinky



1



2



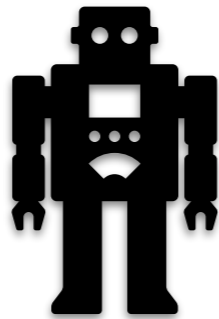
3

**FALSE** Loc(ball,1)

(Loc ball 1)

(Believes blinky (Loc ball 1))

Blinky



1



2



3

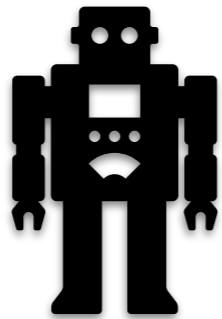


FALSE

(Loc ball 1)

(Believes blinky (Loc ball 1))

Blinky



1



2

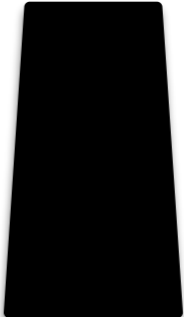
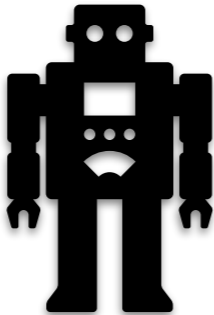


3

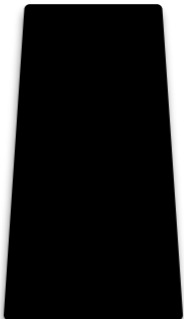
(Loc ball 1)

(Believes blinky (Loc ball 1))

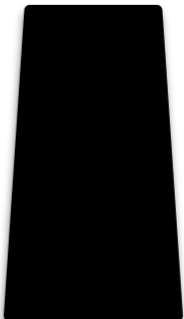
Blinky



1



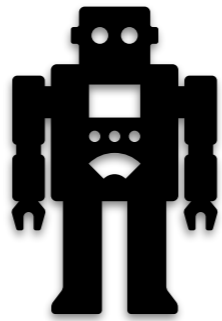
2



3

(Believes blinky (Loc ball 1))

Blinky



1



2

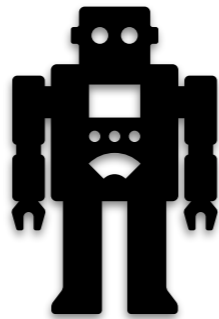


3

The ball is in the cup at location #1 or the ball is at location #3.

(Believes blinky (Loc ball 1))

Blinky



1



2



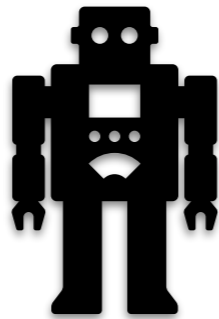
3

The ball is in the cup at location #1 or the ball is at location #3.

$\text{Loc}(\text{ball},1) \vee \text{Loc}(\text{ball},3)$

(Believes blinky (Loc ball 1))

Blinky



1



2



3

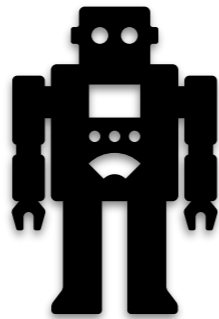
The ball is in the cup at location #1 or the ball is at location #3.

$\text{Loc}(\text{ball},1) \vee \text{Loc}(\text{ball},3)$

(or (Loc ball 1) (Loc ball 3))

(Believes blinky (Loc ball 1))

Blinky



1



2



3

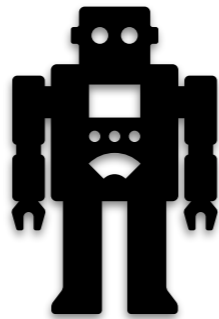
The ball is in the cup at location #1 or the ball is at location #3.

**FALSE**  $\text{Loc}(\text{ball},1) \vee \text{Loc}(\text{ball},3)$

(or (Loc ball 1) (Loc ball 3))

(Believes blinky (Loc ball 1))

Blinky



1



2



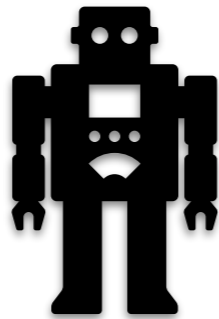
3

**FALSE**  $\text{Loc}(\text{ball},1) \vee \text{Loc}(\text{ball},3)$

(or (Loc ball 1) (Loc ball 3))

(Believes blinky (Loc ball 1))

Blinky



1



2



3

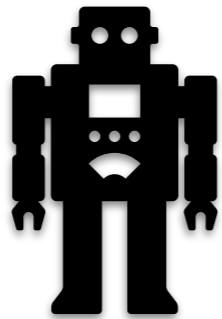


FALSE

(or (Loc ball 1) (Loc ball 3))

(Believes blinky (Loc ball 1))

Blinky



1



2

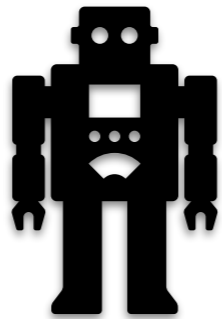


3

FALSE

(Believes blinky (Loc ball 1))

Blinky



1



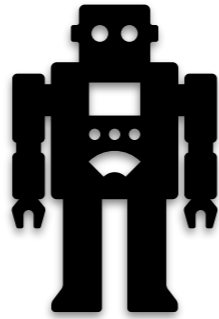
2



3

(Believes blinky (Loc ball 1))

Blinky



1



2

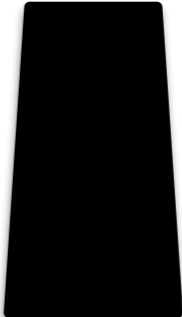
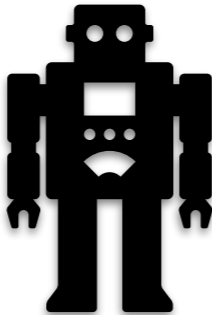


3

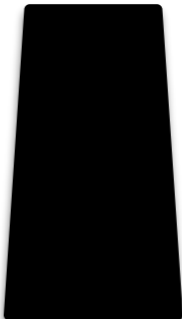
Blinky believes that the ball is in the cup at location #1.

(Believes blinky (Loc ball 1))

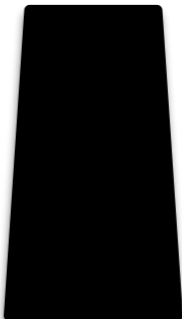
Blinky



1



2



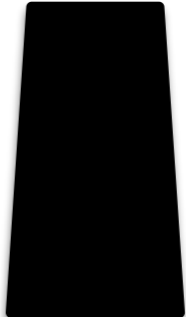
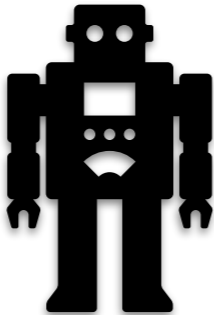
3

Blinky believes that the ball is in the cup at location #1.

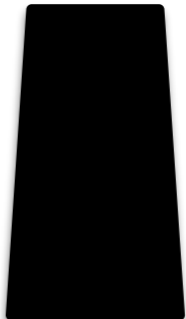
$B(\text{blinky}, \text{Loc}(\text{ball}, 1))$

(Believes blinky (Loc ball 1))

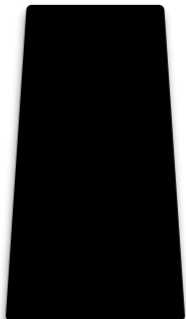
Blinky



1



2



3

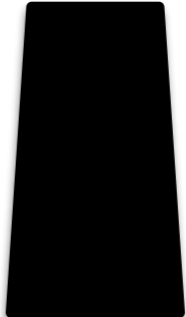
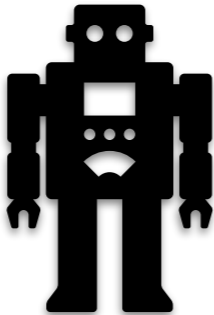
Blinky believes that the ball is in the cup at location #1.

**B(blinky, Loc(ball,1))**

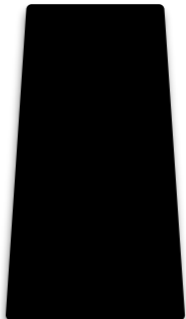
(Believes! blinky (Loc ball 1))

(Believes blinky (Loc ball 1))

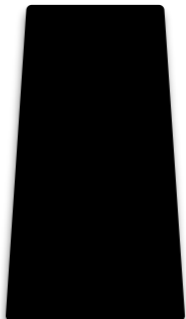
Blinky



1



2



3

Blinky believes that the ball is in the cup at location #1.

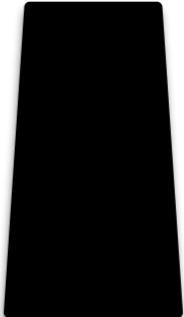
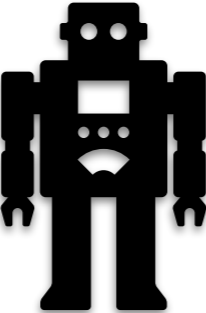
???

B(blinky, Loc(ball,1))

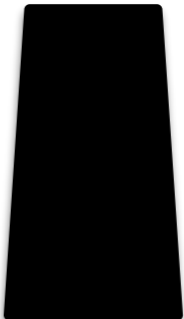
(Believes! blinky (Loc ball 1))

(Believes blinky (Loc ball 1))

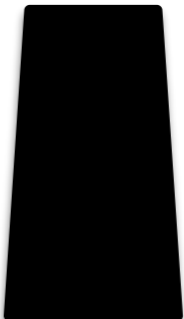
Blinky



1



2



3

Blinky believes that the ball is in the cup at location #1.

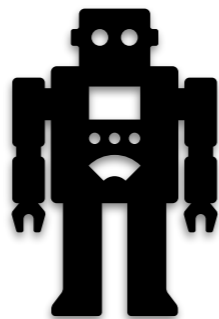
???

B(blinky, Loc(ball,1))

(Believes! blinky (Loc ball 1))

(Believes blinky (Loc ball 1))

Blinky



1



2

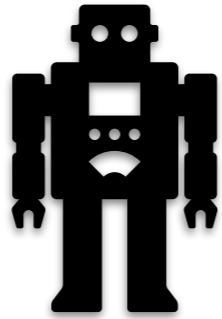


3

In extensional logics, what is denoted is conflated with meaning (the latter being naively compositional), but intensional attitudes like *believes*, *knows*, *hopes*, *fears*, etc cannot be represented and reasoned over smoothly (e.g. without fear of inconsistency rising up).



Blinky



1

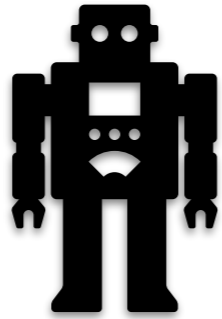


2



3

Blinky



1



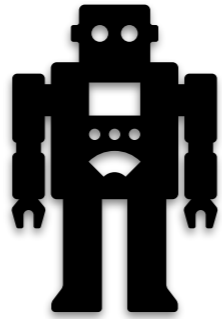
2



3



Blinky



1

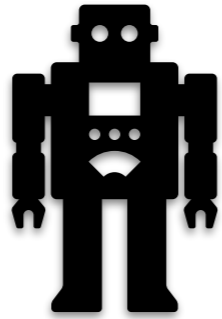


2



3

Blinky



1

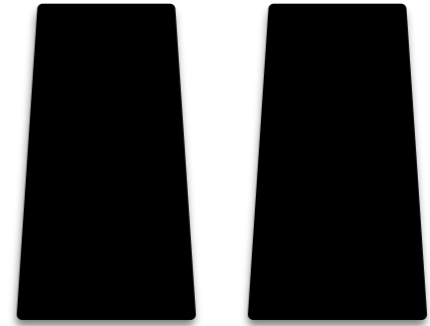
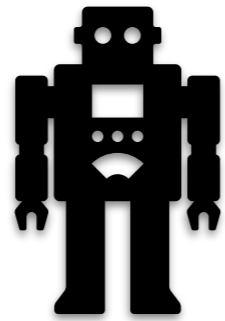


2



3

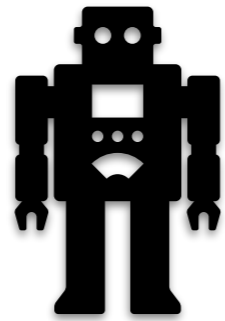
Blinky



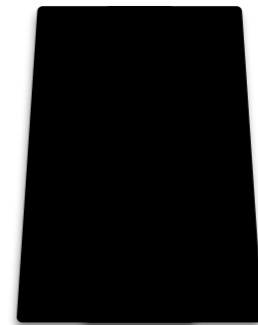
2

3

Blinky



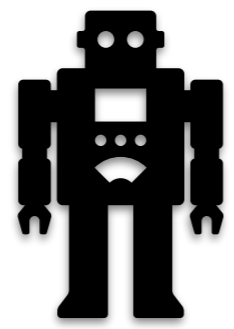
1



2

3

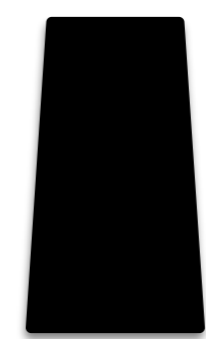
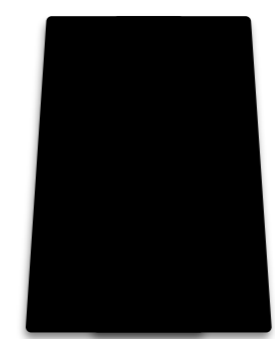
Blinky



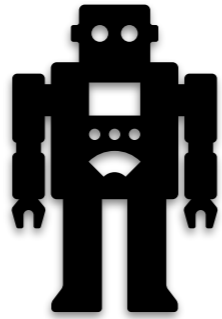
1

2

3



Blinky



1



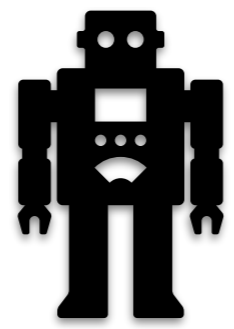
2



3



Blinky



1

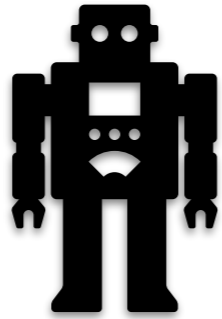


2



3

Blinky



1



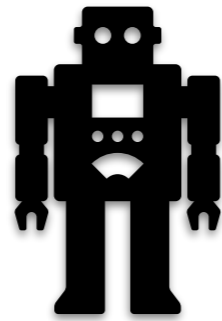
2



3

Blinky believes that the ball is in the cup at location #1.

Blinky



1



2

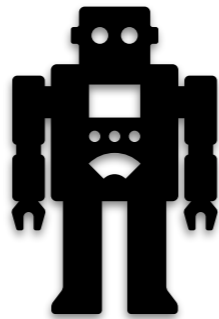


3

Blinky believes that the ball is in the cup at location #1.

$B(\text{blinky}, \text{loc-ball-1})$

Blinky



1



2



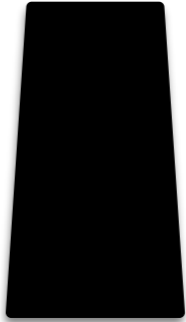
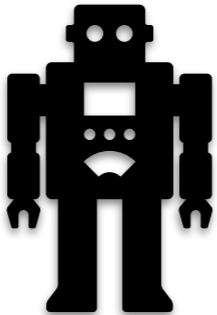
3

Blinky believes that the ball is in the cup at location #1.

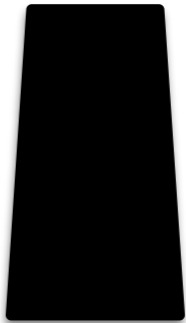
**B(blinky, loc-ball-1)**

(Believes! blinky loc-ball-1)

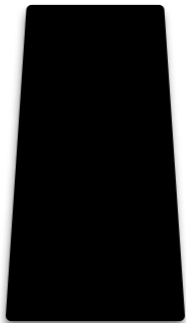
Blinky



1



2



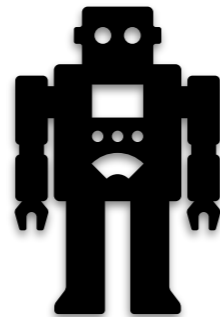
3

Blinky believes that the ball is in the cup at location #1.

$B(\text{blinky}, \text{loc-ball-1})$

(Believes! blinky loc-ball-1)

Blinky



1



2



3

In intensional logics, meaning and designation are separated, and compositionality is abandoned.

Blinky believes that the ball is in the cup at location #1.

$B(\text{blinky}, \text{loc-ball-1})$

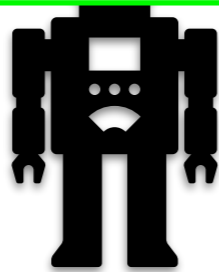
(Believes! blinky loc-ball-1)

Create file

Propositional Calculus   L<sub>0</sub> = Pure Predicate Calculus   L<sub>1</sub> = First-order Logic   L<sub>2</sub> = Second-order Logic   K   T   D   S4   S5

DCEC (fragment)   Hyperlog

Blinky



1



2



3

In intensional logics, meaning and designation are separated, and compositionality is abandoned.

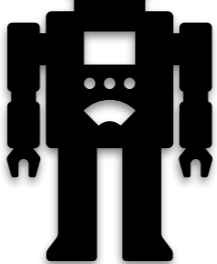
Blinky believes that the ball is in the cup at location #1.

$B(\text{blinky}, \text{loc-ball-1})$

(Believes! blinky loc-ball-1)



Blinky



1

2

3

In intensional logics, meaning and designation are separated, and compositionality is abandoned.



**For Brave Adventurers**

# For Brave Adventurers

“Everything smart knows that everything tinks anything that tinks something identical with something.”

# For Brave Adventurers

“Everything smart knows that everything tinks anything that tinks something identical with something.”

“Blinky is smart.”

# For Brave Adventurers

“Everything smart knows that everything tinks anything that tinks something identical with something.”

“Blinky is smart.”

**Therefore:**

# For Brave Adventurers

“Everything smart knows that everything tinks anything that tinks something identical with something.”

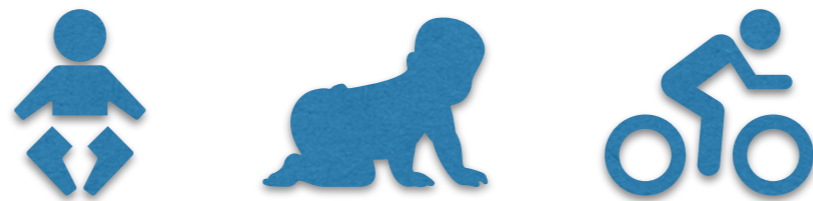
“Blinky is smart.”

**Therefore:**

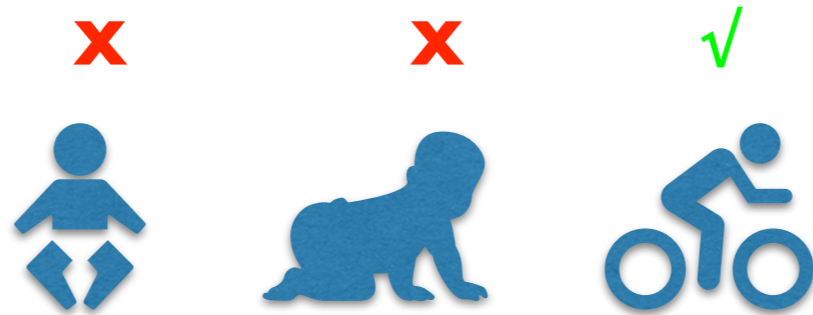
“Everything tinks anything that tinks something identical with something.”

**False Belief Task Demands  
Intensional Logic ...**

# False Belief Task Demands Intensional Logic ...

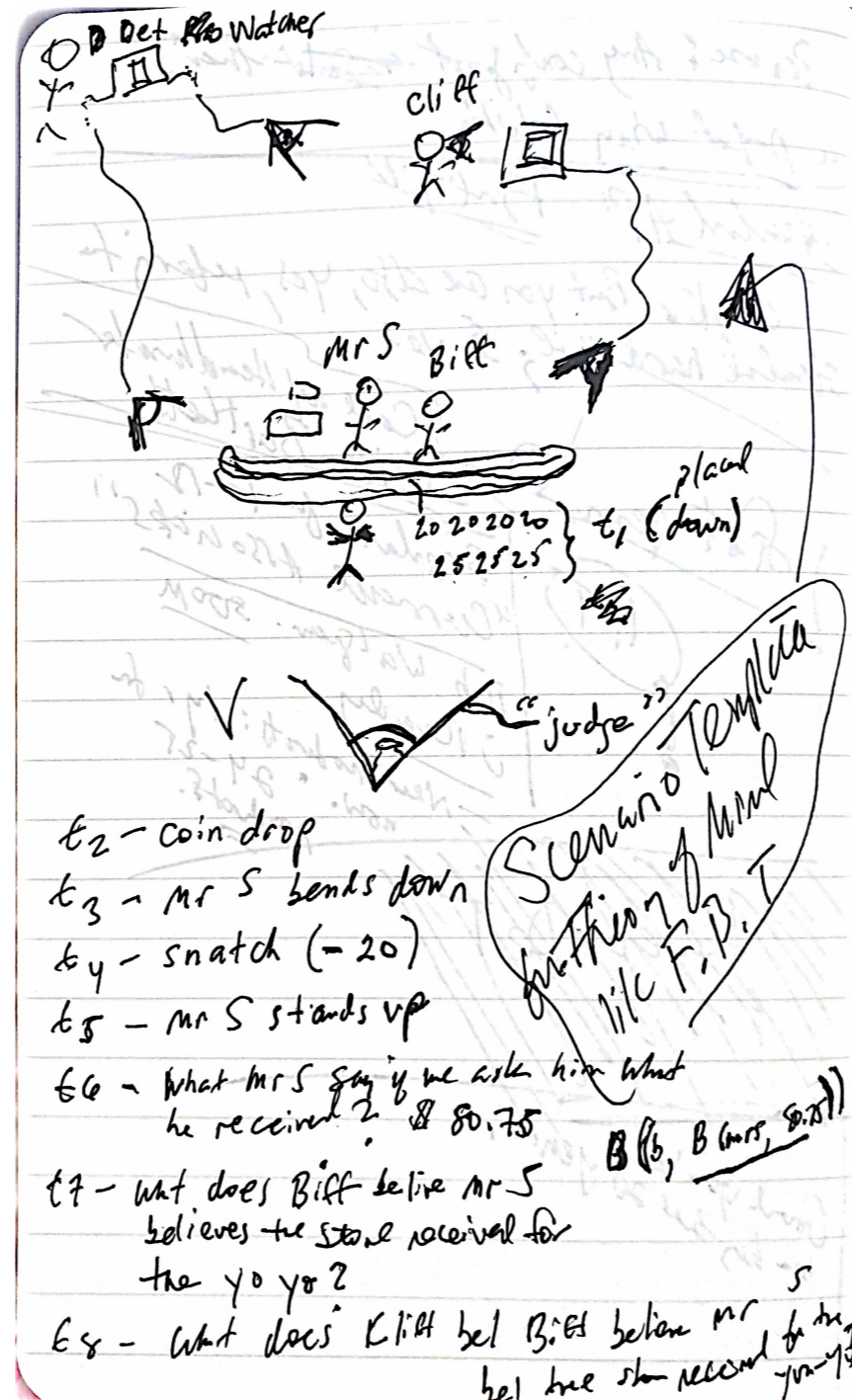


# False Belief Task Demands Intensional Logic ...

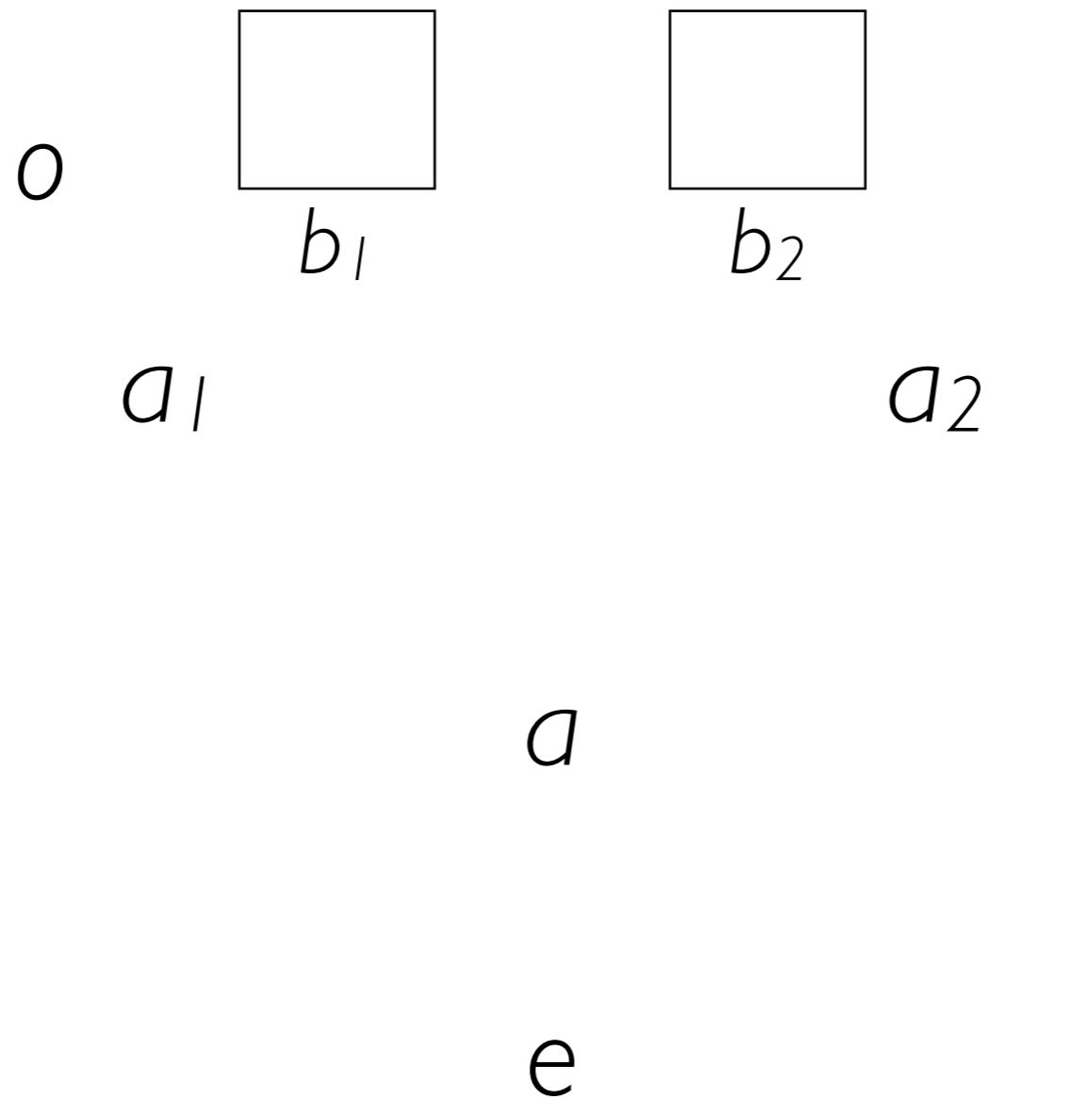




# Better, But Embryonic: The ToM Pawn Shop

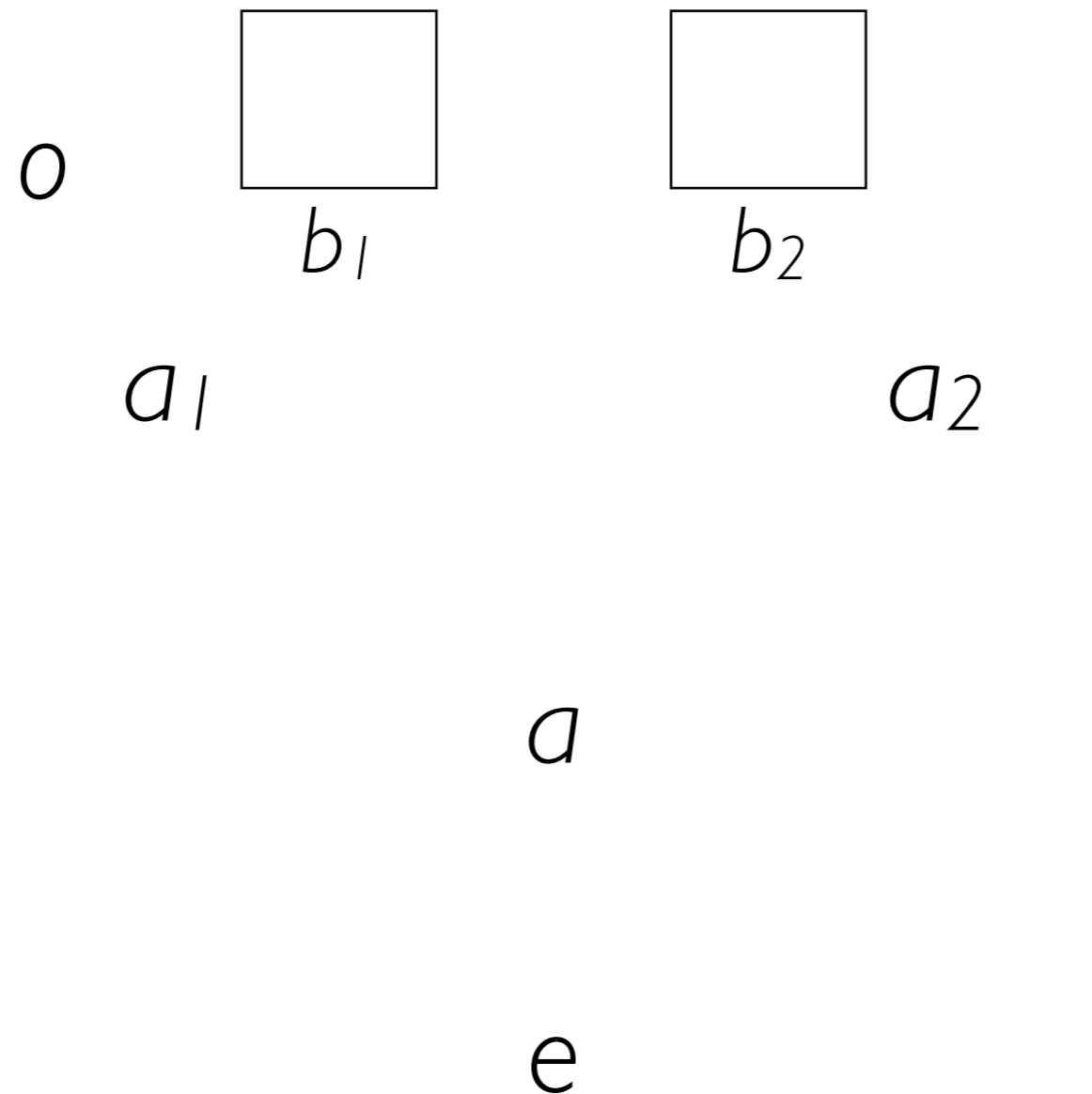


# Framework for $\text{FBT}^0_1$



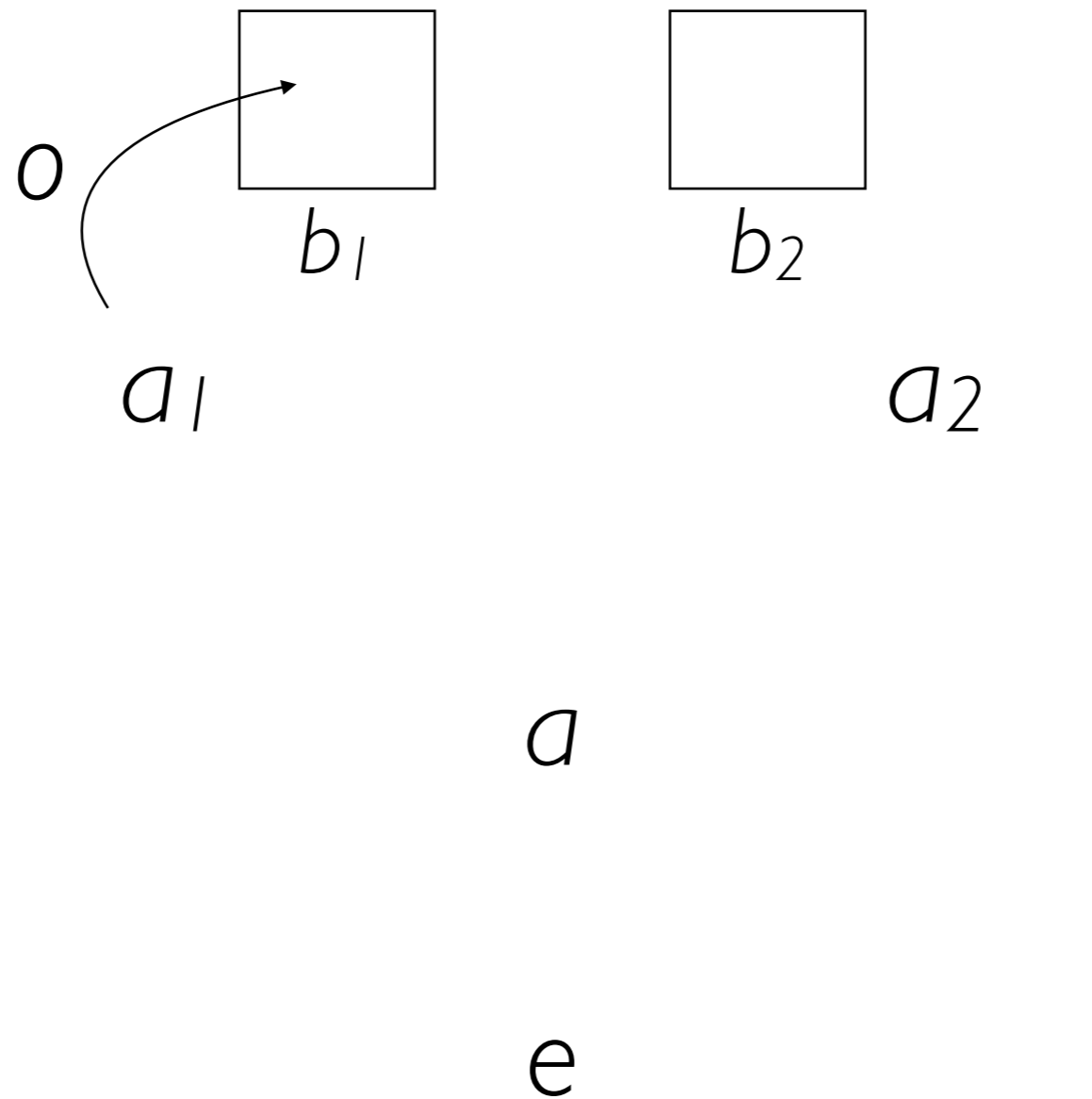
# Framework for $FBT^0_1$

(five timepoints)



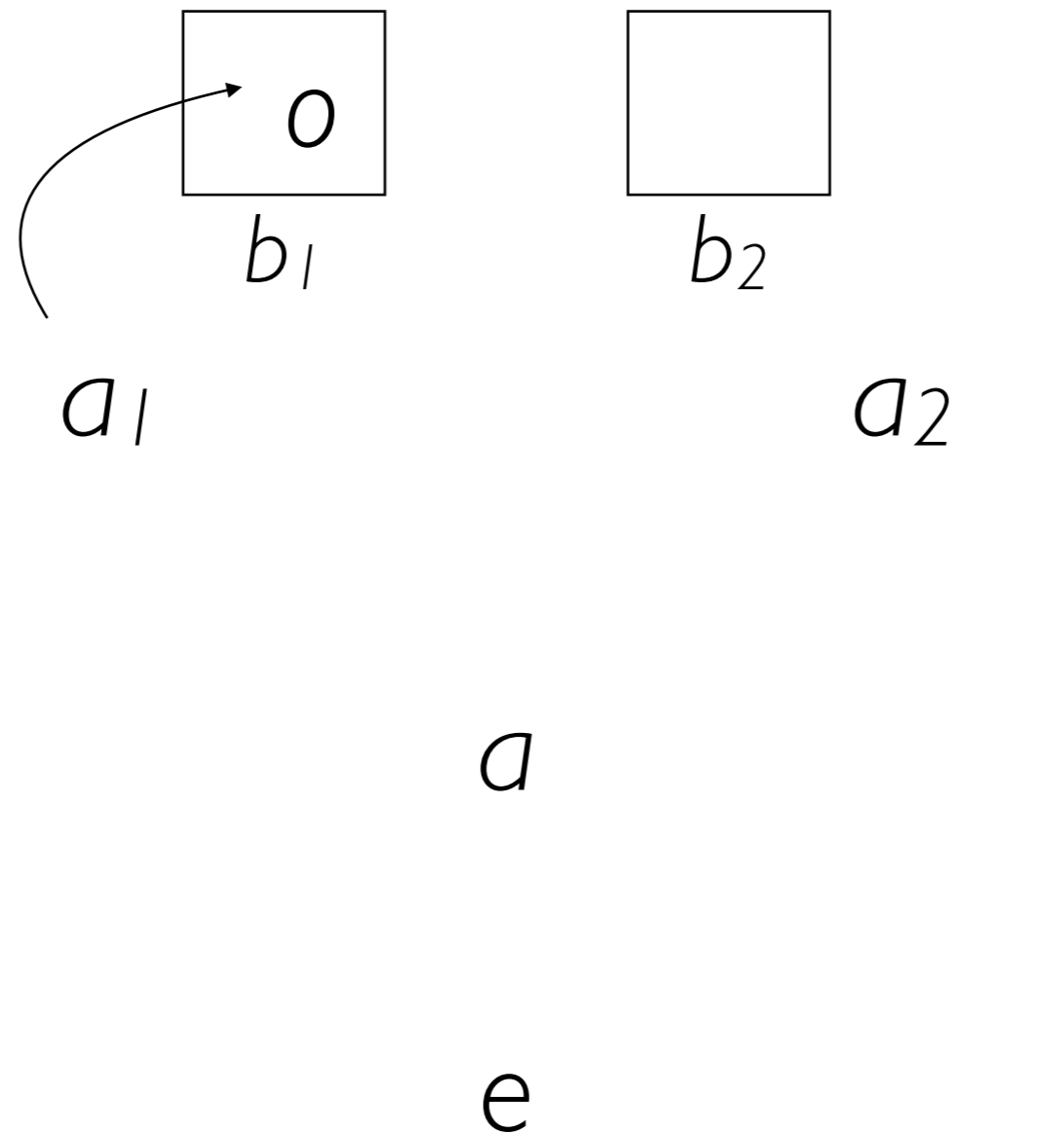
# Framework for $\text{FBT}^0_1$

(five timepoints)



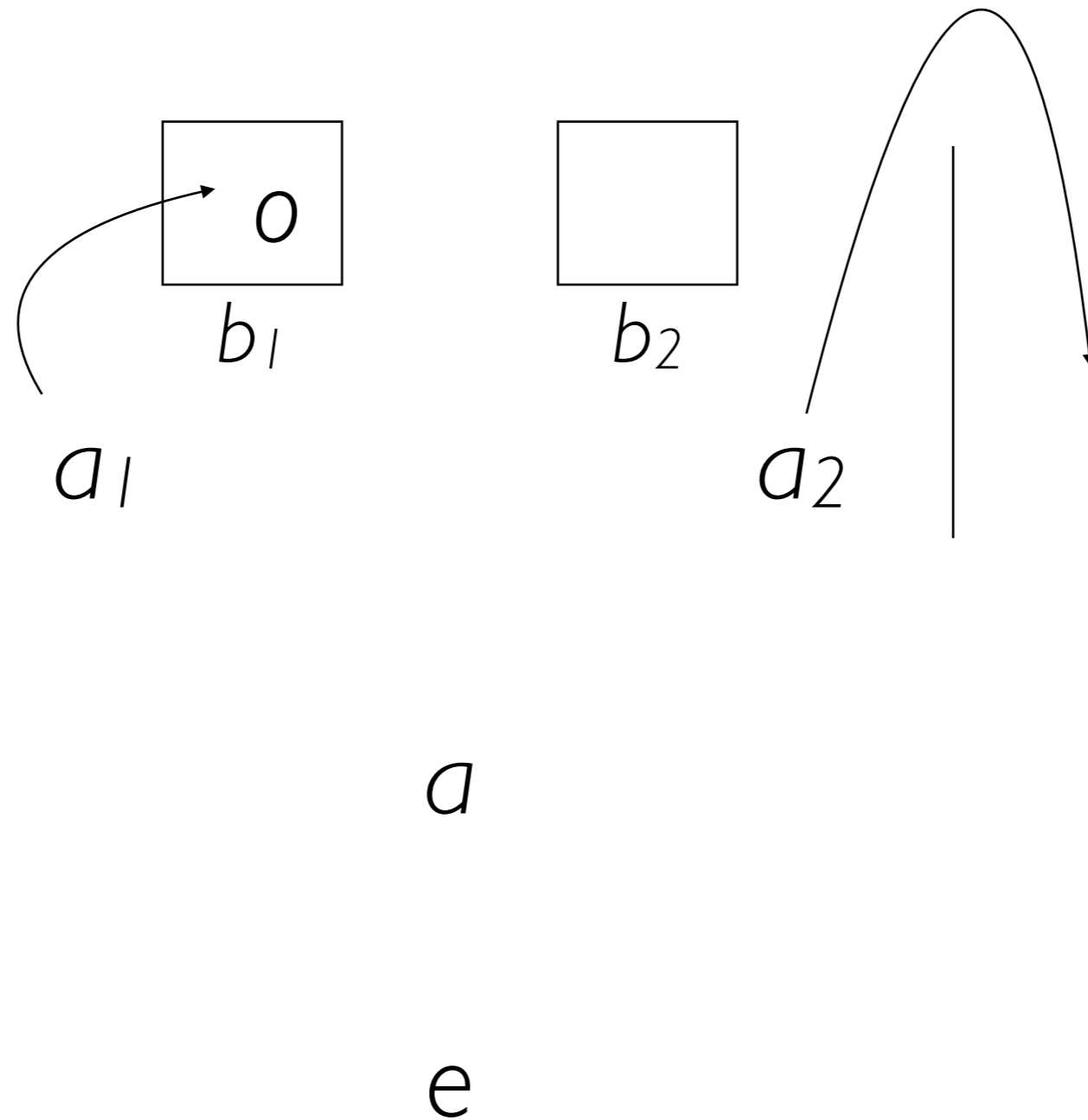
# Framework for $\text{FBT}^0_1$

(five timepoints)



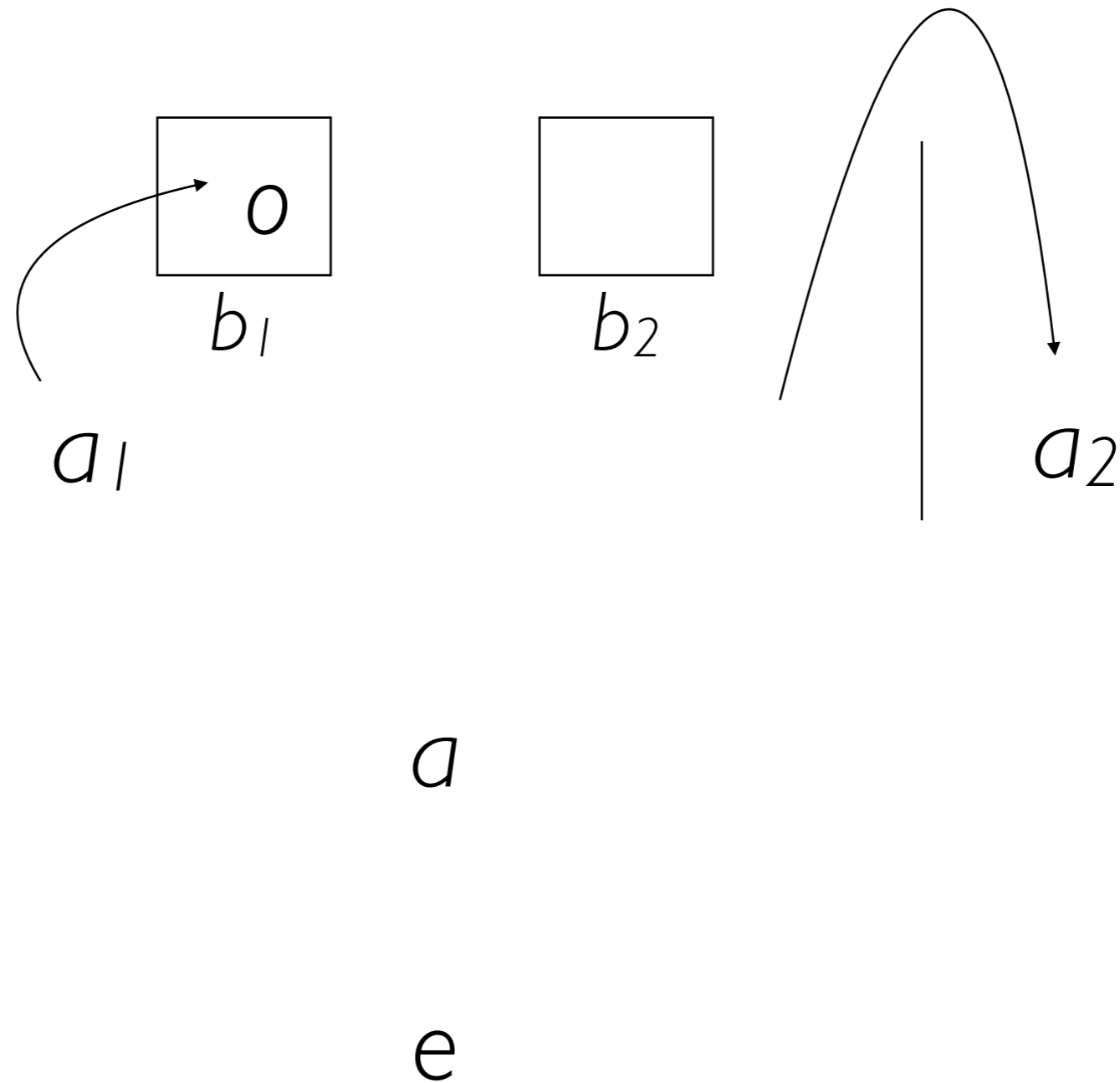
# Framework for $\text{FBT}^0_1$

(five timepoints)



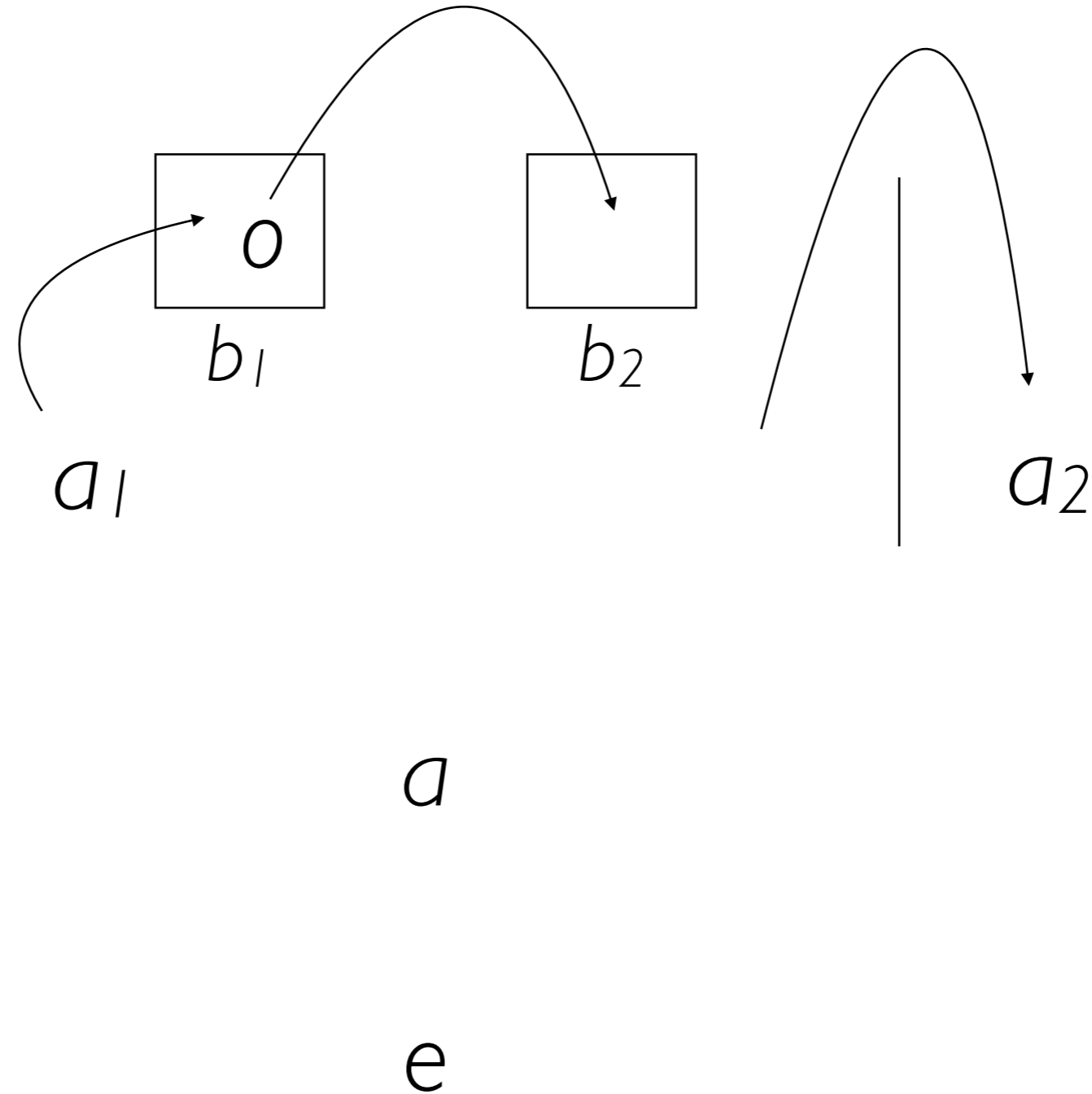
# Framework for $\text{FBT}^0_1$

(five timepoints)



# Framework for $\text{FBT}^0_1$

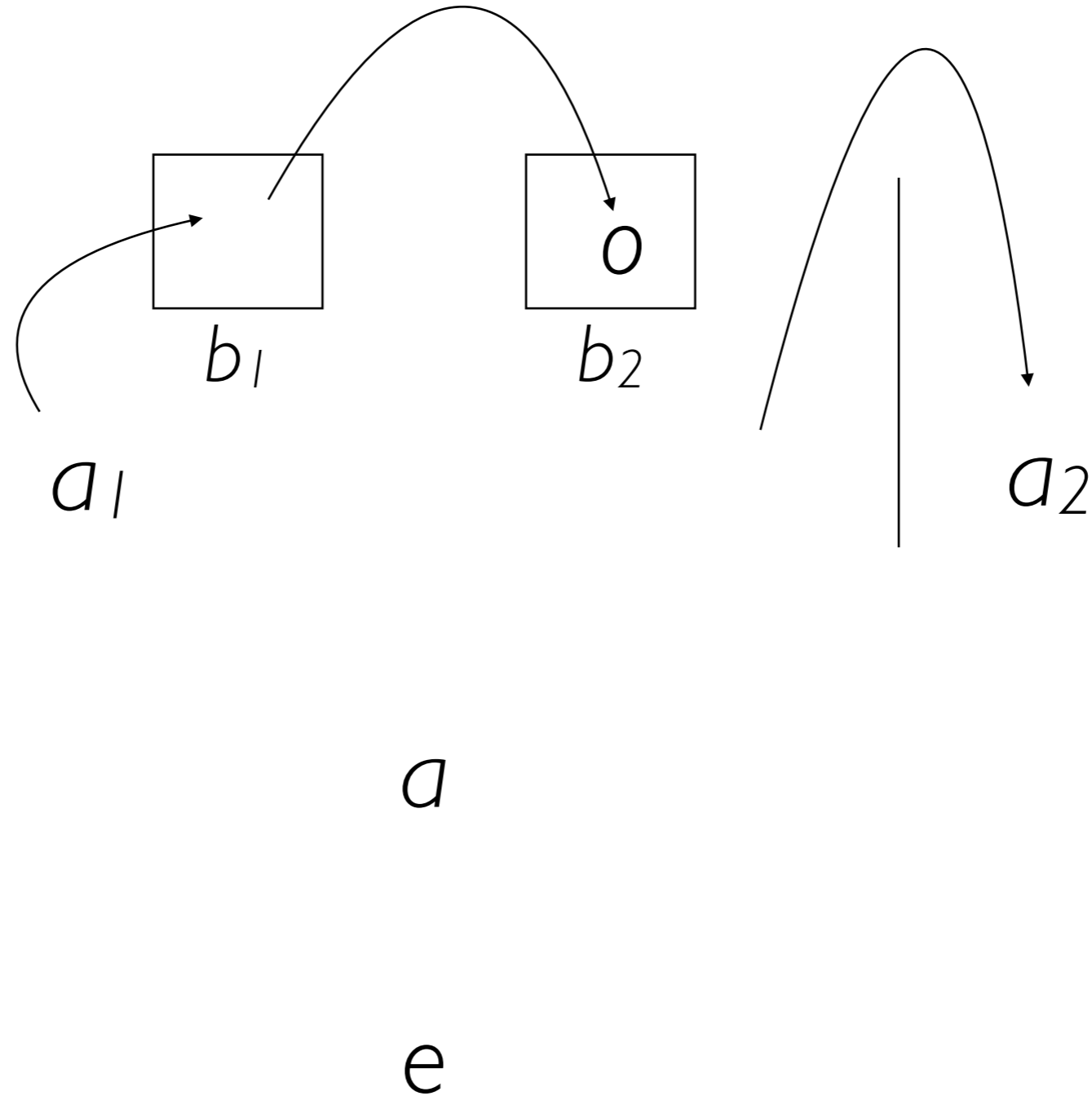
(five timepoints)





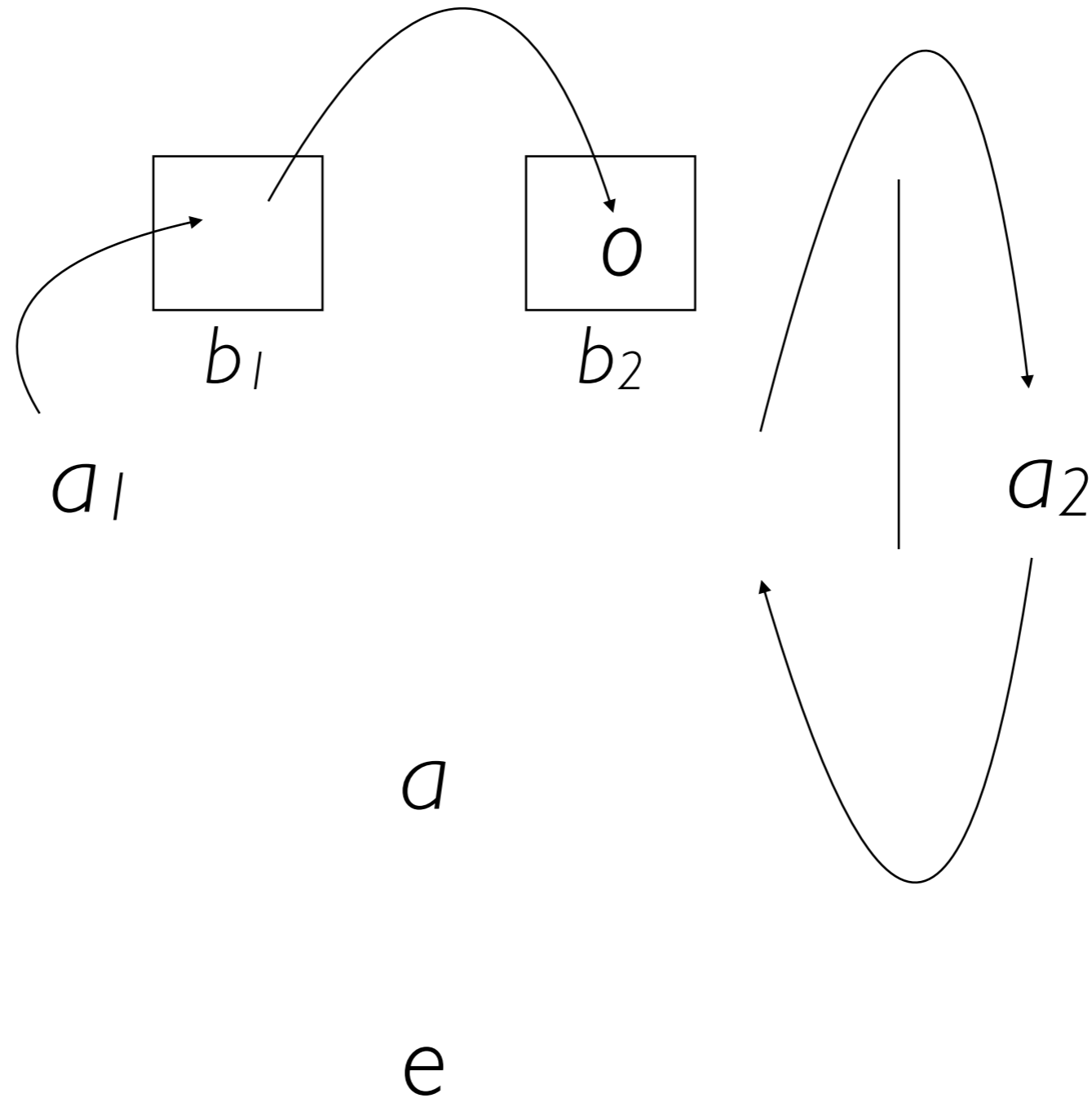
# Framework for $\text{FBT}^0_1$

(five timepoints)



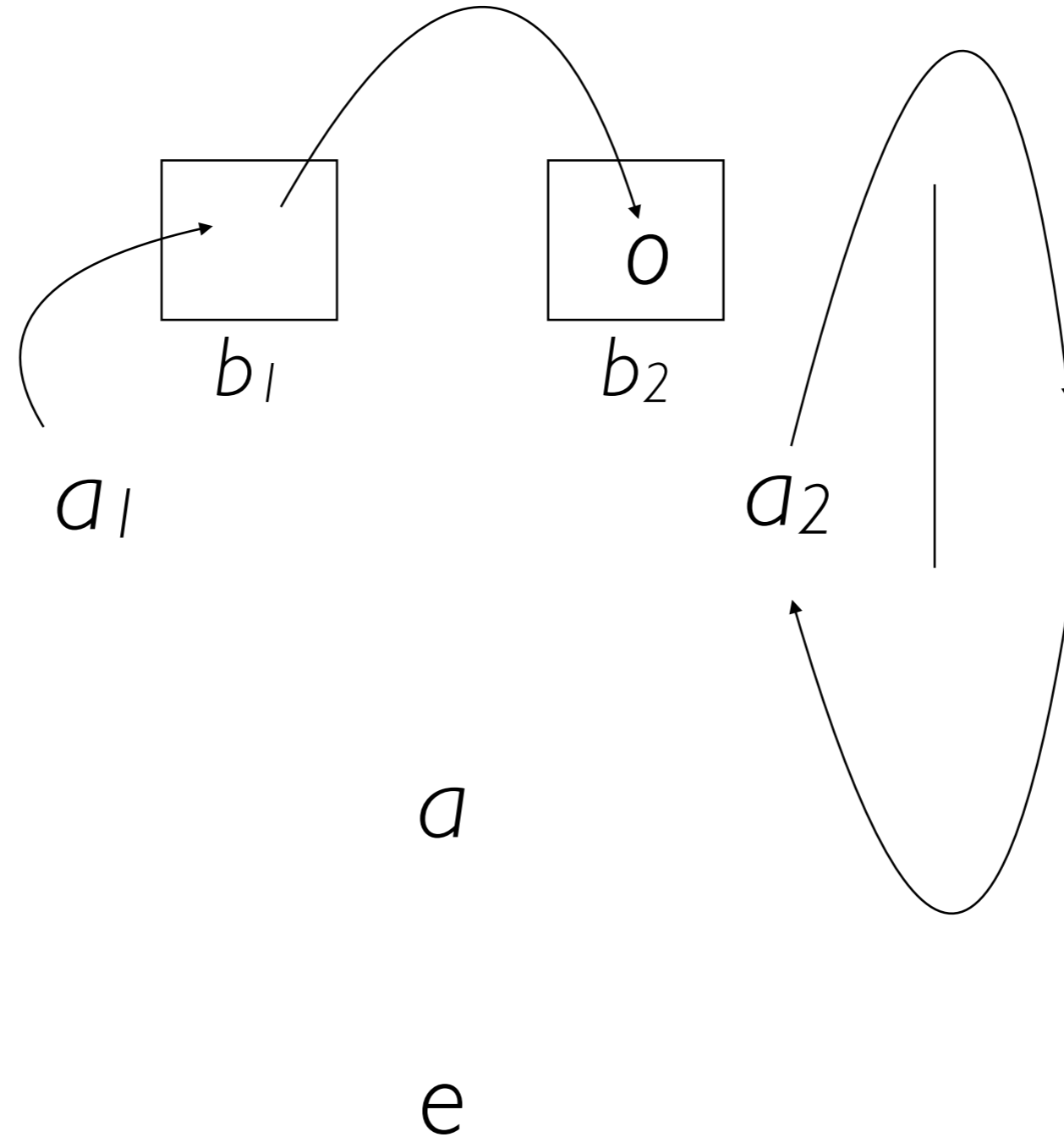
# Framework for $\text{FBT}^0_1$

(five timepoints)

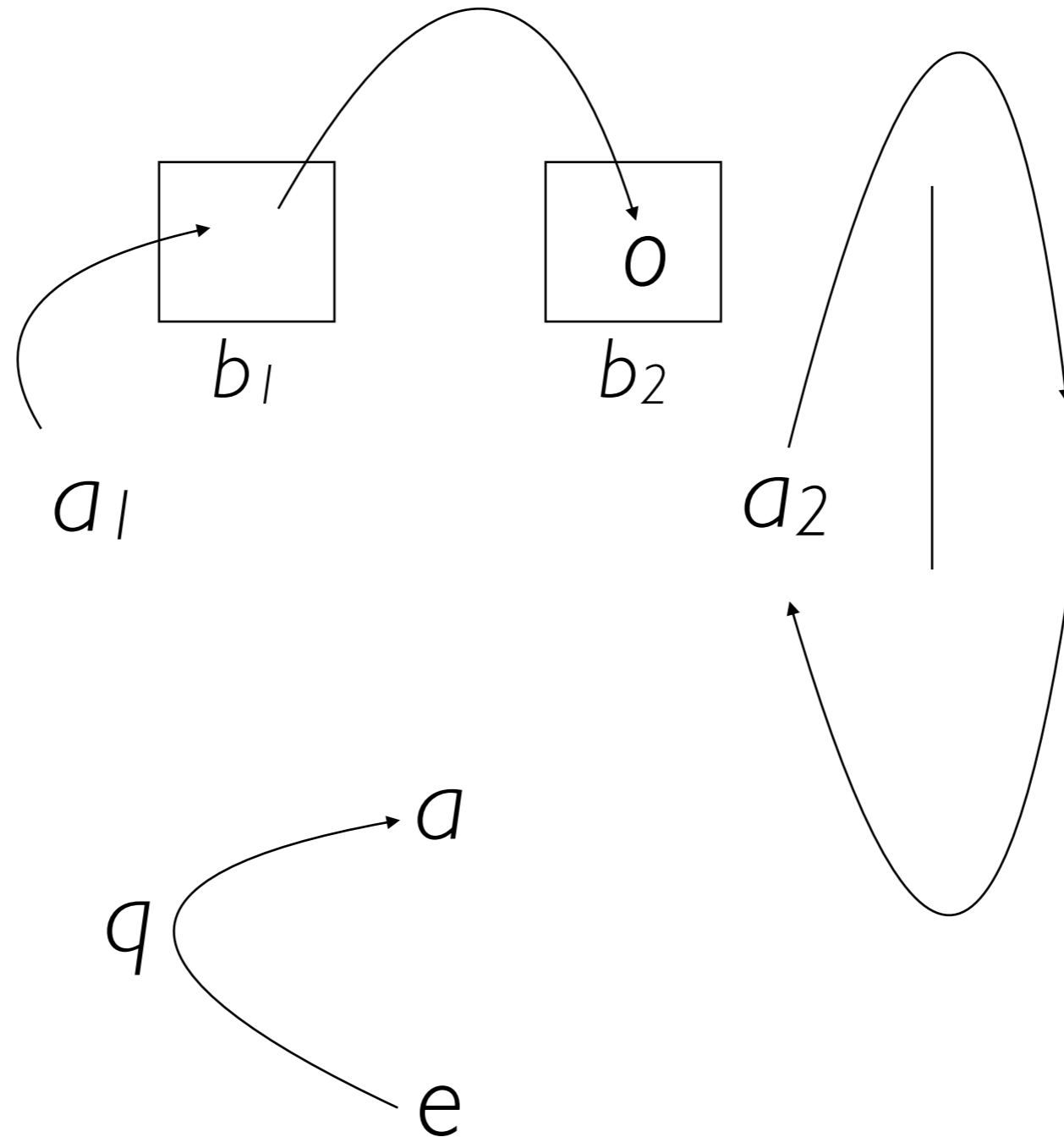


# Framework for $FBT^0_1$

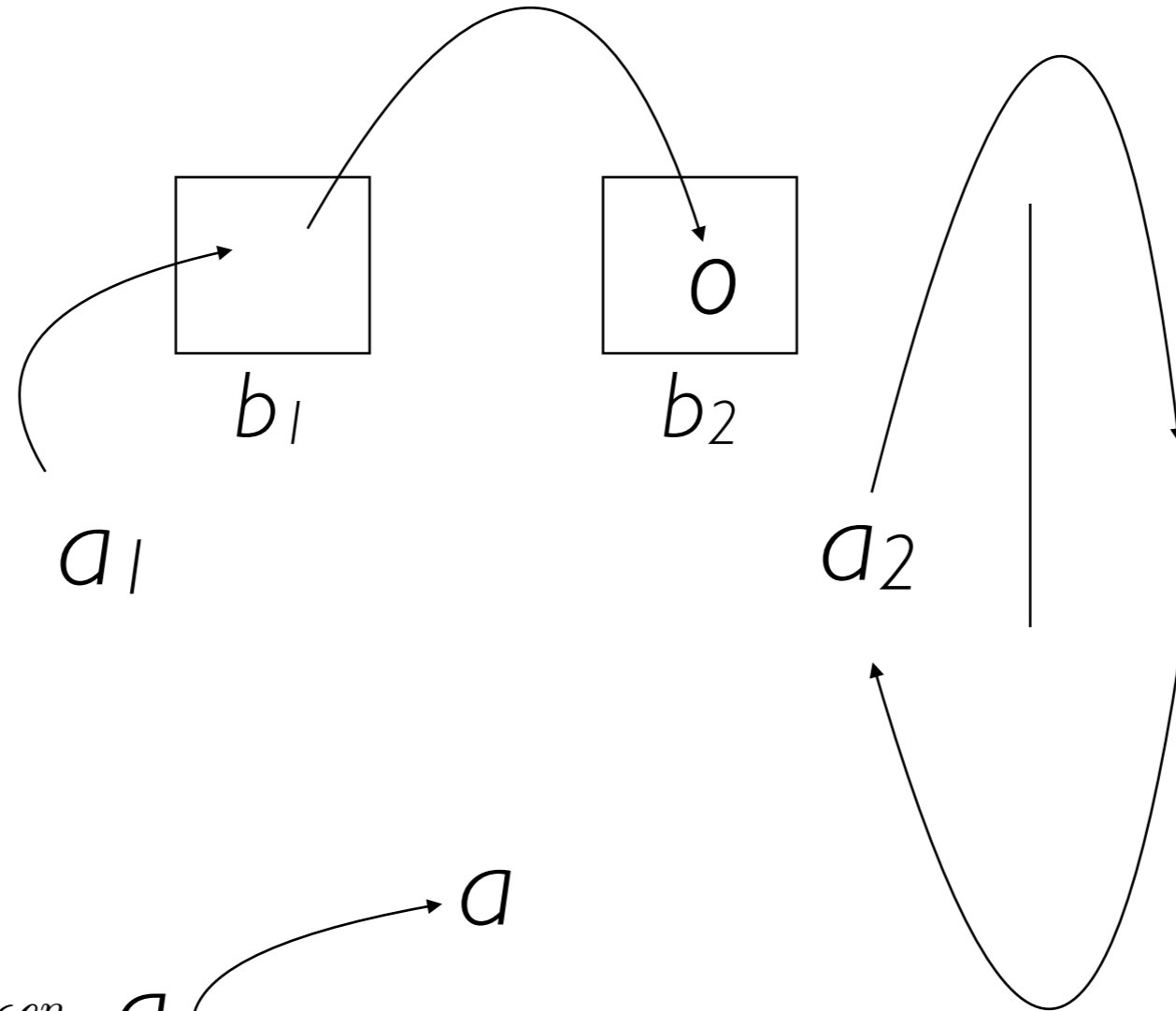
(five timepoints)



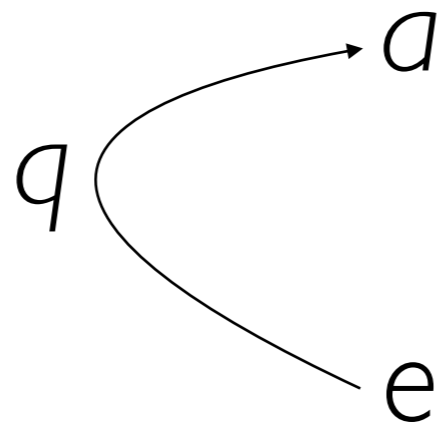
# Framework for $FBT^0_1$ (five timepoints)



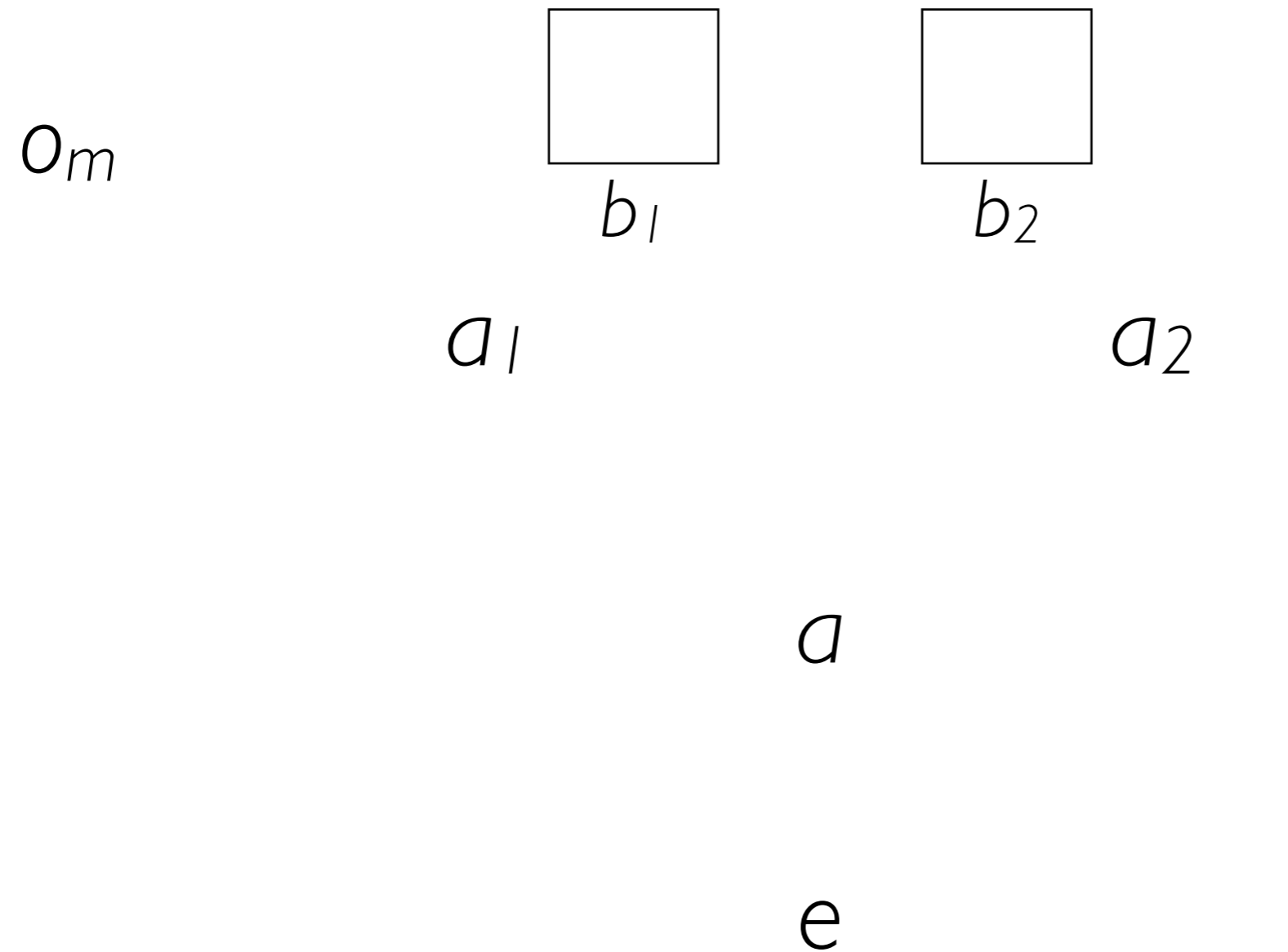
# Framework for $\text{FBT}^0_1$ (five timepoints)



$q$  a formula in modal  $\mathcal{L}^n$

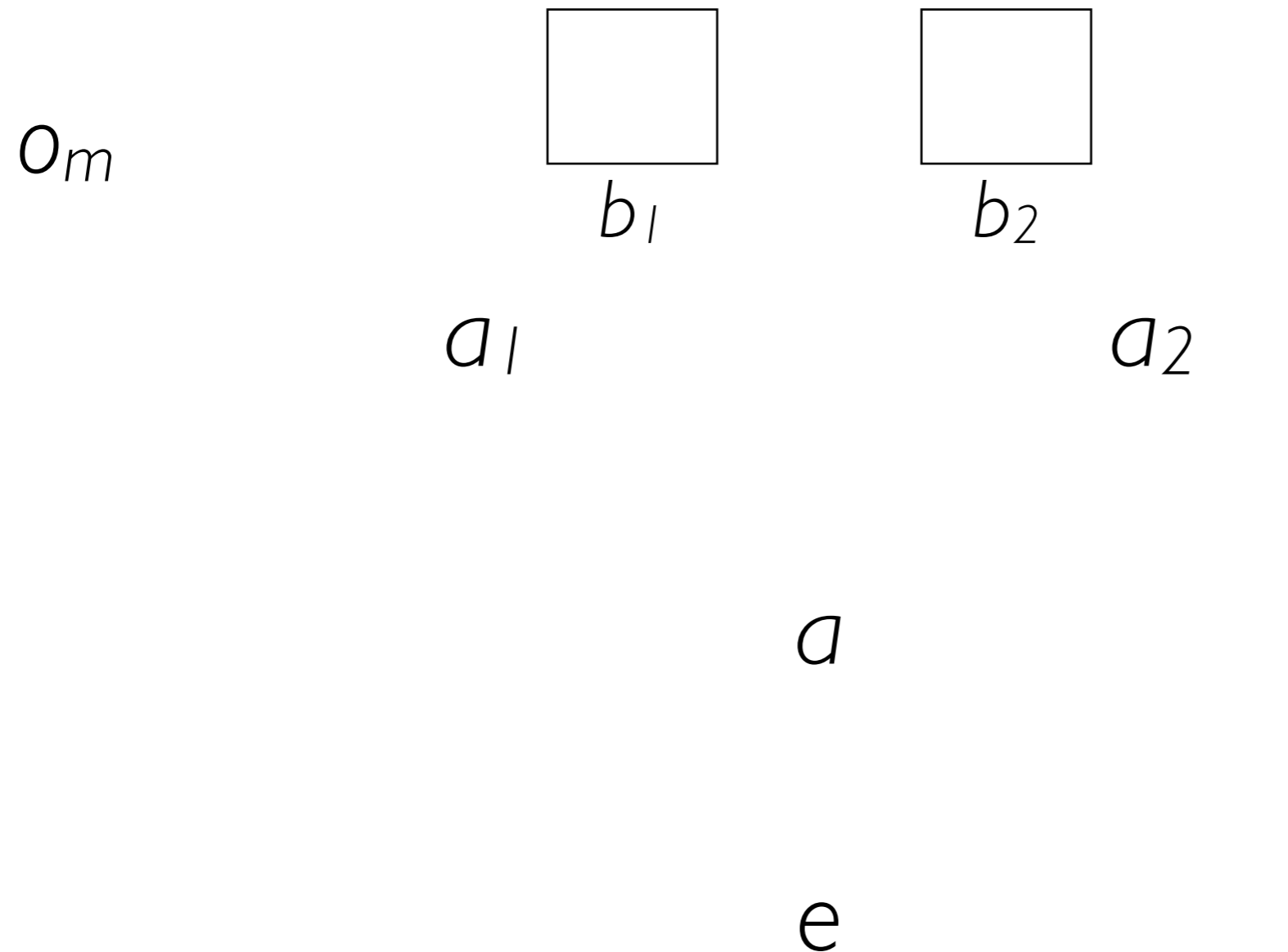


# Framework for FBT<sub>1</sub>



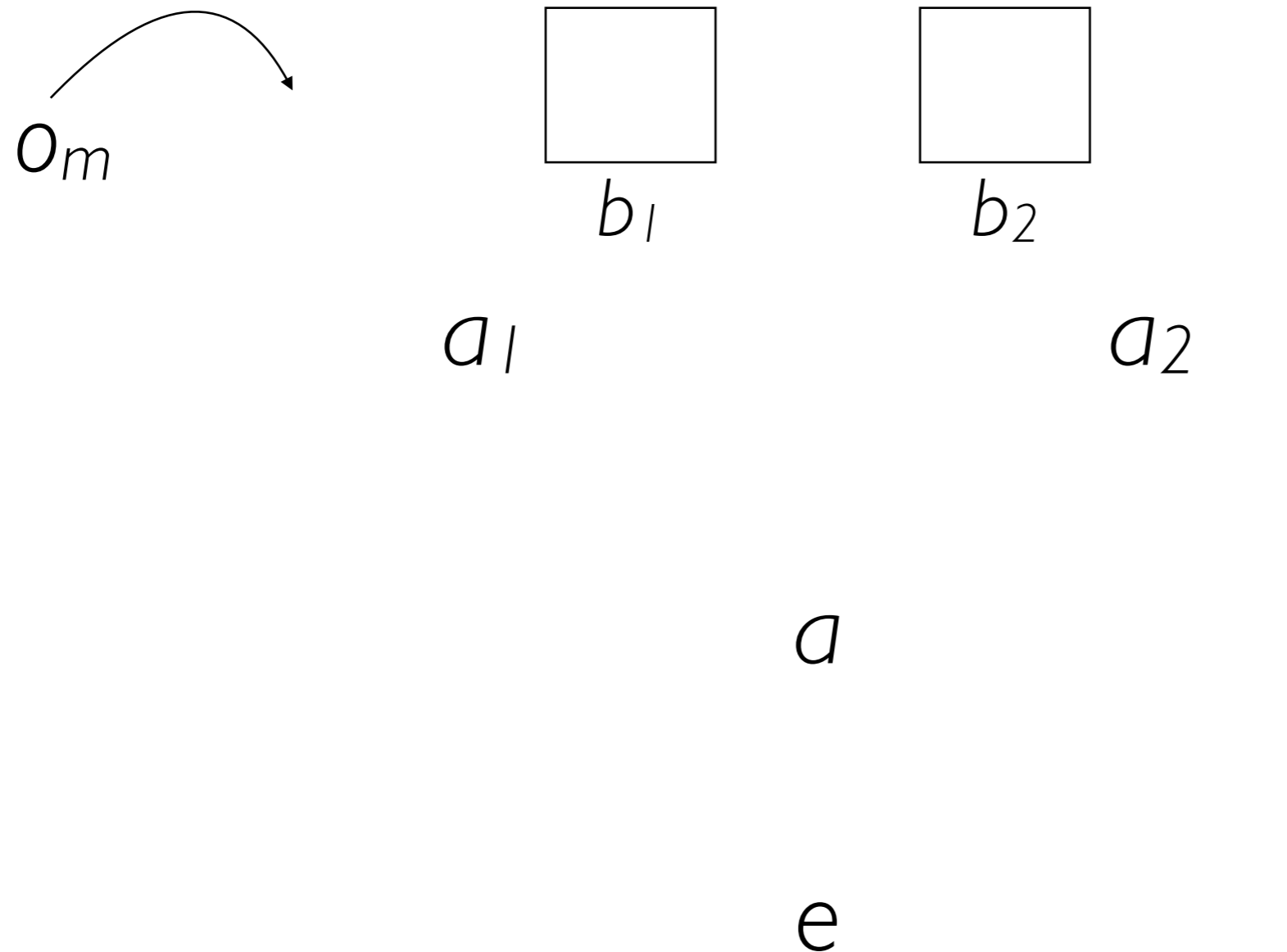
# Framework for FBT<sub>1</sub>

(six timepoints)



# Framework for FBT<sub>1</sub>

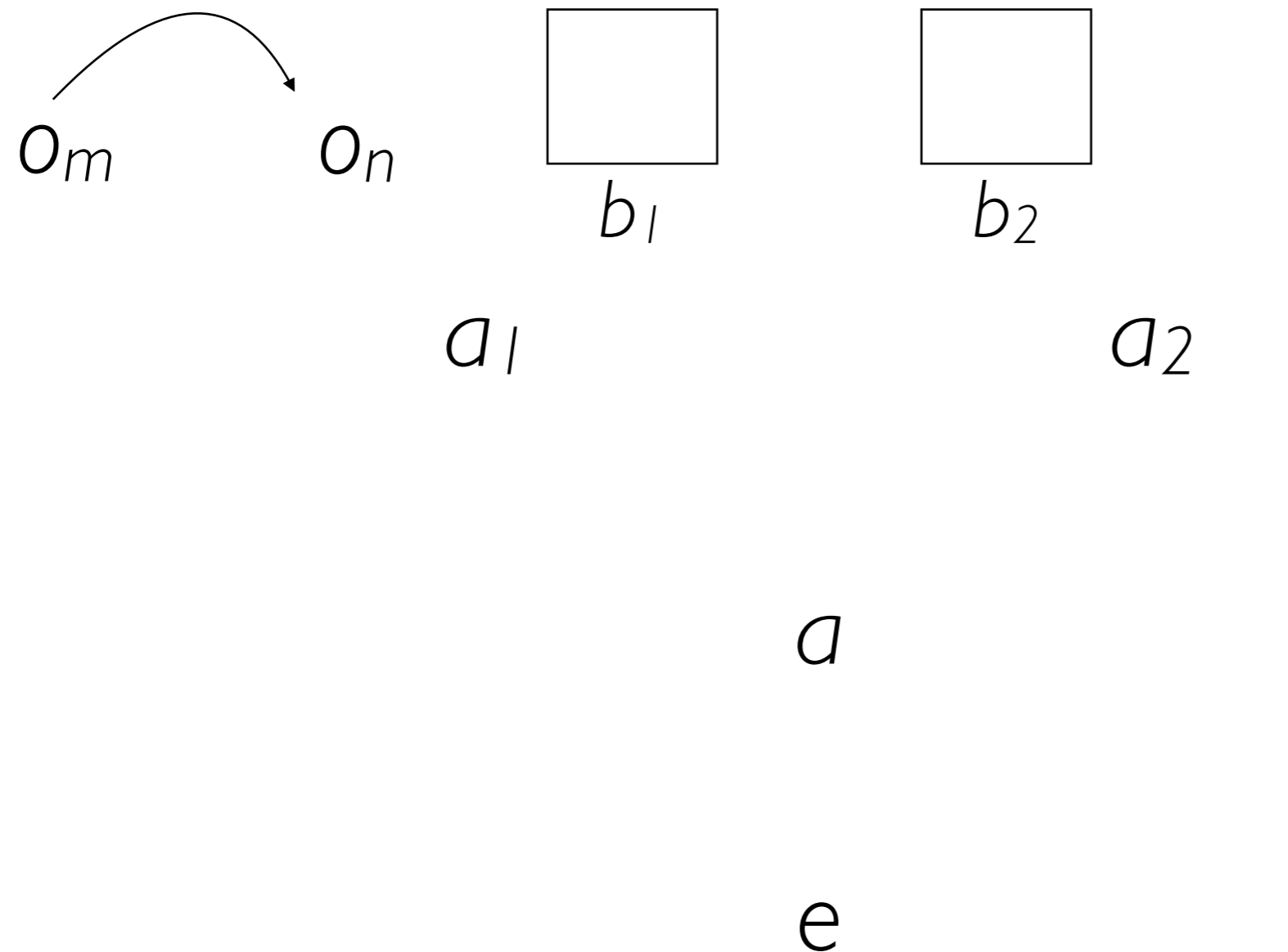
(six timepoints)





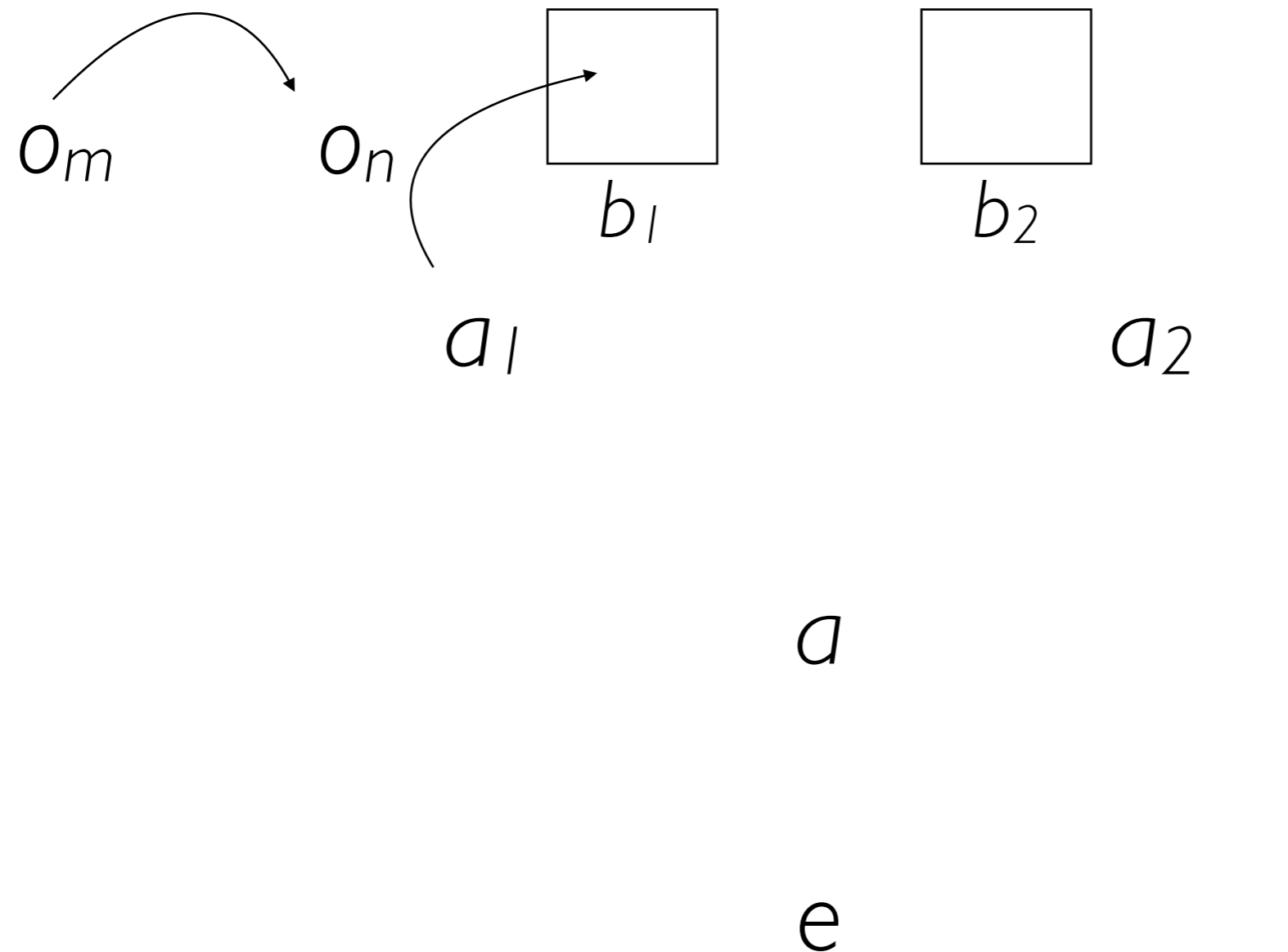
# Framework for FBT<sub>1</sub>

(six timepoints)



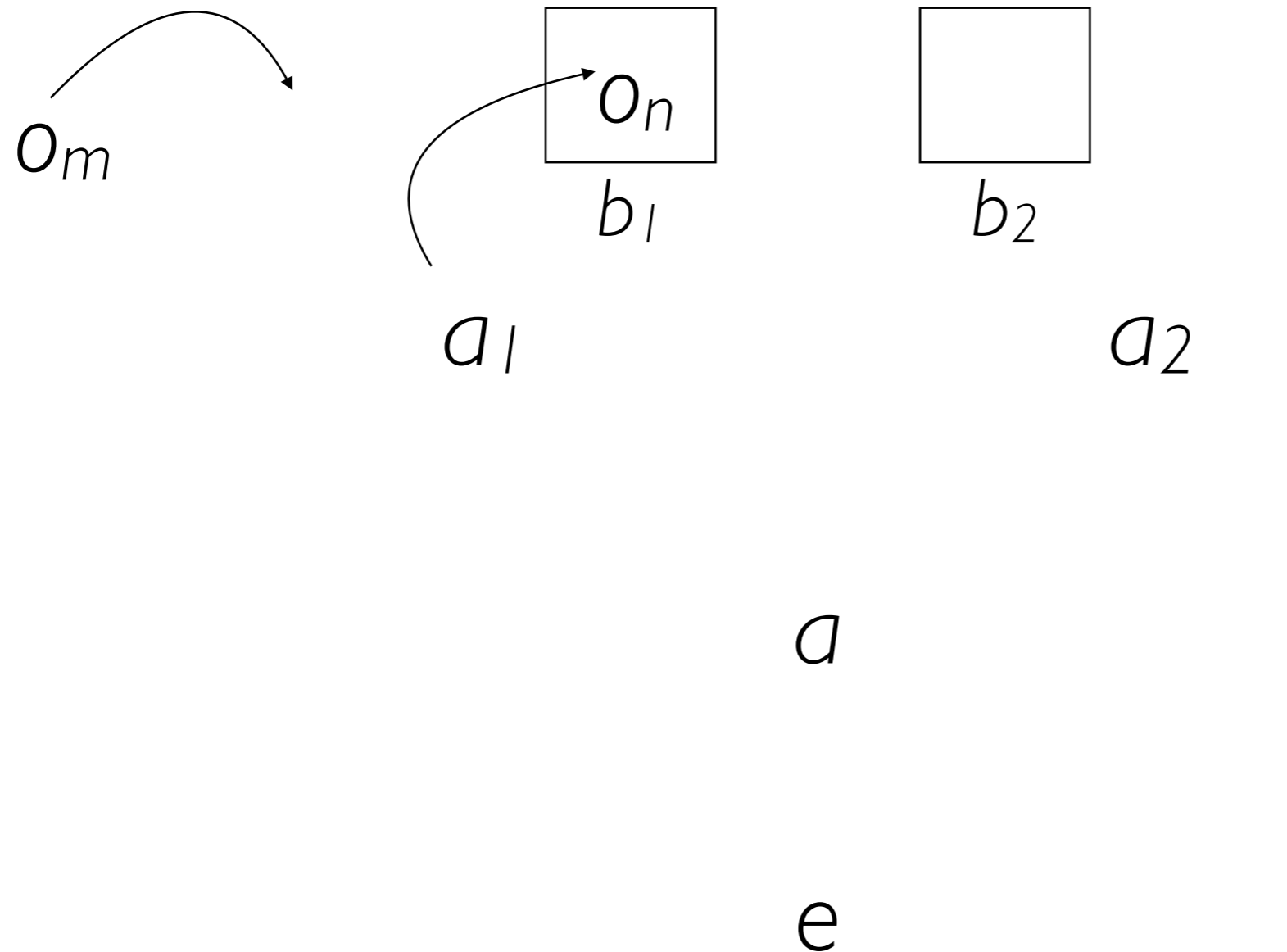
# Framework for FBT<sub>1</sub>

(six timepoints)



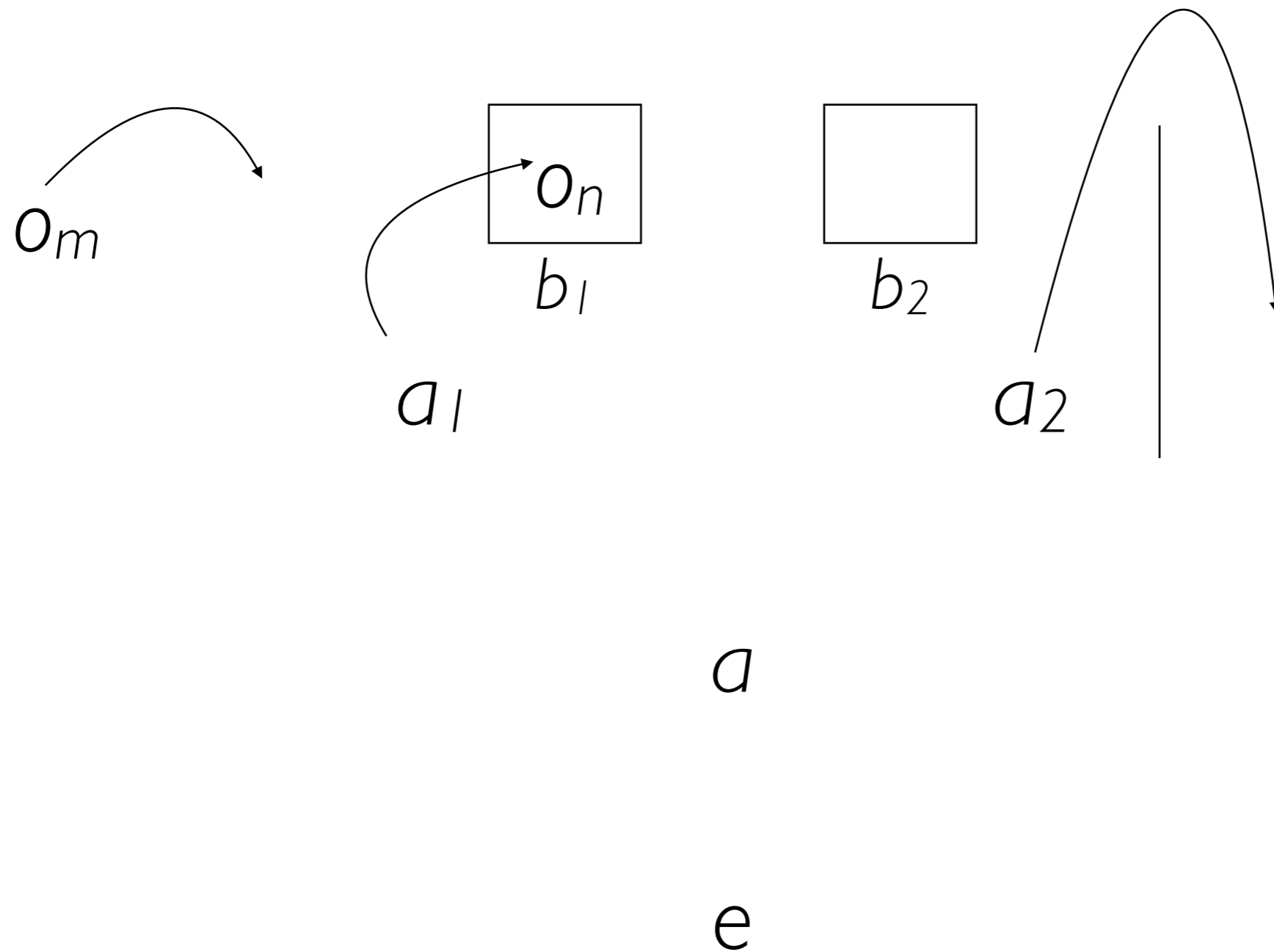
# Framework for FBT<sub>1</sub>

(six timepoints)



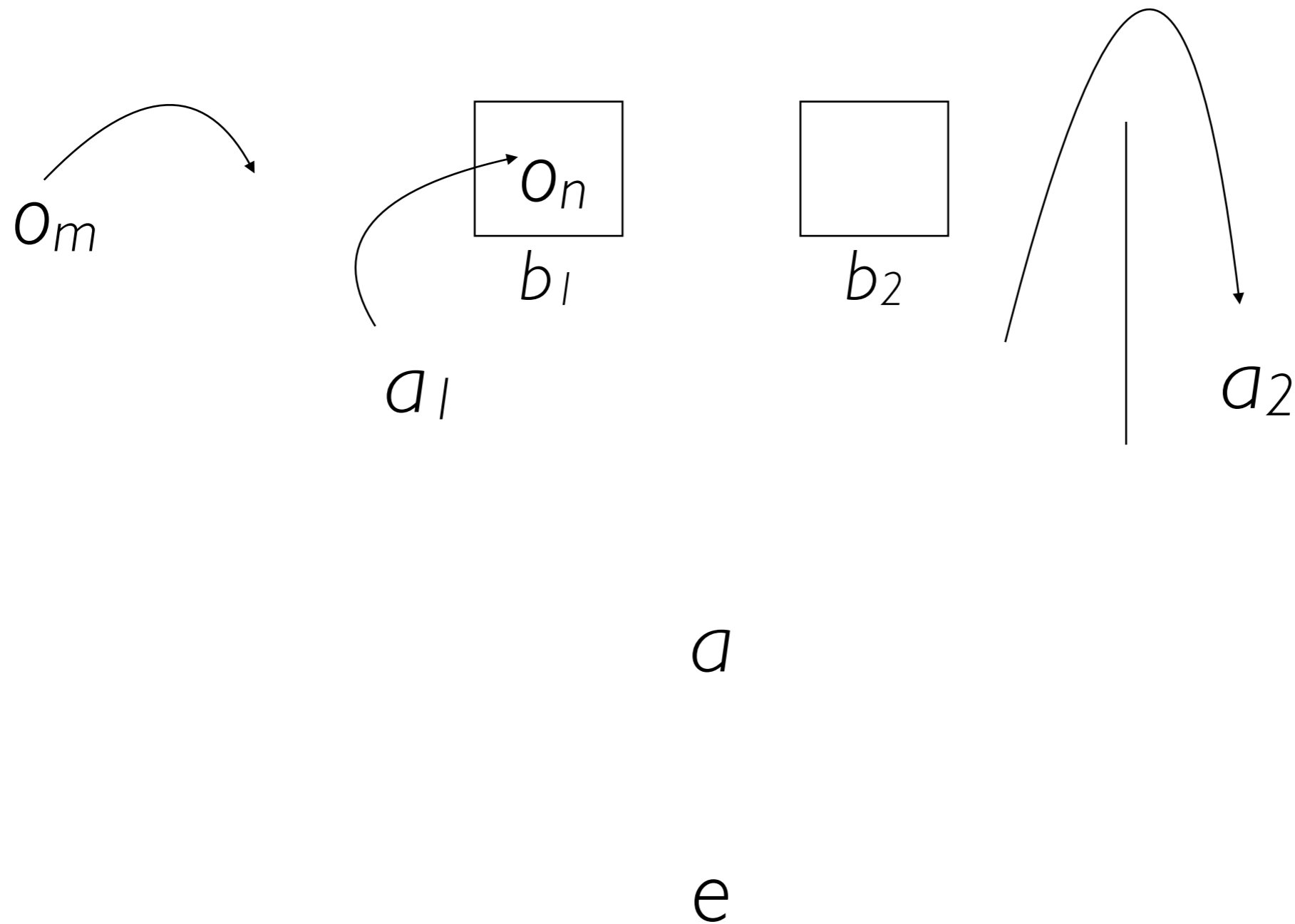
# Framework for FBT<sub>1</sub>

(six timepoints)



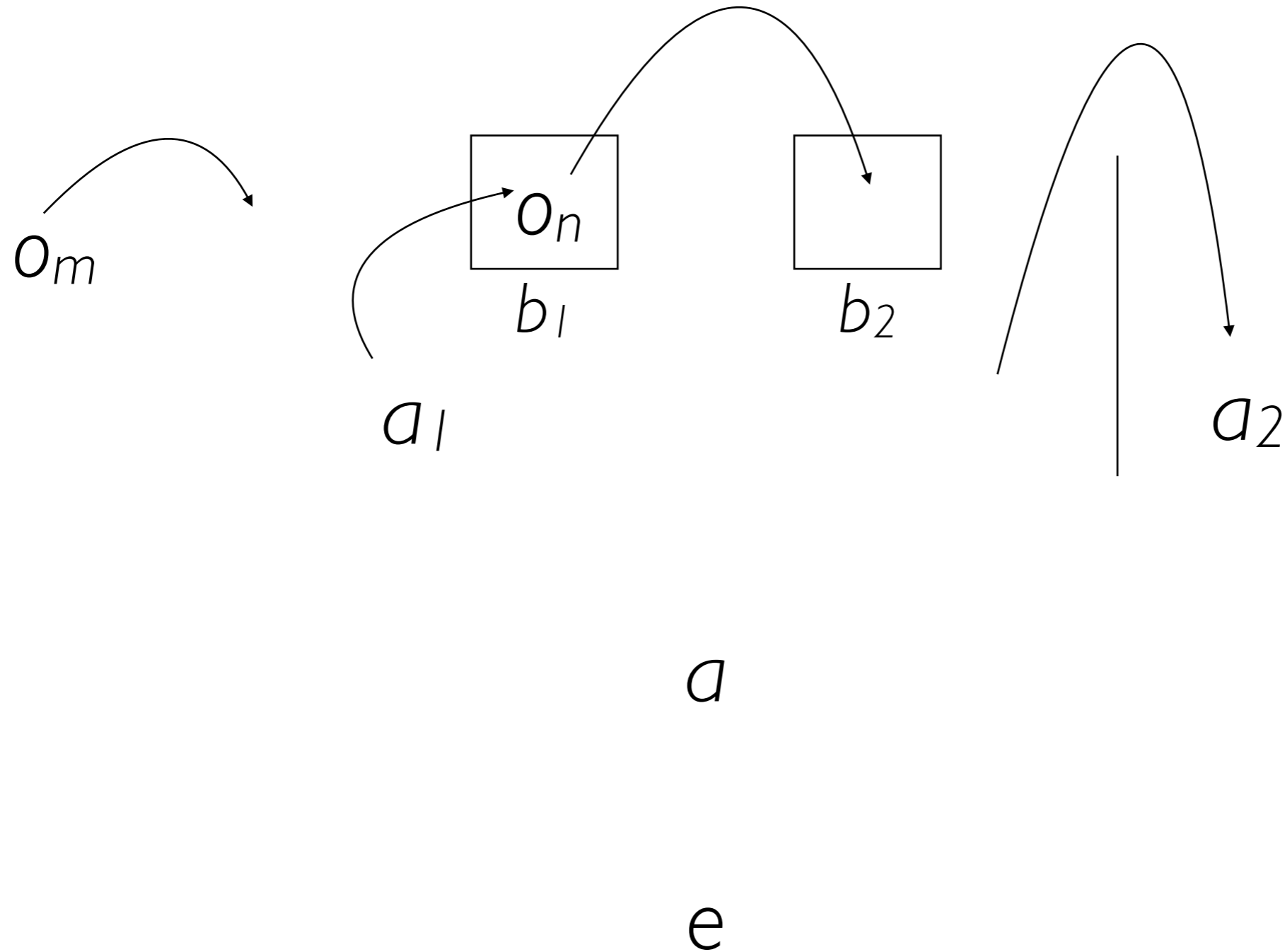
# Framework for FBT<sub>1</sub>

(six timepoints)



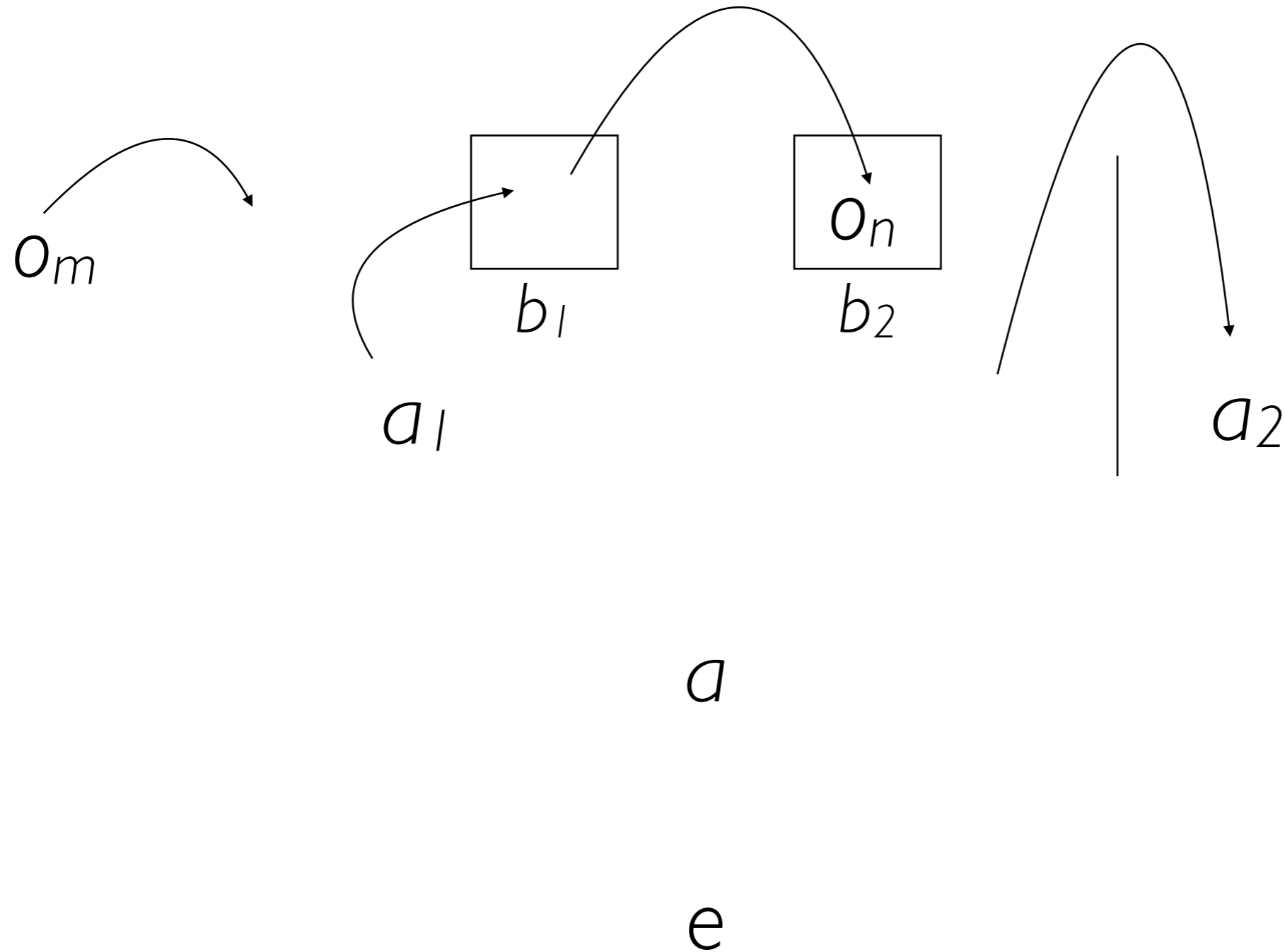
# Framework for FBT<sub>1</sub>

(six timepoints)



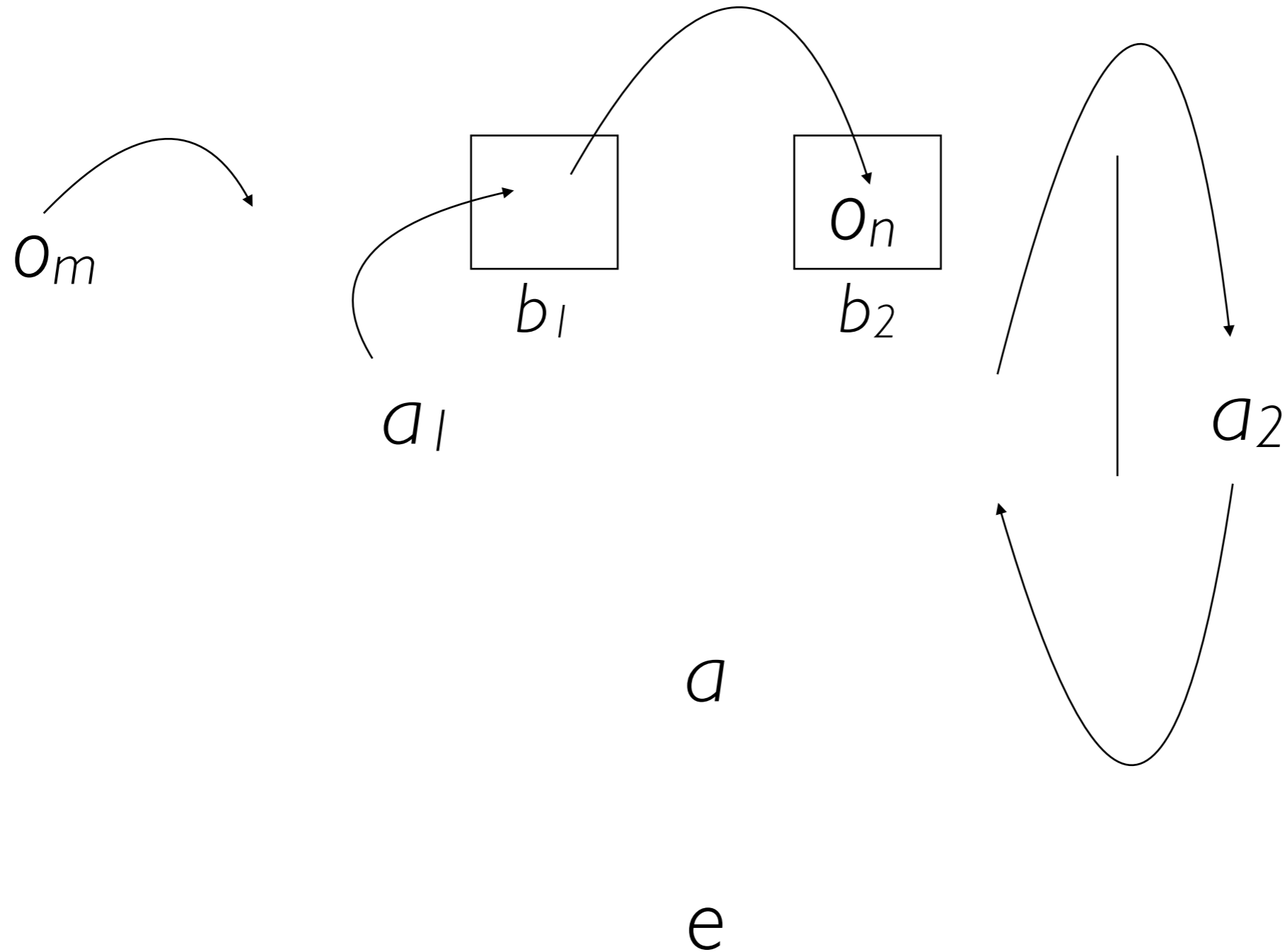
# Framework for FBT<sub>1</sub>

(six timepoints)



# Framework for FBT<sub>1</sub>

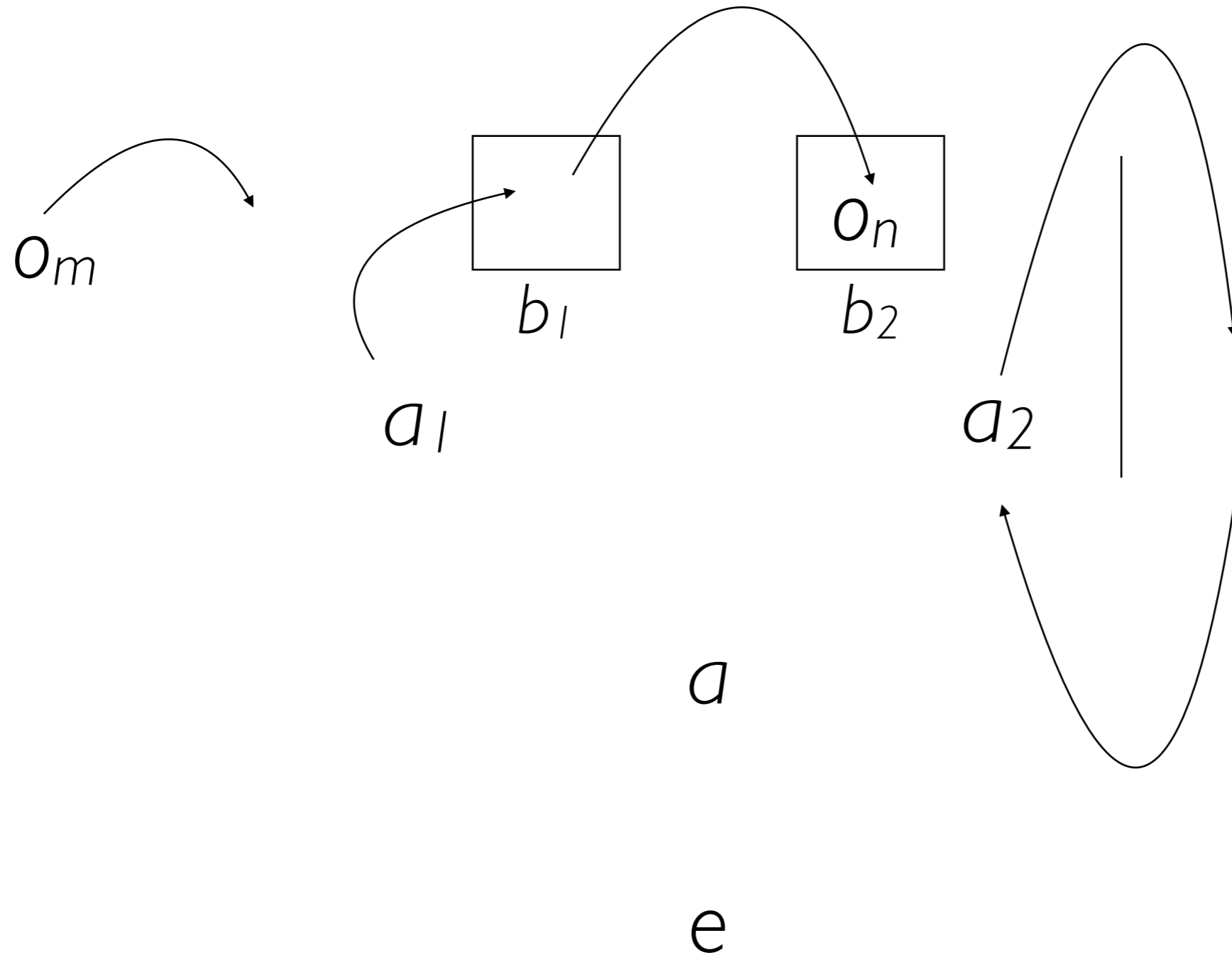
(six timepoints)





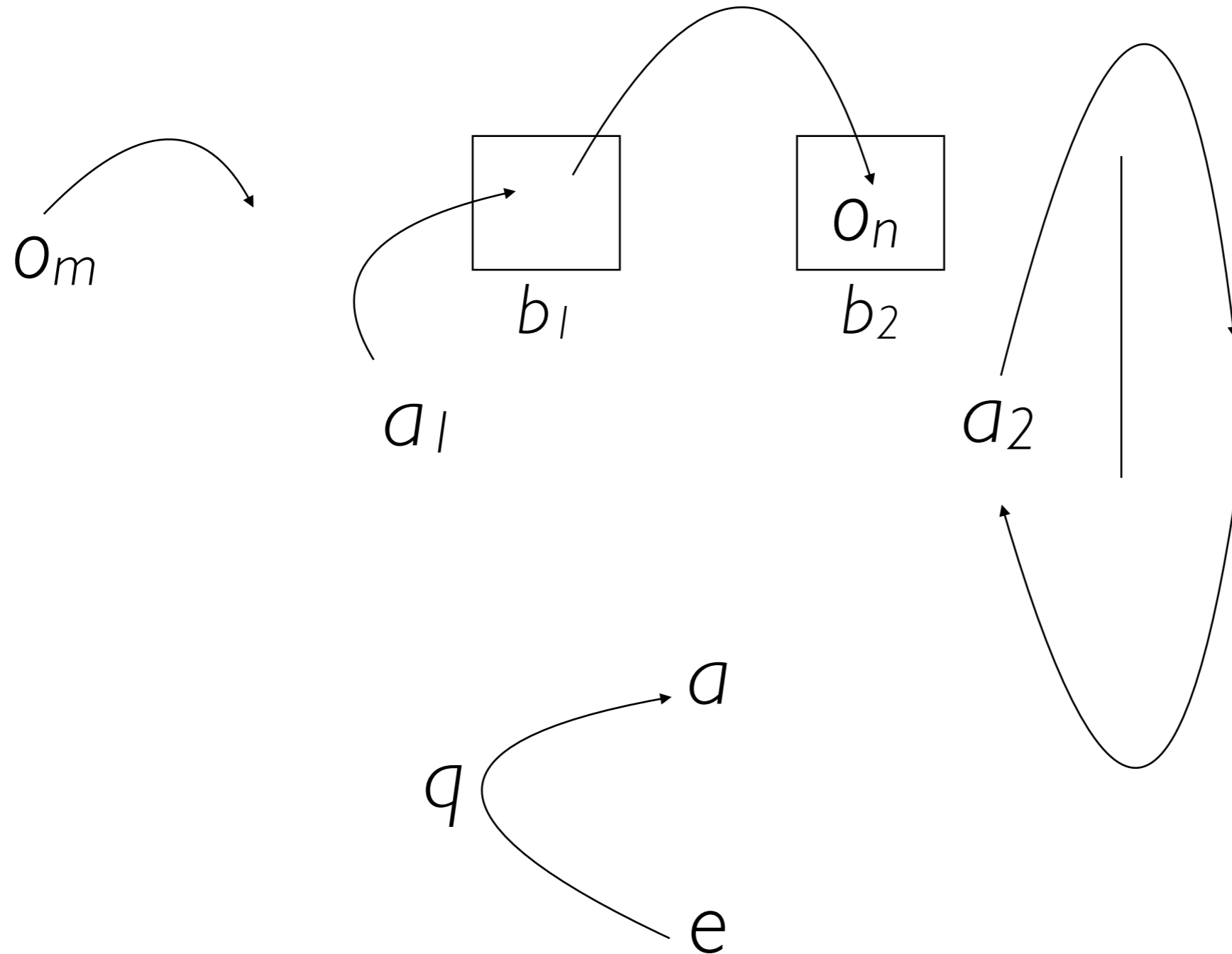
# Framework for FBT<sub>1</sub>

(six timepoints)



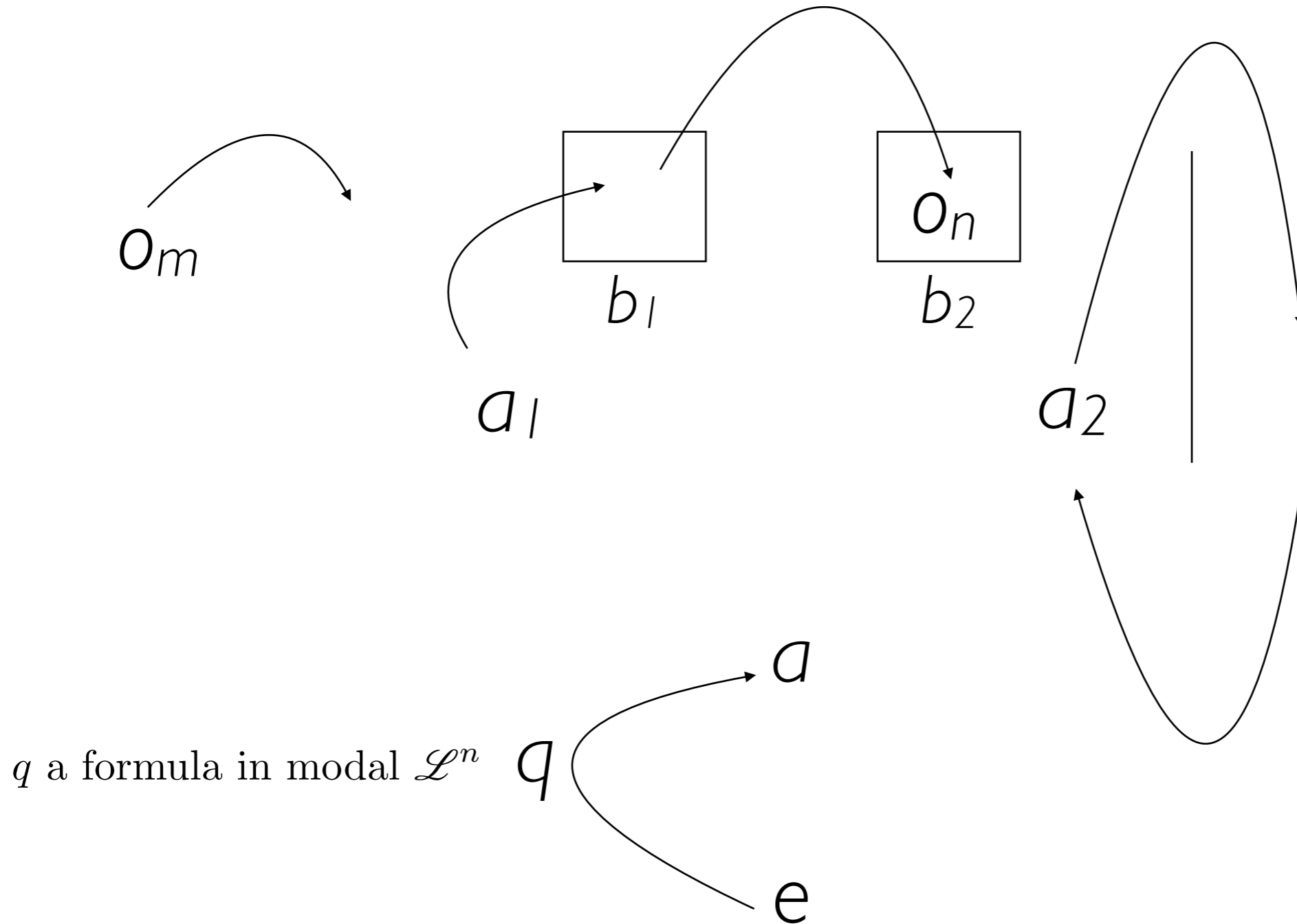
# Framework for FBT<sub>1</sub>

(six timepoints)



# Framework for $\text{FBT}^1_1$

(six timepoints)



# Done, a Decade Ago, Formally & Implementation/Simulation

Arkoudas, K. & Bringsjord, S.  
(2009) “Propositional  
Attitudes and Causation”  
*International Journal of Software  
and Informatics* **3.1**: 47–65.

[http://kryten.mm.rpi.edu/PRICAI\\_w\\_sequentcalc\\_041709.pdf](http://kryten.mm.rpi.edu/PRICAI_w_sequentcalc_041709.pdf)

## Propositional attitudes and causation

Konstantine Arkoudas and Selmer Bringsjord

Cognitive Science and Computer Science Departments, RPI  
arkouk@rpi.edu, brings@rpi.edu

**Abstract.** Predicting and explaining the behavior of others in terms of mental states is indispensable for everyday life. It will be equally important for artificial agents. We present an inference system for representing and reasoning about mental states, and use it to provide a formal analysis of the false-belief task. The system allows for the representation of information about events, causation, and perceptual, doxastic, and epistemic states (vision, belief, and knowledge), incorporating ideas from the event calculus and multi-agent epistemic logic. Unlike previous AI formalisms, our focus here is on mechanized proofs and proof programmability, not on metamathematical results. Reasoning is performed via relatively cognitively plausible inference rules, and a degree of automation is achieved by general-purpose inference methods and by a syntactic embedding of the system in first-order logic.

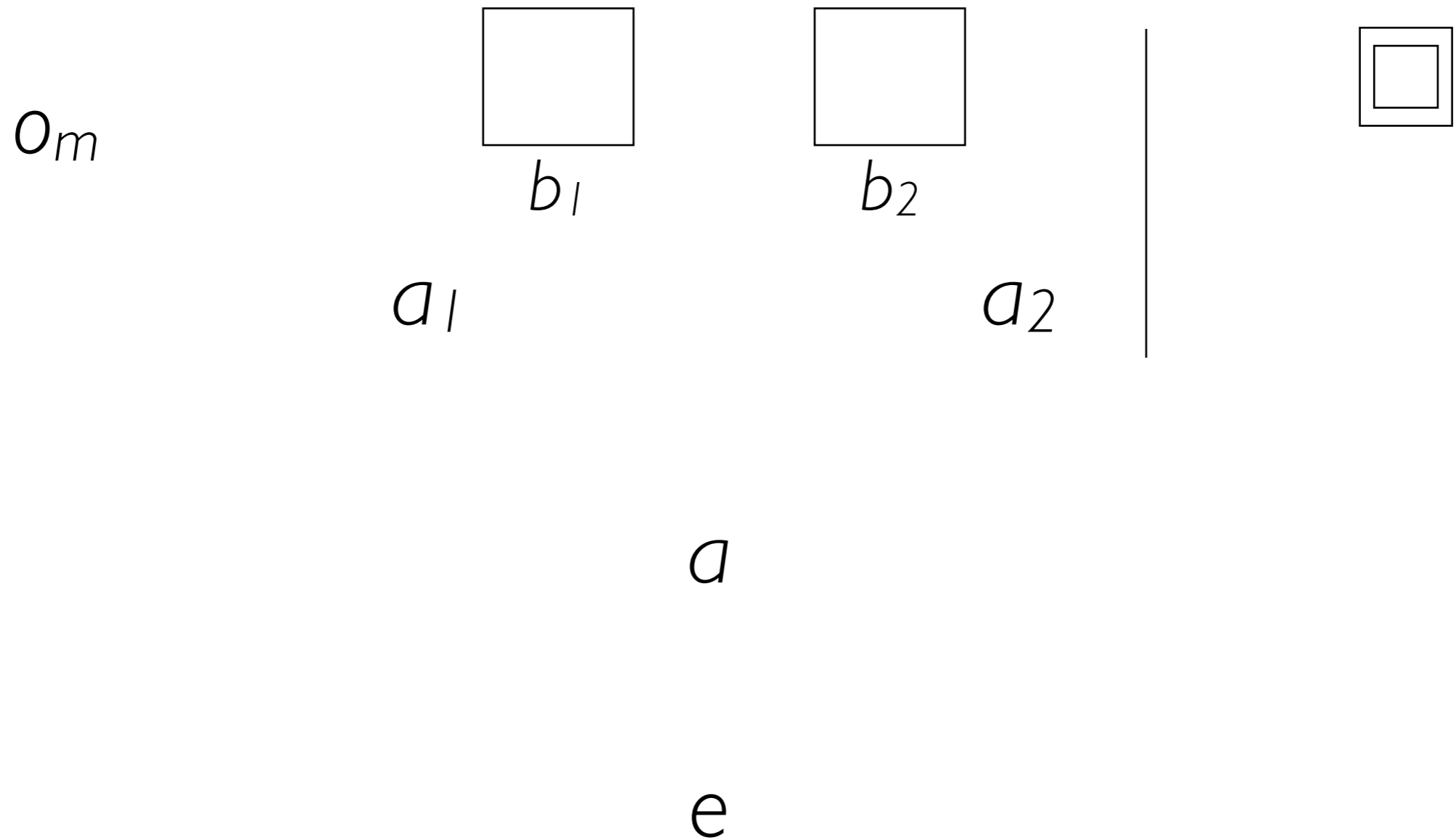
### 1 Introduction

Interpreting the behavior of other people is indispensable for everyday life. It is something that we do constantly, on a daily basis, and it helps us not only to make sense of human behavior, but also to predict it and—to a certain extent—to control it. How exactly do we manage that? That is not currently known, but many have argued that the ability to ascribe mental states to others and to reason about such mental states is a key component of our capacity to understand human behavior. In particular, all social transactions, from engaging in commerce and negotiating to making jokes and empathizing with other people’s pain or joy, appear to require at least a rudimentary grasp of common-sense psychology (CSP), i.e., a large body of truisms such as the following: When an agent  $a$  (1) wants to achieve a certain state of affairs  $p$ , and (2) believes that some action  $c$  can bring about  $p$ , and (3)  $a$  knows how to carry out  $c$ ; then, *ceteris paribus*,<sup>1</sup>  $a$  will carry out  $c$ ; when  $a$  sees that  $p$ ,  $a$  knows that  $p$ ; when  $a$  fears that  $p$  and  $a$  discovers that  $p$  is the case,  $a$  is disappointed; and so on.

Artificial agents without a mastery of CSP would be severely handicapped in their interactions with humans. This could present problems not only for artificial agents trying to interpret human behavior, but also for artificial agents trying to interpret the behavior of one another. When a system exhibits a complex but rational behavior, and detailed knowledge of its internal structure is not

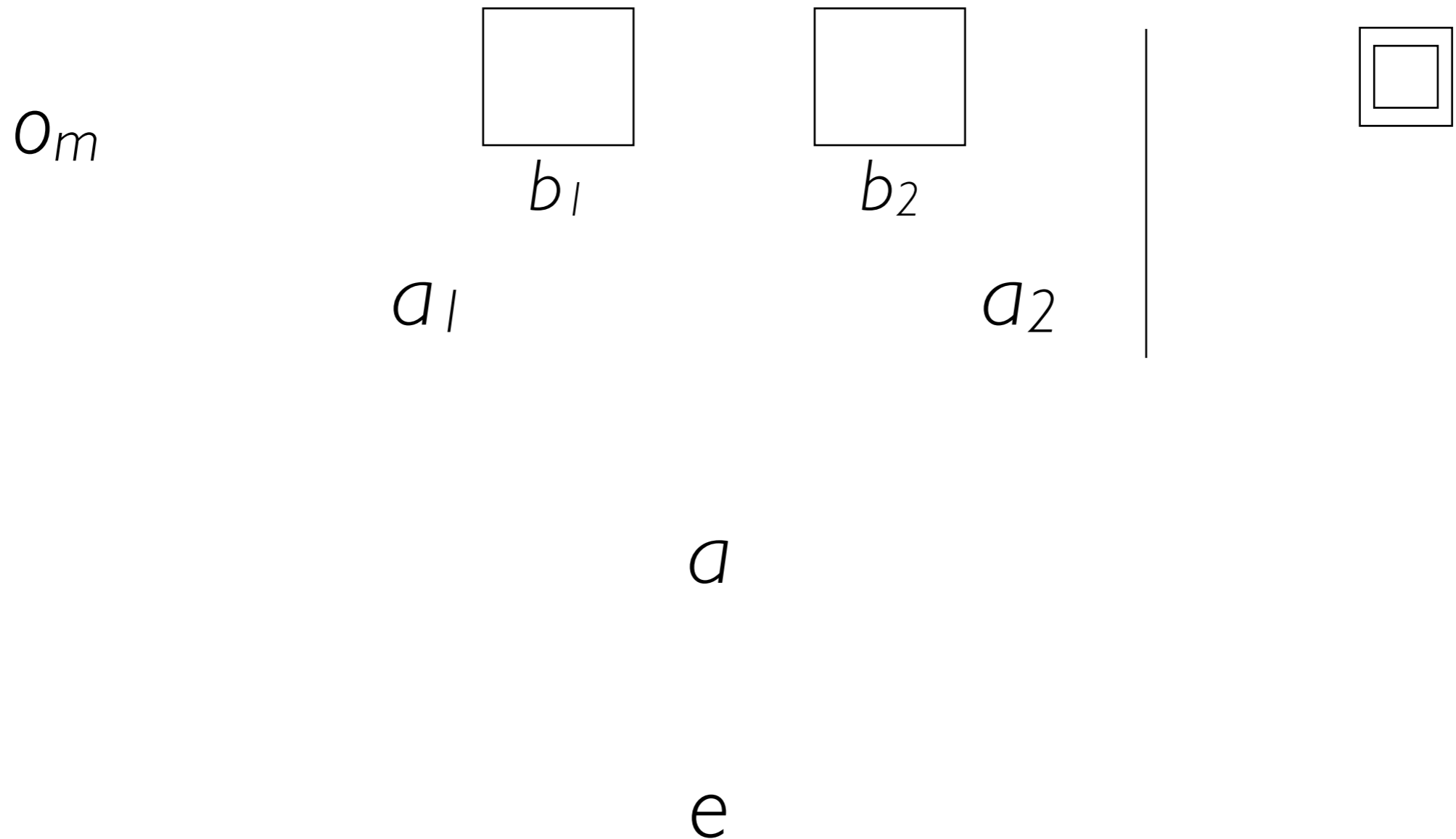
<sup>1</sup> Assuming that  $a$  is able to carry out  $c$ , that  $a$  has no conflicting desires that override his goal that  $p$ ; and so on.

# Framework for FBT<sub>2</sub>



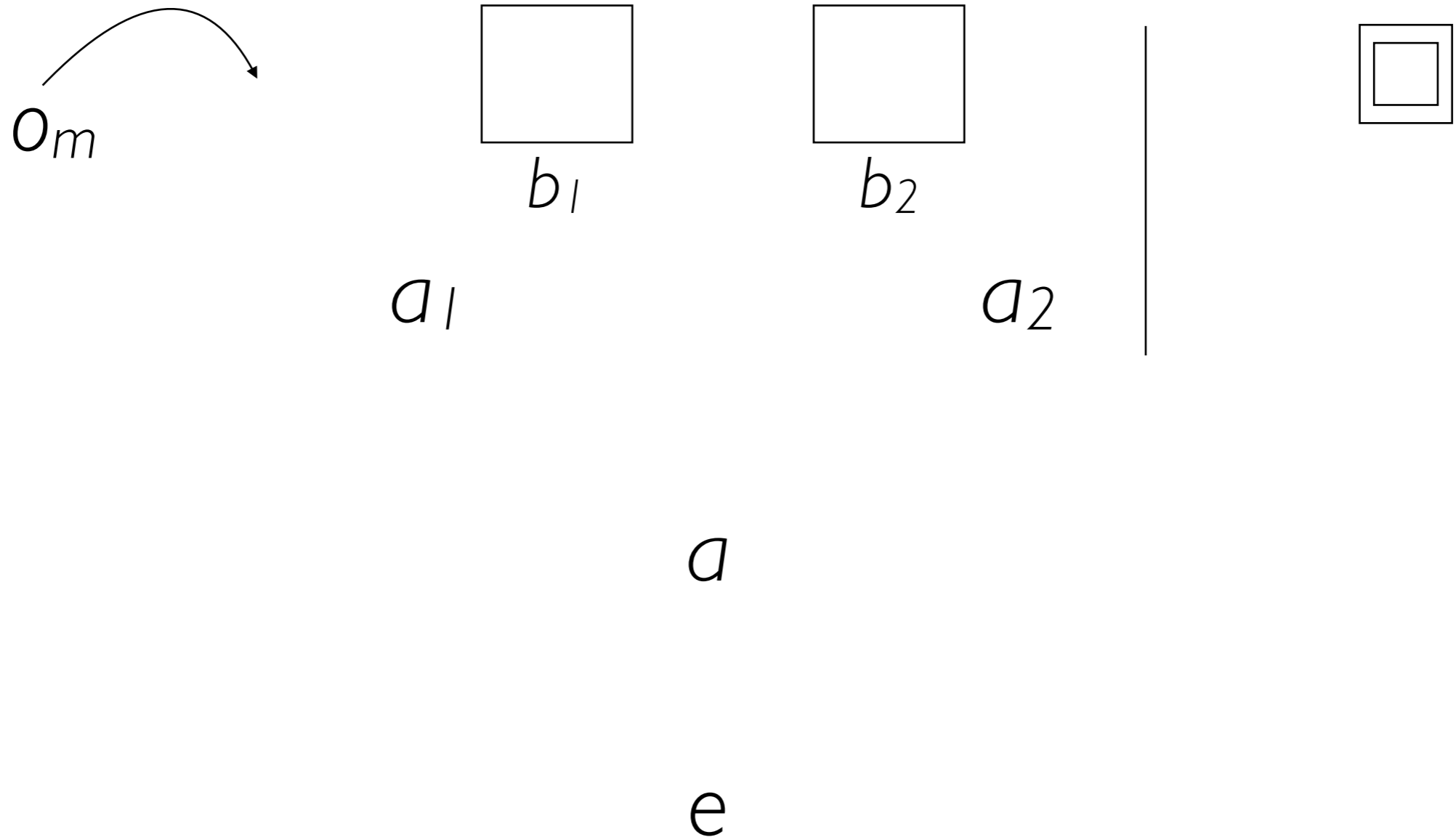
# Framework for $\text{FBT}^1_2$

(seven timepoints)



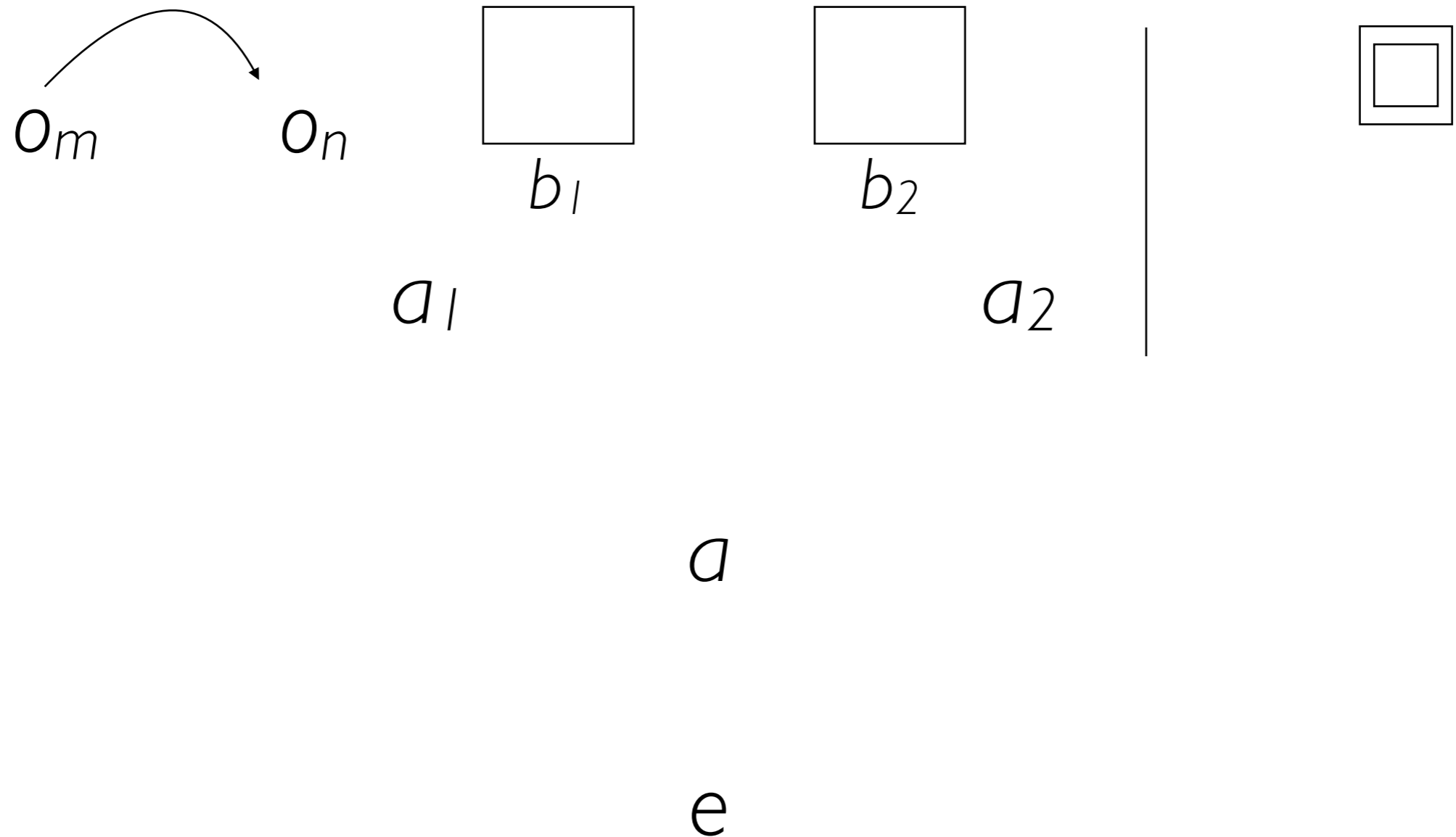
# Framework for FBT<sub>2</sub>

(seven timepoints)



# Framework for FBT<sub>2</sub>

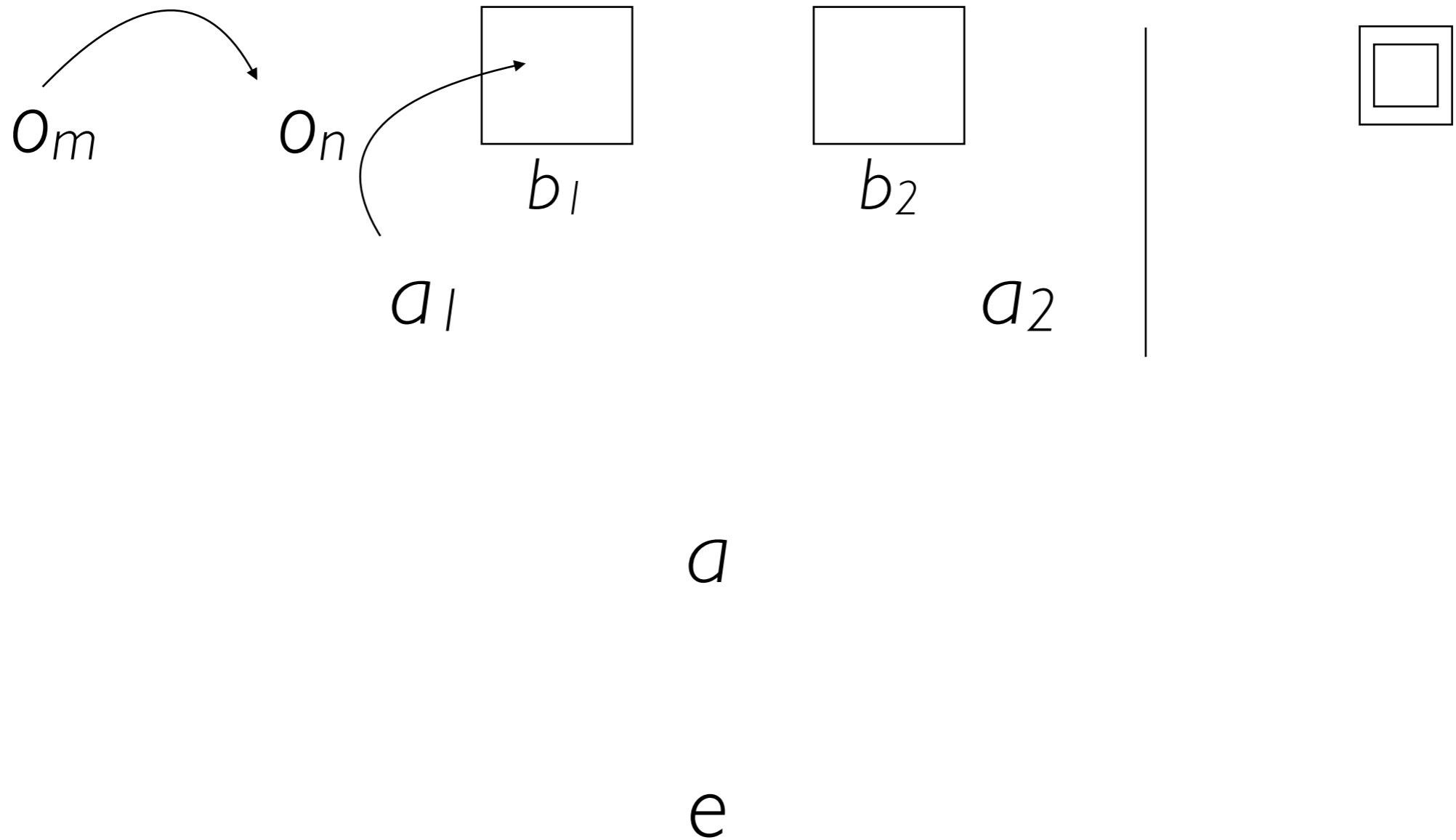
(seven timepoints)





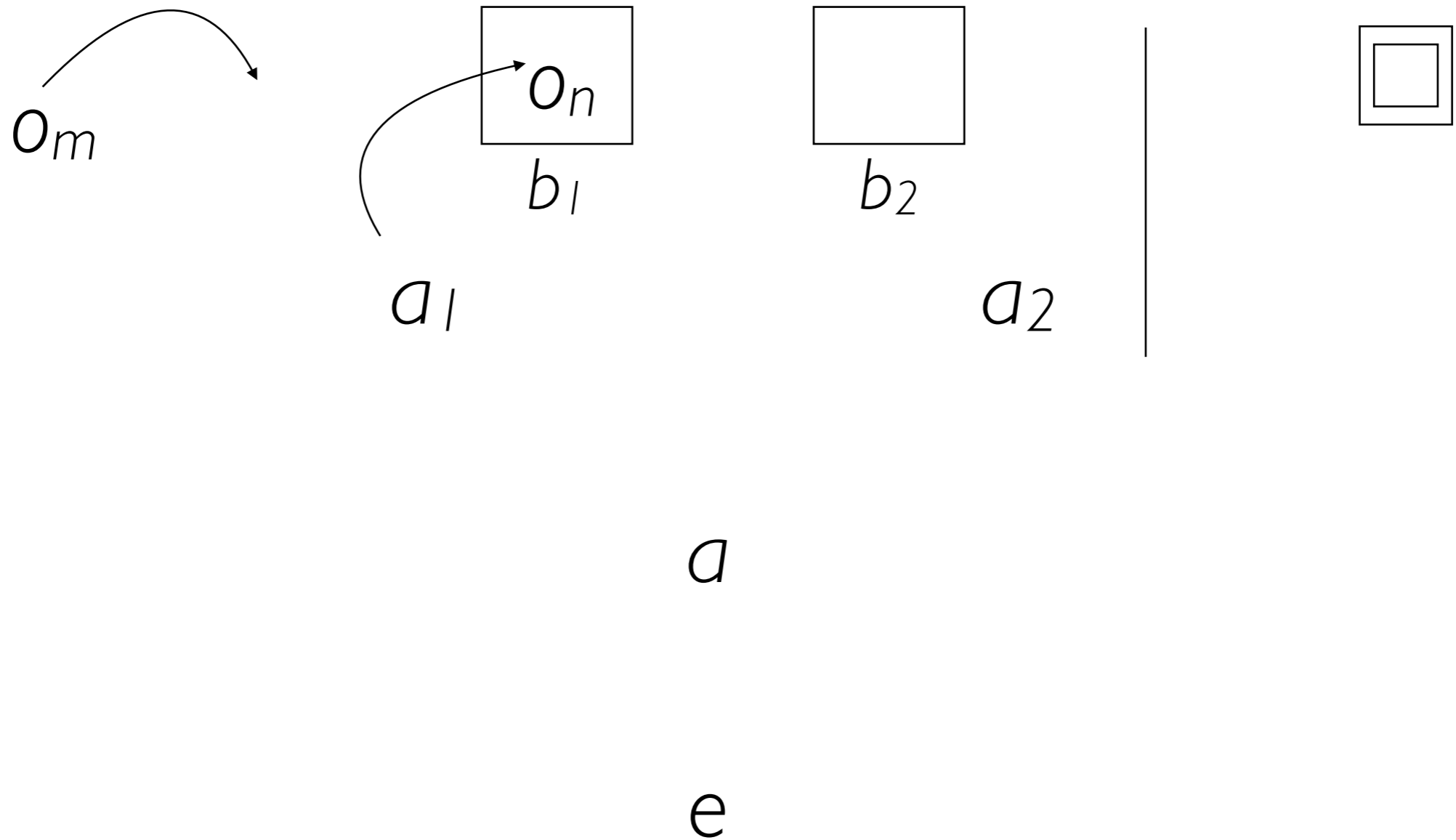
# Framework for $\text{FBT}^1_2$

(seven timepoints)



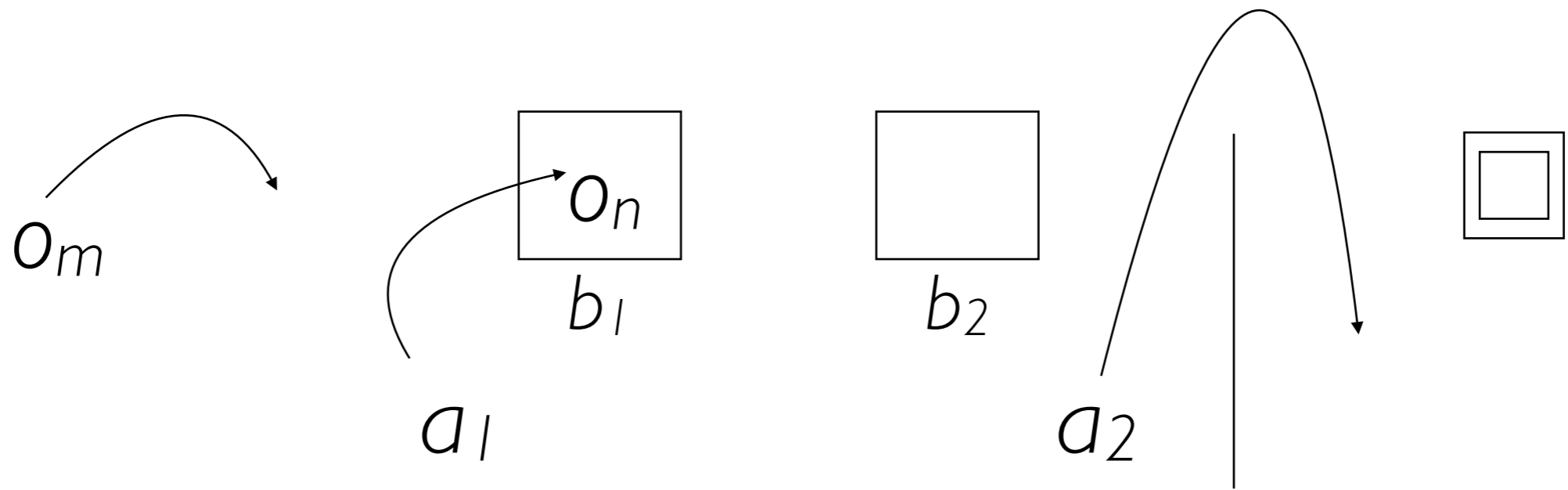
# Framework for $\text{FBT}^1_2$

(seven timepoints)



# Framework for $\text{FBT}^1_2$

(seven timepoints)

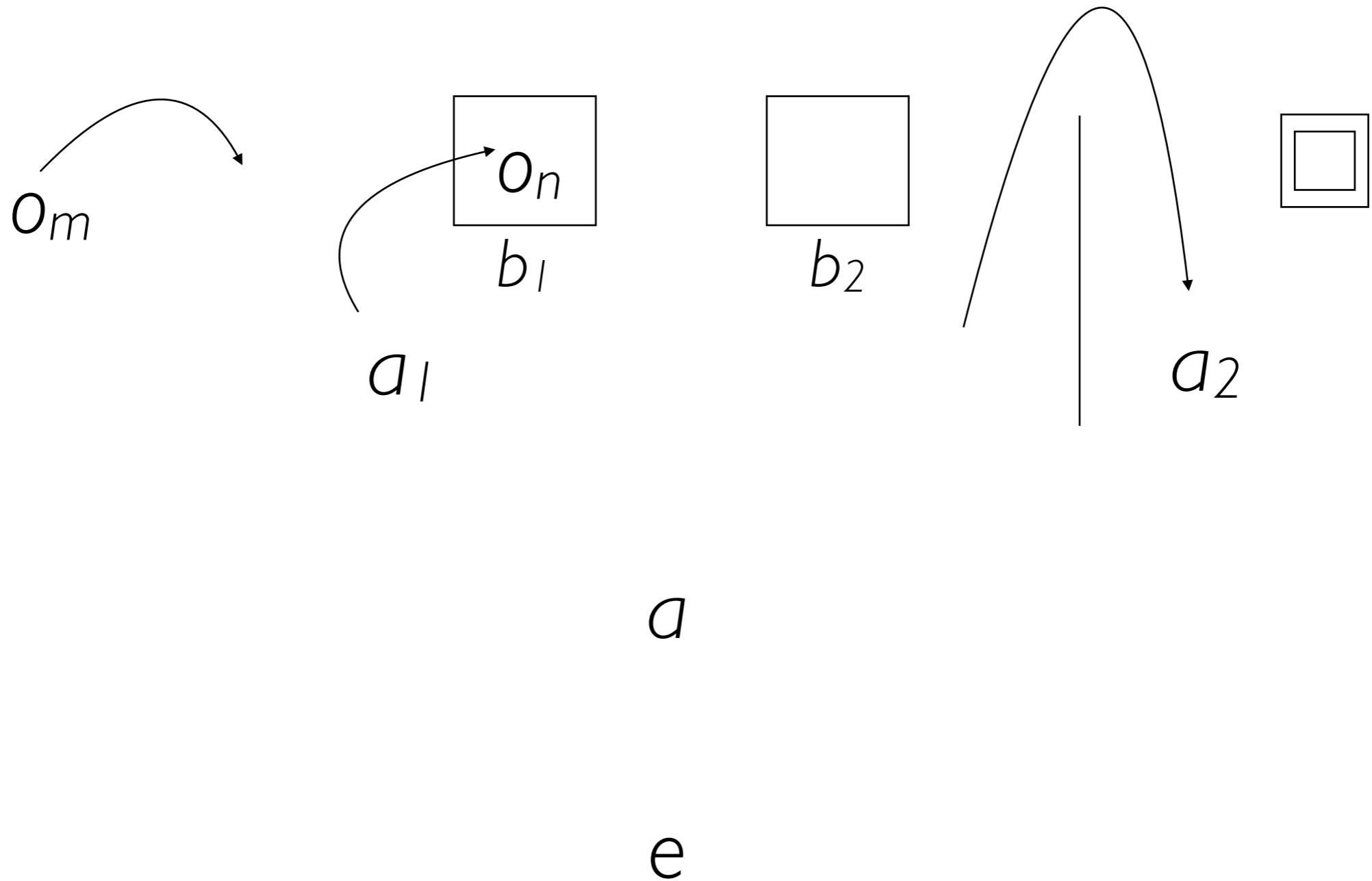


$a$

$e$

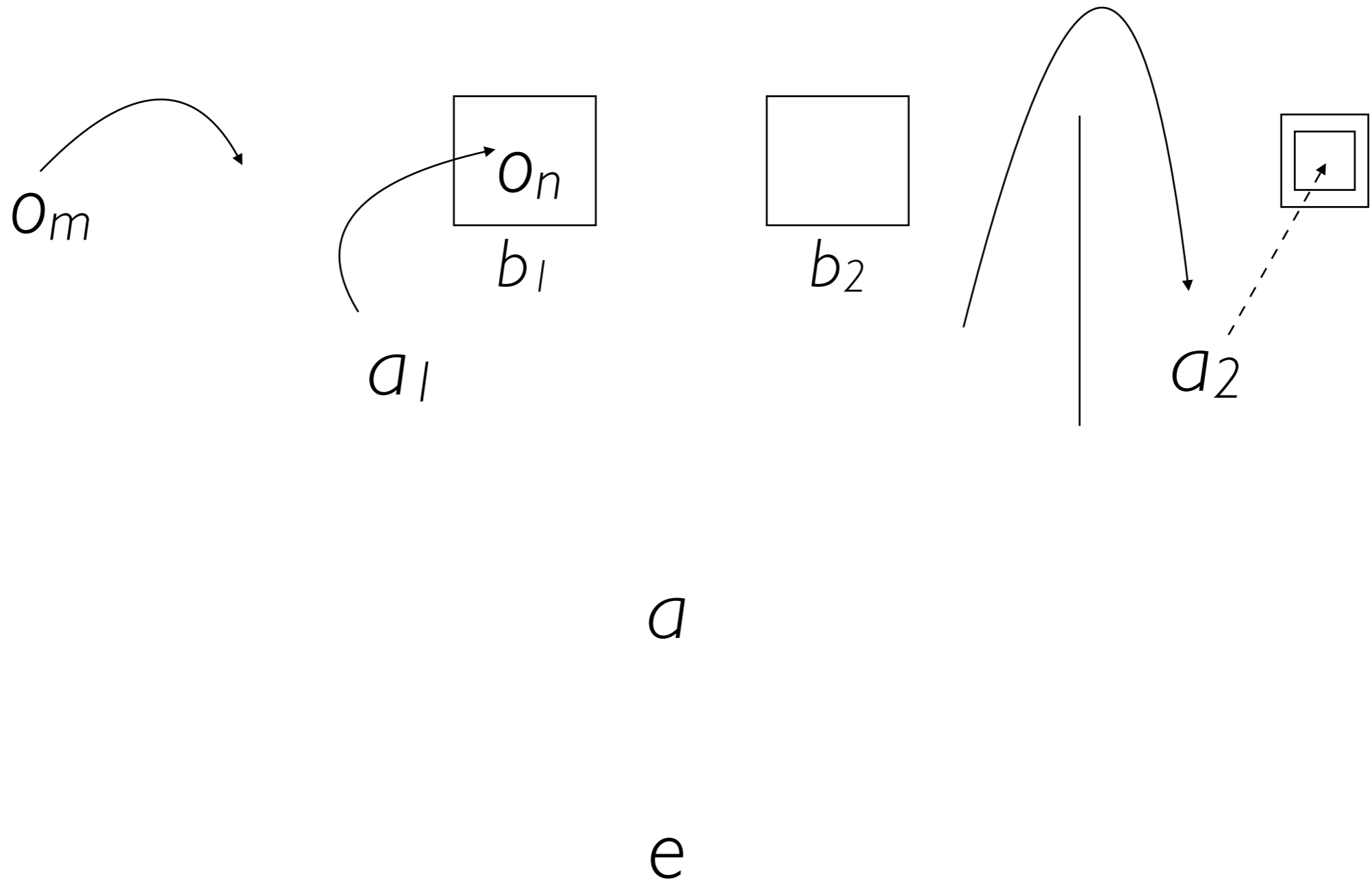
# Framework for $\text{FBT}^1_2$

(seven timepoints)



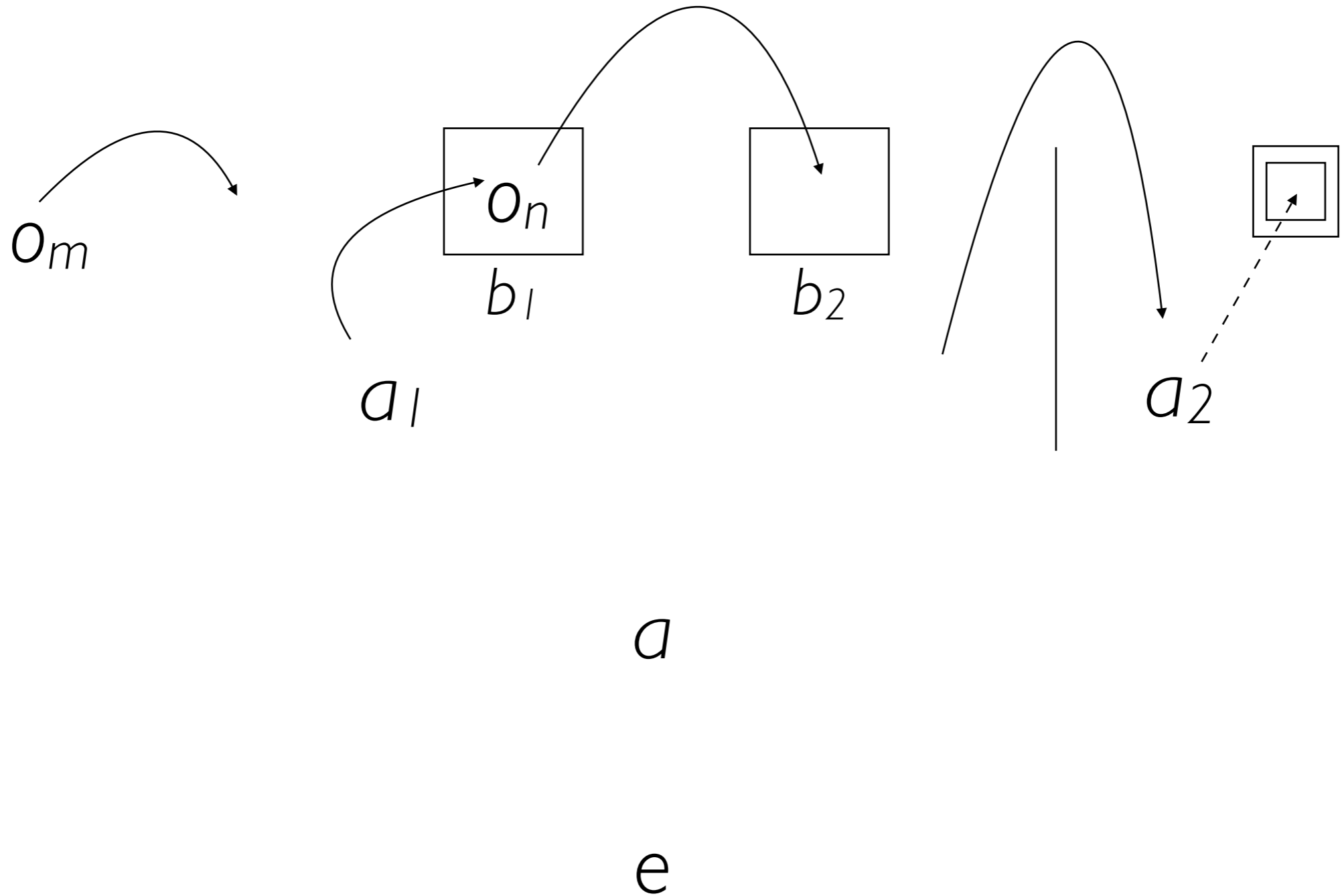
# Framework for $\text{FBT}^1_2$

(seven timepoints)



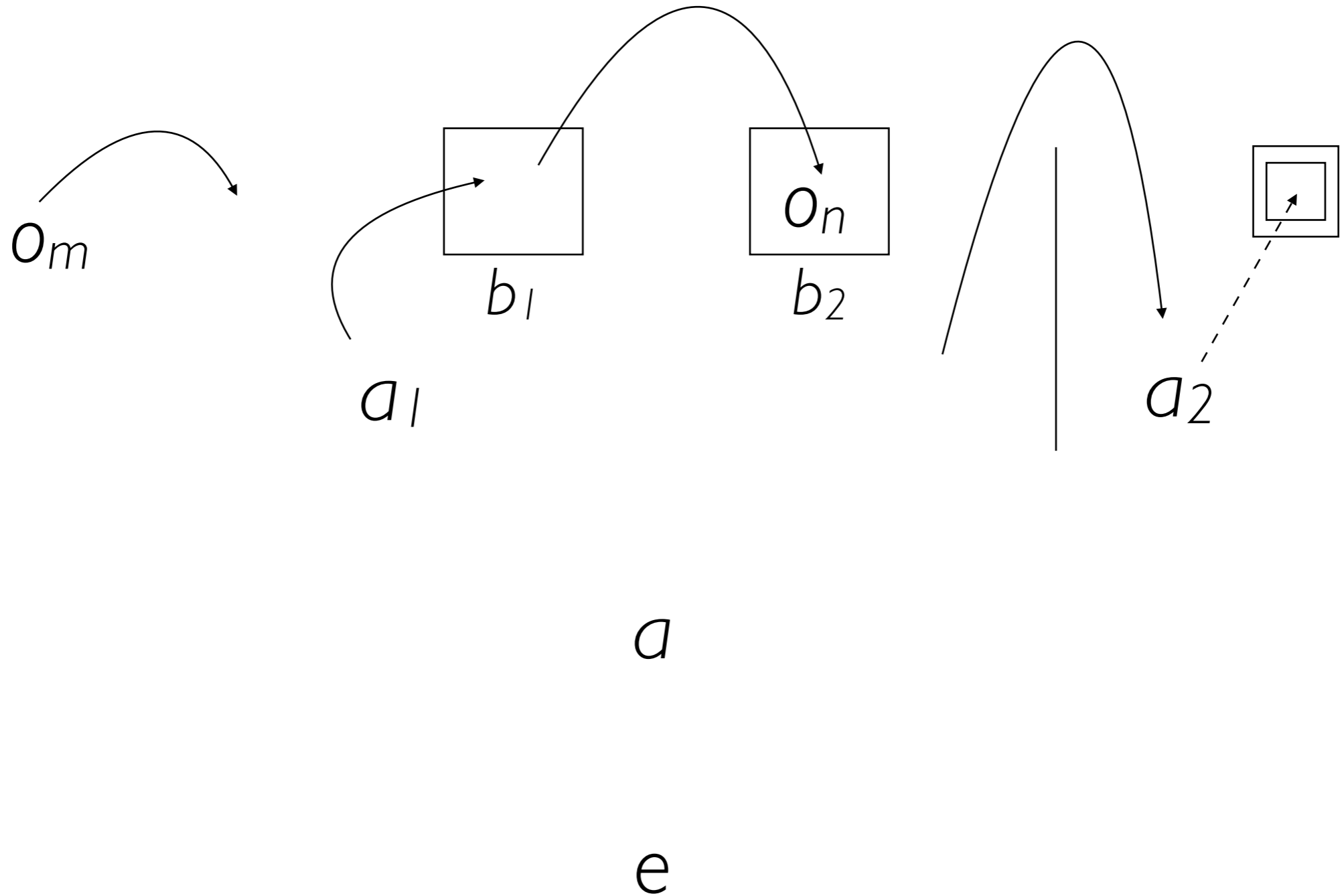
# Framework for $\text{FBT}^1_2$

(seven timepoints)



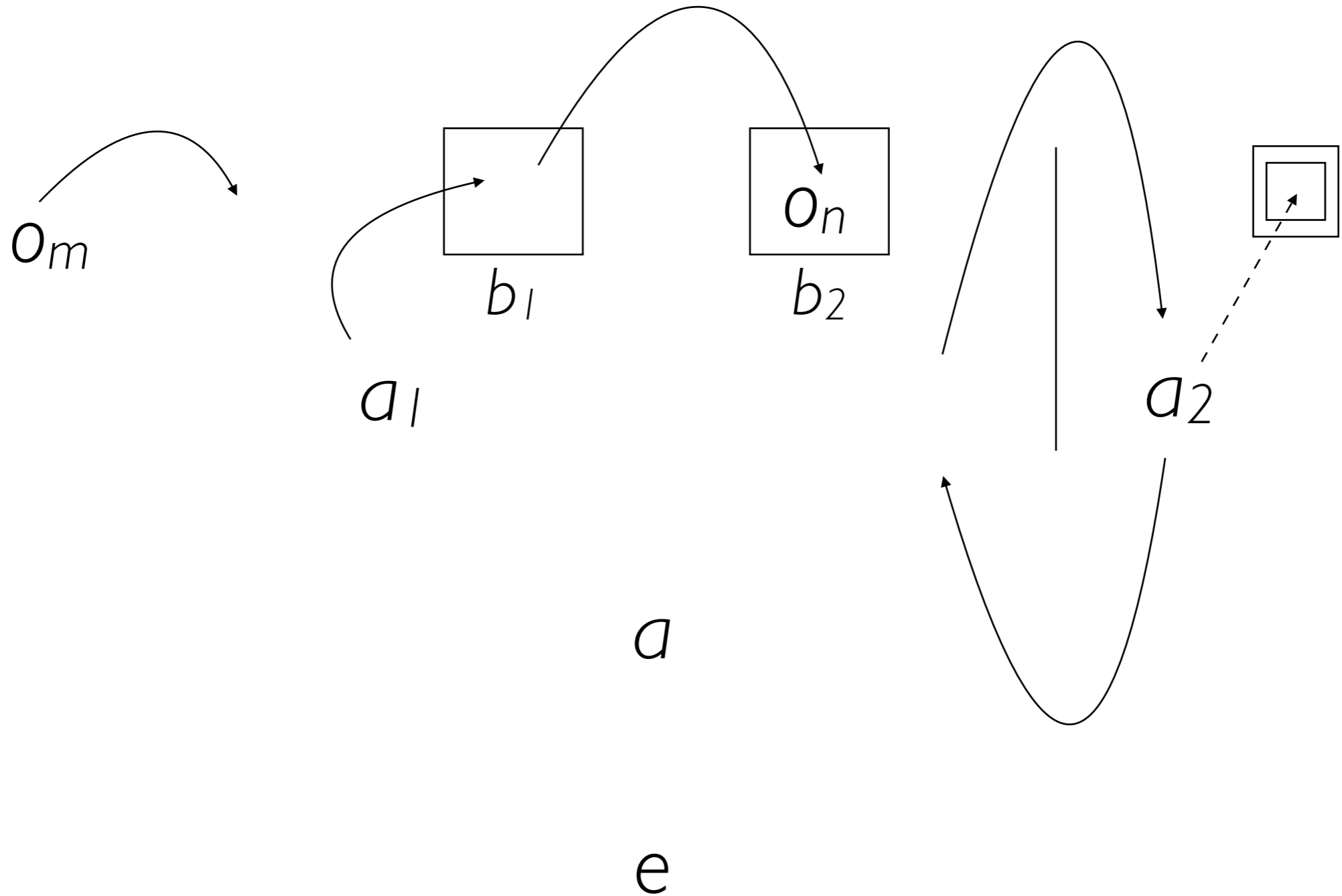
# Framework for $\text{FBT}^1_2$

(seven timepoints)



# Framework for $\text{FBT}^1_2$

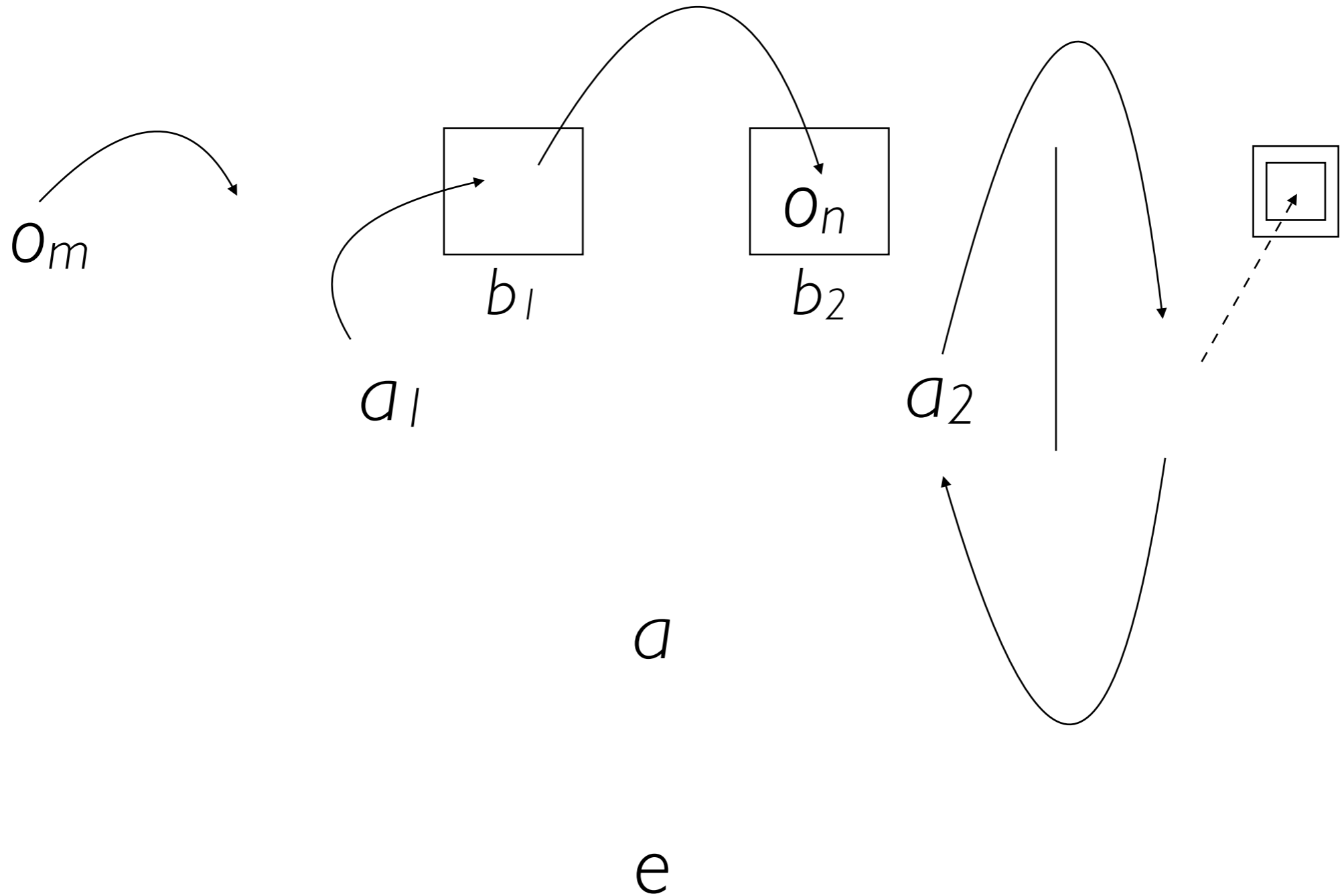
(seven timepoints)





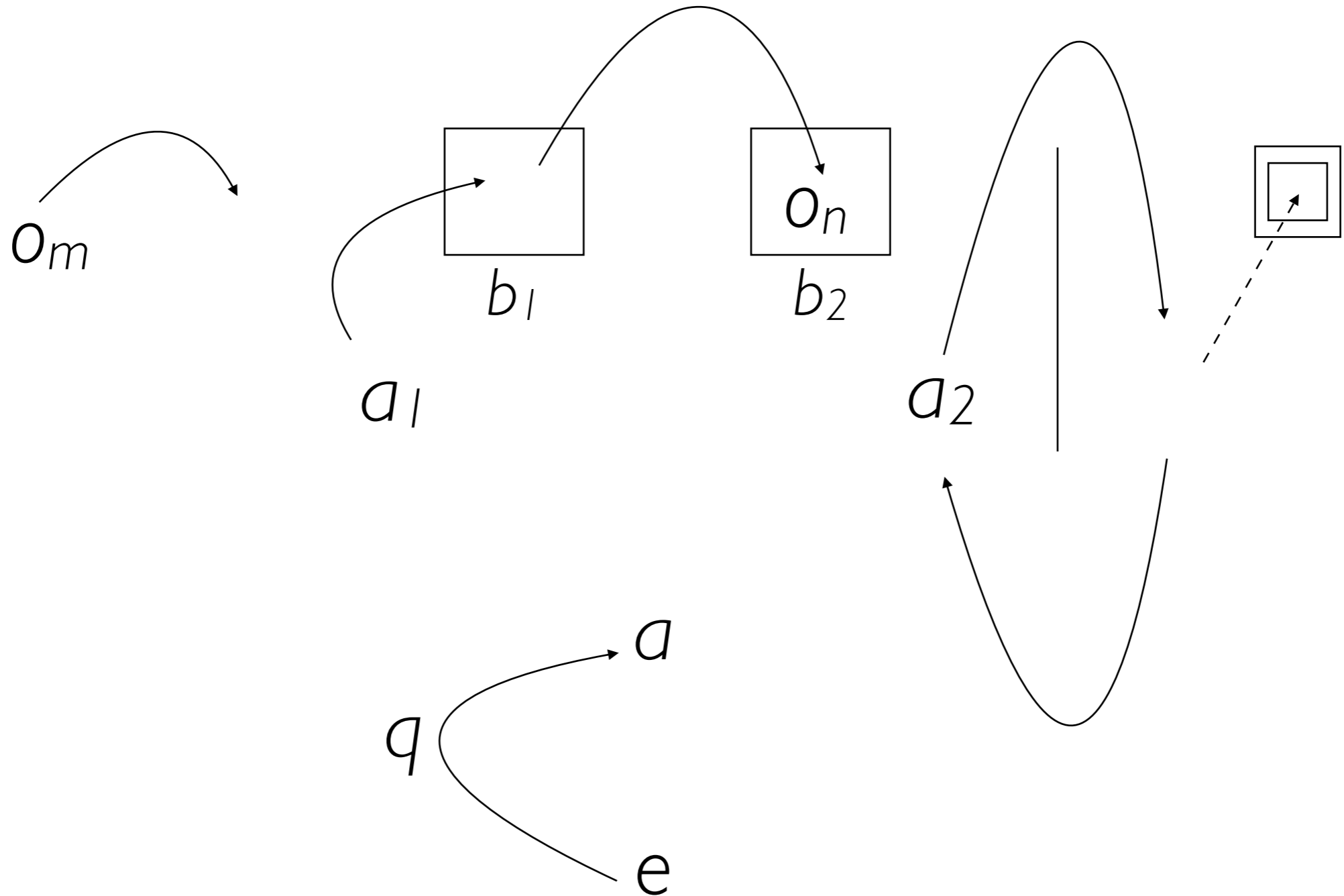
# Framework for $\text{FBT}^1_2$

(seven timepoints)



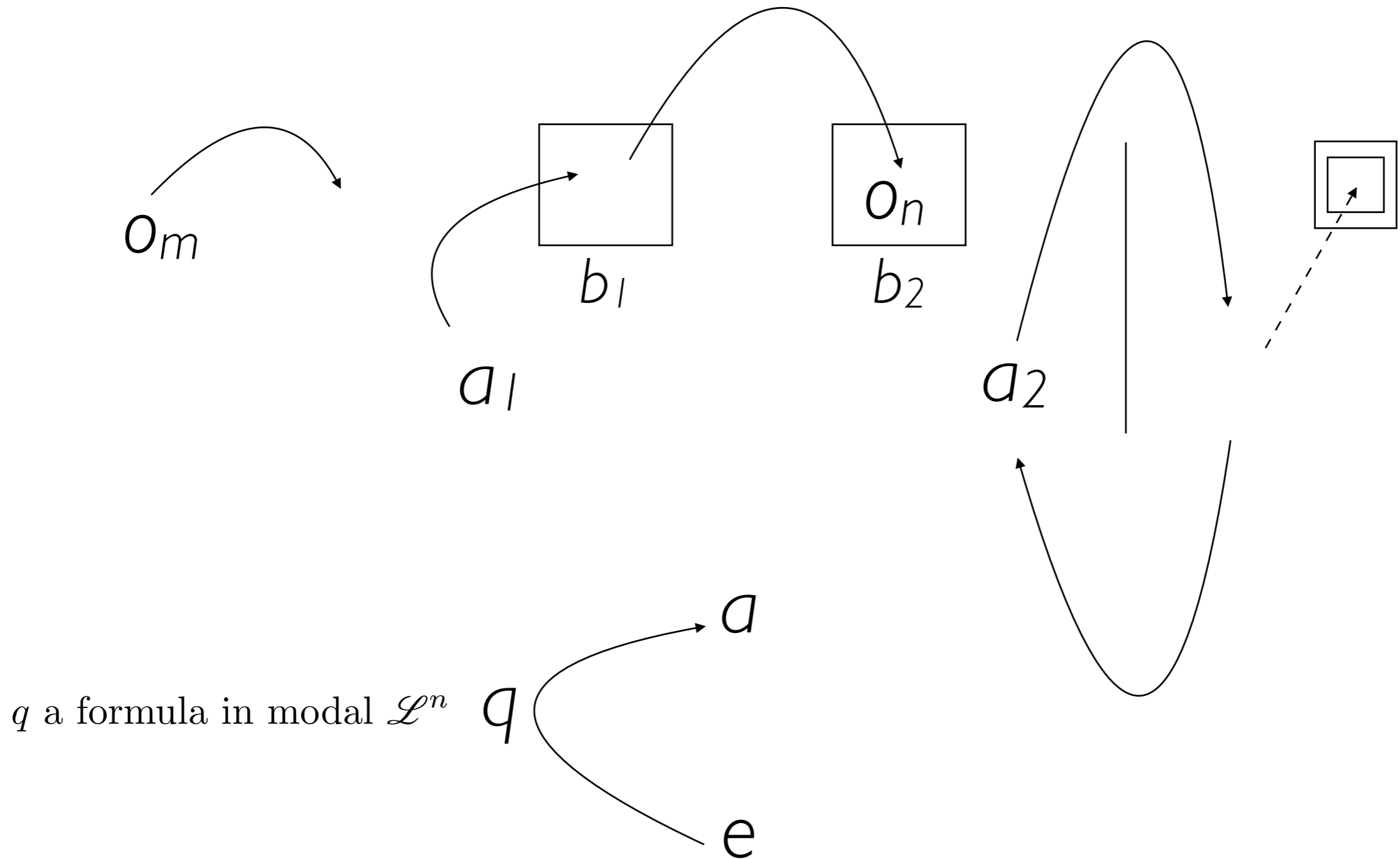
# Framework for $\text{FBT}^1_2$

(seven timepoints)

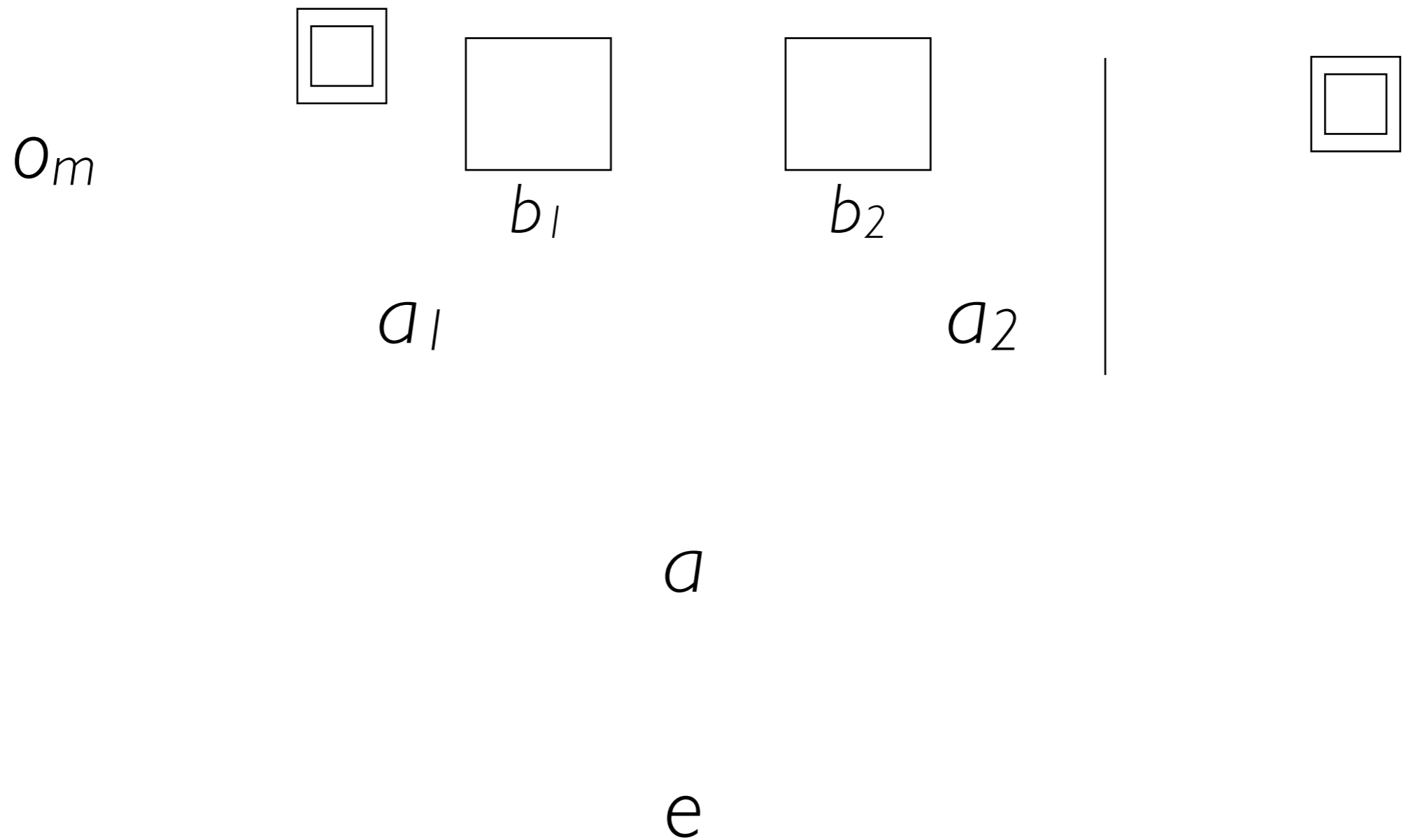


# Framework for $\text{FBT}^1_2$

(seven timepoints)

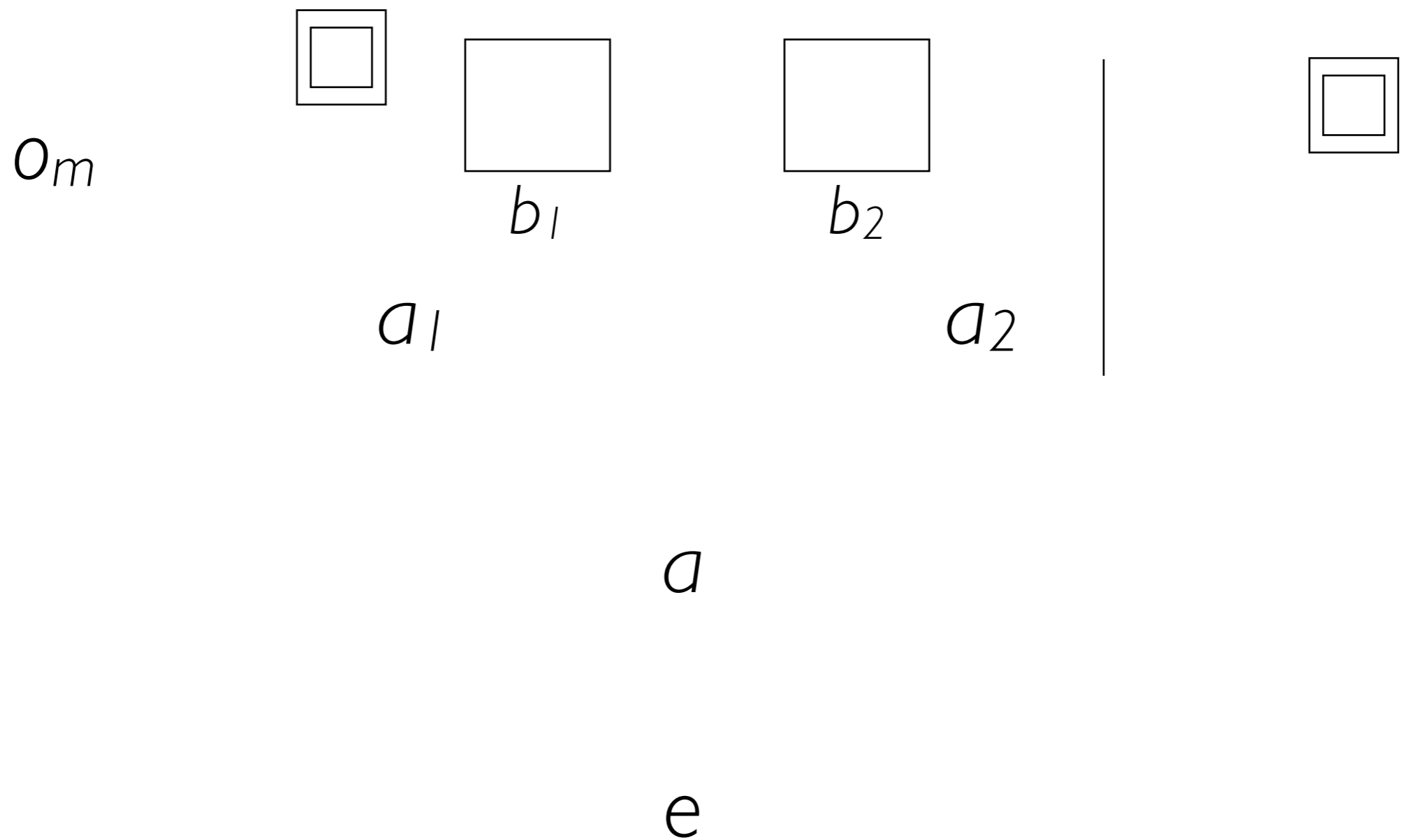


# Framework for $\text{FBT}^1_3$



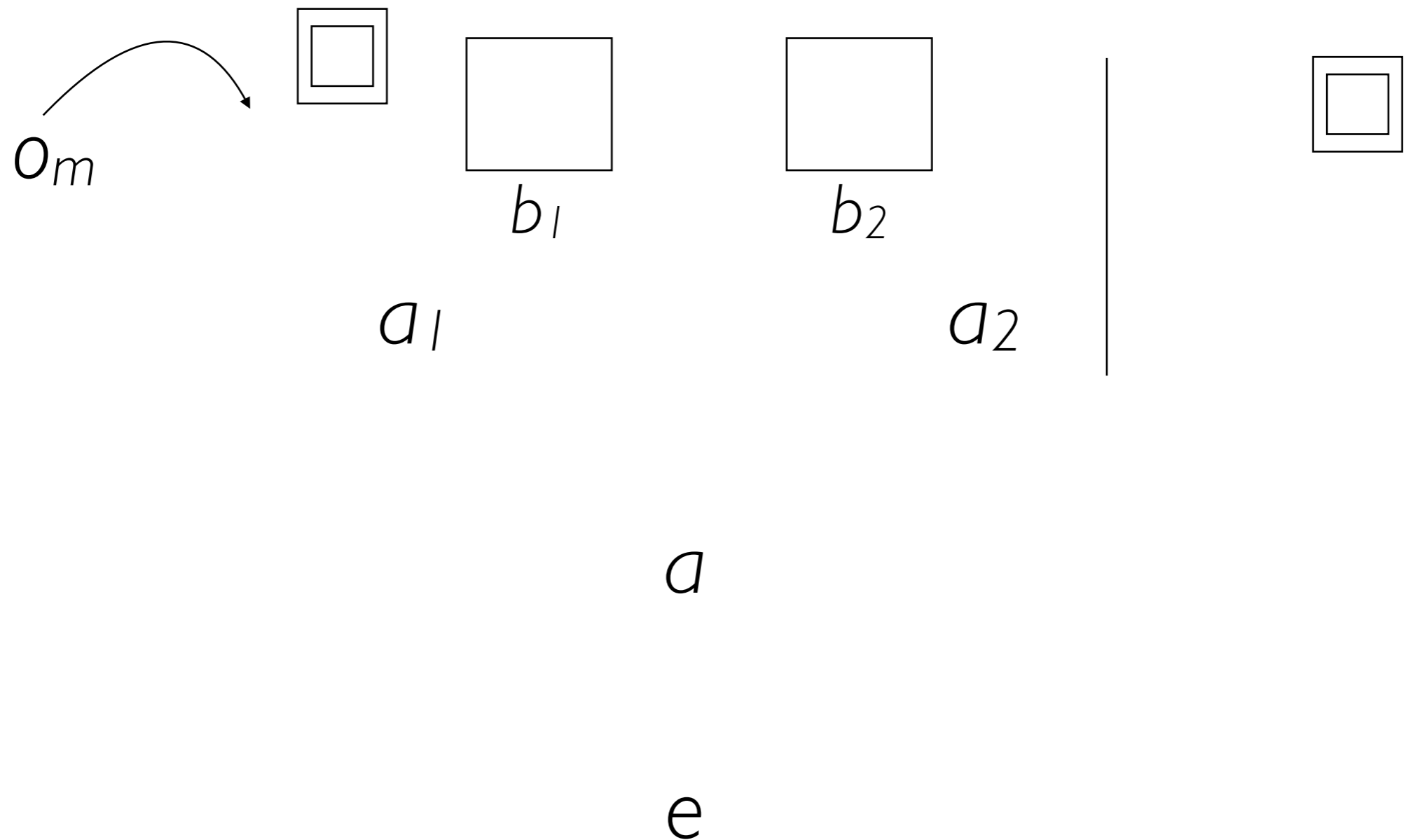
# Framework for $\text{FBT}^1_3$

(eight timepoints)



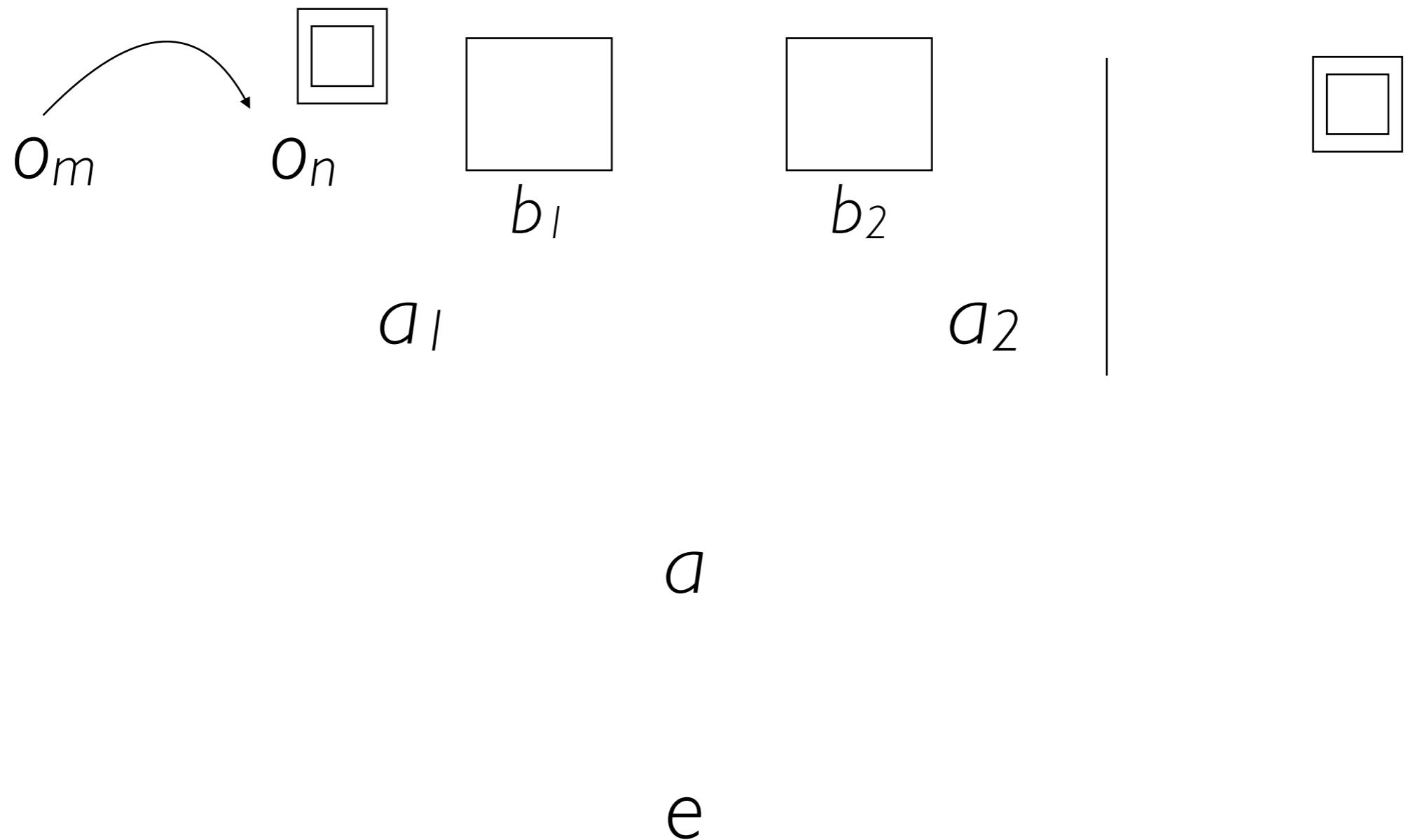
# Framework for $\text{FBT}^1_3$

(eight timepoints)



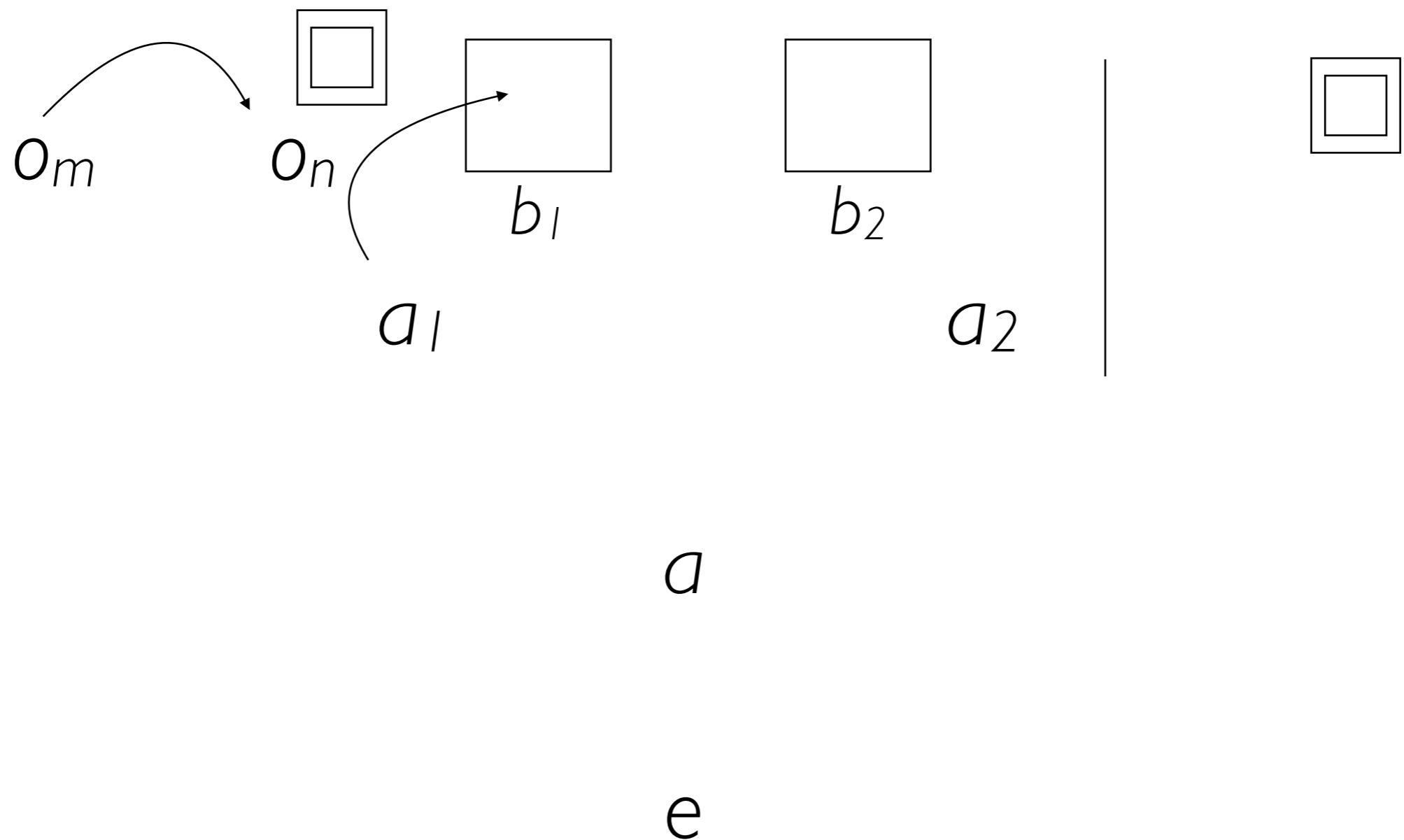
# Framework for $\text{FBT}^1_3$

(eight timepoints)



# Framework for $\text{FBT}^1_3$

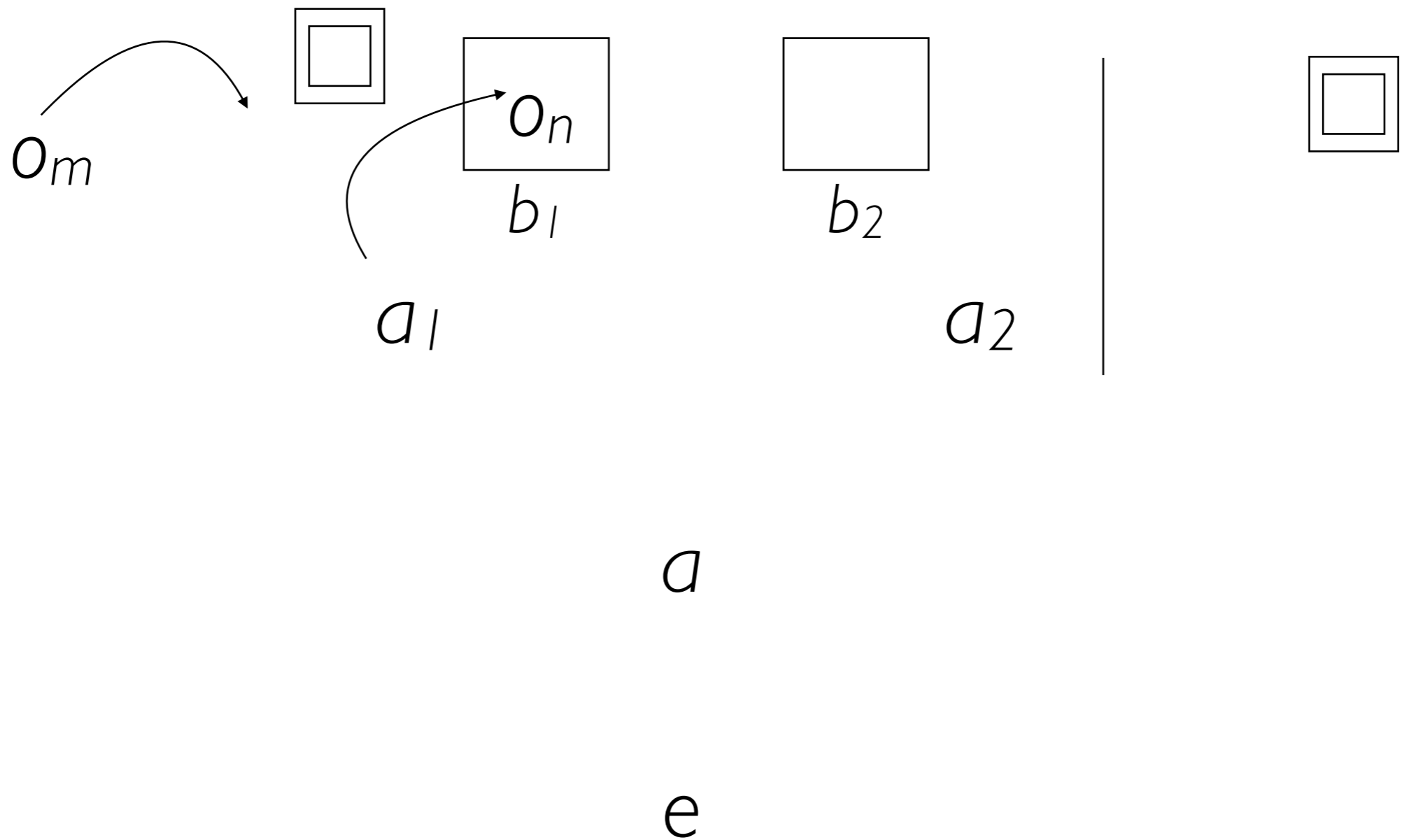
(eight timepoints)





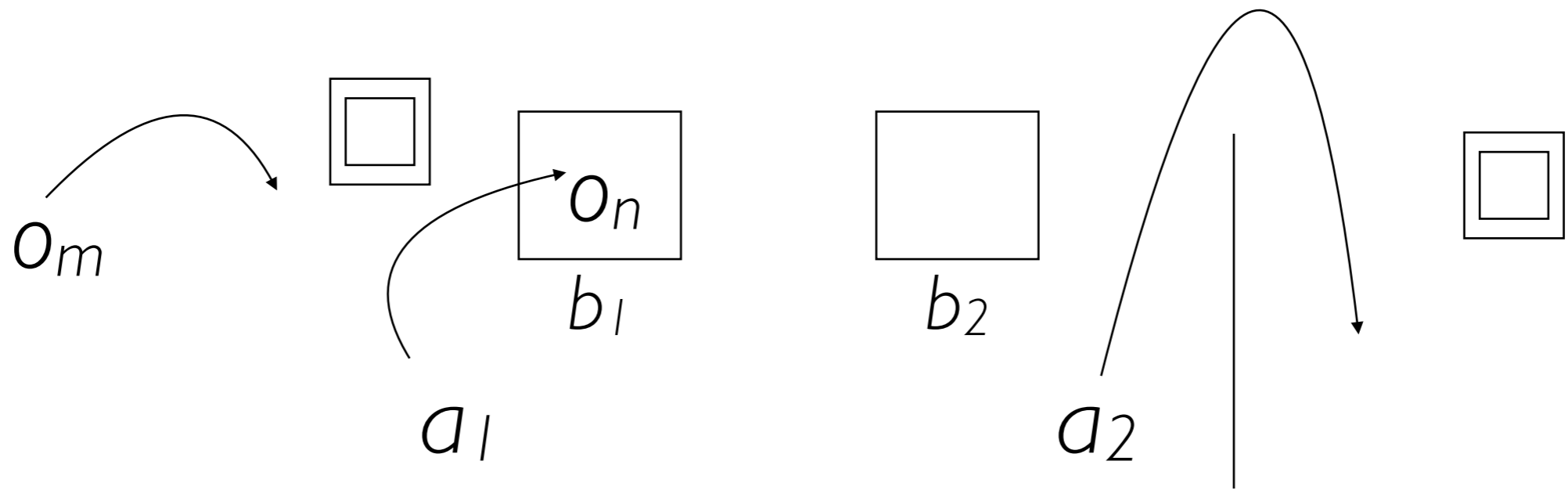
# Framework for $\text{FBT}^1_3$

(eight timepoints)



# Framework for $\text{FBT}^1_3$

(eight timepoints)

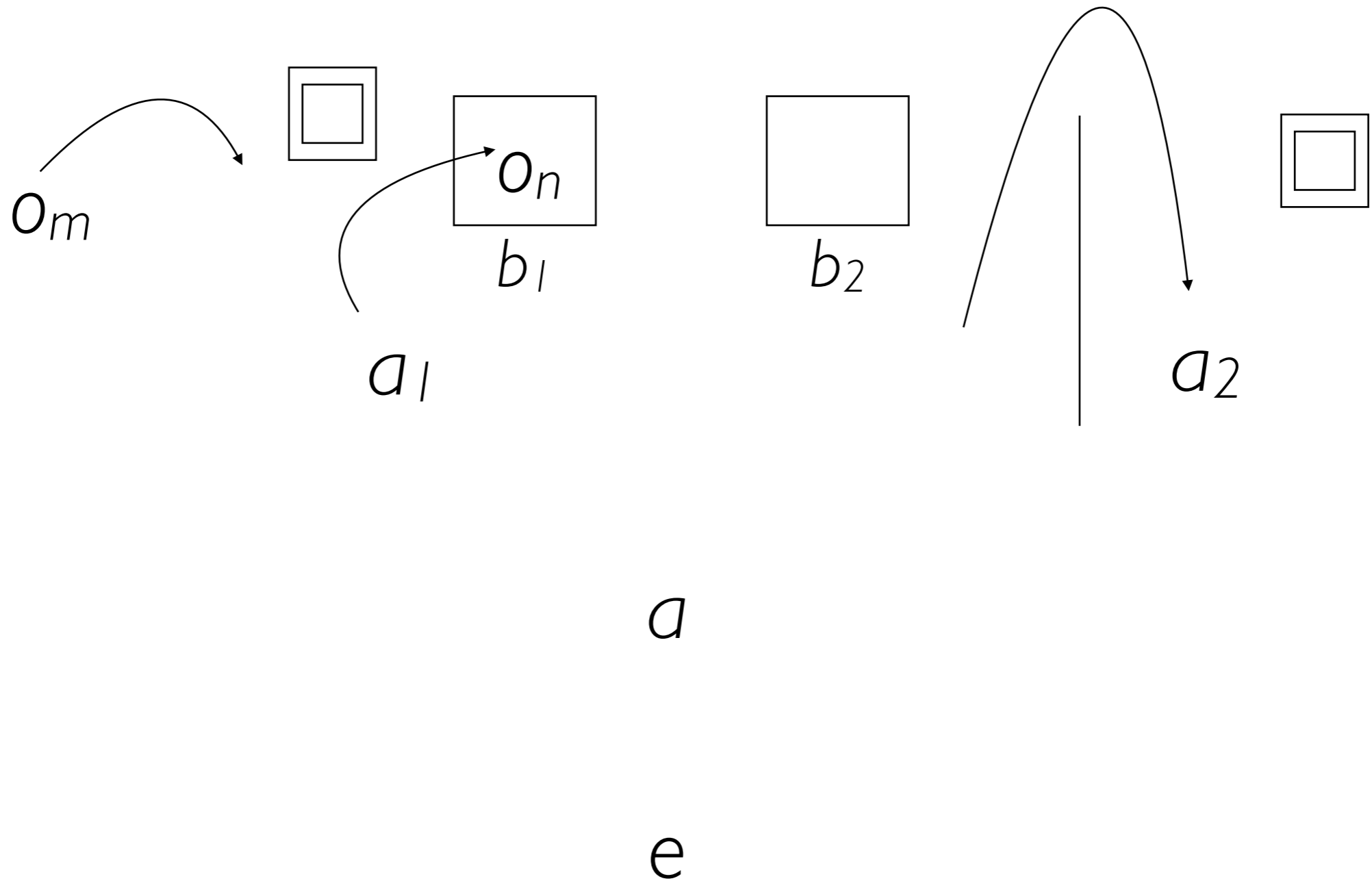


$a$

$e$

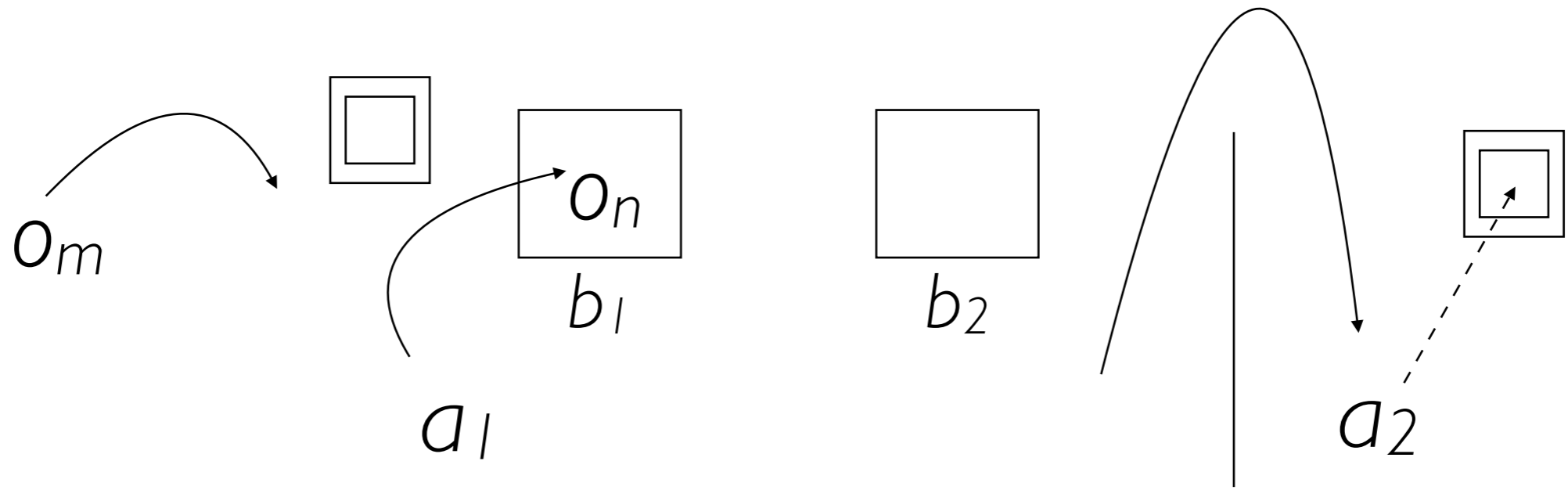
# Framework for $\text{FBT}^1_3$

(eight timepoints)



# Framework for $\text{FBT}^1_3$

(eight timepoints)

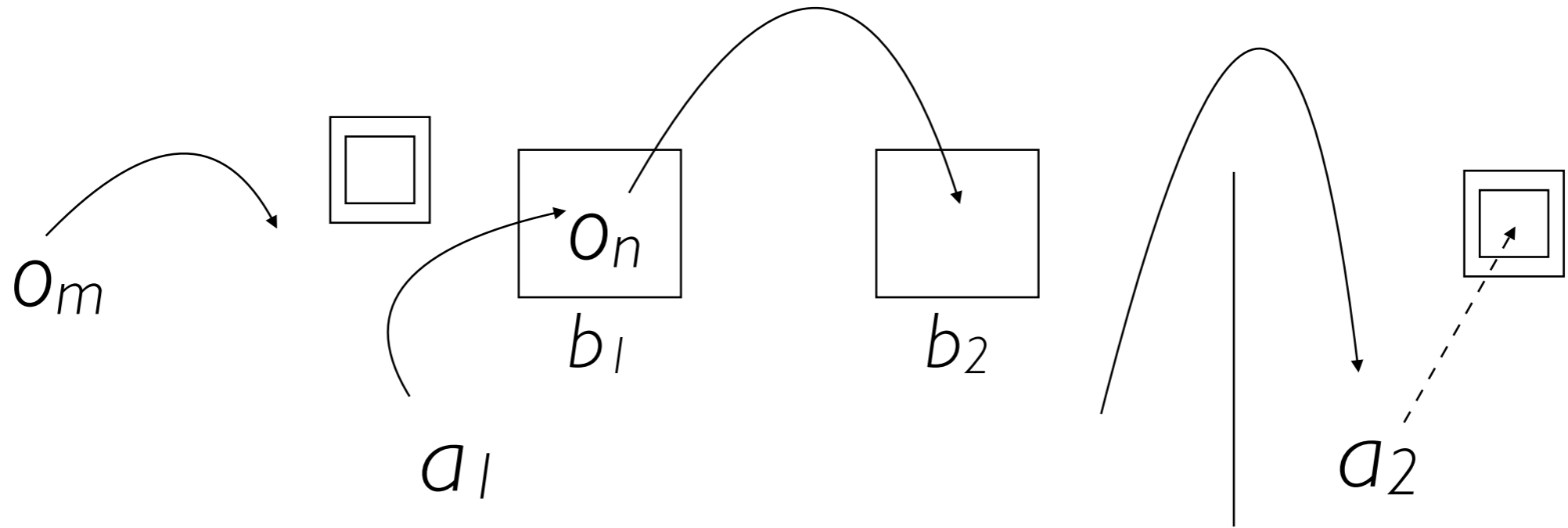


$a$

$e$

# Framework for $\text{FBT}^1_3$

(eight timepoints)

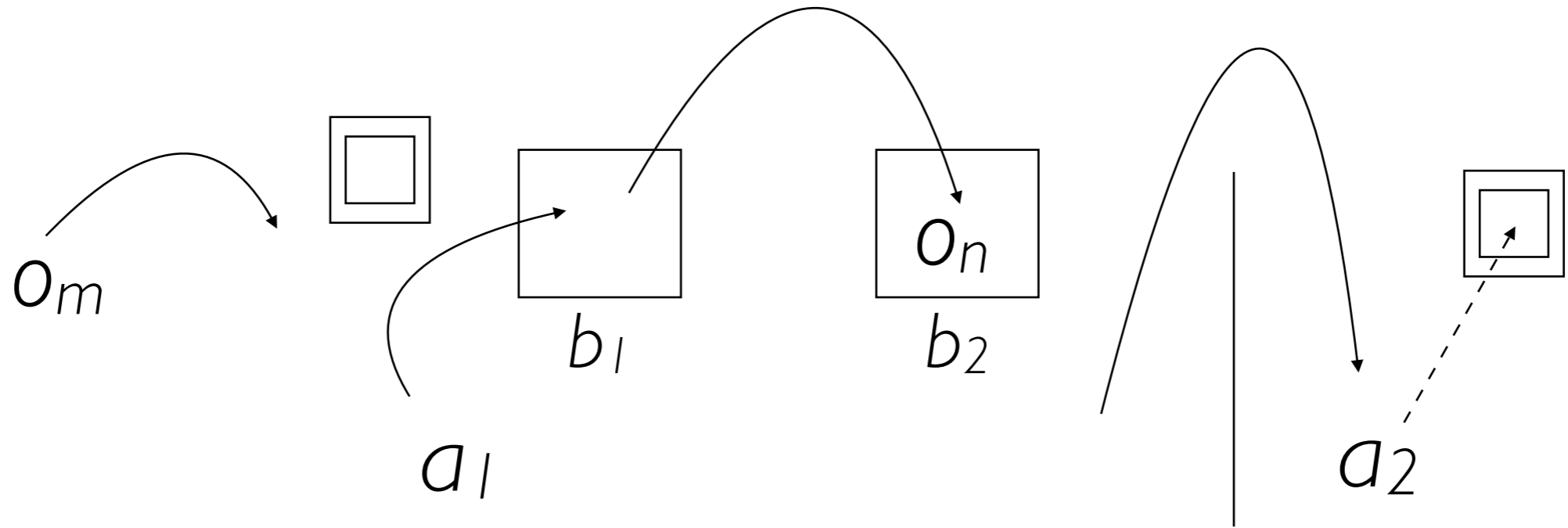


$a$

$e$

# Framework for $\text{FBT}^1_3$

(eight timepoints)

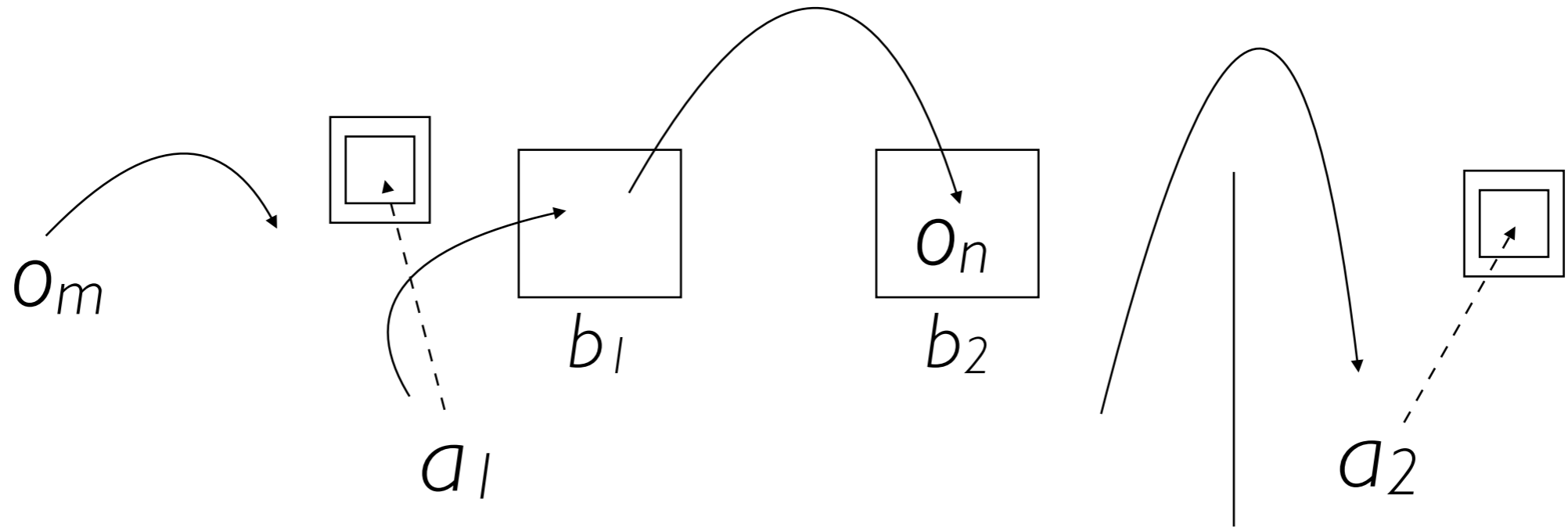


$a$

$e$

# Framework for $\text{FBT}^1_3$

(eight timepoints)

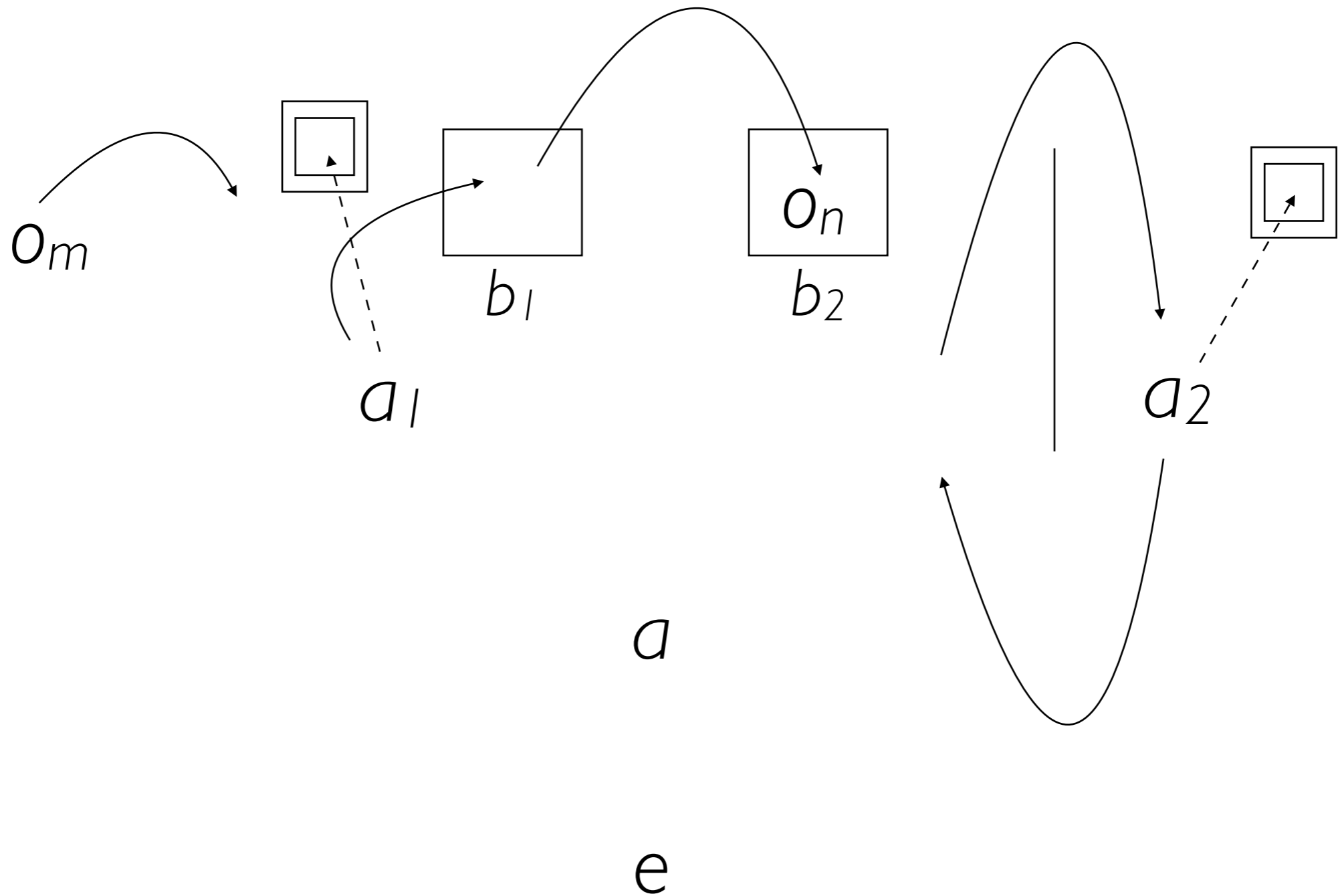


$a$

$e$

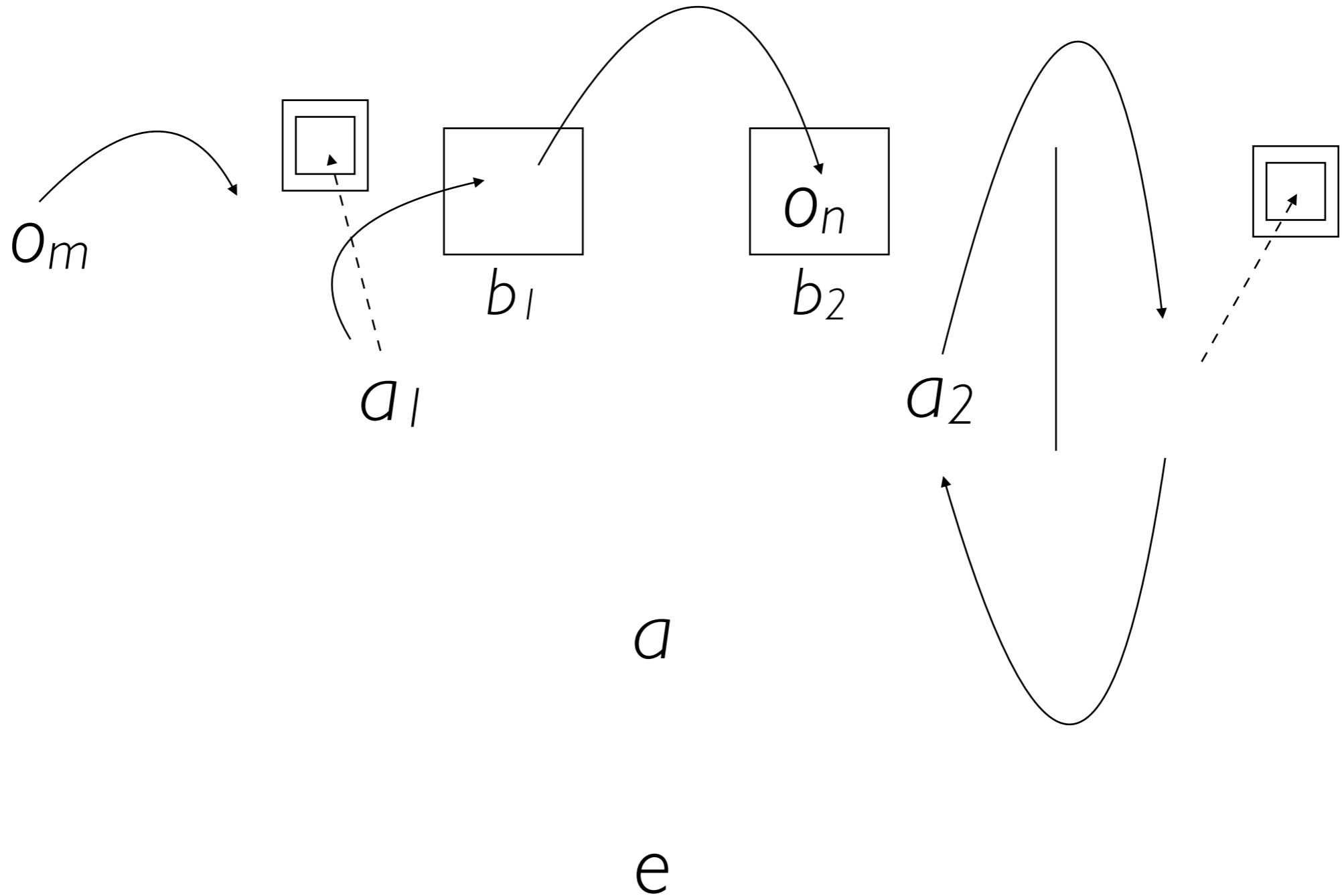
# Framework for $\text{FBT}^1_3$

(eight timepoints)



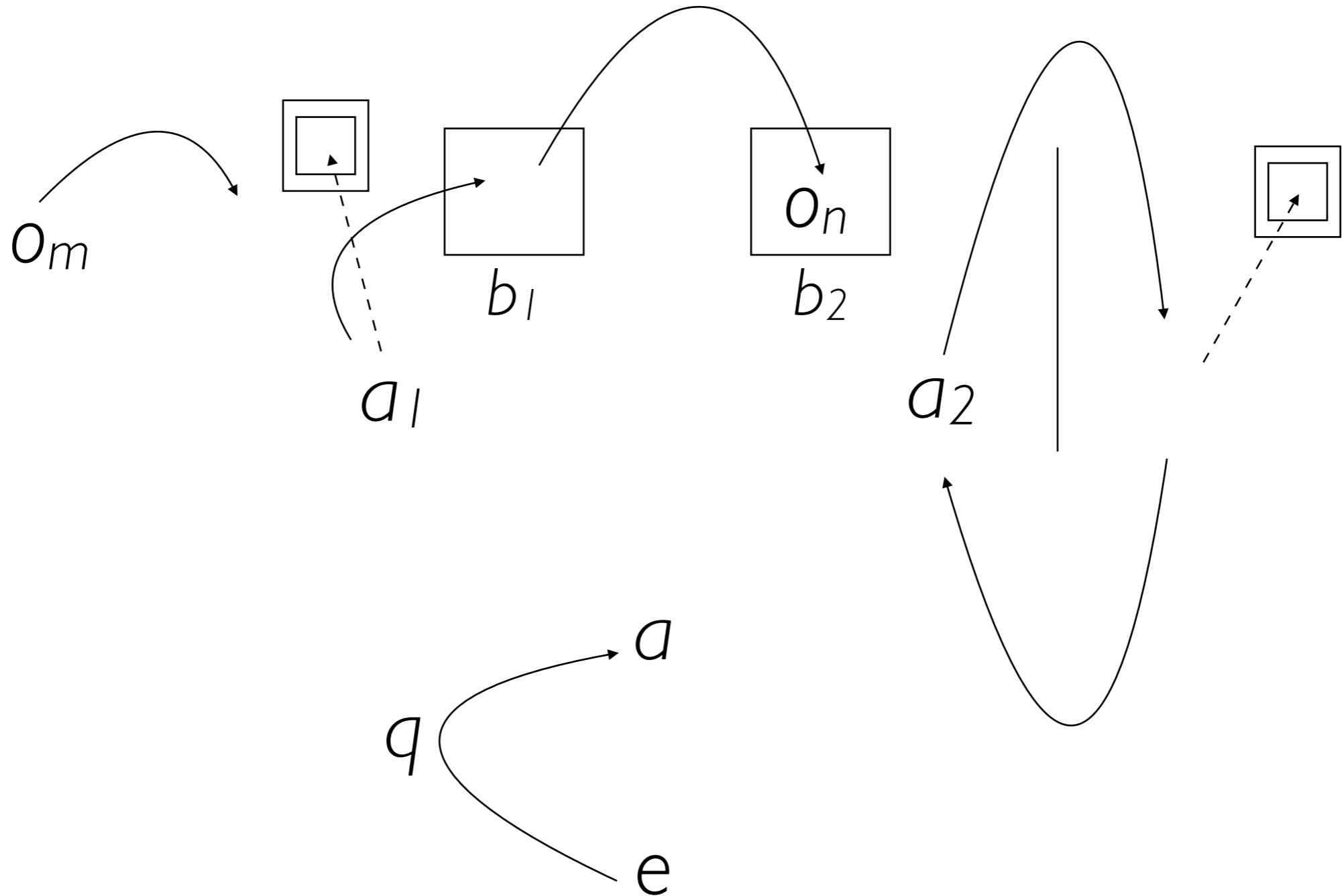


# Framework for $\text{FBT}^1_3$ (eight timepoints)



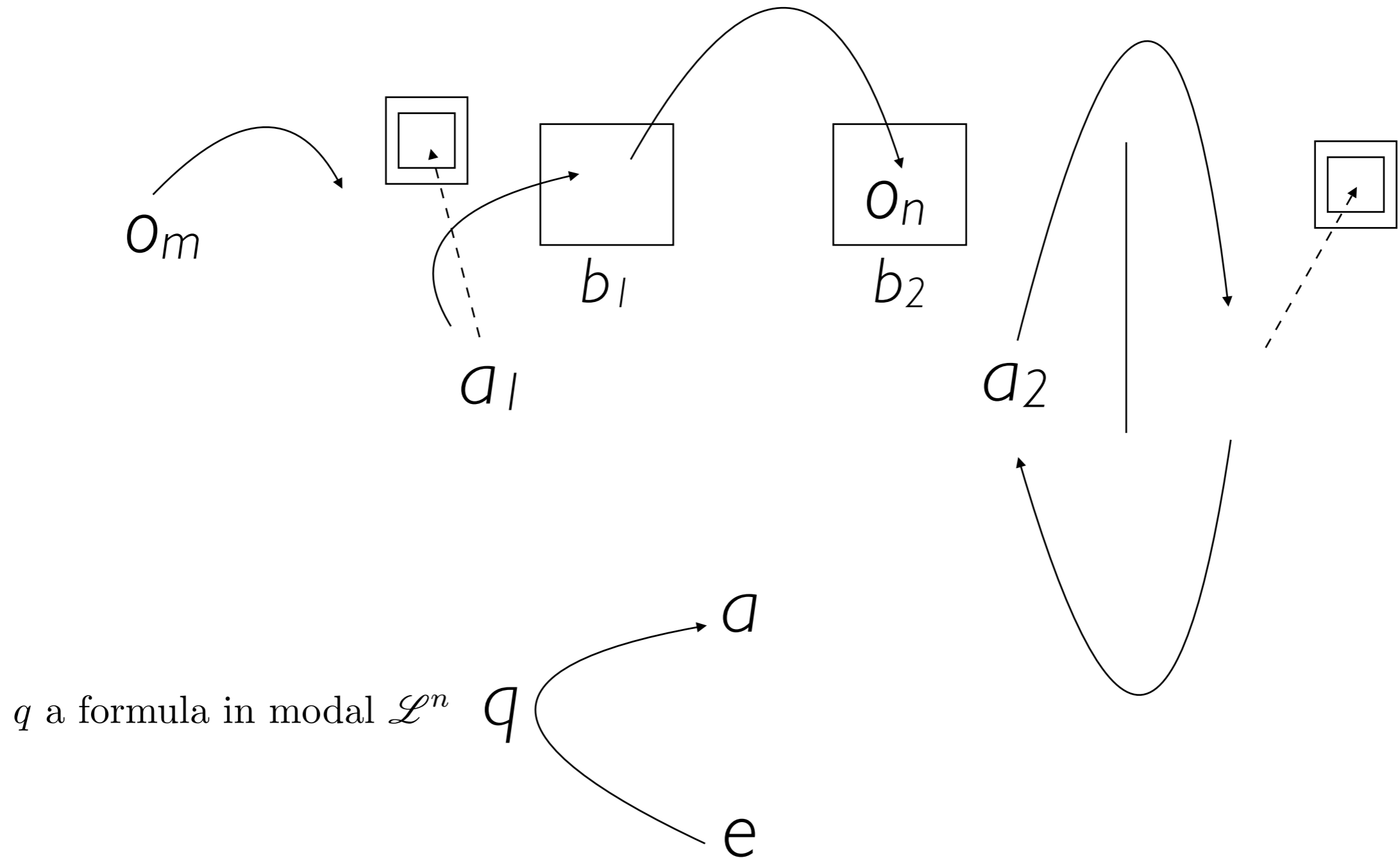
# Framework for $\text{FBT}^1_3$

(eight timepoints)

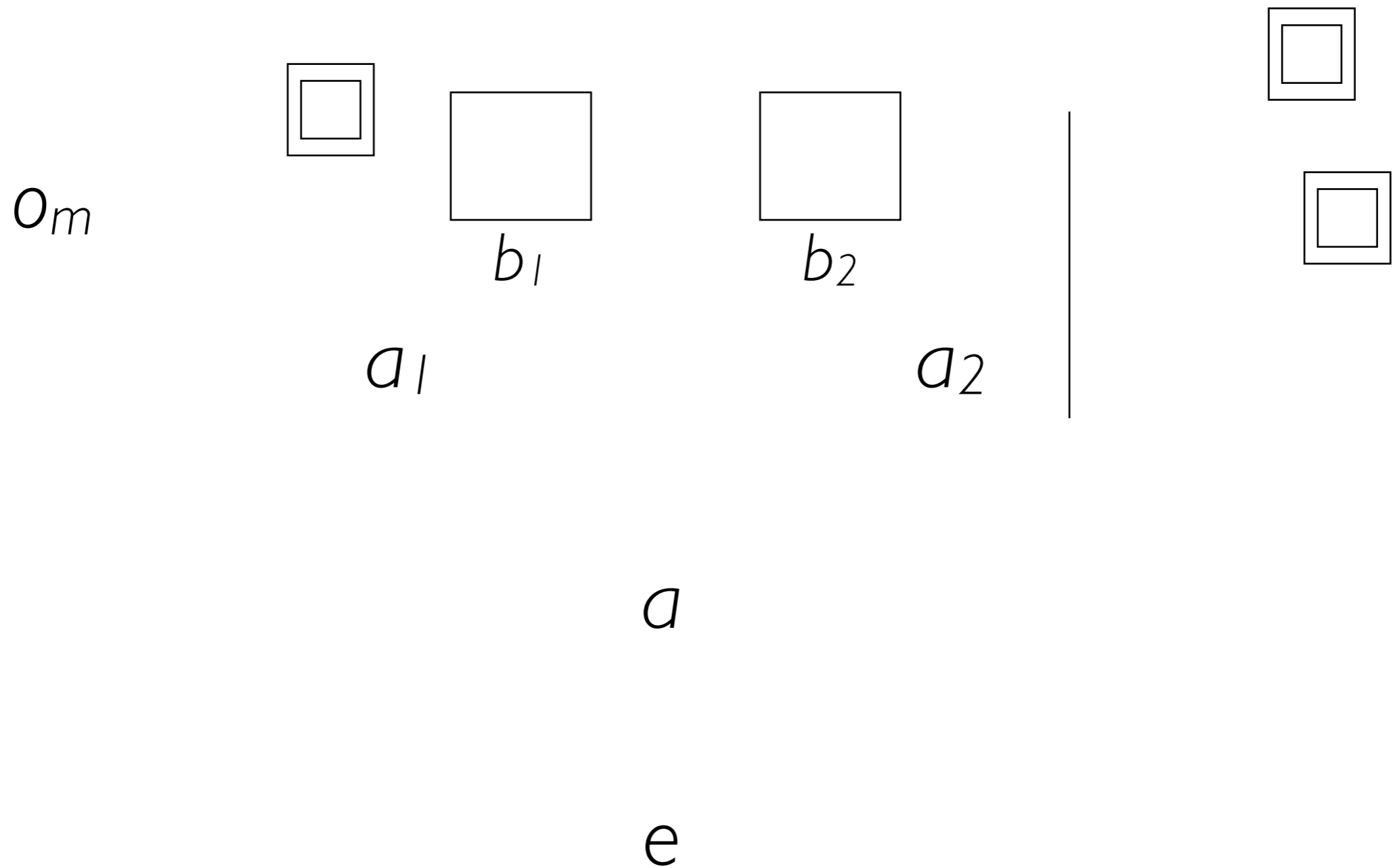


# Framework for $\text{FBT}^1_3$

(eight timepoints)

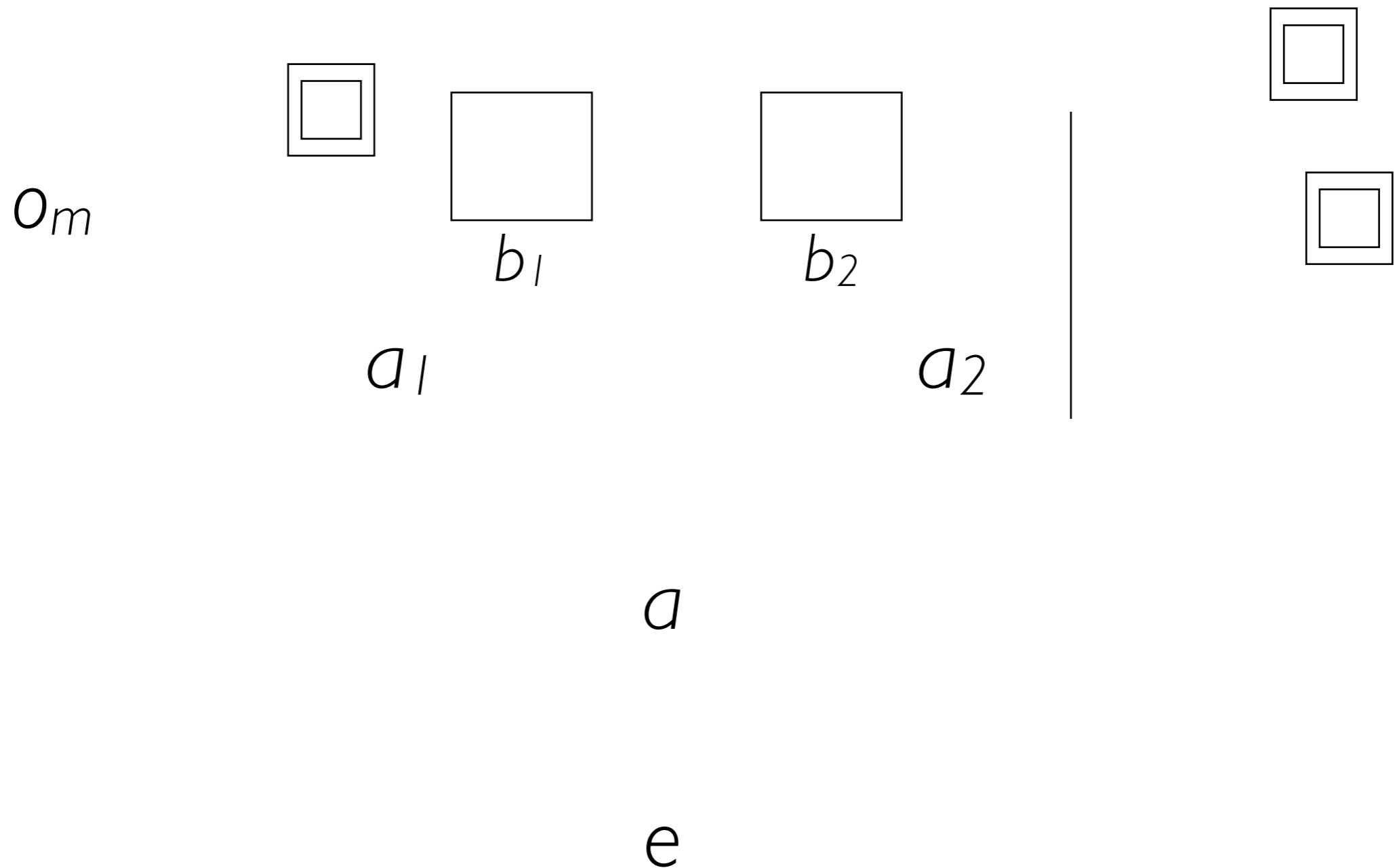


# Framework for $\text{FBT}^1_4$



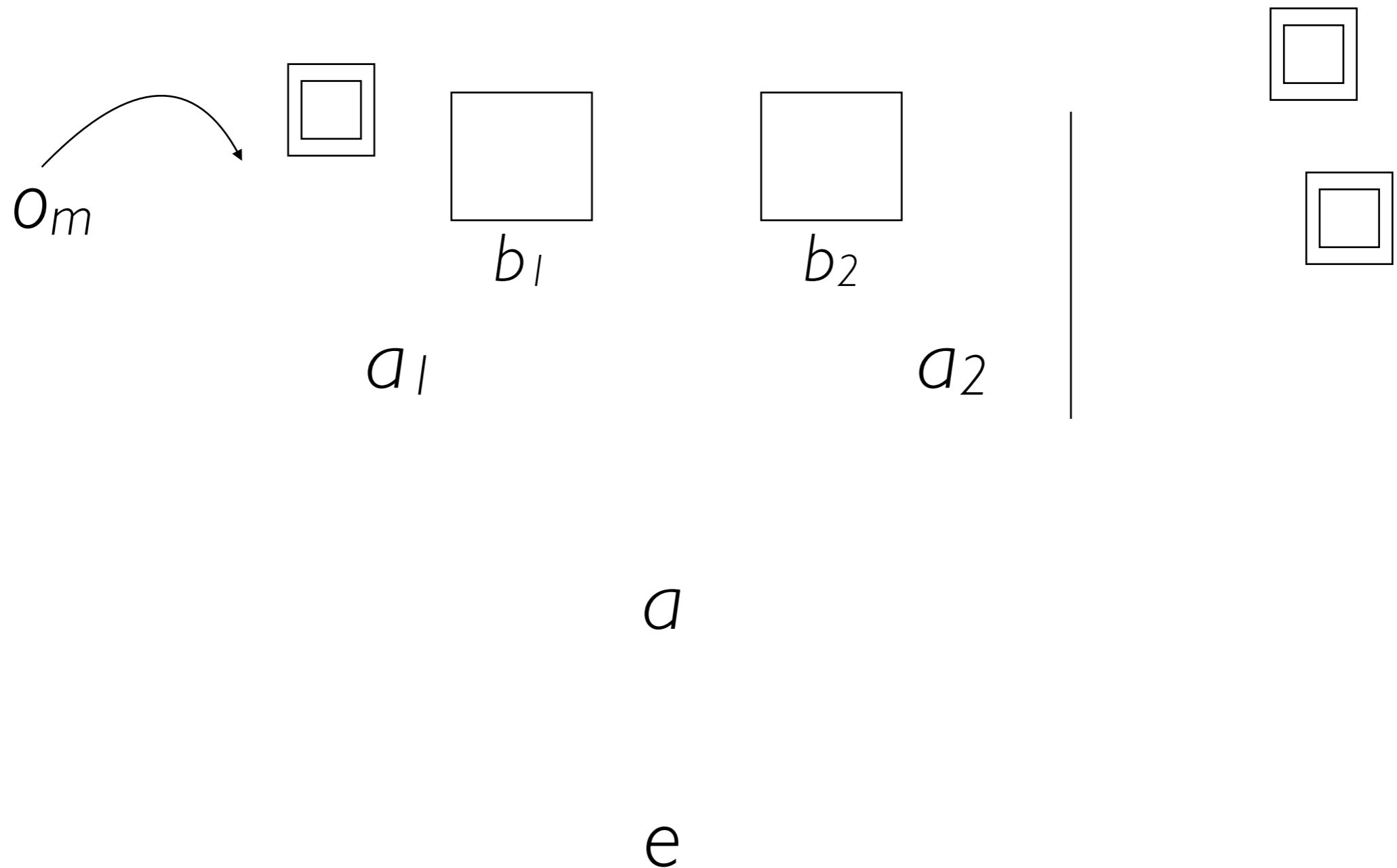
# Framework for $\text{FBT}^1_4$

(nine timepoints)



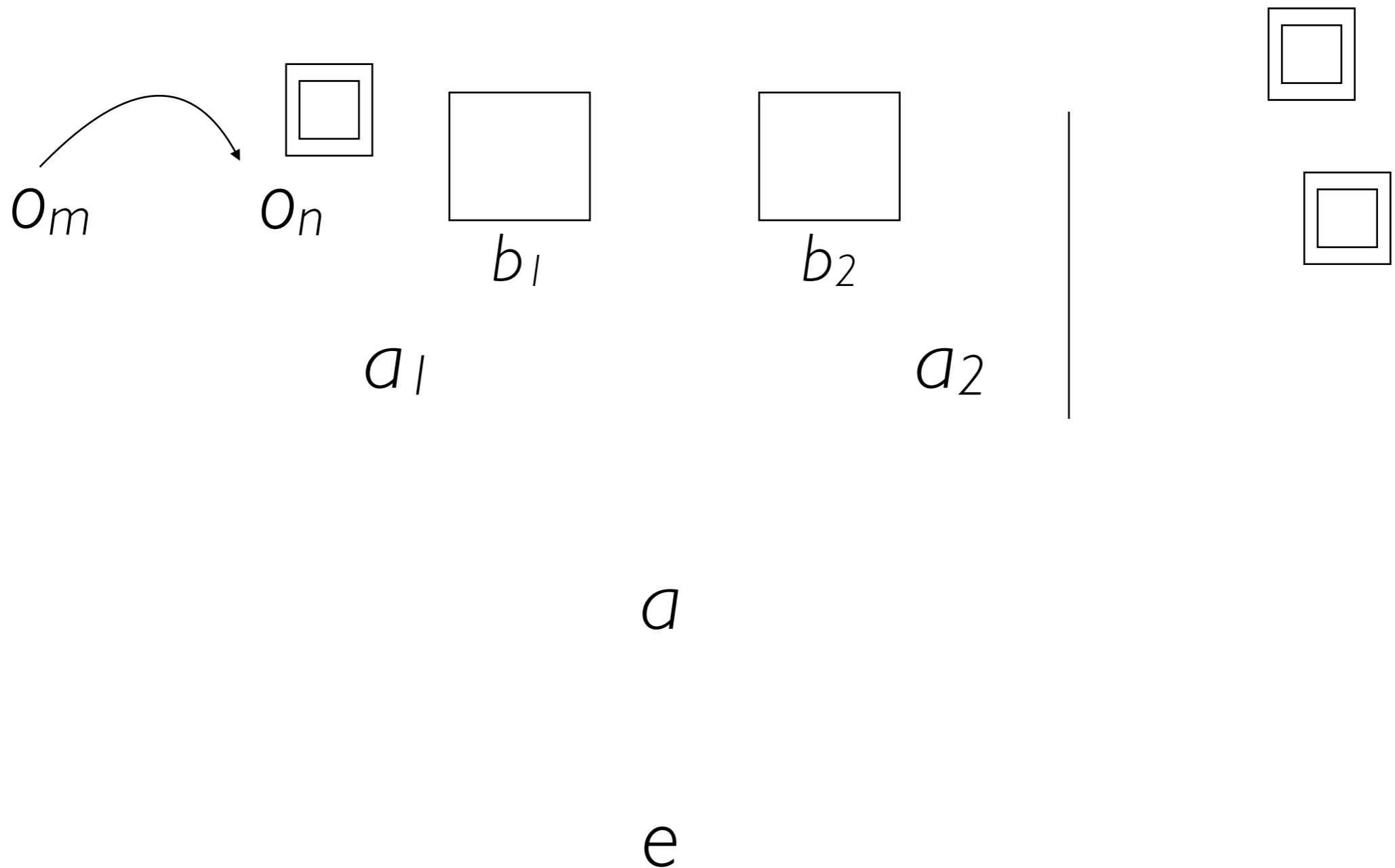
# Framework for $\text{FBT}^1_4$

(nine timepoints)

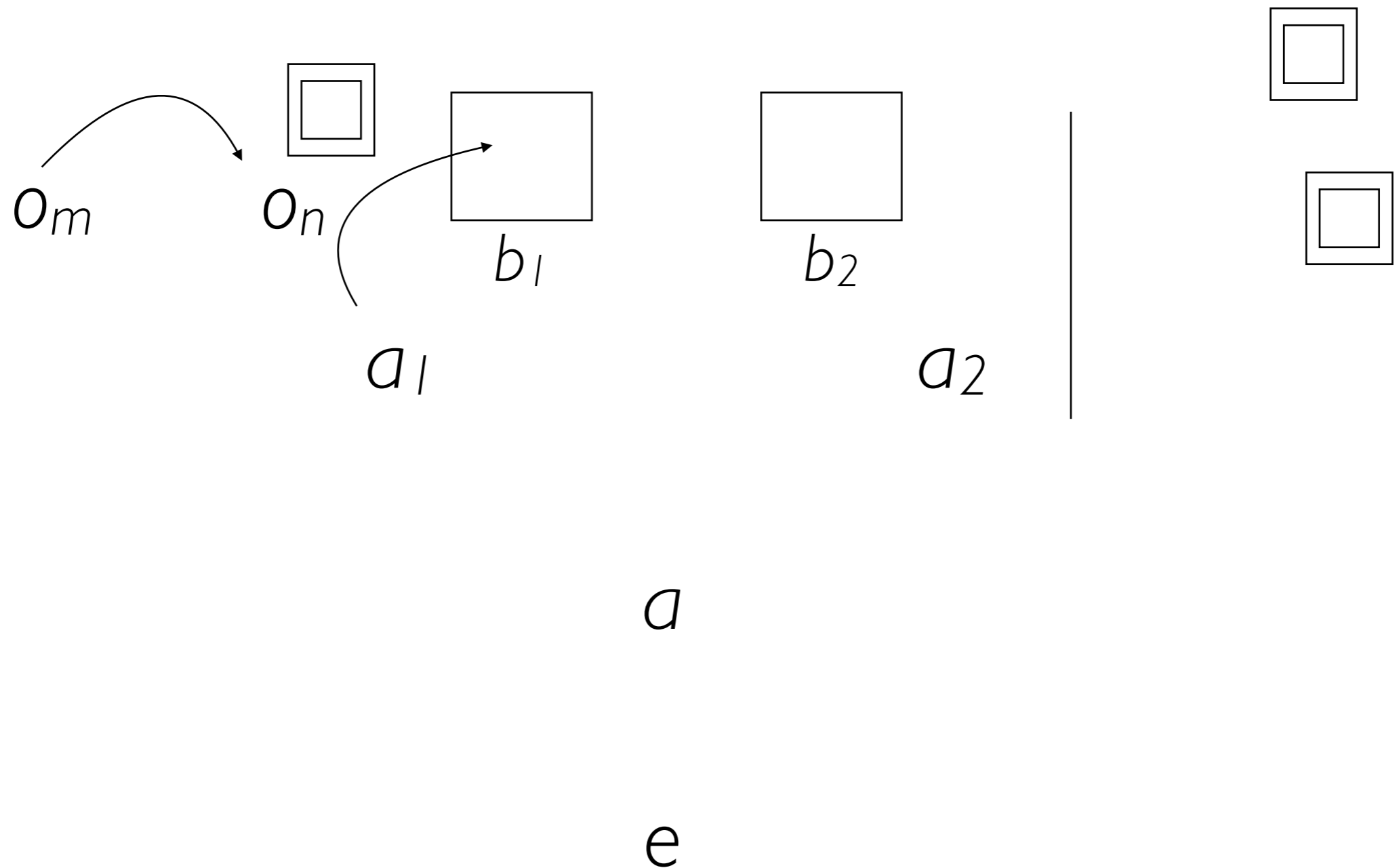


# Framework for $\text{FBT}^1_4$

(nine timepoints)



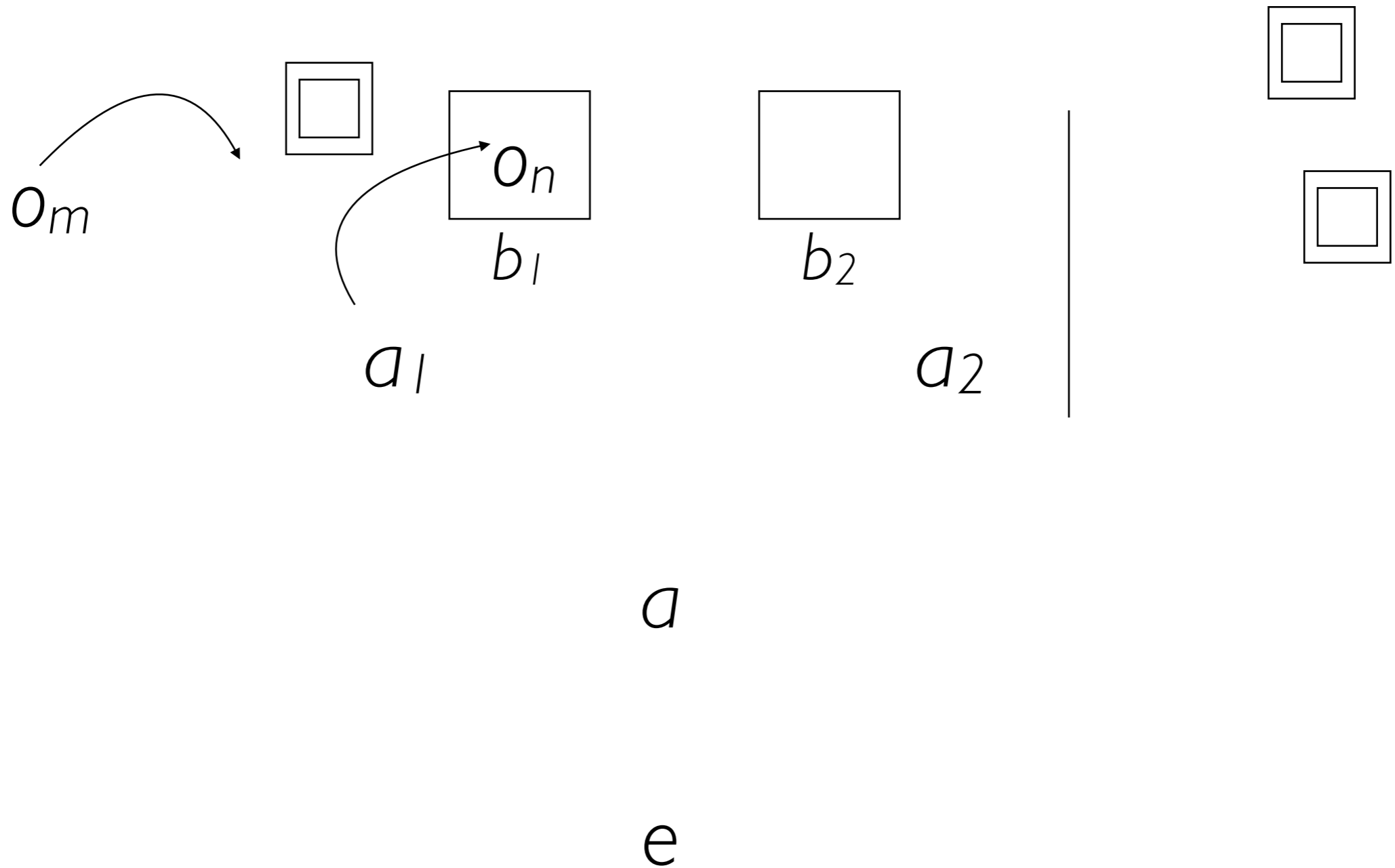
# Framework for $\text{FBT}^4$ (nine timepoints)





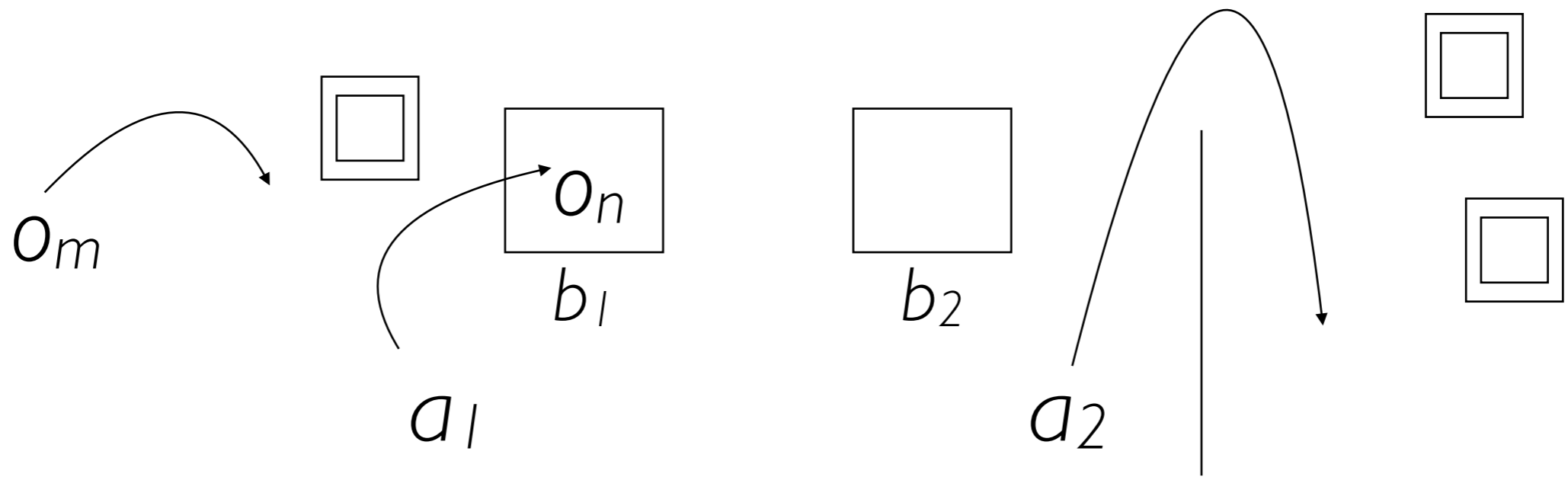
# Framework for $\text{FBT}^1_4$

(nine timepoints)



# Framework for $\text{FBT}^I_4$

(nine timepoints)

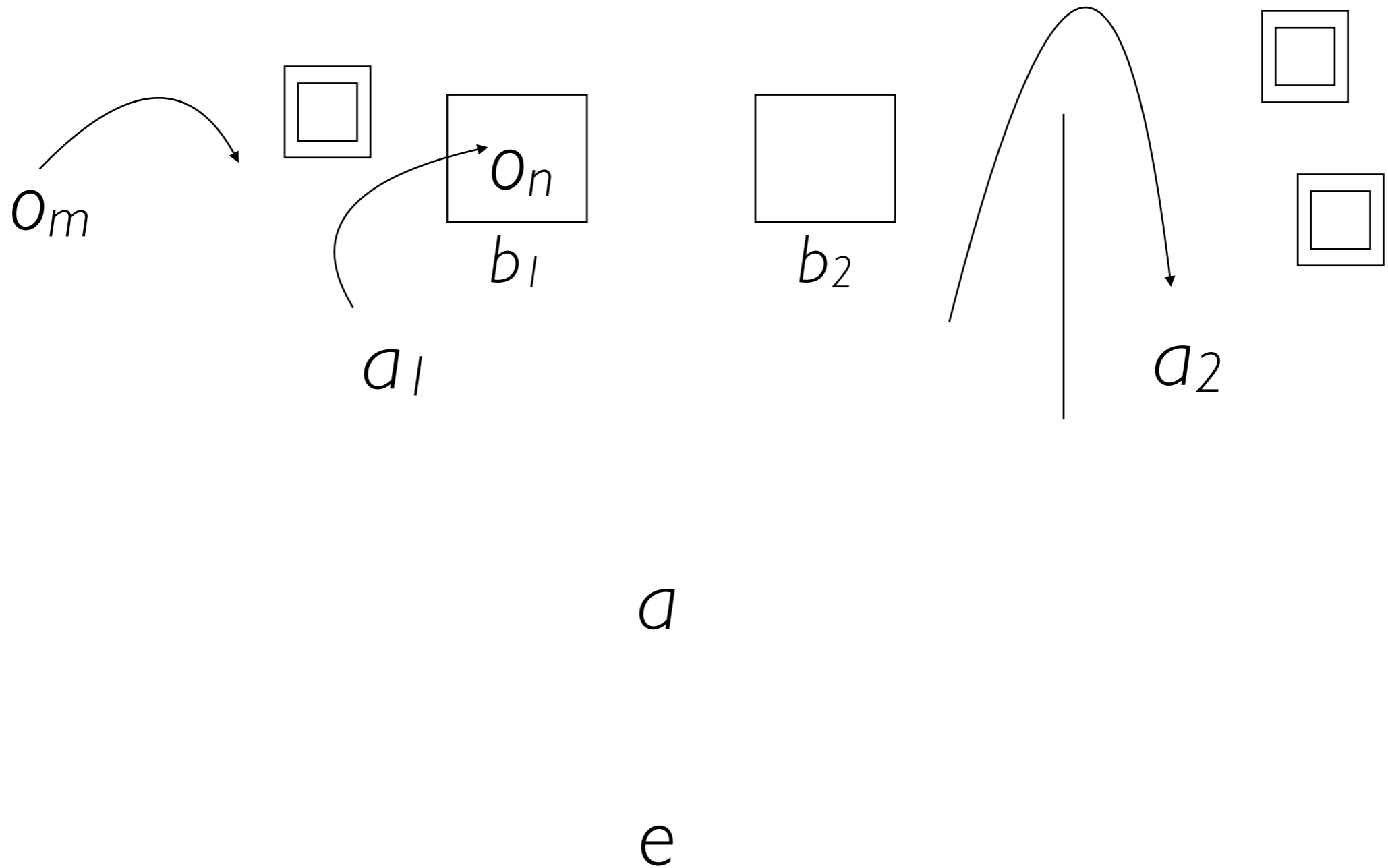


$a$

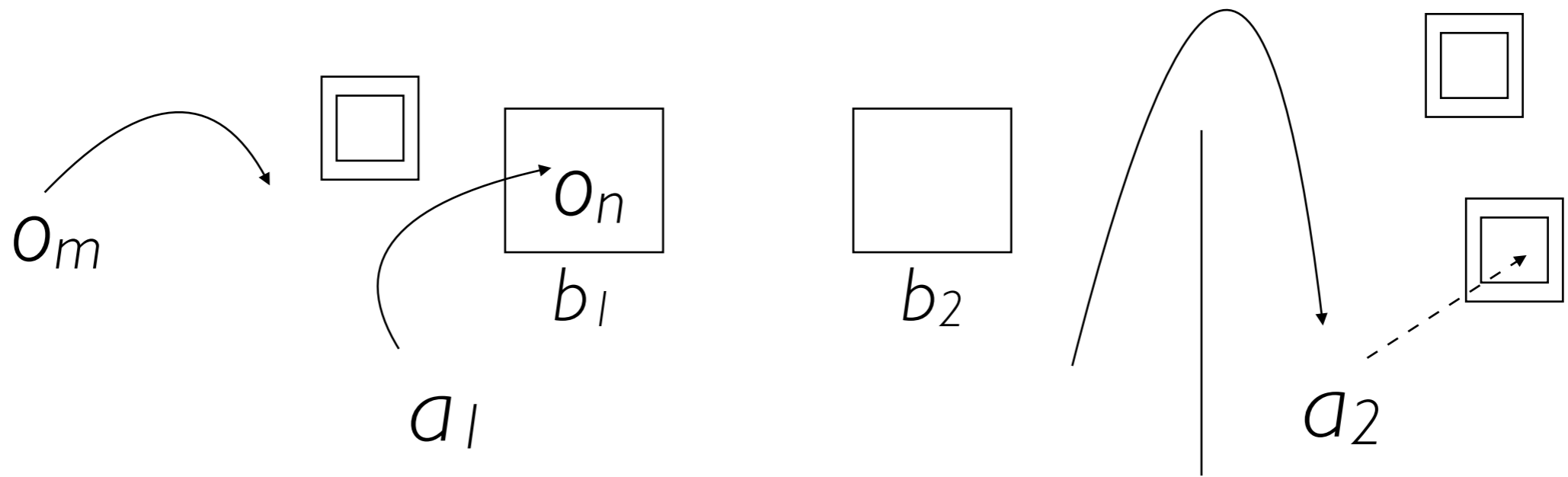
$e$

# Framework for $\text{FBT}^1_4$

(nine timepoints)



# Framework for $\text{FBT}^1_4$ (nine timepoints)

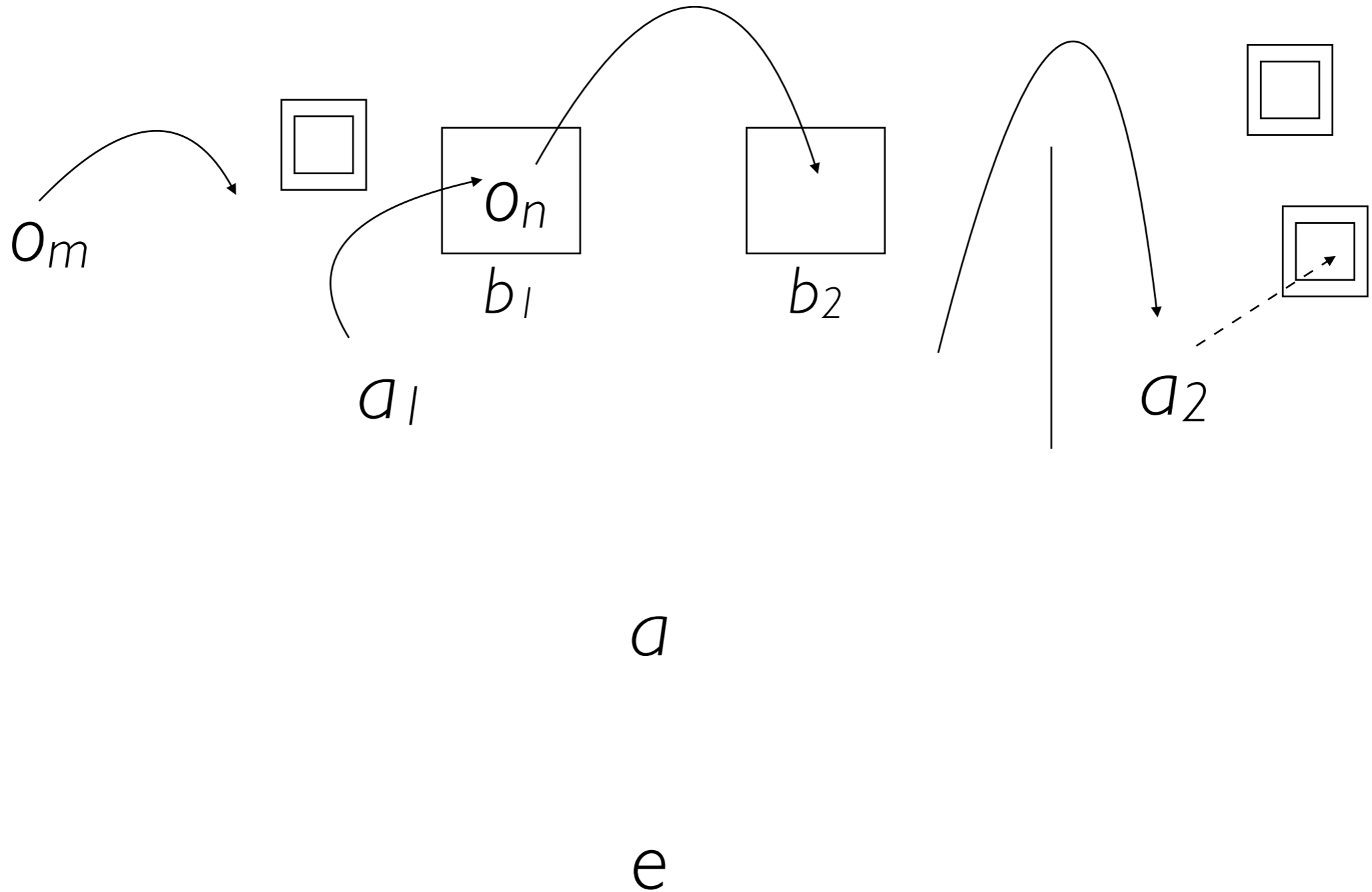


$a$

$e$

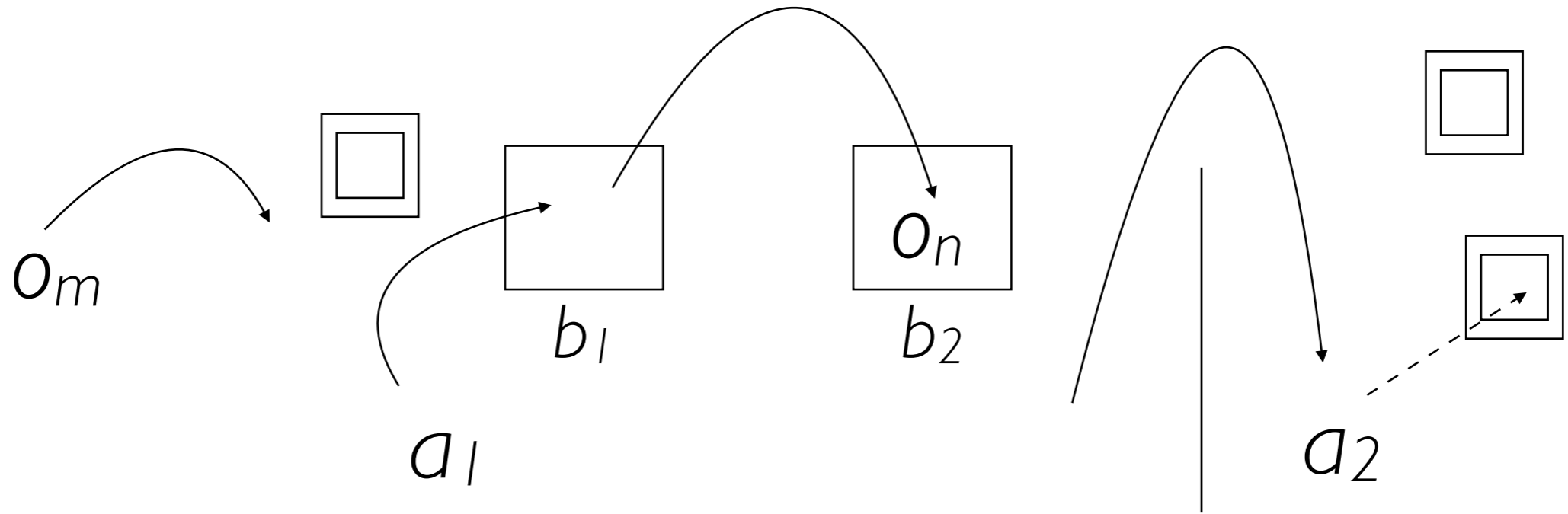
# Framework for $\text{FBT}^1_4$

(nine timepoints)



# Framework for $\text{FBT}^1_4$

(nine timepoints)

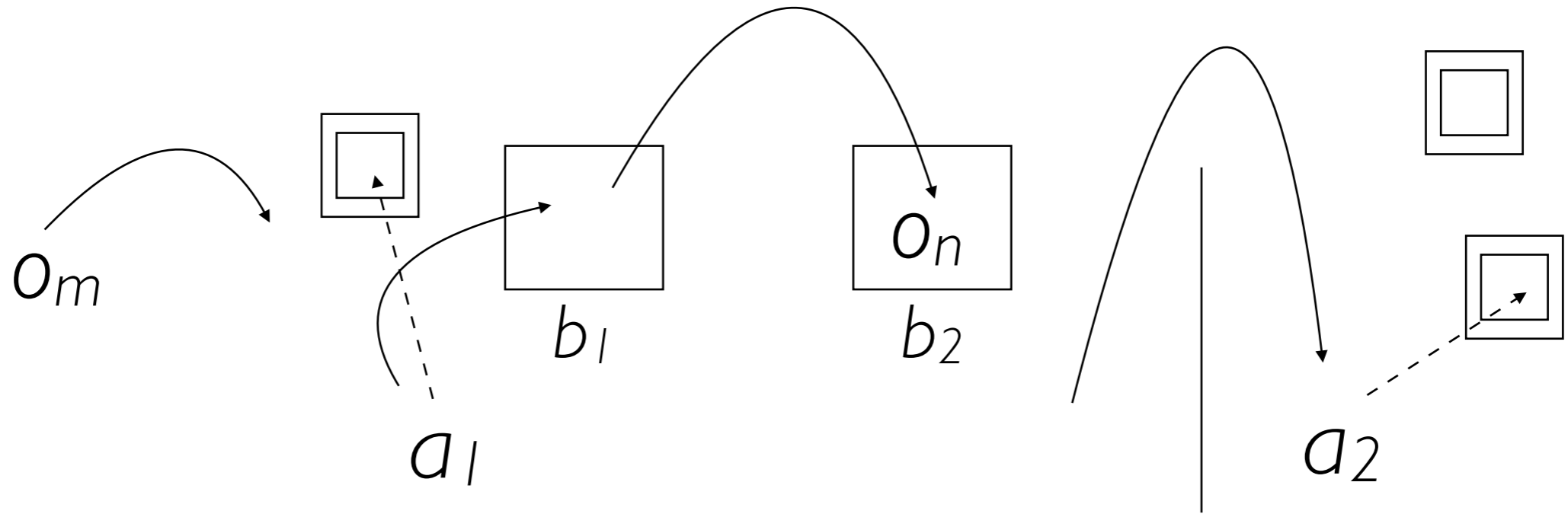


$a$

$e$

# Framework for $\text{FBT}^1_4$

(nine timepoints)

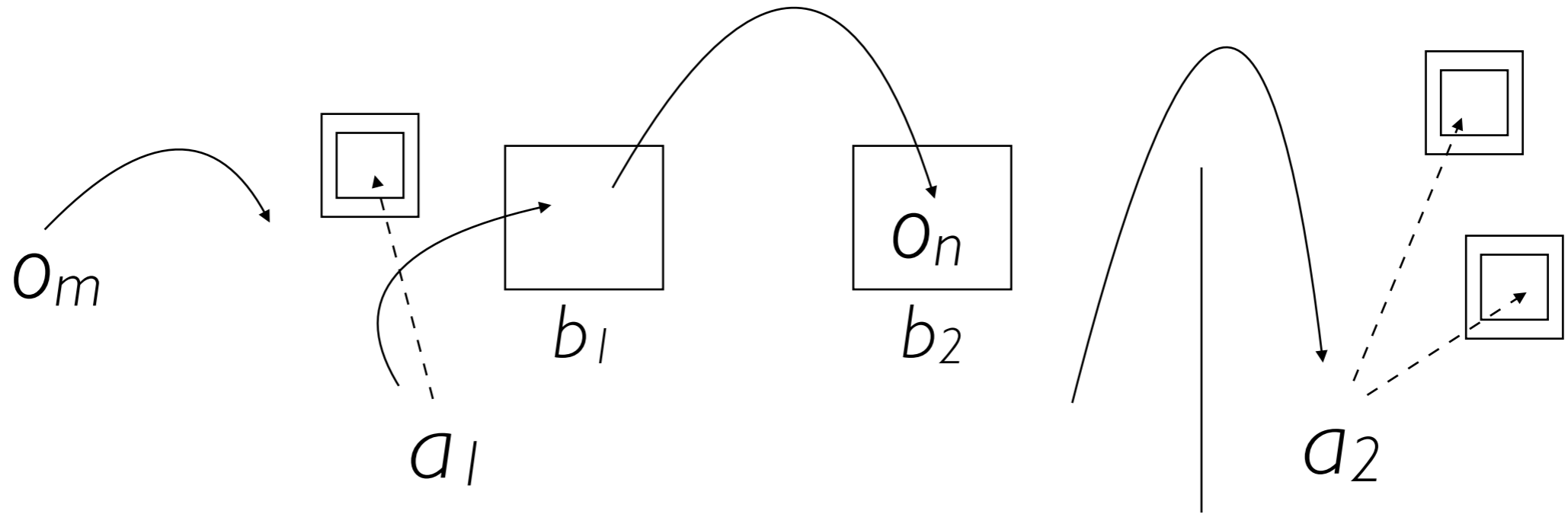


$a$

$e$

# Framework for $\text{FBT}^1_4$

(nine timepoints)



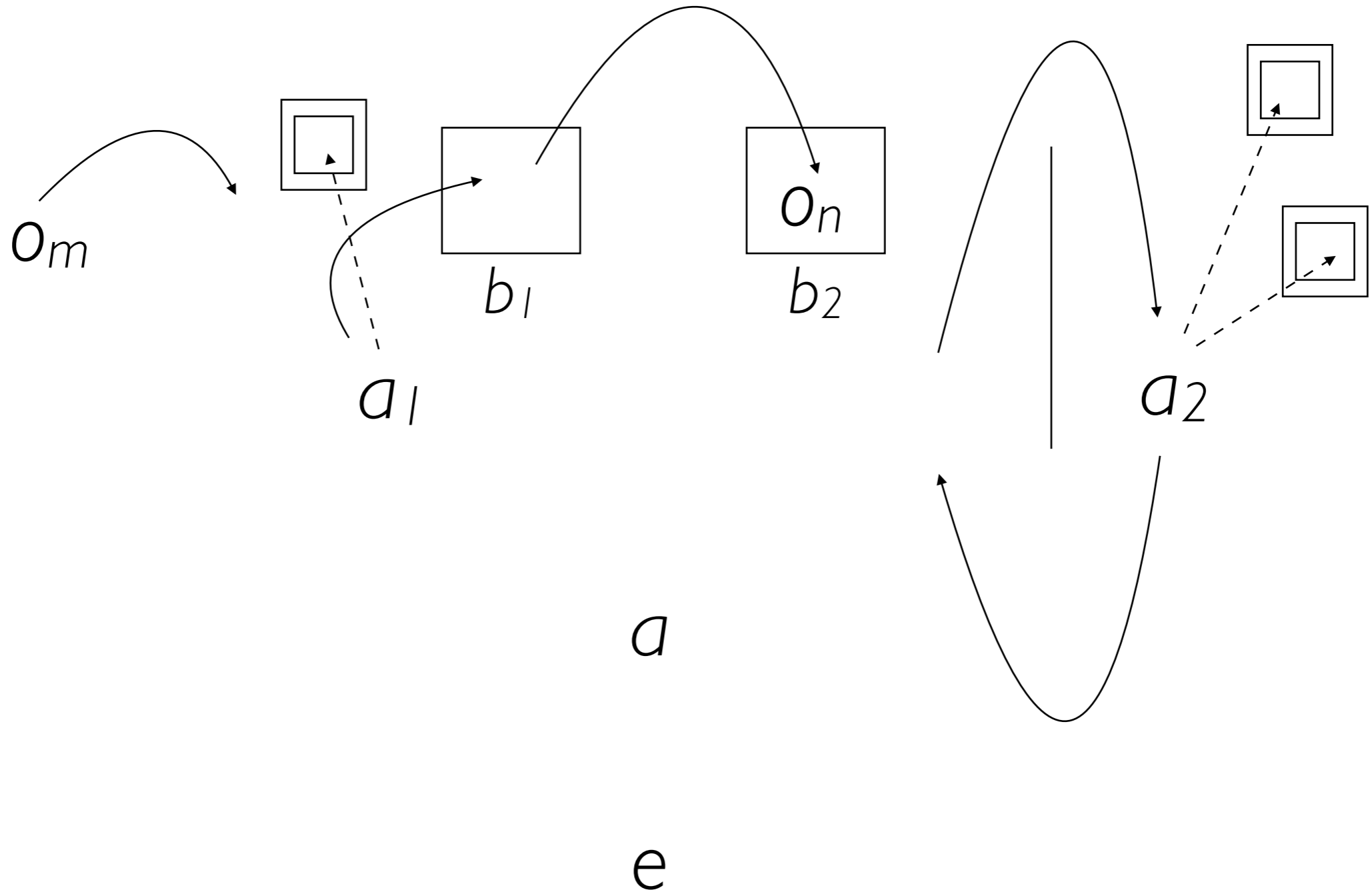
$a$

$e$



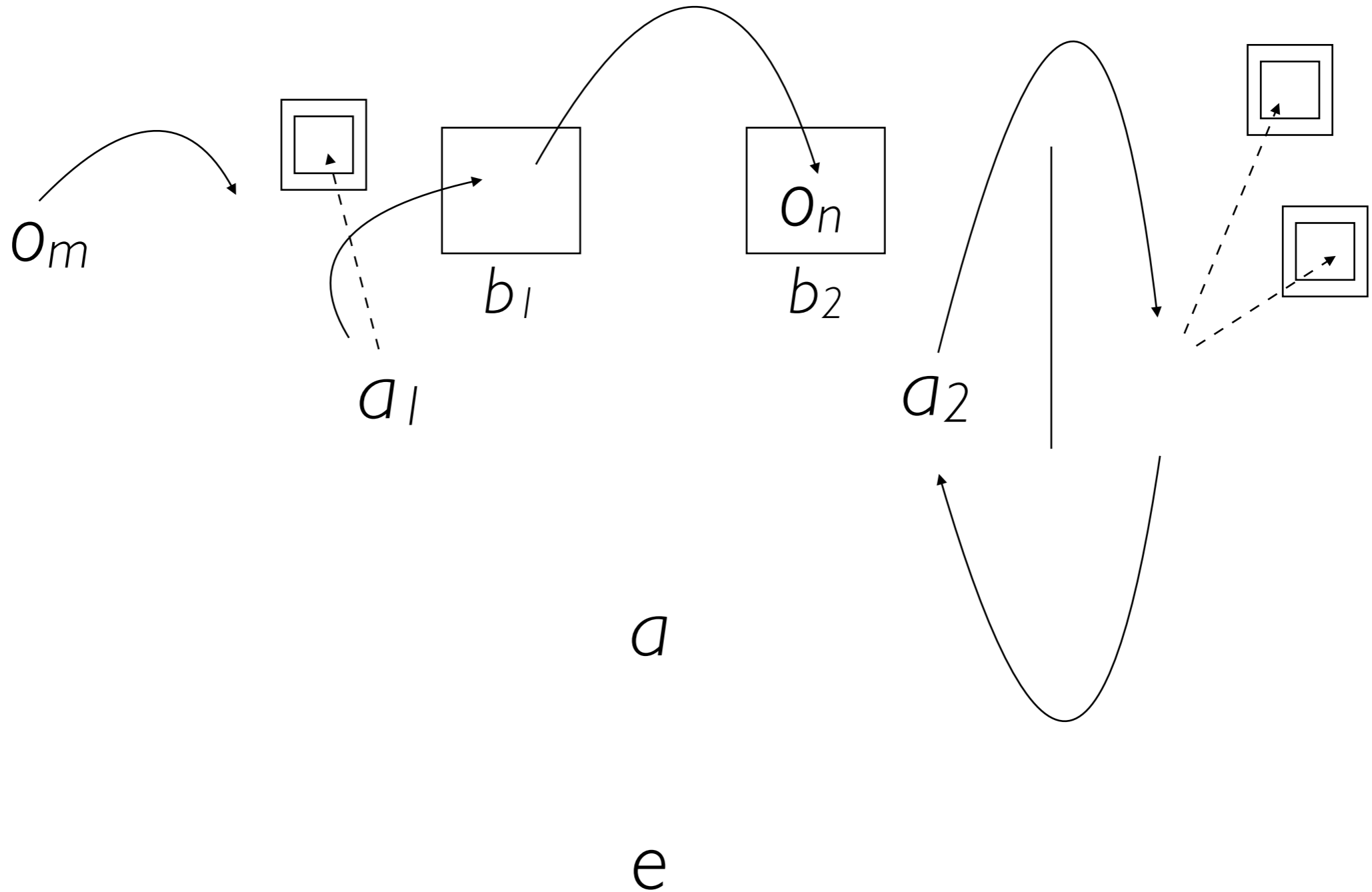
# Framework for $\text{FBT}^1_4$

(nine timepoints)



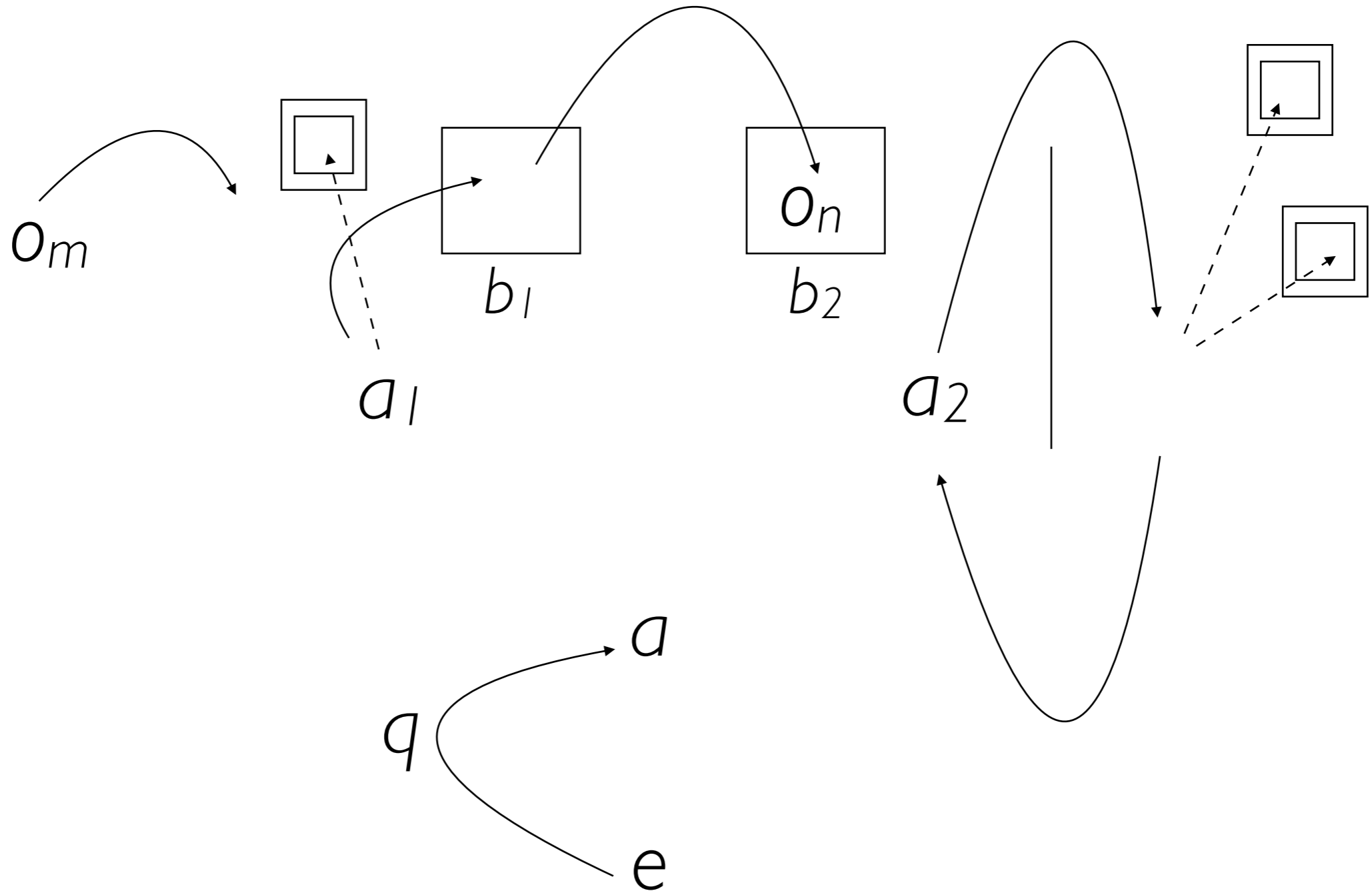
# Framework for $\text{FBT}^1_4$

(nine timepoints)



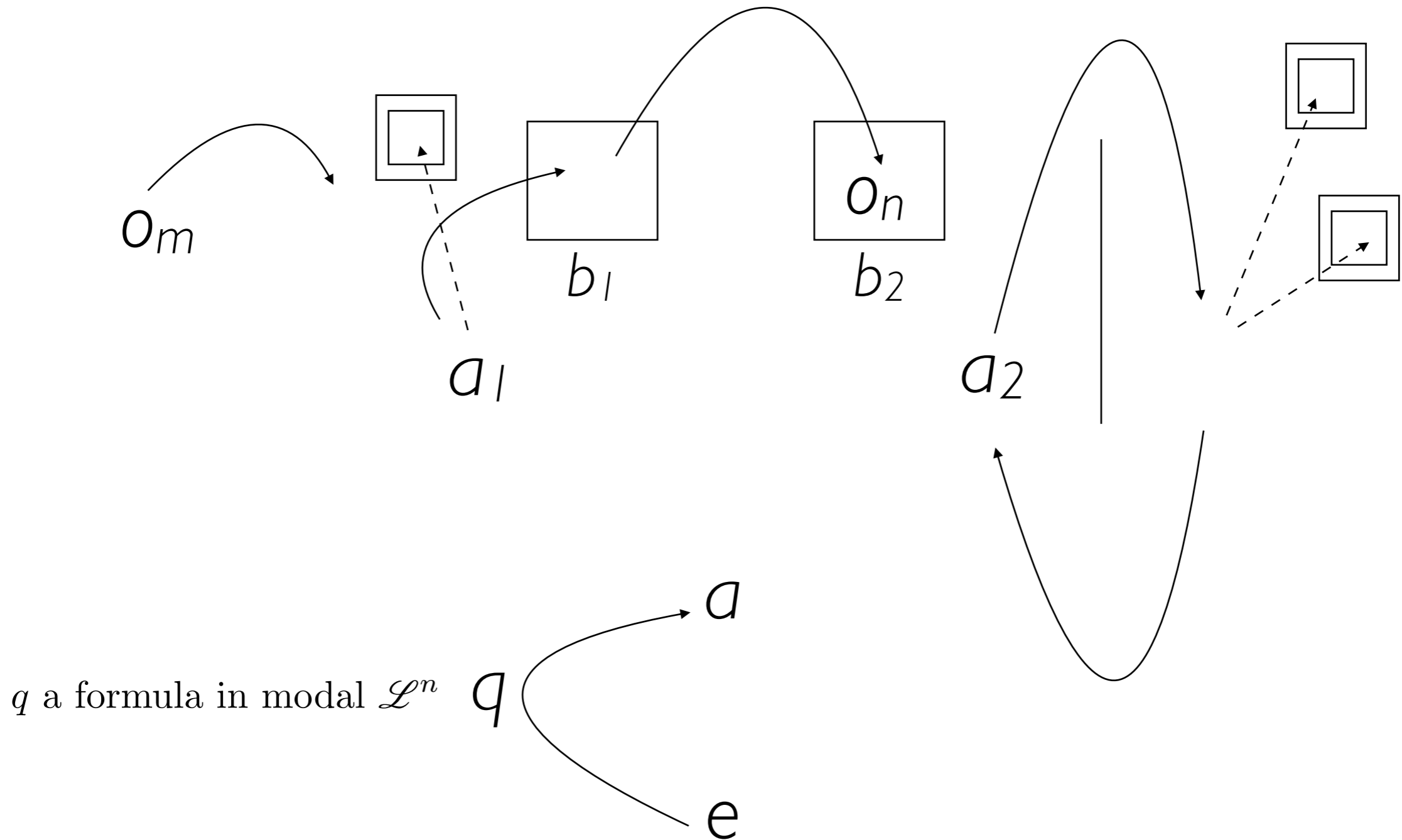
# Framework for $\text{FBT}^I_4$

(nine timepoints)

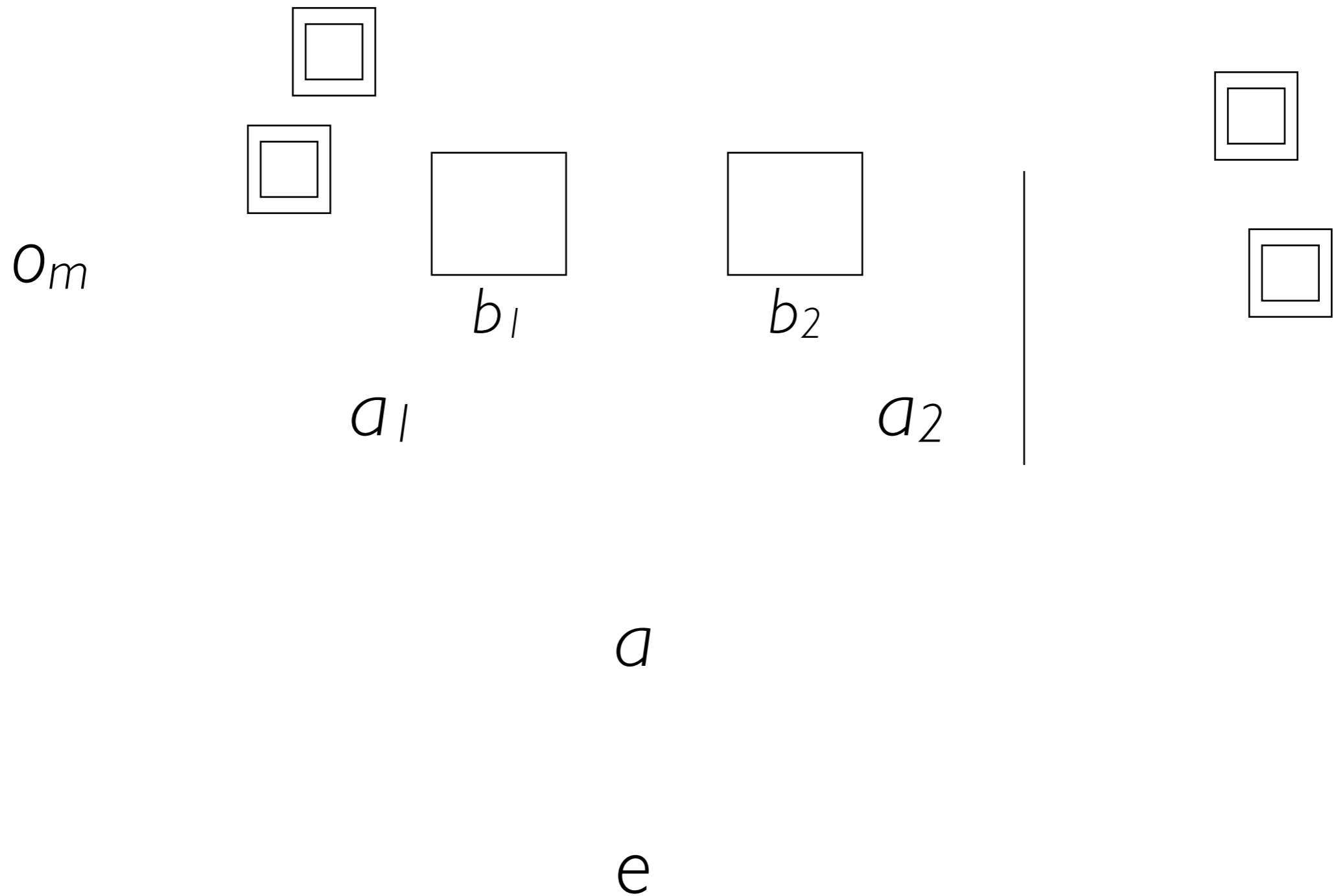


# Framework for $\text{FBT}^1_4$

(nine timepoints)

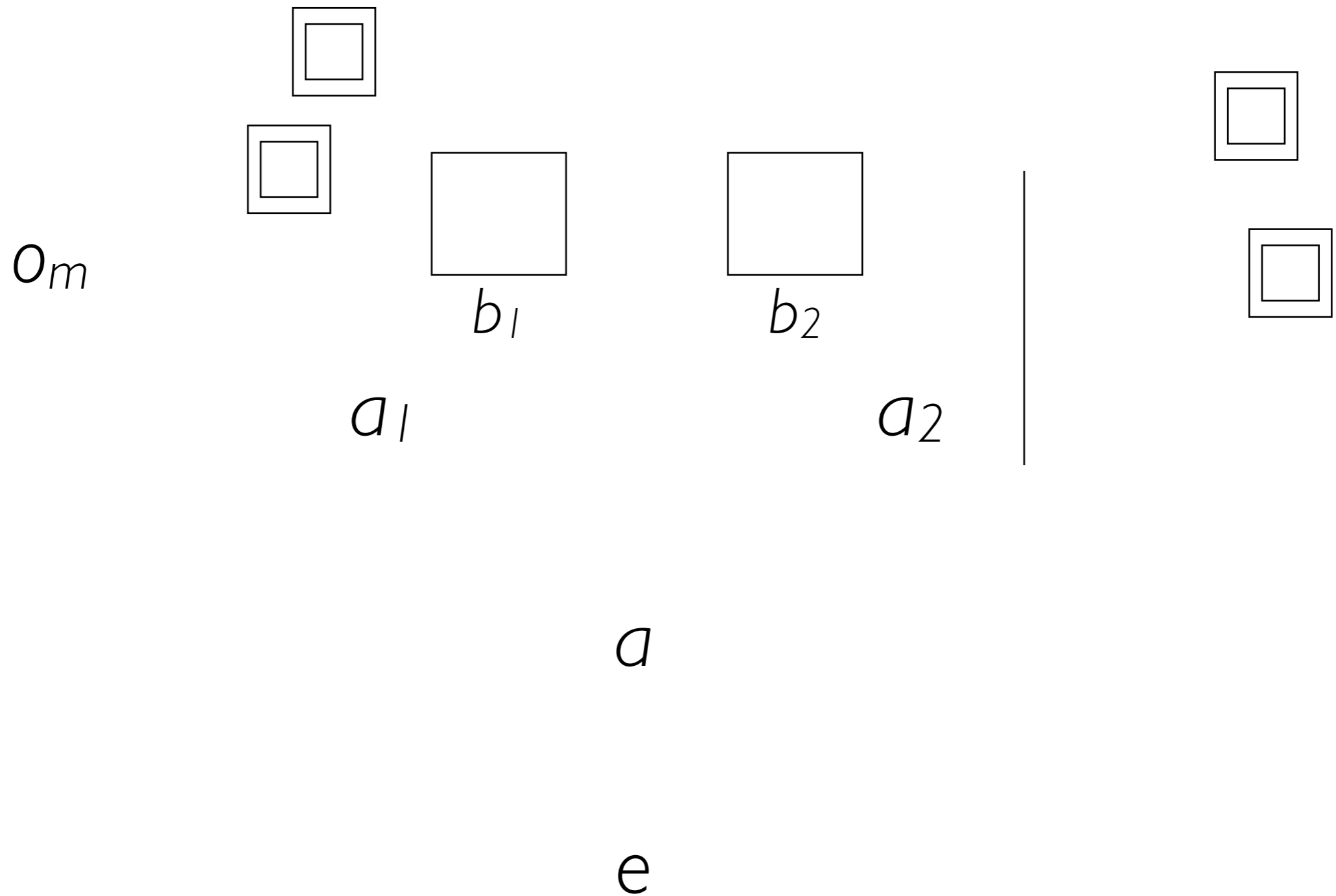


# Framework for $\text{FBT}^1_5$



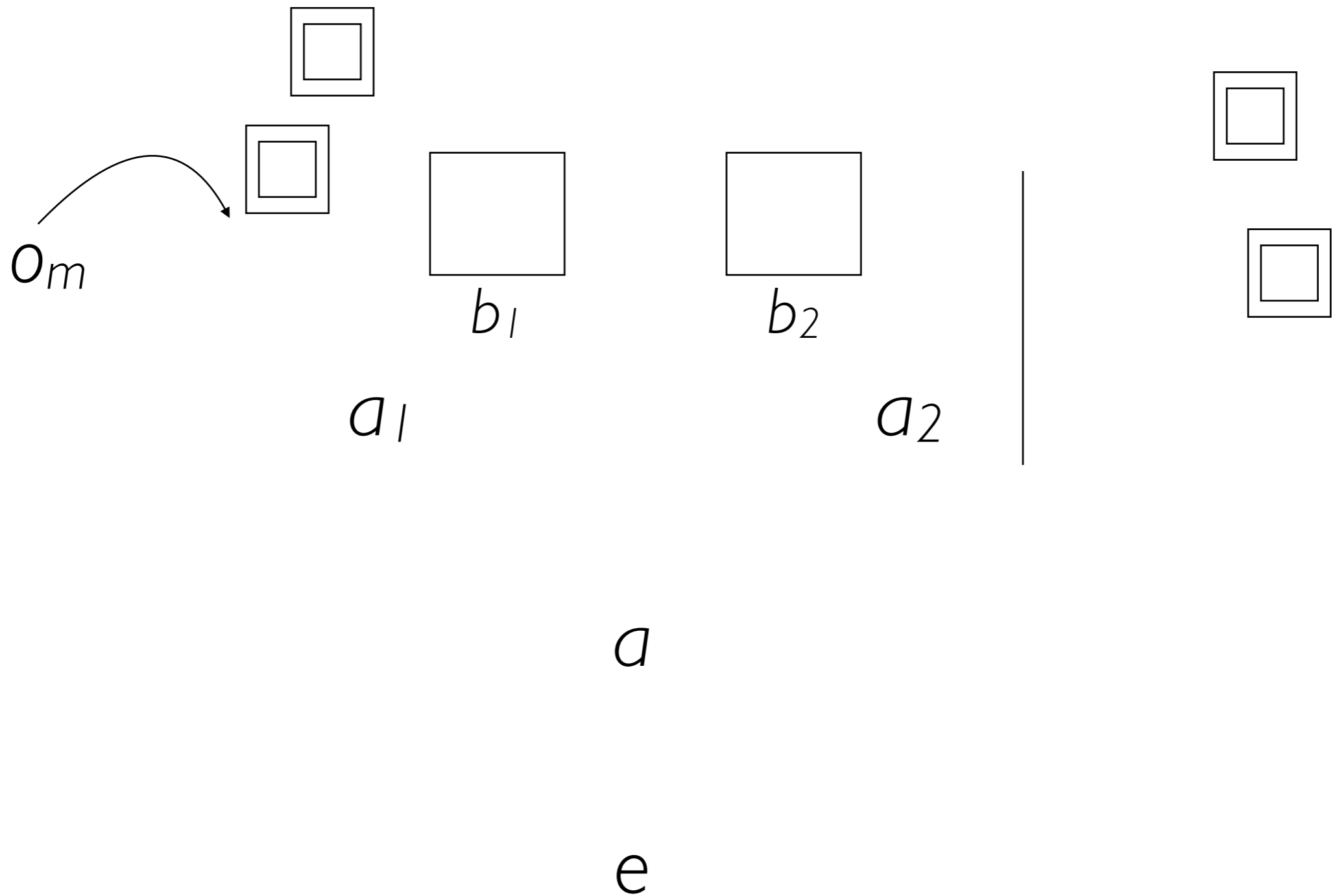
# Framework for $\text{FBT}^1_5$

(ten timepoints)



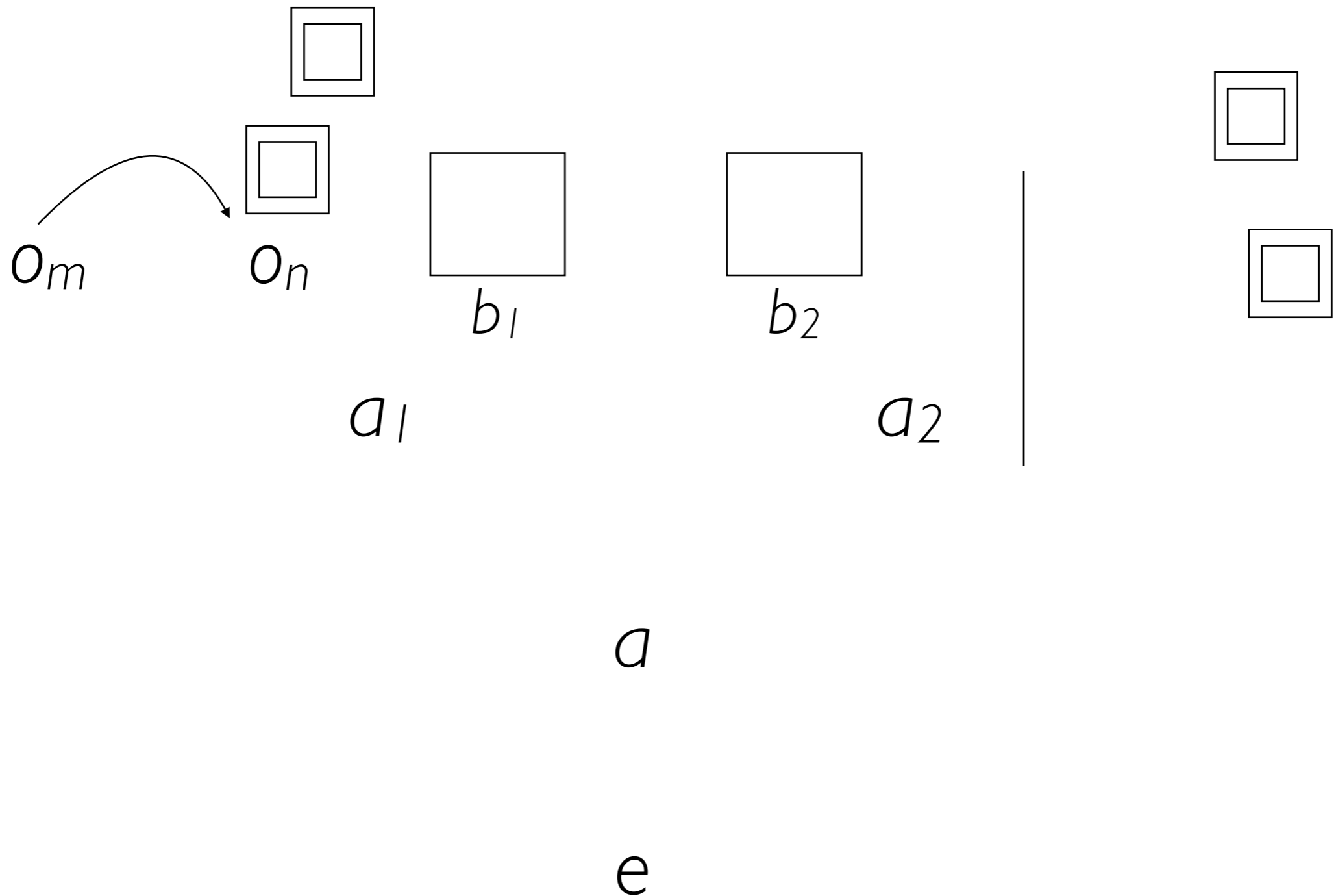
# Framework for FBT<sup>15</sup>

(ten timepoints)



# Framework for FBT<sup>15</sup>

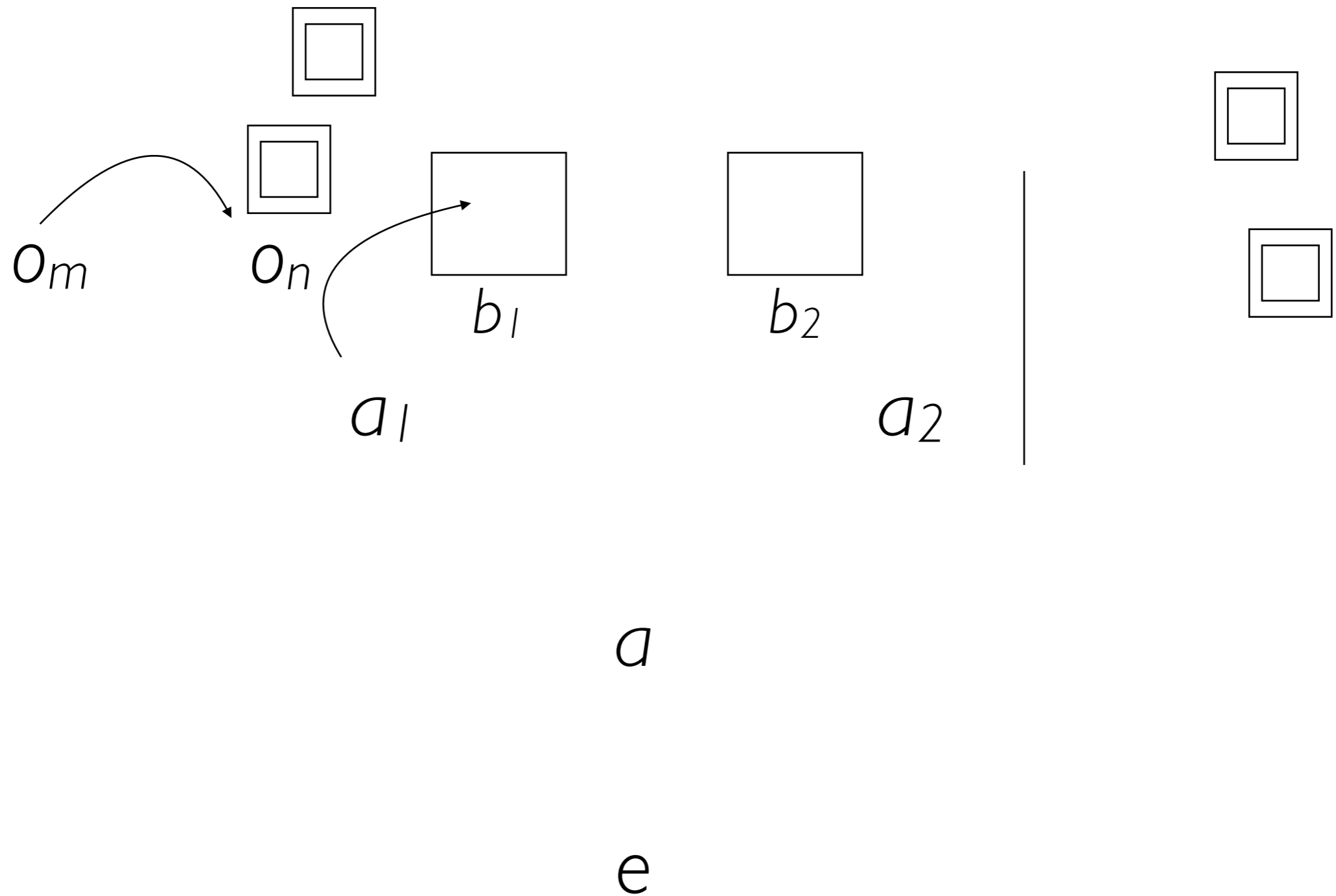
(ten timepoints)





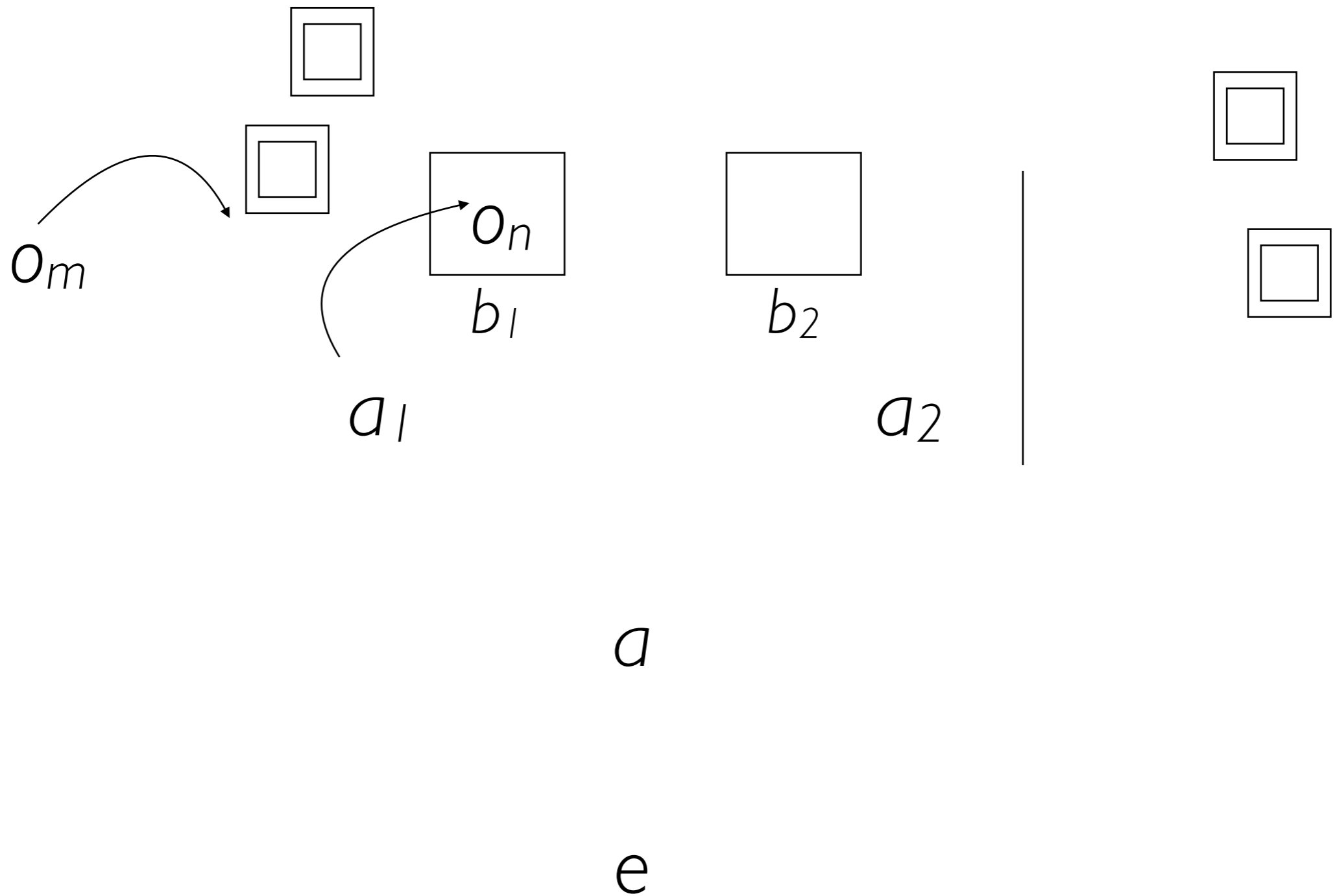
# Framework for FBT<sup>15</sup>

(ten timepoints)



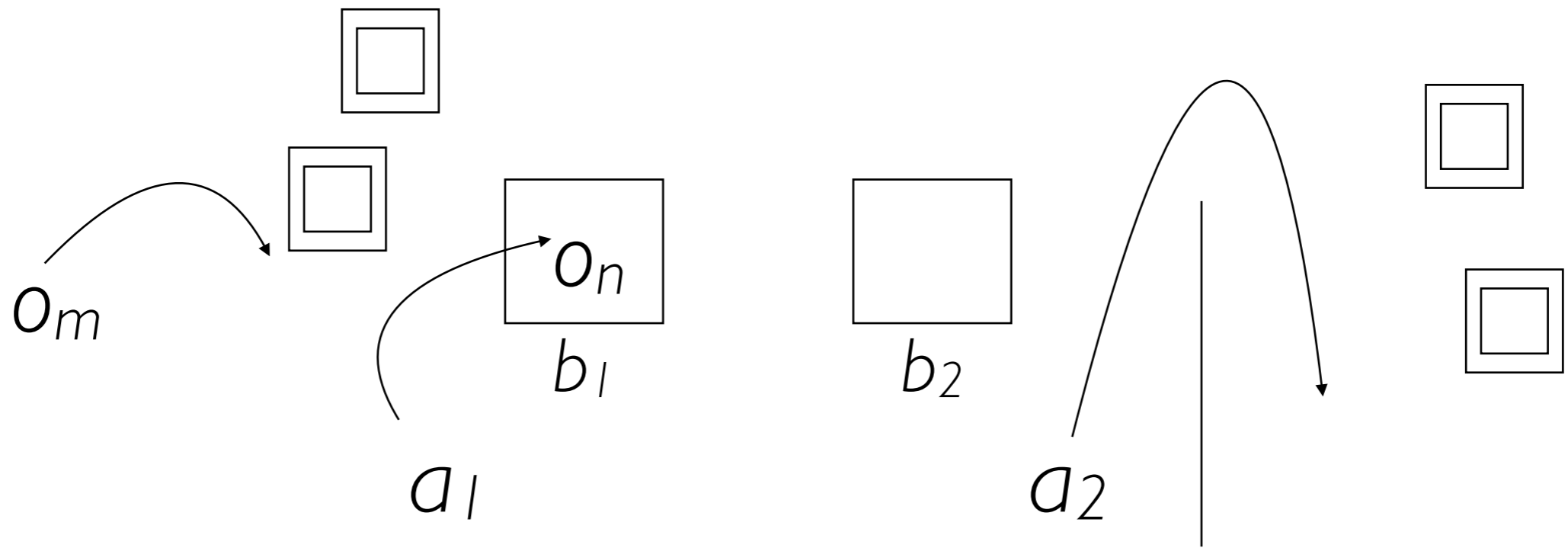
# Framework for $\text{FBT}^1_5$

(ten timepoints)



# Framework for FBT<sup>15</sup>

(ten timepoints)

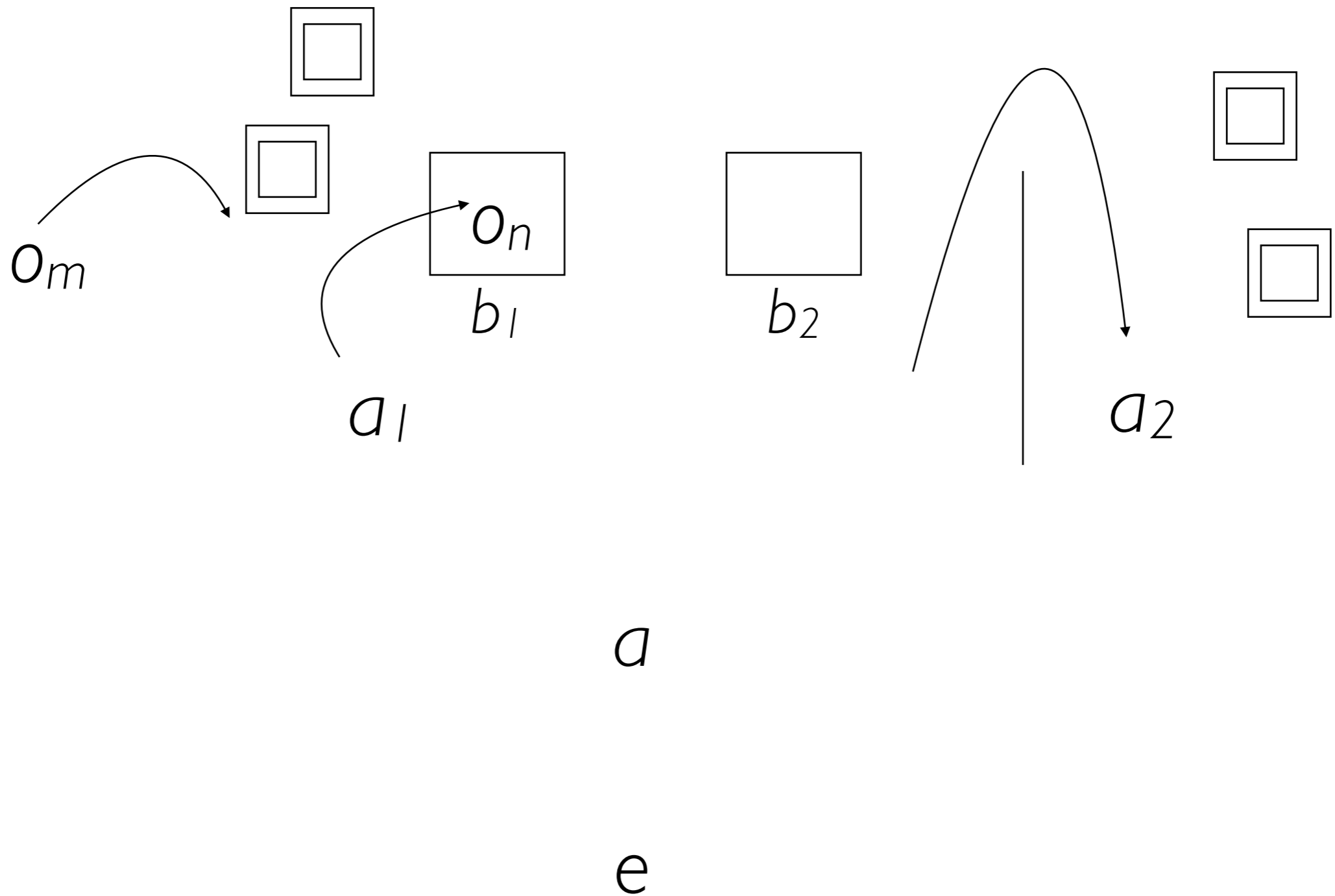


$a$

$e$

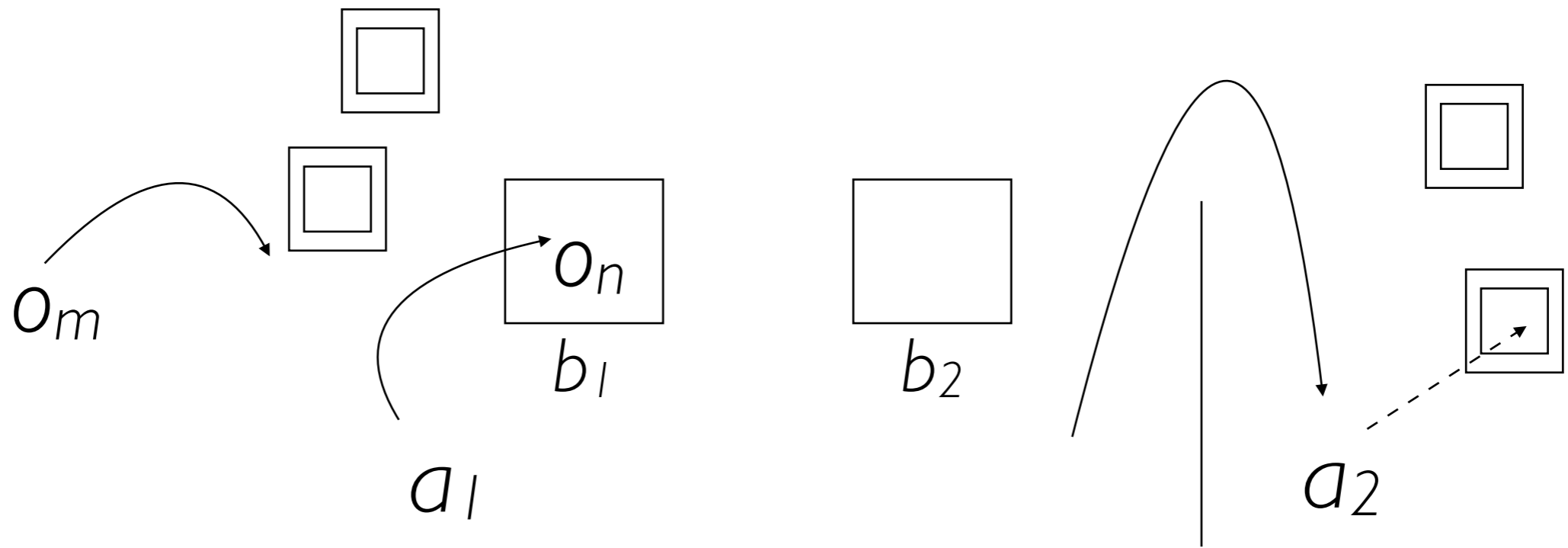
# Framework for FBT<sup>15</sup>

(ten timepoints)



# Framework for FBT<sup>1</sup><sub>5</sub>

(ten timepoints)

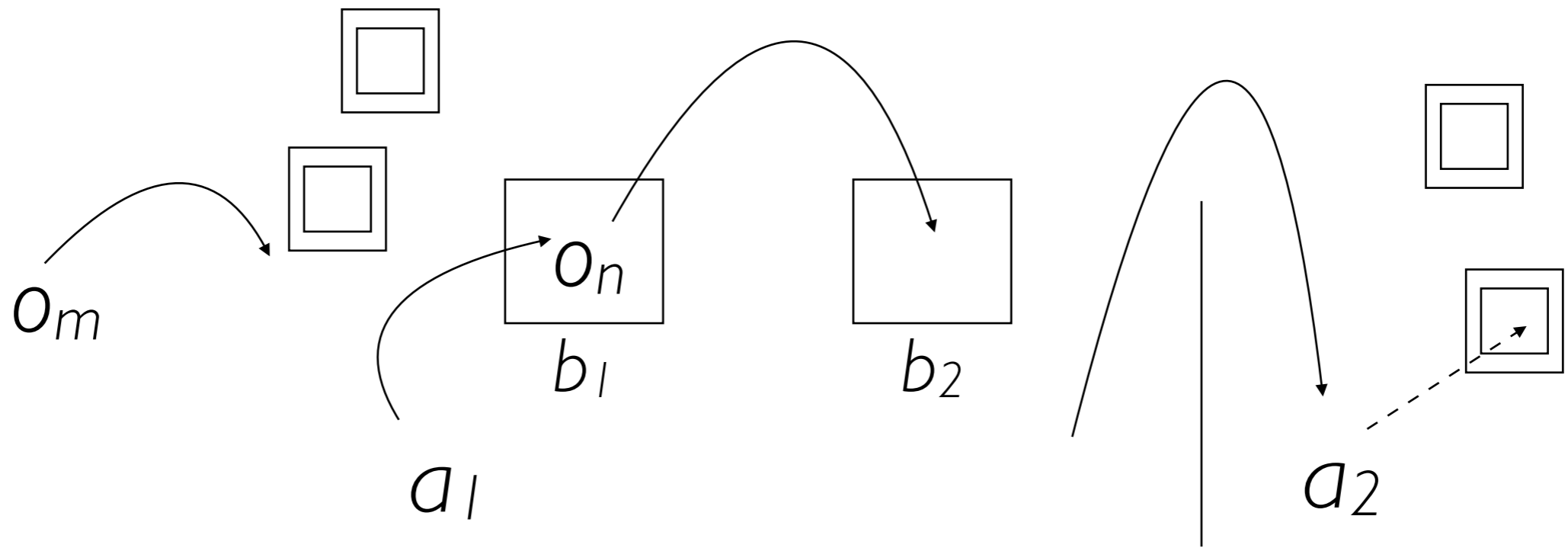


$a$

$e$

# Framework for FBT<sup>1</sup><sub>5</sub>

(ten timepoints)

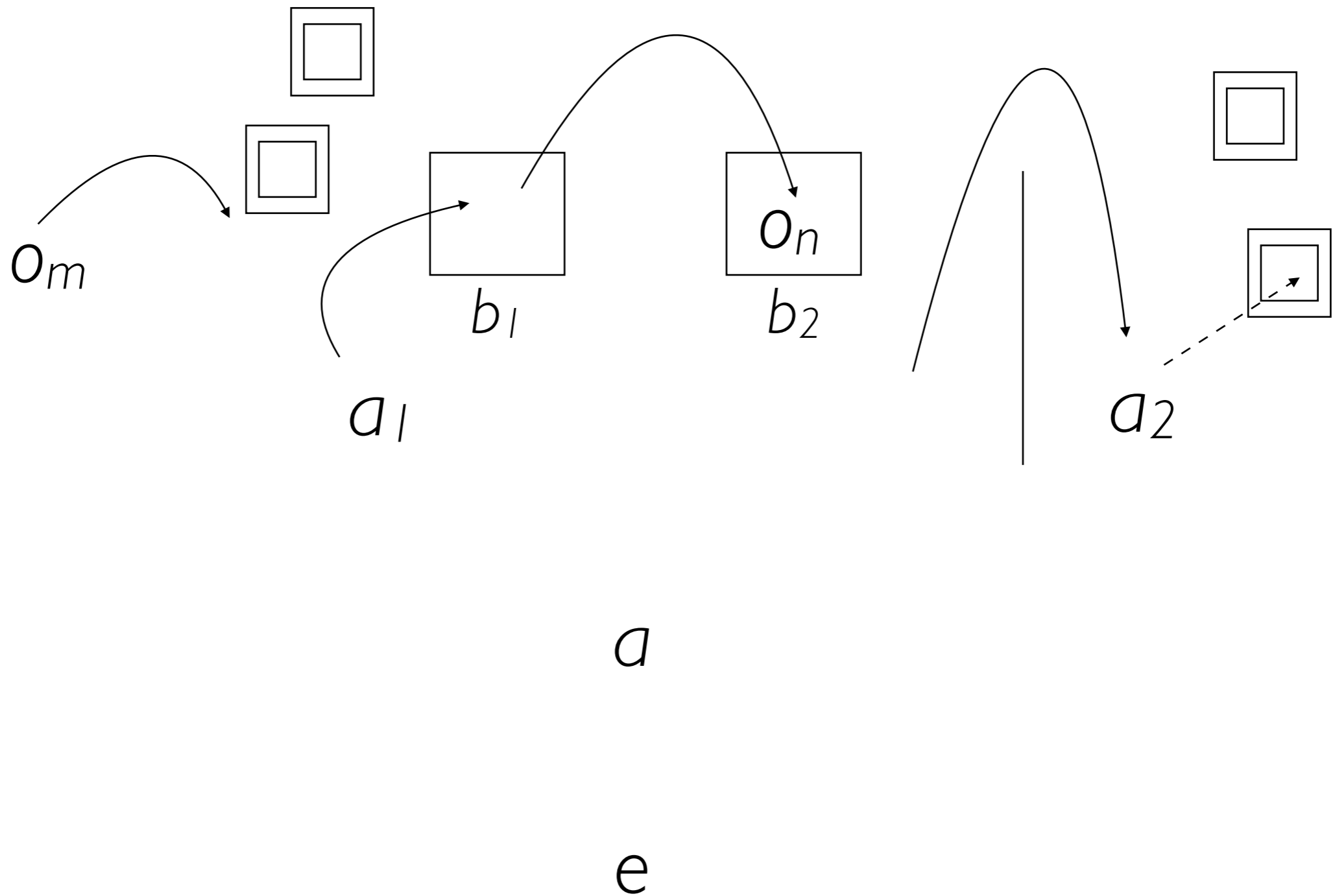


$a$

$e$

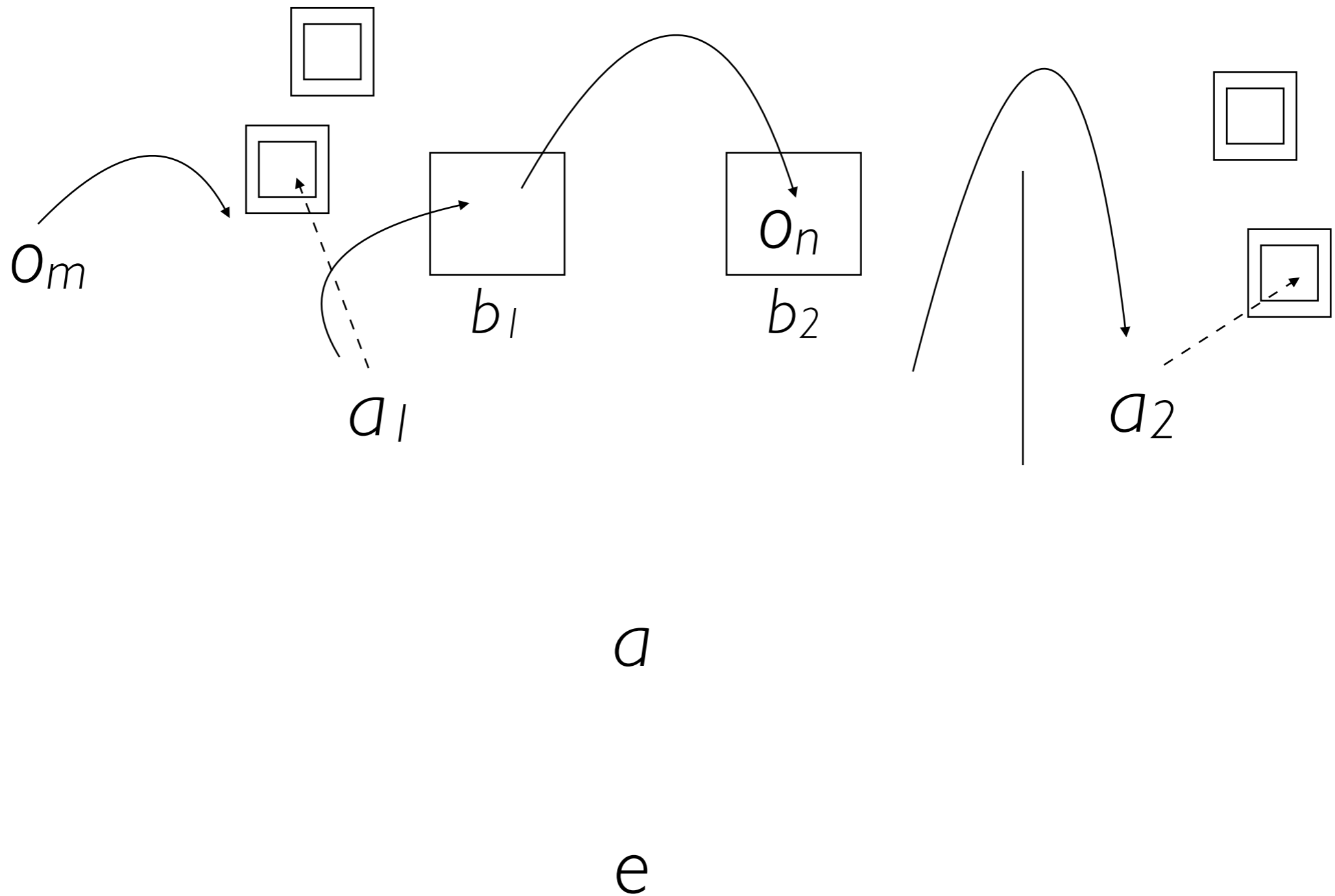
# Framework for FBT<sup>1</sup><sub>5</sub>

(ten timepoints)



# Framework for FBT<sup>1</sup><sub>5</sub>

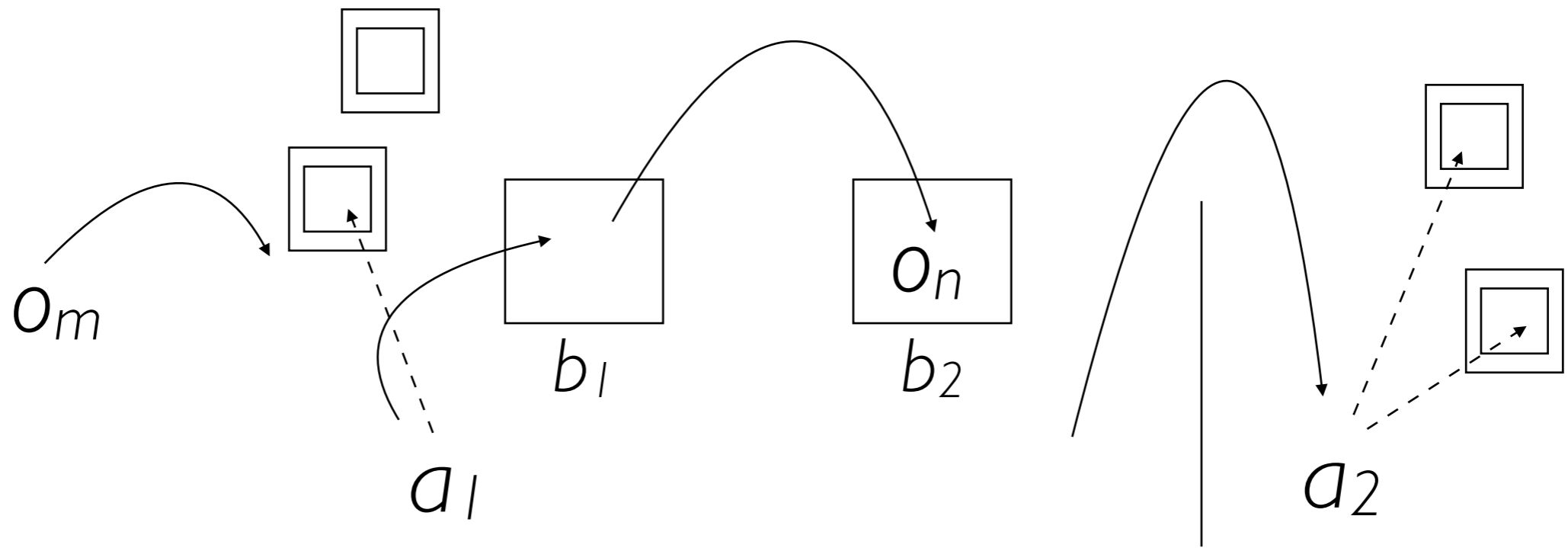
(ten timepoints)





# Framework for FBT<sup>1</sup><sub>5</sub>

(ten timepoints)

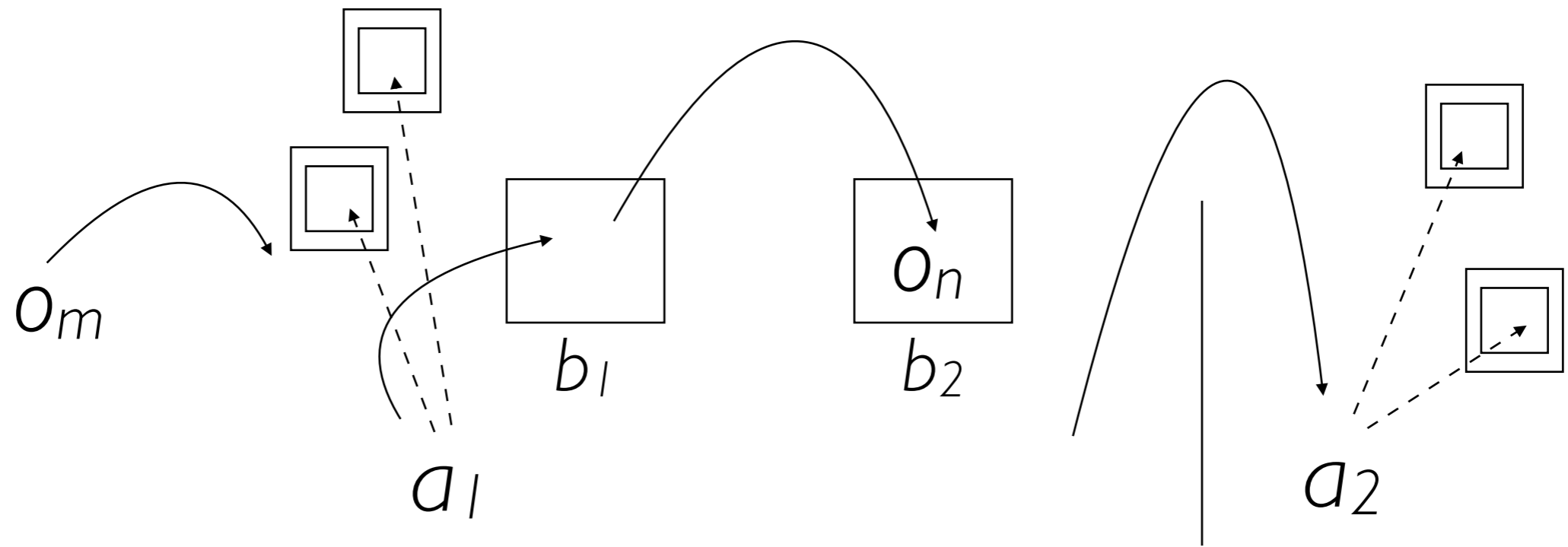


$a$

$e$

# Framework for FBT<sup>1</sup><sub>5</sub>

(ten timepoints)

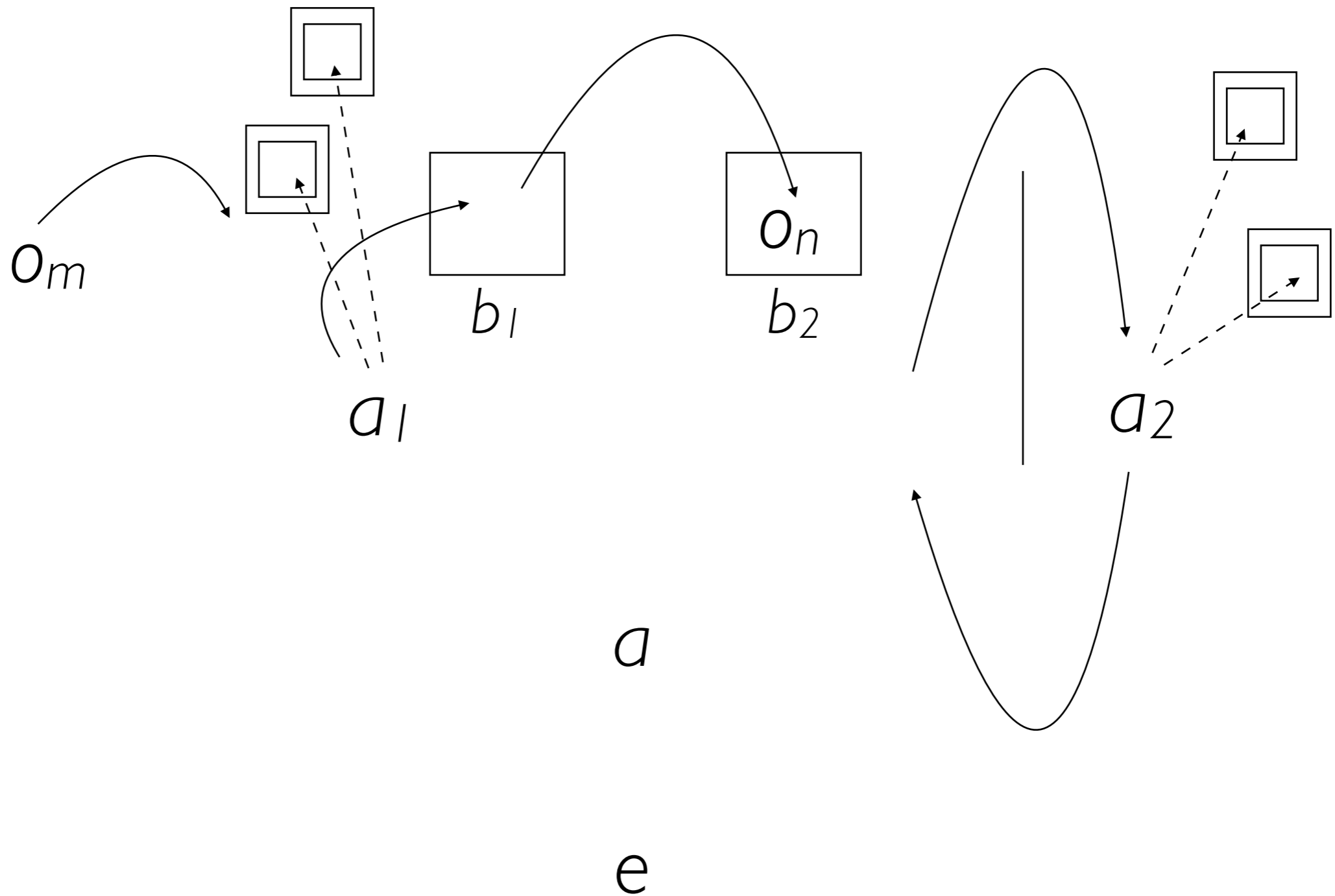


$a$

$e$

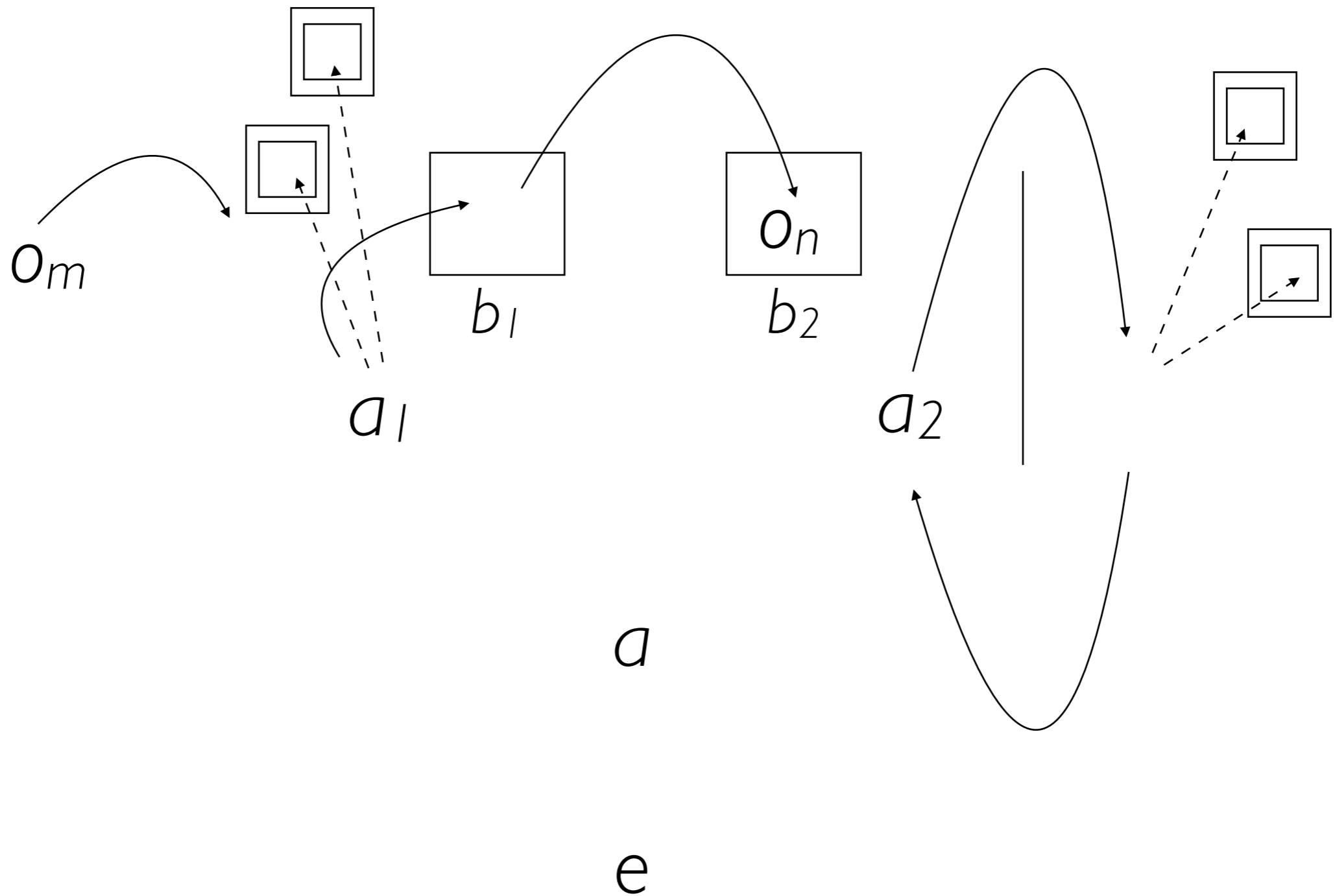
# Framework for FBT<sup>1</sup><sub>5</sub>

(ten timepoints)



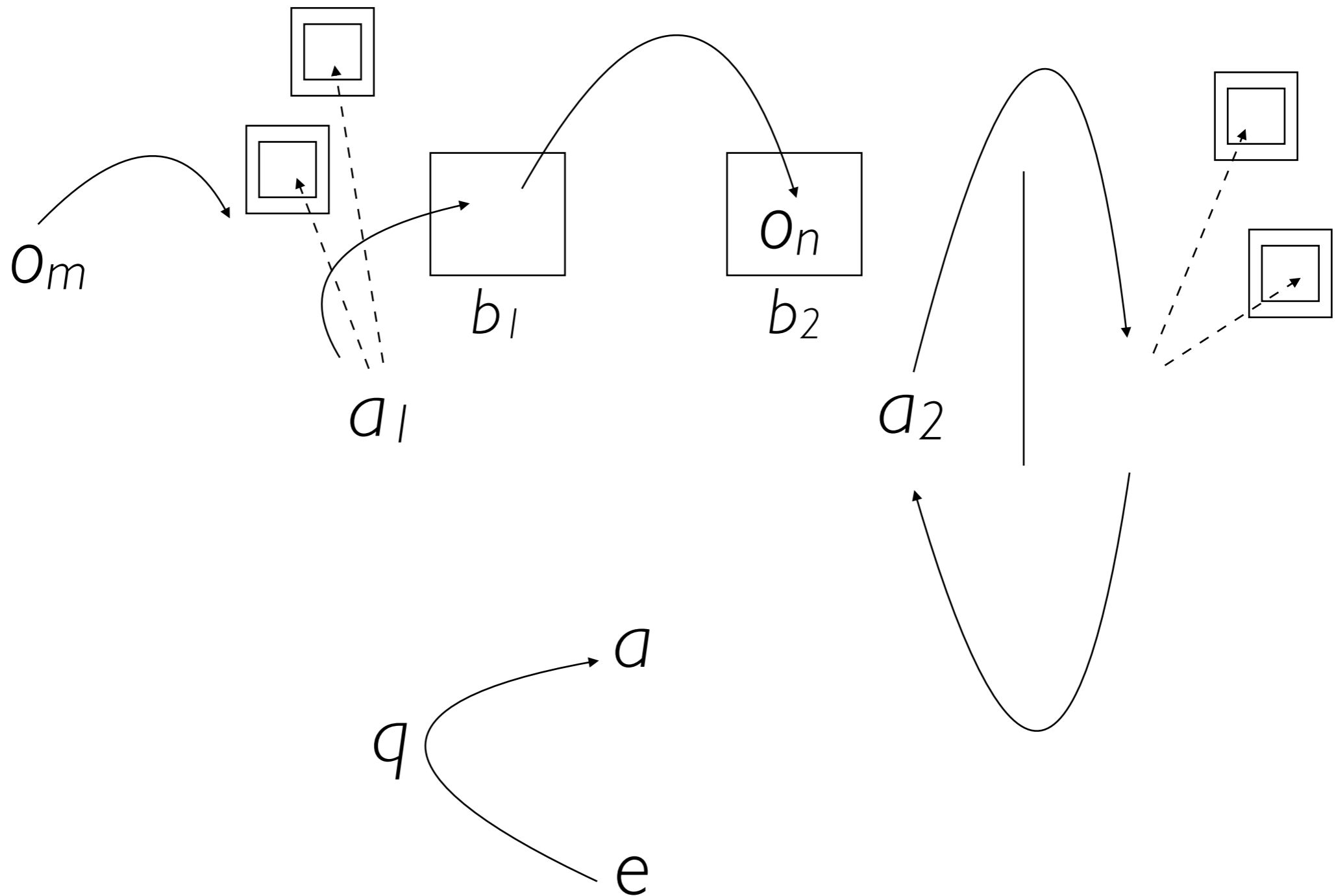
# Framework for FBT<sup>1</sup><sub>5</sub>

(ten timepoints)



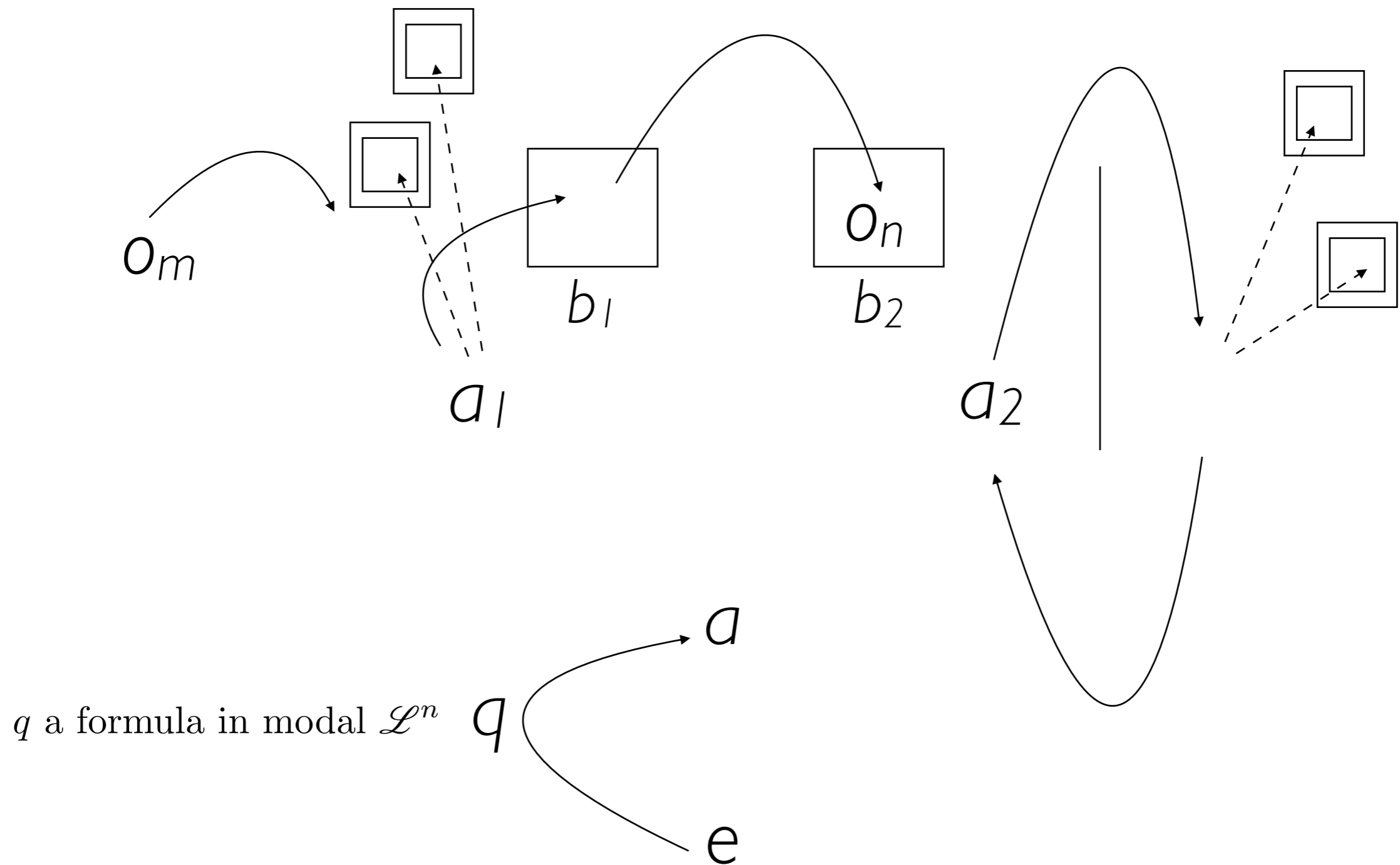
# Framework for $\text{FBT}^1_5$

(ten timepoints)



# Framework for $\text{FBT}^1_5$

(ten timepoints)



# Humans Can Succeed

Neurobiologically normal, nurtured, educated, and sufficiently motivated humans can correctly answer any relevant query  $q$  for the infinite progression, and prove that their answer is correct. For the obvious subclass of queries (the form of which appear in the box below), they can prove and exploit the following lemma.

**Lemma:** Suppose  $\text{FBT}_k, k \in \mathbb{Z}^+$ , holds; (i.e. that level  $k$  of FBT holds). Then, if  $k$  is even,  $\mathbf{B}_2\mathbf{B}_1 \dots \mathbf{B}_2 \iota$ , where there are  $k + 1$  iterated  $\mathbf{B}_i$  operators; otherwise  $\mathbf{B}_1\mathbf{B}_2 \dots \mathbf{B}_1\mathbf{B}_2 \iota$ , where there again there are  $k + 1$  iterated  $\mathbf{B}_i$  operators.

# Passing to Probing Mastery of the Specific Subclass

Experimenter to  $a$ : “At level  $k$ ,  
from which box will  $a_2$  attempt to  
retrieve the objects  $o_n$ ? Prove it!”



# *Theoretical* Machine Success on Infinite FBT!

**Theorem:**  $\forall q \in \mathcal{CC}, \mathfrak{M}$  can correctly answer and justify  $q$ .  
I.e.,  $\mathfrak{M}$  can pass  $\text{FBT}_\omega$ .

Ok, so this logic machine exists in the *mathematical* universe; but does there exist an *implemented* machine with this power?

# *Theoretical* Machine Success on Infinite FBT!

**Theorem:**  $\forall q \in \mathcal{CC}, \mathfrak{M}$  can correctly answer and justify  $q$ .  
I.e.,  $\mathfrak{M}$  can pass  $\text{FBT}_\omega$ .

Ok, so this logic machine exists in the *mathematical* universe; but does there exist an *implemented* machine with this power?

# Simulation Courtesy of ...

**ShadowProver!**



# Level 1

```
:name "Level 1: False Belief Task "  
  
:description "Agent a1 puts an object o into b1 in plain view of a2.  
Agent a2 then leaves, and in the absence of a2, a1 moves o  
from b1 into b2 ; this movement isn't perceived by a2 . Agent  
a2 now returns, and a is asked by the experimenter e: "If a2  
desires to retrieve o, which box will a2 look in?" If younger  
than four or five, a will reply "In b " (which of course fails 2  
the task); after this age subjects respond with the correct "In b1."  
  
Level1 Belief: a1 believes a2 believes o is in b1."  
  
:date "Monday July 22, 2019"  
  
:assumptions {  
  :P1 (Perceives! a1 t1 (Perceives! a2 t1 (holds (In o b1) t1)))  
  
  :P2 (Believes! a1 t2 (Believes! a2 t2 (not (exists [?e] (terminates ?e (In o b1))))))  
  
  :P3 (holds (In o b1) t1)  
  
  :C1 (Common! t0 (forall [?f ?t2 ?t2]  
    (if (and (not (exists [?e] (terminates ?e ?f))) (holds ?f ?t1) (< ?t1 ?t2))  
      (holds ?f ?t2))))  
  
  :C2 (Common! t0 (and (< t1 t2) (< t2 t3) (< t1 t3)))  
}  
  
:goal (Believes! a1 t3 (Believes! a2 t3 (holds (In o b1) t3)))}
```

# Level 2

```
{:name "Level 2: False Belief Task "  
  
:description "Agent a1 puts an object o into b1 in plain view of a2.  
Agent a2 then leaves, and in the absence of a2, a1 moves o  
from b1 into b2 ; this movement isn't perceived by a2 . Agent  
a2 now returns, and a is asked by the experimenter e: "If a2  
desires to retrieve o, which box will a2 look in?" If younger  
than four or five, a will reply "In b " (which of course fails 2  
the task); after this age subjects respond with the correct "In b1."  
  
Level2 Belief: a2 believes a1 believes a2 believes o is in b1."  
  
:date "Monday July 22, 2019"  
  
:assumptions {  
  
:P1 (Perceives! a2 t1 (Perceives! a1 t1 (Perceives! a2 t1 (holds (In o b1) t1))))  
  
:P2 (Believes! a2 t2 (Believes! a1 t2 (Believes! a2 t2 (not (exists [?e] (terminates ?e (In o b1)))))))  
  
:P3 (holds (In o b1) t1)  
  
:C1 (Common! t0  
      (forall [?f ?t2 ?t2]  
              (if (and (not (exists [?e] (terminates ?e ?f))) (holds ?f ?t1) (< ?t1 ?t2))  
                  (holds ?f ?t2))))  
  
:C2 (Common! t0 (and (< t1 t2) (< t2 t3) (< t1 t3)))}  
  
:goal (Believes! a2 t3 (Believes! a1 t3 (Believes! a2 t3 (holds (In o b1) t3))))}
```

# Level 3

```
{:name "Level 3: False Belief Task "  
  
:description "Agent a1 puts an object o into b1 in plain view of a2.  
Agent a2 then leaves, and in the absence of a2, a1 moves o  
from b1 into b2 ; this movement isn't perceived by a2 . Agent  
a2 now returns, and a is asked by the experimenter e: "If a2  
desires to retrieve o, which box will a2 look in?" If younger  
than four or five, a will reply "In b " (which of course fails 2  
the task); after this age subjects respond with the correct "In b1."  
  
Level3 Belief: a2 believes a1 believes a2 believes o is in b1.  
"  
  
:date "Monday July 22, 2019"  
  
:assumptions {  
  
:P1 (Perceives! a1 t1 (Perceives! a2 t1 (Perceives! a1 t1 (Perceives! a2 t1 (holds (In o b1) t1))))  
:P2 (Believes! a1 t2 (Believes! a2 t2 (Believes! a1 t2 (Believes! a2 t2 (not (exists [?e] (terminates ?e (In o b1)))))))  
  
:P3 (holds (In o b1) t1)  
  
:C1 (Common! t0  
| (forall [?f ?t2 ?t2]  
| | (if (and (not (exists [?e] (terminates ?e ?f))) (holds ?f ?t1) (< ?t1 ?t2))  
| | (holds ?f ?t2))))  
  
:C2 (Common! t0 (and (< t1 t2) (< t2 t3) (< t1 t3)))}  
  
:goal (Believes! a1 t3 (Believes! a2 t3 (Believes! a1 t3 (Believes! a2 t3 (holds (In o b1) t3))))))}
```

# Level 4

```
{:name      "Level 4: False Belief Task "
:description "Agent a1 puts an object o into b1 in plain view of a2.
Agent a2 then leaves, and in the absence of a2, a1 moves o
from b1 into b2 ; this movement isn't perceived by a2 . Agent
a2 now returns, and a is asked by the experimenter e: "If a2
desires to retrieve o, which box will a2 look in?" If younger
than four or five, a will reply "In b " (which of course fails 2
the task); after this age subjects respond with the correct "In b1."

Level4 Belief: a2 believes a1 believes a2 believes a1 believes a2 believes o is in b1.
"

:date      "Monday July 22, 2019"

:assumptions {

  :P1 (Perceives! a2 t1 (Perceives! a1 t1 (Perceives! a2 t1 (Perceives! a1 t1 (Perceives! a2 t1 (holds (In o b1) t1))))))
  :P2 (Believes! a2 t2 (Believes! a1 t2 (Believes! a2 t2 (Believes! a1 t2 (Believes! a2 t2 (not (exists [?e] (terminates ?e (In o b1))))))))))
  :P3 (holds (In o b1) t1)

  :C1 (Common! t0
      (forall [?f ?t2 ?t2]
        (if (and (not (exists [?e] (terminates ?e ?f))) (holds ?f ?t1) (< ?t1 ?t2))
            (holds ?f ?t2))))

  :C2 (Common! t0 (and (< t1 t2) (< t2 t3) (< t1 t3)))}

:goal      (Believes! a2 t3 (Believes! a1 t3 (Believes! a2 t3 (Believes! a1 t3 (Believes! a2 t3 (holds (In o b1) t3))))))}
```

# Level 5

```
{:name "Level 5: False Belief Task "

:description "Agent a1 puts an object o into b1 in plain view of a2.
Agent a2 then leaves, and in the absence of a2, a1 moves o
from b1 into b2 ; this movement isn't perceived by a2 . Agent
a2 now returns, and a is asked by the experimenter e: "If a2
desires to retrieve o, which box will a2 look in?" If younger
than four or five, a will reply "In b " (which of course fails 2
the task); after this age subjects respond with the correct "In b1."

Level5 Belief: a1 believes a2 believes a1 believes a2 believes a1 believes a2 believes o is in b1.
"

:date "Monday July 22, 2019"

:assumptions {

:P1 (Perceives! a1 t1 (Perceives! a2 t1 (Perceives! a1 t1 (Perceives! a2 t1 (Perceives! a1 t1 (Perceives! a2 t1 (holds (In o b1) t1))))))
:P2 (Believes! a1 t2 (Believes! a2 t2 (Believes! a1 t2 (Believes! a2 t2 (Believes! a1 t2 (Believes! a2 t2 (not (exists [?e] (terminates ?e (In o b1))))))))))
:P3 (holds (In o b1) t1)

:C1 (Common! t0
      (forall [?f ?t2 ?t2]
              (if (and (not (exists [?e] (terminates ?e ?f))) (holds ?f ?t1) (< ?t1 ?t2))
                  (holds ?f ?t2))))

:C2 (Common! t0 (and (< t1 t2) (< t2 t3) (< t1 t3)))}

:goal (Believes! a1 t3 (Believes! a2 t3 (Believes! a1 t3 (Believes! a2 t3 (Believes! a1 t3 (Believes! a2 t3 (holds (In o b1) t3))))))})
```



gent  
If a2  
If younger  
course fails 2  
in the correct "In b1."  
a1 believes a2 believes a1 believes a2 believes o is in b1.

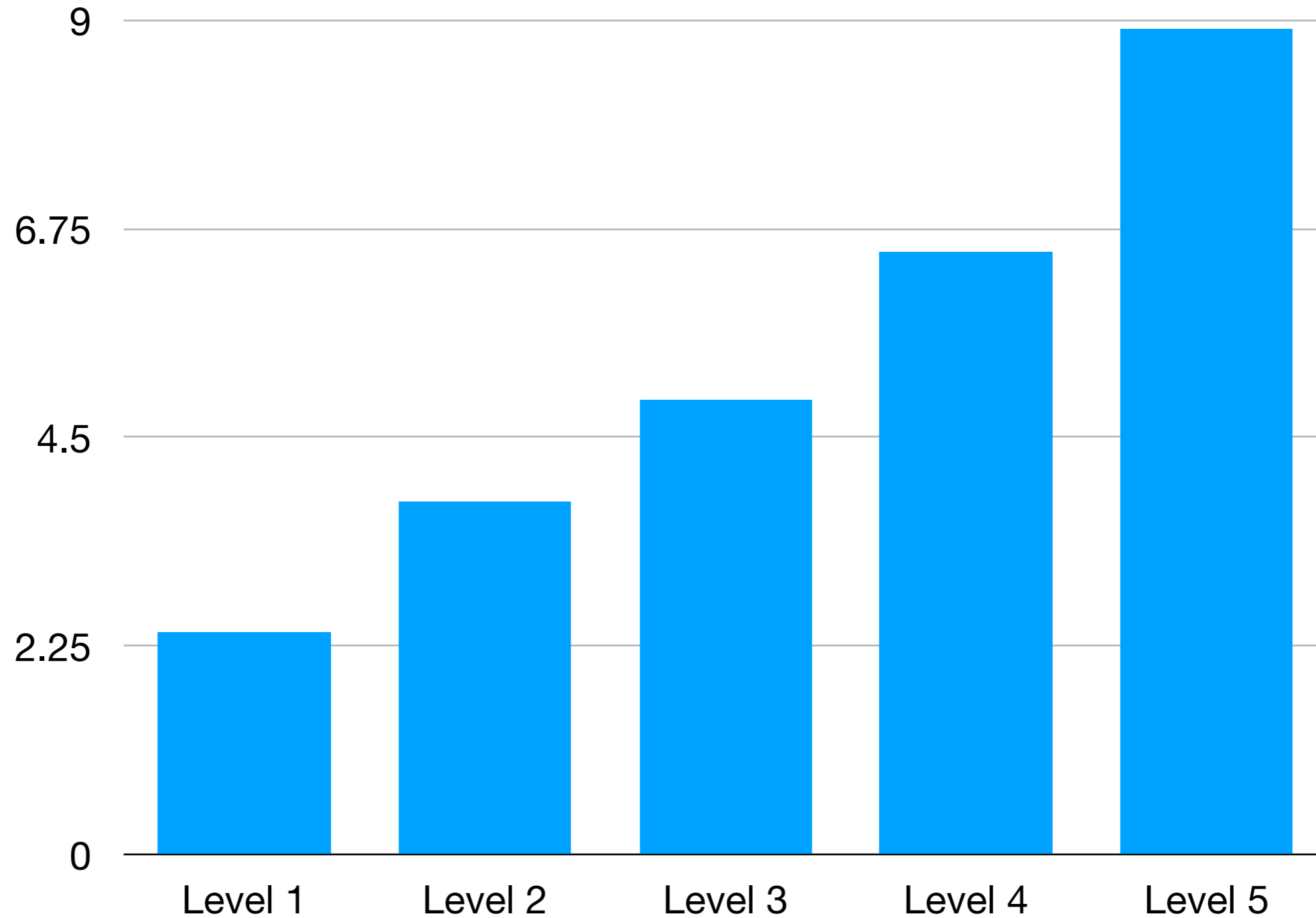
```
(Common! t0 (In o b1) t1)
(forall [?f ?t2 ?t2]
  (if (and (not (exists [?e] (terminates ?e ?f)))
    (holds ?f ?t2)))
    (< t1 t2) (< t2 t3) (< t1 t3)))}
:goal
(C2 (Common! t0 (and (< t1 t2) (< t2 t3) (< t1 t3)))}
(Believes! a1 t3 (Believes! a2 t3 (Believes! a1 t3 (Believes! a2 t3 (Believes! a1 t3 (Believes! a2 t3 (holds (In o b1) t3)))))))))}
```

```

    (In o b1) t1)
  (Common! t0
    (forall [?f ?t2 ?t2]
      (if (and (not (exists [?e] (terminates ?e ?f)))
        (holds ?f ?t2)))
        (holds ?f ?t1) (< ?t1 ?t2))
    (Believes! a1 t3 (Believes! a2 t3 (holds (In o b1) t3))))))}
:goal

```

# Time (in seconds) to Prove



# Simulation of Level 5 in Real Time

```
/Library/Java/JavaVirtualMachines/jdk1.8.0_131.jdk/Contents/Home/bin/java ...
```

```
objc[16653]: Class JavaLaunchHelper is implemented in both /Library/Java/JavaVirtualMachines/jdk1.8.0_131.jdk/Contents/Home/bin/java (0x102a2d4c0) and /Library/Java/JavaVirtualMachines/jdk1.8.0_131.jdk/Contents/Home/jre/lib/libinstrument.dylib (0x102ab94e0)
```

```
----- Level 5 -----
```

# Simulation of Level 5 in Real Time

```
/Library/Java/JavaVirtualMachines/jdk1.8.0_131.jdk/Contents/Home/bin/java ...
```

```
objc[16653]: Class JavaLaunchHelper is implemented in both /Library/Java/JavaVirtualMachines/jdk1.8.0_131.jdk/Contents/Home/bin/java (0x102a2d4c0) and /Library/Java/JavaVirtualMachines/jdk1.8.0_131.jdk/Contents/Home/jre/lib/libinstrument.dylib (0x102ab94e0)
```

```
----- Level 5 -----
```

# Encapsulation

Slate - K.slt

K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ K $\vdash$ ✓ $\infty$ $\Box$	T. $\Box\varphi \rightarrow \varphi$ K $\vdash$ ✗ $\infty$ $\Box$	4. $\Box\varphi \rightarrow \Box\Box\varphi$ K $\vdash$ ✗ $\infty$ $\Box$	5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ K $\vdash$ ✗ $\infty$ $\Box$
--	--	--	--

# Encapsulation

The image shows two windows from a software application named 'Slate'. The top window is titled 'Slate - K.slt' and contains four boxes, each with a modal logic formula and its status in the K system. The bottom window is titled 'Slate - T.slt' and contains the same four boxes, but with their status in the T system.

Formula	K System Status	T System Status
K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$	$K \vdash \checkmark \infty \Box$	$M \vdash \checkmark \infty \Box$
T. $\Box\varphi \rightarrow \varphi$	$K \vdash \times \infty \Box$	$M \vdash \checkmark \infty \Box$
4. $\Box\varphi \rightarrow \Box\Box\varphi$	$K \vdash \times \infty \Box$	$M \vdash \times \infty \Box$
5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$	$K \vdash \times \infty \Box$	$M \vdash \times \infty \Box$

# Encapsulation

The image displays three overlapping windows from the Slate application, each showing a set of modal logic formulas and their validity in various systems. The windows are titled "Slate - K.slt", "Slate - T.slt", and "Slate - D.slt".

**Slate - K.slt**

- K.  $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$   
K  $\vdash \checkmark \infty \Box$
- T.  $\Box\varphi \rightarrow \varphi$   
K  $\vdash \times \infty \Box$
- 4.  $\Box\varphi \rightarrow \Box\Box\varphi$   
K  $\vdash \times \infty \Box$
- 5.  $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$   
K  $\vdash \times \infty \Box$

**Slate - T.slt**

- K.  $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$   
M  $\vdash \checkmark \infty \Box$
- T.  $\Box\varphi \rightarrow \varphi$   
M  $\vdash \checkmark \infty \Box$
- 4.  $\Box\varphi \rightarrow \Box\Box\varphi$   
M  $\vdash \times \infty \Box$
- 5.  $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$   
M  $\vdash \times \infty \Box$

**Slate - D.slt**

- K.  $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$   
D  $\vdash \checkmark \infty \Box$
- T.  $\Box\varphi \rightarrow \varphi$   
D  $\vdash \times \infty \Box$
- D.  $\Box\varphi \rightarrow \Diamond\varphi$   
D  $\vdash \checkmark \infty \Box$
- 4.  $\Box\varphi \rightarrow \Box\Box\varphi$   
D  $\vdash \times \infty \Box$
- 5.  $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$   
D  $\vdash \times \infty \Box$
- INTER.  $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$   
D  $\vdash \checkmark \infty \Box$



# Encapsulation

The image displays four overlapping Slate windows, each showing a set of modal logic formulas and their derivability in various systems. The windows are titled 'Slate - K.slt', 'Slate - T.slt', 'Slate - D.slt', and 'Slate - S4.slt'.

**Slate - K.slt**

- K.  $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$   
K  $\vdash \checkmark \infty \Box$
- T.  $\Box\varphi \rightarrow \varphi$   
K  $\vdash \times \infty \Box$
- 4.  $\Box\varphi \rightarrow \Box\Box\varphi$   
K  $\vdash \times \infty \Box$
- 5.  $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$   
K  $\vdash \times \infty \Box$

**Slate - T.slt**

- K.  $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$   
M  $\vdash \checkmark \infty \Box$
- T.  $\Box\varphi \rightarrow \varphi$   
M  $\vdash \checkmark \infty \Box$
- 4.  $\Box\varphi \rightarrow \Box\Box\varphi$   
M  $\vdash \times \infty \Box$
- 5.  $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$   
M  $\vdash \times \infty \Box$

**Slate - D.slt**

- K.  $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$   
D  $\vdash \checkmark \infty \Box$
- T.  $\Box\varphi \rightarrow \varphi$   
D  $\vdash \times \infty \Box$
- D.  $\Box\varphi \rightarrow \Diamond\varphi$   
D  $\vdash \checkmark \infty \Box$
- 4.  $\Box\varphi \rightarrow \Box\Box\varphi$   
D  $\vdash \times \infty \Box$
- 5.  $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$   
D  $\vdash \times \infty \Box$
- INTER.  $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$   
D  $\vdash \checkmark \infty \Box$

**Slate - S4.slt**

- K.  $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$   
S4  $\vdash \checkmark \infty \Box$
- T.  $\Box\varphi \rightarrow \varphi$   
S4  $\vdash \checkmark \infty \Box$
- D.  $\Box\varphi \rightarrow \Diamond\varphi$   
S4  $\vdash \checkmark \infty \Box$
- 4.  $\Box\varphi \rightarrow \Box\Box\varphi$   
S4  $\vdash \checkmark \infty \Box$
- 5.  $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$   
S4  $\vdash \times \infty \Box$
- INTER.  $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$   
{INTER} Assume  $\checkmark$

# Encapsulation

**K**

**T**

**D**

**4 = S4**

**5 = S5**

The image shows five overlapping Slate windows, each displaying a grid of modal logic formulas and their derivability in different systems. The windows are titled as follows:

- Slate - K.slt**: Shows formulas K, T, 4, and 5. K is derivable in K (K ⊢ ✓ ∞ □). T, 4, and 5 are not derivable in K (K ⊢ ✗ ∞ □).
- Slate - T.slt**: Shows formulas K, T, 4, and 5. K and T are derivable in M (M ⊢ ✓ ∞ □). 4 and 5 are not derivable in M (M ⊢ ✗ ∞ □).
- Slate - D.slt**: Shows formulas K, T, D, 4, 5, and INTER. K, T, and 4 are not derivable in D (D ⊢ ✗ ∞ □). D and 5 are derivable in D (D ⊢ ✓ ∞ □). INTER is derivable in D (D ⊢ ✓ ∞ □).
- Slate - S4.slt**: Shows formulas K, T, D, 4, 5, and INTER. K, T, D, and 4 are derivable in S4 (S4 ⊢ ✓ ∞ □). 5 is not derivable in S4 (S4 ⊢ ✗ ∞ □). INTER is derivable in S4 with the assumption {INTER} (S4 ⊢ ✓ ∞ □).
- Slate - S5.slt**: Shows formulas K, T, D, 4, 5, and INTER. K, T, and 5 are derivable in S5 (S5 ⊢ ✓ ∞ □). D and 4 are not derivable in S5 (S5 ⊢ ✗ ∞ □). D and 4 are derivable in S5 with assumptions {D} and {4} respectively (S5 ⊢ ✓ ∞ □). INTER is derivable in S5 with the assumption {INTER} (S5 ⊢ ✓ ∞ □).

# Encapsulation

**K**

**T**

**D**

**4 = S4**

**5 = S5**

The image shows five Slate windows, each displaying a set of modal logic formulas and their derivability in a specific system. The windows are titled as follows:

- Slate - K.slt**: Shows formulas K, T, 4, and 5. K is derivable (K ⊢ ✓ ∞ □), while T, 4, and 5 are not (K ⊢ ✗ ∞ □).
- Slate - T.slt**: Shows formulas K, T, 4, and 5. K and T are derivable (M ⊢ ✓ ∞ □), while 4 and 5 are not (M ⊢ ✗ ∞ □).
- Slate - D.slt** (highlighted with a red border): Shows formulas K, T, D, 4, 5, and INTER. K, T, and D are derivable (D ⊢ ✓ ∞ □), while 4 and 5 are not (D ⊢ ✗ ∞ □). The INTER formula is derivable (D ⊢ ✓ ∞ □).
- Slate - S4.slt**: Shows formulas K, T, D, 4, 5, and INTER. All formulas are derivable (S4 ⊢ ✓ ∞ □). The INTER formula is derivable with the assumption {INTER} (S4 ⊢ ✓ ∞ □).
- Slate - S5.slt**: Shows formulas K, T, D, 4, 5, and INTER. All formulas are derivable (S5 ⊢ ✓ ∞ □). The INTER formula is derivable with the assumption {INTER} (S5 ⊢ ✓ ∞ □).

# Encapsulation

**K**

**T**

**D**

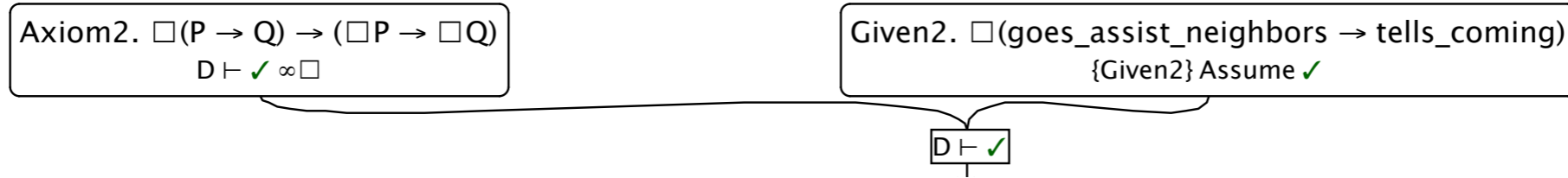
The screenshot displays five windows of the HyperSlate interface, each showing a set of logical formulas and their derivability status in a specific modal logic. The windows are titled as follows:

- Slate - K.slt**: Shows formulas K, T, 4, and 5. K is derivable (K ⊢ ✓ ∞ □), while T, 4, and 5 are not (K ⊢ ✗ ∞ □).
- Slate - T.slt**: Shows formulas K, T, 4, and 5. K and T are derivable (M ⊢ ✓ ∞ □), while 4 and 5 are not (M ⊢ ✗ ∞ □).
- Slate - D.slt** (highlighted with a red border): Shows formulas K, T, D, 4, 5, and INTER. K, T, and D are derivable (D ⊢ ✓ ∞ □), while 4 and 5 are not (D ⊢ ✗ ∞ □). The INTER formula is derivable (D ⊢ ✓ ∞ □).
- Slate - S4.slt**: Shows formulas K, T, D, 4, 5, and INTER. All formulas are derivable (S4 ⊢ ✓ ∞ □). The INTER formula is marked with "{INTER} Assume ✓".
- Slate - S5.slt**: Shows formulas K, T, D, 4, 5, and INTER. All formulas are derivable (S5 ⊢ ✓ ∞ □). The D and 4 formulas are marked with "{D} Assume ✓" and "{4} Assume ✓" respectively. The INTER formula is marked with "{INTER} Assume ✓".

**4 = S4**

**5 = S5**

# Chisholm's Paradox

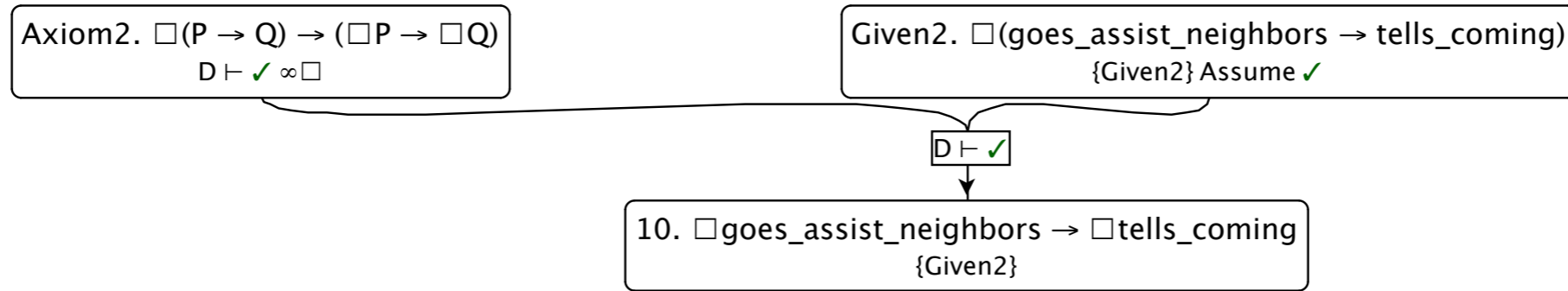


Axiom4. "Modus ponens for provability."  
{Axiom4} Assume ✓

Axiom5. "Theorems are obligatory."  
{Axiom5} Assume ✓

Axiom1. "All theorems of the propositional calculus."  
{Axiom1} Assume ✓

# Chisholm's Paradox

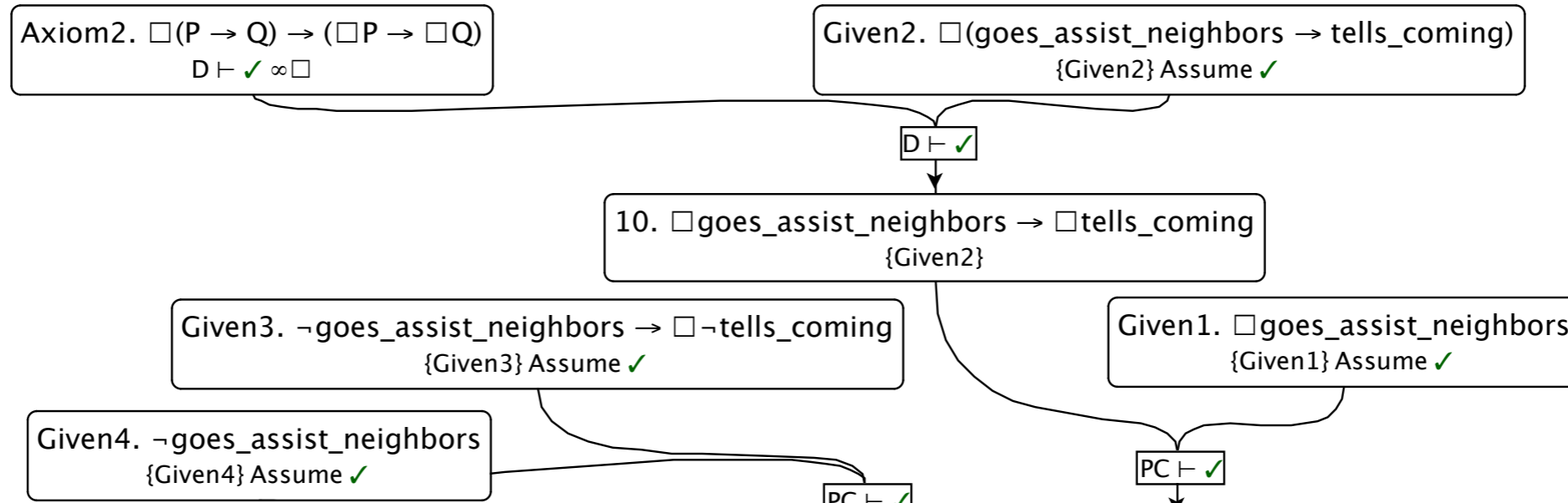


Axiom4. "Modus ponens for provability."  
{Axiom4} Assume  $\checkmark$

Axiom5. "Theorems are obligatory."  
{Axiom5} Assume  $\checkmark$

Axiom1. "All theorems of the propositional calculus."  
{Axiom1} Assume  $\checkmark$

# Chisholm's Paradox

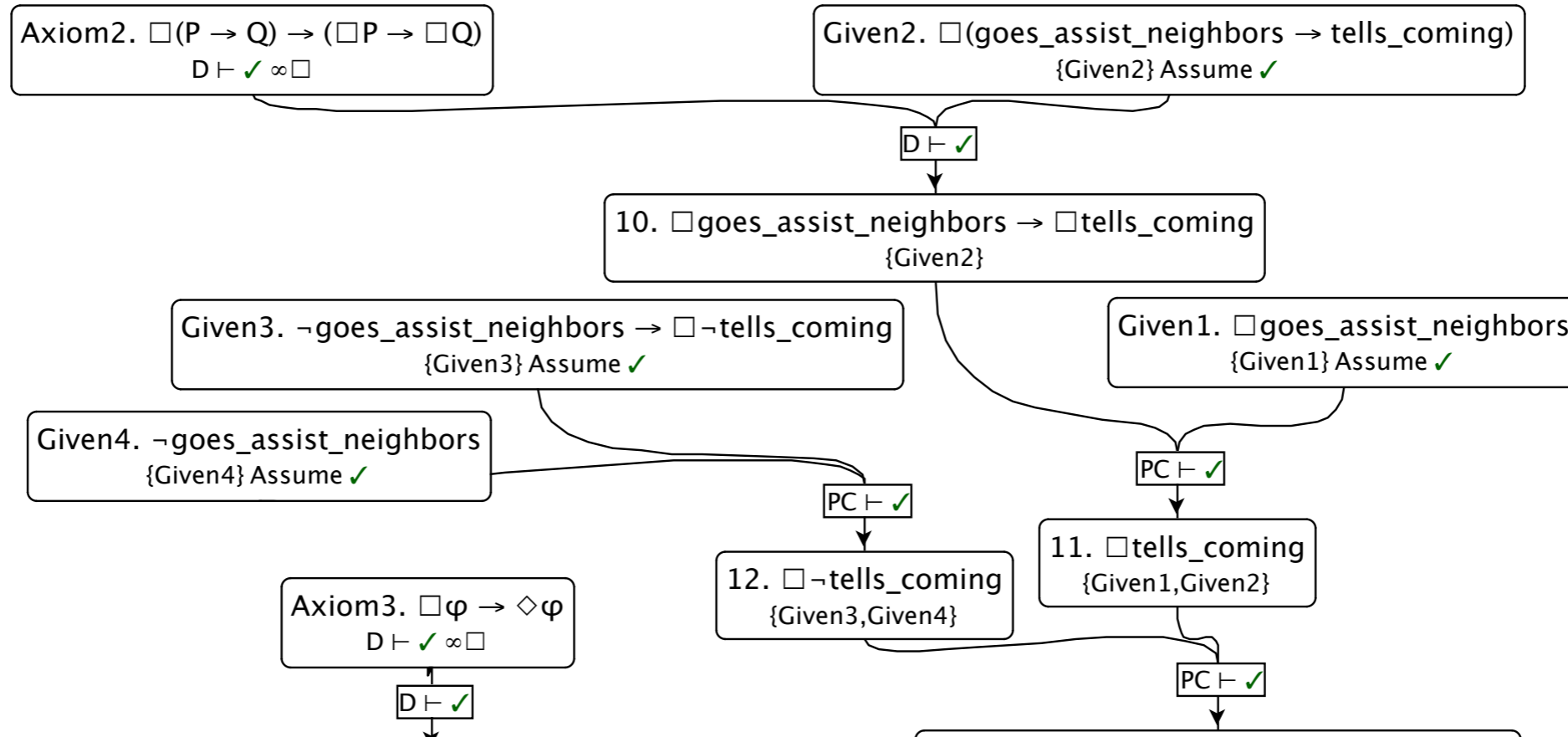


Axiom4. "Modus ponens for provability."  
 $\{\text{Axiom4}\} \text{Assume } \checkmark$

Axiom5. "Theorems are obligatory."  
 $\{\text{Axiom5}\} \text{Assume } \checkmark$

Axiom1. "All theorems of the propositional calculus."  
 $\{\text{Axiom1}\} \text{Assume } \checkmark$

# Chisholm's Paradox



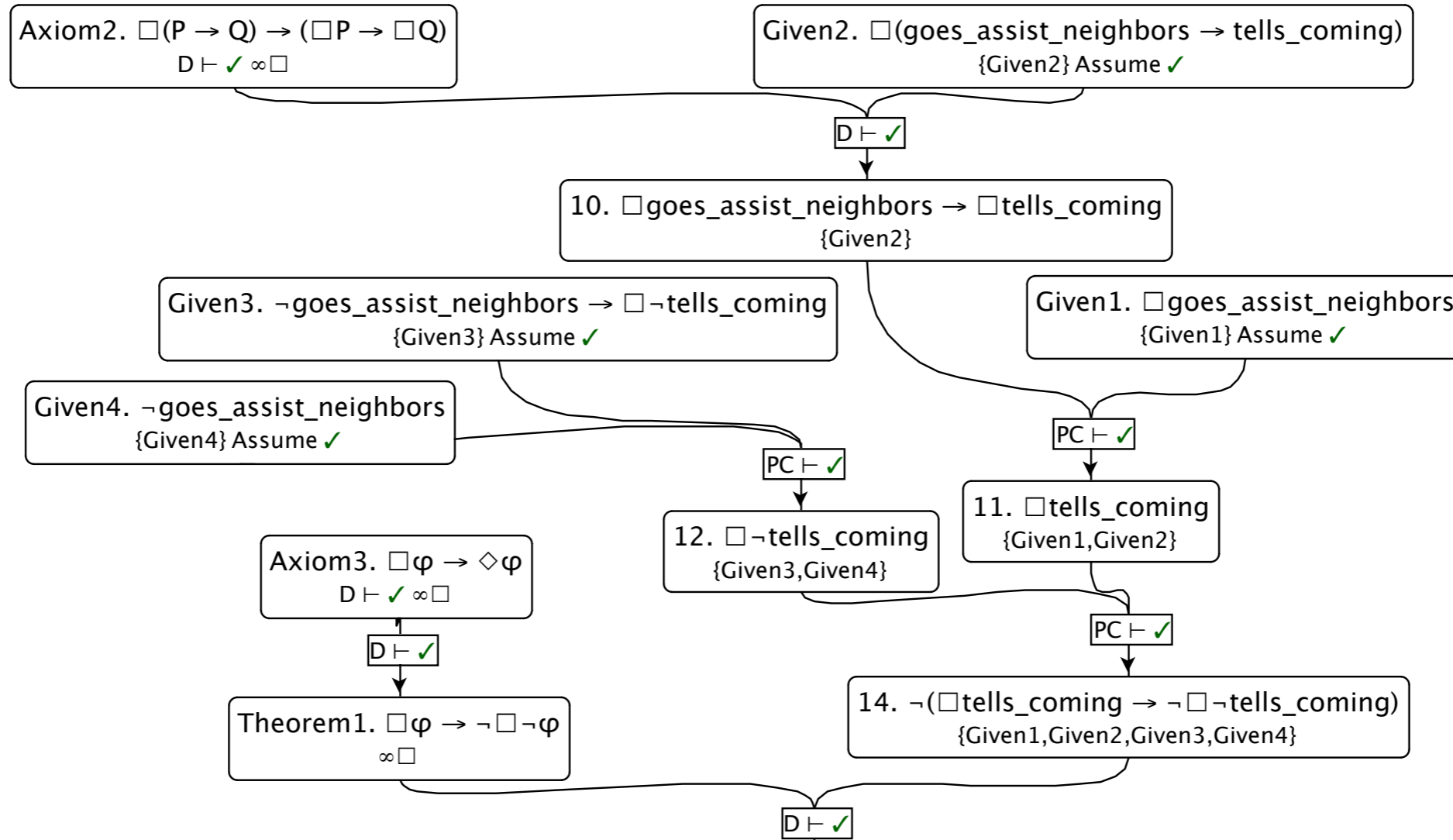
Axiom4. "Modus ponens for provability."  
 $\{\text{Axiom4}\} \text{Assume } \checkmark$

Axiom5. "Theorems are obligatory."  
 $\{\text{Axiom5}\} \text{Assume } \checkmark$

Axiom1. "All theorems of the propositional calculus."  
 $\{\text{Axiom1}\} \text{Assume } \checkmark$



# Chisholm's Paradox

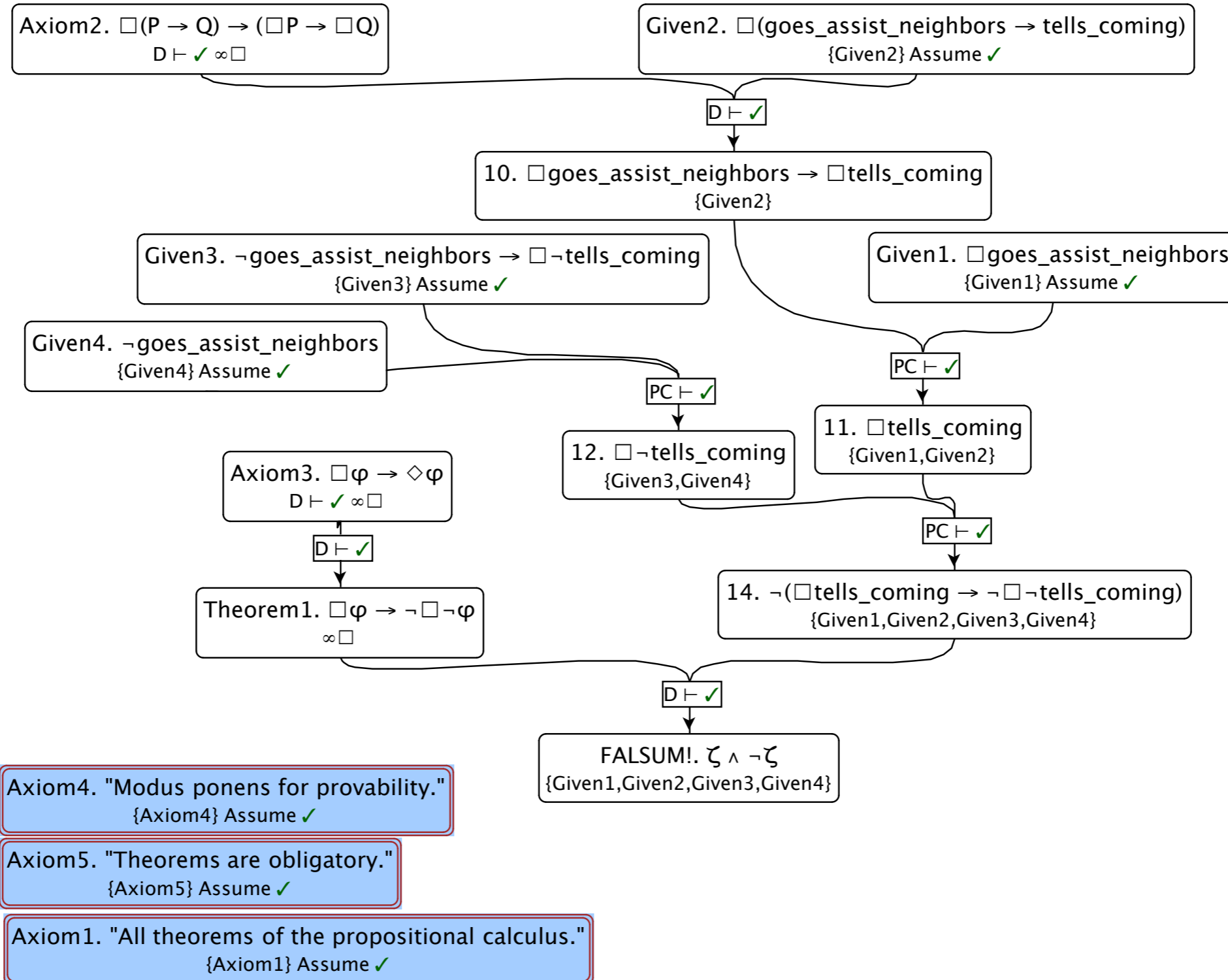


Axiom4. "Modus ponens for provability."  
 $\{\text{Axiom4}\} \text{Assume } \checkmark$

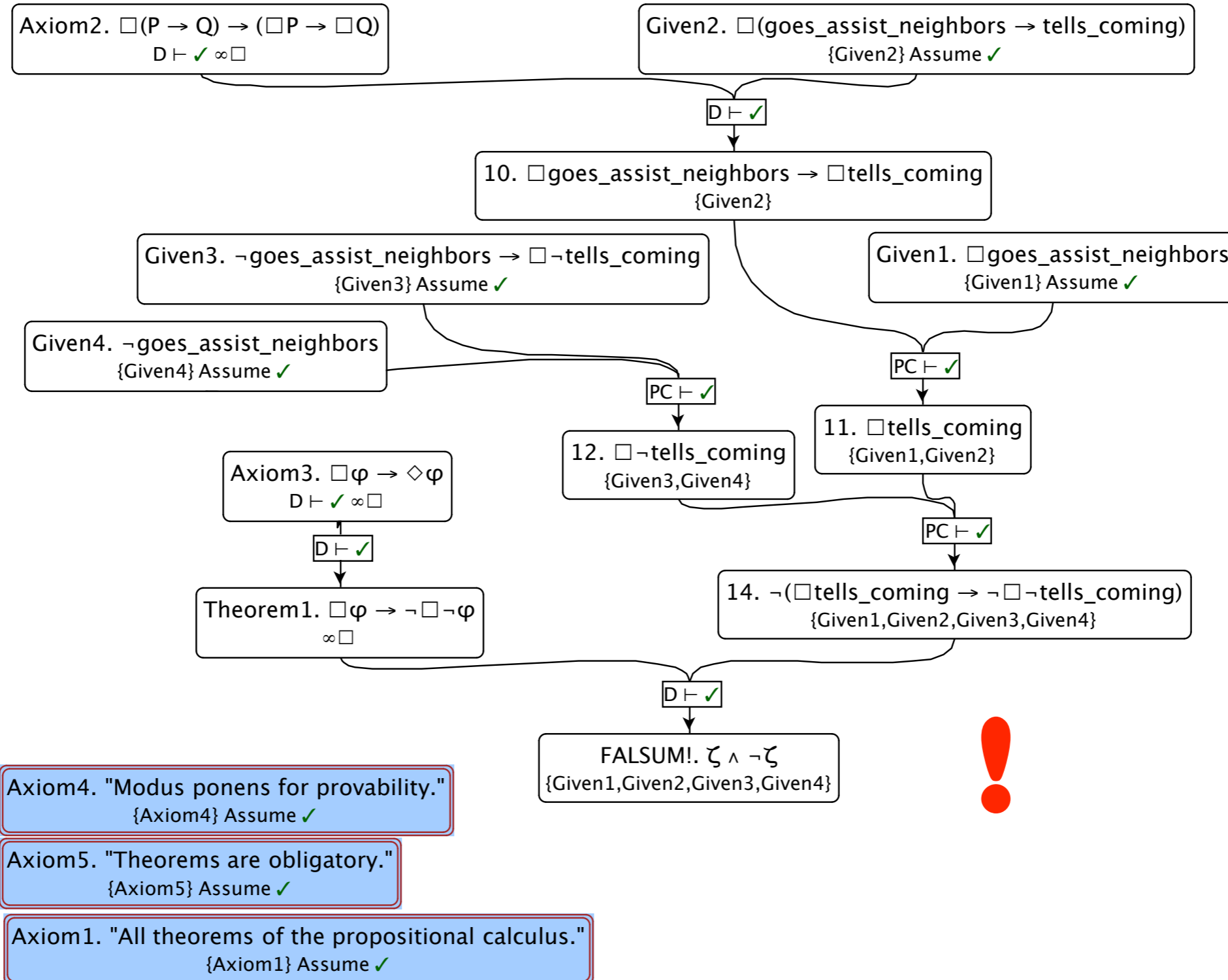
Axiom5. "Theorems are obligatory."  
 $\{\text{Axiom5}\} \text{Assume } \checkmark$

Axiom1. "All theorems of the propositional calculus."  
 $\{\text{Axiom1}\} \text{Assume } \checkmark$

# Chisholm's Paradox



# Chisholm's Paradox



# Review: Encapsulation

**K**

**T**

**D**

**4 = S4**

**5 = S5**

The screenshot displays five windows, each showing a grid of logical formulas and their derivability status in a specific modal logic. The status is indicated by a checkmark (✓) for derivability and a red 'X' for non-derivability.

- Slate - K.slt:**
  - K.  $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$  (K  $\vdash \checkmark \infty \Box$ )
  - T.  $\Box\varphi \rightarrow \varphi$  (K  $\vdash \times \infty \Box$ )
  - 4.  $\Box\varphi \rightarrow \Box\Box\varphi$  (K  $\vdash \times \infty \Box$ )
  - 5.  $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$  (K  $\vdash \times \infty \Box$ )
- Slate - T.slt:**
  - K.  $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$  (M  $\vdash \checkmark \infty \Box$ )
  - T.  $\Box\varphi \rightarrow \varphi$  (M  $\vdash \checkmark \infty \Box$ )
  - 4.  $\Box\varphi \rightarrow \Box\Box\varphi$  (M  $\vdash \times \infty \Box$ )
  - 5.  $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$  (M  $\vdash \times \infty \Box$ )
- Slate - D.slt:**
  - K.  $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$  (D  $\vdash \checkmark \infty \Box$ )
  - T.  $\Box\varphi \rightarrow \varphi$  (D  $\vdash \times \infty \Box$ )
  - D.  $\Box\varphi \rightarrow \Diamond\varphi$  (D  $\vdash \checkmark \infty \Box$ )
  - 4.  $\Box\varphi \rightarrow \Box\Box\varphi$  (D  $\vdash \times \infty \Box$ )
  - 5.  $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$  (D  $\vdash \times \infty \Box$ )
  - INTER.  $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$  (D  $\vdash \checkmark \infty \Box$ )
- Slate - S4.slt:**
  - K.  $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$  (S4  $\vdash \checkmark \infty \Box$ )
  - T.  $\Box\varphi \rightarrow \varphi$  (S4  $\vdash \checkmark \infty \Box$ )
  - D.  $\Box\varphi \rightarrow \Diamond\varphi$  (S4  $\vdash \checkmark \infty \Box$ )
  - 4.  $\Box\varphi \rightarrow \Box\Box\varphi$  (S4  $\vdash \checkmark \infty \Box$ )
  - 5.  $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$  (S4  $\vdash \times \infty \Box$ )
  - INTER.  $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$  ({INTER} Assume ✓)
- Slate - S5.slt:**
  - K.  $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$  (S5  $\vdash \checkmark \infty \Box$ )
  - T.  $\Box\varphi \rightarrow \varphi$  (S5  $\vdash \checkmark \infty \Box$ )
  - D.  $\Box\varphi \rightarrow \Diamond\varphi$  ({D} Assume ✓)
  - 4.  $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$  ({4} Assume ✓)
  - 5.  $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$  (S5  $\vdash \checkmark \infty \Box$ )
  - INTER.  $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$  ({INTER} Assume ✓)

# Review: Encapsulation

**K**

**T**

**D**

The screenshot displays five windows of the HyperSlate interface, each showing logical formulas and their provability status in different modal logics. The windows are titled as follows:

- Slate - K.slt**: Shows formulas for K, T, 4, and 5. K is provable (K ⊢ ✓ ∞ □), while T, 4, and 5 are not (K ⊢ ✗ ∞ □).
- Slate - T.slt**: Shows formulas for K, T, 4, and 5. K and T are provable (M ⊢ ✓ ∞ □), while 4 and 5 are not (M ⊢ ✗ ∞ □).
- Slate - D.slt**: Shows formulas for K, T, D, 4, 5, and INTER. K and T are not provable (D ⊢ ✗ ∞ □), while D and INTER are provable (D ⊢ ✓ ∞ □). 4 and 5 are not provable (D ⊢ ✗ ∞ □).
- Slate - S4.slt**: Shows formulas for K, T, D, 4, 5, and INTER. K, T, D, and 4 are provable (S4 ⊢ ✓ ∞ □), while 5 is not (S4 ⊢ ✗ ∞ □). INTER is provable with the assumption {INTER} (S4 ⊢ ✓ ∞ □).
- Slate - S5.slt**: Shows formulas for K, T, D, 4, 5, and INTER. K, T, D, and 5 are provable (S5 ⊢ ✓ ∞ □), while 4 is not (S5 ⊢ ✗ ∞ □). INTER is provable with the assumption {INTER} (S5 ⊢ ✓ ∞ □).

The bottom two windows (S4.slt and S5.slt) are highlighted with a red border. On the left side, there are two boxes with the following text:

- 4 = S4**
- 5 = S5**

# Review: Encapsulation

K

T

D

4 = S4

5 = S5

The screenshot displays the HyperSlate interface with several windows showing logical formulae and their derivability in different modal logics. A green box highlights the 'Create file' menu, and a red box highlights the S4 and S5 logic windows.

**Windows and Formulae:**

- Slate - K.slt:**
  - K.  $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$  (K  $\vdash \checkmark \infty \Box$ )
  - T.  $\Box\varphi \rightarrow \varphi$  (K  $\vdash \times \infty \Box$ )
  - 4.  $\Box\varphi \rightarrow \Box\Box\varphi$  (K  $\vdash \times \infty \Box$ )
  - 5.  $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$  (K  $\vdash \times \infty \Box$ )
- Slate - T.slt:**
  - K.  $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$  (M  $\vdash \checkmark \infty \Box$ )
  - T.  $\Box\varphi \rightarrow \varphi$  (M  $\vdash \checkmark \infty \Box$ )
  - 4.  $\Box\varphi \rightarrow \Box\Box\varphi$  (M  $\vdash \times \infty \Box$ )
  - 5.  $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$  (M  $\vdash \times \infty \Box$ )
- Create file (Green Box):**
  - Propositional Calculus
  - L<sub>0</sub> = Pure Predicate Calculus
  - L<sub>1</sub> = First-order Logic
  - L<sub>2</sub> = Second-order Logic
  - K
  - T
  - D
  - S4
  - S5
  - DCEC (fragment)
  - Hyperlog
- Slate - S4.slt (Red Box):**
  - K.  $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$  (S4  $\vdash \checkmark \infty \Box$ )
  - T.  $\Box\varphi \rightarrow \varphi$  (S4  $\vdash \checkmark \infty \Box$ )
  - D.  $\Box\varphi \rightarrow \Diamond\varphi$  (S4  $\vdash \checkmark \infty \Box$ )
  - 4.  $\Box\varphi \rightarrow \Box\Box\varphi$  (S4  $\vdash \checkmark \infty \Box$ )
  - 5.  $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$  (S4  $\vdash \times \infty \Box$ )
  - INTER.  $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$  ({INTER} Assume  $\checkmark$ )
- Slate - S5.slt (Red Box):**
  - K.  $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$  (S5  $\vdash \checkmark \infty \Box$ )
  - T.  $\Box\varphi \rightarrow \varphi$  (S5  $\vdash \checkmark \infty \Box$ )
  - D.  $\Box\varphi \rightarrow \Diamond\varphi$  ({D} Assume  $\checkmark$ )
  - 4.  $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$  ({4} Assume  $\checkmark$ )
  - 5.  $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$  (S5  $\vdash \checkmark \infty \Box$ )
  - INTER.  $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$  ({INTER} Assume  $\checkmark$ )

# Review: Encapsulation

K

T

D

The screenshot displays the HyperSlate interface with several windows showing logic calculi and their verification results. A green box highlights the 'Create file' dialog, and a red box highlights the 'Slate - S4.slt' and 'Slate - S5.slt' windows. A green arrow points to the 'S5' button in the 'Create file' dialog.

**Create file dialog:**

- Propositional Calculus
- $L_0$  = Pure Predicate Calculus
- $L_1$  = First-order Logic
- $L_2$  = Second-order Logic
- K
- T
- D
- S4
- S5
- DCEC (fragment)
- Hyperlog

**Slate - K.slt:**

- K.  $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$   $K \vdash \checkmark \infty \Box$
- T.  $\Box\varphi \rightarrow \varphi$   $K \vdash \times \infty \Box$
- 4.  $\Box\varphi \rightarrow \Box\Box\varphi$   $K \vdash \times \infty \Box$
- 5.  $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$   $K \vdash \times \infty \Box$

**Slate - T.slt:**

- K.  $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$   $M \vdash \checkmark \infty \Box$
- T.  $\Box\varphi \rightarrow \varphi$   $M \vdash \checkmark \infty \Box$
- 4.  $\Box\varphi \rightarrow \Box\Box\varphi$   $M \vdash \times \infty \Box$
- 5.  $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$   $M \vdash \times \infty \Box$

**Slate - S4.slt:**

- K.  $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$   $S4 \vdash \checkmark \infty \Box$
- T.  $\Box\varphi \rightarrow \varphi$   $S4 \vdash \checkmark \infty \Box$
- D.  $\Box\varphi \rightarrow \Diamond\varphi$   $S4 \vdash \checkmark \infty \Box$
- 4.  $\Box\varphi \rightarrow \Box\Box\varphi$   $S4 \vdash \checkmark \infty \Box$
- 5.  $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$   $S4 \vdash \times \infty \Box$
- INTER.  $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$   $\{INTER\} \text{ Assume } \checkmark$

**Slate - S5.slt:**

- K.  $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$   $S5 \vdash \checkmark \infty \Box$
- T.  $\Box\varphi \rightarrow \varphi$   $S5 \vdash \checkmark \infty \Box$
- D.  $\Box\varphi \rightarrow \Diamond\varphi$   $\{D\} \text{ Assume } \checkmark$
- 4.  $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$   $\{4\} \text{ Assume } \checkmark$
- 5.  $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$   $S5 \vdash \checkmark \infty \Box$
- INTER.  $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$   $\{INTER\} \text{ Assume } \checkmark$

4 = S4

5 = S5





*Det er en logikk for  
hvert problem!*