An Introduction to Non-Axiomatic Logic And Some Quantification

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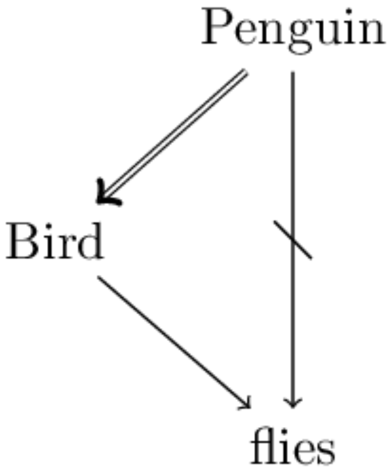
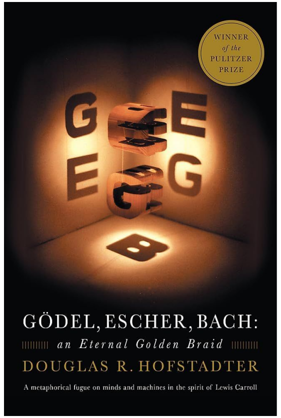
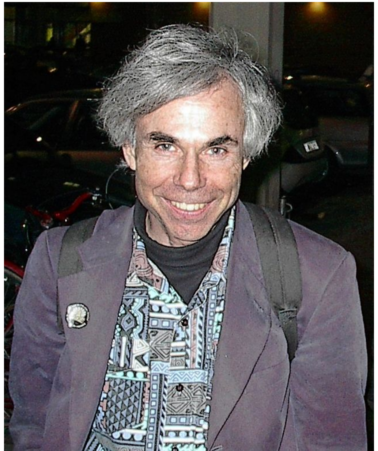


Part 1: Background

Pei Wang

- Contributor for the official Chinese translation of Godel-Escher-Bach as an undergraduate at Peking University while researching AI (1980s)
- Finished his PhD under Douglas Hofstadter at Indiana University (1995)
- Founded the AGI Conference Series with Ben Goertzel (2008)
- Founding Editor of the Journal of AGI (2011)

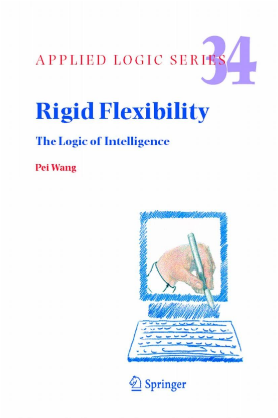




Timeline

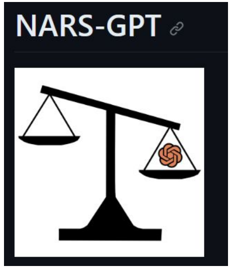
Wang's Dissertation

Wang publishes his dissertation, "Non-Axiomatic Reasoning System", Providing the first complete account of basic NAL built up over the early 1990s. Builds up to NAL Level 3.

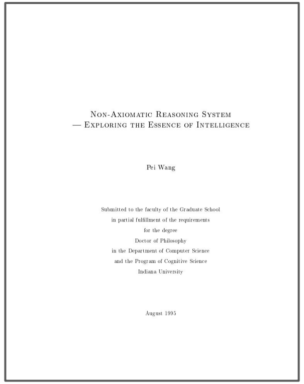


"Non Axiomatic Logic"

Builds up to NAL Level 9, adding support in NAL for Events, Goals, Tasks, Operations.



1995 — **2006** — **2012** — **2023**


Non-Axiomatic Reasoning System
— Exploring the Essence of Intelligence

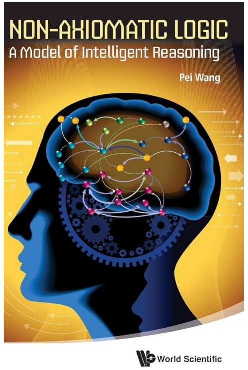
Pei Wang

Submitted to the faculty of the Graduate School
in partial fulfillment of the requirements
for the degree
Doctor of Philosophy
in the Department of Computer Science
and the Program of Cognitive Science
Indiana University

August 1995

"Rigid Flexibility, The Logic Of Intelligence"

Formalizes NAL, provides better comparisons with other systems.

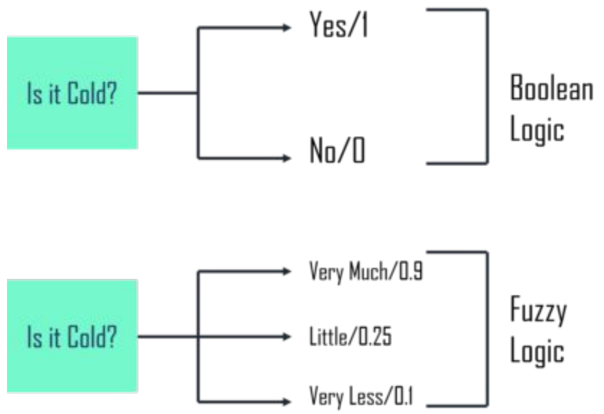


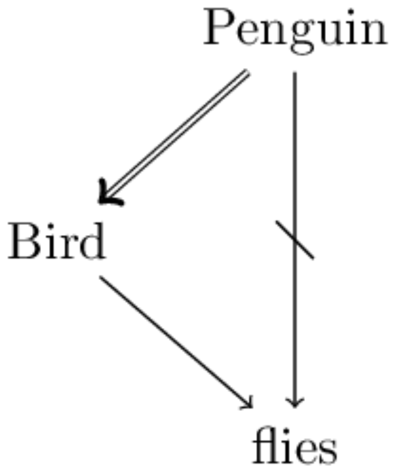
Still in Active Research and Development
~300 items on GS with NAL as title or topic
~400 items on GS with NARS as title or topic

2nd only paradigm to Hutter's AIXI in AGI circles.
Far more pragmatic than AIXI.

Features of Non-Axiomatic Logic

- Term Based (Syllogistic/Aristotelian)
  - Propositions take the form of relations between two objects, a subject and object (terms)
- Fuzzy
  - Statements are not just true or false but can take real numbered truth values between zero and one.
  - In NAL truth value corresponds to an agents belief in the truth of the statement.
- Non-monotonic (Defeasible)
  - Allows retraction of past inferences.
  - Can handle inconsistent information and perform belief revision over time as knowledge improves.





Levels of Non-Axiomatic Logic

- NAL 1: Fuzzy non-monotonic inheritance logic on atoms
- NAL 2: More expressive term relations (similarity, properties)
- NAL 3: Composite Terms
- NAL 4: Arbitrary relations
- NAL 5: Higher order reasoning on terms with predicates as terms.
- NAL 6: Quantification
- NAL 7: …

Each Level has a corresponding “Idealized Logic” removing non-monotonicity and fuzziness.

Part 2: Wang’s Inheritance Logic (IL-1)

Basic Aristotelian Term Logic

An **assertion** is a statement that is true or false (affirmed or denied).

James is Fat, Cats are green, Selmer is not an Elephant

Each assertion contains a **subject** and a **predicate**

A **term** is either a subject or predicate of an assertion:

Terms from the assertions: {James, Fat, Cats, Green, Selmer, Elephant}

Basic Aristotelian Term Logic (Cont.)

A term is either **universal** or **individual**

Individuals are objects, universals are categories or sets of individuals.

The subject of an assertion of an can either be individual or universal, while the predicate must be a universal.

James is Fat, Cats are green, Selmer is not an Elephant

Individuals: {James, Selmer}

Universals: {Cat, Fat, Green, not Elephant}

Inheritance Logic (IL-1)

IL-1 is a term logic, who's only assertions are **inheritance relation (→)**.

Assertions in IL-1 are called **statements** and take the form:

Subject → Predicate

This is read as

(1) "Subject is a special type of Predicate"

(2) "Predicate is a generalization of Subject"

Table 2.1. The grammar rules of IL-1.

$\langle statement \rangle$	$::=$	$\langle term \rangle \langle copula \rangle \langle term \rangle$
$\langle copula \rangle$	$::=$	$\rightarrow$
$\langle term \rangle$	$::=$	$\langle word \rangle$

Examples of IL-1 statements

Robin → Bird (Robins are a special case of birds)

Water → Liquid (Water is a type of liquid)

Bird → Animal (Bird is a special case of animal)

Semantic Properties of →

Wang defines the inheritance relation to be **reflexive** and **transitive**. From this we can derive two theorems concerning truth in the meta-logic of IL-1.

(Reflexivity of →)

For any term T: T → T is true

(Transitivity of →)

For any terms T1, T2, T3, if T1 → T2 and T2 → T3 are true then T1 → T3 is true

A **tautology** in IL-1 is any statement in the form T → T

Defines a weak semantics, but so far the class of true statements is quite boring and we don't have anything we can use our transitivity theorem with.

Semantics of IL-1
Wang's Experience Grounded Semantics

Semantics of classical logic typically is only concerned with discovering the truth value of statements.

Experience Grounded Semantics of IL-1 care **BOTH** about defining truth of statements **AND** the meanings of terms within those statements.

Example: given the statement Bird → Animal we would like to know not only if this statement is true, but would also like to know what a bird and what an animal is.

*This only makes sense in the context of knowledge we ground semantics in this prior knowledge.

Semantics of IL-1: Experience and Knowledge

An **experience** is a finite non-empty set of statements in IL-1 that does not include any tautologies. We will refer to this experience as K. For example:

K = {robin → bird, bird → animal, water → liquid}

knowledge is defined as the transitive closure (with respect to →) of an **experience** not containing any tautologies, we use * to denote this transformation.

K* = {robin → bird, bird → animal, water → liquid, robin → animal}

Note that if we have K2 = {T1 → T2, T2 → T1} than K2 = K2*

Semantics of IL-1: Truth of Statements

Truth is then defined with respect to the knowledge derived from an experience.

A statement φ in IL-1 is said to be true given an experience K iff φ is in K* or it is a tautology, otherwise it is false.

In metalogic we can express this semantic definition of truth as:

$K \vDash_{IL} \varphi \overset{\text{def}}{=} (\varphi \in K^*) \vee (\exists T: \varphi = (T \rightarrow T))$

This dichotomy in the definition of truth leads to two types of truth

- **<u>synthetically true</u>** statements derived from experience $(\varphi \in K^*)$
- **<u>analytically true</u>** statements considered to be “True by definition” $(\exists T: \varphi = (T \rightarrow T))$.

Semantics of IL-1: Meaning of Terms

The meaning of a term is defined as its relation with other terms according to experience.

Let V(K) be the set of terms in an experience K.

Ex. $V(\{T1 \rightarrow T2, T2 \rightarrow T3\}) = \{T1, T2, T3\}$

The **extension** of a term T with respect to K is $\{x \in V(K) \mid x \rightarrow T \in K\} \cup \{T\}$.

The **intention** of a term T with respect to K is $\{x \in V(K) \mid T \rightarrow x \in K\} \cup \{T\}$.

“Known specializations of T in K” vs “Known generalizations of T in K”

Wang asserts that the meaning of a term **IS** its intension and extension. In NAL-1, meaning of terms is used to help derive fuzzy truth values.

Inference Rule of IL-1

The single inference rule of IL-1 is “inheritance deduction”. (Transitivity of →)

$premise_1$	$premise_2$	$conclusion$
$M \rightarrow P$	$S \rightarrow M$	$S \rightarrow P$

It should be clear that we have a metalogical completeness result for semantic and syntactic truth in IL-1. (Note that we assume T→T is true for all T for $\vdash_{IL}$)

$K \vDash_{IL} \varphi \Leftrightarrow K \vdash_{IL} \varphi$

Can also be called a syllogism rather than inference rule, conforms to Aristotle’s definition.

A Brief Stop at IL-5:
Statements as Terms

IL-5 Syntax

IL-5 lets us make basic logical statements about IL-1 statements right in the logic, by considering some statements as terms. Adds Copulae for implication (not material implication) and equivalence.

$$
\begin{aligned}
\langle term \rangle &::= (\langle statement \rangle) \\
\langle statement \rangle &::= \langle term \rangle \\
&\mid (\neg \langle statement \rangle) \\
&\mid (\wedge \langle statement \rangle \langle statement \rangle^{+}) \\
&\mid (\vee \langle statement \rangle \langle statement \rangle^{+}) \\
\langle copula \rangle &::= \Rightarrow \mid \Leftrightarrow
\end{aligned}
$$

IL-5 Semantics: Desires for Implication

Formally we want to define semantics of $\Rightarrow$ to function as it does in natural deduction, such that $(\varphi \Rightarrow \psi)$ is true only if you can derive ψ from φ in a finite number of steps.

$K \vDash_{IL} (\varphi \Rightarrow \psi) \Leftrightarrow K \cup \{\varphi\} \vdash_{IL} \psi$

Since IL is a term logic we also want the restriction that it is impossible to derive $\varphi \Rightarrow \psi$ unless they are related in content, this rules out material implication.

IL-5 Semantics:

Solution: Borrow from our definition of inheritance. Define semantics of implication with respect to experience. If the sufficient conditions for φ are a subset of the sufficient conditions for ψ then $\varphi \Rightarrow \psi$. Forces φ, ψ to be derivable and "related".

The **sufficient conditions** of a of a term T with respect to K is defined

$T^S_K = \{x \in V(K) \mid x \Rightarrow T \in K\}$

The **necessary conditions** of a term T with respect to K is is defined

$T^N_K = \{x \in V(K) \mid T \Rightarrow x \in K\}$

Then assuming $\varphi, \psi \in K$ we can define implication as

$K \vDash_{IL} (\varphi \Rightarrow \psi) \stackrel{\text{def}}{=} \varphi^S_K \subseteq \psi^S_K$ or $K \vDash_{IL} (\varphi \Rightarrow \psi) \stackrel{\text{def}}{=} \psi^N_K \subseteq \varphi^N_K$

Partial Semantics of IL-5

Function exactly how they do in propositional logic, semantic entailment can be seen as a recursively defined proposition down to terminal cases, Atomic Terms, inheritance, and implication.

$K \vDash_{IL} \neg\varphi \stackrel{\text{def}}{=} \neg K \vDash_{IL} \varphi$

$K \vDash_{IL} (\varphi \wedge \psi) \stackrel{\text{def}}{=} (K \vDash_{IL} \varphi) \wedge (K \vDash_{IL} \psi)$

$K \vDash_{IL} (\varphi \vee \psi) \stackrel{\text{def}}{=} (K \vDash_{IL} \varphi) \vee (K \vDash_{IL} \psi)$

$K \vDash_{IL} (\varphi \rightarrow \psi) \stackrel{\text{def}}{=} (\varphi \rightarrow \psi) \in K^* \vee \varphi = \psi$ (Where K^* = transitive closure of K under $\rightarrow$)

$K \vDash_{IL} (\varphi \Rightarrow \psi) \stackrel{\text{def}}{=} (\varphi \Rightarrow \psi) \in K^\dagger \vee \varphi = \psi$ (Where $K^\dagger$ = transitive closure of K under $\Rightarrow$)

Quantification with IL-6

Quantifiers in Term Logic, The Square of Opposition

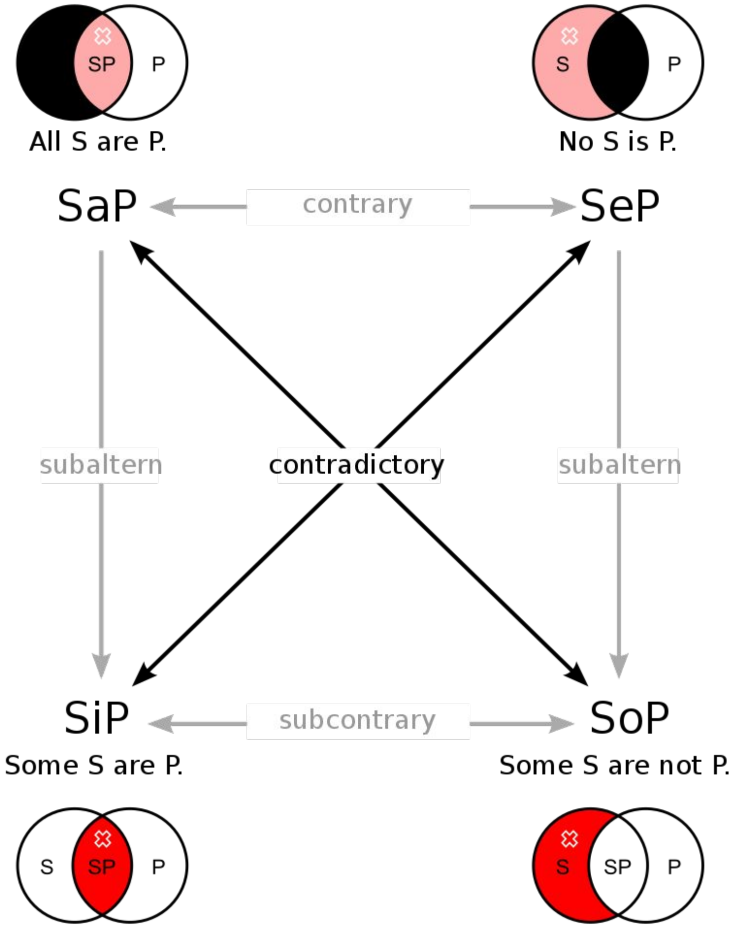
Four types of assertions. Four types of relationships between them.

Contradictory: Can not both be true

Subaltern: A $\Rightarrow$ I, $\neg$ I $\Rightarrow$ $\neg$ A

Contrary: If one is true the other must be false, can both be false.

Subcontrary: If one is false the other must be true, can both be true.



Quantification in IL-6

Can’t quantify like FOL, IL-5 has already built in the ability to represent other statements and predicates things that traditionally minimally need Higher Order Logic (HOL) to quantify over.

IL-6 introduces named term variables for dependent and independent terms. Roughly correspond to existentially and universally quantified variables.

This allows us to mimic the behavior of some first order quantification, and allows us to represent the classic Aristotelian quantifiers.

Getting to the Aristotelian Quantifiers

Two basic binary relations between sets we may wish to know about for extensions and intentions: Inclusion and common element.

- Inclusion: "If a term is in the extension of S than it is in the extension of P"

  $(? \rightarrow S) \Rightarrow (? \rightarrow P)$

- Common Element: "There is a term in both the extension of S and the Extension of P"

  $(? \rightarrow S) \land (? \rightarrow P)$

Both of these require a notion of variables.

But quantification is being used in different senses…

IL-6 Term Variables

An **independent variable term** is a term prefixed with a # that represents any arbitrary unspecified term under restriction. (Roughly represents a universally quantified variable)

#x → dog

#x is an arbitrary restricted term such that dog generalizes it (dog is in its intention)

A **dependent variable term** is a term prefixed with a # that represents a certain term under restriction followed by (). (Roughly represents an existentially quantified variable).

#x() → dog

#x() is a certain term such that dog generalizes it

Quantifiers

$(\#x \rightarrow S) \Rightarrow (\#x \rightarrow P)$ truth value corresponds to the A quantifier, All S are P.

$(\#x() \rightarrow S) \land (\#x() \rightarrow P)$ truth value corresponds to the I quantifier, Some S are P

We can use NOT to construct E and O

$(\#x \rightarrow S) \Rightarrow (\neg(\#x \rightarrow P))$ E quantifier, No S are P

$(\#x() \rightarrow S) \land (\neg(\#x() \rightarrow P))$ O quantifier, Some S are not P

Differences Between FOL Variables and IL Variables

- In IL, variables represent a term under a restriction, while in FOL, they represent a term in the entire domain.

- IL variables range over terms, thus can be either instances or properties (universals) normally need HOL to do this.

- In IL variables relate two or more terms. Independent variables can only appear on two sides of an implication or equivalence statement. Dependents can only appear on two sides of a conjunction.

Fin