Could AI Ever Match Gödel's Greatness? Part II

Selmer Bringsjord

IFLAI2 2020

12/10/20

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RAIR

Rensselaer AI and Reasoning Lab

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Monographic Context
(yet again!)

. . .

Gödel's Great Theorems (OUP)

by Selmer Bringsjord

- √ Introduction ("The Wager")
- √ Brief Preliminaries (e.g. the propositional calculus & FOL)
- √ The Completeness Theorem
- √ The First Incompleteness Theorem
- √ The Second Incompleteness Theorem
- √ The Speedup Theorem
- √ The Continuum-Hypothesis Theorem
- √ The Time-Travel Theorem
- √ Gödel's "God Theorem"
- Could a Finite Machine Match Gödel's Greatness?

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Part II

Gödel's Either/Or …

The Question

Q* Is the human mind more powerful than the class of standard computing machines?

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(= finite machines)

The Question

Q* Is the human mind more powerful than the class of standard computing machines?

(= finite machines)

(= Turing machines)

(= register machines)

…

Gödel's Either/Or

"[E]ither … the human mind (even within the realm of pure mathematics) infinitely surpasses the power of any finite machine, or else there exist absolutely unsolvable diophantine problems."
— Gödel, 1951, Providence RI

PT as a Diophantine Equation

Equations of this sort were introduced to you in middle-school, when you were asked to find the hypotenuse of a right triangle when you knew its sides; the familiar equation, the famous Pythagorean Theorem that most adults will remember at least echoes of into their old age, is:

$$\text{(PT)} \quad a^2 + b^2 = c^2,$$

and this is of course equivalent to

$$\text{(PT')} \quad a^2 + b^2 - c^2 = 0,$$

which is a Diophantine equation. Such equations have at least two unknowns (here, we of course have three: a, b, c), and the equation is solved when positive integers for the unknowns are found that render the equation true. Three positive integers that render (PT') true are

$$a = 4,\ b = 3,\ c = 5.$$

It is mathematically impossible that there is a finite computing machine capable of solving any Diophantine equation given to it as a challenge.

… which means that the 10th of Hilbert's Problems is settled:

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Hilbert's problems

From Wikipedia, the free encyclopedia

David Hilbert

Hilbert's problems are twenty-three problems in mathematics published by German mathematician David Hilbert in 1900. The problems were all unsolved at the time, and several of them were very influential for 20th-century mathematics. Hilbert presented ten of the problems (1, 2, 6, 7, 8, 13, 16, 19, 21, and 22) at the Paris conference of the International Congress of Mathematicians, speaking on August 8 in the Sorbonne. The complete list of 23 problems was published later, most notably in English translation in 1902 by Mary Frances Winston Newson in the *Bulletin of the American Mathematical Society*.[1]

Contents [hide]

… which means that the 10th of Hilbert's Problems is settled:

10th	Find an algorithm to determine whether a given polynomial Diophantine equation with integer coefficients has an integer solution.	Resolved. Result: Impossible; Matiyasevich's theorem implies that there is no such algorithm.	1970

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It was a *team* effort, actually; it's not due solely to Matiyasevich, and is often denoted as the 'MRDP Theorem.'

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Julia **R**obinson   Martin **D**avis   Hilary **P**utnam

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Julia **R**obinson   Martin **D**avis   Hilary **P**utnam

Background

problem?[7] In his lecture, Gödel precisely defines diophantine problems, but we don't need to bother with all of the details here; we only need to appreciate the general structure of such a problem, and that can be achieved quickly as follows, given what was introduced in Chapter 2.

Each diophantine problem has at its core a polynomial $\mathcal{P}$ whose variables are comprised by two lists, $x_1, x_2, \ldots, x_n$ and $y_1, y_2, \ldots, y_m$; all variables must be integers, and the same for subscripts n and m. To represent a polynomial in a manner that announces its variables, we can write

$$\mathcal{P}(x_1, x_2, \ldots, x_k, y_1, y_2, \ldots, y_j).$$

But Gödel was specifically interested in whether, for all integers that can be set to the variables x_i, there are integers that can be set to the y_j, such that the polynomial equals 0. To make this clearer, first, here are two particular, simple equations that employ polynomials that are both instances of the needed form:

E1 $3x - 2y = 0$

E2 $2x^2 - y = 0$

All we need to do now is prefix these equations with quantifiers in the pattern Gödel gave. This pattern is quite simple: universally quantify over each x_i variable (using the now-familiar $\forall$), after which we existentially quantify over each y_i variable (using the also-now-familiar $\exists$). Thus, here are the two diophantine problems that correspond to the pair E1 and E2 from just above:

P1 Is it true that $\forall x \exists y (3x - 2y = 0)$?

P2 Is it true that $\forall x \exists y 2x^2 - y = 0$?

Great Paper

Hilbert's Tenth Problem is Unsolvable
Author(s): Martin Davis
Source: *The American Mathematical Monthly*, Vol. 80, No. 3 (Mar., 1973), pp. 233-269
Published by: Mathematical Association of America
Stable URL: http://www.jstor.org/stable/2318447

Great Paper

MAA
MATHEMATICAL ASSOCIATION OF AMERICA

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Great Paper

1. Diophantine Sets. In this article the usual problem of Diophantine equations will be inverted. Instead of being given an equation and seeking its solutions, one will begin with the set of "solutions" and seek a corresponding Diophantine equation. More precisely:

DEFINITION. A set S of ordered n-tuples of positive integers is called **Diophantine** if there is a polynomial $P(x_1, \cdots, x_n, y_1, \cdots, y_m)$, where $m \geqq 0$, with integer coefficients such that a given n-tuple $\langle x_1, \cdots, x_n \rangle$ belongs to S if and only if there exist positive integers $y_1, \cdots, y_m$ for which

$$P(x_1, \cdots, x_n, y_1, \cdots, y_m) = 0.$$

Borrowing from logic the symbols "$\exists$" for "there exists" and "$\Leftrightarrow$" for "if and only if", the relation between the set S and the polynomial P can be written succinctly as:

$$\langle x_1, \cdots, x_n \rangle \in S \Leftrightarrow (\exists y_1, \cdots, y_m)\ [P(x_1, \cdots, x_n, y_1, \cdots, y_m) = 0],$$

or equivalently:

$$S = \{\langle x_1, \cdots, x_n \rangle \mid (\exists y_1, \cdots, y_m)\ [P(x_1, \cdots, x_n, y_1, \cdots, y_m) = 0]\}.$$

Note that P may (and in non-trivial cases always will) have negative coefficients. *The word* "**polynomial**" *should always be so construed in the article* except where the contrary is explicitly stated. Also *all numbers in this article are positive integers* unless the contrary is stated.

Great Paper

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Great Paper

Notice that this is a perfect fit with how we used formal logic to present and understand the Polynomial Hierarchy and the Arithmetic Hierarchy.

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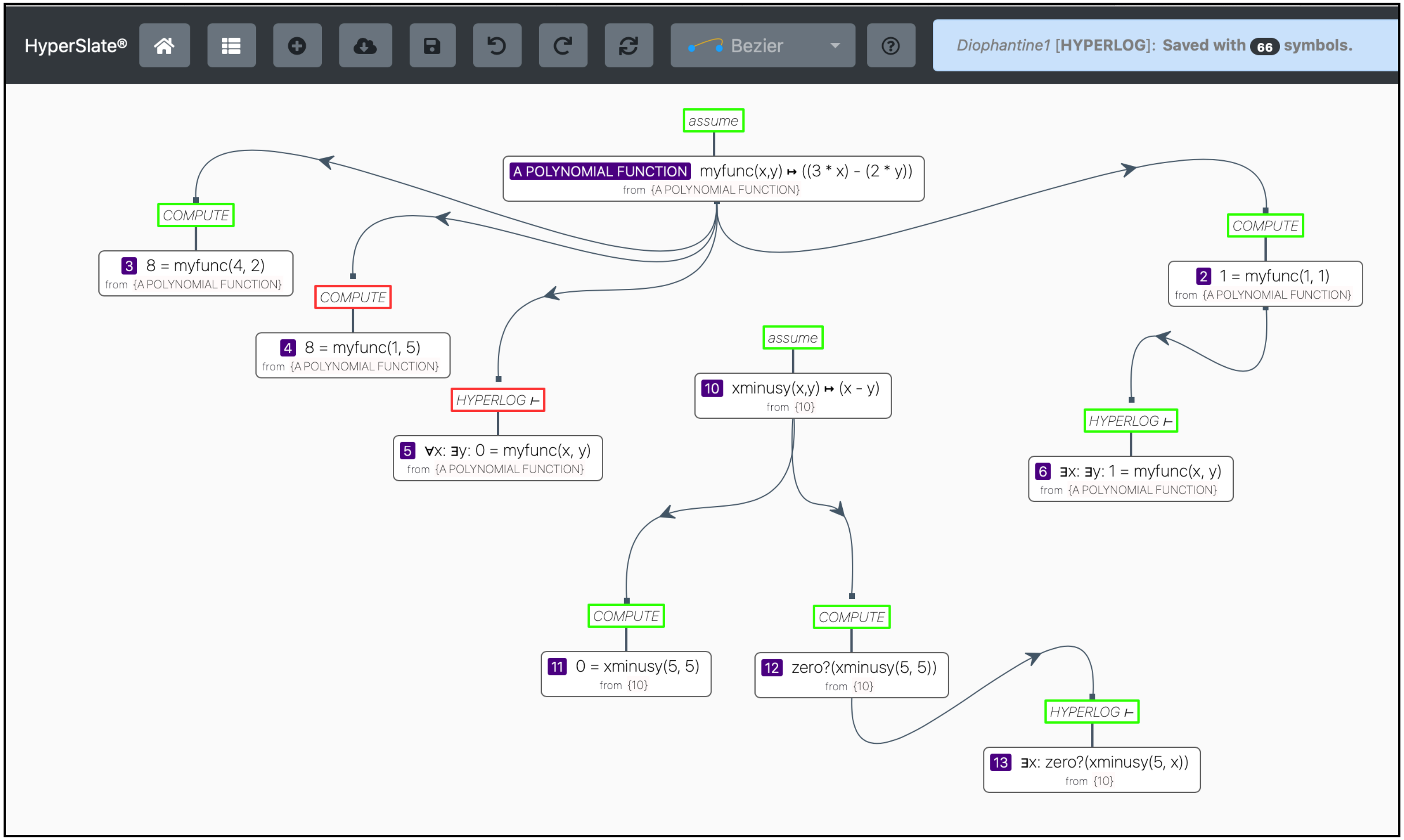
Diophantine “Threat”



The Crux

$\exists \mathcal{P}$ s.t. no human mind could ever decide $\forall x_1 \forall x_2 \cdots \forall x_k \exists y_1 \exists y_2 \cdots \exists x_j (\mathcal{P}(x_1, x_2, \ldots, x_k, y_1, y_2, \ldots, y_j)$?

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Yes.

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Yes.

↓

The human mind is *not* infinitely more powerful than any standard computing machine.

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Yes. No.

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Yes.

No.

The human mind *is* infinitely more powerful than any standard computing machine.

The human mind is *not* infinitely more powerful than any standard computing machine.

Earlier Gödelian Argument for the "No."

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Applied Mathematics and Computation

Volume 176, Issue 2, 15 May 2006, Pages 516-530

A new Gödelian argument for hypercomputing minds based on the busy beaver problem ☆

Selmer Bringsjord, Owen Kellett, Andrew Shilliday, Joshua Taylor, Bram van Heuveln, Yingrui Yang, Jeffrey Baumes, Kyle Ross

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https://doi.org/10.1016/j.amc.2005.09.071 Get rights and content

Abstract

Do human persons hypercompute? Or, as the doctrine of *computationalism* holds, are they information processors at or below the Turing Limit? If the former, given the essence of hypercomputation, persons must in some real way be capable of infinitary information processing. Using as a springboard Gödel's little-known assertion that the human mind has a power "converging to infinity", and as an anchoring problem Rado's [T. Rado, On non-computable functions, Bell System Technical Journal 41 (1963) 877–884] Turing-uncomputable "busy beaver" (or Σ) function, we present in this short paper a new argument that, in fact, human persons can hypercompute. The argument is intended to be formidable, not conclusive: it brings Gödel's intuition to a greater level of precision, and places it within a sensible case against computationalism.

A New One Coming! — in …

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Will AI Match (Or Even Exceed) Human Intellligence?

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Yes.

A New One Coming! — in …

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No. Yes.

Will AI Match (Or Even Exceed) Human Intellligence?

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Yes.

Will AI Match (Or Even Exceed) Human Intellligence?

No. Yes.

I: "Negative" enumerative induction for $\neg\exists year_k(\mathrm{AI} = \mathrm{HI}@year_k)$ from $\mathrm{AI} \neq \mathrm{HI}@year_{1958} \wedge \ldots \wedge \mathrm{AI} \neq \mathrm{HI}@year_{2020}$.

Will AI Match (Or Even Exceed) Human Intellligence?

No. **Yes.**

1: “Negative” enumerative induction for $\neg \exists year_k(\mathrm{AI} = \mathrm{HI} @ year_k)$ from $\mathrm{AI} \neq \mathrm{HI} @ year_{1958} \wedge \ldots \wedge \mathrm{AI} \neq \mathrm{HI} @ year_{2020}$.

2: There is no absolutely unsolvable-for-humans Diophantine problem. Hence as Gödel explained, we get “No.”

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3: Amundsen and The Explorer Argument.

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3: Amundsen and The Explorer Argument.

4: And finally, the sledgehammer is used: *phenomenal consciousness*.

Og på det glade merknaden for Selmer (men ikke for Bill), er forelesningene våre nå fullført .. men . . .

Finally, finally, …

Gödel v. AI "Scorecard"

The Particular Work	Nutshell Diagnosis	Beyond AI?

Gödel v. AI "Scorecard"

The Particular Work	Nutshell Diagnosis	Beyond AI?
Completeness Thm. (Ch. 3)	Reduction lemma impressive.	Likely Not

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First Incomp. Thm. (Ch. 4)	Arithmetization seminal.	Likely Not

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Second Incomp. Thm. (Ch. 5)	Easy with G1 in hand.	Not

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Speedup Thm. (Ch. 6)	*Some* versions quick w/ G1 in hand.	Not

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Continuum Hyp. Thm. (Ch. 7)	Stunning *tour de force*; fully *ab initio*.	Yes

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Continuum Hyp. Thm. (Ch. 7)	Stunning *tour de force*; fully *ab initio*.	Yes
Time-Travel Thm. (Ch. 8)	Unqualified to even guess.	Unknown.

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"God Theorem" (Ch. 9)	An ancient trajectory from Anselm.	Yes

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⋆On Intuitionistic Logic	Beyond our scope.	Likely Not
⋆Philosophical Reasoning	Undeniably beyond foreseeable AI.	Yes

Med nok penger, kan logikk løse alle problemer.