Could AI Ever Match Gödel's Greatness? Part I

(With brief remarks on skipped "God Theorem.")

Selmer Bringsjord

IFLAI2 2020

12/7/20

Selmer.Bringsjord@gmail.com

RAIR

Rensselaer AI and Reasoning Lab

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Monographic Context

(yet again!)

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Gödel's Great Theorems (OUP)

by Selmer Bringsjord

- Introduction ("The Wager")
- Brief Preliminaries (e.g. the propositional calculus & FOL)
- The Completeness Theorem
- The First Incompleteness Theorem
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Re Gödel’s “God Theorem” …

Recommended Podcast :)

https://mindmatters.ai/podcast/ep81

The Ontological/Modal Argument Meets AI

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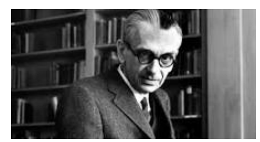

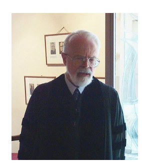

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Automating Gödel's Ontological Proof of God's Existence with Higher-order Automated Theorem Provers

Christoph Benzmüller[1] and **Bruno Woltzenlogel Paleo**[2]

Abstract. Kurt Gödel's ontological argument for God's existence has been formalized and automated on a computer with higher-order automated theorem provers. From Gödel's premises, the computer proved: necessarily, there exists God. On the other hand, the theorem provers have also confirmed prominent criticism on Gödel's ontological argument, and they found some new results about it.

The background theory of the work presented here offers a novel perspective towards a *computational theoretical philosophy*.

1 INTRODUCTION

Kurt Gödel proposed an argumentation formalism to prove the existence of God [23, 30]. Attempts to prove the existence (or non-existence) of God by means of abstract, ontological arguments are an old tradition in western philosophy. Before Gödel, several prominent philosophers, including St. Anselm of Canterbury, Descartes and Leibniz, have presented similar arguments. Moreover, there is an impressive body of recent and ongoing work (cf. [31, 19, 18] and the references therein). Ontological arguments, for or against the existence of God, illustrate well an essential aspect of metaphysics: some (necessary) facts for our existing world are deduced by purely a priori, analytical means from some abstract definitions and axioms.

What motivated Gödel as a logician was the question, whether it

A1 Either a property or its negation is positive, but not both: $\forall\phi[P(\neg\phi) \equiv \neg P(\phi)]$

A2 A property necessarily implied by a positive property is positive: $\forall\phi\forall\psi[(P(\phi) \land \Box\forall x[\phi(x) \supset \psi(x)]) \supset P(\psi)]$

T1 Positive properties are possibly exemplified: $\forall\phi[P(\phi) \supset \Diamond\exists x\phi(x)]$

D1 A *God-like* being possesses all positive properties: $G(x) \equiv \forall\phi[P(\phi) \supset \phi(x)]$

A3 The property of being God-like is positive: $P(G)$

C Possibly, God exists: $\Diamond\exists xG(x)$

A4 Positive properties are necessarily positive: $\forall\phi[P(\phi) \supset \Box\, P(\phi)]$

D2 An *essence* of an individual is a property possessed by it and necessarily implying any of its properties: $\phi\ ess.\ x \equiv \phi(x) \land \forall\psi(\psi(x) \supset \Box\forall y(\phi(y) \supset \psi(y)))$

T2 Being God-like is an essence of any God-like being: $\forall x[G(x) \supset G\ ess.\ x]$

D3 *Necessary existence* of an individ. is the necessary exemplification of all its essences: $NE(x) \equiv \forall\phi[\phi\ ess.\ x \supset \Box\exists y\phi(y)]$

A5 Necessary existence is a positive property: $P(NE)$

T3 Necessarily, God exists: $\Box\exists xG(x)$

Figure 1. Scott's version of Gödel's ontological argument [30].

Proceedings of the Twenty-Fifth International Joint Conference on Artificial Intelligence (IJCAI-16)

The Inconsistency in Gödel's Ontological Argument: A Success Story for AI in Metaphysics

Christoph Benzmüller*
Freie Universität Berlin & Stanford University
c.benzmueller@gmail.com

Bruno Woltzenlogel Paleo
Australian National University
bruno.wp@gmail.com

Abstract

This paper discusses the discovery of the inconsistency in Gödel's ontological argument as a success story for artificial intelligence. Despite the popularity of the argument since the appearance of Gödel's manuscript in the early 1970's, the inconsistency of the axioms used in the argument remained unnoticed until 2013, when it was detected automatically by the higher-order theorem prover

on the proof [Fuhrmann, 2016].

The in-depth analysis presented here substantially extends previous computer-assisted studies of Gödel's ontological argument. Similarly to the related work [Benzmüller and Woltzenlogel-Paleo, 2013a; 2014] the analysis has been conducted with automated theorem provers for classical higher-order logic (HOL; cf. [Andrews, 2014] and the references therein), even though Gödel's proof is actually formulated in higher-order *modal* logic (HOML; cf. [Muskens, 2006]

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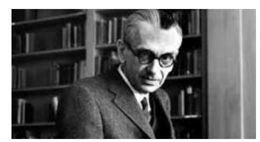

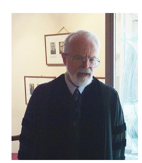

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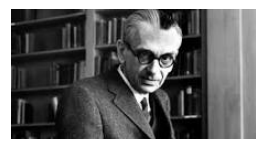

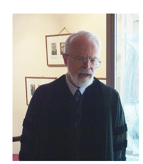

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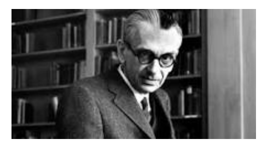

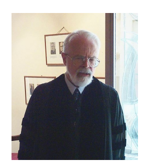

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For a wonderfully economical, non-technical overview that includes this observation, see "Chapter 7: Gödel" by Alexander Pruss, in *Ontological Arguments*, G. Oppy, ed. (Cambridge, UK: Cambridge University Press).

Are there invariants? Apparently.

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$$(\mathbf{Pos1}^{\star}) \quad \forall R \forall \delta \neq 0[GPPos(R^{\delta}) \rightarrow \neg GPPos(\bar{R})]$$

Gödel's Great Theorems (OUP)

by Selmer Bringsjord

- √ Introduction ("The Wager")
- √ Brief Preliminaries (e.g. the propositional calculus & FOL)
- √ The Completeness Theorem
- √ The First Incompleteness Theorem
- √ The Second Incompleteness Theorem
- √ The Speedup Theorem
- √ The Continuum-Hypothesis Theorem
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- Gödel's "God Theorem"
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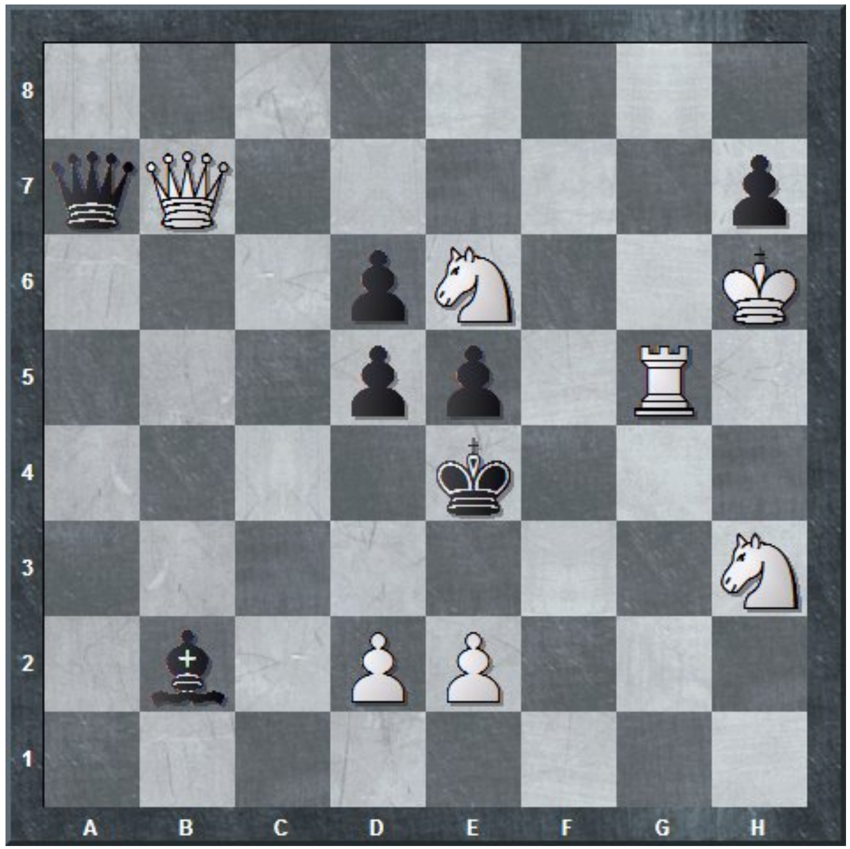
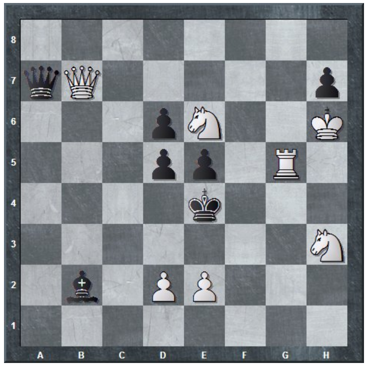
Gödel’s Greatness & Games

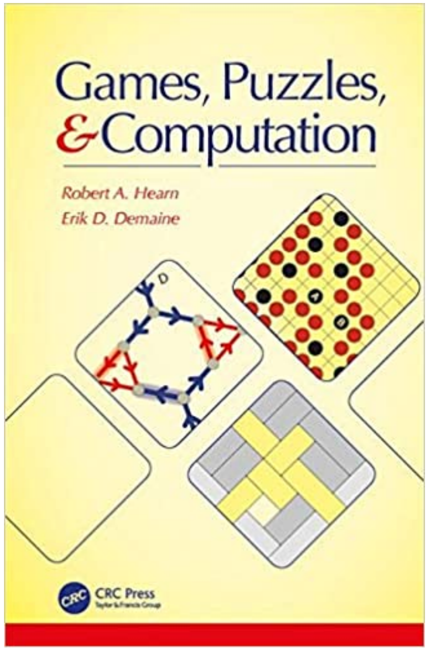
. . .

Mate in 2 Problem



Mate in 2 Problem





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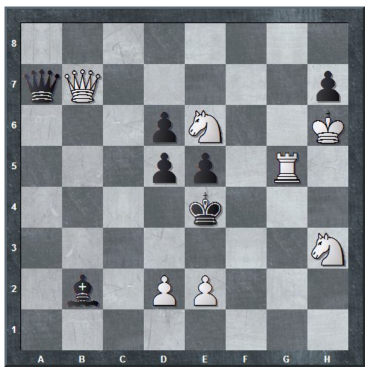

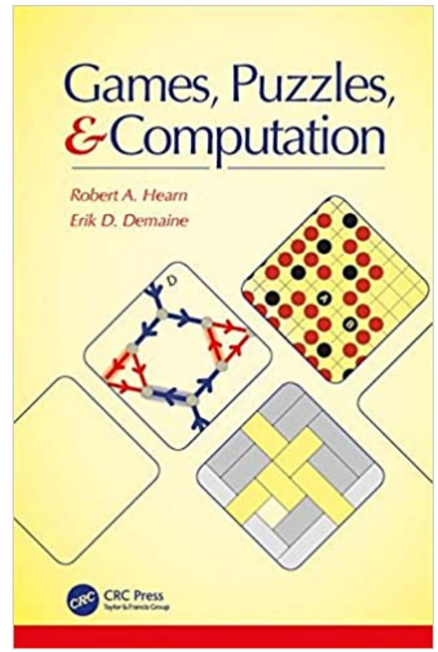

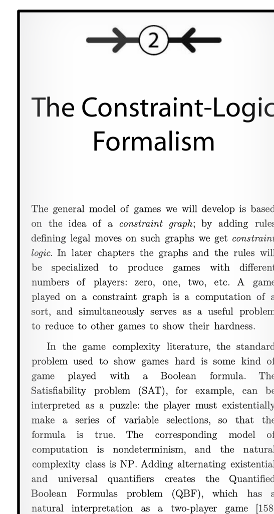
2

The Constraint-Logic Formalism

The general model of games we will develop is based on the idea of a *constraint graph*; by adding rules defining legal moves on such graphs we get *constraint logic*. In later chapters the graphs and the rules will be specialized to produce games with different numbers of players: zero, one, two, etc. A game played on a constraint graph is a computation of a sort, and simultaneously serves as a useful problem to reduce to other games to show their hardness.

In the game complexity literature, the standard problem used to show games hard is some kind of game played with a Boolean formula. The Satisfiability problem (SAT), for example, can be interpreted as a puzzle: the player must existentially make a series of variable selections, so that the formula is true. The corresponding model of computation is nondeterminism, and the natural complexity class is NP. Adding alternating existential and universal quantifiers creates the Quantified Boolean Formulas problem (QBF), which has a natural interpretation as a two-player game [158,

Logico-Mathematical Landscape that Has Gödel Turning in His Grave

Super-Serious Human Cognitive Power

Serious Human Cognitive Power

Entscheidungsproblem

Mere Calculative Cognitive Power

Logico-Mathematical Landscape that Has Gödel Turning in His Grave

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Podcast: The Turing Test is Dead. Long Live the Lovelace Test.

Gödel

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Logico-Mathematical Landscape that Has Gödel Turning in His Grave

Analytical Hierarchy

Serious Human Cognitive Power

Gödel

Entscheidungsproblem

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Analytical Hierarchy

Arithmetical Hierarchy

Gödel

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Logico-Mathematical Landscape that Has Gödel Turning in His Grave

Analytical Hierarchy

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Gödel

Entscheidungsproblem

Polynomial Hierarchy

Logico-Mathematical Landscape that Has Gödel Turning in His Grave

Analytical Hierarchy

Arithmetical Hierarchy

Gödel

Entscheidungsproblem

Polynomial Hierarchy

$$\mathbf{P} \subseteq \mathbf{NP} \subseteq \mathbf{PSPACE} = \mathbf{NPSPACE} \subseteq \mathbf{EXPTIME} \subseteq \mathbf{NEXPTIME} \subseteq \mathbf{EXPSPACE}$$

Logico-Mathematical Landscape that Has Gödel Turning in His Grave

Analytical Hierarchy

Arithmetical Hierarchy

Gödel

$$\vdots$$
$$\Pi_2$$
$$\Sigma_2$$
$$\Pi_1$$
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$$\Sigma_0$$

Entscheidungsproblem

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Logico-Mathematical Landscape that Has Gödel Turning in His Grave

Analytical Hierarchy

Arithmetical Hierarchy

Gödel

Go: AlphaGo

$\vdots$

Π_2

Σ_2

Π_1

Σ_1

Σ_0

Entscheidungsproblem

Polynomial Hierarchy

$$\mathbf{P} \subseteq \mathbf{NP} \subseteq \mathbf{PSPACE} = \mathbf{NPSPACE} \subseteq \mathbf{EXPTIME} \subseteq \mathbf{NEXPTIME} \subseteq \mathbf{EXPSPACE}$$

Logico-Mathematical Landscape that Has Gödel Turning in His Grave

Analytical Hierarchy

Arithmetical Hierarchy

Gödel

Jeopardy!:

Go: AlphaGo

$\vdots$

Π_2

Σ_2

Π_1

Σ_1

Σ_0

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Logico-Mathematical Landscape that Has Gödel Turning in His Grave

Analytical Hierarchy

Arithmetical Hierarchy

Gödel

Chess: Deep Blue

Jeopardy!:

Go: AlphaGo

$\vdots$

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Σ_2

Π_1

Σ_1

Σ_0

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Logico-Mathematical Landscape that Has Gödel Turning in His Grave

Analytical Hierarchy

Arithmetical Hierarchy

Gödel

Checkers: Chinook

Chess: Deep Blue

Jeopardy!:

Go: AlphaGo

$\vdots$

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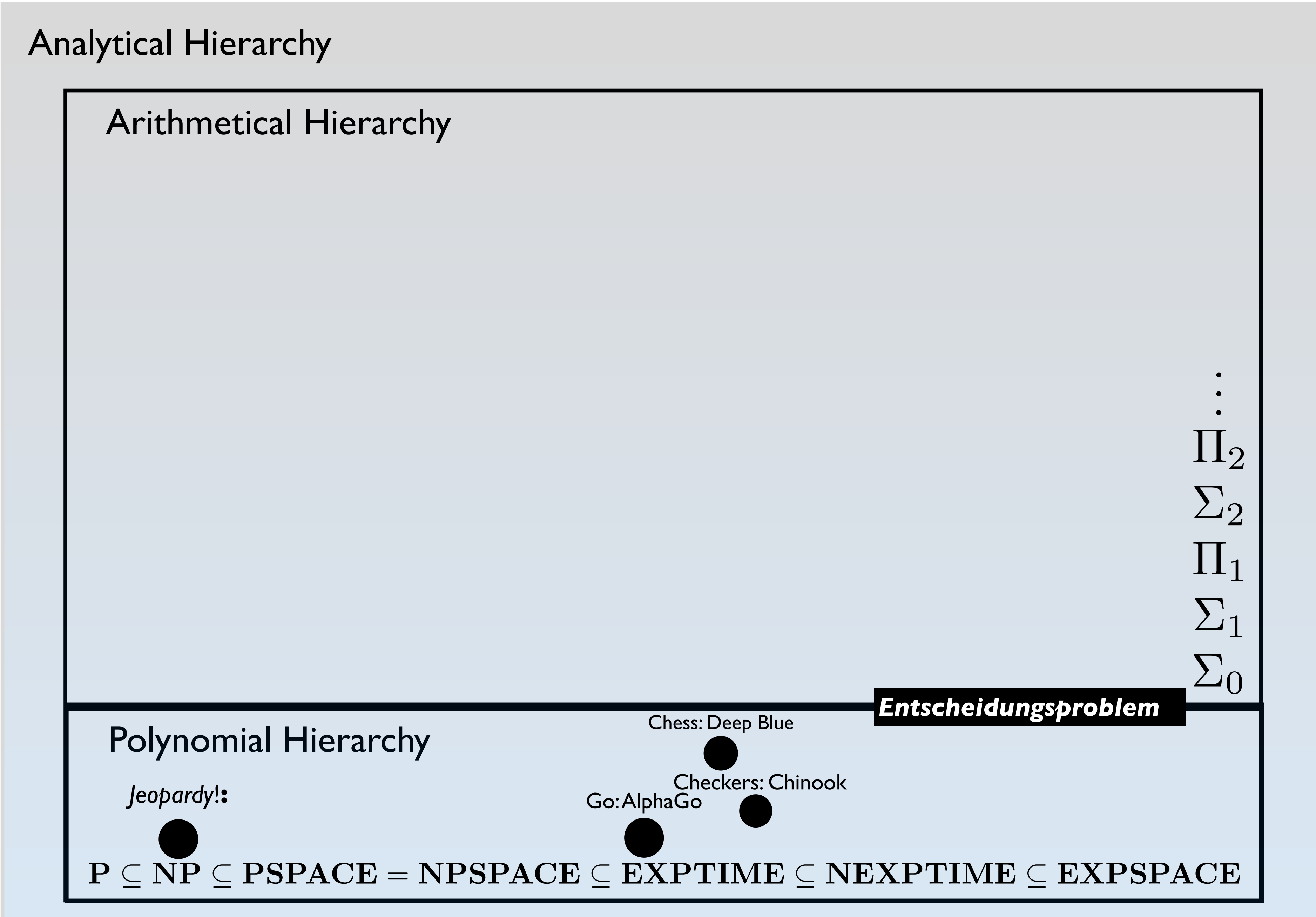
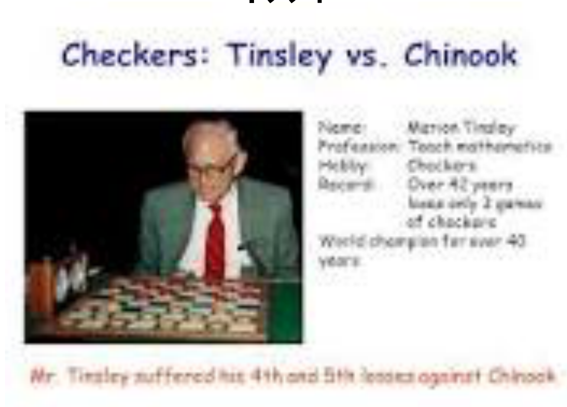
Logico-Mathematical Landscape that Has Gödel Turning in His Grave



1994


Checkers: Tinsley vs. Chinook
Name: Marion Tinsley
Profession: Teach mathematics
Hobby: Checkers
Record: Over 42 years
loses only 3 games
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World champion for over 40 years
Mr. Tinsley suffered his 4th and 5th losses against Chinook

1994


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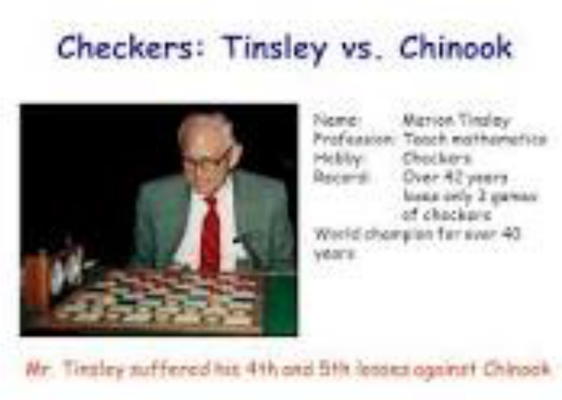
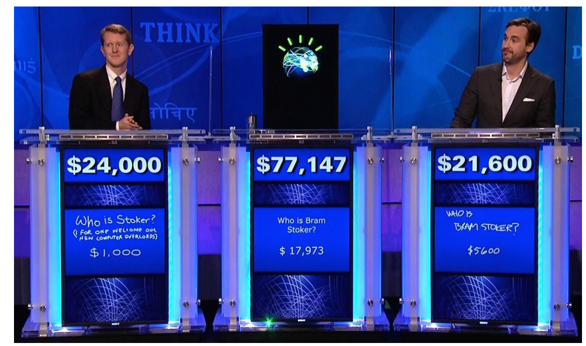
1997

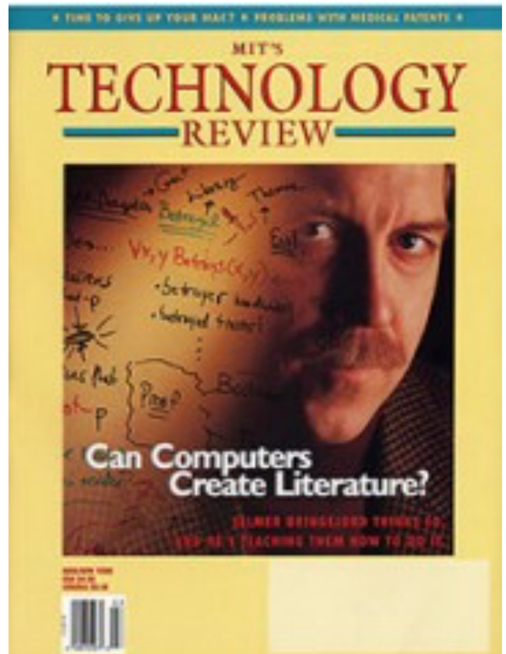
1994



1997

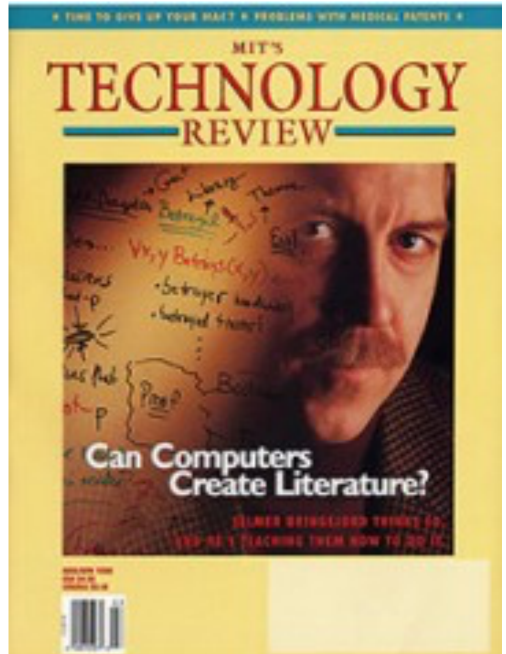
2011





1998

“Chess is Too Easy”



1998

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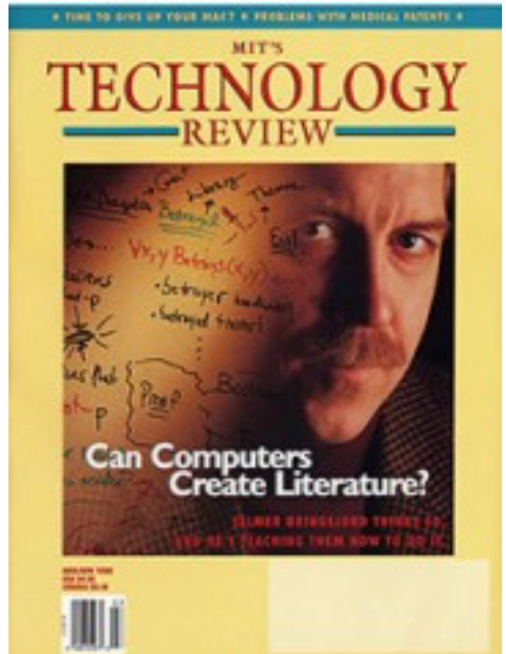

1998

Some of Gödel’s great work is at the level of chess.

But to *fully* "gamify" Gödel, we need a harder game! …

Rengo Kriegspiel

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US Go Congress Goes a Little Crazy

Wednesday August 13, 2014

"White plays capturing black, putting herself and black into atari," calls Crazy Go TD Terry Benson. He officiated several games of Rengo Kriegspiel on Tuesday evening – a pair go game in which all four players face away from the main board and play their stones on their own empty board in front of them; the only clues about where their opponents — and even their partner — have played comes when they make an illegal move, or play where their own team or their opponents already have stones. Rengo Kriegspiel is only one of dozens of variants on the game of go that were played by an enthusiastic crowd of around 100 players. Familiar games include Magnetic Go, 4 Color Go, Tessellation Go, 3D Go, Spiral Go, and Blind Go. "After all these years, it's still crazy," said TD and Crazy Go founder Terry Benson. New Crazy Go games, never before played at a Go Congress, were even invented on the spot. Four players donned sleeping masks to block their vision and transformed Blind Go into Rengo Blind Go, and a few other players added the fundamentals of Tiddlywinks to their go game. Spectators and players alike are enthusiastic about the creativity of the games and the fun of adding a little Crazy to Go; "Crazy Go is my favorite part of the Congress!" said Bob Crites.

- report/photos by Karoline Li

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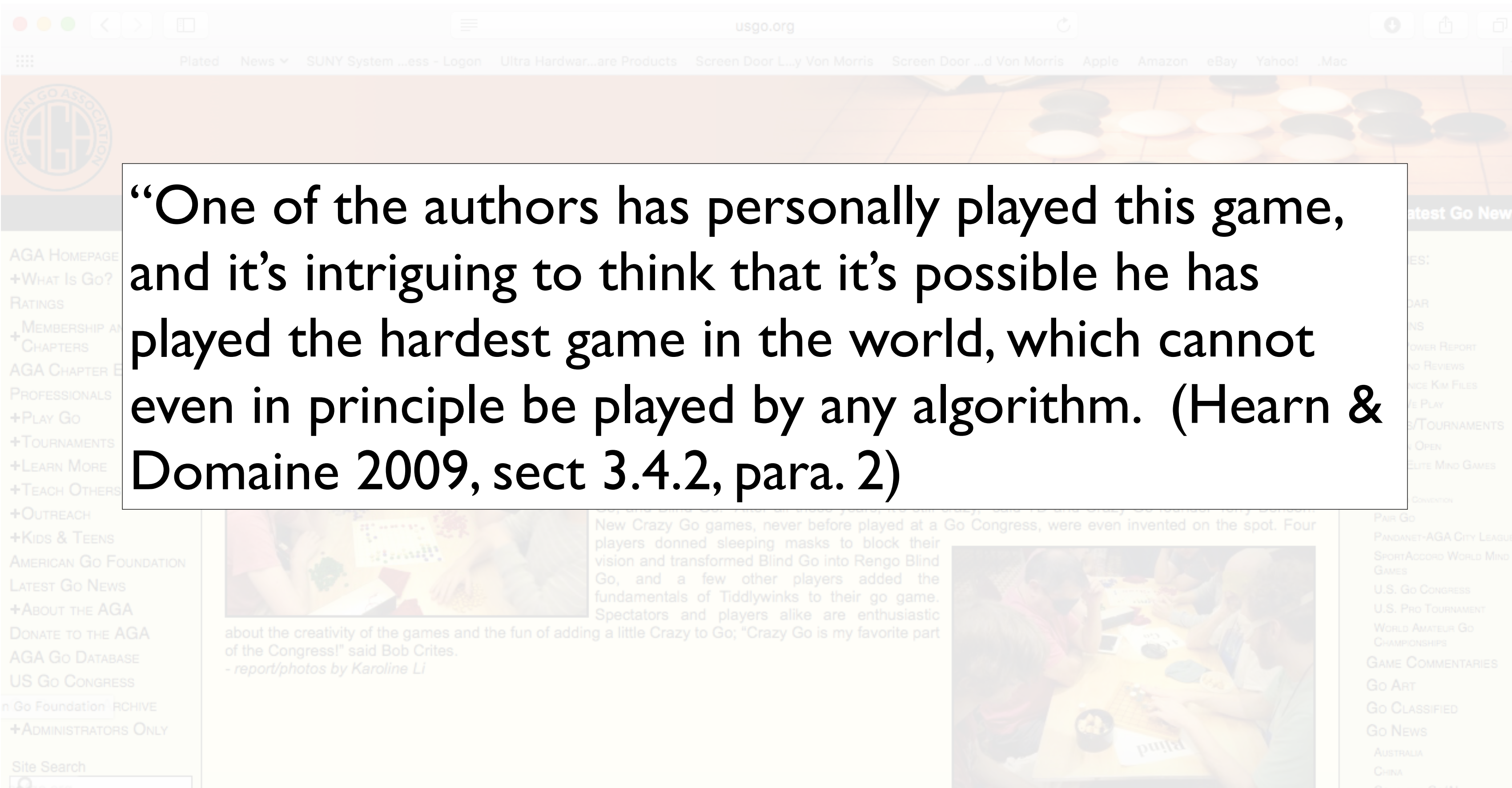
Rengo Kriegspiel



"One of the authors has personally played this game, and it's intriguing to think that it's possible he has played the hardest game in the world, which cannot even in principle be played by any algorithm. (Hearn & Domaine 2009, sect 3.4.2, para. 2)

The Gödel Game