

On Quantificational Modal Logic (S5-centric)

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Intro to Logic
4/12/2020
ver 1112202100NY



Logistics ...

Status? Some discussion ...

The screenshot shows a LaTeX editor interface with the following components:

- Top Bar:** Includes a menu icon, an upward arrow, the document title "IFLAI2F20_PAPERTOPICS", and several utility icons: a question mark, a letter 'L', a review icon, a share icon, a submit icon, a history icon, and a chat icon.
- Editor Tabs:** Two tabs are visible: "Source" (active) and "Rich Text".
- Source Code (Left Panel):** Displays LaTeX code for a document class and preamble. The code includes package loading for `inputenc`, `fullpage`, `amssymb`, and `colorlinks`. The document title is `\textbf{IFLAI2F20 Paper Topics}`, the author is `Selmer Bringsjord`, and the date is `\texttt{ver 1109201500NY}`. The main text begins with a comment explaining the document's purpose and provides instructions for listing paper topics and specific claims.
- Rendered Document (Right Panel):** Shows the output of the LaTeX compilation, titled "The List". It contains a bulleted list of three entries, each with a "Topic Area" and a "Specific Claim":
 - Mike Giancola ✓**
 - Topic Area:** Paternalistic Taxation of Machine Learning.
 - Specific Claim:** The proposal to tax corporate ML activity made recently by S Bringsjord would face four major roadblocks to successful implementation: (1) passage into law; (2) enforcement; and efficacy, both in terms of (3a) reducing harm and (3b) shifting research towards logic-based methods.
 - Jasper Covey ✓**
 - Topic Area:** Modeling Taxation, Effort, and Wealth.
 - Specific Claim:** The taxation model, *S*, proposed in class by S Bringsjord lacks an account of the effects of capital on effort that, when implemented, would necessitate a progressive tax scheme.
 - Joe Halasz ✓**
 - Topic Area:** The Argument for God's Existence from AI
 - Specific Claim:** The argument for God's Existence proposed by S Bringsjord, specifically section 4.1 about premise 4 vulnerabilities, does not take new studies on canine ability into account that could remove the discontinuity between the human mind and the canine mind, and premise 5 in The Argument does not take into account the fact that other natural forces still having to do with physical science could have caused it to be the case that we have this level of cognitive power.
- John Slowik**
 - Topic Area:** Modeling Taxation, Effort, and Wealth.
 - Specific Claim:** The proposed tax model fails to afford the taxed individuals ethical standards of living, promotes counterproductive behavior in the taxed population, and stifles competition and innovation, contrary to its claims that such a model is required for the respective promotion or suppression of the same. I intend to model this using an ordinal set of activities *A* which citizens can participate in only if they satisfy some requirement, e.g. having sufficient capital. The set being ordinal means that a citizen will choose to participate in activities in order until they cannot perform further activities due to exhausted means (again noting that each activity maintains its own satisfaction conditions).

Recall: Schedule Switcheroo

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- **Nov 5:** *Pure General Logic Programming, Functional Programming, Turing-Completeness, and Beyond.* We review the basic paradigms of computer programming. For the imperative case, we use the simple imperative language of (Davis, Sigal & Weyuker 1994), and also discuss register machines, Turing machines (again), KU machines. We also discuss whether programming beyond the Turing Limit makes sense and can be pursued.
- **Nov 9:** *Hypergraphical Proof and Programming in HyperSlate[®].* We here introduce the availability of writing Clojure functions in the context of proofs in HyperSlate[®].
- **Nov 12:** *Quantified Modal Logic.* We here explore quantified **S5**, the infamous Barcan Formula. HyperSlate[®] is used.
- **Nov 16:** *Killer Robots, D, and Beyond in HyperSlate[®] to DC $\mathcal{E}\mathcal{C}$.* We begin here by stating the “PAID Problem,” and then the approach to it from Bringsjord et al. advocates.

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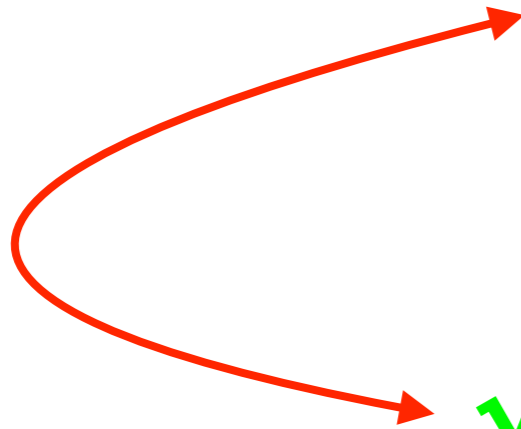
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Q3

Consider a new propositional modal logic: *propositional provability logic*, or for short, **PPL**. We here make use of the familiar “box” and “diamond” we have seen in our propositional modal logics so far, which of course are available in HS[®]. In **PPL** we read $\Box\phi$ as saying that ϕ is provable, and $\Diamond\phi$ is simply an abbreviation for $\neg\Box\neg\phi$. In order to have **PPL** available to us for exploration in HS[®], we simply use **K**, and add to our workspace a formula that expresses this new principle:

(Löb) If it's provable that (if ϕ is provable, then ϕ), then ϕ is provable.

Let $\langle\text{Löb}\rangle$ denote this formula. Now here are the two tasks for you in Q3:

(i) Can the characteristic axiom of **S4** be proved in **PPL**? Prove your answer. (Max one page.)

(ii) It would seem that a more interesting and (given what those in the business of proving things do) accurate logic would be *quantified provability logic* (**QPL**), since after all, all interesting theorems have quantifiers and relation symbols in them. After you are clear on what **QPL** amounts to formally, answer the following question, and justify your answer with cogent argumentation.

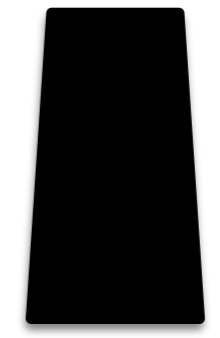
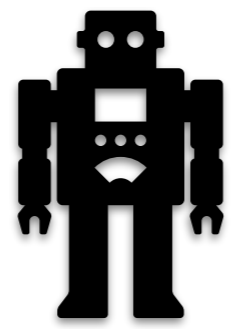
Question: Can an artificial agent can be engineered which productively uses **QPL**? Max one page.

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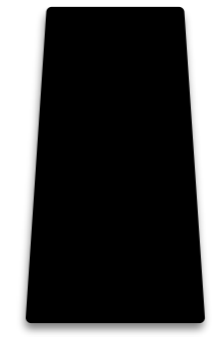
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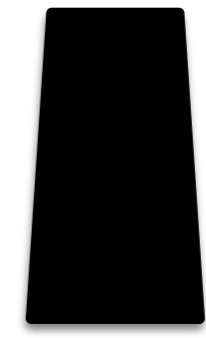
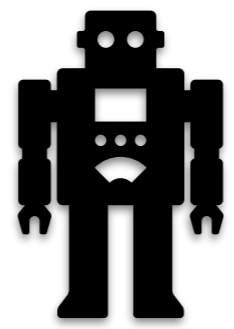
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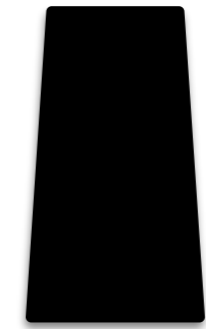
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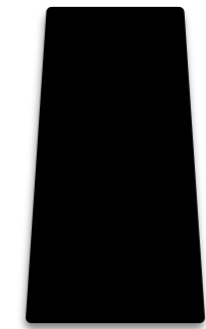
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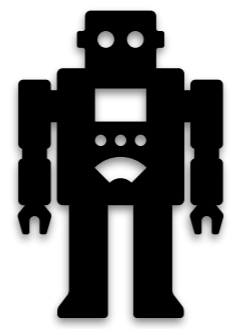


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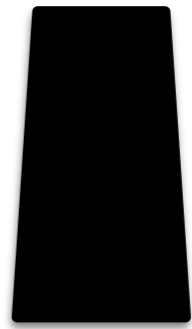




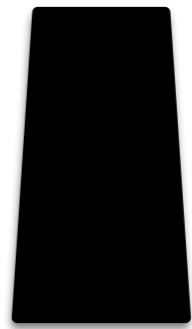
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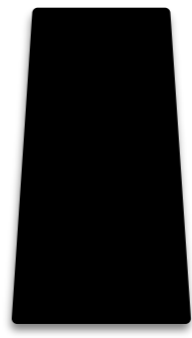
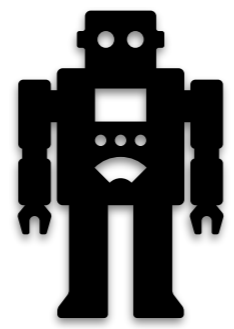
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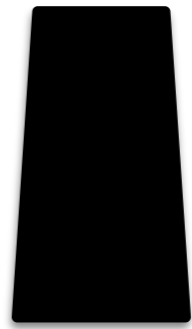
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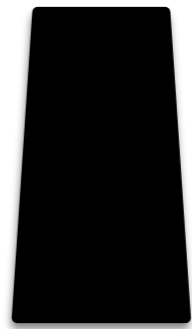
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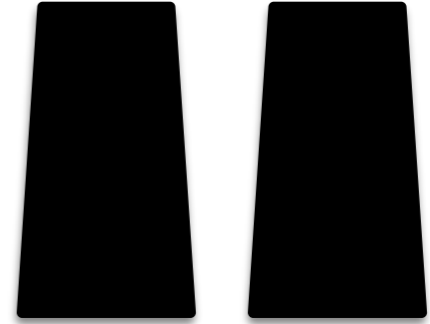
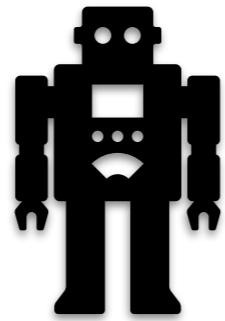
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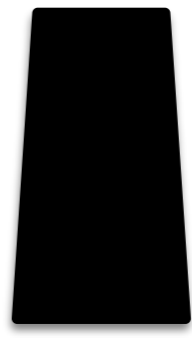
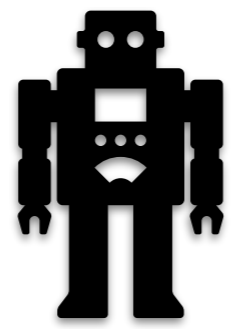


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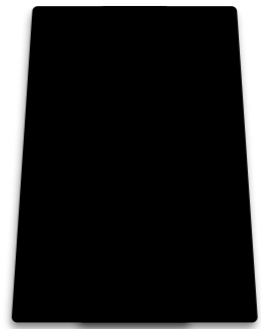
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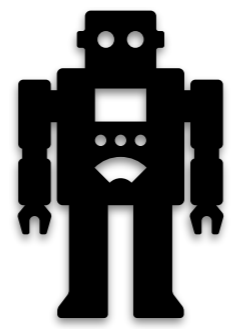


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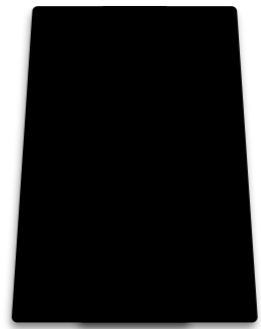
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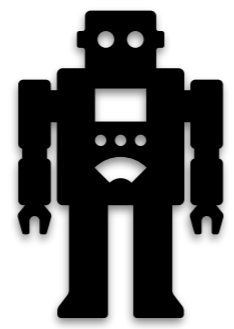
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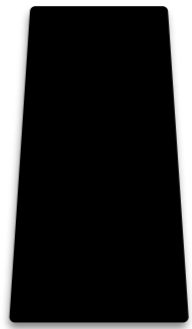
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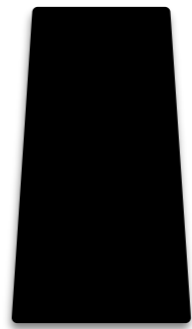
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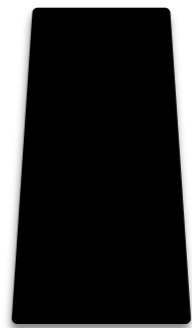
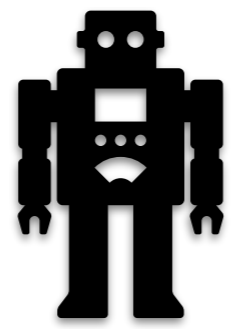
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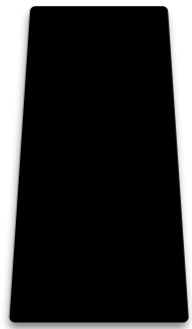
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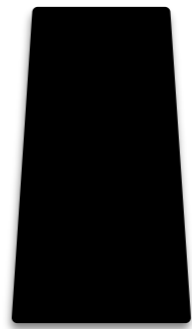
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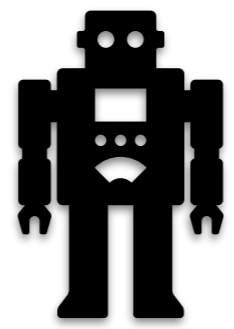
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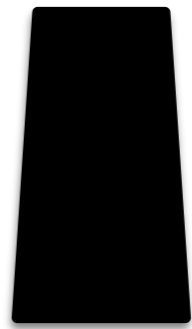
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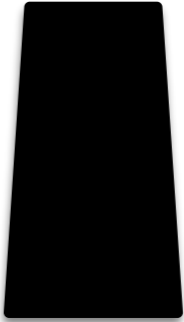
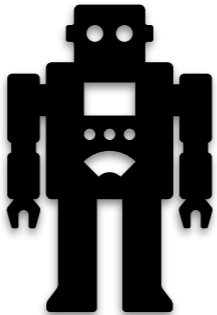


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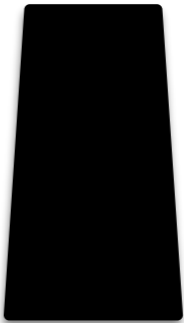


The ball is in the cup at location #1.

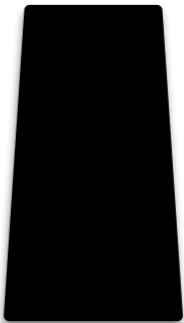
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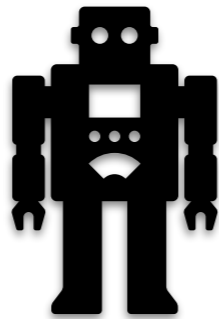
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The ball is in the cup at location #1.

Loc(ball,1)

Blinky



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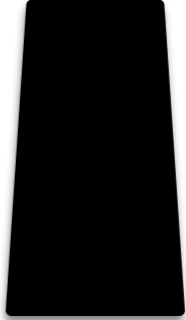
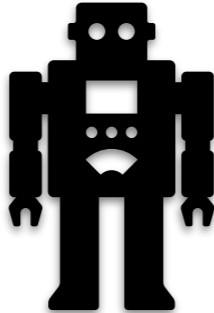


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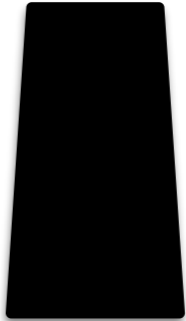
Loc(ball,1)

(Loc ball 1)

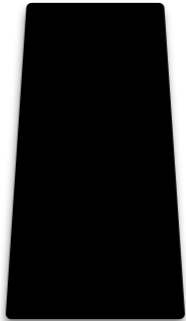
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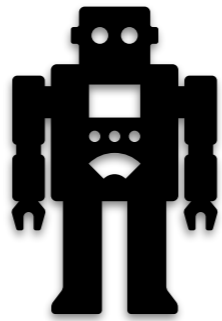


The ball is in the cup at location #1.

FALSE Loc(ball,1)

(Loc ball 1)

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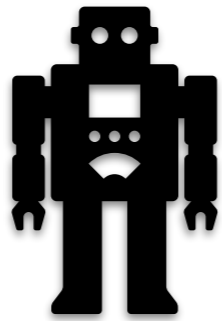
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FALSE Loc(ball,1)

(Loc ball 1)

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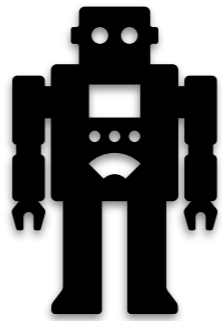
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FALSE

(Loc ball 1)

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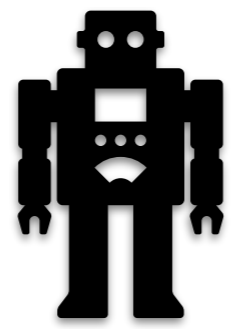


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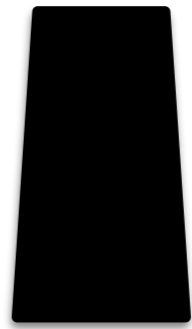


(Loc ball 1)

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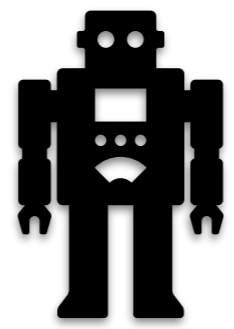
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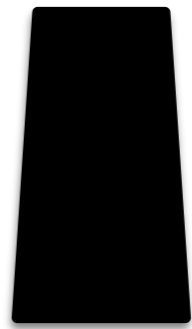
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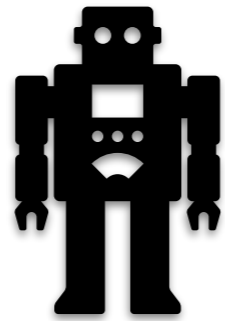


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The ball is in the cup at location #1 or the ball is at location #3.

Blinky



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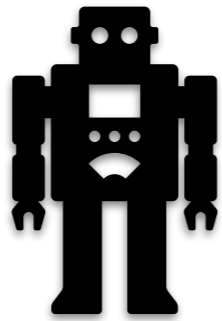
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The ball is in the cup at location #1 or the ball is at location #3.

$\text{Loc}(\text{ball}, 1) \vee \text{Loc}(\text{ball}, 3)$

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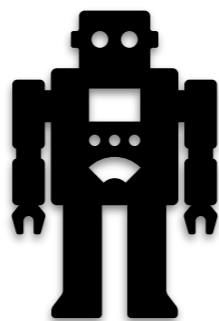


The ball is in the cup at location #1 or the ball is at location #3.

$\text{Loc}(\text{ball},1) \vee \text{Loc}(\text{ball},3)$

(or (Loc ball 1) (Loc ball 3))

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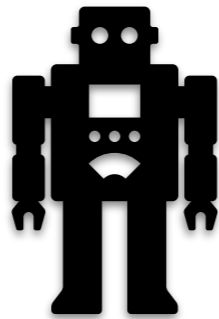


The ball is in the cup at location #1 or the ball is at location #3.

FALSE $\text{Loc}(\text{ball},1) \vee \text{Loc}(\text{ball},3)$

(or (Loc ball 1) (Loc ball 3))

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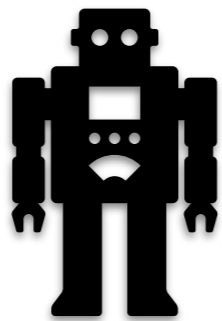
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FALSE $\text{Loc}(\text{ball}, 1) \vee \text{Loc}(\text{ball}, 3)$

(or (Loc ball 1) (Loc ball 3))

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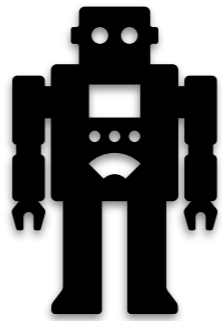
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FALSE

(or (Loc ball 1) (Loc ball 3))

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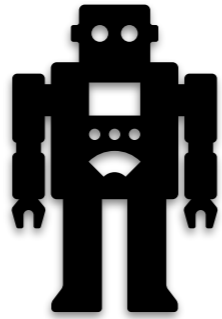


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FALSE

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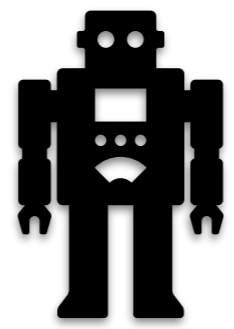
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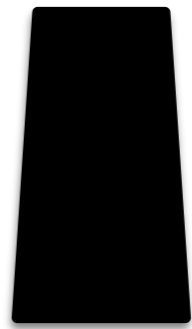
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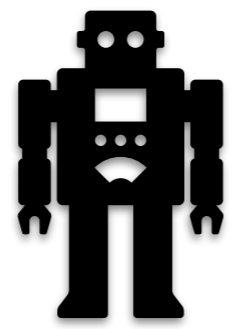


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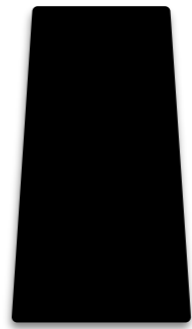


Blinky believes that the ball is in the cup at location #1.

Blinky



1



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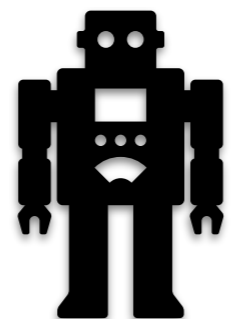
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Blinky believes that the ball is in the cup at location #1.

$B(\text{blinky}, \text{Loc}(\text{ball}, 1))$

Blinky



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3

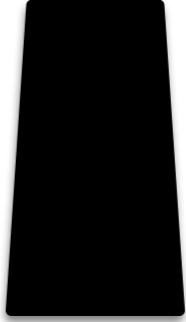
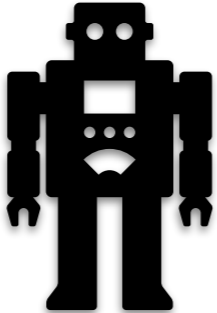


Blinky believes that the ball is in the cup at location #1.

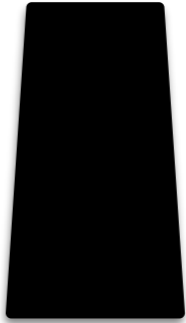
$B(\text{blinky}, \text{Loc}(\text{ball}, 1))$

(Believes! t blinky (Loc ball 1))

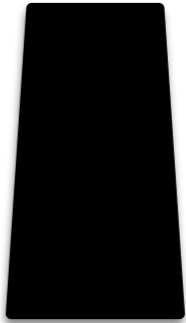
Blinky



1



2



3



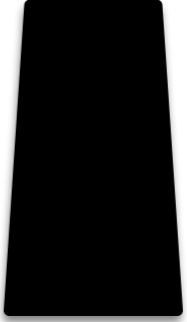
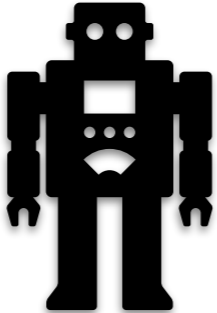
Blinky believes that the ball is in the cup at location #1.

???

B(blinky, Loc(ball,1))

(Believes! t blinky (Loc ball 1))

Blinky



1



2



3



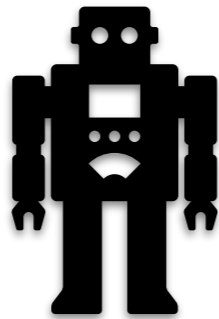
Blinky believes that the ball is in the cup at location #1.

???

$B(\text{blinky}, \text{Loc}(\text{ball}, 1))$

(Believes! t blinky (Loc ball 1))

Blinky



1



2



3

In extensional logics, what is denoted is conflated with meaning (the latter being naively compositional), and intensional attitudes like *believes*, *knows*, *hopes*, *fears*, etc cannot be represented and reasoned over smoothly (e.g. without fear of inconsistency rising up).

Review: Encapsulation

Slate - K.slt

K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $K \vdash \checkmark \infty \Box$	T. $\Box\varphi \rightarrow \varphi$ $K \vdash \times \infty \Box$	4. $\Box\varphi \rightarrow \Box\Box\varphi$ $K \vdash \times \infty \Box$	5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $K \vdash \times \infty \Box$
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Review: Encapsulation

The image shows two overlapping windows from the Slate application. The top window is titled "Slate - K.slt" and the bottom window is titled "Slate - T.slt". Each window contains four boxes, each representing a modal logic formula and its validity in a specific system.

Slate - K.slt

- Box 1: $K. \Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$
 $K \vdash \checkmark \infty \Box$
- Box 2: $T. \Box\varphi \rightarrow \varphi$
 $K \vdash \times \infty \Box$
- Box 3: $4. \Box\varphi \rightarrow \Box\Box\varphi$
 $K \vdash \times \infty \Box$
- Box 4: $5. \neg\Box\varphi \rightarrow \Box\neg\Box\varphi$
 $K \vdash \times \infty \Box$

Slate - T.slt

- Box 1: $K. \Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$
 $M \vdash \checkmark \infty \Box$
- Box 2: $T. \Box\varphi \rightarrow \varphi$
 $M \vdash \checkmark \infty \Box$
- Box 3: $4. \Box\varphi \rightarrow \Box\Box\varphi$
 $M \vdash \times \infty \Box$
- Box 4: $5. \neg\Box\varphi \rightarrow \Box\neg\Box\varphi$
 $M \vdash \times \infty \Box$

Review: Encapsulation

The image shows three overlapping windows from the Slate application, each displaying a set of modal logic formulas and their validity in different frame types. The windows are titled "Slate - K.slt", "Slate - T.slt", and "Slate - D.slt".

Slate - K.slt

- K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$
K $\vdash \checkmark \infty \Box$
- T. $\Box\varphi \rightarrow \varphi$
K $\vdash \times \infty \Box$
- 4. $\Box\varphi \rightarrow \Box\Box\varphi$
K $\vdash \times \infty \Box$
- 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$
K $\vdash \times \infty \Box$

Slate - T.slt

- K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$
M $\vdash \checkmark \infty \Box$
- T. $\Box\varphi \rightarrow \varphi$
M $\vdash \checkmark \infty \Box$
- 4. $\Box\varphi \rightarrow \Box\Box\varphi$
M $\vdash \times \infty \Box$
- 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$
M $\vdash \times \infty \Box$

Slate - D.slt

- K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$
D $\vdash \checkmark \infty \Box$
- T. $\Box\varphi \rightarrow \varphi$
D $\vdash \times \infty \Box$
- D. $\Box\varphi \rightarrow \Diamond\varphi$
D $\vdash \checkmark \infty \Box$
- 4. $\Box\varphi \rightarrow \Box\Box\varphi$
D $\vdash \times \infty \Box$
- 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$
D $\vdash \times \infty \Box$
- INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$
D $\vdash \checkmark \infty \Box$

Review: Encapsulation

The image shows four overlapping Slate windows, each displaying a set of modal logic formulas and their provability status in a specific system. The windows are titled 'Slate - K.slt', 'Slate - T.slt', 'Slate - D.slt', and 'Slate - S4.slt'.

Slate - K.slt

- K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $K \vdash \checkmark \infty \Box$
- T. $\Box\varphi \rightarrow \varphi$ $K \vdash \times \infty \Box$
- 4. $\Box\varphi \rightarrow \Box\Box\varphi$ $K \vdash \times \infty \Box$
- 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $K \vdash \times \infty \Box$

Slate - T.slt

- K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $M \vdash \checkmark \infty \Box$
- T. $\Box\varphi \rightarrow \varphi$ $M \vdash \checkmark \infty \Box$
- 4. $\Box\varphi \rightarrow \Box\Box\varphi$ $M \vdash \times \infty \Box$
- 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $M \vdash \times \infty \Box$

Slate - D.slt

- K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $D \vdash \checkmark \infty \Box$
- T. $\Box\varphi \rightarrow \varphi$ $D \vdash \times \infty \Box$
- D. $\Box\varphi \rightarrow \Diamond\varphi$ $D \vdash \checkmark \infty \Box$
- 4. $\Box\varphi \rightarrow \Box\Box\varphi$ $D \vdash \times \infty \Box$
- 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $D \vdash \times \infty \Box$
- INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$ $D \vdash \checkmark \infty \Box$

Slate - S4.slt

- K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $S4 \vdash \checkmark \infty \Box$
- T. $\Box\varphi \rightarrow \varphi$ $S4 \vdash \checkmark \infty \Box$
- D. $\Box\varphi \rightarrow \Diamond\varphi$ $S4 \vdash \checkmark \infty \Box$
- 4. $\Box\varphi \rightarrow \Box\Box\varphi$ $S4 \vdash \checkmark \infty \Box$
- 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $S4 \vdash \times \infty \Box$
- INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$ $\{INTER\} \text{ Assume } \checkmark$

Review: Encapsulation

K

T

D

4 = S4

5 = S5

The image shows five Slate windows, each displaying a grid of modal logic formulas and their derivability in different systems. The windows are titled 'Slate - K.slt', 'Slate - T.slt', 'Slate - D.slt', 'Slate - S4.slt', and 'Slate - S5.slt'.

- Slate - K.slt:**
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $K \vdash \checkmark \infty \Box$
 - T. $\Box\varphi \rightarrow \varphi$ $K \vdash \times \infty \Box$
 - 4. $\Box\varphi \rightarrow \Box\Box\varphi$ $K \vdash \times \infty \Box$
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $K \vdash \times \infty \Box$
- Slate - T.slt:**
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $M \vdash \checkmark \infty \Box$
 - T. $\Box\varphi \rightarrow \varphi$ $M \vdash \checkmark \infty \Box$
 - 4. $\Box\varphi \rightarrow \Box\Box\varphi$ $M \vdash \times \infty \Box$
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $M \vdash \times \infty \Box$
- Slate - D.slt:**
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $D \vdash \checkmark \infty \Box$
 - T. $\Box\varphi \rightarrow \varphi$ $D \vdash \times \infty \Box$
 - D. $\Box\varphi \rightarrow \Diamond\varphi$ $D \vdash \checkmark \infty \Box$
 - 4. $\Box\varphi \rightarrow \Box\Box\varphi$ $D \vdash \times \infty \Box$
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $D \vdash \times \infty \Box$
 - INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$ $D \vdash \checkmark \infty \Box$
- Slate - S4.slt:**
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $S4 \vdash \checkmark \infty \Box$
 - T. $\Box\varphi \rightarrow \varphi$ $S4 \vdash \checkmark \infty \Box$
 - D. $\Box\varphi \rightarrow \Diamond\varphi$ $S4 \vdash \checkmark \infty \Box$
 - 4. $\Box\varphi \rightarrow \Box\Box\varphi$ $S4 \vdash \checkmark \infty \Box$
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $S4 \vdash \times \infty \Box$
 - INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$ $\{INTER\} \text{ Assume } \checkmark$
- Slate - S5.slt:**
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $S5 \vdash \checkmark \infty \Box$
 - T. $\Box\varphi \rightarrow \varphi$ $S5 \vdash \checkmark \infty \Box$
 - D. $\Box\varphi \rightarrow \Diamond\varphi$ $\{D\} \text{ Assume } \checkmark$
 - 4. $\Box\phi \rightarrow \Box\Box\phi$
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $S5 \vdash \checkmark \infty \Box$
 - INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$ $\{INTER\} \text{ Assume } \checkmark$

Review: Encapsulation

K

T

D

4 = S4

5 = S5

The image shows five Slate windows, each displaying a grid of modal logic formulas and their derivability in different systems. The windows are titled as follows:

- Slate - K.slt**: Shows formulas K, T, 4, and 5. K is derivable in K (K ⊢ ✓ ∞ □). T, 4, and 5 are not derivable in K (K ⊢ ✗ ∞ □).
- Slate - T.slt**: Shows formulas K, T, 4, and 5. K and T are derivable in M (M ⊢ ✓ ∞ □). 4 and 5 are not derivable in M (M ⊢ ✗ ∞ □).
- Slate - D.slt**: Shows formulas K, T, D, 4, 5, and INTER. K, T, and 4 are not derivable in D (D ⊢ ✗ ∞ □). D and 5 are derivable in D (D ⊢ ✓ ∞ □). INTER is derivable in D (D ⊢ ✓ ∞ □).
- Slate - S4.slt**: Shows formulas K, T, D, 4, 5, and INTER. K, T, D, and 4 are derivable in S4 (S4 ⊢ ✓ ∞ □). 5 is not derivable in S4 (S4 ⊢ ✗ ∞ □). INTER is derivable in S4 with the assumption {INTER} (S4 ⊢ ✓ ∞ □).
- Slate - S5.slt** (highlighted with a red border): Shows formulas K, T, D, 4, 5, and INTER. K, T, and 5 are derivable in S5 (S5 ⊢ ✓ ∞ □). D and 4 are not derivable in S5 (S5 ⊢ ✗ ∞ □). INTER is derivable in S5 with the assumption {D} (S5 ⊢ ✓ ∞ □).

S5 ...

The Characteristic Axiom

$$\diamond\phi \rightarrow \square\diamond\phi$$

The Characteristic Axiom

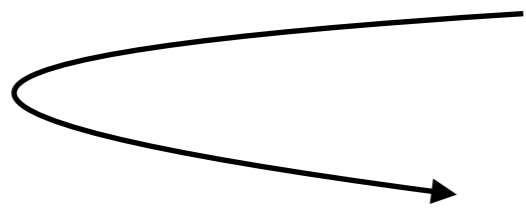
$$\diamond\phi \rightarrow \square\diamond\phi$$

$$\neg\square\psi \rightarrow \square\neg\square\psi$$

The Characteristic Axiom

$$\diamond\phi \rightarrow \square\diamond\phi$$

$$\neg\square\psi \rightarrow \square\neg\square\psi$$

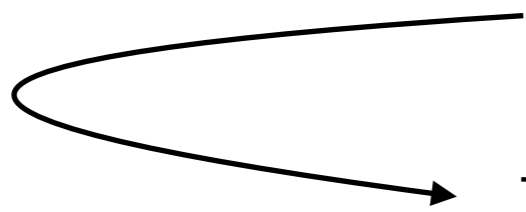


The Characteristic Axiom

$$\diamond\phi \rightarrow \square\diamond\phi$$

$$\neg\square\psi \rightarrow \square\neg\square\psi$$

$$\neg\square\neg\phi \rightarrow \square\neg\square\neg\phi$$

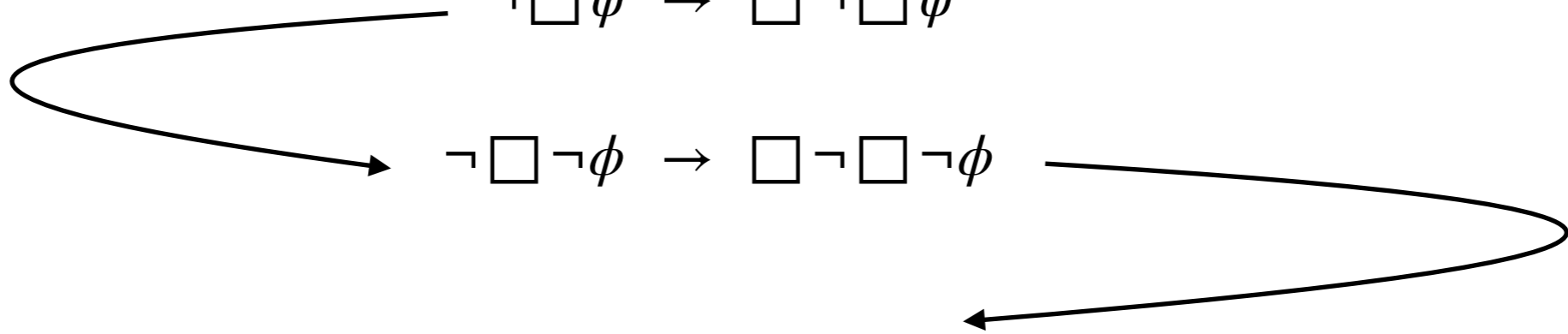


The Characteristic Axiom

$$\diamond\phi \rightarrow \square\diamond\phi$$

$$\neg\square\psi \rightarrow \square\neg\square\psi$$

$$\neg\square\neg\phi \rightarrow \square\neg\square\neg\phi$$



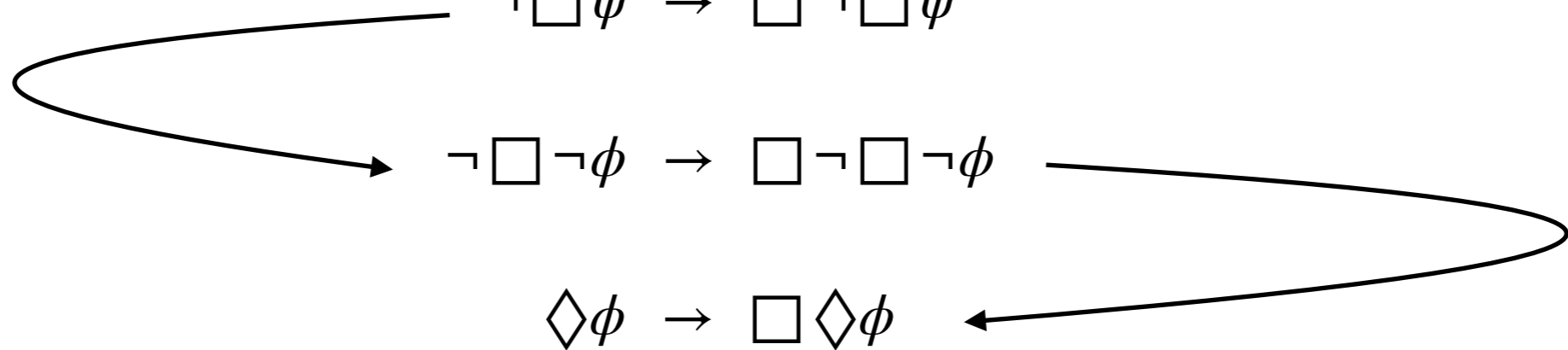
The Characteristic Axiom

$$\diamond\phi \rightarrow \square\diamond\phi$$

$$\neg\square\psi \rightarrow \square\neg\square\psi$$

$$\neg\square\neg\phi \rightarrow \square\neg\square\neg\phi$$

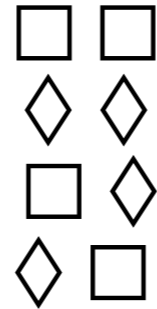
$$\diamond\phi \rightarrow \square\diamond\phi$$



Nice S5 Reduction Lemmas

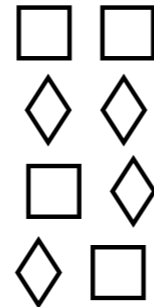
Nice S5 Reduction Lemmas

The Four Possible Pairs



Nice S5 Reduction Lemmas

The Four Possible Pairs



The Four Reduction Principles

$$\Box \phi \leftrightarrow \Box \Box \phi$$

$$\Diamond \phi \leftrightarrow \Diamond \Diamond \phi$$

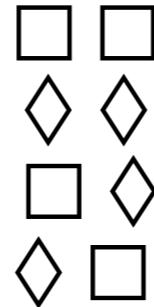
$$\Box \phi \leftrightarrow \Diamond \Box \phi$$

$$\Diamond \phi \leftrightarrow \Box \Diamond \phi$$

(where $\phi \in \mathcal{L}_{pc}$)

Nice S5 Reduction Lemmas

The Four Possible Pairs



The Four Reduction Principles

$$\Box \phi \leftrightarrow \Box \Box \phi$$

$$\Diamond \phi \leftrightarrow \Diamond \Diamond \phi$$

$$\Box \phi \leftrightarrow \Diamond \Box \phi$$

$$\Diamond \phi \leftrightarrow \Box \Diamond \phi$$

(where $\phi \in \mathcal{L}_{pc}$)

(verify in HS[®])

Quantificational S5, ...

Quantificational S5, ...

Quantificational S5, ...

Easy peasy: Marry **PS5** + \mathcal{L}_1 !

Quantificational S5, ...

Easy peasy: Marry **PS5** + \mathcal{L}_1 !

Theorem: $\forall x \diamond R(x) \rightarrow \forall x \square \diamond R(x)$

Quantificational S5, ...

Easy peasy: Marry **PS5** + \mathcal{L}_1 !

Theorem: $\forall x \Diamond R(x) \rightarrow \forall x \Box \Diamond R(x)$

Theorem: $\Diamond \exists x R(x) \leftrightarrow \exists x \Diamond R(x)$

Quantificational S5, ...

Easy peasy: Marry **PS5** + \mathcal{L}_1 !

Theorem: $\forall x \diamond R(x) \rightarrow \forall x \square \diamond R(x)$

Theorem: $\diamond \exists x R(x) \leftrightarrow \exists x \diamond R(x)$

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Quantificational S5, ...

Easy peasy: Marry **PS5** + \mathcal{L}_1 !

Theorem: $\forall x \diamond R(x) \rightarrow \forall x \square \diamond R(x)$

(verify in HS[®])

Theorem: $\diamond \exists x R(x) \leftrightarrow \exists x \diamond R(x)$

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The Notorious Barcans

The Notorious Barcan

Barcan Formula: $\vdash_{QS5_1} \Diamond \exists x \phi(x) \rightarrow \exists x \Diamond \phi(x)$

The Notorious Barcan

Barcan Formula: $\vdash_{QS5_1} \Diamond \exists x \phi(x) \rightarrow \exists x \Diamond \phi(x)$


Converse Barcan Formula: $\vdash_{QS5_1} \exists x \Diamond \phi(x) \rightarrow \Diamond \exists x \phi(x)$

The Notorious Barcan

Barcan Formula: $\vdash_{QS5_1} \Diamond \exists x \phi(x) \rightarrow \exists x \Diamond \phi(x)$

Converse Barcan Formula: $\vdash_{QS5_1} \exists x \Diamond \phi(x) \rightarrow \Diamond \exists x \phi(x)$

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 CrossMark

An Argument for P=NP

Selmer Bringsjord^{1,2}

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Abstract I articulate a novel modal argument for P=NP.

Keywords P=NP · Modal logic · Digital physics

The Clay Mathematics Institute offers a \$1 million prize for a solution to the P=?NP problem.¹ I look forward to receiving my award—but concede that the expected format of a solution is an *object-level* proof, not a meta-level argument like what I provide. On the other hand, certainly the winner needn't provide a *constructive* proof that P=NP.² Despite Gödel's recently discovered position on the

¹ See <http://www.claymath.org/millennium>. There are six other “millennium” problems; each of these is also associated with a \$1M prize.


² As many readers know, the history of the problem is littered with failed attempts to provide non-constructive substantiation of the received view that P≠NP.

I'm greatly indebted to Michael Zenzen for many valuable discussions about the P=?NP problem and physics (*simpliciter* and digital), and to Jim Fahey for discussions about such physics and mixed-mode dual-diamond operators in modal logic. The presentation of the core arguments herein to editions of Bringsjord's graduate seminar, *Logic & Artificial Intelligence*, and his guest lectures on P=?NP in *Formal Foundations of Cognitive Science* graduate seminars, sparked a number of helpful objections and suggestions, for which I'm grateful. I'm indebted as well to two anonymous referees for trenchant comments. Though the two arguments herein (the second of which seems to establish P=NP) are for weal or woe Bringsjord's, Joshua Taylor's astute objections catalyzed much thought and crucial refinements.

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