Portal to Inductive Logic: The Paradox of Grue

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Intermediate Formal Logic & AI (IFLAI2)
10/15/2020 (ver 1016201100NY)

The Grue Paradox Constructed

source of minerals

Mineralogist

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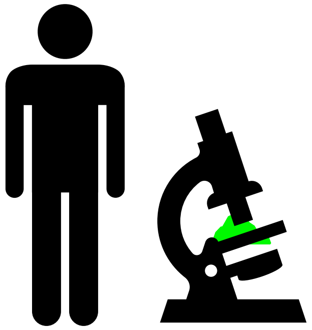

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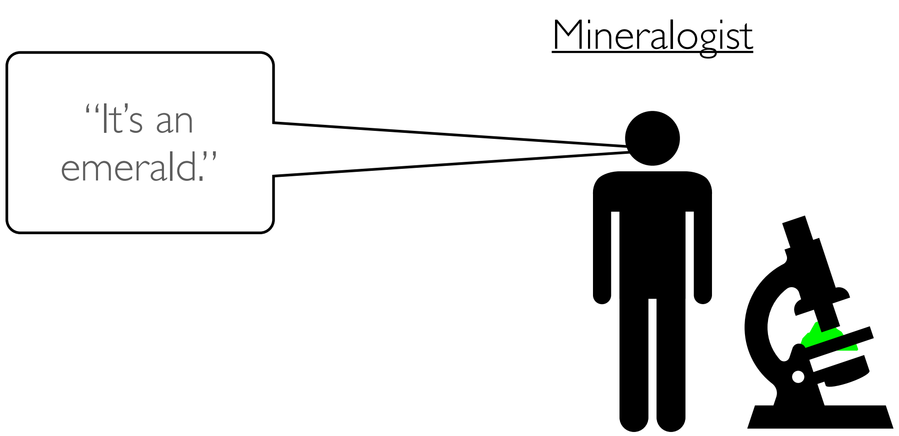

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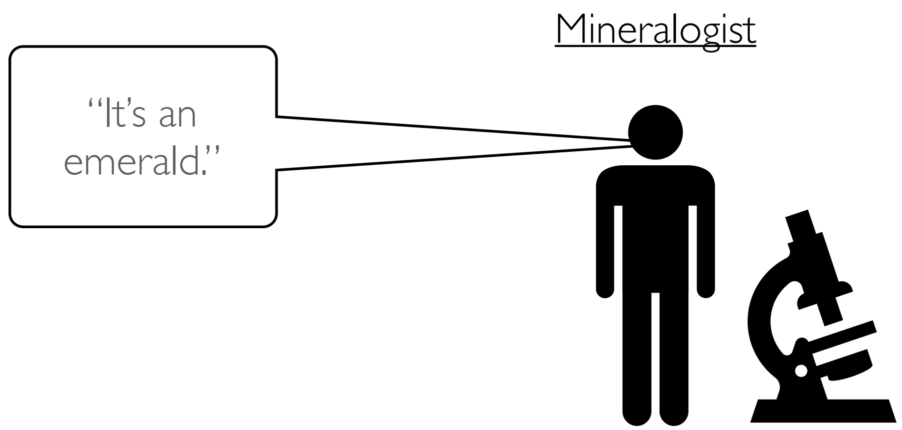

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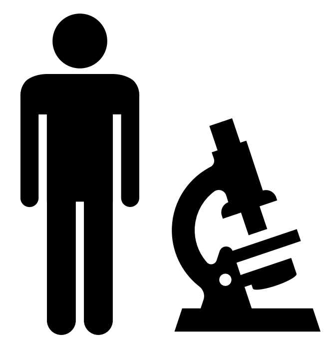

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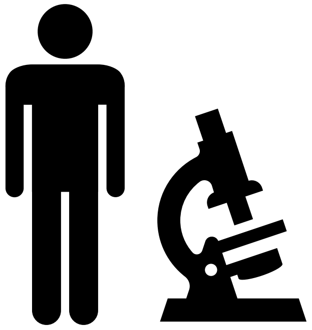

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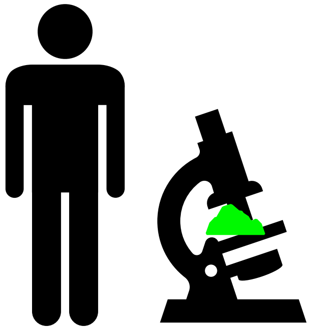

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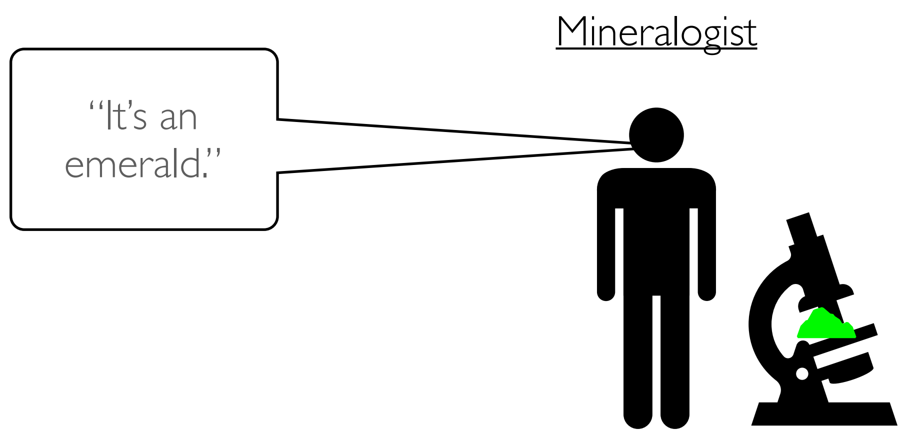

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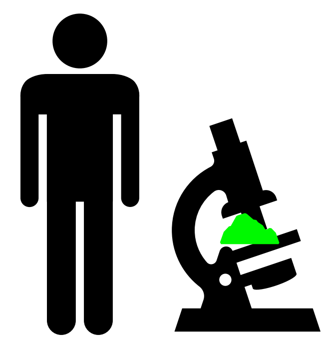

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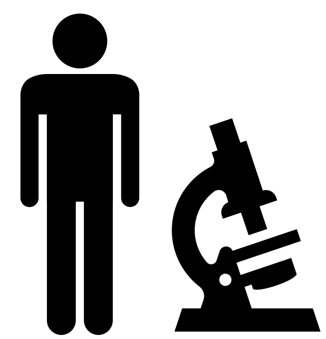

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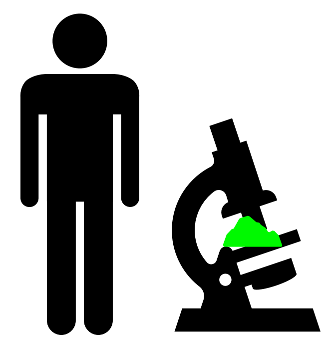

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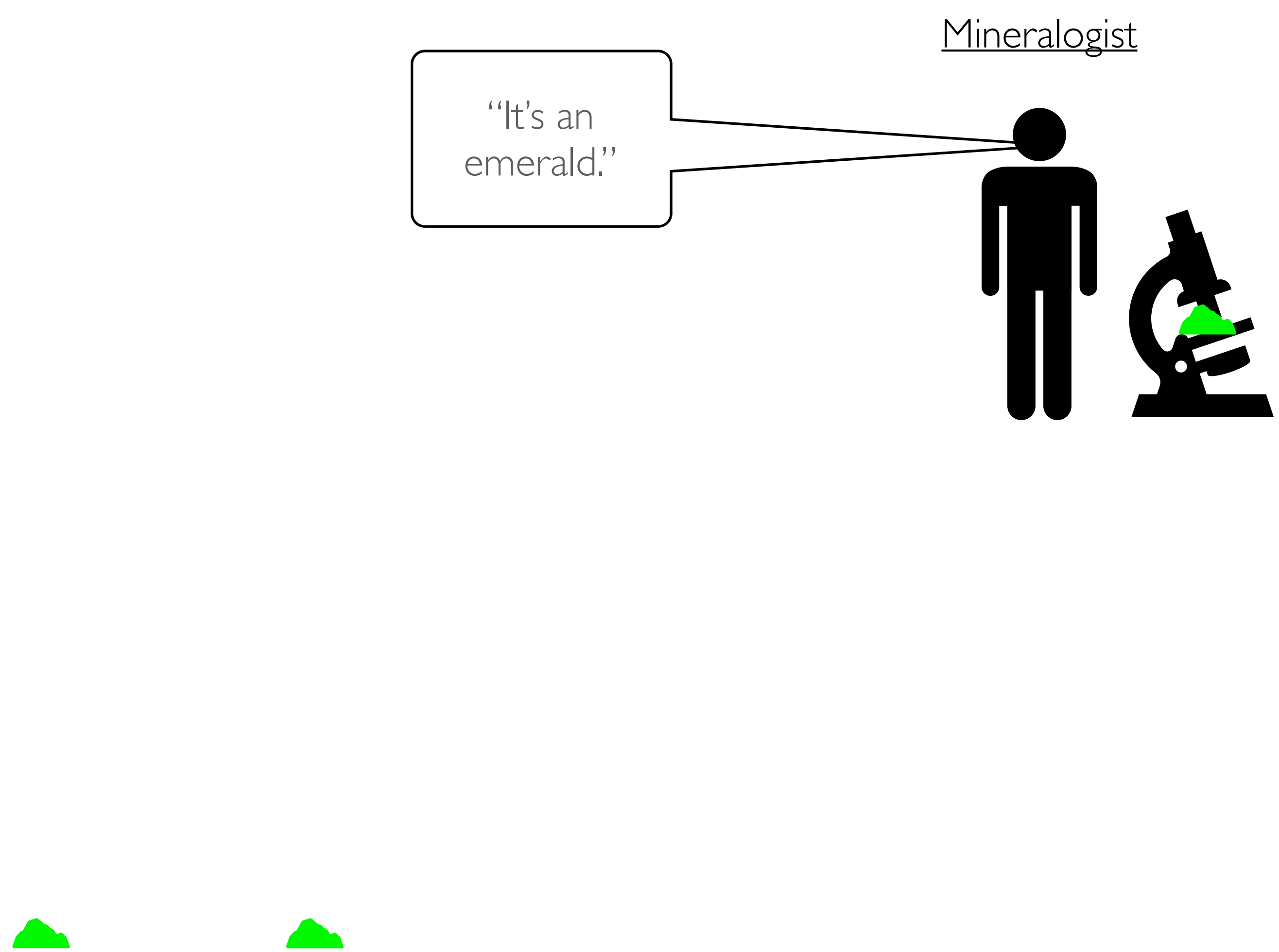

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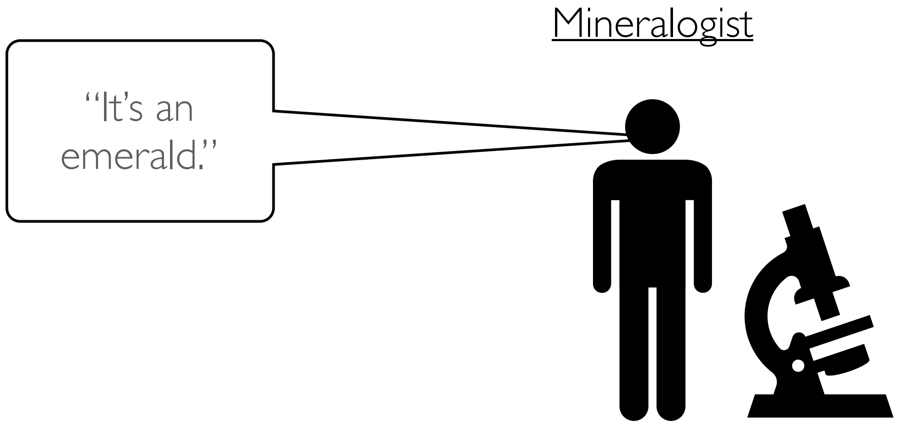

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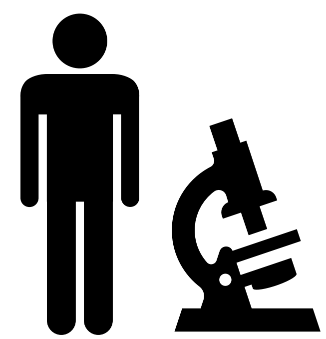

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$\ldots\ k, k \in \mathbb{Z}^{+},$ and all before time $t^{\star}$

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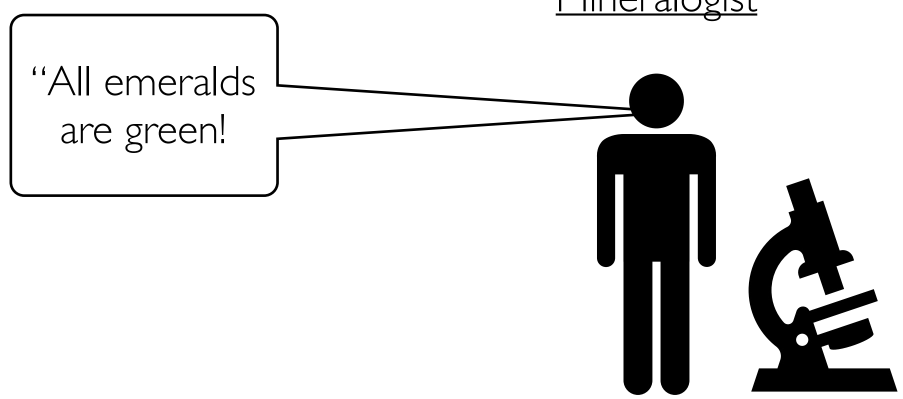

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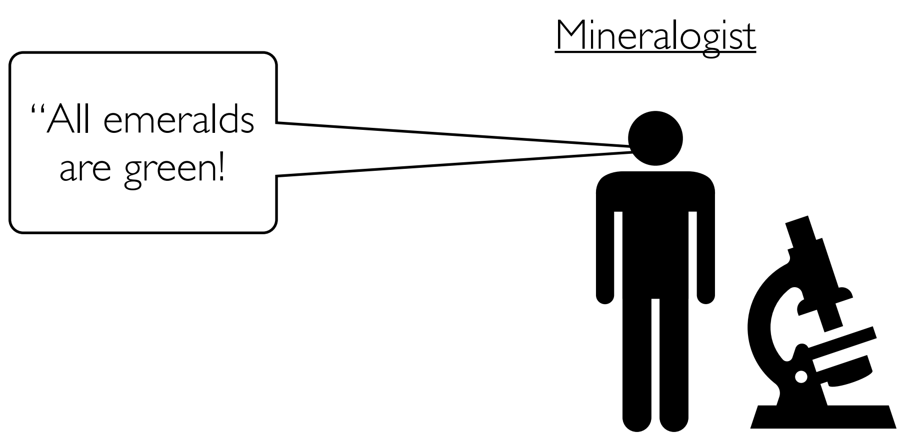
The Paradox: First, we define the property *Grue*:



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The Paradox: First, we define the property *Grue*:

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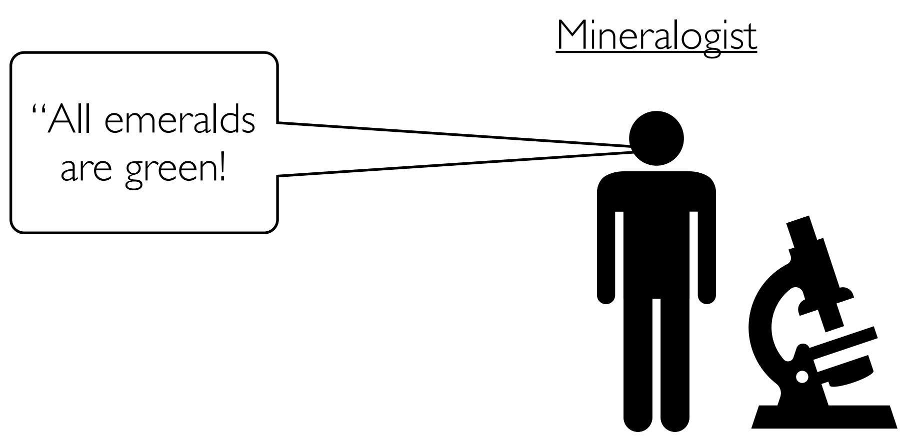

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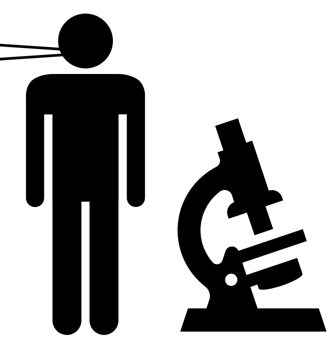
Mineralogist

"All emeralds are green!



$Green(o_1, t_1)$

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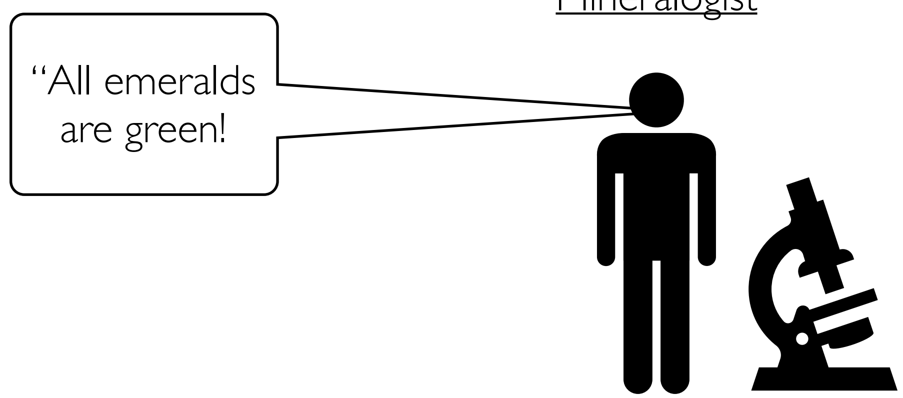

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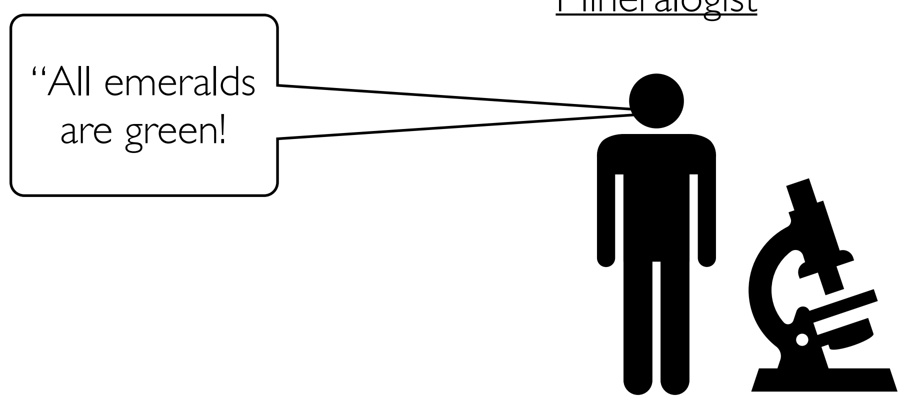

$Green(o_1, t_1)$ $Green(o_2, t_2)$ $Green(o_3, t_k)$ …

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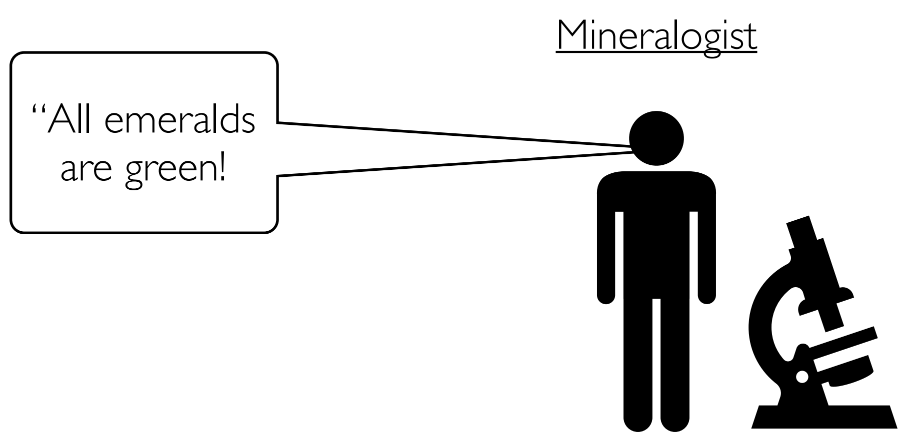
The k atomic formulae, together, support the general mineralogical law affirmed by our mineralogist, who we assume to be a rational empirical scientist. But this scientist must *also* affirm the proposition that all emeralds are grue, since the very same atomic formulae support this proposition, and to the same degree. But surely this is absurd!



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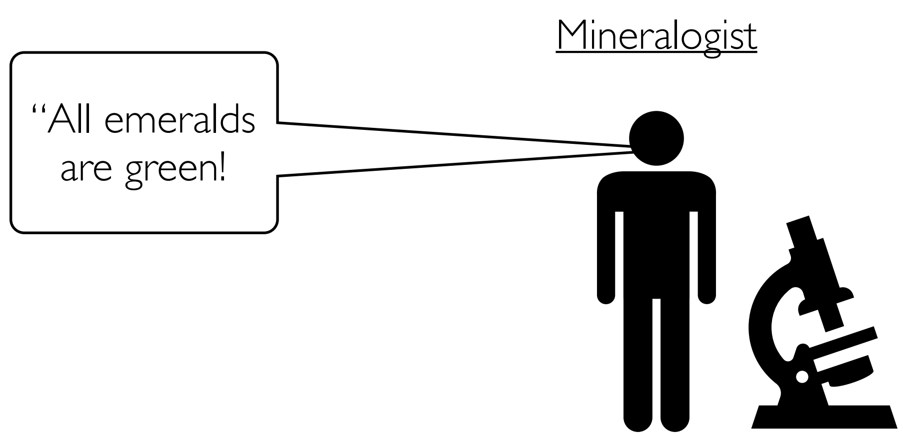

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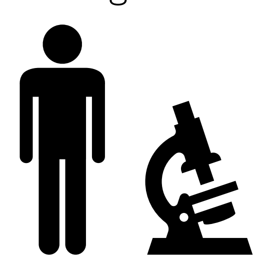

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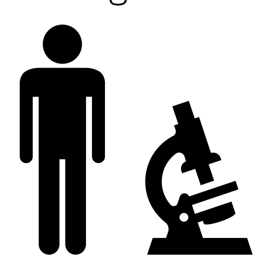

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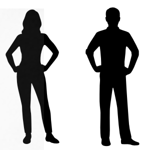
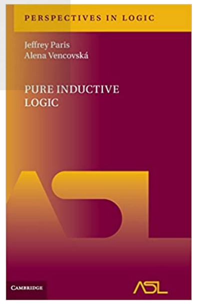
The Grue Paradox Divides Hearts and Minds

The Mathematicians/Logicians





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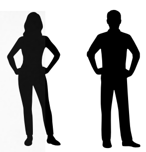

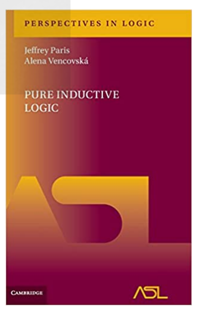

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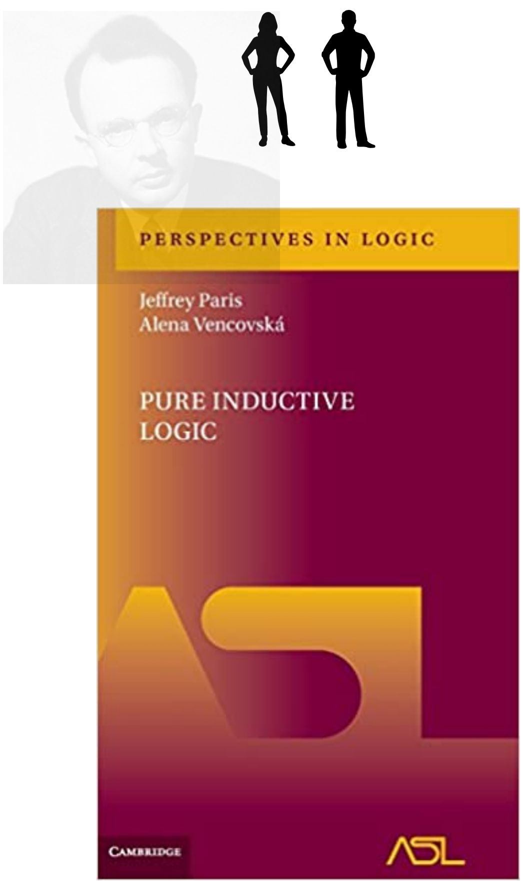

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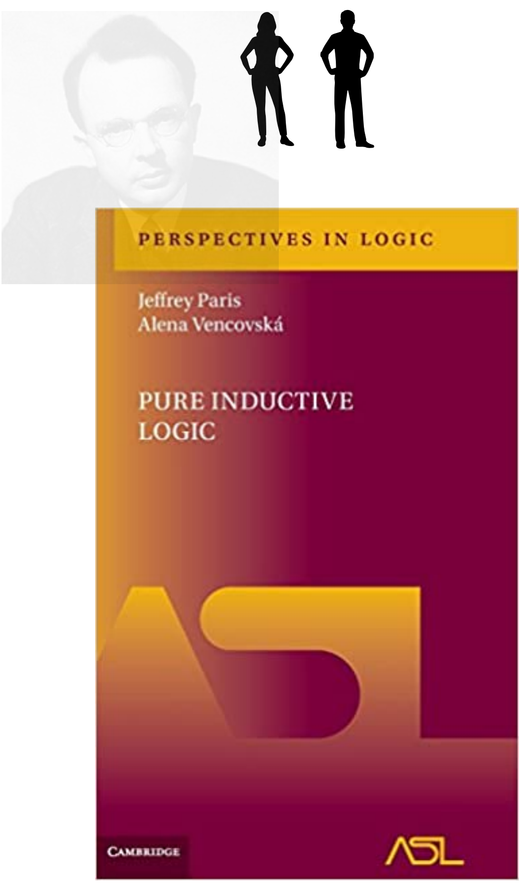

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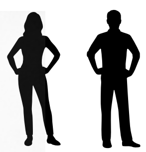
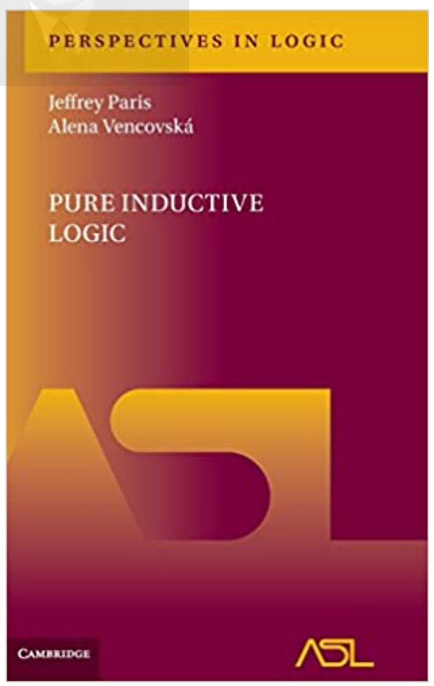
The Philosophers





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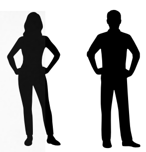

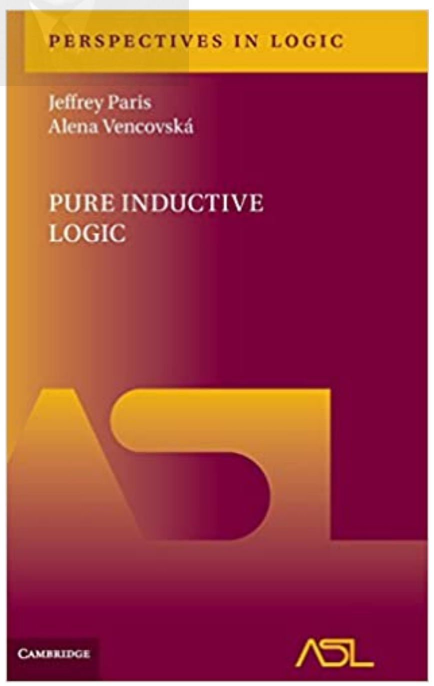

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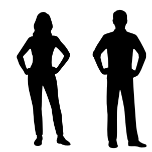

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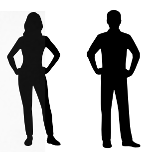

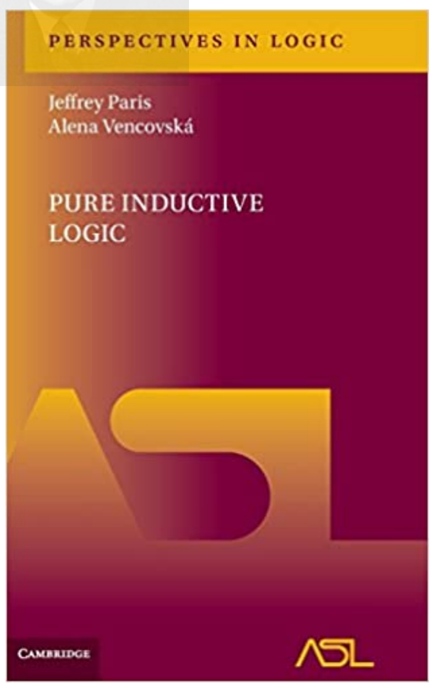

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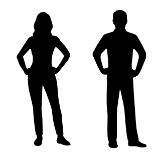

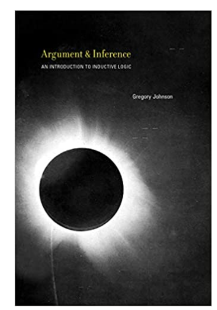

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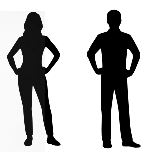

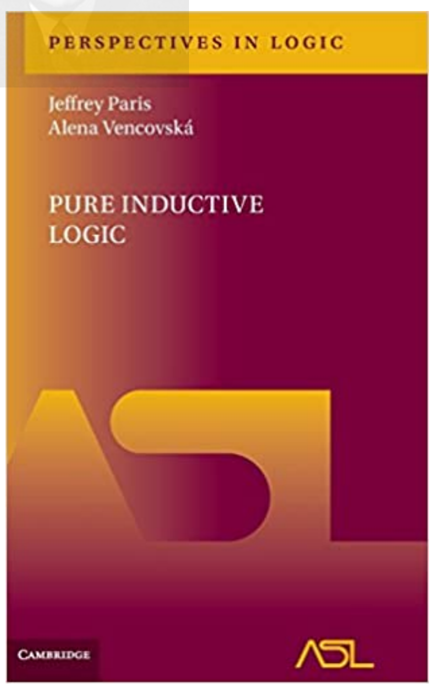

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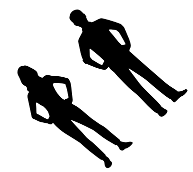

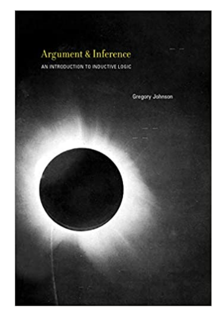

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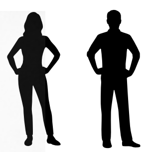

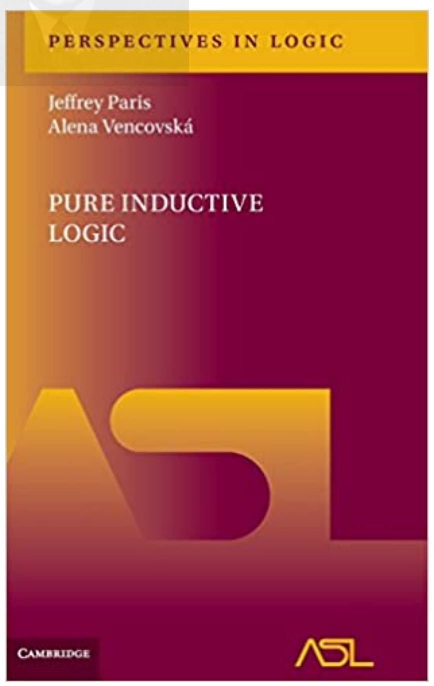

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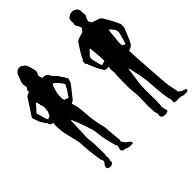

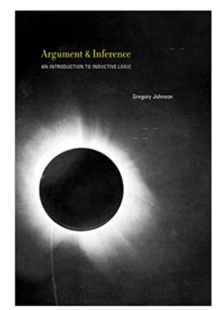

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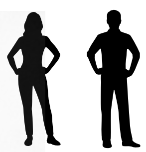
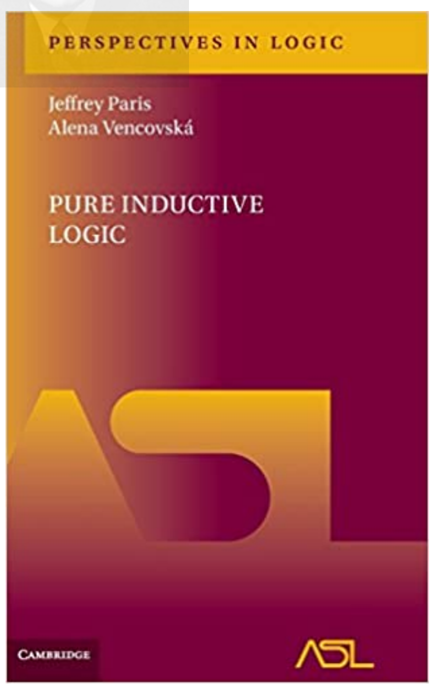

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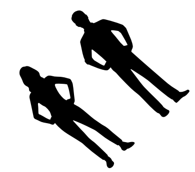
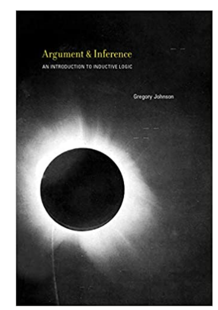

The Logicist AI Folks/
Computational Logicians

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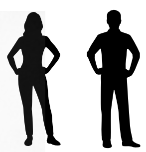

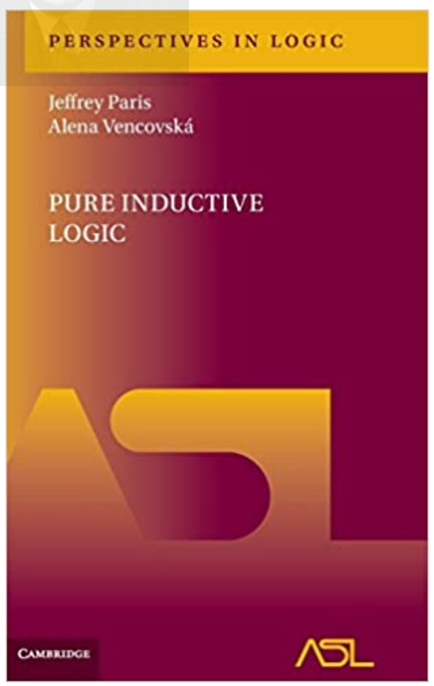

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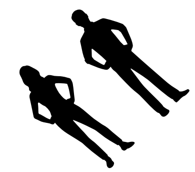

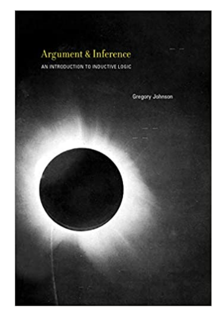

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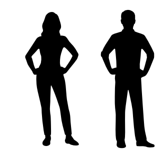

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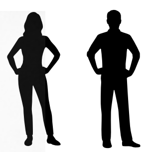

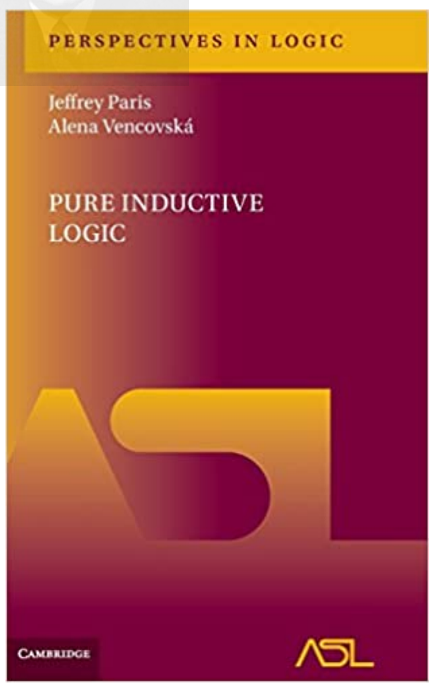

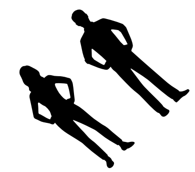

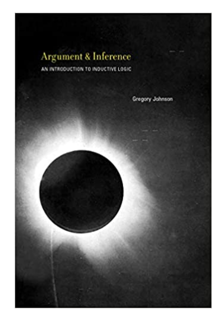

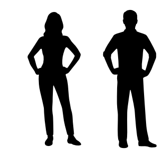

The Grue Paradox Divides Hearts and Minds

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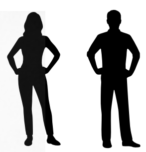
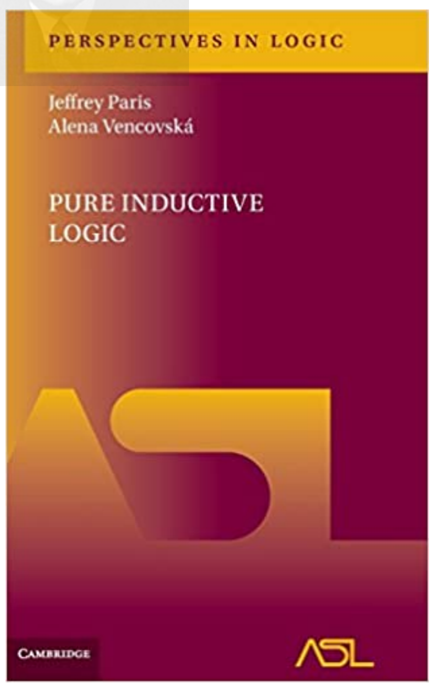
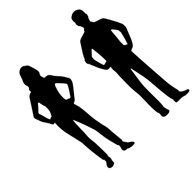
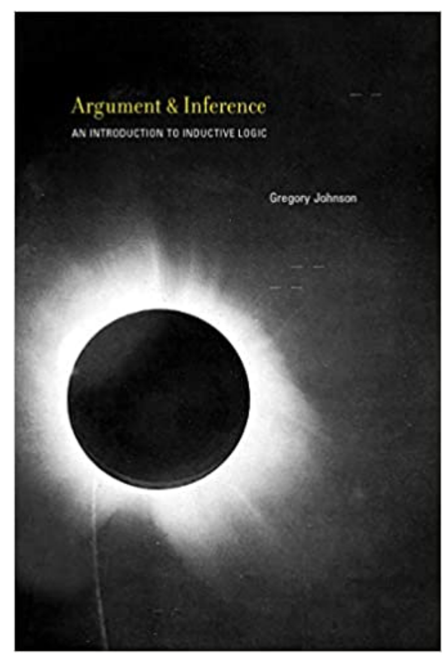




The Philosophers





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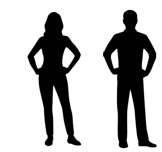

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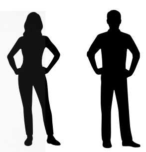

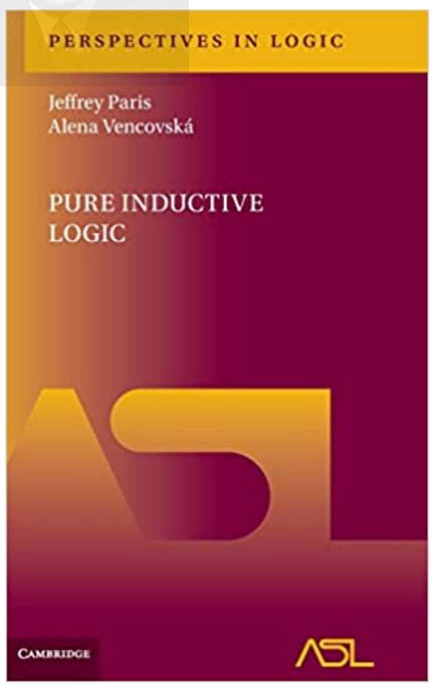

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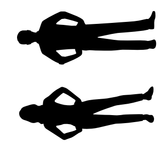

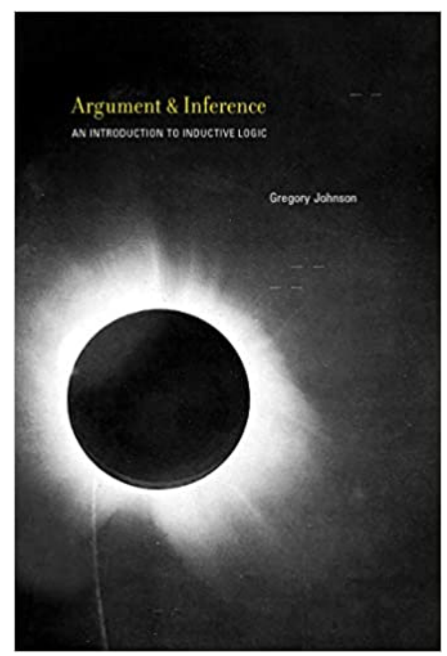

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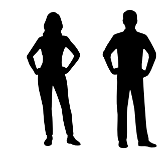

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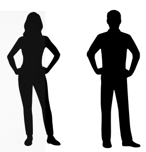

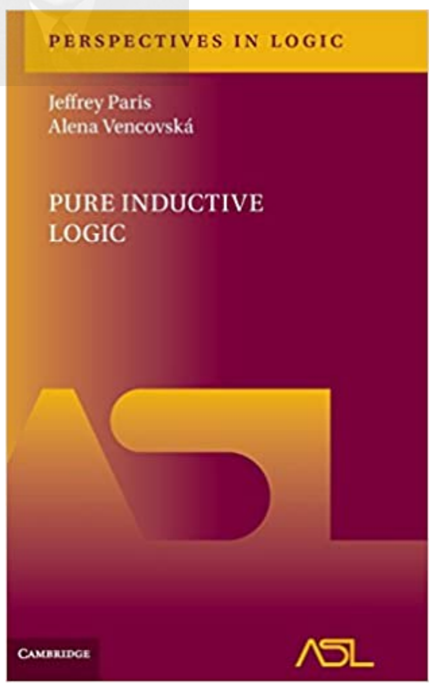

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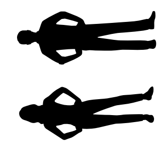

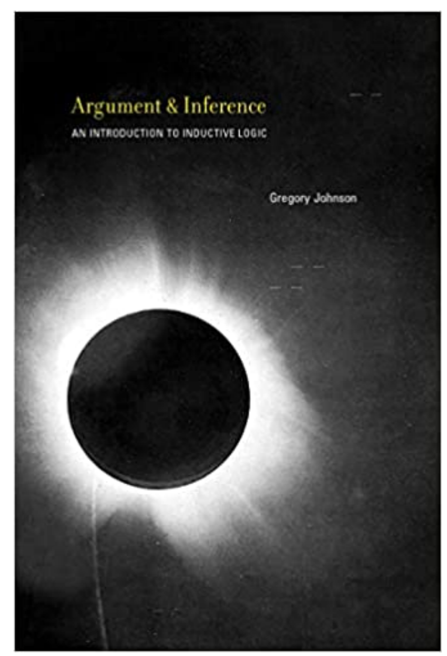

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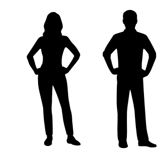

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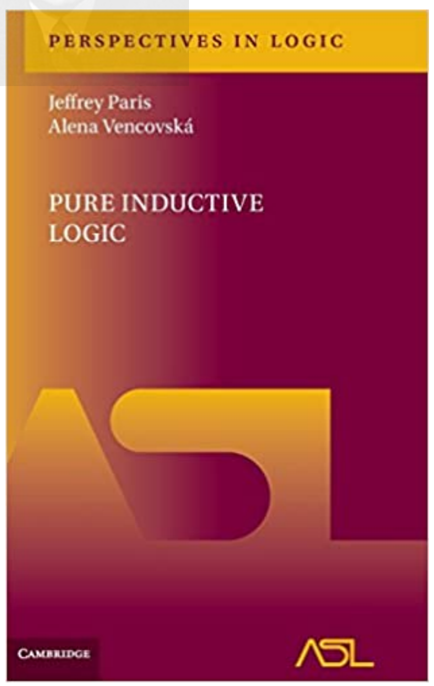

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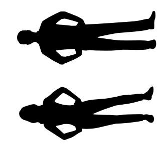

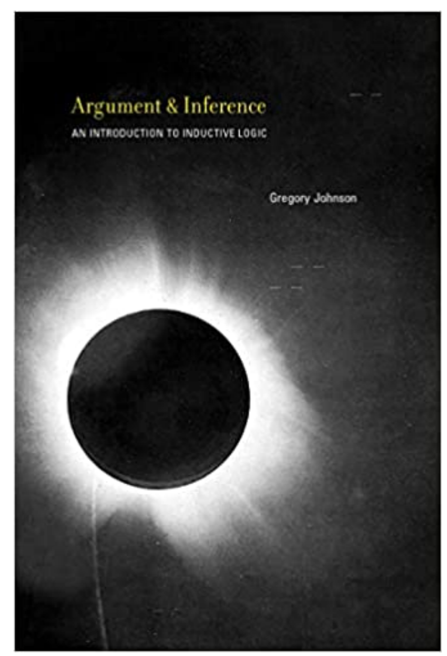

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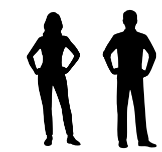

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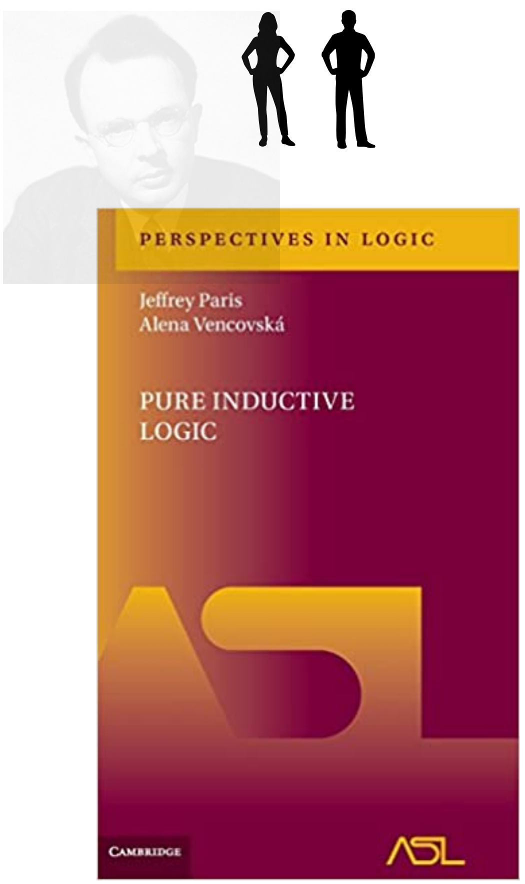

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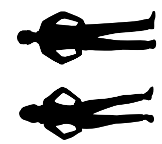

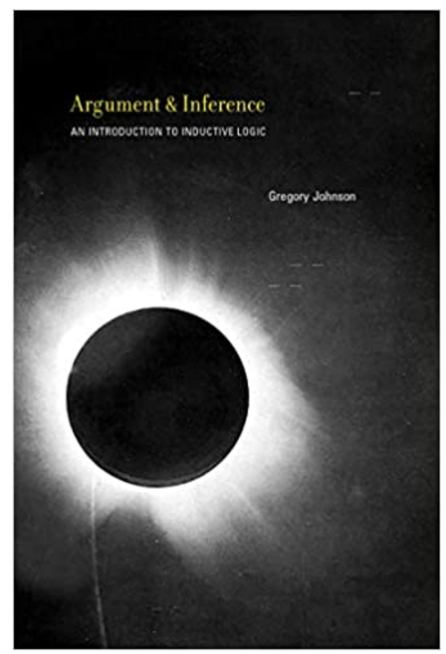

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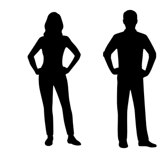

Background Reading …

The Original Publication Introducing The Grue Paradox

Goodman, N. (1955) *Fact, Fiction, and Forecast* (Cambridge, MA: Harvard University Press). 4th edition 1983, also HUP.

From "Nelson Goodman" in SEP (https://plato.stanford.edu/entries/goodman)

Here comes the riddle. Suppose that your research is in gemology. Your special interest lies in the color properties of certain gemstones, in particular, emeralds. All emeralds you have examined before a certain time t were green (your notebook is full of evidence statements of the form "Emerald x found at place y date $z(z \leq t)$ is green"). It seems that, at t, this supports the hypothesis that all emeralds are green (L3).

Now Goodman introduces the predicate "grue". This predicate applies to all things examined before some future time t just in case they are green but to other things (observed at or after t) just in case they are blue:

(DEF1) x is grue $=_{df}$ x is examined before t and green $\vee$ x is not so examined and blue

Until t it is obviously the case that for each statement in your notebook, there is a parallel statement asserting that the emerald x found at place y date $z(z \leq t)$ is grue. Each of these statements is analytically equivalent with the corresponding one in your notebook. All these grue-evidence statements taken together confirm the hypothesis that all emeralds are grue (L4), and they confirm this hypothesis to the exact same degree as the green-evidence statements confirmed the hypothesis that all emeralds are green. But if that is the case, then the following two predictions are also confirmed to the same degree:

(P1) The next emerald first examined after t will be green.
(P2) The next emerald first examined after t will be grue.

However, to be a grue emerald examined after t is not to be a green emerald. An emerald first examined after t is grue iff it is blue. We have two mutually incompatible predictions, both confirmed to the same degree by the past evidence. We could obviously define infinitely many grue-like predicates that would all lead to new, similarly incompatible predictions.

The immediate lesson is that we cannot use all kinds of weird predicates to formulate hypotheses or to classify our evidence. Some predicates (which are the ones like "green") can be used for this; other predicates (the ones like "grue") must be excluded, if induction is supposed to make any sense. This already is an interesting result. For valid inductive inferences the choice of predicates matters.

It is not just that we lack justification for accepting a general hypothesis as true only on the basis of positive instances and lack of counterinstances (which was the old problem), or to define what rule we are using when accepting a general hypothesis as true on these grounds (which was the problem after Hume). The problem is to explain why some general statements (such as L3) are confirmed by their instances, whereas others (such as L4) are not. Again, this is a matter of the lawlikeness of L3 in contrast to L4, but how are we supposed to tell the lawlike regularities from the illegitimate generalizations?

Wikipedia Entry
"New Riddle of Induction" Isn't Half Bad!

(https://en.wikipedia.org/wiki/New_riddle_of_induction)

Tutorial by Paris on Pure Inductive Logic:

http://fitelson.org/few/paris_notes.pdf

(Paris explains that the mathematicians just assumed the reasoning in the grue paradox is invalid, and then continued on their way to erect upon Carnap's work a robust formal edifice (= *pure inductive logic*).)

See "Inductive Logic" in SEP for an excellent overview, and in particular nice coverage of Carnap's seminal contributions, which PIL extends. (https://plato.stanford.edu/entries/logic-inductive)

Inductive Logic

First published Mon Sep 6, 2004; substantive revision Mon Mar 19, 2018

An inductive logic is a logic of evidential support. In a deductive logic, the premises of a valid deductive argument *logically entail* the conclusion, where *logical entailment* means that every logically possible state of affairs that makes the premises true *must* make the conclusion truth as well. Thus, the premises of a valid deductive argument provide *total support* for the conclusion. An inductive logic extends this idea to weaker arguments. In a good inductive argument, the truth of the premises provides some *degree of support* for the truth of the conclusion, where this *degree-of-support* might be measured via some numerical scale. By analogy with the notion of deductive entailment, the notion of inductive degree-of-support might mean something like this: among the logically possible states of affairs that make the premises true, the conclusion must be true in (at least) proportion *r* of them—where *r* is some numerical measure of the support strength.

If a logic of *good inductive arguments* is to be of any real value, the measure of support it articulates should be up to the task. Presumably, the logic should at least satisfy the following condition:

> **Criterion of Adequacy (CoA)**:
> The logic should make it likely (as a matter of logic) that as evidence accumulates, the total body of true evidence claims will eventually come to indicate, via the logic's *measure of support*, that false hypotheses are probably false and that true hypotheses are probably true.

The CoA stated here may strike some readers as surprisingly strong. Given a specific logic of evidential support, how might it be shown to satisfy such a condition? Section 4 will show precisely how this condition is satisfied by the logic of evidential support articulated in Sections 1 through 3 of this article.

This article will focus on the kind of the approach to inductive logic most widely studied by epistemologists and logicians in recent years. This approach employs conditional probability functions to represent measures of the degree to which evidence statements support hypotheses. Presumably, hypotheses should be empirically evaluated based on what they *say* (or imply) about the likelihood that evidence claims will be true. A straightforward theorem of probability theory, called Bayes' Theorem, articulates the way in which what hypotheses *say* about the likelihoods of evidence claims influences the degree to which hypotheses are supported by those evidence claims. Thus, this approach to the logic of evidential support is often called a *Bayesian Inductive Logic* or a *Bayesian Confirmation Theory*. This article will first provide a detailed explication of a Bayesian approach to inductive logic. It will then examine the extent to which this logic may pass muster as an adequate logic of evidential support for hypotheses. In particular, we will see how such a logic may be shown to satisfy the Criterion of Adequacy stated above.

Med nok penger, kan logikk løse alle problemer.