Intermediate Formal Logic and AI

(= IFLAI2, rhymes with "eye fly boo")

Crosslistings:
30520 CSCI-4972-01
30503 MATH-4961-01
30502 PHIL-4140-01
30404 PHIL-4961-01
Fall 20, Mo Th 4:45p–6:35 NY time; Sage 3303

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1 General Orientation

This course is an intermediate class in formal logic, with substantive connections made to logicist AI, within the LAMA® paradigm.[1] (While emphasis is on *deductive* formal logic, there is some coverage of formal *in*ductive and heterogeneous formal logic).[2] AI plays a significant role in advancing learning in the class; and the class includes some coverage of logicist AI and logicist computer programming.[3]

We have referred above to "the LAMA® paradigm." What is that? This question will be answered in more detail later, but we do say here that while the LAMA® paradigm is based upon a number of pedagogical principles, first and foremost among them is what can be labelled the Driving Dictum:[4]

If you can't prove it, you don't get it.

Turning back to the nature of formal logic, it can accurately be said that it's the science and engineering of reasoning,[5] but even this supremely general slogan fails to convey the flexibility and enormity of the field. For example, all of classical mathematics can be deductively derived from a small set of formulae (e.g., **ZFC** set theory, with which you should already have some familiarity with, and indeed experimenting with in the HyperSlate® system) expressed in the formal logic known as 'first-order logic' (= FOL = $\mathscr{L}_1$, with which you are *also* familiar, and, as we shall review and discuss in class, computer science emerged from and is in large part based upon logic (for cogent coverage of this emergence, see Glymour 1992, Halpern, Harper, Immerman, Kolaitis, Vardi & Vianu 2001). Logic is indeed the foundation for *all* at once rational-and-rigorous intellectual pursuits. (If you can find a counter-example, i.e. such a pursuit that doesn't directly and crucially

[1]'LAMA®' is an acronym for 'Logic: A Modern Approach," and is pronounced to rhyme with 'llama' in contemporary English, the name of the exotic and sure-footed camelid whose binomial name is *Lama glama*, and has in fact been referred to in the past by the single-l 'lama.'

[2]Sometimes 'symbolic' is used in place of 'formal,' but that's a bad practice, since — as students in this class will soon see — formal logic includes the representation of and systematic reasoning over *pictorial* information, and such information is decidedly *not* symbolic. For a discussion of the stark difference between the pictorial vs. the symbolic, and presentation of a formal logic that enables representation of and reasoning over both, see (Arkoudas & Bringsjord 2009), which directly informs Chapter 8 of LAMA-BDLAHGHS.

[3]We use 'logicist computer programming' to denote a general approach to computer programming based on formal logic; this general approach covers what is called 'logic programming,' which is connected specifically to such languages as Prolog.

[4]It's profitable to ponder a variant of this dictum, applicable in venues [e.g. legal hearings, courtrooms, reports by analysts in various domains that are not exclusively formal (e.g. fundamental investing, intelligence, etc.)] in which reasoning is not only deductive, but inductive, viz. "If you can't show by explicit argument that it's likelihood reaches a sufficient level, you don't get it."

[5]Warning: Increasingly, the term 'reasoning' is used by some who don't *really* do anything related to reasoning, as traditionally understood, to nonetheless label what they do. Fortunately, it's easy to verify that some reasoning is that which is covered by formal logic: *If the reasoning is explicit, links declarative statements or declarative formulae together via explicit, abstract reasoning schemata or rules of inference (giving rise to at least explicit arguments, and often proofs), is surveyable and inspectable, and ultimately machine-checkable, then the reasoning in question is what formal logic is the science and engineering of.* In order to characterize *in*formal logic, one can remove from the previous sentence the requirements that the links must conform to explicit reasoning schemata or rules of inference, and machine-checkability. It follows that so-called informal logic would revolve around arguments, but not proofs. An excellent overview of informal logic, which will be completely ignored in this class and its LAMA-BDLAHGHS textbook, is provided in "Informal Logic" in the Stanford Encyclopedia of Philosophy. In this article, it's made clear that, yes, informal logic concentrates on the nature and uses of argument.

partake of logic, S Bringsjord would be very interested to see it.)

2 Prerequisites

Students are expected to have taken a serious university-level introductory formal-logic course (which hopefully included second-order logic and some propositional modal logic), and to have a significant degree of logico-mathematical maturity. Phase I of the course will include a review of some introductory formal logic, via problems in HyperSlate®.

3 Reading/Videos/Textbook/Courseware

Slide decks and video lectures/tutorials are part of the crucial content for this course, and will be linked-to from the course web page; in this regard we have a parallel situation to IFLAI1.

Papers that are required reading will be made available to students as we proceed.

Students will purchase access to the inseparable and symbiotic triadic combination published by Motalen:

- the e-textbook *Logic: A Modern Approach; Beginning Deductive Logic, Advanced via HyperSlate® and HyperGrader®* (LAMA-BDLAHSHG);
- access to and use of the HyperGrader® AI system (for, among other things, assessing student work); and
- access to and use of the HyperSlate® AI system (for, among other things, engineering proofs in collaboration with AI);

All three items will be available after purchase in the RPI Bookstore of an envelope with a personalized starting code for registration. Students who previously registered for a version of the online software and ebook will be able to present their email address used in the system and receive a discount. Logistics of the purchase, and the contents of the envelope that purchase will secure, will be encapsulated in the first class meeting, Aug 31 2020, and then gone over in more detail on Sep 3 2020, soon after which the envelopes in question will soon be on sale in and — for remote students — from the Rensselaer Bookstore. The first use in earnest of HyperSlate® and HyperGrade® will happen in class on Sep 3 2020, so by the start of class on that day students should have LAMA-BDLAHSHG, and be able to open both HyperSlate® and HyperGrader® on their laptops in class. Updates to LAMA-BDLAHGHS, and additional exercises, will be provided by listing on the course web page (and sometimes by email) through the course of the semester. You will need to manage many electronic files in the course of this course, and e-housekeeping and e-orderliness are of paramount importance. You will specifically need to assemble a library of completed and partially completed proofs/arguments/truth-trees etc. so that you can use them as building blocks in harder proofs; in other words, building up your own "logical library" will be crucial.

Please note that HyperSlate® and HyperGrader® are trademarked, copyrighted, and Pat. Pend. software: copying and/or reverse-engineering and/or distributing this software to others is strictly prohibited. You will need to submit online a signed version of a License Agreement. This agreement will also reference the textbook, which is copyrighted as well, and since it's an ebook, cannot be copied or distributed or resold in any way.

In addition, occasionally papers may be assigned as reading. Two background ones, indeed, are hereby assigned: (Bringsjord, Taylor, Shilliday, Clark & Arkoudas 2008, Bringsjord 2008).

Finally, slide decks used in class will contain crucial additional content above and beyond LAMA-BDLA and HyperSlate® and HyperGrader® content, and will be available on the web site for course for study. Along with slide decks, video and audio tutorials and mini-lectures will be provided as well.

4 Schedule

4.1 The Four Coverage Areas

This class is divided into four I–IV coverage areas:

I *Review*$^+$. We use HyperSlate® and HyperGrader® to review the logics $\mathscr{L}_{\mathrm{PC}}, \mathscr{L}_1$, $\mathscr{L}_2$, and the propositional modal logics **K**, **T**, **D**. For some students, modal logics may be new; these students will want to pay close attention to, and expend some genuine effort exploring, these logics. Our review will also include brief description of the modal logics **S4** and **S5**; both are still only *propositional* modal logics. Importantly, students who took "IFLAI1" will not have used a version of HyperSlate® that included **S4** or **S5**. This the reason for the use of '+' after 'Review' in giving a label for Phase I of the class.

II *Metalogic, Including Gödel's Great Theorems.* The standard bulk of intermediate formal logic consists in a series of metatheorems that can be viewed as showing that certain metaproperties hold of certain formal logics and parts thereof. For instance, "completeness" (COMP) is often one of these properties. As we will e.g. see:

- **Theorem**: COMP[$\mathscr{L}_{\mathrm{PC}}$]
- **Gödel's Completeness Theorem/GCT**: COMP[$\mathscr{L}_1$]

One of the distinctive aspects of IFLAI2 is that its coverage of metalogic will include nearly all the great theorems of the greatest logician (Kurt Gödel); this coverage will be from Bringsjord's forthcoming *Gödel's Great Theorems.*

III *Advanced Topics in* HyperSlate®. Recall that as said above Phase I includes some new formal logics to be explored in HyperSlate®. In Phase III of the course we take a jump to *quantified* modal logic, and we also explore, to a degree, a more advanced version of HyperSlate® in which the user if allowed to write Clojure functions that play a role in proofs. (We will only consider relatively simple Clojure functions because of time limitations in our schedule.)

IV *Big Questions: e.g. Will AI Match, or Even Exceed, Human Intelligence?.* Our fourth area of coverage includes "big questions" that AI forces us to consider, if we're thoughtful. One example is given in the heading just above, but there are many others, as the student will see. The first big question we'll ponder is whether The Singularity is going to happen or not.

4.2 Fine-Grained Schedule

A more fine-grained schedule now follows.[6]

[6] Note that the Rensselaer Academic Calendar is available here.

- **Aug 31**: *General Orientation to the LAMA® Paradigm, Logistics, Mechanics.* The syllabus is reviewed in detail. It's made clear to the students that, in this class, there is a very definite, comprehensive, theoretical position on formal logic and the teaching thereof; this position corresponds to the affirmation of the LAMA® (= Logic: A Modern Approach) paradigm, and that in lockstep with this position the tightly integrated trio of

    1. the LAMA-BDLAHSHG textbook,
    2. HyperSlate® proof and program construction system, and
    3. HyperGrader® system for (among other things) automated assessment of proofs,

  are used. Students wishing to learn intermediate formal logic under e.g. the "Stanford" paradigm are encouraged to drop this LAMA®-based course and take *Intermediate Logic* at Rensselaer from another instructor.

- **Sep 3**: *Tutorials, Mechanics; Historical and Scientific Context re. Formal Logic, AI, and Logic Machines.* A rapid overview of the relevant history and background of formal logic and AI is provided; this content forms the context for our investigations and learning.

- **Sep 8**: *Review of Extensional Logics.* Note, this is a *Tuesday.* The day before is Labor Day: no classes are held. Students by this point should have HyperSlate® running on their laptops, have their codes registered, and have signed and accepted their LA. We here first explain the core difference between extensional logics such as $\mathscr{L}_1$ and intensional logics, using Blinky the robot and a cup-switching challenge, and the infinitary False Belief Task. We then proceed to explore

  `SpecialLlamasDisjunction`

  in HyperSlate®.

- **Sep 10**: *Review of Intensional/Modal Logics.* We focus on modal logic **D** in review, and include (for most) new coverage of modal logics **S4** and **S5**.

- **Sep 14**: *Church's Theorem, the Halting Problem, and The Singularity.* Church's Theorem tells us that theoremhood for $\mathscr{L}_1$ is Turing-undecidable (= that the *Entscheidungsproblem* is Turing-unsolvable). How do we prove this? And what implications does Church's Theorem have for the future of AI, and The Singularity in particular?

- **Sep 17**: *Completeness Theorems.* We here cover the first of Gödel's great theorems, which says that $\mathscr{L}_1$ is complete (in the sense that every necessary truth has a proof). He consider Henkin's version of this theorem as well, albeit briefly.

- **Sep 21**: *Gödel's First Incompleteness Theorem (G1).*

- **Sep 24**: *Gödel's Second Incompleteness Theorem (G2).*

- **Sep 28**: *Gödel's Speedup Theorem and Artificial Superintelligence.*

- **Oct 1**: *Formal Logic, AI, Computer Science, and the Immaterial.* Formal logic, AI, and computer science all at least appear to entail some non-physical things (e.g. algorithms, infinite cardinal numbers, etc.) exist. Does the entailment go through? And if it does, does this in turn entail that we are immaterial as well? Affirmative answers to both questions are defended by Bringsjord.

- **Oct 5**: *AI and Consciousness.* This class if based on work by Bringsjord & Govindarajulu in which a new theory of machine consciousness is set out and associated with a scheme (Λ) for measuring this consciousness is set out. B&G also here articulate and analyze purported refutations of the Integrated Information Theory of consciousness advanced by Tononi & Koch, its associated scheme (Φ) for measuring consciousness.

- **Oct 8**: *Real Learning ($\mathcal{RL}$).* AI of today has given the world so-called "machine learning," or just 'ML' for short. But do machines doing ML actually learn? A negative answer is given, and defended; and a genuine form of learning (for natural and artificial agents), $\mathcal{RL}$, is introduced and defended.

- **Oct 12**: *No class: Columbus Day.*

- **Oct 15**: *What is Formal Inductive Logic?* This class includes compressed coverage of so-called "pure inductive logic" (PIL), which has become nearly the sole province of mathematicians and logicians, with AI activity nearly zero. Why? One reason, which we find compelling, is that PIL is devoid of proofs and arguments build on the basis of the formal structures involved.

- **Oct 19**: *Defeasible/Nonmonotonic Logic and AI.* Here, argument-based defeasible/nonmonotonic logic is introduced. This serves as a way to do inductive logic, included automated inductive logic, that is superior to PIL.
- **Oct 22**: *AI to Surmount Arrow's Impossibility Theorem.*
- **Oct 26**: *The Argument for God's Existence from AI.*
- **Oct 29**: *What is the Brain, Computationally and AI Speaking?* We here begin by considering the claim that the human brain is fundamentally *less* than a Turing machine (and of course thus its equivalents, e.g. a register machine).
- **Nov 2**: *Logic, AI, and Tax Technology.* Can the U.S. federal tax code be captured in formal logic? If so, wouldn't that allow AI to compute minimal tax payments, and certify such payments as minimal?
- **Nov 5**: *Pure General Logic Programming, Functional Programming, Turing-Completeness, and Beyond.* We review the basic paradigms of computer programming. For the imperative case, we use the simple imperative language of (Davis, Sigal & Weyuker 1994), and also discuss register machines, Turing machines (again), KU machines. We also discuss whether programming beyond the Turing Limit makes sense and can be pursued.
- **Nov 9**: *Hypergraphical Proof and Programming in HyperSlate®.* We here introduce the availability of writing Clojure functions in the context of proofs in HyperSlate®.
- **Nov 12**: *Quantified Modal Logic.* We here explore quantified **S5**, the infamous Barcan Formula. HyperSlate® is used.
- **Nov 16**: *Killer Robots, **D**, and Beyond in HyperSlate® to $\mathcal{DCEC}$.* We begin here by stating the "PAID Problem," and then the approach to it from Bringsjord et al. advocates. We review that modal logic **D** is painfully inadequate, but now move to some exploration of a version of $\mathcal{DCEC}$ in HyperSlate®.
- **Nov 19**: *The Logicist AI-ification of the Doctrines of N Effect to Solve the PAID Problem.*
- **Nov 23**: ***ZFC***. We review and expand our understanding of axiomatic set theory, and of the relative size of infinite sets. **ZFC** in HyperSlate® is visited and explored. **Note**: This is the last day of any in-person instruction.
- **Nov 26**: **No Class** (Thanksgiving).
- **Nov 30**: *Gödel's Greatest Theorem: The Continuum Hypothesis.* We turn to Sherlock Holmes for help in understanding Gödel's result that CH cannot be proved false on the basis of **ZFC**.
- **Dec 3**: *Gödel's Time-Travel Theorem.* We here visit the world known as "Flatland," and use it to articulate a visual Gödelian proof that backwards time travel is possible. We also consider the the Paradox of Proust/Looping Painter Paradox, and Bringsjord provides his analysis and solution.
- **Dec 7**: *Gödel's "God Theorem".* Did Gödel prove that God exists? We discuss this question, and look in some detail at his attempt to do so, which has become an active area in AI of today.
- **Dec 10**: *The "Games" of Gödel and His "Diophantine Disjunction".* We here assess the theorems of Gödel by considering them in connection with games measured computationally and logically, so as to answer the question (Q) as to whether an AI could ever match Gödel. We also consider Gödel's view on this question, which he connected to a certain disjunction involving Diophantine problems. Bringsjord answers Q in the negative, and provide supporting argument for this position. His position is contrasted with Bill Rapaport's contrary position.

5 Grading

Grades are based on four factors:

1. All required problems in HyperSlate®, when completed and certified correct by HyperGrader®, earns the student an A/4 for 50% of her/his final grade. Students cannot pass the course unless all these required problems are solved and certified correct. It is not expected that passing all of these problems will be onerous; in this regard, IFLAI2 is perhaps quite different than IFLAI1.

2. Answers to questions regarding metalogic/metatheorems covered. These questions will go out be email; answers will need to come back as pdfs. This will constitute 20% of one's grade. Usually the questions will ask for (informal) proofs or proof-sketches, or — a concept to be explained — sub-proofs of steps that are/seem mysterious in metaproofs that you are presented with.

3. A 3-page paper written as a critique of a position on formal logic, AI, and the mind advanced by Bringsjord. (It will be easy to find a position that you vehemently disagree with. The topic must be pre-accepted by Bringsjord.) This paper will be submitted in two versions, a first version on which feedback is given, and then a final version submitted after that that takes account of this feedback. This paper is 20% of one's grade. Bringsjord's positions are expressed as declarative propositions, and will often have a philosophical dimension. As an example, here is a position that will be advanced:

   $\overline{\mathrm{AI}^{=\mathrm{HI}}}$ It is logically/mathematically impossible for AI (as defined in today in the textbooks and primary literature of the field of AI) to match (let alone exceed) human intelligence.

4. Finally, the remaining 10% of one's grade is based on participation through discussion and email, etc. Cogent critique from students of Bringsjord's positions on "big questions" re. AI and the mind.

6 Some Learning Outcomes

There are three desired outcomes. *One*: Students will be able to/refresh their ability to carry out formal proofs and disproofs, within the HyperSlate® system and its workspaces, at the level of the propositional and predicate calculi, and propositional modal logic (the aforementioned systems **T**, **S4**, **D**, and **S5**). *Two*: Students will understand the main metatheorems of intermediate formal logic, and all of those achieved by Gödel (as enumerated above). *Three*, students will be able to debate some of the profound questions raised by AI (questions enumerated above).

7 Academic Honesty

Student-teacher relationships are built on mutual respect and trust. Students must be able to trust that their teachers have made responsible decisions about the structure and content of the course, and that they're conscientiously making their best effort to help students learn. Teachers must be able to trust that students do their work conscientiously and honestly, making their best effort to learn. Acts that violate this mutual respect and trust undermine the educational process; they counteract and contradict our very reason for being at Rensselaer and will not be tolerated. Any student who engages in any form of academic dishonesty will receive an F in this course and will be reported to the Dean of Students for further disciplinary action. (The *Rensselaer Handbook* defines various forms of Academic Dishonesty and procedures for responding to them. All of these forms are violations of trust between students and teachers. Please familiarize yourself with this portion of the handbook.) In particular, all solutions submitted to HyperGrader® for course credit under a student id are to be the work of the student associated with that id alone, and not in any way copied or based on the work of anyone else.

References

Arkoudas, K. & Bringsjord, S. (2009), 'Vivid: An AI Framework for Heterogeneous Problem Solving', *Artificial Intelligence* **173**(15), 1367–1405.
URL: *http://kryten.mm.rpi.edu/KA_SB_Vivid_offprint_AIJ.pdf*

Bringsjord, S. (2008), Declarative/Logic-Based Cognitive Modeling, *in* R. Sun, ed., 'The Handbook of Computational Psychology', Cambridge University Press, Cambridge, UK, pp. 127–169.
URL: [7]

Bringsjord, S., Taylor, J., Shilliday, A., Clark, M. & Arkoudas, K. (2008), Slate: An Argument-Centered Intelligent Assistant to Human Reasoners, *in* F. Grasso, N. Green, R. Kibble & C. Reed, eds, 'Proceedings of the 8th International Workshop on Computational Models of Natural Argument (CMNA 8)', University of Patras, Patras, Greece, pp. 1–10.
URL: *http://kryten.mm.rpi.edu/Bringsjord_etal_Slate_cmna_crc_061708.pdf*

Davis, M., Sigal, R. & Weyuker, E. (1994), *Computability, Complexity, and Languages: Fundamentals of Theoretical Computer Science*, Academic Press, New York, NY. This is the second edition, which added Sigal as a co-author.

Glymour, C. (1992), *Thinking Things Through*, MIT Press, Cambridge, MA.

Halpern, J., Harper, R., Immerman, N., Kolaitis, P., Vardi, M. & Vianu, V. (2001), 'On the Unusual Effectiveness of Logic in Computer Science', *The Bulletin of Symbolic Logic* **7**(2), 213–236.

[7] http://kryten.mm.rpi.edu/sb_lccm_ab-toc_031607.pdf