Could AI Ever Match Gödel's Greatness?

(With brief remarks on G's "God Theorem.")

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Rensselaer AI and Reasoning Lab

Monographic Context

(yet again!)

. . .

STOP & REVIEW IF NEEDED!

Gödel's Great Theorems (OUP)

by Selmer Bringsjord

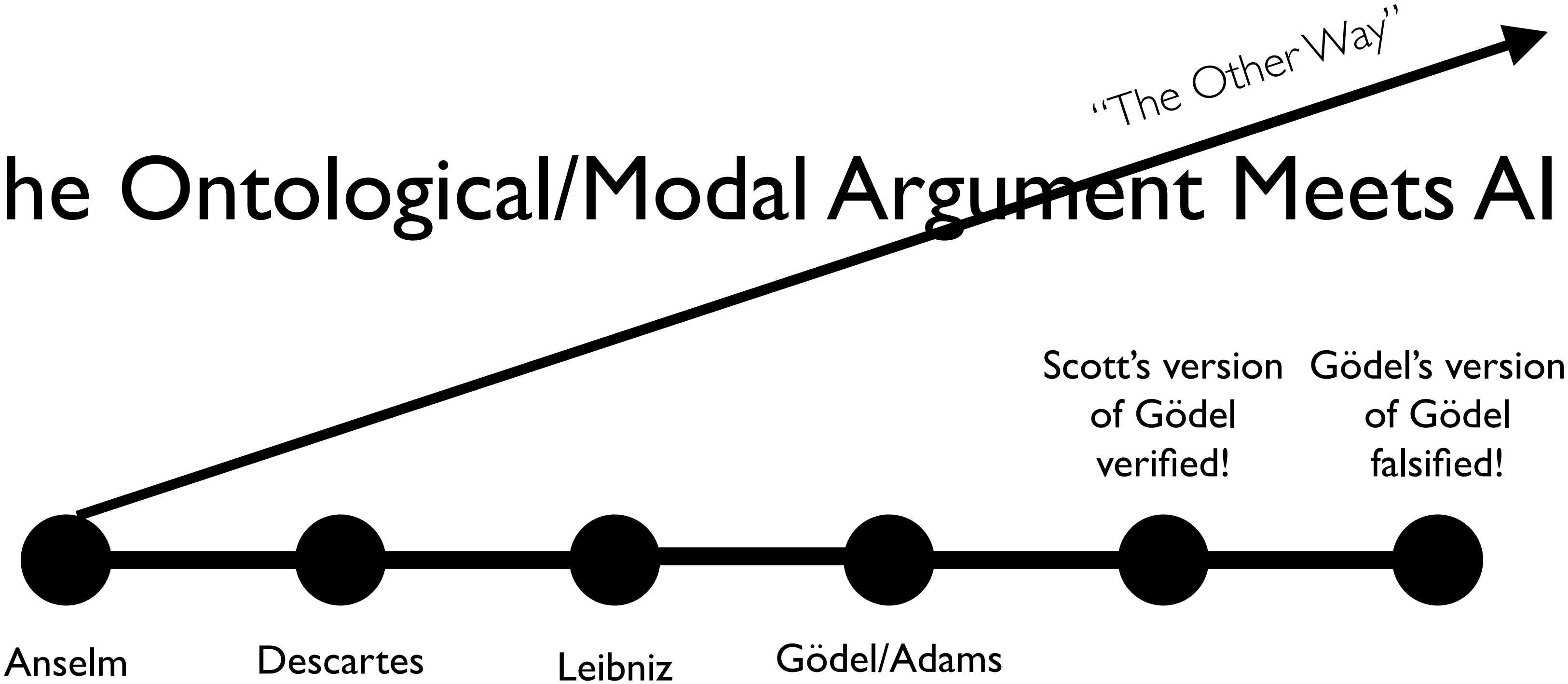
√ • Introduction ("The Wager")
√ • Brief Preliminaries (e.g. the propositional calculus & FOL)
√ • The Completeness Theorem
√ • The First Incompleteness Theorem
√ • The Second Incompleteness Theorem
√ • The Speedup Theorem
√ • The Continuum-Hypothesis Theorem
√ • The Time-Travel Theorem
• Gödel's "God Theorem"
• Could a Finite Machine Match Gödel's Greatness?

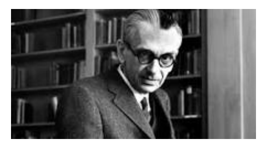
Re Gödel’s “God Theorem” …

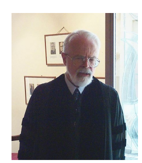
Recommended Podcast :)

https://mindmatters.ai/podcast/ep81

The Ontological/Modal Argument Meets AI







?

Proceedings of the Twenty-Fifth International Joint Conference on Artificial Intelligence (IJCAI-16)

The Inconsistency in Gödel's Ontological Argument: A Success Story for AI in Metaphysics

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Abstract

This paper discusses the discovery of the inconsistency in Gödel's ontological argument as a success story for artificial intelligence. Despite the popularity of the argument since the appearance of Gödel's manuscript in the early 1970's, the inconsistency of the axioms used in the argument remained unnoticed until 2013, when it was detected automatically by the higher-order theorem prover

on the proof [Fuhrmann, 2016].

The in-depth analysis presented here substantially extends previous computer-assisted studies of Gödel's ontological argument. Similarly to the related work [Benzmüller and Woltzenlogel-Paleo, 2013a; 2014] the analysis has been conducted with automated theorem provers for classical higher-order logic (HOL; cf. [Andrews, 2014] and the references therein), even though Gödel's proof is actually formulated in higher-order *modal* logic (HOML; cf. [Muskens, 2006]

some (necessary) facts for our existing world are deduced by purely a priori, analytical means from some abstract definitions and axioms.

What motivated Gödel as a logician was the question, whether it

T3 Necessarily, God exists: $\Box\exists x G(x)$

Figure 1. Scott's version of Gödel's ontological argument [30].

Some Key Arguments/Results Up to Present

- Analysis of Leibniz's (& Descartes') version: *Leibniz* by Robert Adams (1998). This book explains what Gödel took as "input," & sought to improve.

- Benzmüller & Paleo (2014) "Automating Gödel's Ontological Proof of God's Existence with Higher-order Automated Theorem Provers." An AI-based formal verification of the validity of the argument.
  - https://page.mi.fu-berlin.de/cbenzmueller/papers/C40.pdf

- "The Inconsistency in Gödel's Ontological Argument: A Success Story for AI in Metaphysics," also by B&P, 2016. An AI-based formal refutation of the argument.
  - https://www.ijcai.org/Proceedings/16/Papers/137.pdf

- Fitting, M. (2002) *Types, Tableaus, and Gödel's God*. Includes full formalization of (a version of) Gödel's proof of God's existence.
  - https://www.amazon.com/Types-Tableaus-Gödels-Trends-Logic/dp/9401039127

- "A (Simplified) Supreme Being Necessarily Exists — Says the Computer!," by C. Benzmüller.
  - https://www.groundai.com/project/a-simplified-supreme-being-necessarily-exists-says-the-computer/3

- "Computer-Supported Analysis of Positive Properties … In Variants of Gödel's Ontological Argument," Benzmüller & Fuenmayor, 2020.
  - https://arxiv.org/pdf/1910.08955.pdf

Are there invariants? Apparently.

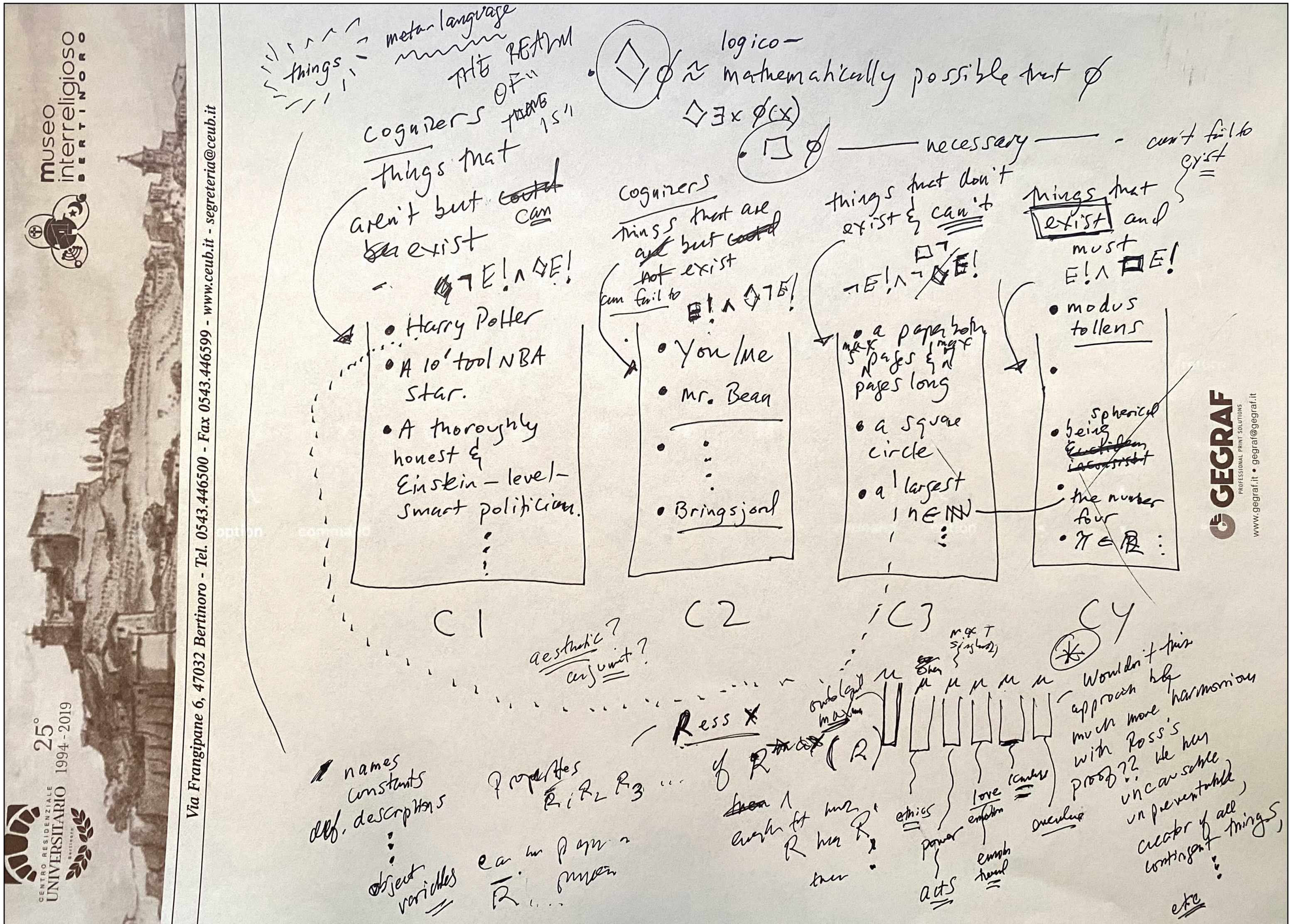
$\frac{1}{2}$ of A1 $\quad (\mathbf{Pos1}) \quad \forall R(Pos(R) \rightarrow \neg Pos(\bar{R}))$

A2 $\quad (\mathbf{Pos2}) \quad \forall R[(Pos(R) \wedge \Box \forall x \forall R'(R(x) \rightarrow R'(x))) \rightarrow Pos(R')]$

For a wonderfully economical, non-technical overview that includes this observation, see "Chapter 7: Gödel" by Alexander Pruss, in *Ontological Arguments*, G. Oppy, ed. (Cambridge, UK: Cambridge University Press).

$$(\mathbf{Pos1^{\star}}) \quad \forall R \forall \delta \neq 0[GPPos(R^{\delta}) \rightarrow \neg GPPos(\bar{R})]$$

"The Other Way"



Gödel's Either/Or . . .

The Question

Q* Is the human mind more powerful than the class of standard computing machines?

(= finite machines)

(= Turing machines)

(= register machines)

(= KU machines)

…

Gödel's Either/Or

"[E]ither … the human mind (even within the realm of pure mathematics) infinitely surpasses the power of any finite machine, or else there exist absolutely unsolvable diophantine problems."
— Gödel, 1951, Providence RI

PT as a Diophantine Equation

Equations of this sort were introduced to you in middle-school, when you were asked to find the hypotenuse of a right triangle when you knew its sides; the familiar equation, the famous Pythagorean Theorem that most adults will remember at least echoes of into their old age, is:

$$\text{(PT)} \quad a^2 + b^2 = c^2,$$

and this is of course equivalent to

$$\text{(PT')} \quad a^2 + b^2 - c^2 = 0,$$

which is a Diophantine equation. Such equations have at least two unknowns (here, we of course have three: a, b, c), and the equation is solved when positive integers for the unknowns are found that render the equation true. Three positive integers that render (PT') true are

$$a = 4,\ b = 3,\ c = 5.$$

It is *mathematically impossible* that there is a finite computing machine capable of solving any Diophantine equation given to it as a challenge (!).

… which means that the 10th of Hilbert's Problems is settled:

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Hilbert's problems

From Wikipedia, the free encyclopedia

Hilbert's problems are twenty-three problems in mathematics published by German mathematician David Hilbert in 1900. The problems were all unsolved at the time, and several of them were very influential for 20th-century mathematics. Hilbert presented ten of the problems (1, 2, 6, 7, 8, 13, 16, 19, 21, and 22) at the Paris conference of the International Congress of Mathematicians, speaking on August 8 in the Sorbonne. The complete list of 23 problems was published later, most notably in English translation in 1902 by Mary Frances Winston Newson in the *Bulletin of the American Mathematical Society*.[1]

David Hilbert

Contents [hide]

… which means that the 10th of Hilbert’s Problems is settled:

10th	Find an algorithm to determine whether a given polynomial Diophantine equation with integer coefficients has an integer solution.	Resolved. Result: Impossible; Matiyasevich's theorem implies that there is no such algorithm.	1970

It was a *team* effort, actually; it’s not due solely to Matiyasevich, and is often denoted as the ‘MRDP Theorem.’

Julia **R**obinson   Martin **D**avis   Hilary **P**utnam

Background

problem?[7] In his lecture, Gödel precisely defines diophantine problems, but we don't need to bother with all of the details here; we only need to appreciate the general structure of such a problem, and that can be achieved quickly as follows, given what was introduced in Chapter 2.

Each diophantine problem has at its core a polynomial $\mathcal{P}$ whose variables are comprised by two lists, $x_1, x_2, \ldots, x_n$ and $y_1, y_2, \ldots, y_m$; all variables must be integers, and the same for subscripts n and m. To represent a polynomial in a manner that announces its variables, we can write

$$\mathcal{P}(x_1, x_2, \ldots, x_k, y_1, y_2, \ldots, y_j).$$

But Gödel was specifically interested in whether, for all integers that can be set to the variables x_i, there are integers that can be set to the y_j, such that the polynomial equals 0. To make this clearer, first, here are two particular, simple equations that employ polynomials that are both instances of the needed form:

E1 $\quad 3x - 2y = 0$

E2 $\quad 2x^2 - y = 0$

All we need to do now is prefix these equations with quantifiers in the pattern Gödel gave. This pattern is quite simple: universally quantify over each x_i variable (using the now-familiar $\forall$), after which we existentially quantify over each y_i variable (using the also-now-familiar $\exists$). Thus, here are the two diophantine problems that correspond to the pair E1 and E2 from just above:

P1 $\quad$ Is it true that $\forall x \exists y (3x - 2y = 0)$?

P2 $\quad$ Is it true that $\forall x \exists y 2x^2 - y = 0$?

Great Paper!

1. Diophantine Sets. In this article the usual problem of Diophantine equations will be inverted. Instead of being given an equation and seeking its solutions, one will begin with the set of "solutions" and seek a corresponding Diophantine equation. More precisely:

DEFINITION. A set S of ordered n-tuples of positive integers is called **Diophantine** if there is a polynomial $P(x_1, \cdots, x_n, y_1, \cdots, y_m)$, where $m \geqq 0$, with integer coefficients such that a given n-tuple $\langle x_1, \cdots, x_n \rangle$ belongs to S if and only if there exist positive integers $y_1, \cdots, y_m$ for which

$$P(x_1, \cdots, x_n, y_1, \cdots, y_m) = 0.$$

from logic the symbols "∃" for "there exists" and "⇔" for "if and ...lation between the set S and the polynomial P can be written succinctly as:

$$\langle x_1, \cdots, x_n \rangle \in S \Leftrightarrow (\exists y_1, \cdots, y_m) \, [P(x_1, \cdots, x_n, y_1, \cdots, y_m) = 0],$$

or equivalently:

$$S = \{\langle x_1, \cdots, x_n \rangle \mid (\exists y_1, \cdots, y_m) \, [P(x_1, \cdots, x_n, y_1, \cdots, y_m) = 0]\}.$$

Note that P may (and in non-trivial cases always will) have negative coefficients. *The word* "**polynomial**" *should always be so construed in the article* except where the contrary is explicitly stated. Also *all numbers in this article are positive integers* unless the contrary is stated.

Notice that this is a perfect fit with how we use formal logic to present and understand the Polynomial Hierarchy and the Arithmetic Hierarchy — but this presentation is for IFLAI**2**.

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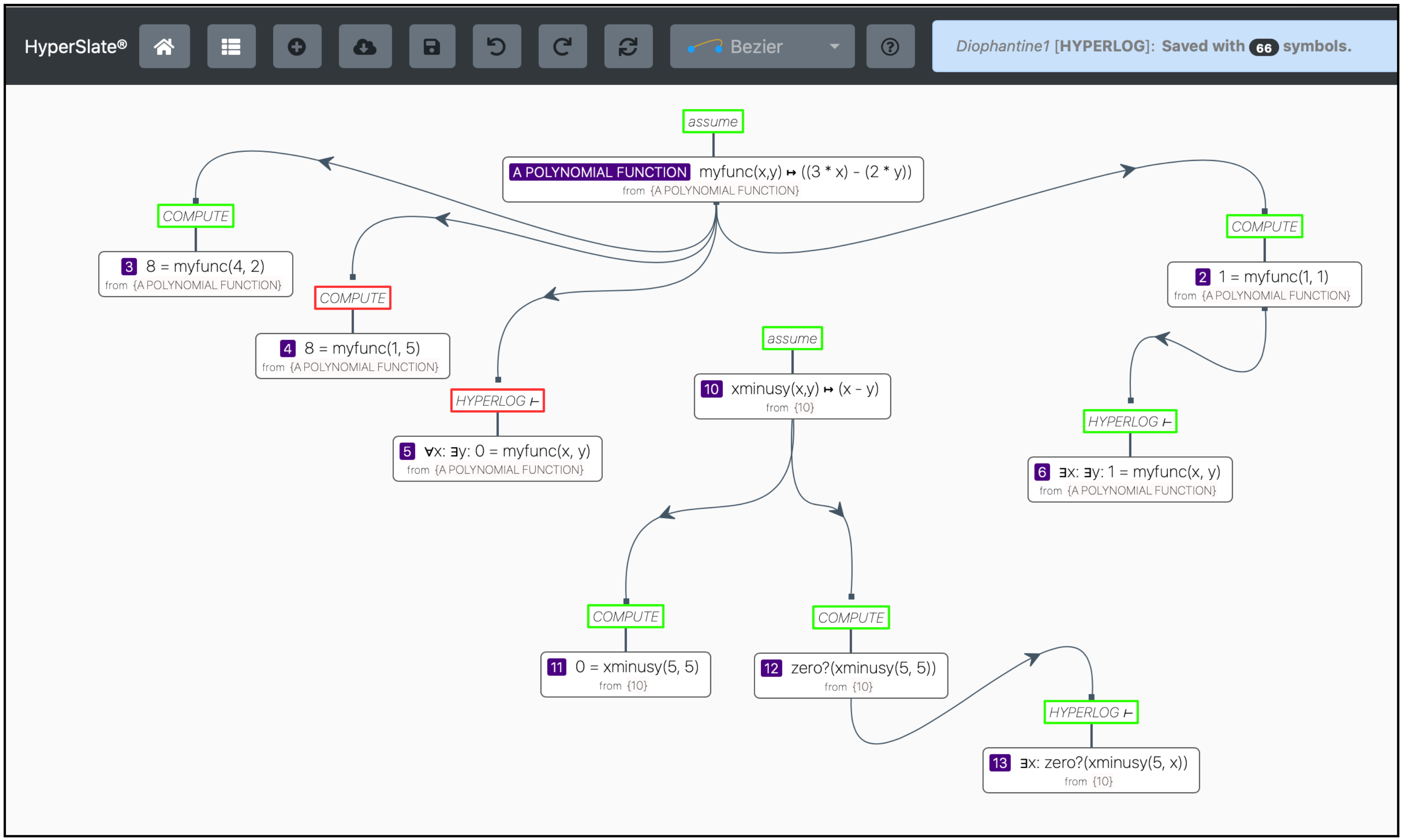
Diophantine "Threat" in the New Programming Language Hyperlog®

(Another IFLAI**2** Topic/Technology)



The Crux

$\exists \mathcal{P}$ s.t. no human mind could ever decide $\forall x_1 \forall x_2 \cdots \forall x_k \exists y_1 \exists y_2 \cdots \exists x_j (\mathcal{P}(x_1, x_2, \ldots, x_k, y_1, y_2, \ldots, y_j)$?

Yes.

No.

The human mind *is* infinitely more powerful than any standard computing machine.

The human mind is *not* infinitely more powerful than any standard computing machine.

Earlier Gödelian Argument for the "No."

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Applied Mathematics and Computation

Volume 176, Issue 2, 15 May 2006, Pages 516-530

A new Gödelian argument for hypercomputing minds based on the busy beaver problem ☆

Selmer Bringsjord, Owen Kellett, Andrew Shilliday, Joshua Taylor, Bram van Heuveln, Yingrui Yang, Jeffrey Baumes, Kyle Ross

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https://doi.org/10.1016/j.amc.2005.09.071

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Abstract

Do human persons hypercompute? Or, as the doctrine of *computationalism* holds, are they information processors at or below the Turing Limit? If the former, given the essence of hypercomputation, persons must in some real way be capable of infinitary information processing. Using as a springboard Gödel's little-known assertion that the human mind has a power "converging to infinity", and as an anchoring problem Rado's [T. Rado, On non-computable functions, Bell System Technical Journal 41 (1963) 877–884] Turing-uncomputable "busy beaver" (or Σ) function, we present in this short paper a new argument that, in fact, human persons can hypercompute. The argument is intended to be formidable, not conclusive: it brings Gödel's intuition to a greater level of precision, and places it within a sensible case against computationalism.

A New One Coming! — in …

Will AI Match (Or Even Exceed) Human Intellligence?

No. Yes.

Will AI Match (Or Even Exceed) Human Intellligence?

No. **Yes.**

1: "Negative" enumerative induction for $\neg \exists year_k(\text{AI} = \text{HI}@year_k)$ from $\text{AI} \neq \text{HI}@year_{1958} \wedge \ldots \wedge \text{AI} \neq \text{HI}@year_{2021}$. Plus the proposition that AI is in fact not improving — relative to the intellectual stuff that matters most.

2: There is no absolutely unsolvable-for-humans Diophantine problem. Hence as Gödel explained, we get "No."

3: Amundsen and The Explorer Argument.

4: And finally, the sledgehammer is used: *phenomenal consciousness*.

Og på det glade merknaden for Selmer (men ikke for Bill), er forelesningene våre nå fullført ... men ...

Finally, finally, …

Gödel-vs-AI "Scorecard"

The Particular Work	Nutshell Diagnosis	Beyond AI?

Test-3 Grading Scheme

Test-3 problems will begin to be published today, & grades on Test 3 will be dynamically reported out in HG® in your **My Progression** page (which, importantly, also shows your progress on **Required** problems!) as we get closer to the deadline of end of day May 11:

- C: Easier personalized ZOL problem + easier personalized FOL problem
- B = C + harder personalized ZOL problem
- A = B + harder personalized FOL problem + `Datalog1`
- A+ = A + `Variant1LeibnizsLaw` + the Bonus problem (hardest "non-professional" SOL problem so far)

$$\boxed{\{\forall X(Xa \rightarrow \forall y(y \neq a \rightarrow \neg Xy)), Qa\} \vdash_2 \exists Z \neg \exists x \exists y(x \neq y \wedge (Zx \wedge Zy))}$$

Med nok penger, kan logikk løse alle problemer.