

# Logic, AI, and Tax Technology

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*Intermediate Formal Logic & AI (IFLAI2)*  
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# Scenario S (informal)

Suppose there are three agents,  $a$ ,  $b$ , and  $c$ , with annual incomes of \$20K, \$60K, and \$200K, resp. The ability of each agent on a scale of 1 (lowest) to 10 (highest) is:  $a:3$ ;  $b:4$ ;  $c:8$ . Each agent has been earning their income for each year five years running with a level of effort, during each year, of their choosing, on a scale of 1 (doing next to nothing) to 4 (an 80-hr work week). At present each agent is at level 4. The lower the effort, the lower the probability that any agent will remain employed; but here we assume a binary function from both effort and ability such that, the higher the ability, the less downward effect the function registers for probability of employment. Unemployment means a productivity of zero, and with lowered effort comes lower productivity as well. An unemployed agent generates no income and hence no revenue by taxation. Currently the probability that a member of the trio will remain employed is .8; this probability, again, is a function of both effort and ability. As effort declines, enjoyment from non-compensatory activity increases.

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Suddenly there arises a cost for protecting the trio from an exogenous malicious agent **D** who desires to destroy the trio and the system in which this trio live, and will likely do so unless protection is purchased. If protection is put in place, the odds of **D**'s success is zero. The levying of an income tax that annually generates funds to purchase (successful) protection from **D** on an ongoing basis can be instituted; it must generate at least \$60K/yr. What do you suggest as a rational, optimal ongoing income tax system?

# Some Options (informal)

Consider a few simple calculations based on three different types of income-tax systems

**Lump-sum** tax:

$$60000/3 = 20000$$

What do you think?

**Linear/Flat** Tax:

Eg 10%  $\Rightarrow$  2K + 8K + 20K; insufficient.

Eg 20%  $\Rightarrow$  4K + 16K + 40K = 60K; sufficient.

Is this okay? What do you think?

**Progressive** tax scheme:

Eg:

Above 100K: 50%

50–100: 25%

20–49: 10%

0–19: 0%

Then: 2K + 20K + 100K = 122K

How about a progression leaving: 0K, 20K, 100K?

Or why not a progression leaving: 0K, 0K, 100K?

Or for that matter: 0K, 0K, 60K?



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How about a progression leaving: 0K, 20K, 100K?

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Can you see a road forward to any full formalization and theorems, using formal logic?

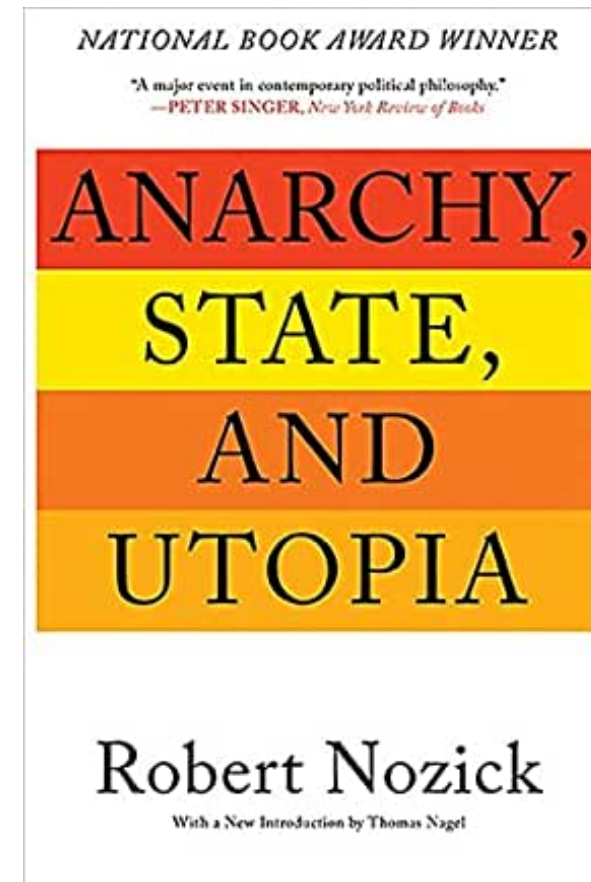
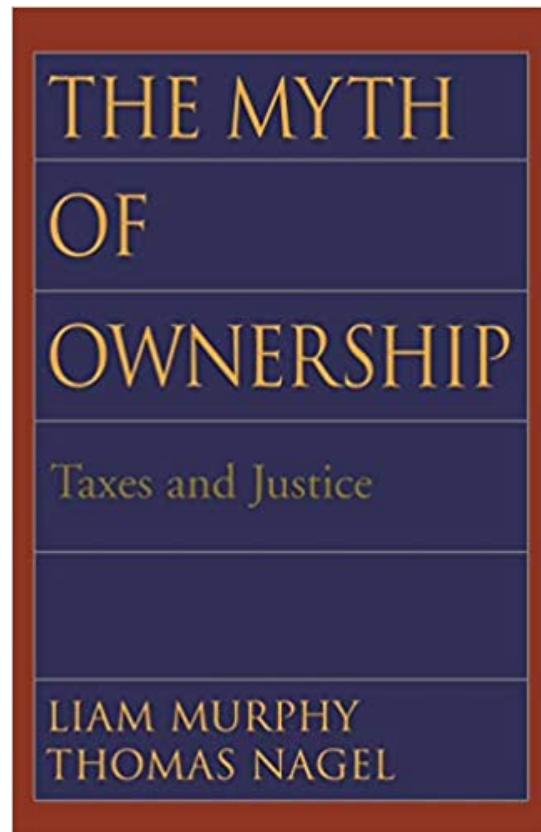
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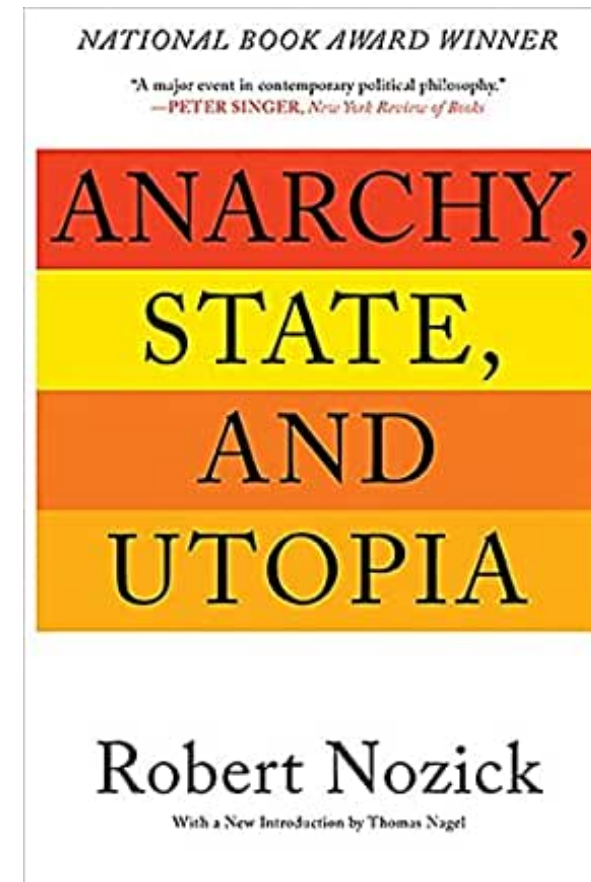
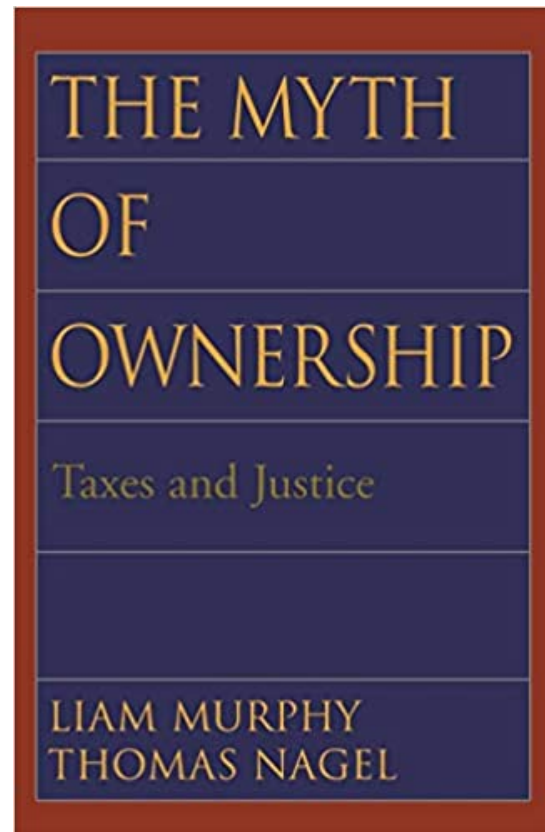
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...

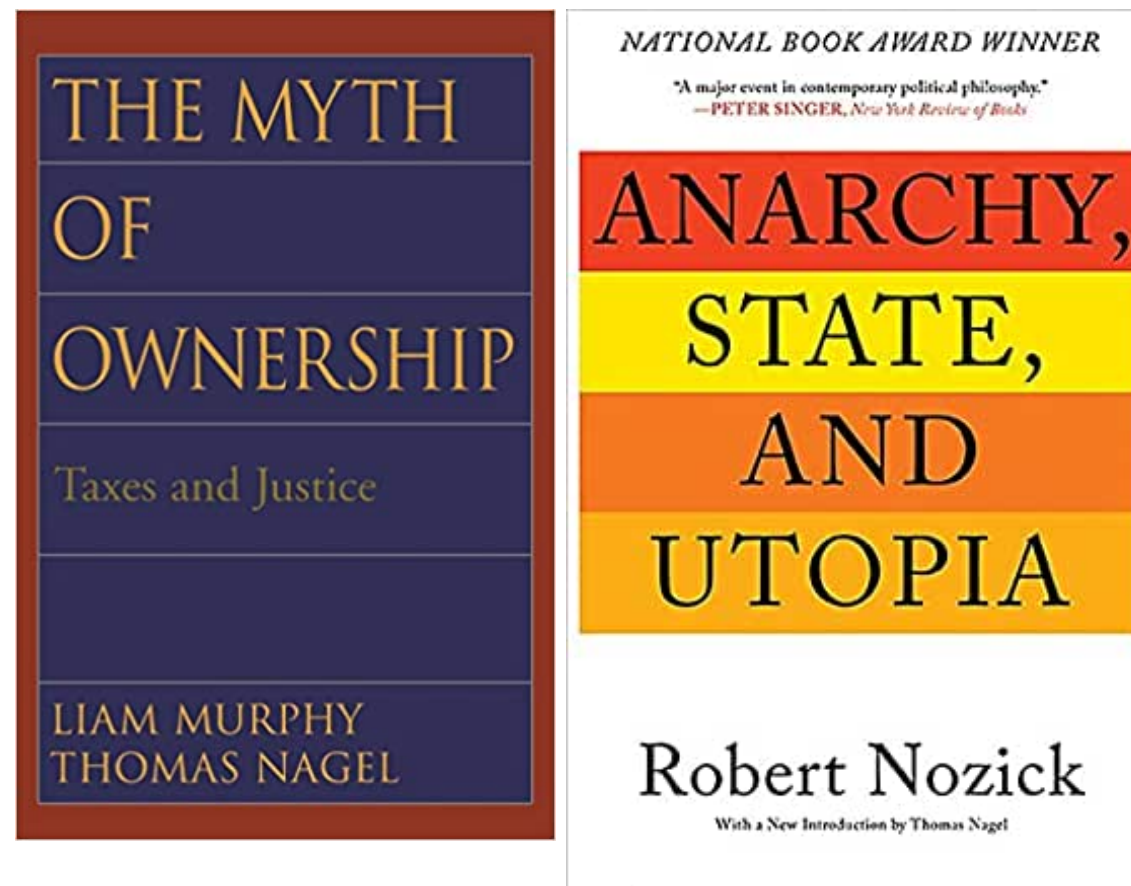
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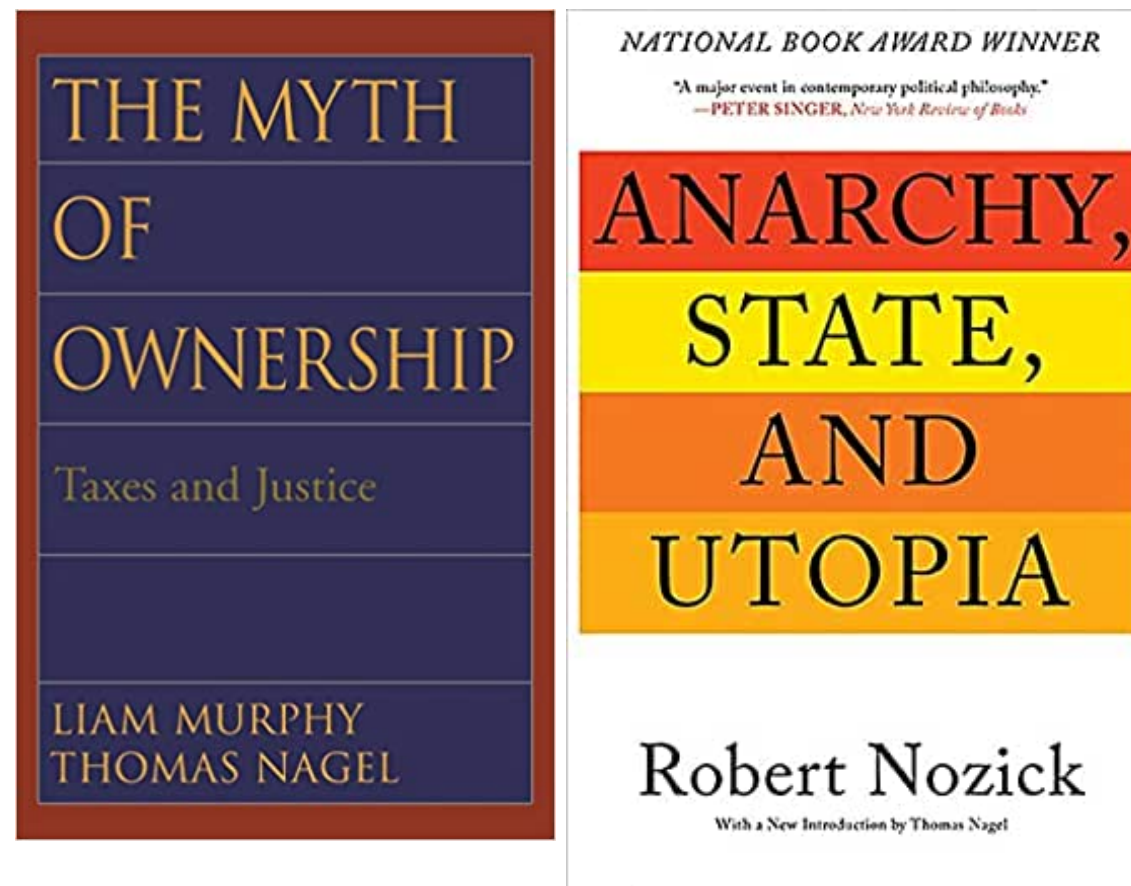
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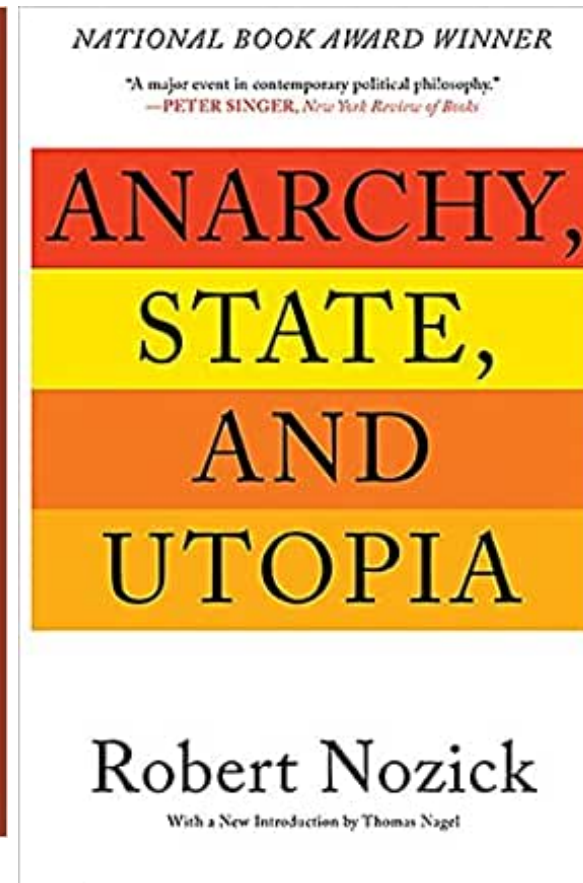
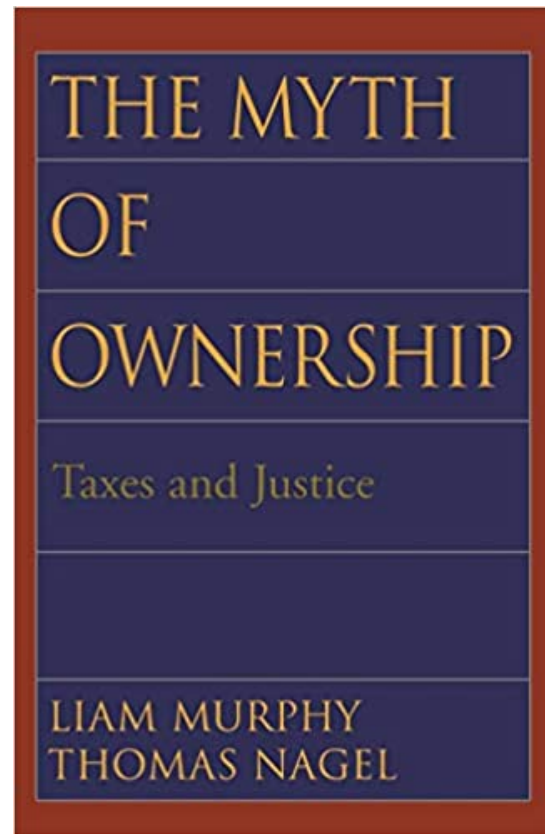
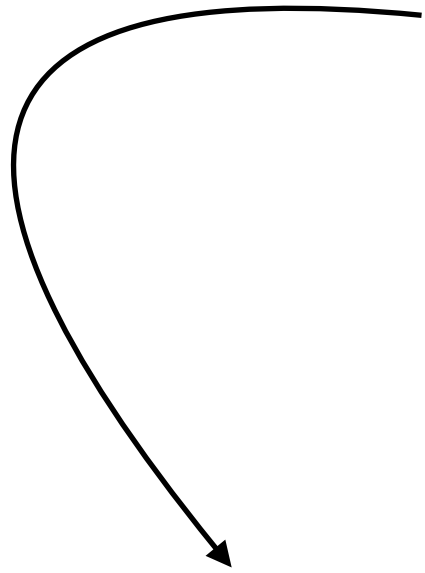


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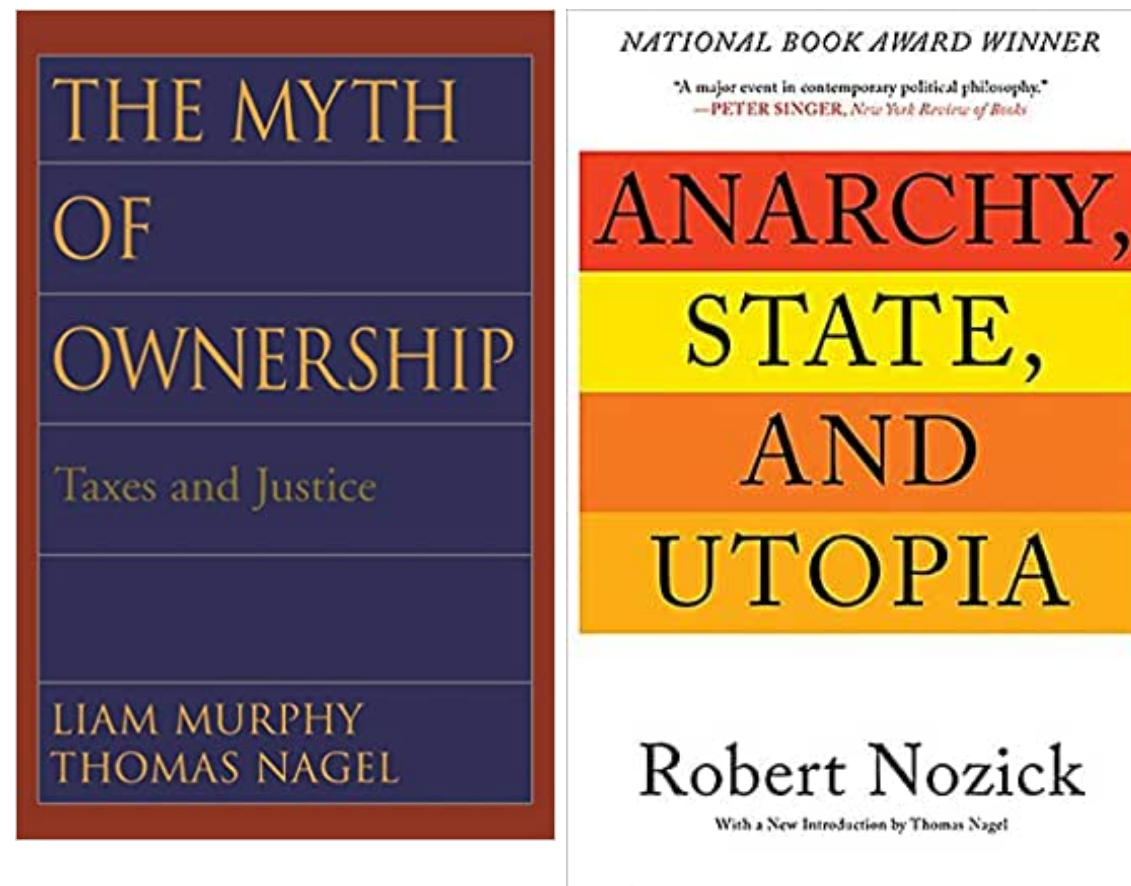




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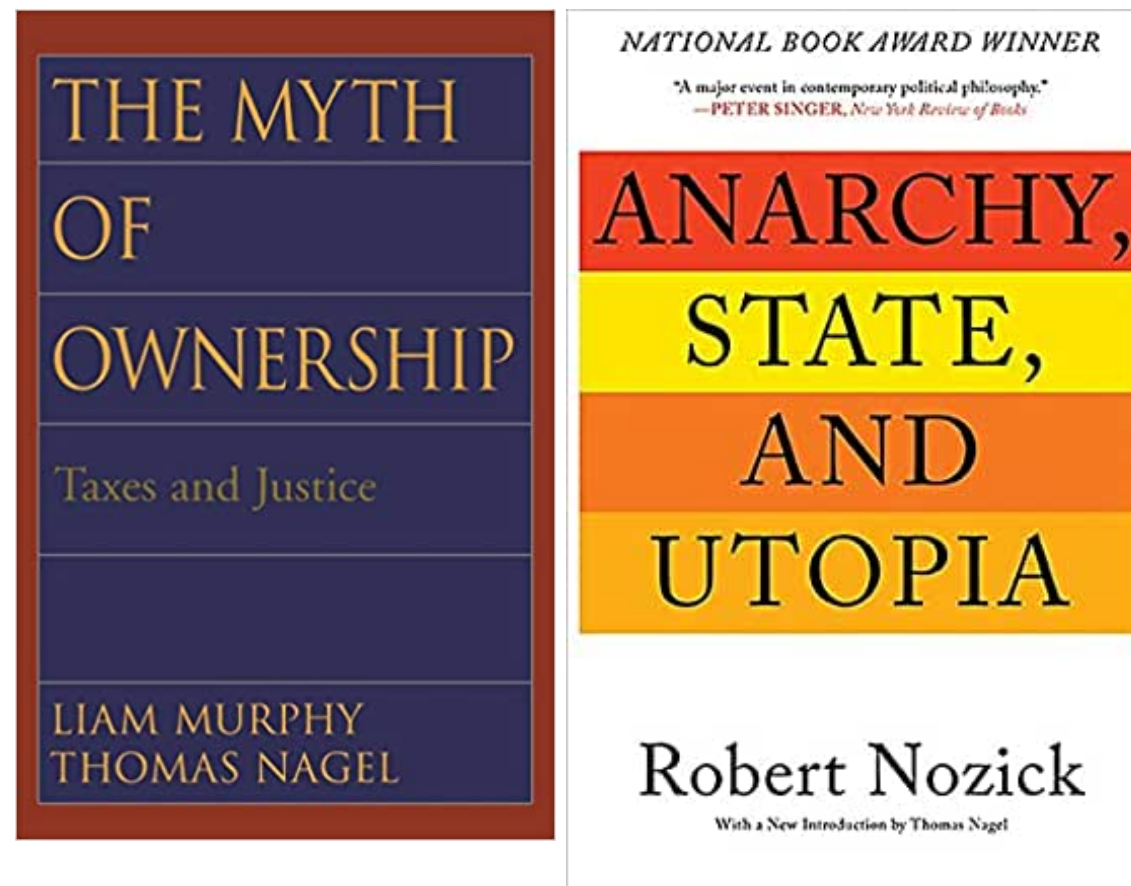


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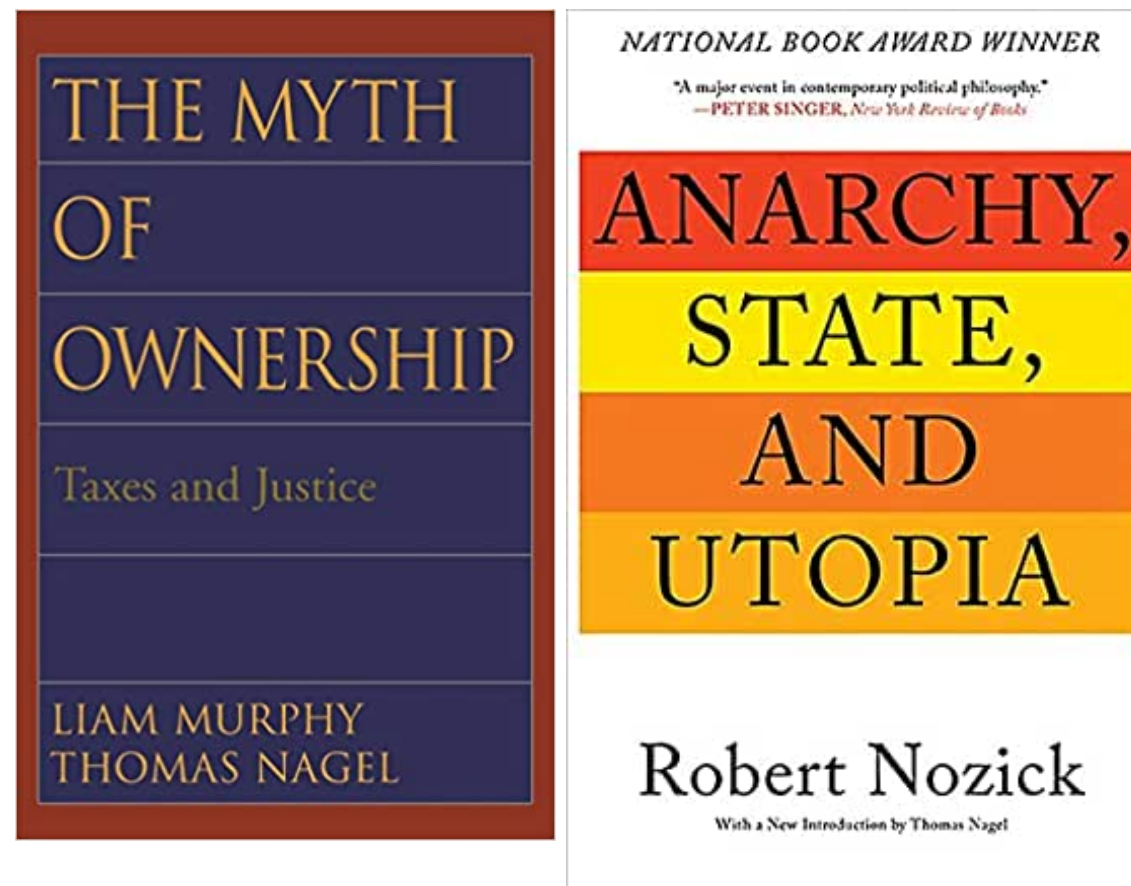
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Since you don't own your pre-tax income (that you do is a myth), pretty much *any* level of income tax is ethically permissible.

Any level of taxation beyond a minimum required for Defense+ is the moral equivalent of forced labor.

**But for IFLAI2 ...**

But for  $|FLA|_2 \dots$

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The original Mirrlees argument runs as follows. Suppose there is a positive marginal tax rate on the individual earning the top income in an economy, and suppose that income is  $y$ . The positive marginal tax rate has a discouraging effect on the individual's effort, generating an efficiency cost. If the marginal tax rate on that earner was reduced to zero for any income beyond  $y$ , then the same amount of revenue would be collected and the efficiency costs would be avoided. Thus, a positive marginal tax on the top earner cannot be optimal.

—Mankiw



# An Exploration in the Theory of Optimum Income Taxation<sup>1,2</sup>

J. A. MIRRLEES  
*Nuffield College, Oxford*

## 1. INTRODUCTION

One would suppose that in any economic system where equality is valued, progressive income taxation would be an important instrument of policy. Even in a highly socialist economy, where all who work are employed by the State, the shadow price of highly skilled labour should surely be considerably greater than the disposable income actually available to the labourer. In Western Europe and America, tax rates on both high and low incomes are widely and lengthily discussed<sup>3</sup>: but there is virtually no relevant economic theory to appeal to, despite the importance of the tax.

Redistributive progressive taxation is usually related to a man's income (or, rather, his estimated income). One might obtain information about a man's income-earning potential from his apparent I.Q., the number of his degrees, his address, age or colour: but the natural, and one would suppose the most reliable, indicator of his income-earning potential is his income. As a result of using men's economic performance as evidence of their economic potentialities, complete equality of social marginal utilities of income ceases to be desirable, for the tax system that would bring about that result would completely discourage unpleasant work. The questions therefore arise what principles should govern an optimum income tax; what such a tax schedule would look like; and what degree of inequality would remain once it was established.

The problem seems to be a rather difficult one even in the simplest cases. In this paper, I make the following simplifying assumptions:

(1) Intertemporal problems are ignored. It is usual to levy income tax upon each year's income, with only limited possibilities of transferring one year's income to another for tax purposes. In an optimum system, one would no doubt wish to relate tax payments to the whole life pattern of income,<sup>4</sup> and to initial wealth; and in scheduling payments one would wish to pay attention to imperfect personal capital markets and imperfect foresight. The economy discussed below is timeless. Thus the effects of taxation on saving are ignored. One might perhaps regard the theory presented as a theory of "earned income" taxation (i.e. non-property income).

(2) Differences in tastes, in family size and composition, and in voluntary transfers, are ignored. These raise rather different kinds of problems, and it is natural to assume them away.

<sup>1</sup> First version received Aug. 1970; final version received October 1970 (Eds.).

<sup>2</sup> Work on this paper and its continuation was begun during a stimulating and pleasurable visit to the Department of Economics, M.I.T. The influence of Peter Diamond is particularly great, and his comments have been very useful. Earlier versions were presented at the Cowles Foundation, to the Economic Study Society, at the London School of Economics, and to CORE. I am grateful to the members of these seminars and to A. B. Atkinson for valuable comments. I am also greatly indebted to P. G. Hare and J. R. Broome for the computations.

<sup>3</sup> Discussions on (usually) orthodox lines, including many important points neglected in the present paper, can be found in [7], [1], [5, Chapters 5, 7, 8], and [6, Chapters 11 and 12]. [2] is close in spirit to what is attempted here.

<sup>4</sup> Cf. [7, Chapter 6].

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## Now a battle of AI simulations in the empirical realm?

# Challenge #1:

Engineer artificial agent to excel on CPA Exam,  
and provide proofs/justifications for answers!

What is the most likely opportunity for theft or fraud by employees?

- ☐ The belief that the theft is a common practice
- ☐ Needlessly complex transactions
- ☐ Access to assets that are easily traced
- ☐ Stock options that expire soon after the release of financial statements

3 / 24

CLOSE

**Correct**

**Next Question**

*When transactions are complex, many individuals within the entity will not understand the intricacies and, as a result, it becomes easier to deceive others, creating an opportunity to commit fraud. Ineffective oversight by governance also creates an opportunity for individuals to commit fraud but does not provide an incentive. A belief that the theft is a common practice is a rationalization, not an opportunity. Access that are easily traced tend to result in apprehension and prosecution of the perpetrator of a theft, discouraging such theft. Stock options are due to expire shortly after financial statements are issued create an incentive to overstate results in order to increase the value of the options, but it does not provide an opportunity.*



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## Teaching logic to auditing students: can training in logic reduce audit judgment errors?

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Graeme J. Mitchell<sup>c</sup>

<sup>a</sup>*School of Accountancy, Utah State University, Logan, UT 84322-3540, USA*

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### Abstract

Recent audit failures of unprecedented magnitude and their effects upon the capital markets have resulted in heightened public and regulatory concern towards the auditing profession. Increased scrutiny and a possible movement to principles-based accounting standards are creating an auditing environment in which “critical thinking” skills will be increasingly important. Consequently, rule based auditing courses may be insufficient to prepare students for the emerging discipline. Logic is an important component of “critical thinking;” in fact, the two are considered synonymous in the philosophy literature. Although logic has been called “the mother discipline of auditing” (Mautz & Sharaf, 1961. *Philosophy of auditing*. Sarasota, FL: American Accounting Association), training in logic is conspicuously absent from accounting curricula, while research in logic is almost non-existent in both the accounting education and audit judgment literatures. Students in this study were taught formal and informal logic in an auditing course. They studied valid and invalid argument forms within the specific context of auditing services. These students, others without training, and a sample of professional auditors were then tested with a series of real-world auditing vignettes requiring critical reasoning and judgment. Students trained in logic outperformed students without such training. Furthermore, students trained in logic outperformed experienced auditors in their abilities to discern valid versus invalid argument forms. Conversely, experienced auditors outperformed trained students in their abilities to discern believable versus less believable argument premises. The results of the study have important implications both for university education and for continuing professional education for auditors.

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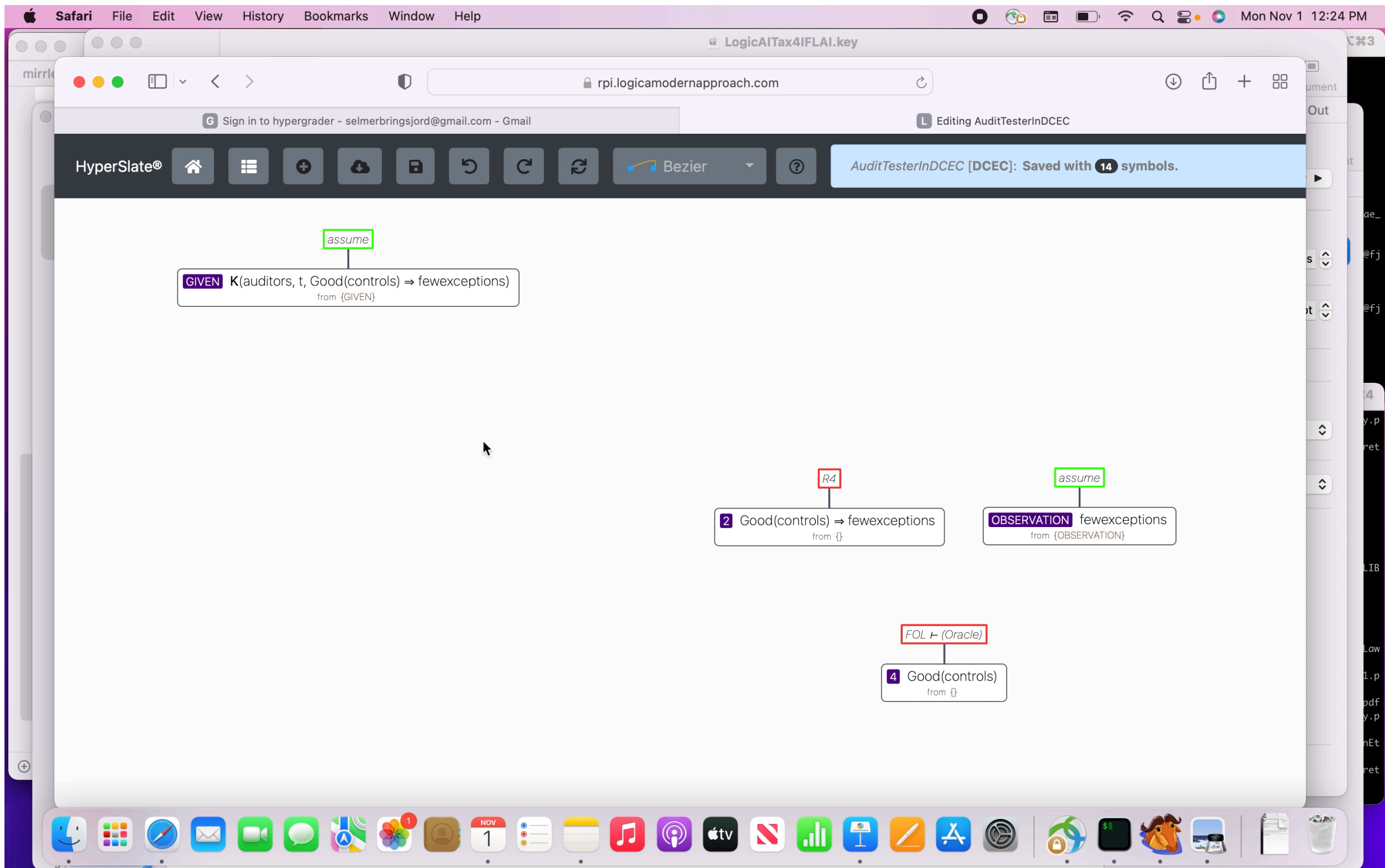
Auditors' conclusion: Controls over the processing of loan applications are good.

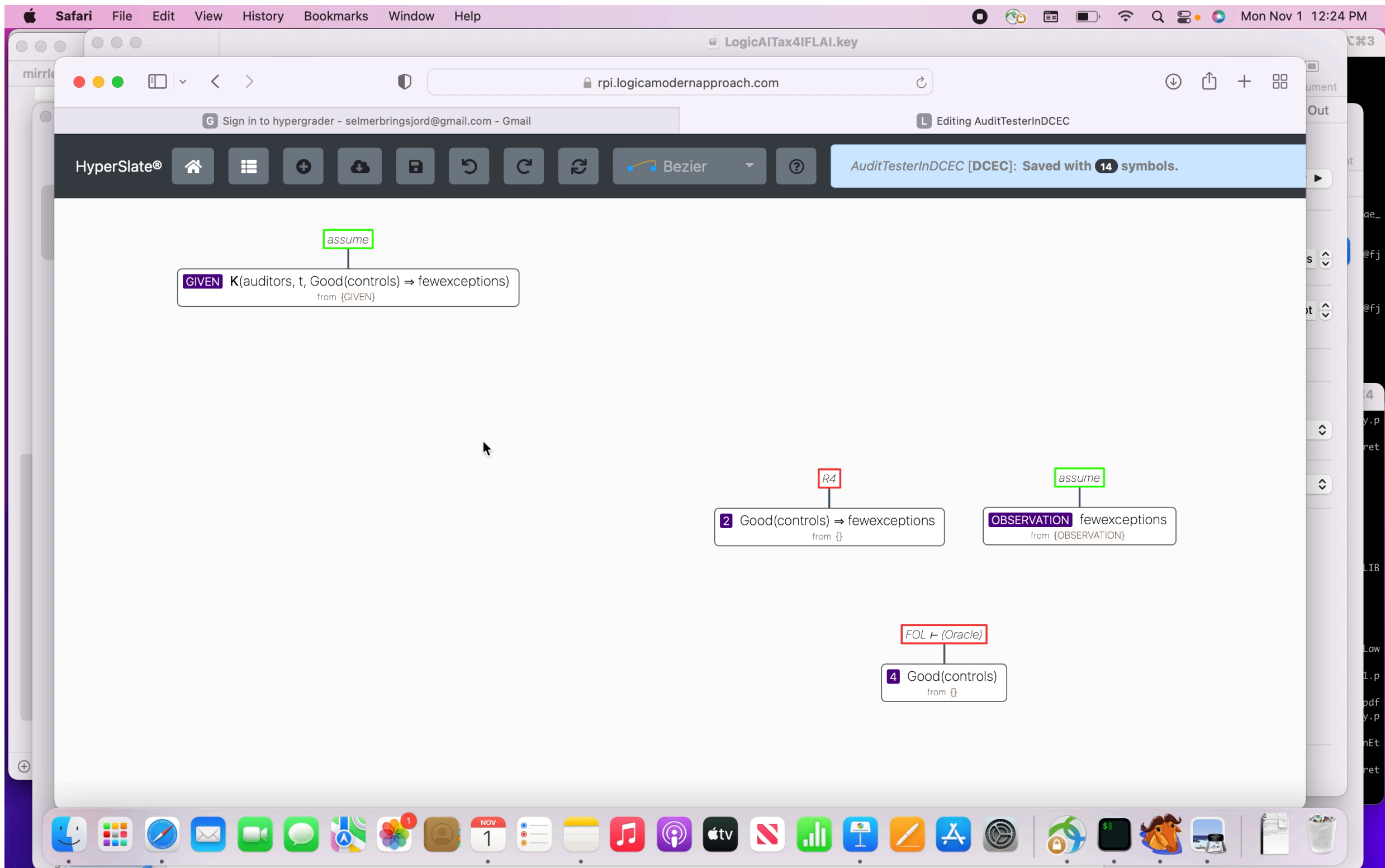
How certain are you of your answer? \_\_\_\_\_ I am certain  
\_\_\_\_\_ I am pretty sure, but not certain  
\_\_\_\_\_ I think so, but have significant doubts  
\_\_\_\_\_ I am guessing

The instrument was administered to individuals in the following natural groups:

- <sup>4</sup> Twenty were chosen for pragmatic reasons: that was the number we could include and have the instrument take less than 20 min to complete. The basis for scenario selection was not random. We desired to include a variety of valid and invalid argument forms, and strong and weak inductions. Another consideration was the length of the vignettes, with shorter ones being favored, again to keep instrument administration time at a reasonable length.

<sup>6</sup> The instruction covered part of Chapter 7, focusing on validity, truth, and valid versus invalid argument forms.





1. Nineteen undergraduate students in an auditing course, upon completion of instruction in logic for seven 75-min class periods.<sup>5</sup>
2. Fifteen undergraduate students at the end of their first course in auditing, with two class periods devoted to a study of logic.<sup>6</sup>
3. Thirteen masters' students in accounting at the end of their second course in auditing, with no instruction in logic. (None of these students reported having taken a course in formal logic in any of their studies.)

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A bank's auditors were examining the controls over the processing of loan applications. They knew that if the controls were good, then they would discover few, if any, exceptions in their tests of controls. Indeed, their tests revealed only a few minor exceptions.

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\_\_\_\_\_ Justified    \_\_\_\_\_ Not Justified

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<sup>5</sup> The instruction was based on Chapters 7 and 8 from *Introduction to Auditing: Logic, Principles, and Techniques* (Ratliff & Reding, 2002). Covered topics included evidence, structure of audit arguments, validity, truth, deduction, induction, eight valid argument forms, sixteen invalid argument forms and informal fallacies, epistemic probability, truth tables, and tests of causality.

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\_\_\_\_ Justified **X** Not Justified

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# Challenge #2:

Can the U.S. federal tax code (= IRC of 1986, as amended) be captured by some group  $\Gamma$  of formulae in some formal logic  $\mathcal{L}$ ?

# Challenge #3:

Can an artificial agent able to create new and effective tax strategies to minimize tax bills be engineered?

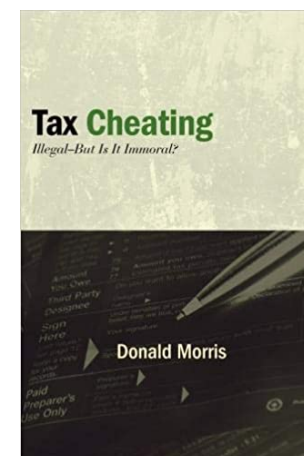


# Challenge(s) #4:

Can an artificial agent automatically prove that some tax filing is illegal? Immoral? How about automatically proving that some tax code *itself* is immoral?!

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# Returning now to $S$ ...

- Paradigm: *Logicist Agent-based Economics* (LABE)
- Formalize  $S$  completely.
- Then, what theorems can be obtained re what tax frameworks are good or bad and in between?

# Microsimulation

Orcutt's Vision, 50 years on

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October 2, 2007

# Microsimulation

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Broadly defined, microsimulation is a methodology used in a large variety of scientific fields to simulate the states and behaviors of different *units* - e.g. individuals, households, firms - as they evolve in a given *environment* - a market, a state, an institution. Very often it is motivated by a policy interest, so that narrower definitions are generally provided. For instance, [Martini and Trivellato, 1997] define microsimulation models as

computer programs that simulate aggregate and distributional effects of a policy, by implementing the provisions of the policy on a representative sample of individuals and families, and then summing up the results across individual units (p. 85).

MSM can answer relevant policy questions by handling simultaneously a large number of data, and calculating both individual and aggregate outcomes emerging from the *complex interaction* of several explanatory levels: the macro level, including e.g. demographic or labor market trends, the institutional level, including e.g. the tax and benefit system or a certain normative environment, and the micro level, including e.g. the characteristics, choices and actions of basic behavioral units such as households or firms.

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# Microsimulation

## 2. Direct Taxation and Behavioural Microsimulation: A Review of Applications in Italy and Norway

*Rolf Aaberge*  
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*Ugo Colombino*  
Turin University

### 2.1. Introduction

In this contribution we illustrate various applications of a behavioural microsimulation model that we have been developed during the last few years. Behavioural models are complex and costly tools to develop, use and maintain, but also very powerful ones as we wish to show through the examples that follow. In section 2.2 we present the main features of the microeconomic model. In section 2.3 we comment upon the labour supply elasticities implied by the estimates. In section 2.4 we illustrate a simulation of behavioural and welfare effects of some tax reform proposals. In section 2.5 we report on an exercise where we look for the optimal tax system. In section 2.6 we report on an ongoing project aimed at integrating the microeconomic model and a Computable General Equilibrium model. Lastly, in section 2.7, we show an out-of-sample test of the model, where we compare predictions of a model estimated on 1994 data to the observed effects of reform in 2001.

### 2.2. The microeconomic model

Over the last ten years, together with other colleagues, we have developed a structural model of labour supply<sup>1</sup> which features: si-

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$$\begin{aligned} & \max_{h,w,j} U(C,h,z) \\ & s.t. \\ & C = f(wh,I) \\ & (h,w,z) \in B, \end{aligned} \tag{2.1}$$

where  $I$  is an unearned income,  $C$  is a net income and  $f(\cdot)$  is the tax-benefit rule that transforms gross income into net income.

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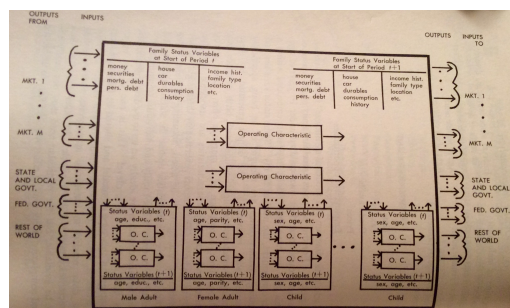
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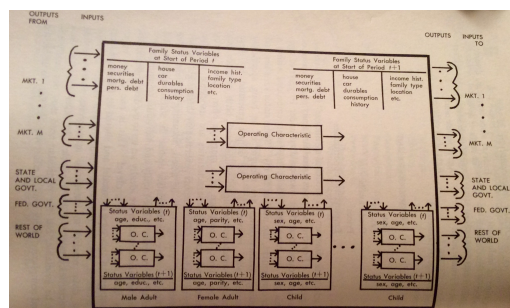
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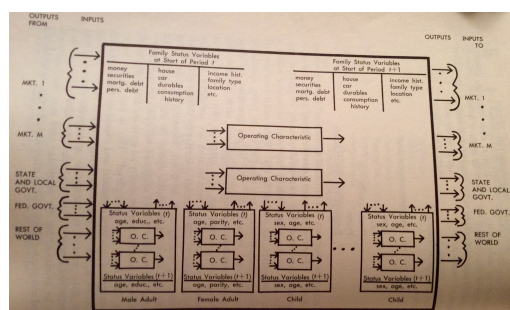


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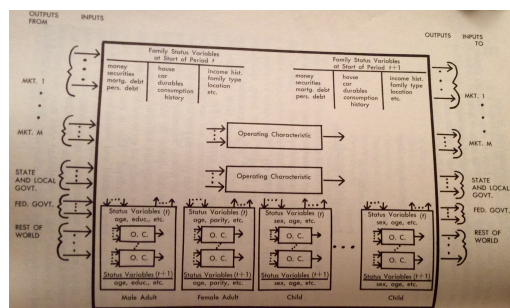
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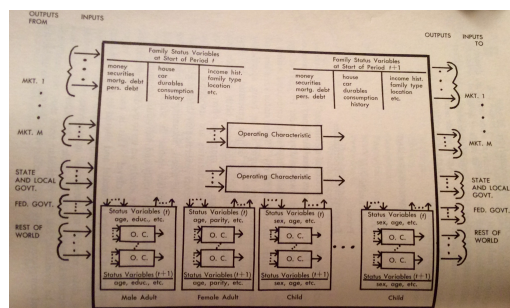
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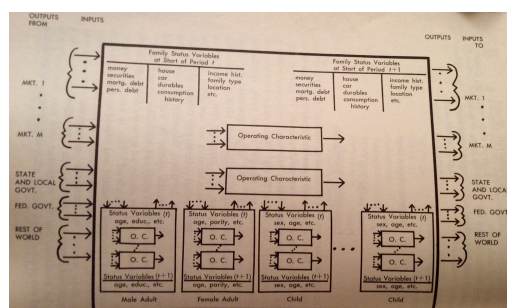
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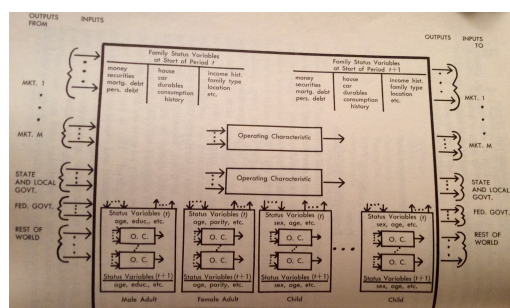
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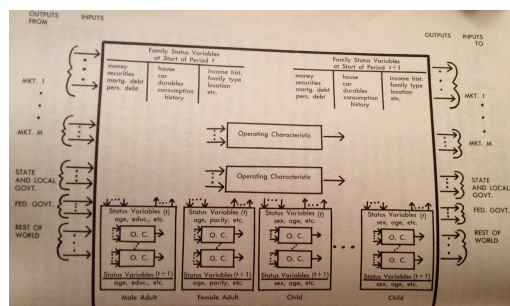
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Oh, & no communicative capacity!

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# Same Holds for Standard ABM

## Agent-Based Modelling in Economics

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Author(s): Lynne Hamill, Nigel Gilbert

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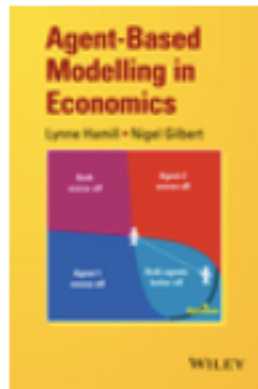
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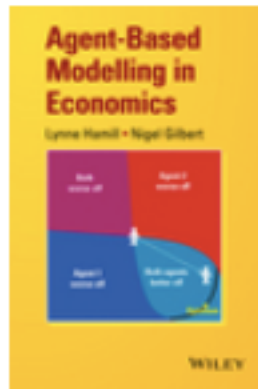
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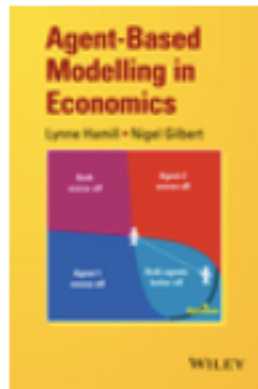
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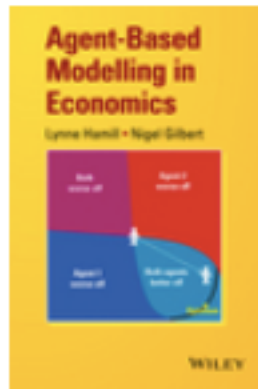
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# Some Key Papers

Ramsey, F. (1927) “A Contribution to the Theory of Taxation” *The Economic Journal* **37.145**: 47–61.

<https://eml.berkeley.edu/~saez/course131/Ramsey27.pdf>

Mirrlees, J. (1971) “An Exploration in the Theory of Optimal Income Taxation” *Review of Economic Studies* **38**: 175–208.

“Optimal Taxation in Theory and Practice” by N. Gregory Mankiw, Matthew Weinzierl, and Danny Yagan.

[https://scholar.harvard.edu/files/mankiw/files/optimal\\_taxation\\_in\\_theory.pdf](https://scholar.harvard.edu/files/mankiw/files/optimal_taxation_in_theory.pdf)





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