From the Lottery Paradox to Defeasible/Nonmonotonic Logic and AI

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Paradoxes are engines of progress in formal logic.

E.g., Russell's Paradox — now up as an exercise on HG®.

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- A perfectly rational person can never believe *P* and believe ¬*P* at the same time.

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- A perfectly rational person can never believe *P* and believe $\neg P$ at the same time.
- The Lottery Paradox (apparently) shows, courtesy of its two Sequences (of Reasoning), that a perfectly rational person can indeed have such a belief (upon considering a fair, large lottery).

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- The Lottery Paradox (apparently) shows, courtesy of its two Sequences (of Reasoning), that a perfectly rational person can indeed have such a belief (upon considering a fair, large lottery).
- Contradiction! — and hence a paradox!

Types of Paradoxes

- Deductive Paradoxes. The reasoning in question is exclusively deductive.
  - Russell's Paradox/Barber Paradox
  - The Liar Paradox (see knight-knave problems)
  - Richard's Paradox (anyone looking for a formal project?)
- Inductive Paradoxes Some of the reasoning in question uses non-deductive reasoning (e.g., probabilistic reasoning, abductive reasoning, analogical reasoning, etc.).

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*In*ductive Logic

the hallmark of deductive logic is *proof*, the hallmark of inductive logic is the concept of an *argument*. An exceptionally strong kind of argument is a proof, but plenty of arguments fall short of being proofs — and yet still have considerable force. For instance, consider the following argument α_1:

	(1)	Tweety is bird.
	(2)	Most birds can fly.
∴	(3)	Tweety can fly.

For start contrast, consider as well this argument (α_2):

	(1′)	3 is a positive integer.
	(2′)	All positive integers are greater than 0.
∴	(3′)	3 is greater than 0.

The second of these arguments qualifies as an outright proof. That is, using the notation much employed before the present chapter:

$$\{(1'),(2')\} \vdash (3')$$

But in stark contrast, argument α_1 is not a proof that Tweety can fly. The reason is obvious: (3) isn't deduced from the combination of (1) and (2); that is,

$$\{(1),(2)\} \nvdash (3)$$

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Inductive-Reasoning Example from Pollock — for Peek Ahead

Imagine the following:

Keith tells you that the morning news predicts rain in Troy today. However, Alvin tells you that the same news report predicted sunshine.

Imagine the following: Keith tells you that the morning news predicts rain in Tucson today. However, Alvin tells you that the same news report predicted sunshine.

Without any other source of information, it would be irrational to place belief in either Keith's or Alvin's statements.

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Further, suppose you happened to watch the noon news report, and that report predicted rain. Then you should believe that it will rain despite your knowledge of Alvin's argument.

Defeasible Reasoning in OSCAR

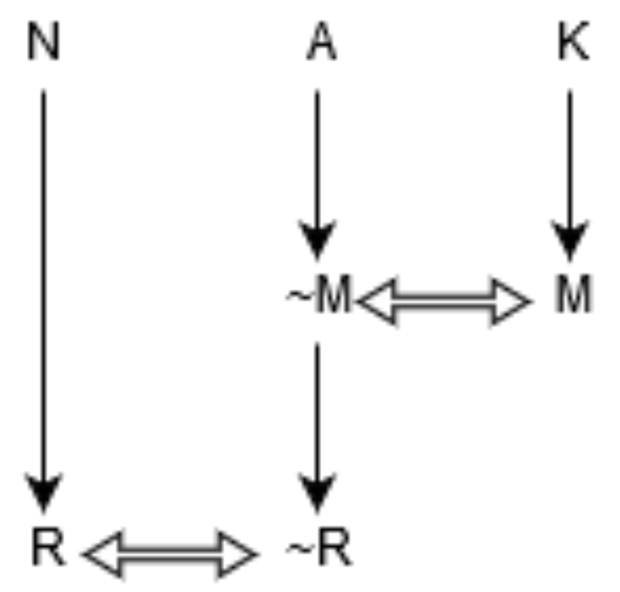
K- Keith says that M

A- Alvin says that ~M

M- The morning news said that R

R- It is going to rain this afternoon

N- The noon news says that R



All such can be absorbed into the RAIR Lab's inductive logics and our automated inductive reasoners (= our AI).

In Our *IDCEC**

(1)	$\mathbf{K}(you, \mathbf{S}(keith, \mathbf{S}(m, rain)))$	fact
(2)	$\mathbf{K}(you, \mathbf{S}(alvin, \mathbf{S}(m, \neg rain)))$	fact

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	(1)	$\mathbf{K}(you, \mathbf{S}(keith, \mathbf{S}(m, rain)))$	fact
	(2)	$\mathbf{K}(you, \mathbf{S}(alvin, \mathbf{S}(m, \neg rain)))$	fact
$\therefore$	(3)	$\mathbf{S}(keith, \mathbf{S}(m, rain))$	?
$\therefore$	(4)	$\mathbf{S}(alvin, \mathbf{S}(m, \neg rain)))$	?

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$\therefore$	(3)	$\mathbf{S}(keith, \mathbf{S}(m, rain))$	?
$\therefore$	(4)	$\mathbf{S}(alvin, \mathbf{S}(m, \neg rain)))$	?
	(5)	$\mathbf{S}(keith, \phi) \rightarrow \mathbf{B}^2(you, \phi)$	Testimonial P1

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$\therefore$	(3)	$\mathbf{S}(keith, \mathbf{S}(m, rain))$	?
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	(5)	$\mathbf{S}(keith, \phi) \rightarrow \mathbf{B}^2(you, \phi)$	Testimonial P1
$\therefore$	(6)	$\mathbf{B}^2(you, \mathbf{S}(m, rain)) \wedge \mathbf{B}^2(you, \mathbf{S}(m, \neg rain))$	

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$\therefore$	(6)	$\mathbf{B}^2(you, \mathbf{S}(m, rain)) \wedge \mathbf{B}^2(you, \mathbf{S}(m, \neg rain))$	
$\therefore$	(7)	$\neg\mathbf{B}^2(you, \mathbf{S}(m, rain)) \wedge \neg\mathbf{B}^2(you, \mathbf{S}(m, \neg rain))$	“Clash” Principle

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	(8)	$\mathbf{K}(you, \mathbf{S}(noonnews, rain))$	

In Our *IDCEC**

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	(10)	$\mathbf{S}(noonnews, \phi) \rightarrow \mathbf{B}^3(you, \phi)$	Testimonial P2

In Our *IDCEC**

$$
\begin{array}{c|l|l|l}
 & (1) & \mathbf{K}(you, \mathbf{S}(keith, \mathbf{S}(m, rain))) & \text{fact} \\
 & (2) & \mathbf{K}(you, \mathbf{S}(alvin, \mathbf{S}(m, \neg rain))) & \text{fact} \\
\therefore & (3) & \mathbf{S}(keith, \mathbf{S}(m, rain)) & ? \\
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\therefore & (7) & \neg\mathbf{B}^2(you, \mathbf{S}(m, rain)) \wedge \neg\mathbf{B}^2(you, \mathbf{S}(m, \neg rain)) & \text{``Clash'' Principle} \\
 & (8) & \mathbf{K}(you, \mathbf{S}(noonnews, rain)) & \\
\therefore & (9) & \mathbf{S}(noonnews, rain) & ? \\
 & (10) & \mathbf{S}(noonnews, \phi) \rightarrow \mathbf{B}^3(you, \phi) & \text{Testimonial P2} \\
\therefore & (11) & \mathbf{B}^3(you, rain) &
\end{array}
$$

In what's coming, 2 = *likely*.

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$\therefore$	(3)	$\mathbf{S}(keith, \mathbf{S}(m, rain))$	?
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$\therefore$	(9)	$\mathbf{S}(noonnews, rain)$	?
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In what's coming, 3 = highly *likely*.

The Lottery Paradox …

Stewart's Shops

WE ARE CLOSER TO YOU!

E: “Please go down to Stewart’s & get the T U.”

Stewart's
Shops
WE ARE CLOSER TO YOU!

E: “Please go down to Stewart’s & get the T U.”

S: “I’m sorry, E, I’m afraid I can’t do that.
It would be irrational.”

Stewart's Shops
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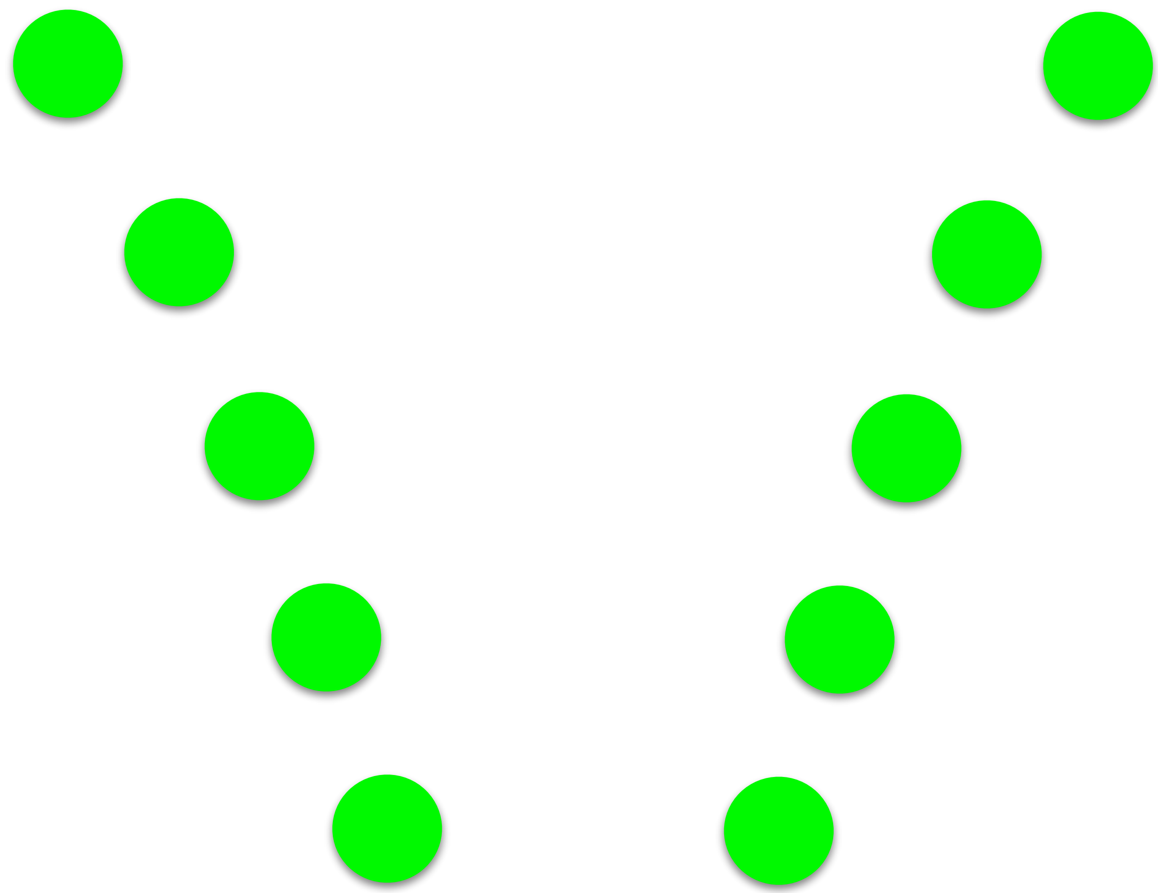

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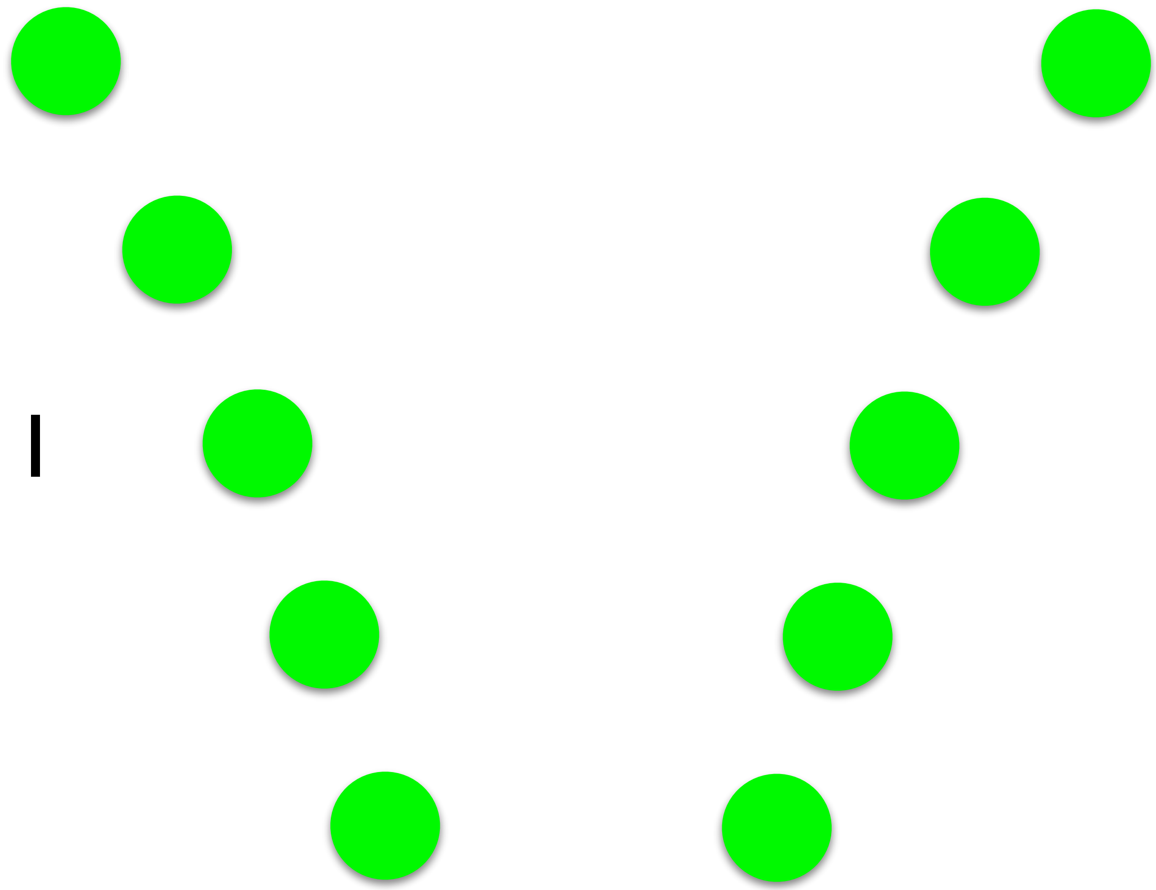

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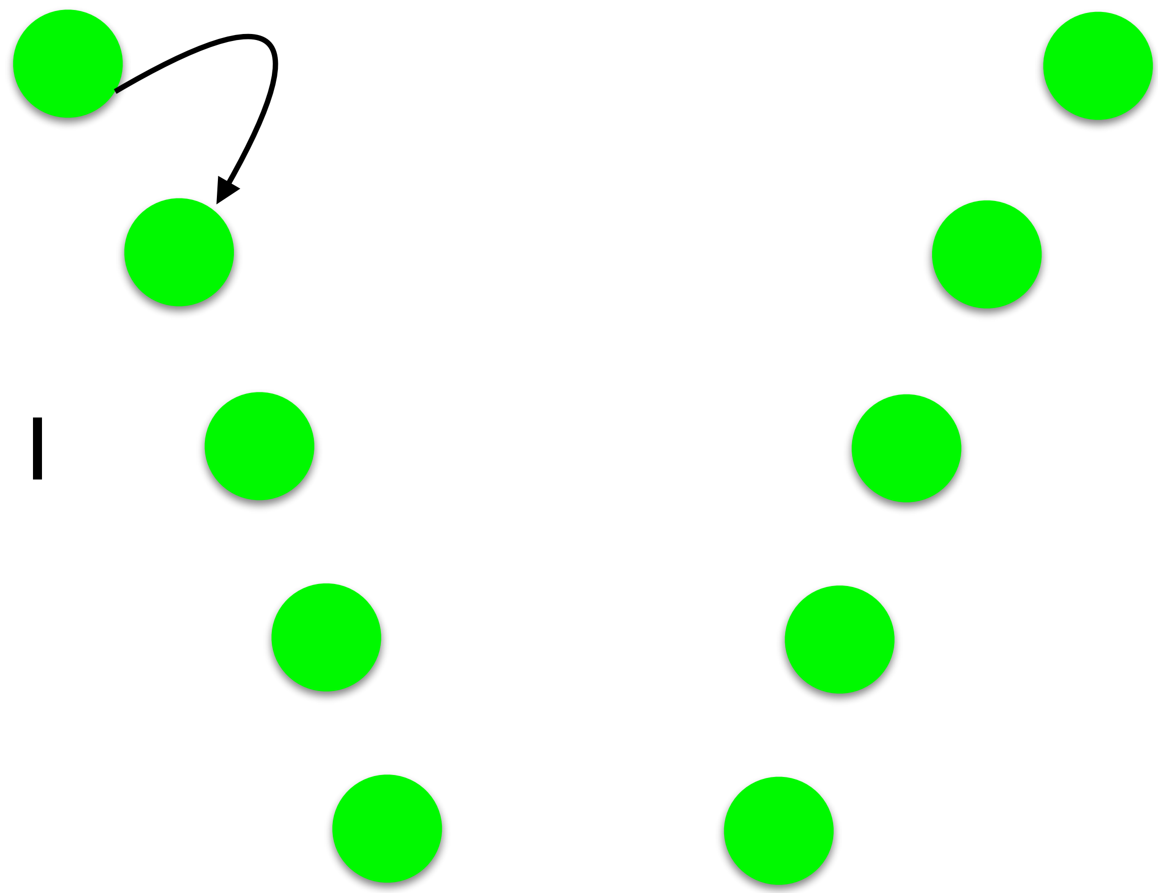

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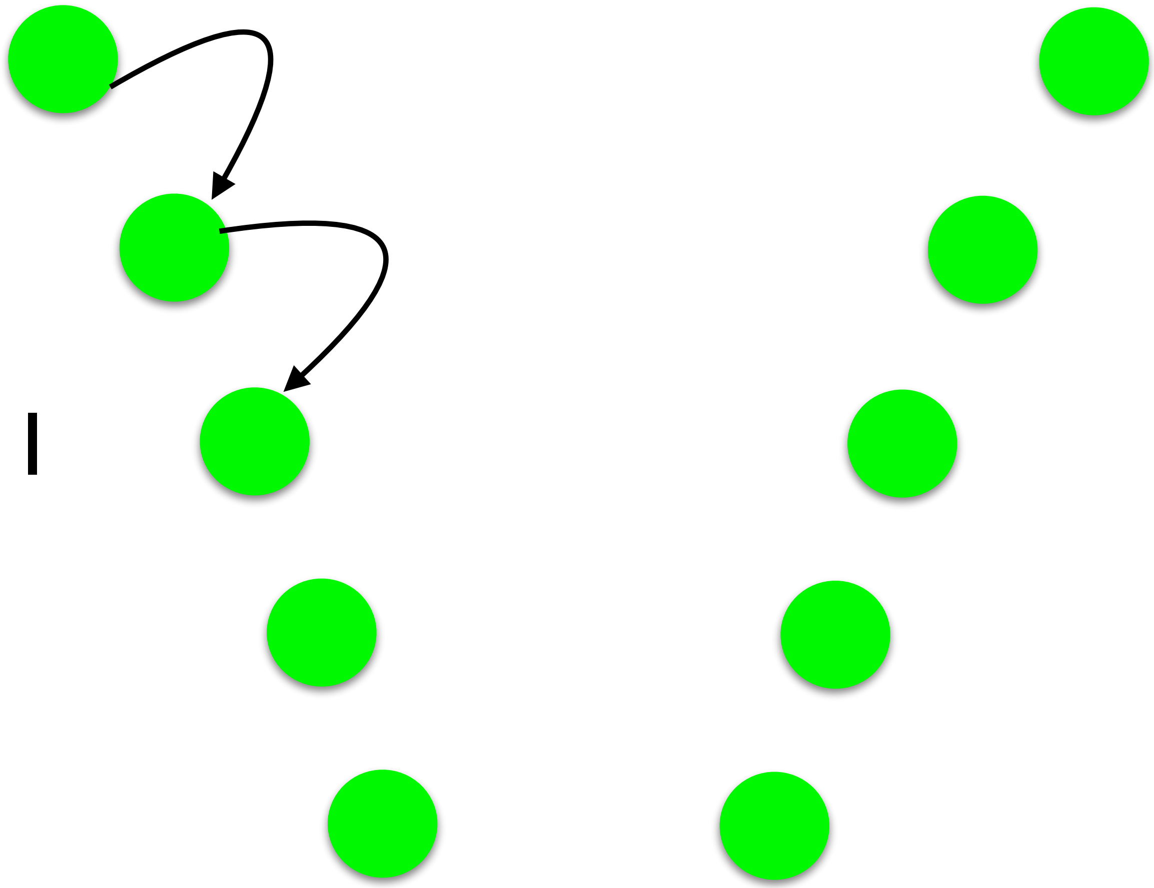

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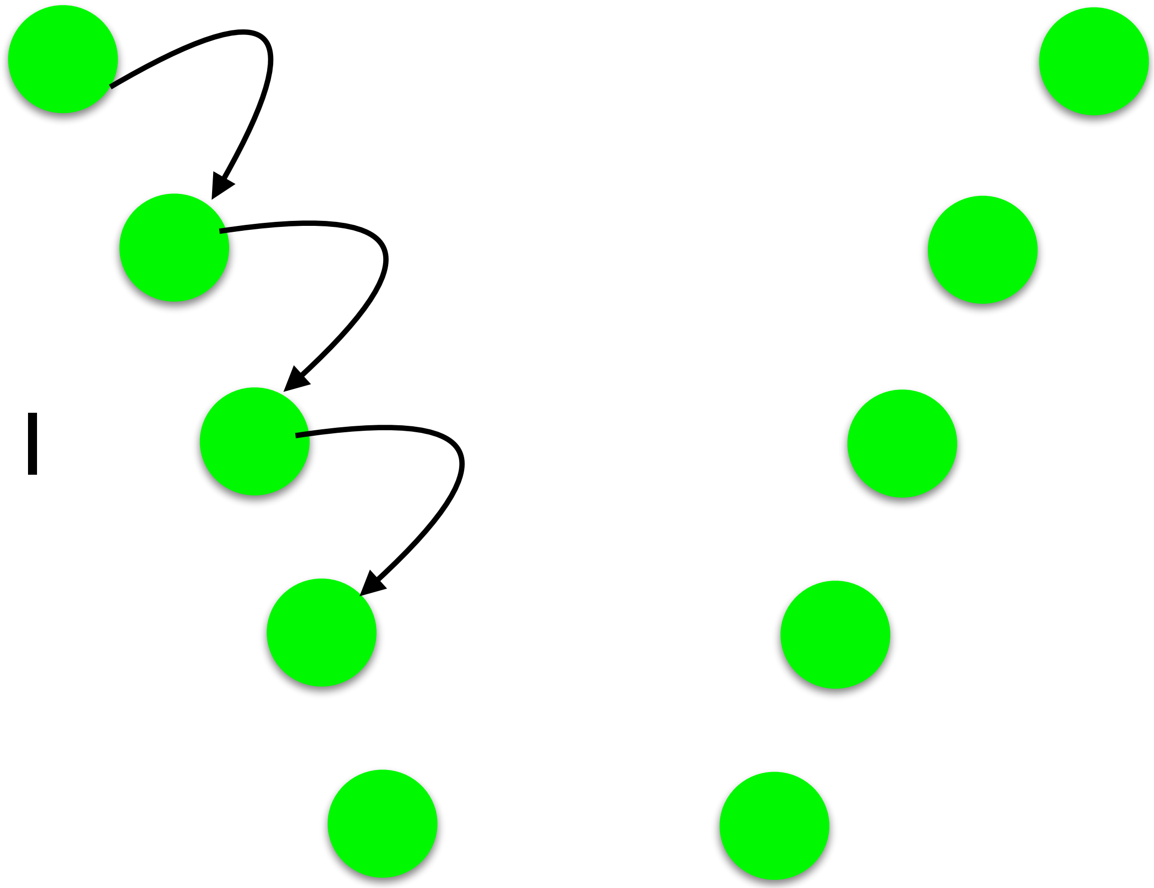

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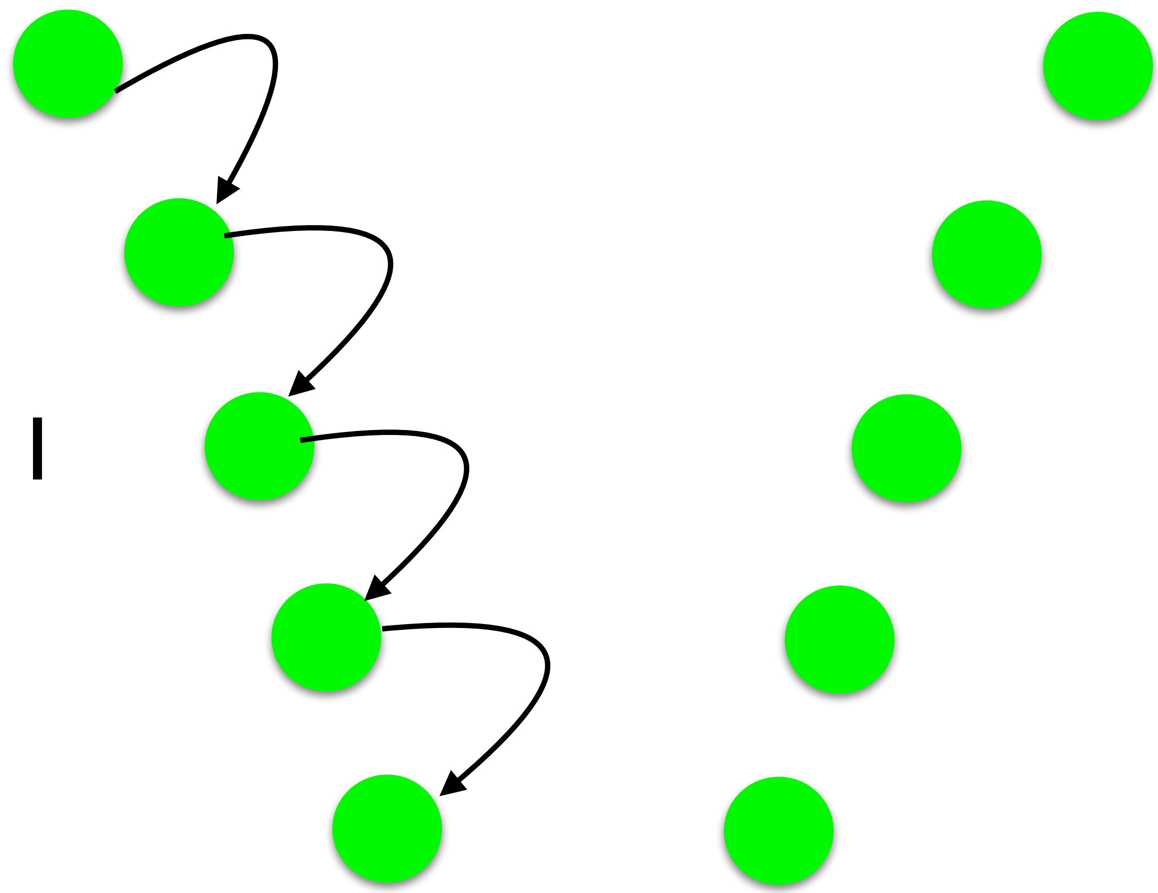

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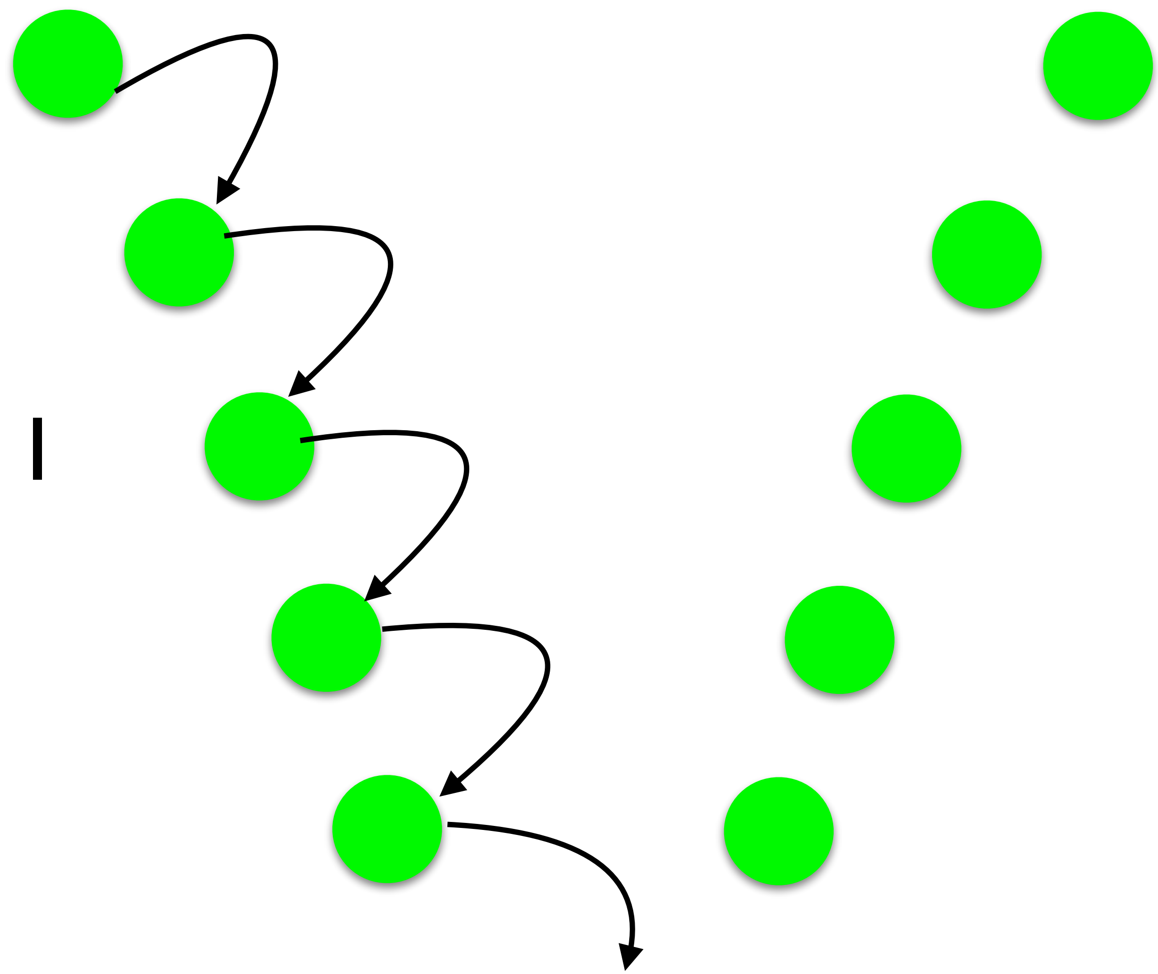
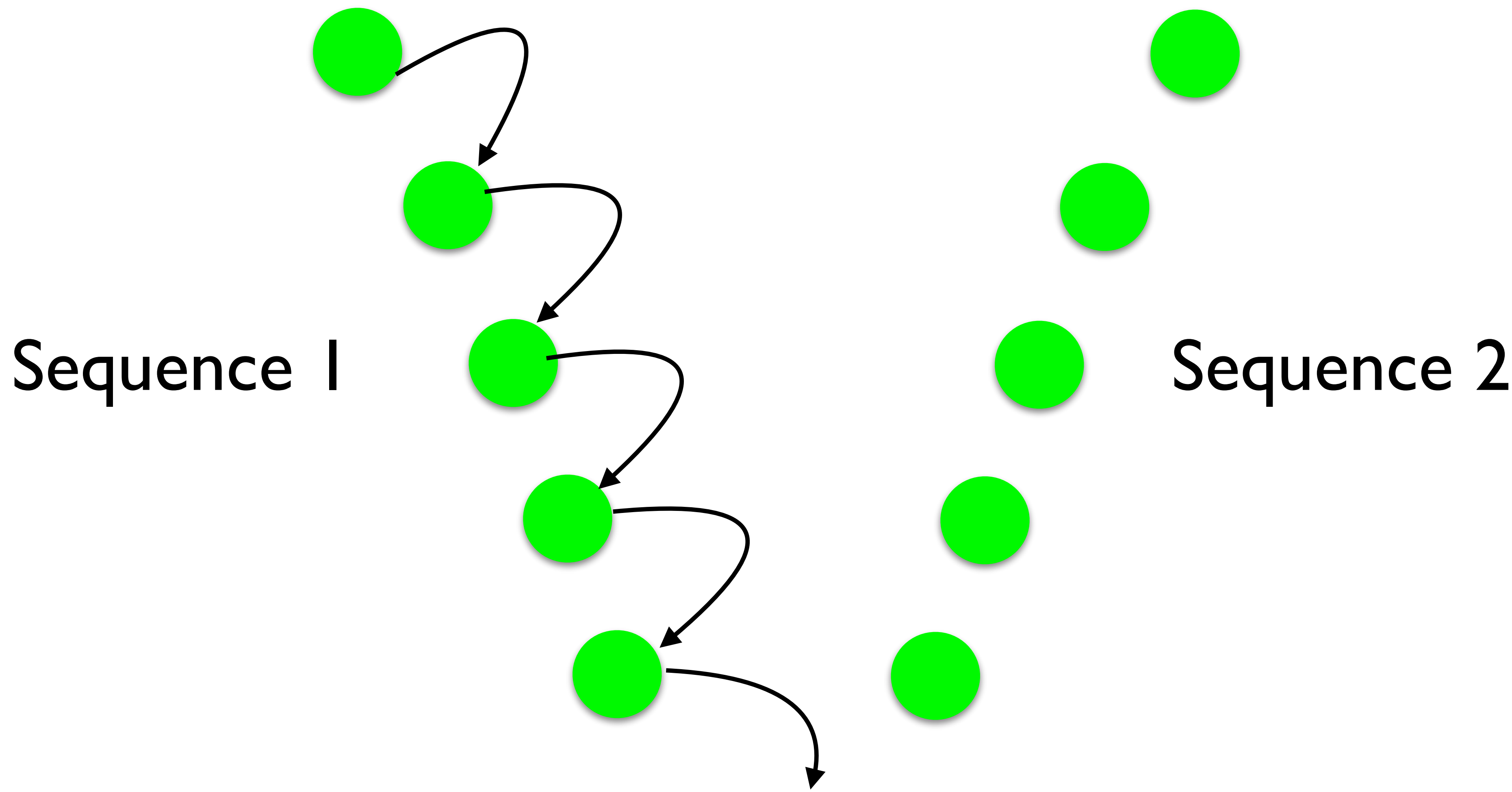
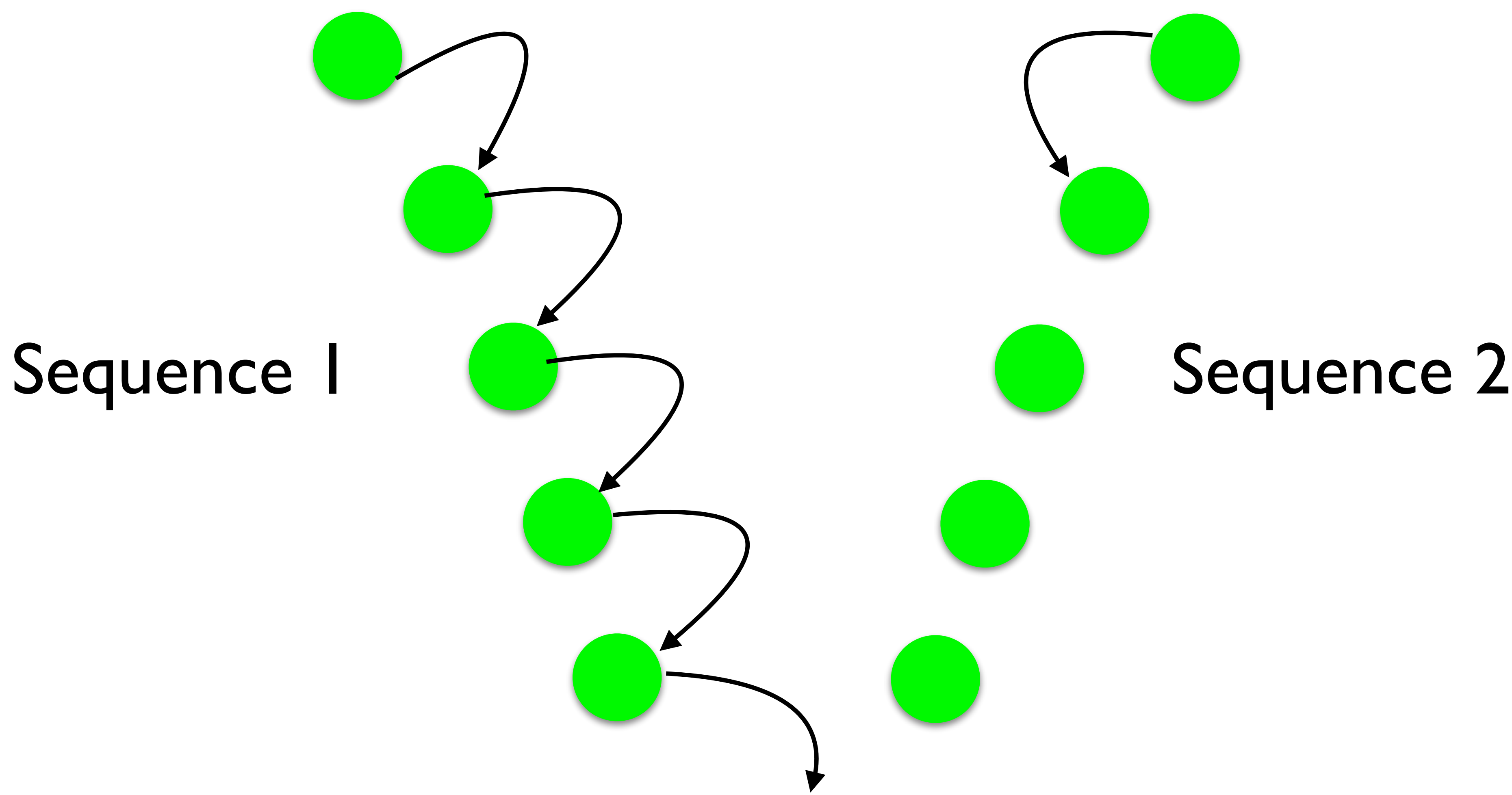



Sequence 1
Sequence 2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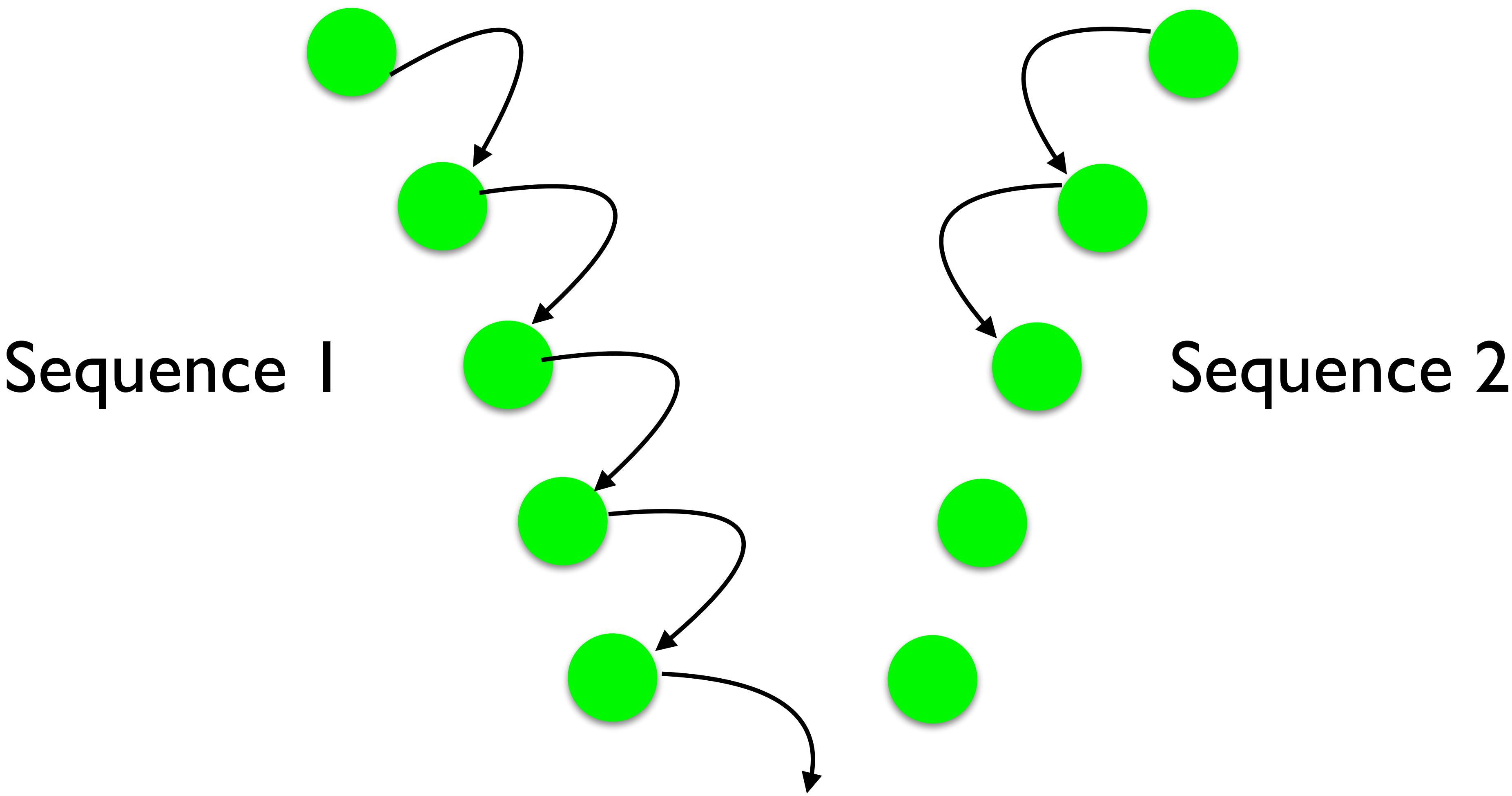
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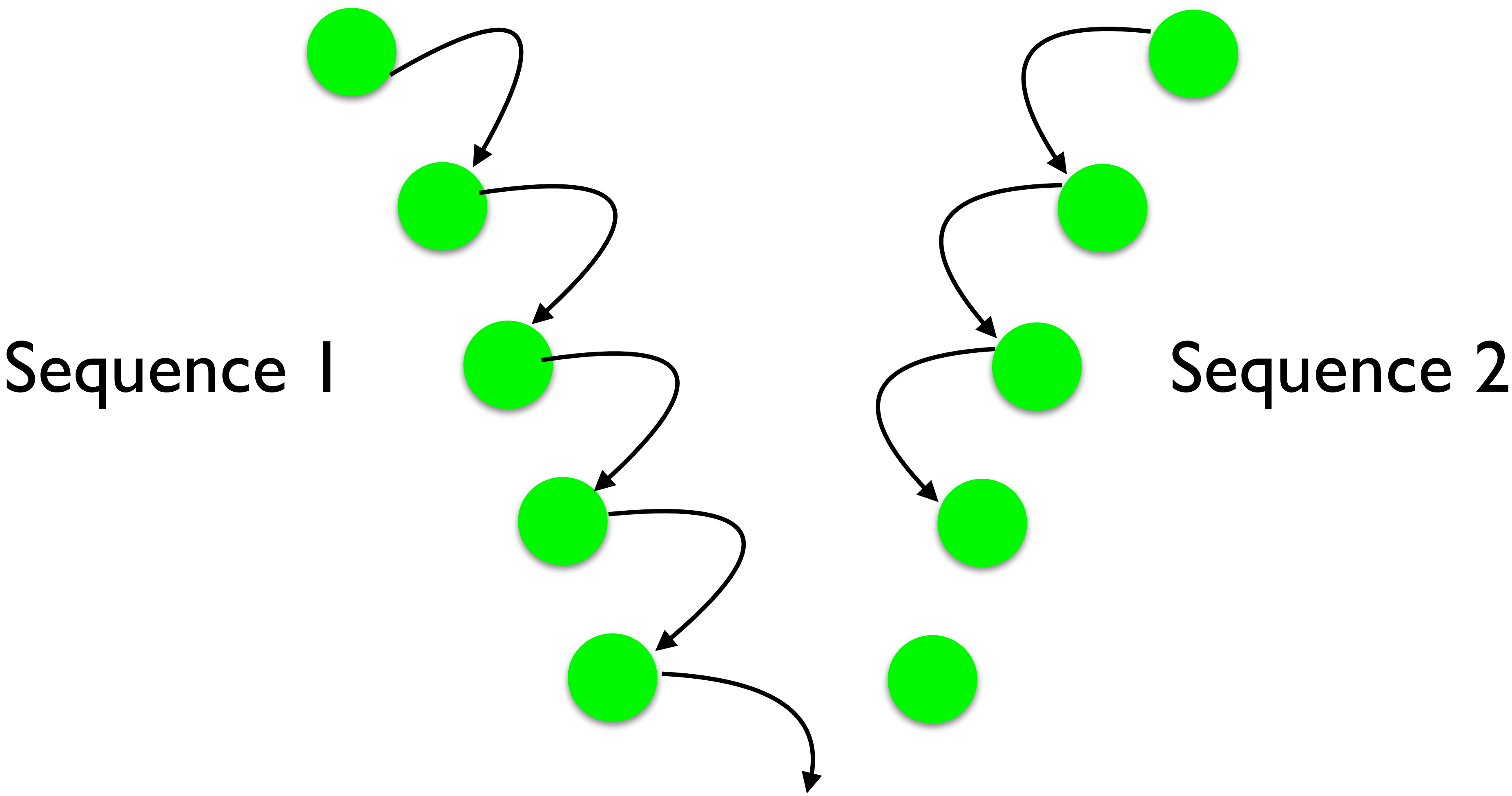
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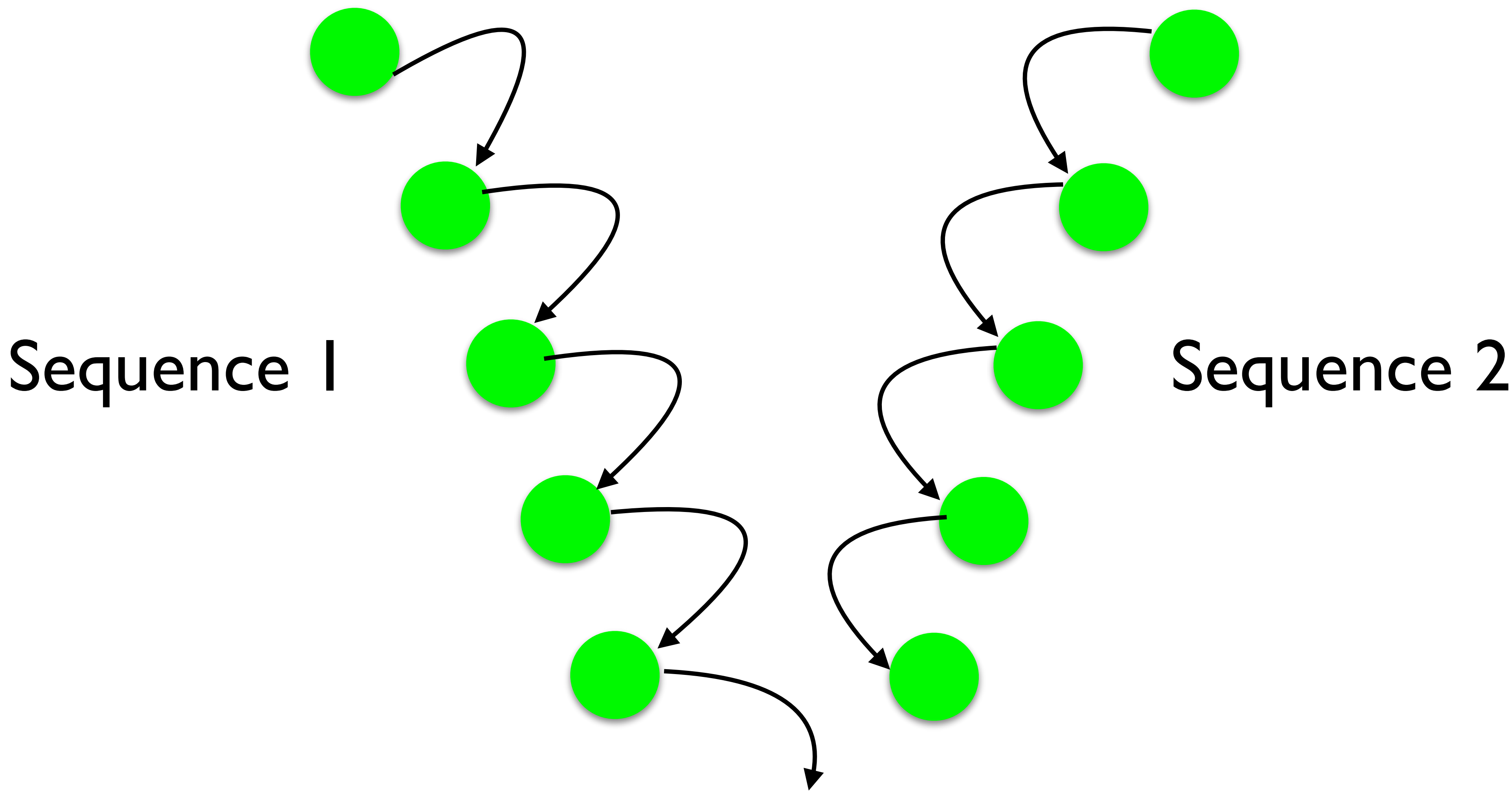
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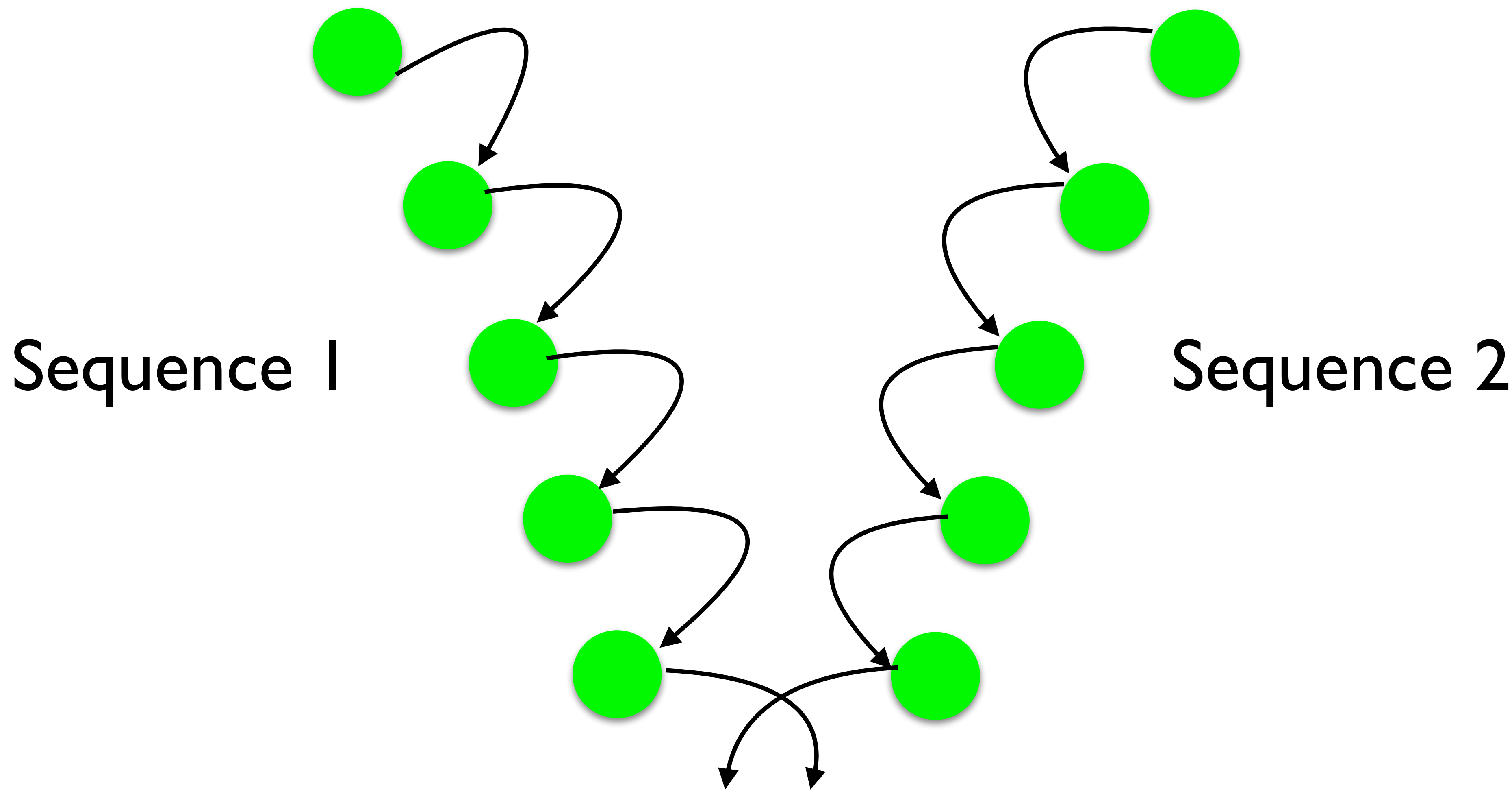

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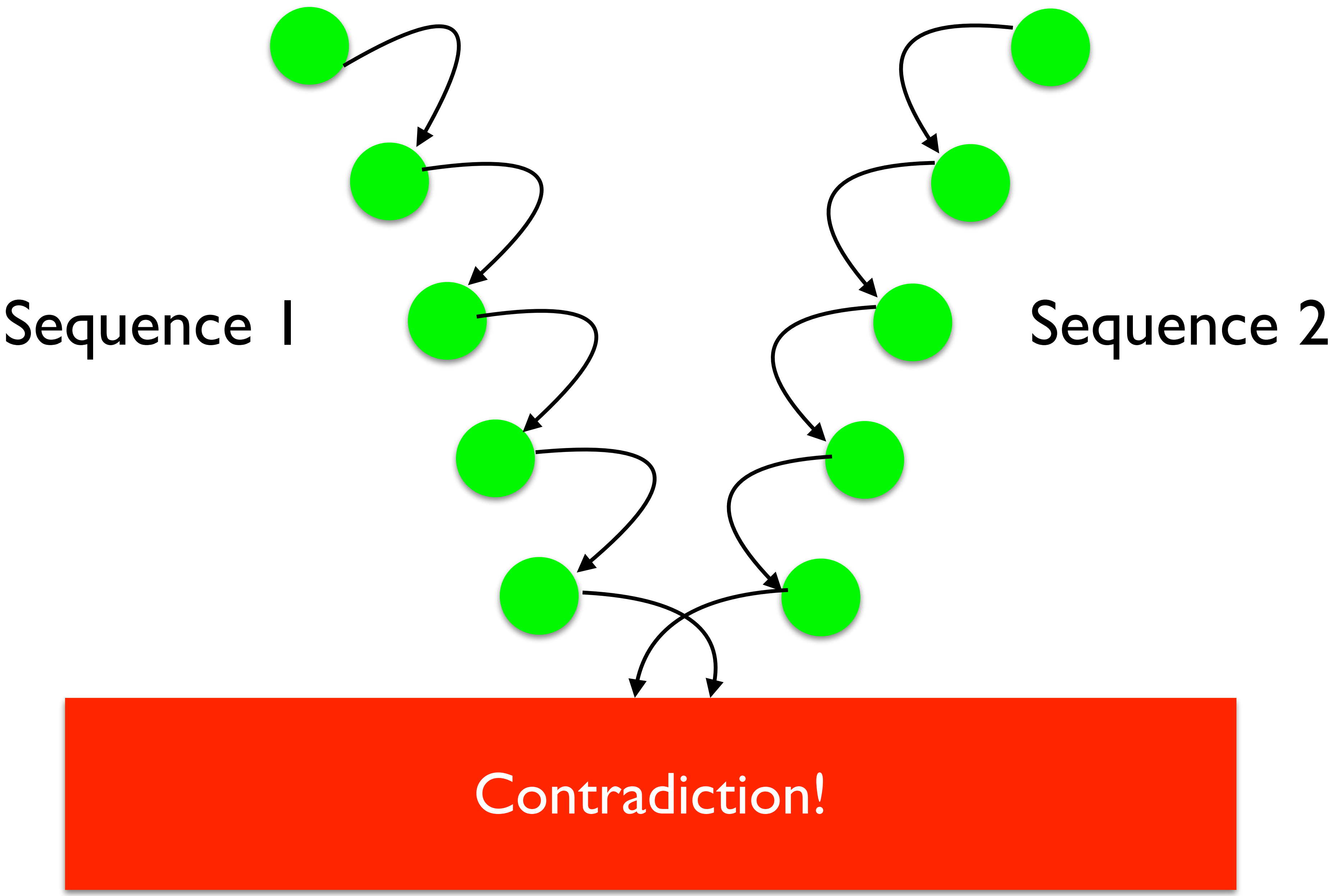
Sequence 1
Sequence 2
Contradiction!

Sequence 1

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Let **D** be a meticulous and perfectly accurate description of a 1,000,000,000,000-ticket lottery, of which rational agent *a* is fully apprised.

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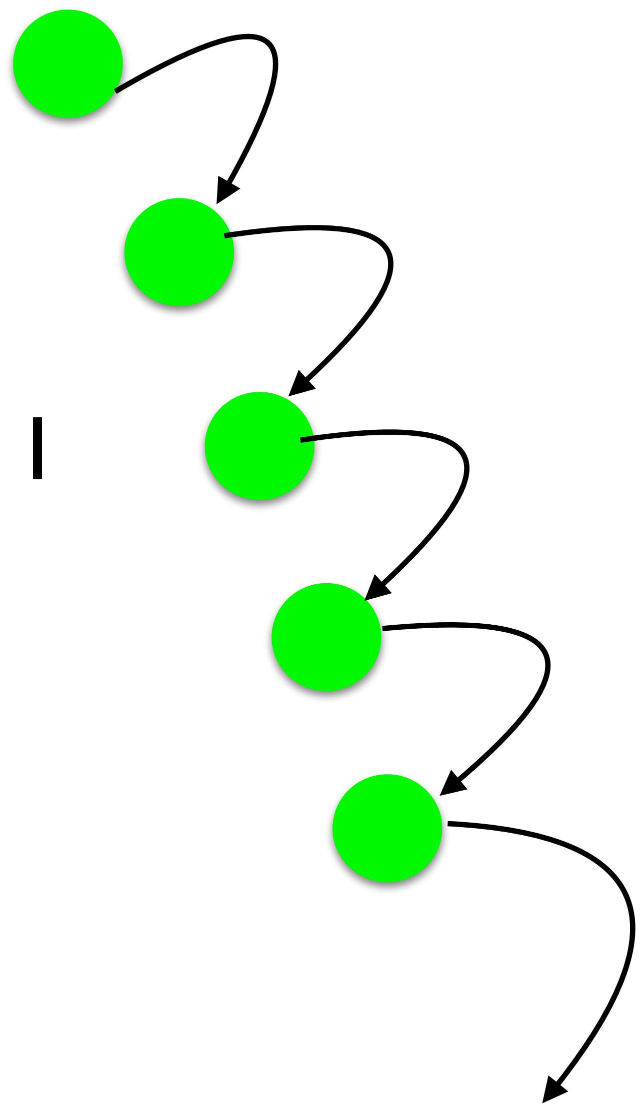

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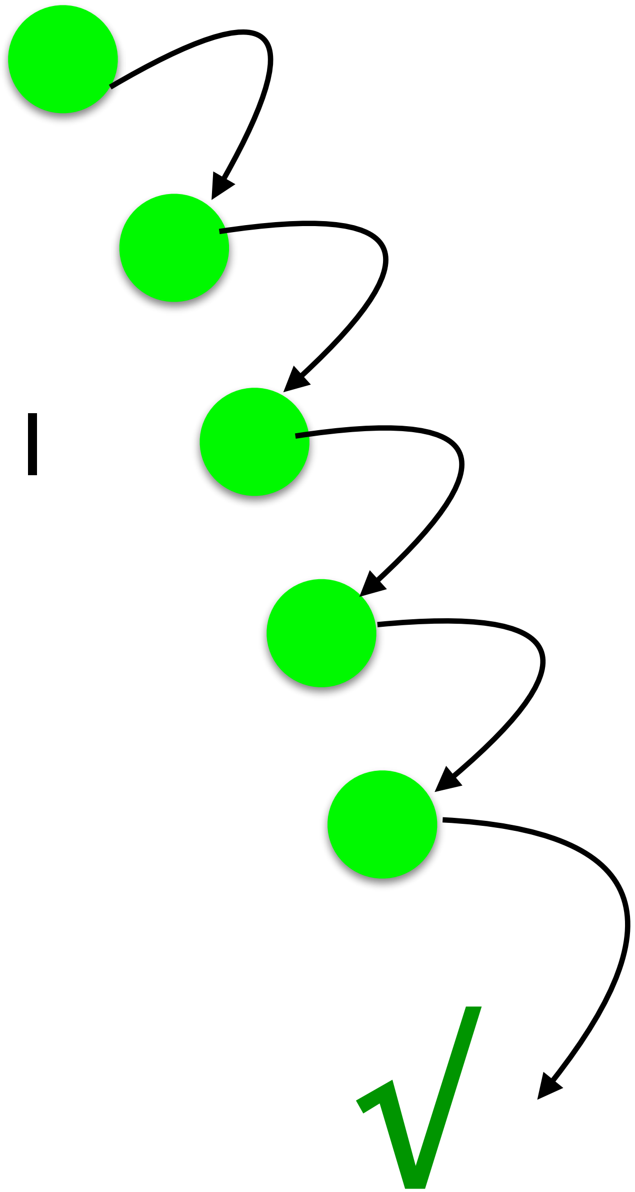

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But (3) is logically equivalent to the statement that there doesn't exist a winning ticket; i.e., (3) is logically equivalent to this result from Sequence 2:

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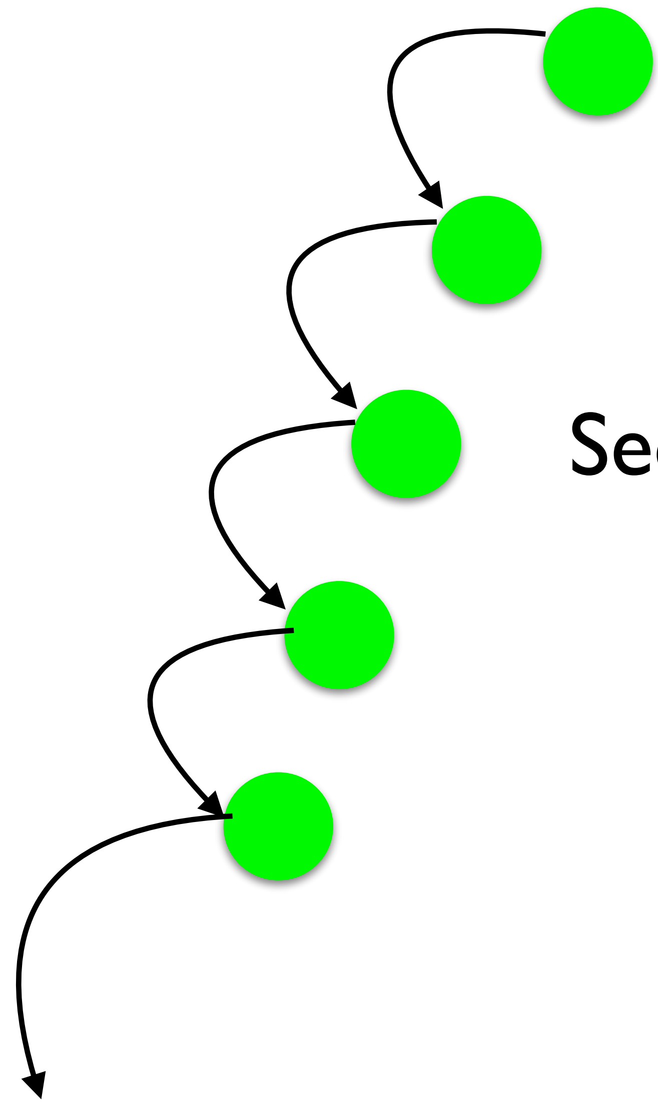
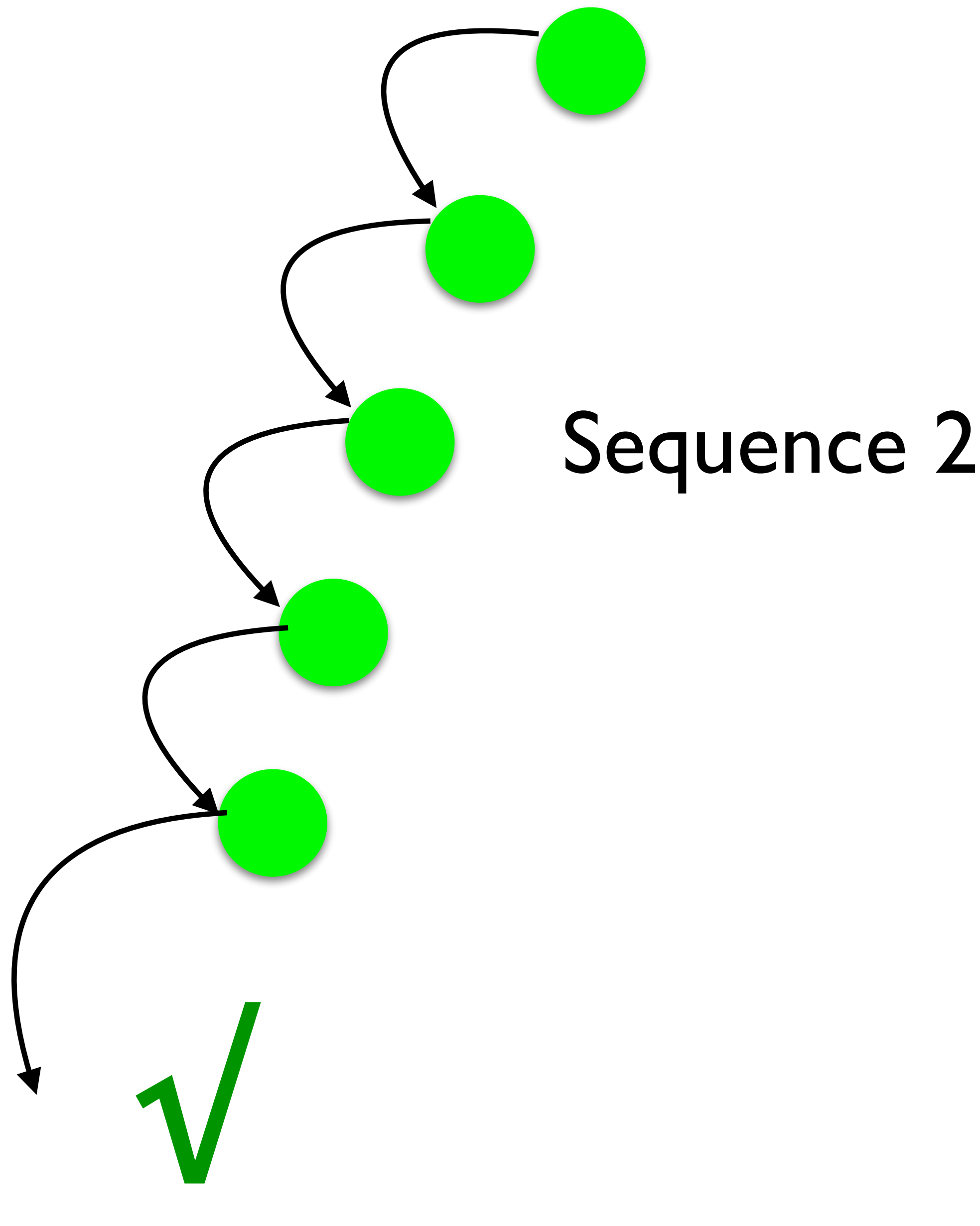
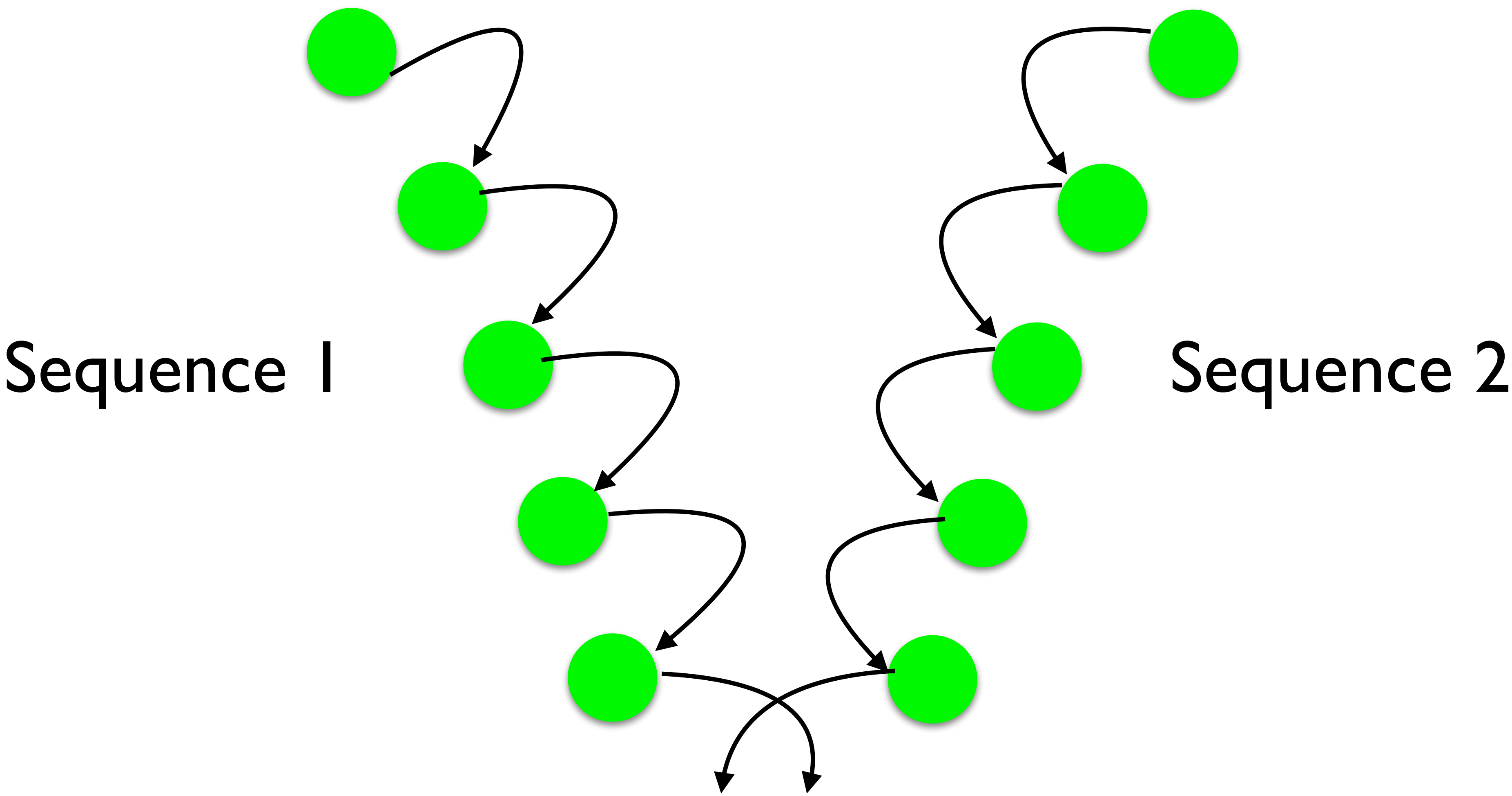


Sequence 2


Sequence 2
√


Sequence 1
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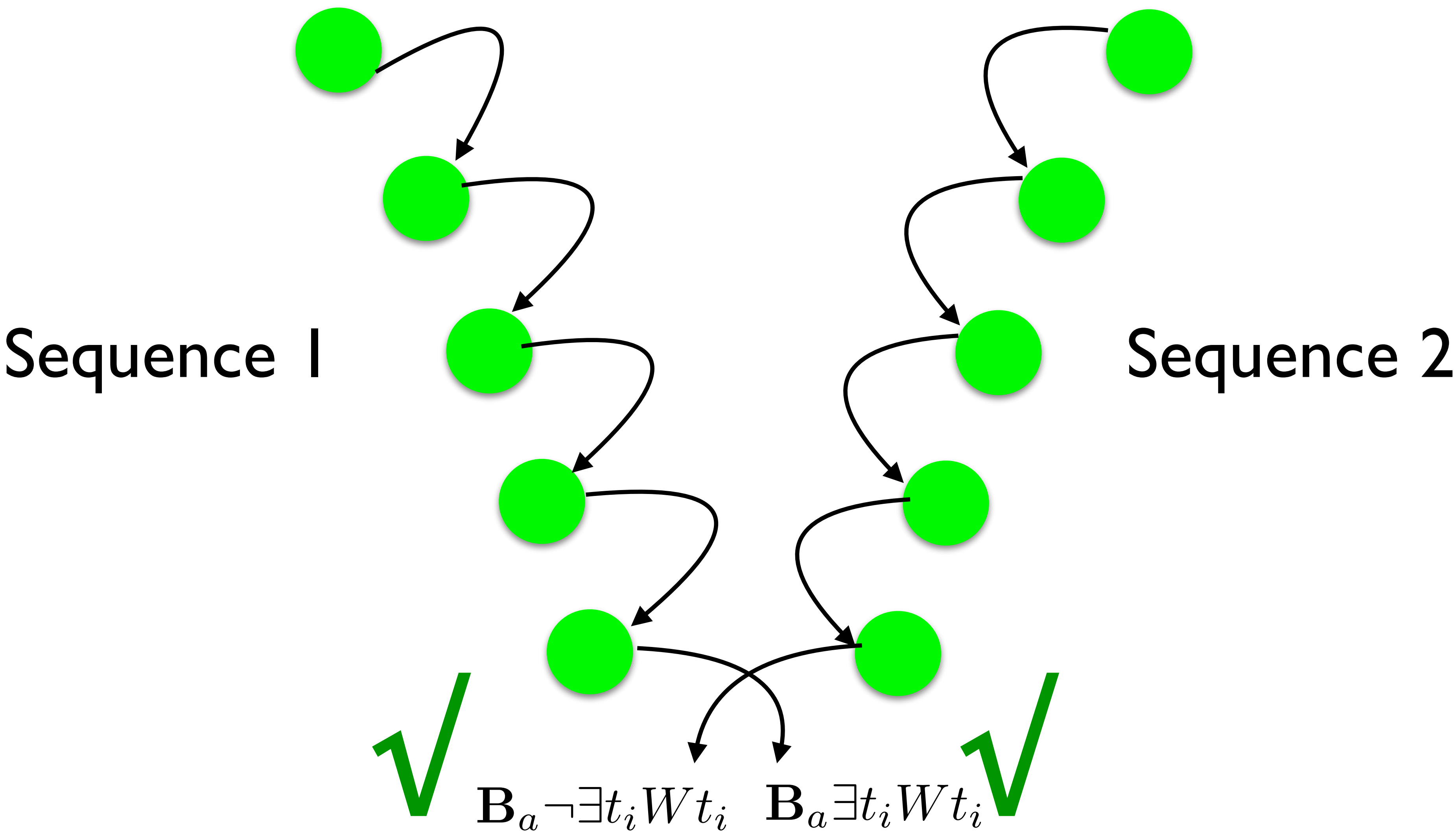
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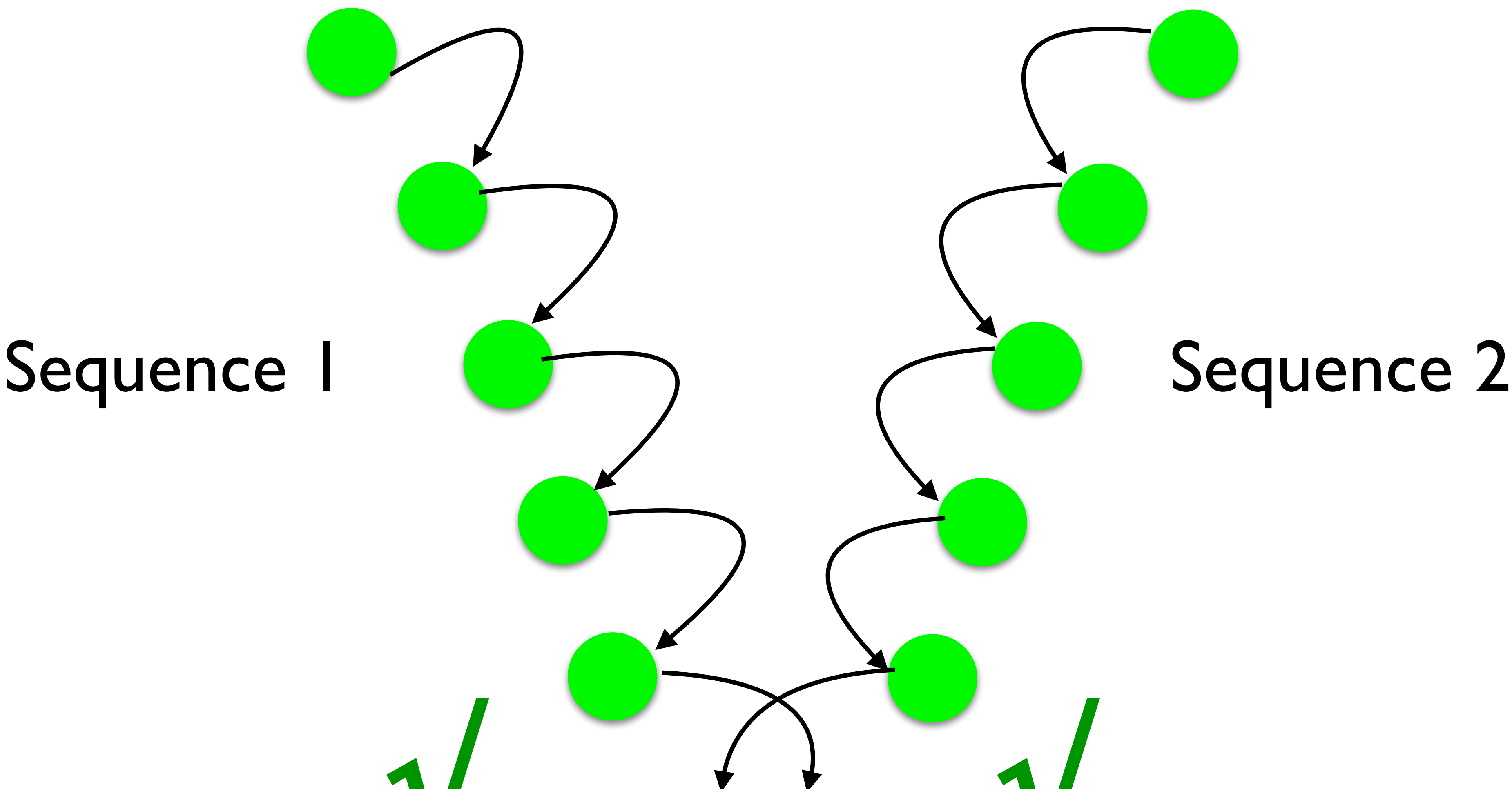

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(The contradiction we sketched earlier has arrived.)

A Solution to The Lottery Paradox …

Strength-Factor Continuum

Certain

Improbable

Evidently False

Likely

Beyond Reasonable Belief

Certainly False

Counterbalanced

Evident

Beyond Reasonable Doubt

Strength-Factor Continuum

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Counterbalanced

Unlikely

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Actually, now …

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English	**Value**
certain	6
evident	5
overwhelmingly likely = "beyond reasonable doubt" = "one in a million"	4
very likely	3
likely	2
more likely than not	1
counterbalanced	0

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English	Value
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… but let’s use the simpler scheme.

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Certain

Evident

Beyond Reasonable Doubt

Likely

Counterbalanced

Unlikely

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Certainly False

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Likely

Counterbalanced

Unlikely

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Evidently False

Certainly False

Strength-Factor Continuum

Epistemically Positive

Certain

Evident

Beyond Reasonable Doubt

Likely

Counterbalanced

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Strength-Factor Continuum

Epistemically Positive

(4) Certain

(3) Evident

(2) Beyond Reasonable Doubt

(1) Likely

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(-1) Unlikely

(-2) Beyond Reasonable Belief

(-3) Evidently False

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Strength-Factor Continuum

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Key Principles

Epistemically Positive

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Key Principles

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(4) Certain

Deduction preserves strength.

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Any proposition *p* such that *prob*(*p*) < 1 is at most evident.

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Key Principles

Epistemically Positive

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(3) Evident

(2) Beyond Reasonable Doubt

Clashes are resolved in favor of higher strength.

(1) Probable

Any proposition *p* such that *prob*(*p*) < 1 is at most evident.

(-1) Improbable

Any rational belief that *p*, where the basis for *p* is at most evident, is at most an evident (= level 3) belief.

(-3) Evidently False

Epistemically Negative

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Sequence 1, "Rigorized"

Let **D** be a meticulous and perfectly accurate description of a 1,000,000,000,000-ticket lottery, of which rational agent *a* is fully apprised.

From **D** it obviously can be proved that either ticket 1 will win or ticket 2 will win or … or ticket 1,000,000,000,000 will win. Let's write this (exclusive) disjunction as follows:

$$Wt_1 \oplus Wt_2 \oplus \ldots \oplus Wt_{1T} \quad (1)$$

We then deduce from this that there is at least one ticket that will win, a proposition represented — using standard notation — as:

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Very well; perfectly clear so far. And now we can add another deductive step: Since our rational agent *a* can follow this deduction sequence to this point, and since **D** is assumed to be indubitable, it follows that our rational agent in turn believes (2); i.e., we conclude Sequence 1 by obtaining the following:

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Sequence 2, "Rigorized"

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Sequence 2, "Rigorized"

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Paradox Solved!

Deduction preserves strength.

Clashes are resolved in favor of higher strength; clashes propagate backwards through inverse deduction; if no higher-strength factors, suspend belief.

Any proposition *p* such that *prob*(*p*) < 1 is at most evident.

Any rational belief that *p*, where the basis for *p* is at most evident, is at most an evident (= level 3) belief.

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This is why, to Mega Millions ticket holder:
"Sorry. I'm rational, and I believe you won't win."

To be clear about the effects of the first principle:

$$\vdash \mathbf{B}_a^3 \neg \cancel{\exists x} W x \wedge \mathbf{B}_a^3 \exists x W x!$$

$$\vdash \mathbf{B}_a^2 \neg \cancel{\exists x} W x \wedge \mathbf{B}_a^2 \exists x W x!$$

$$\vdash \mathbf{B}_a^1 \neg \cancel{\exists x} W x \wedge \mathbf{B}_a^1 \exists x W x!$$

Clashes are resolved in favor of higher strength; clashes propagate backwards through inverse deduction, preserving affirmation/belief of premises as far as is possible; if no higher-strength factors, suspend belief. (This means that in this case belief at level 4 also shoots down belief at level 2, and level 1. This is sort of bizarre, because to retain the belief (at levels 3, 2, 1) that every particular ticket won't win, the step that gets to believing the existential formula is blocked. Pollock doesn't have steps in his "arguments." Our agents thus ends up believing at all levels that some ticket will win, and believing at all levels 3 and down, of each particular ticket, that it won't win.)

slutten