On Quantificational Modal Logic

(S5-centric)

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Troy, New York 12180 USA

IFLAI2
11/15/2021
ver 1115211230NY

Rensselaer AI and Reasoning Lab

Logistics …

Status? Some discussion …

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%% TODO

%% [ ]

\documentclass[11pt]{article}

\usepackage[utf8]{inputenc}

\usepackage{fullpage} %% <= why not use this in your own paper?
\usepackage{setspace}
%% Toggle the following on for doublespacing:
%% \doublespacing


%% Some standard package calls by S:
 \usepackage{amssymb}
 \usepackage[colorlinks]{hyperref}
 \usepackage{harvard}  %% Selmer's preference for citations/References.
 \usepackage{color}
 \usepackage{marvosym}
 \usepackage{mathrsfs}
 \usepackage{verbatim}
 \usepackage{eufrak}


\begin{document}

\title{\textbf{IFLAI2F21 Paper Topics}}
\author{Prof Selmer Bringsjord}
\date{\texttt{ver 1115211415NY}}
\maketitle

\begin{small}
 \tableofcontents
\end{small}


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\section{General Orientation}
\label{sect:gen_orientation}
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Recall: Schedule Switcheroo

KU machines. We also discuss whether programming beyond the Turing Limit makes sense and can be pursued. In this connection we explore the hierarchy $\mathfrak{LM}$.

- **Nov 8**: *Hypergraphical Proof and Programming in HyperLog®*. We here introduce the availability of writing Clojure functions in the context of proofs in HyperLog®.
- **Nov 11**: *Quantified Modal Logic.* We here explore quantified **S5**, including the the infamous Barcan Formula. HyperSlate® is used.
- **Nov 15**: *Killer Robots, **D**, and Beyond in HyperSlate® to $\mathcal{DCEC}$.* We begin here by stating the "PAID Problem," and then the approach to it from Bringsjord et al. advocates. We review that modal logic **D** is painfully inadequate, but now move to some exploration of a version of $\mathcal{DCEC}$ in HyperSlate®.
- **Nov 18**: *The Logicist AI-ification of the Doctrines of N Effect to Solve the PAID Problem.*
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Q2?

Return to Return-to-Blinky

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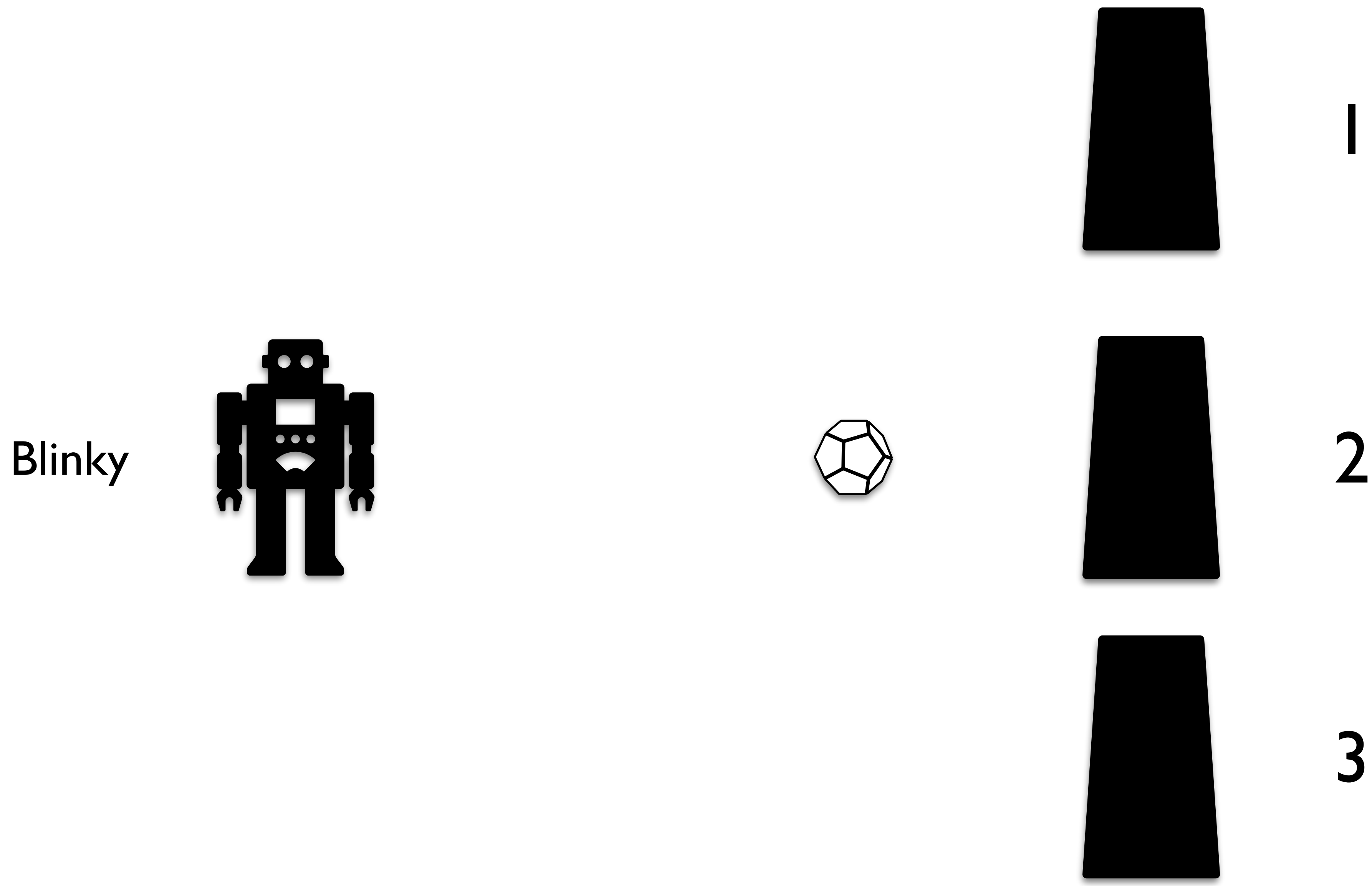
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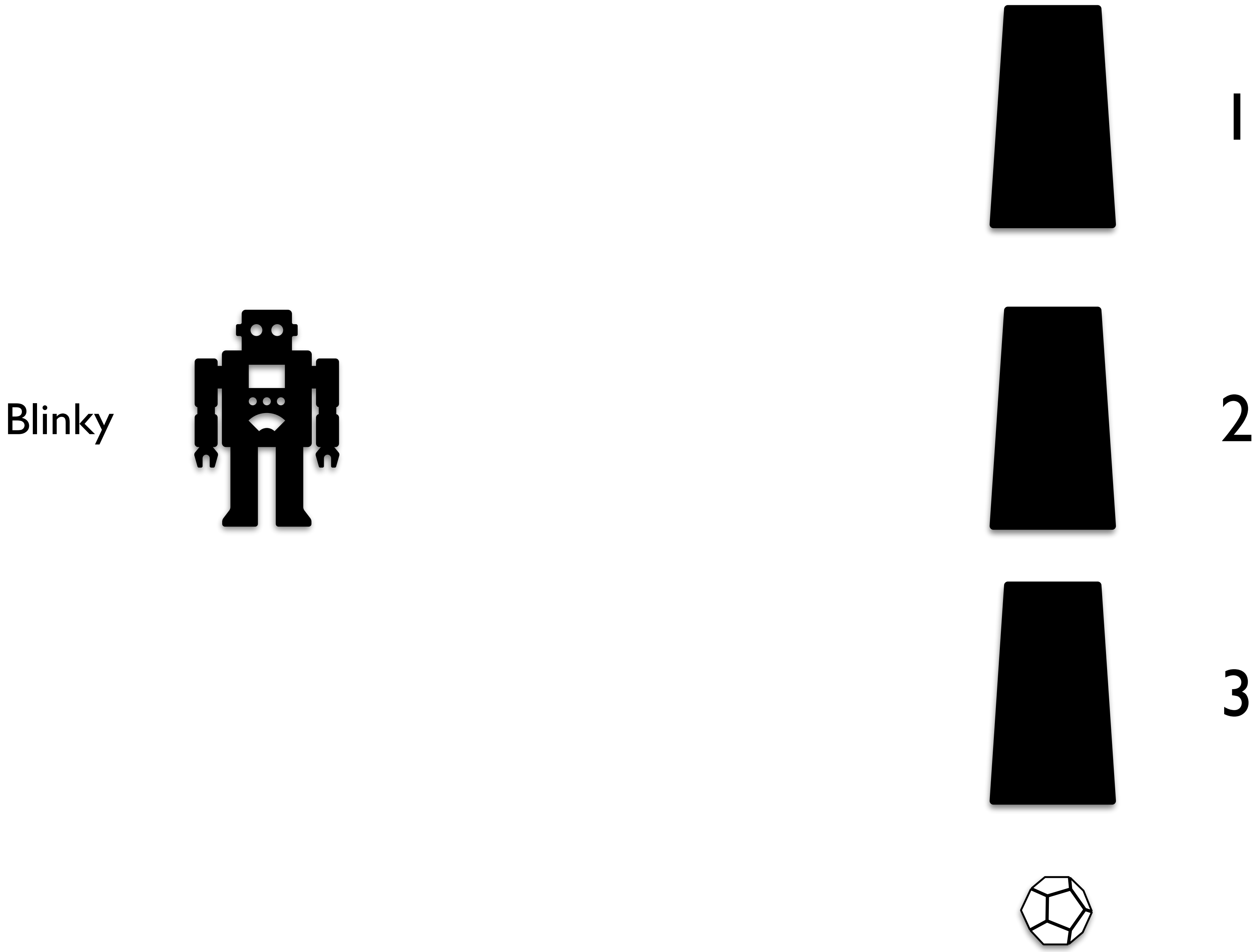
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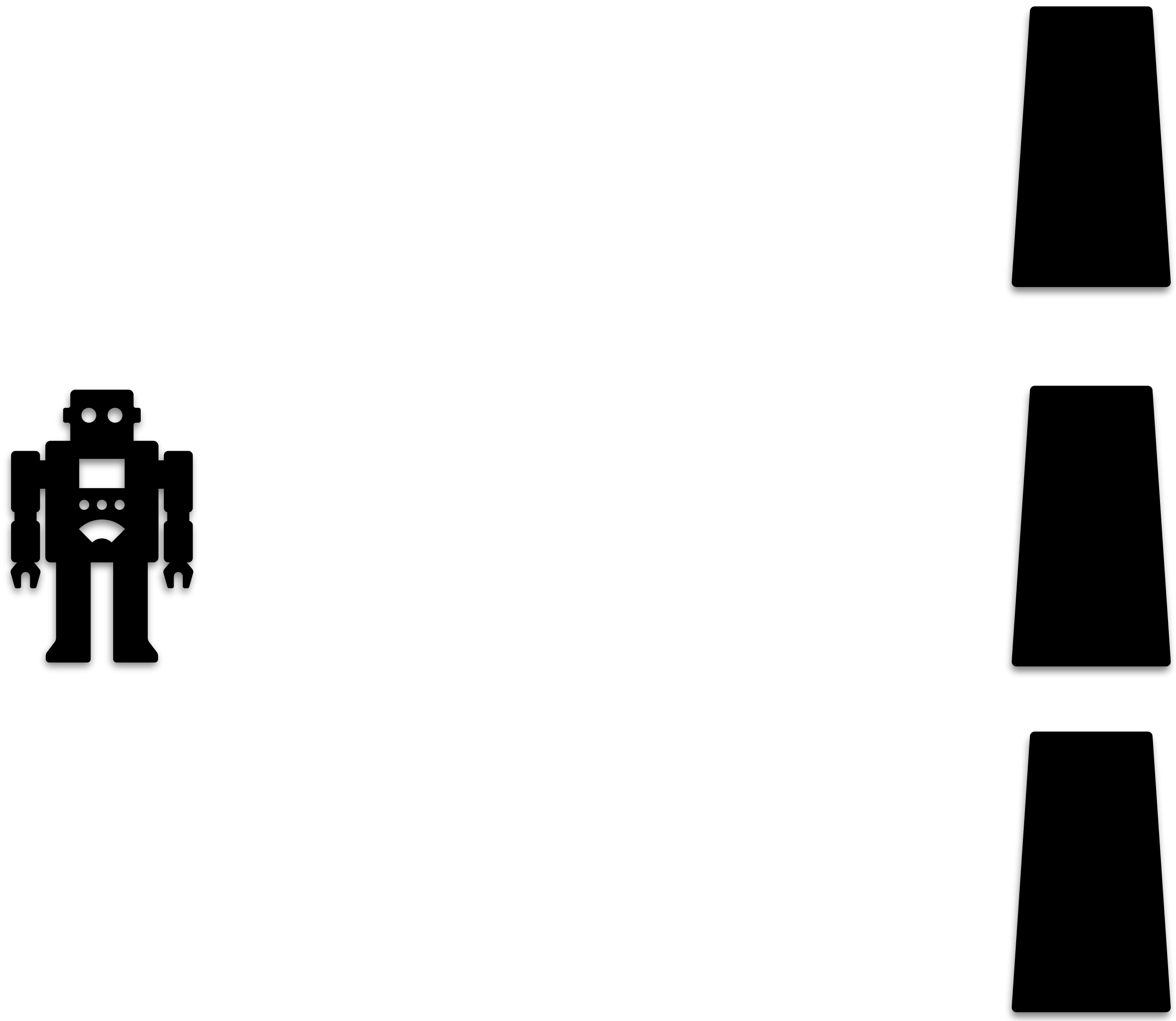
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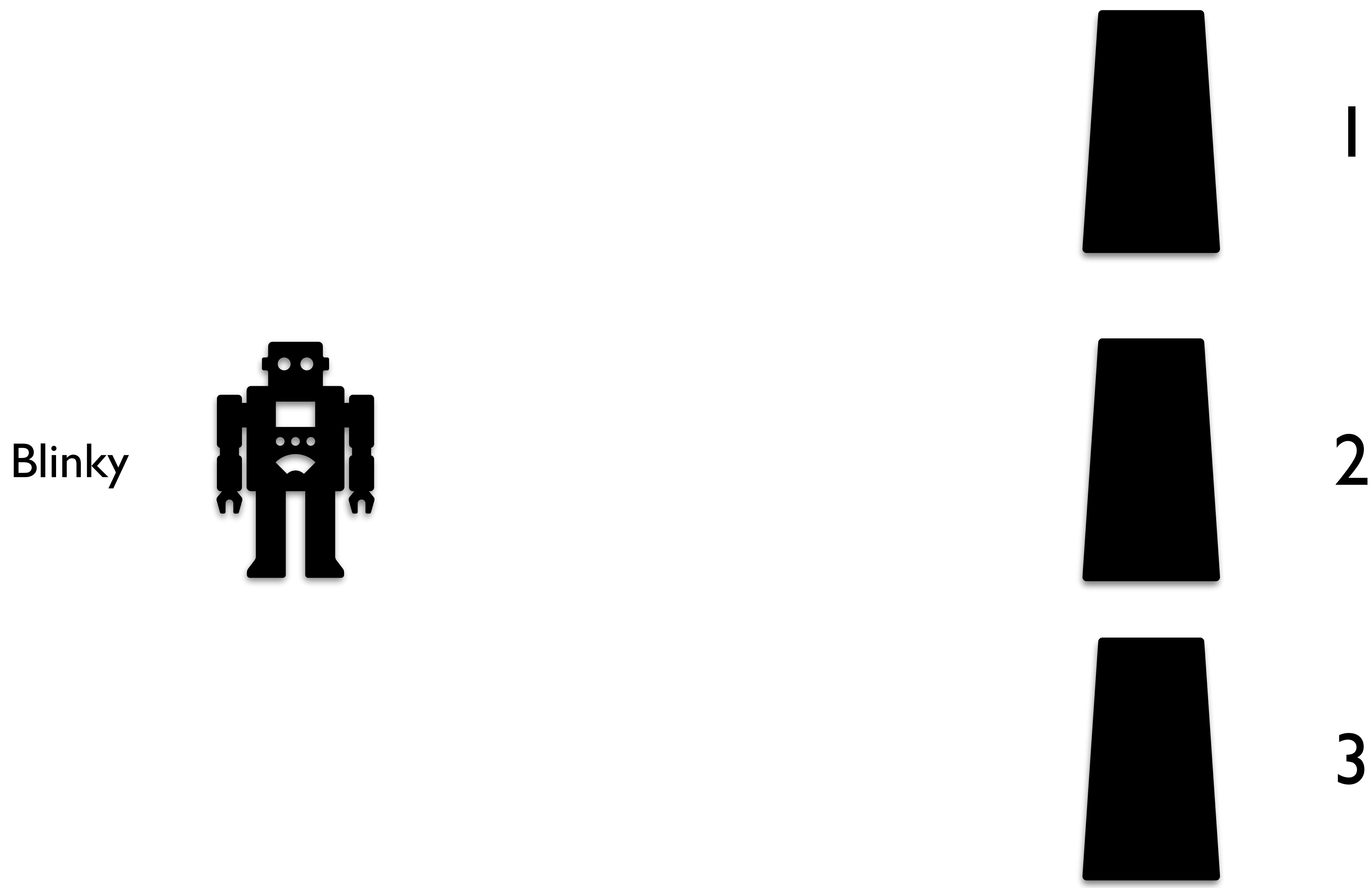
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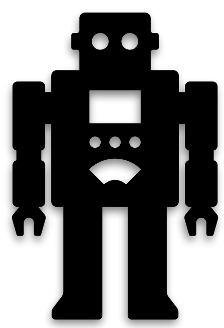

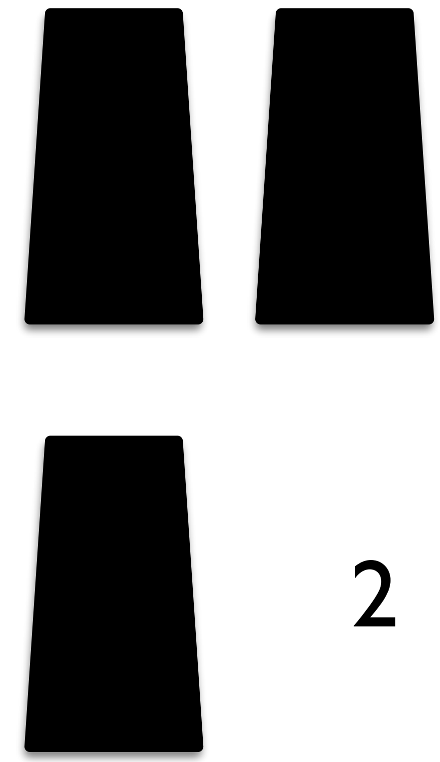

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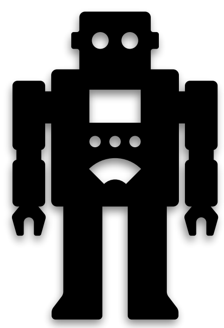

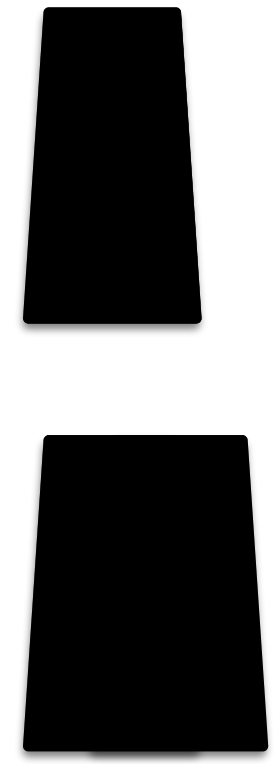

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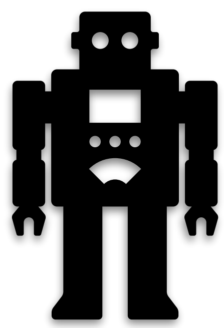

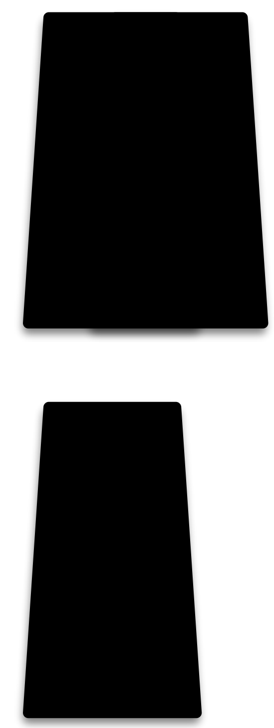

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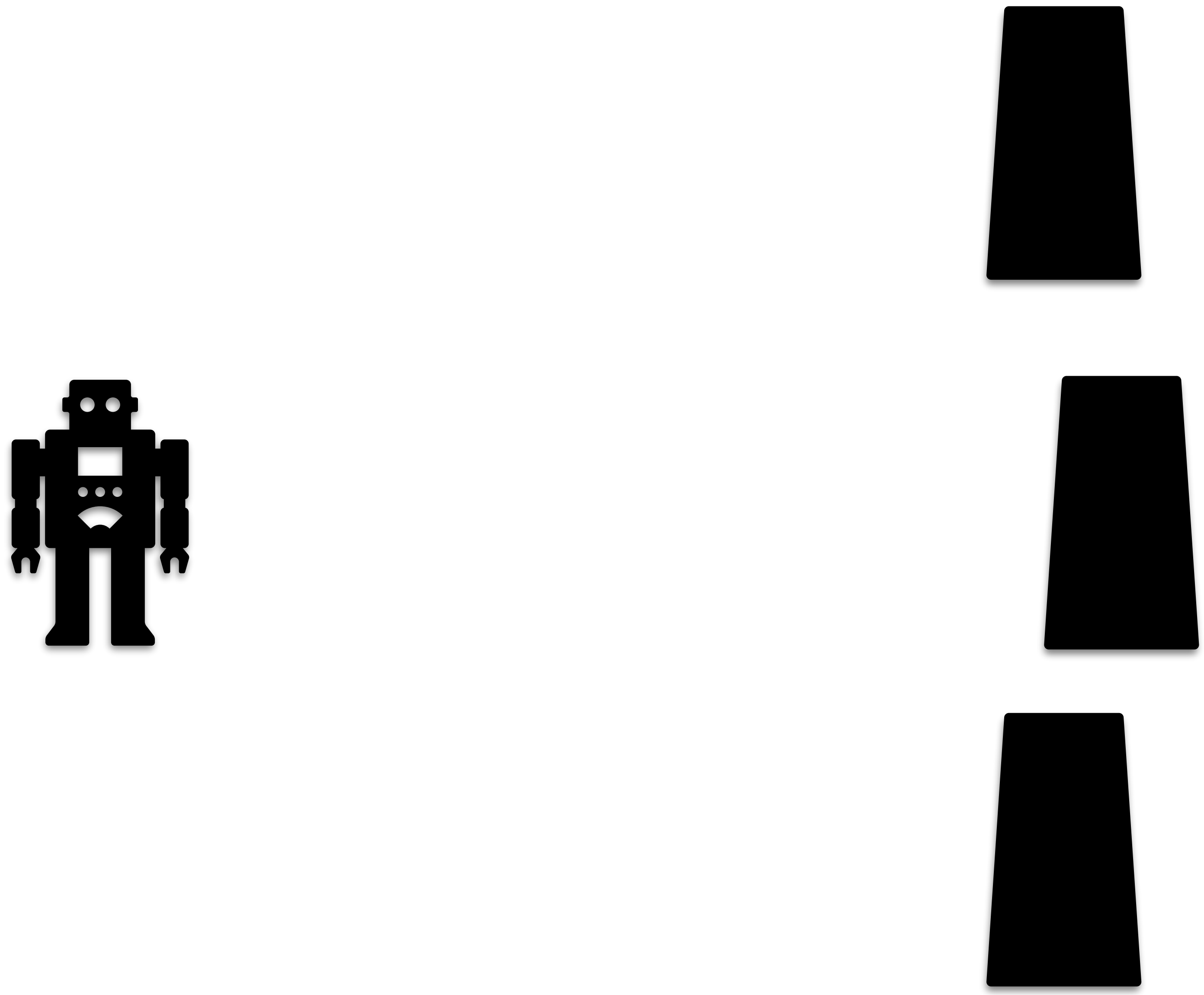

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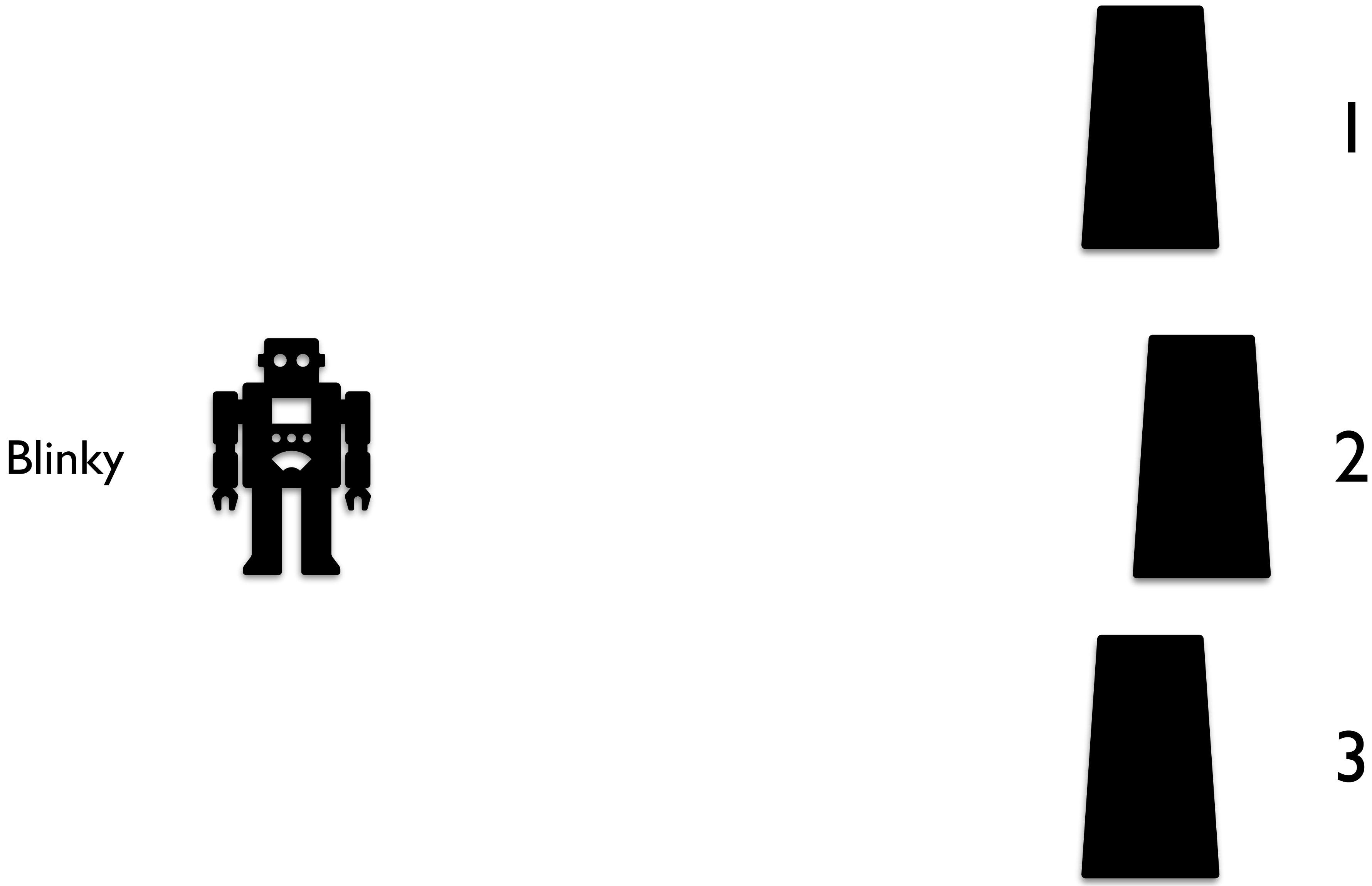
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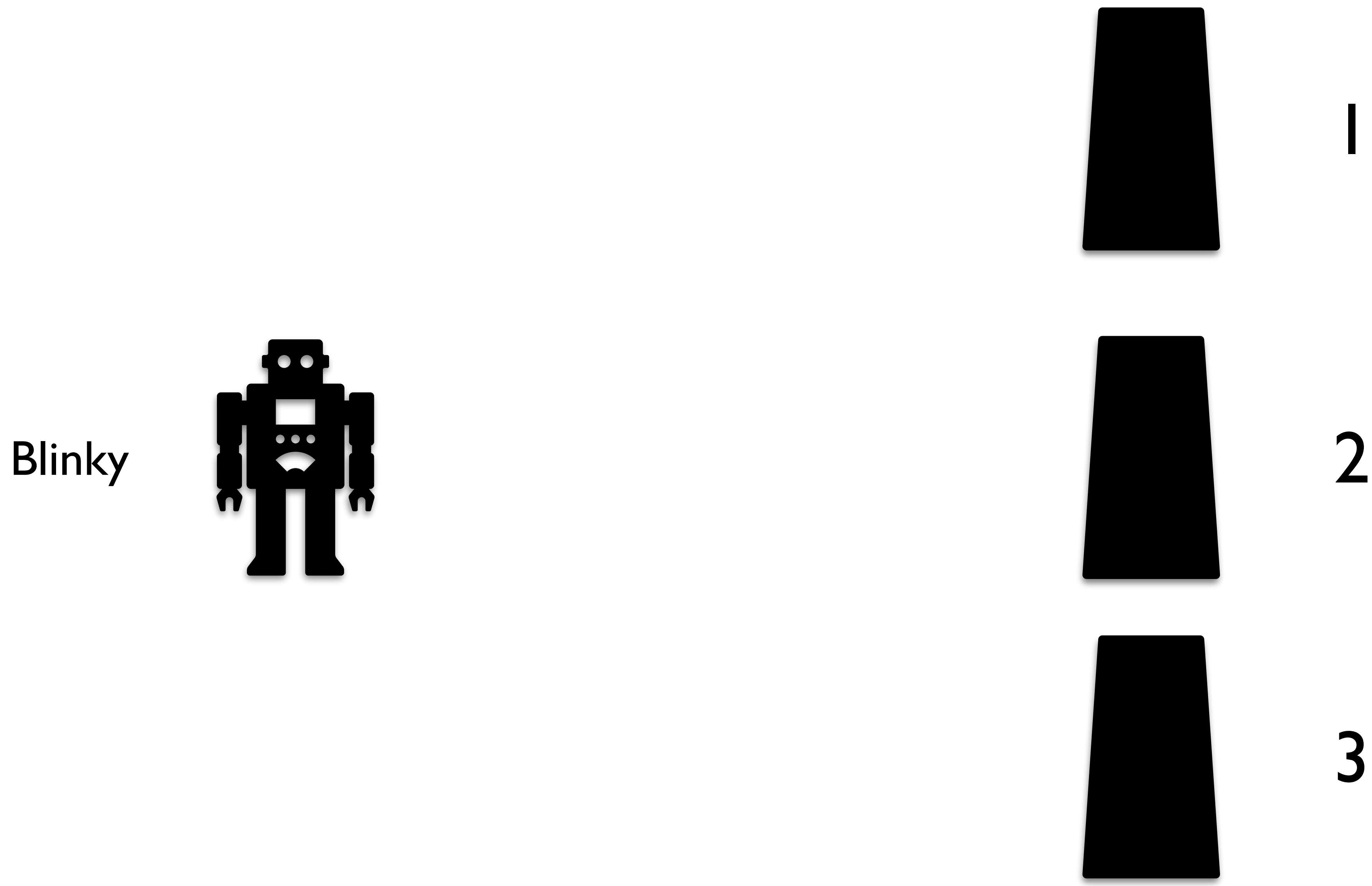
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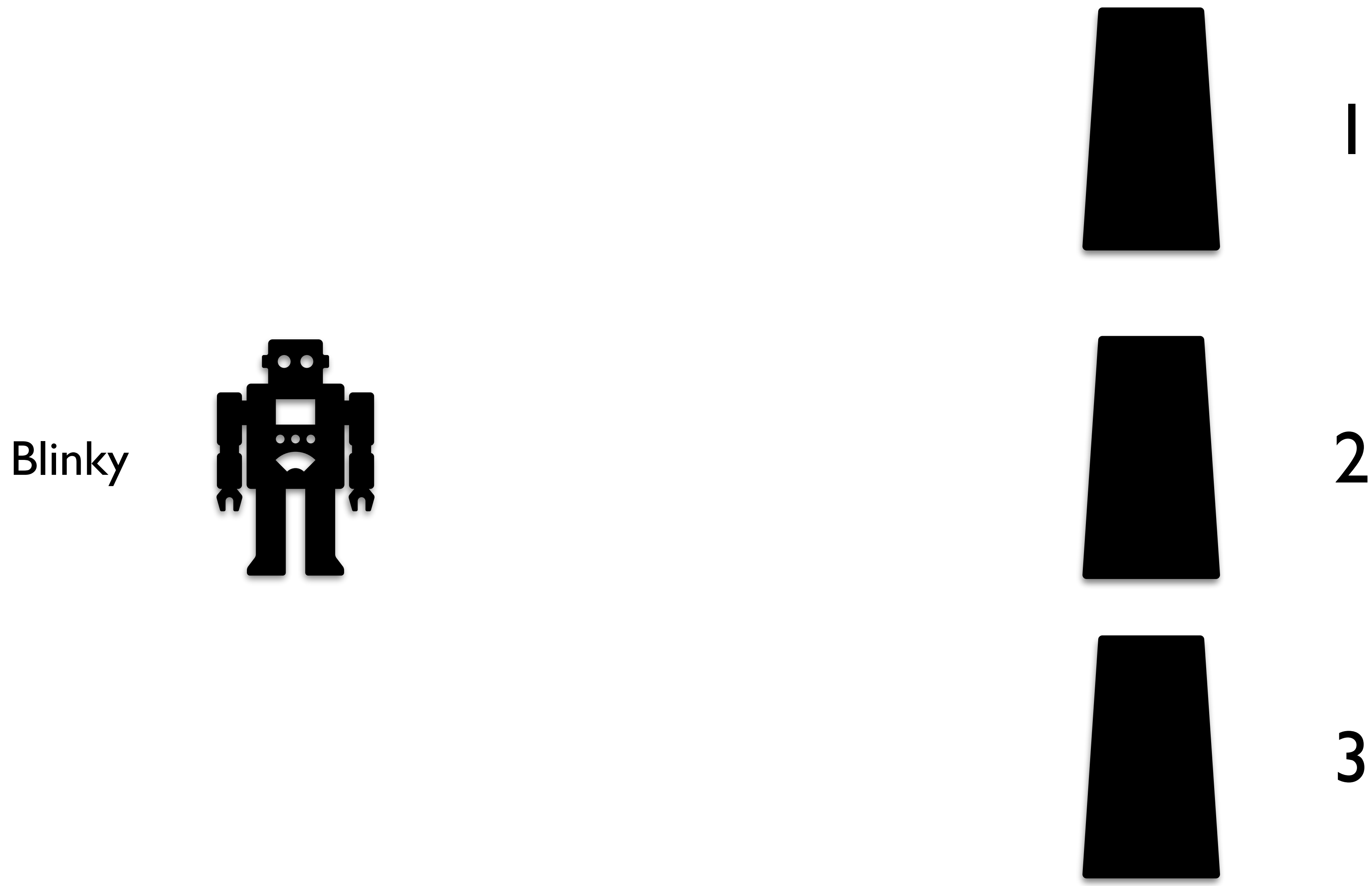
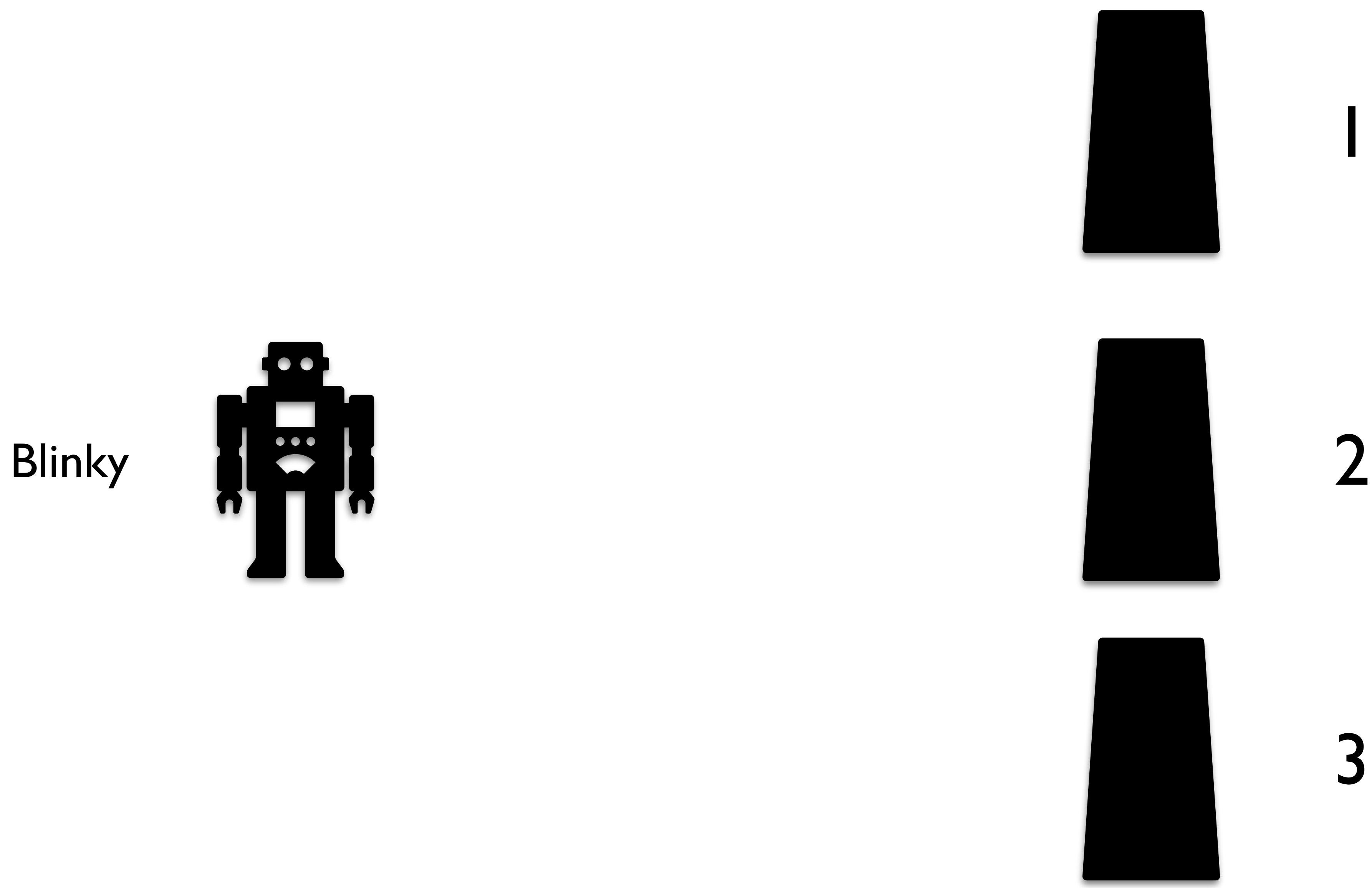
The ball is in the cup at location #1.



The ball is in the cup at location #1.

$$Loc(ball, 1)$$



The ball is in the cup at location #1.

Loc(ball,1)

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(Loc ball 1)
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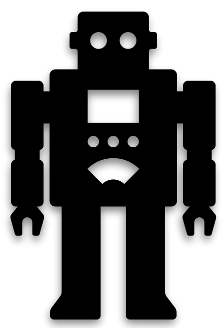

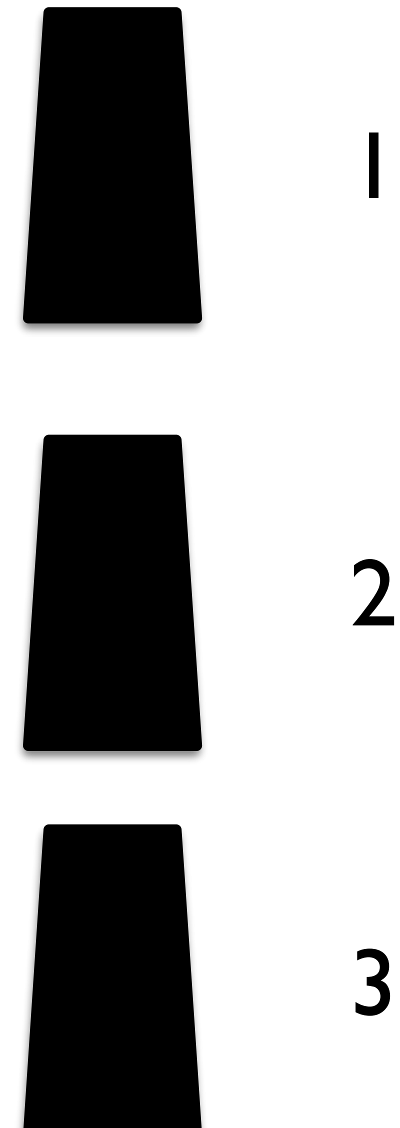

The ball is in the cup at location #1.

FALSE $\text{Loc}(\text{ball}, 1)$

```
(Loc ball 1)
```

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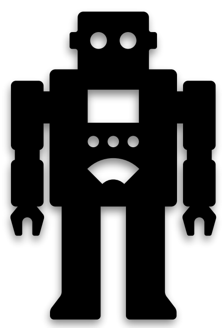

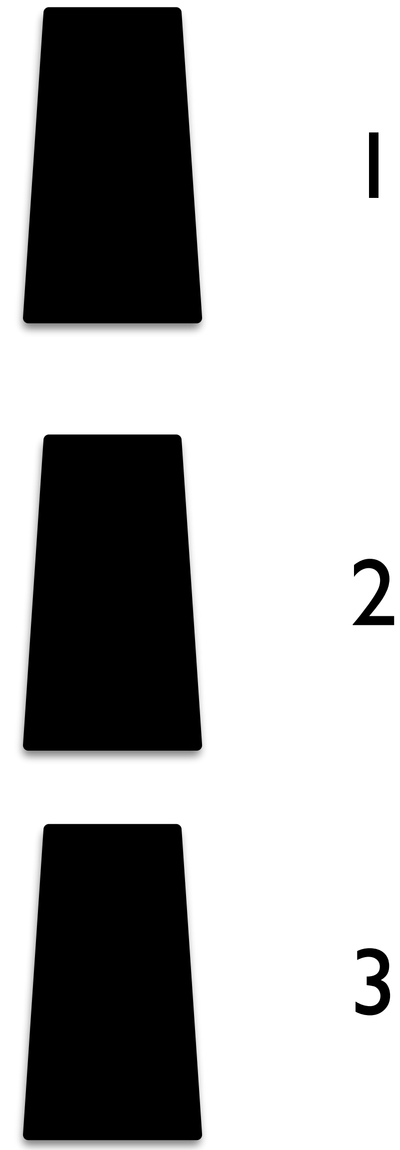

FALSE Loc(ball, 1)

`(Loc ball 1)`

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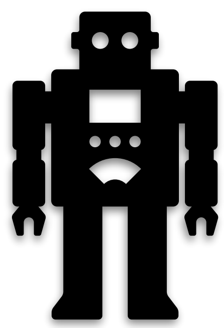

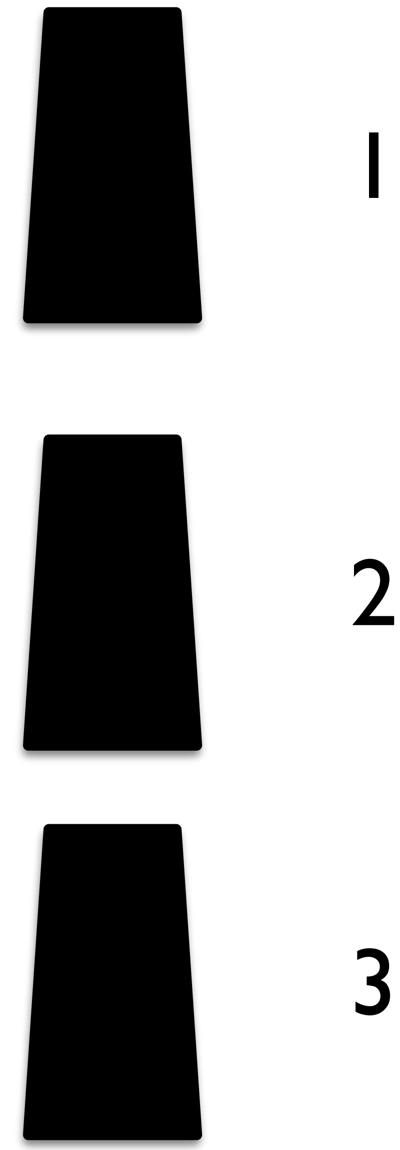

FALSE

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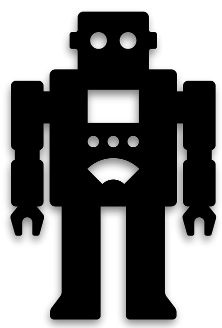

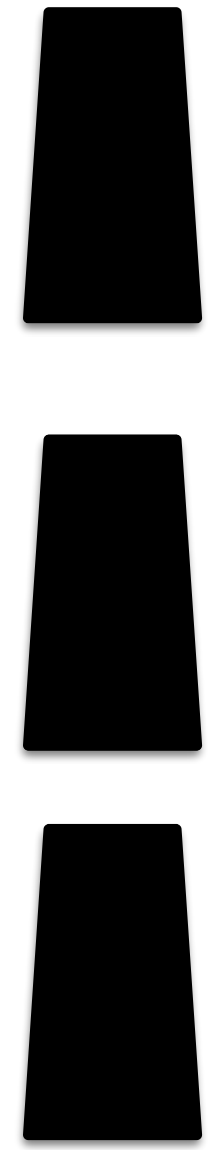

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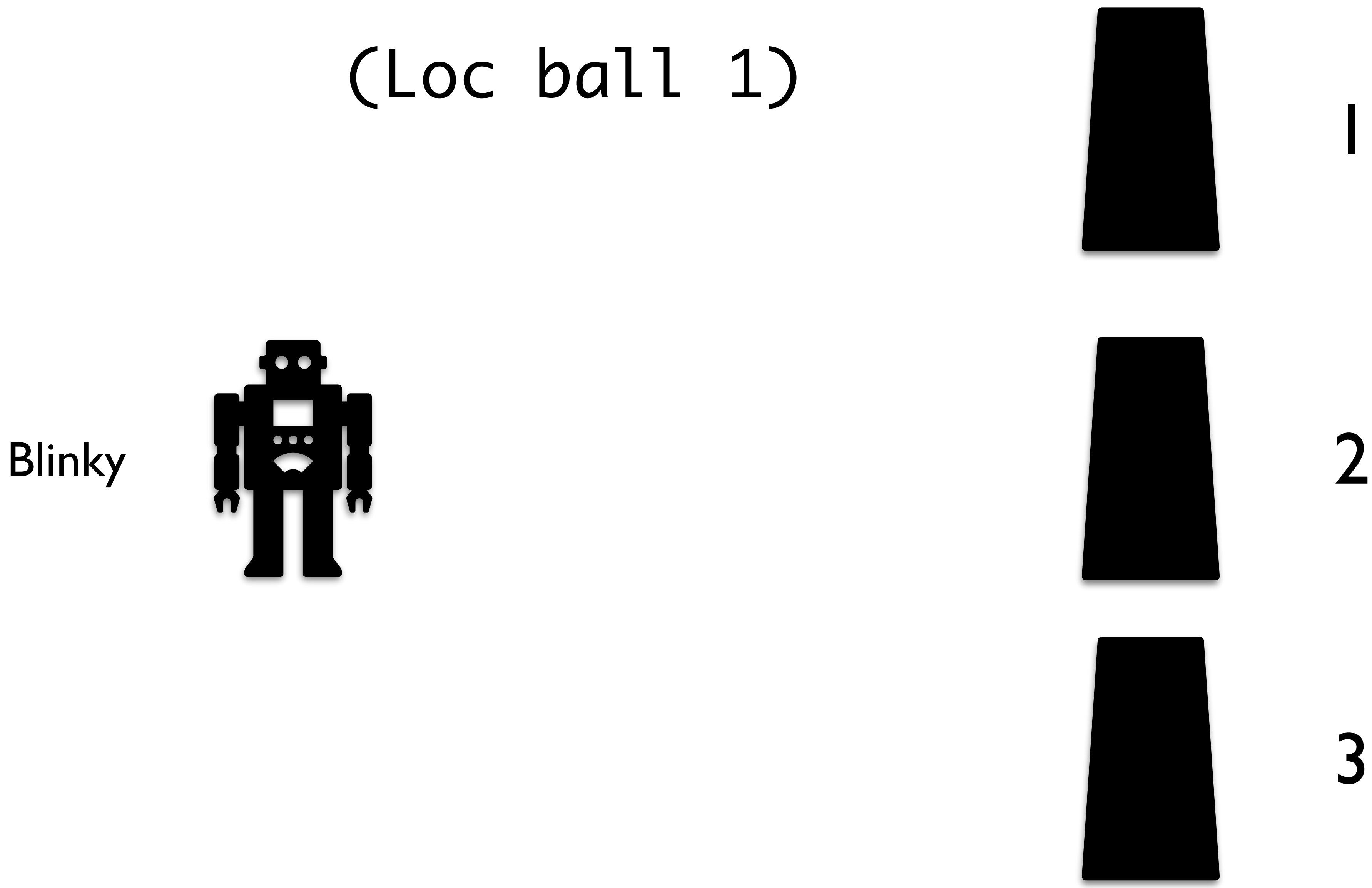

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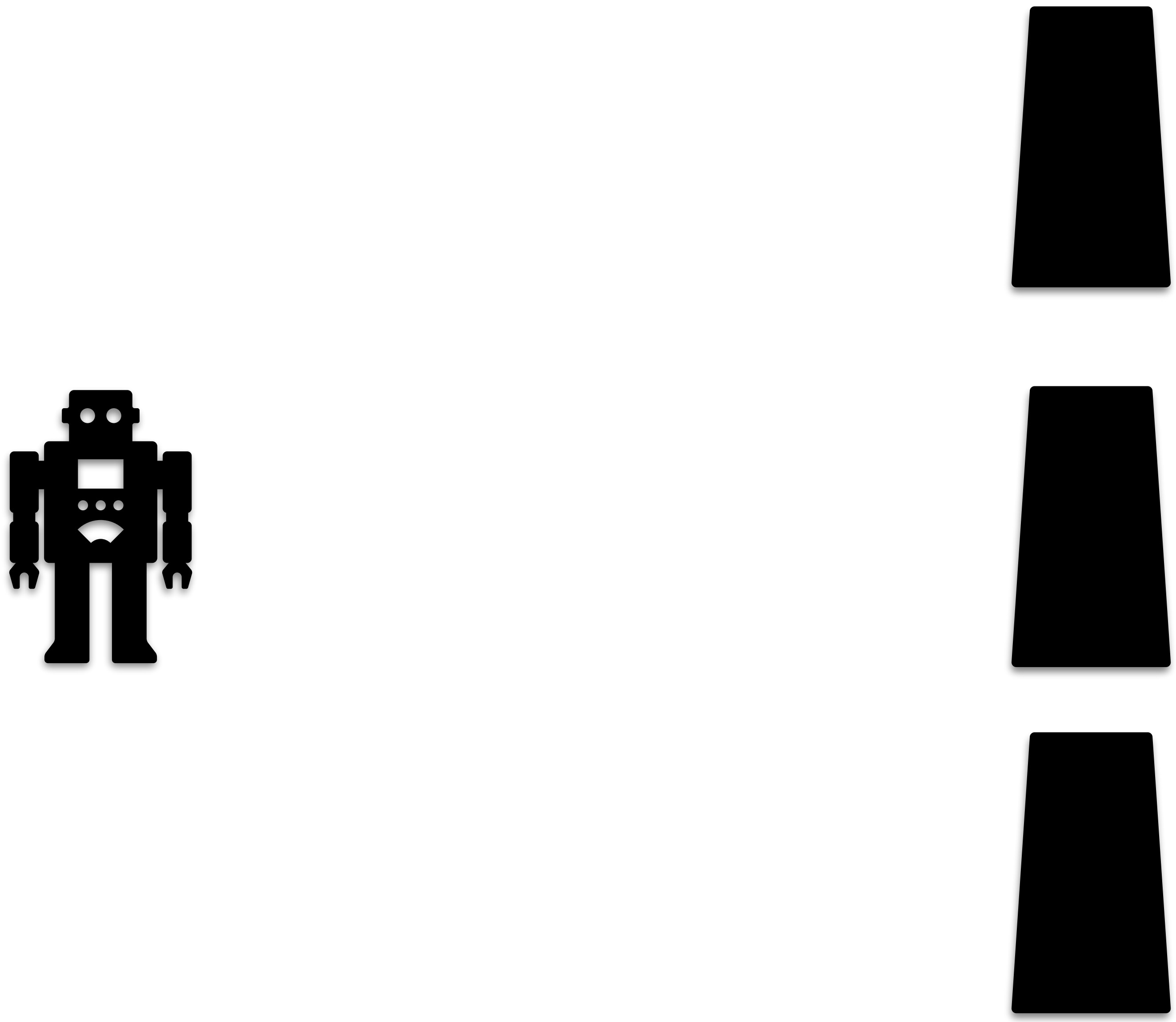

The ball is in the cup at location #1 or the ball is at location #3.

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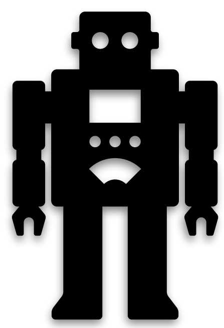
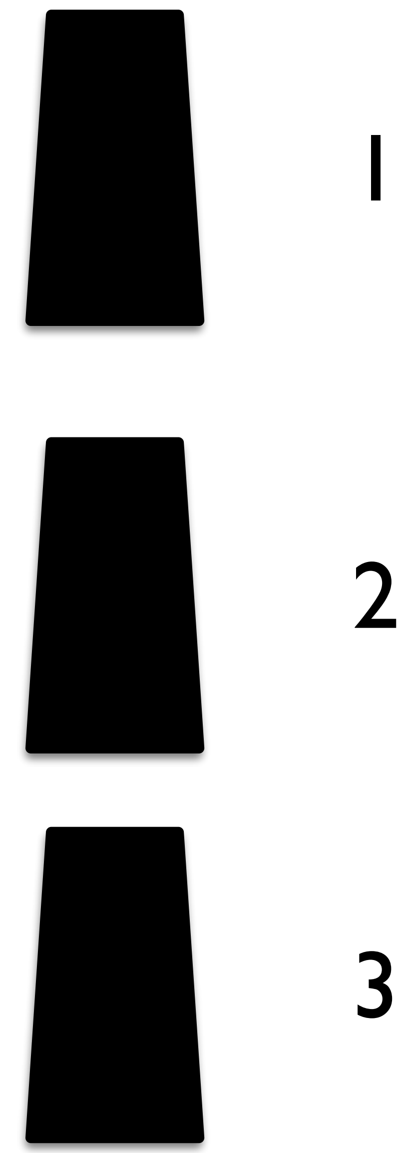
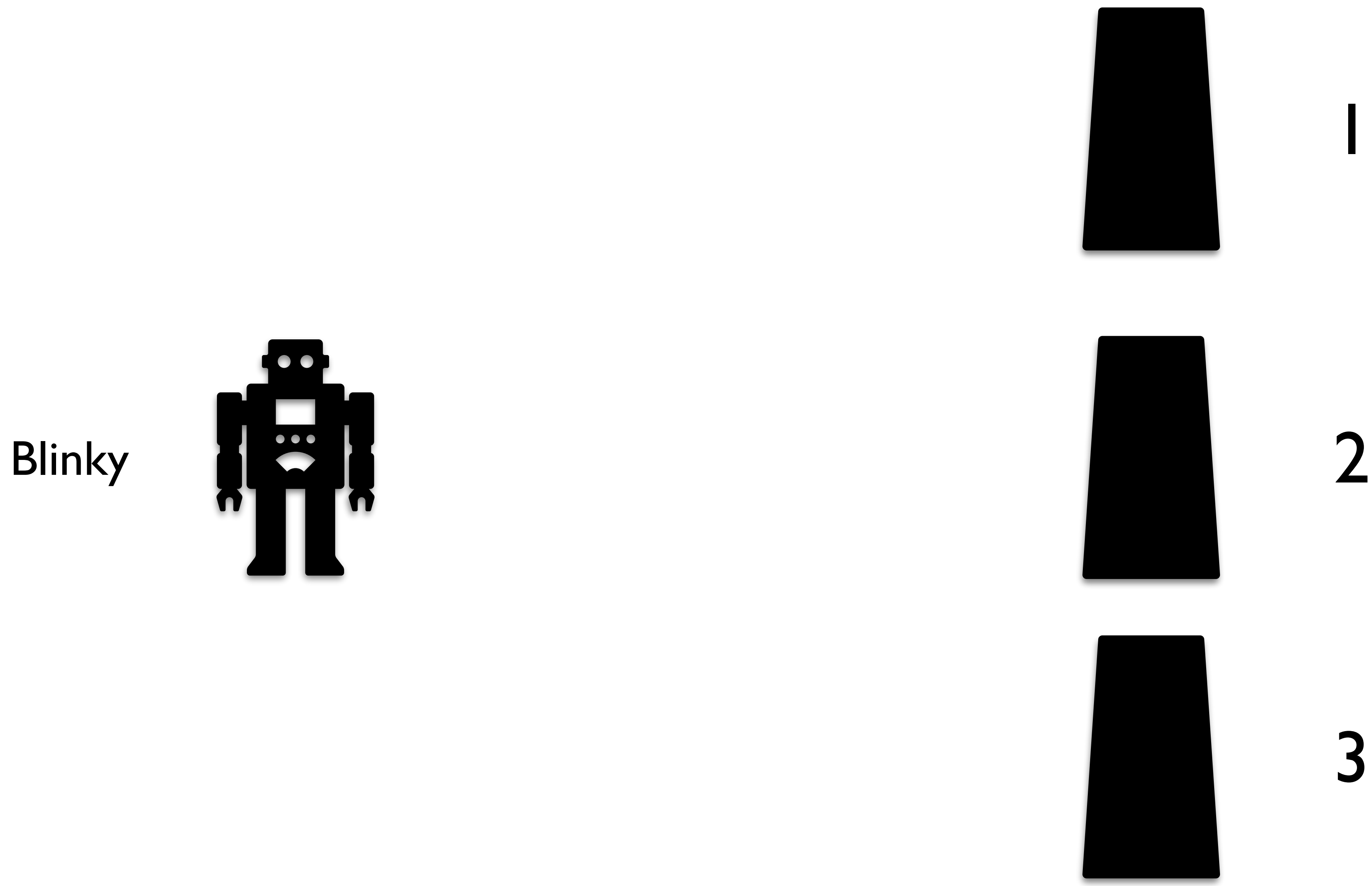




The ball is in the cup at location #1 or the ball is at location #3.

$$Loc(ball,1) \vee Loc(ball,3)$$



The ball is in the cup at location #1 or the ball is at location #3.

$$\text{Loc}(\text{ball}, 1) \vee \text{Loc}(\text{ball}, 3)$$

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(or (Loc ball 1) (Loc ball 3))
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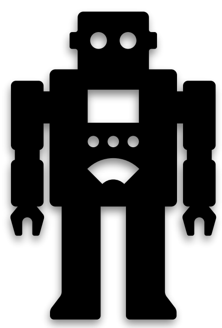
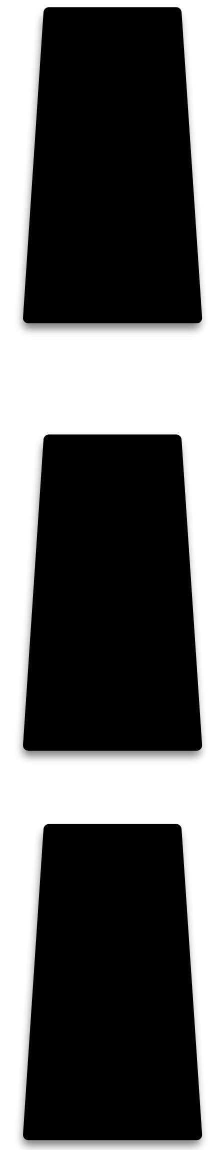
Blinky





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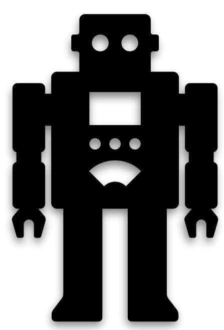

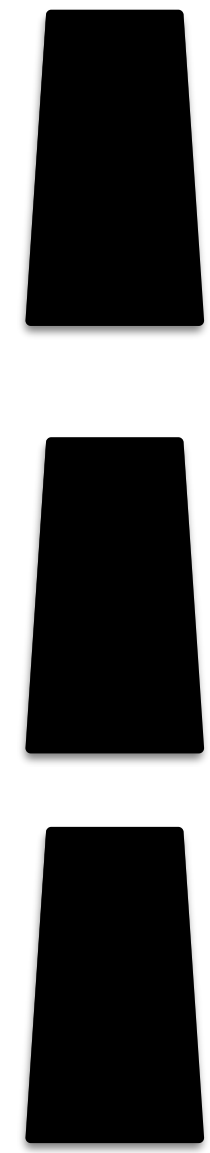

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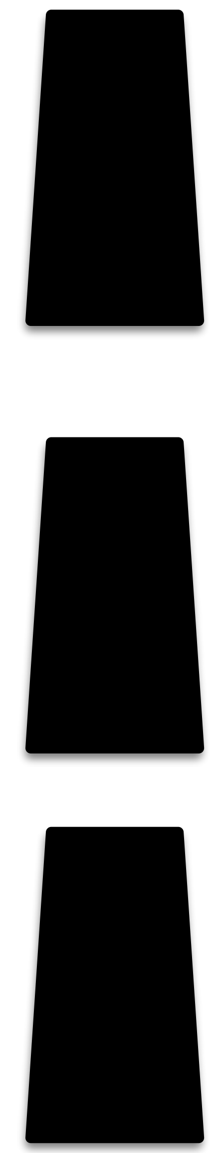

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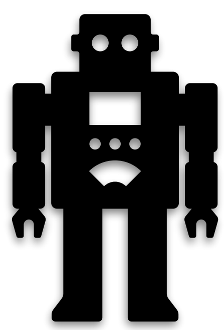

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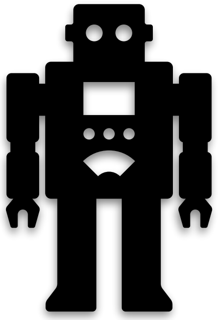

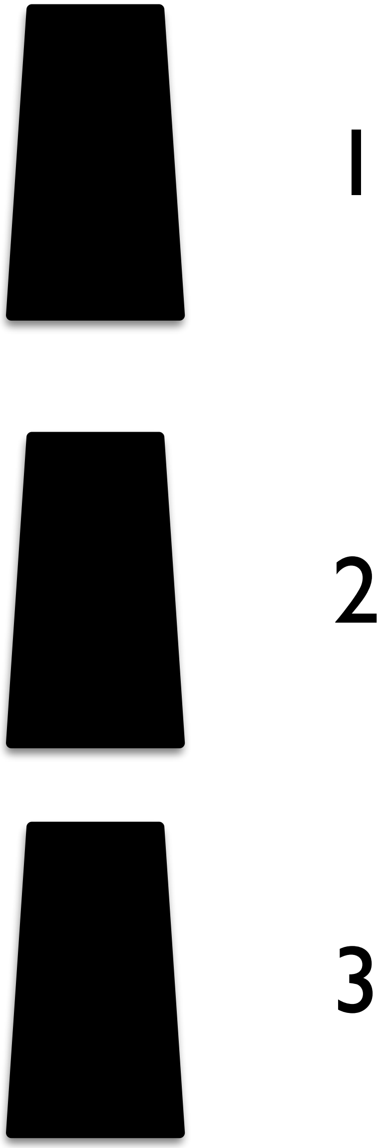

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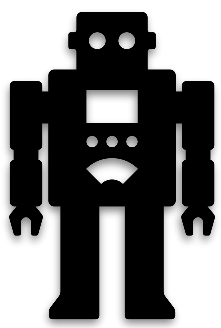

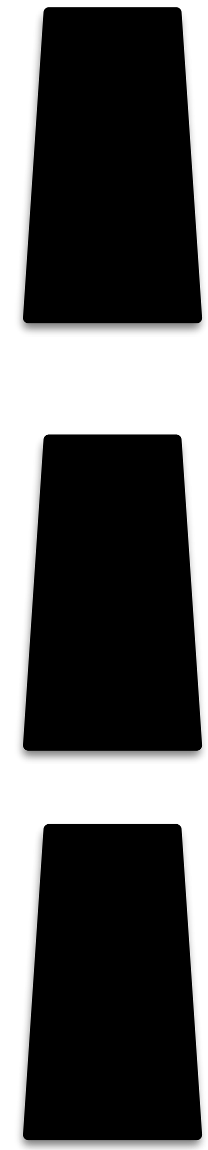

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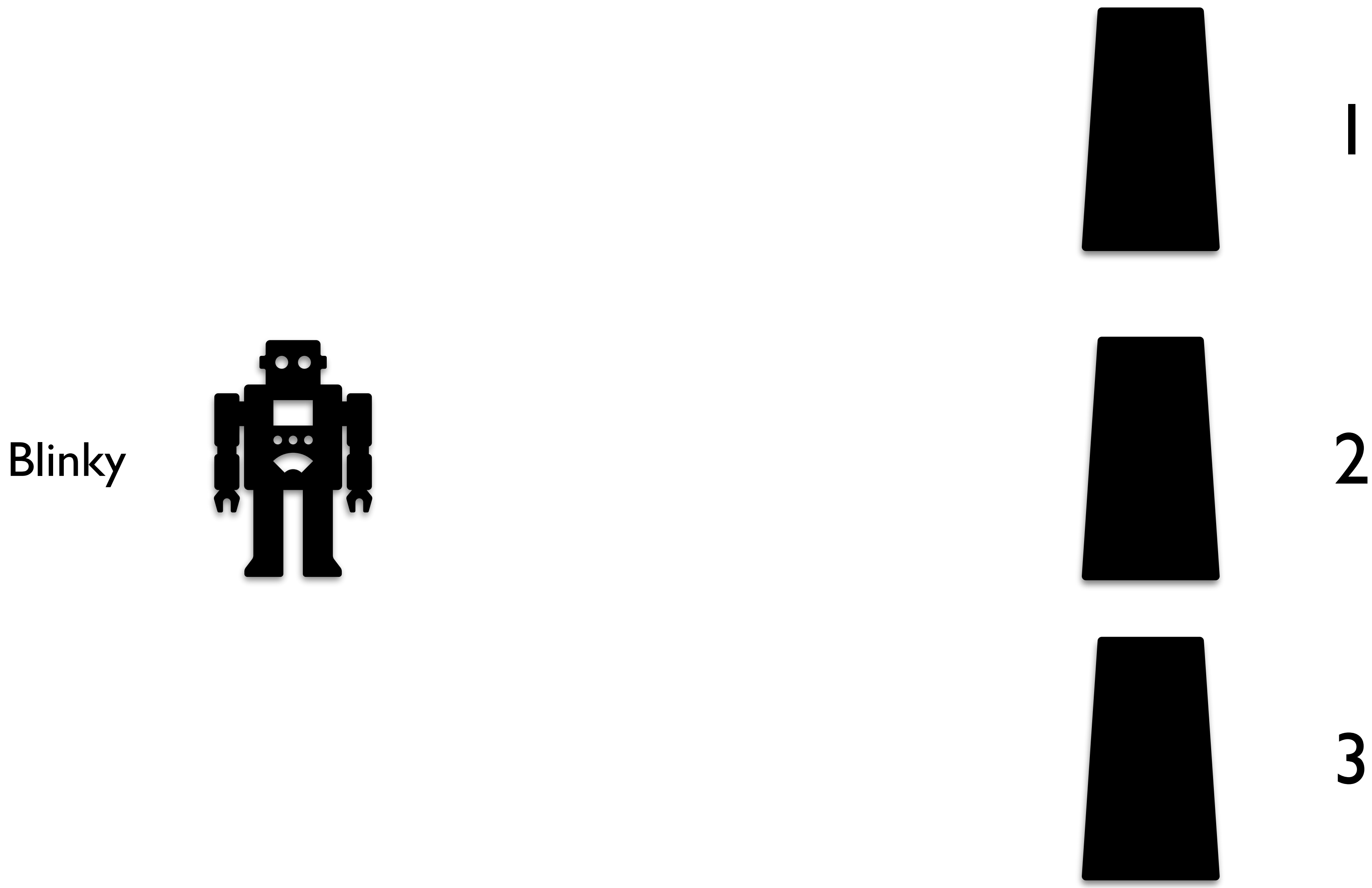
Blinky
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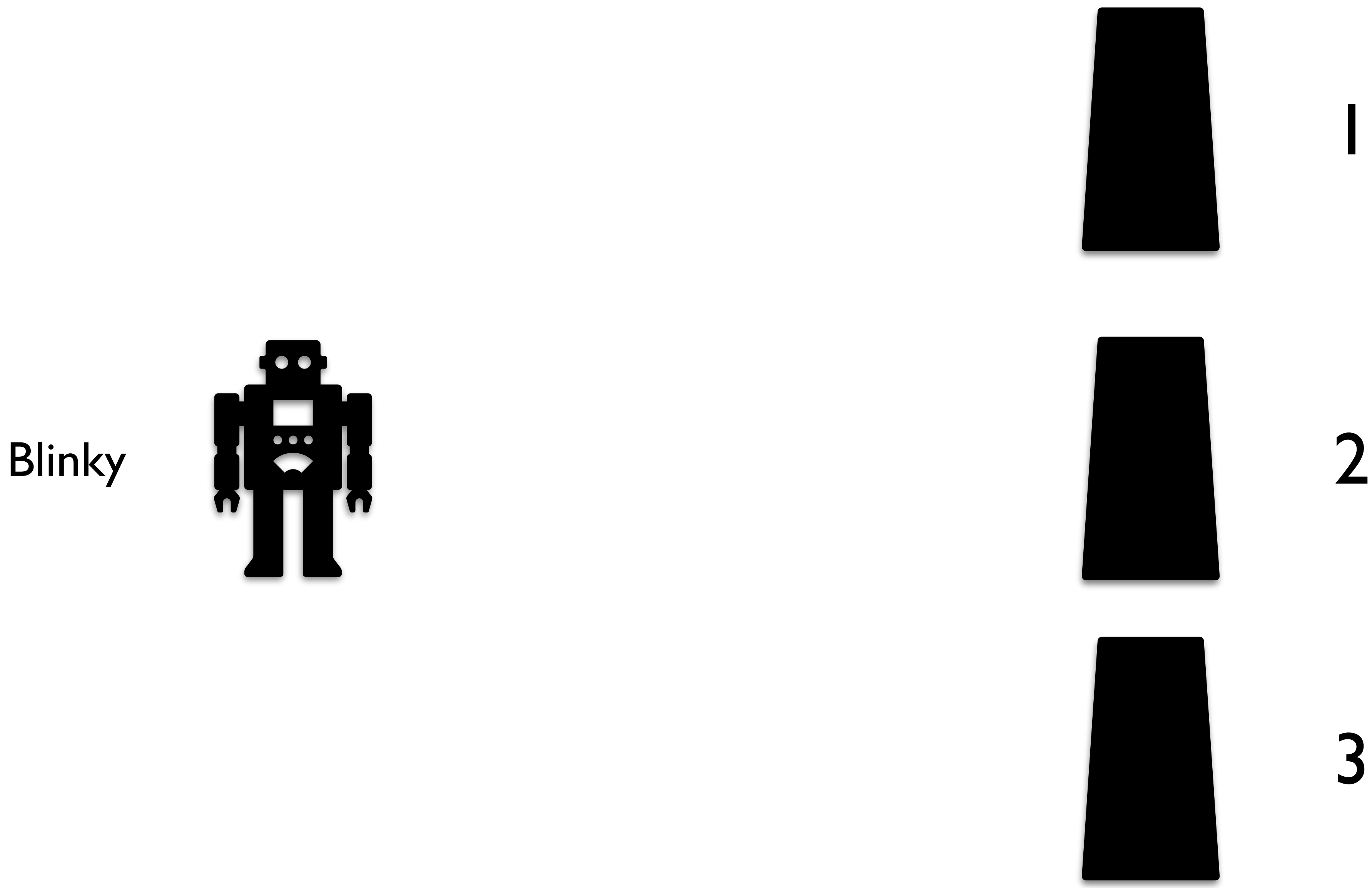
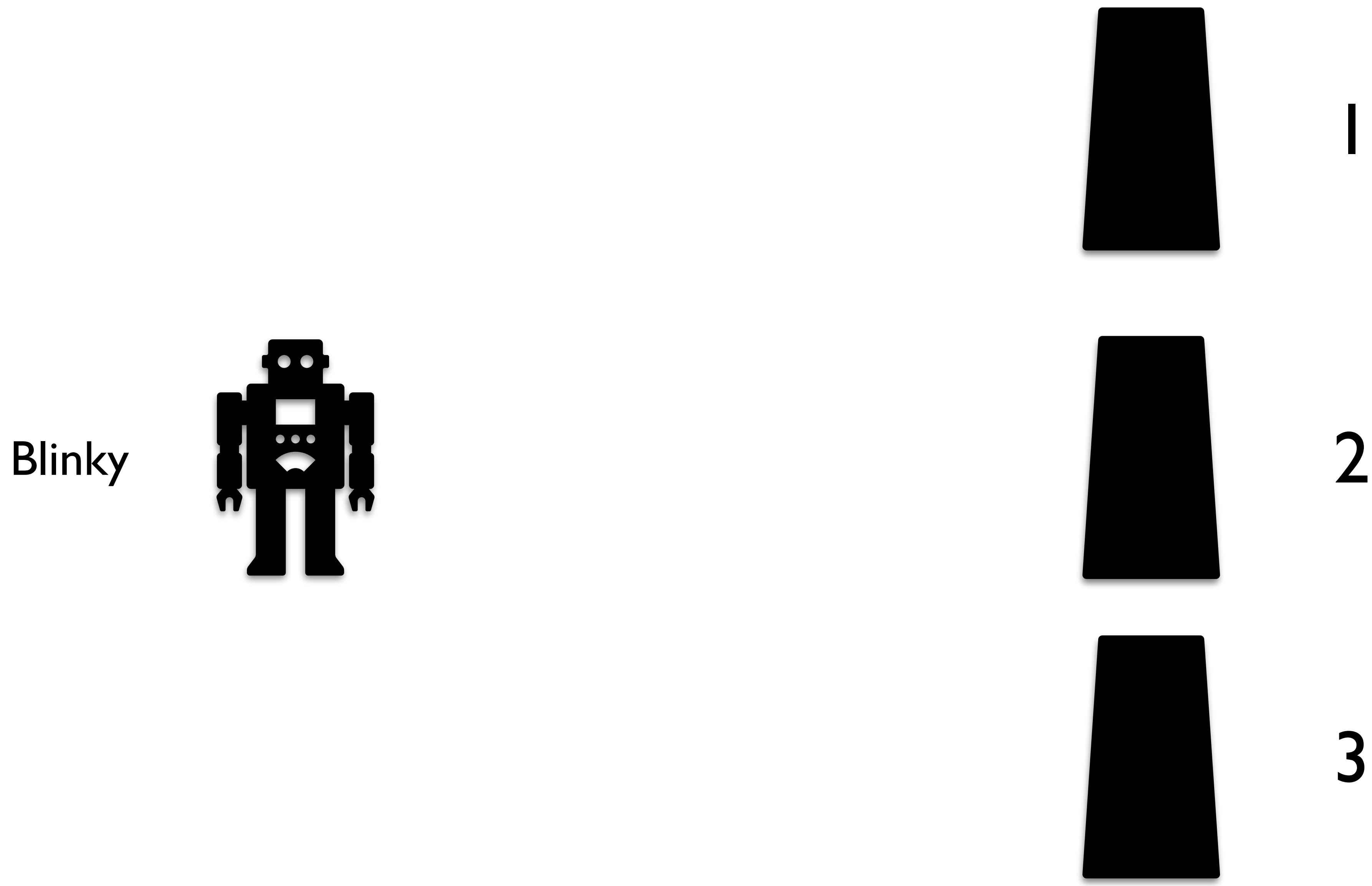
Blinky believes that the ball is in the cup at location #1.



Blinky believes that the ball is in the cup at location #1.

$$B(blinky, Loc(ball, 1))$$



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```
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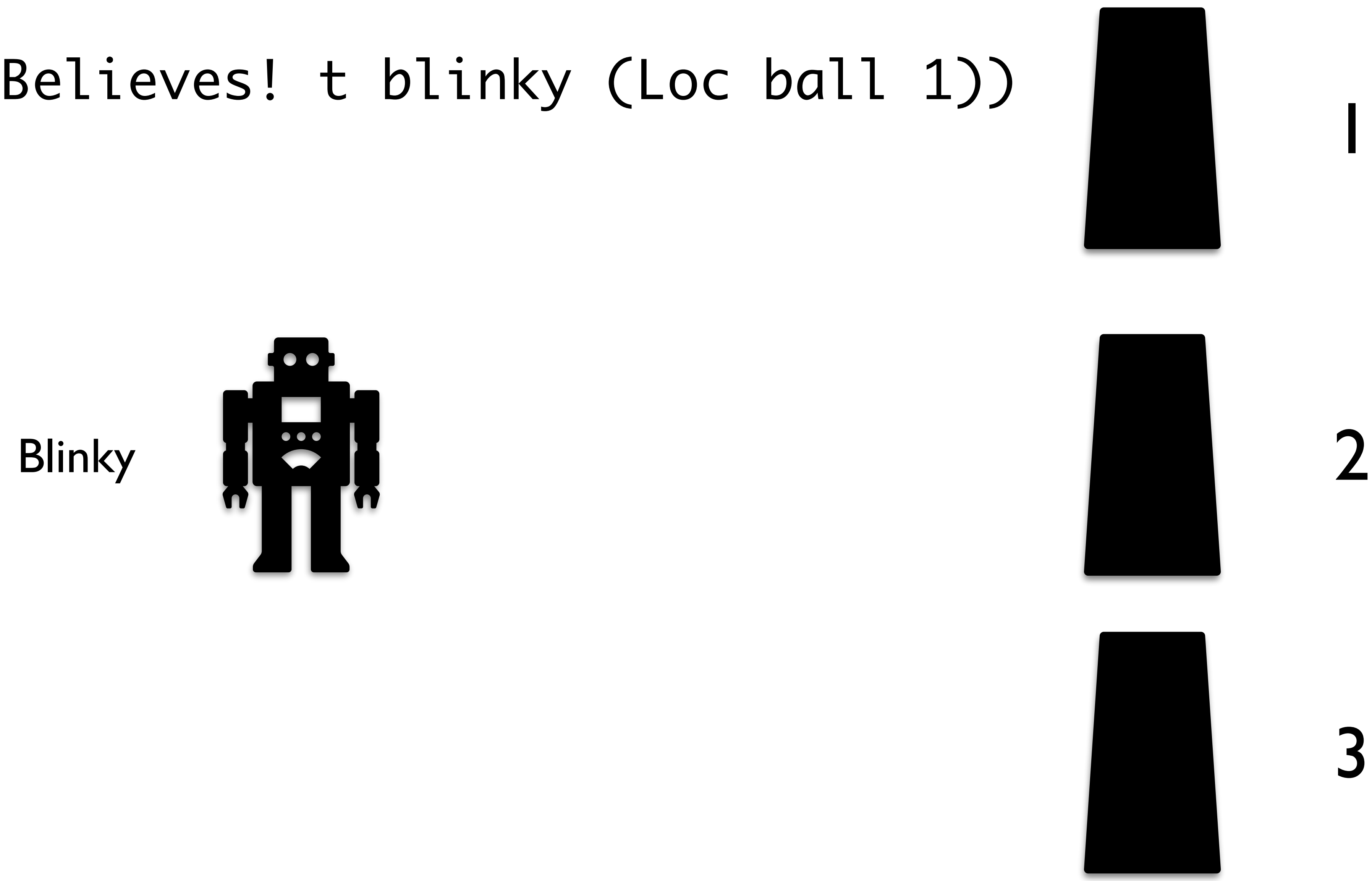

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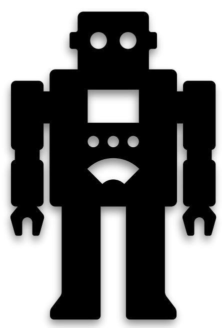

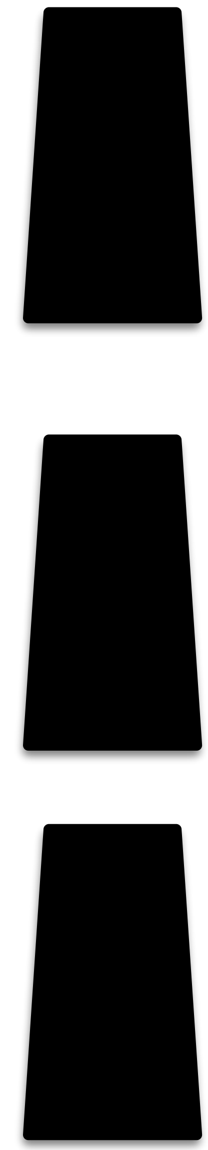

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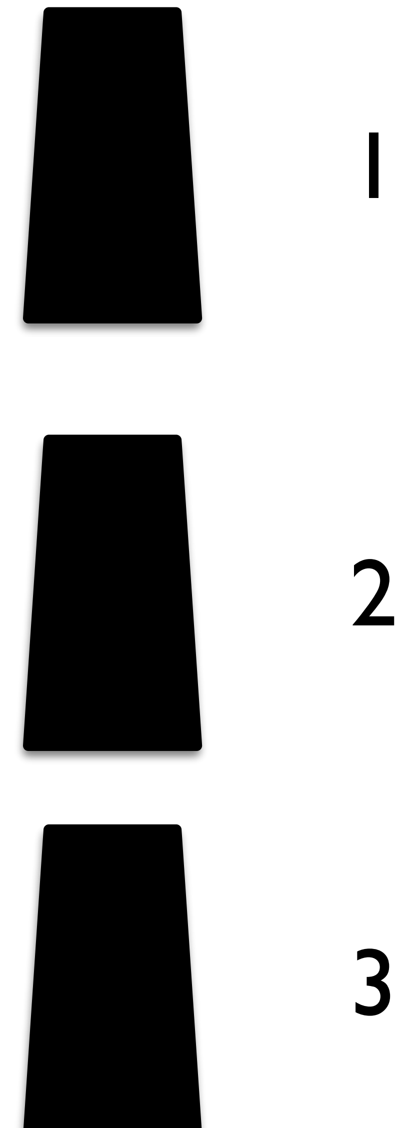

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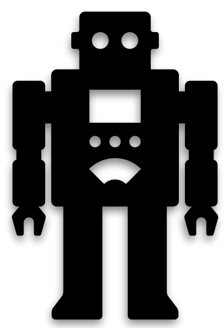

In extensional logics, what is denoted is conflated with meaning (the latter being naïvely compositional), and intensional attitudes like *believes*, *knows*, *hopes*, *fears*, etc cannot be represented and reasoned over smoothly (e.g. without fear of inconsistency rising up).

Review: Encapsulation

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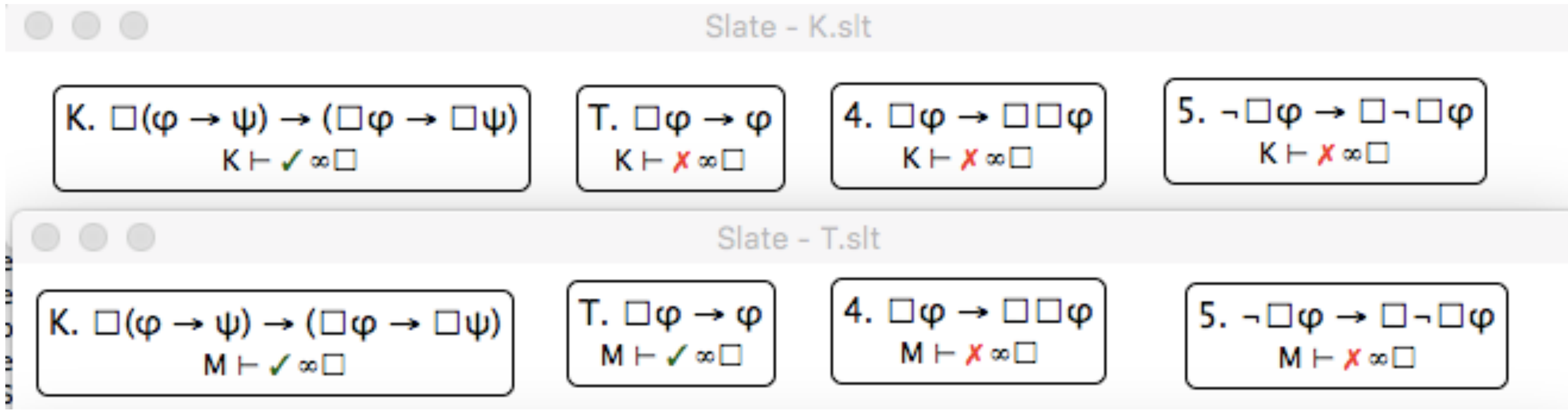

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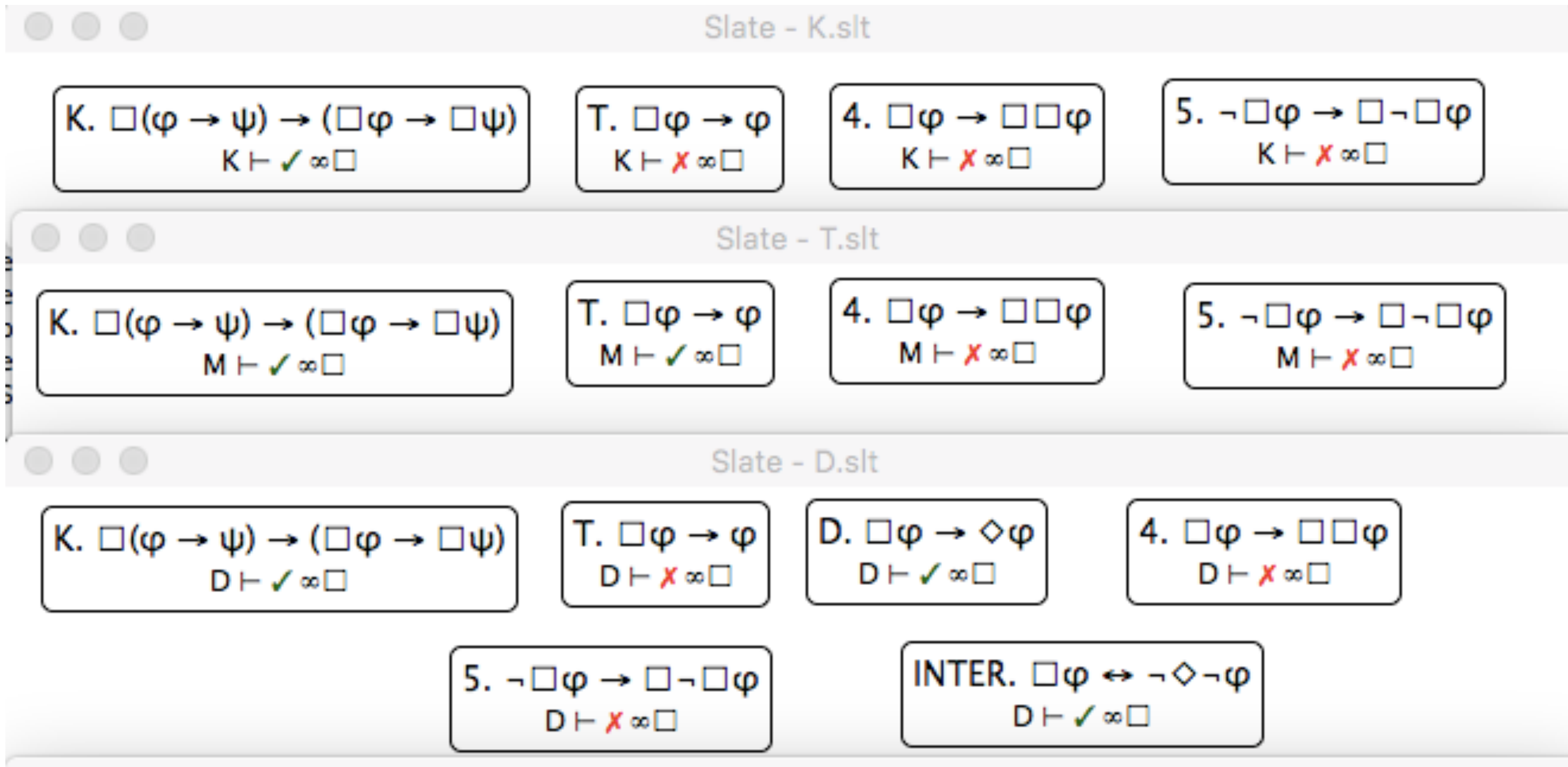

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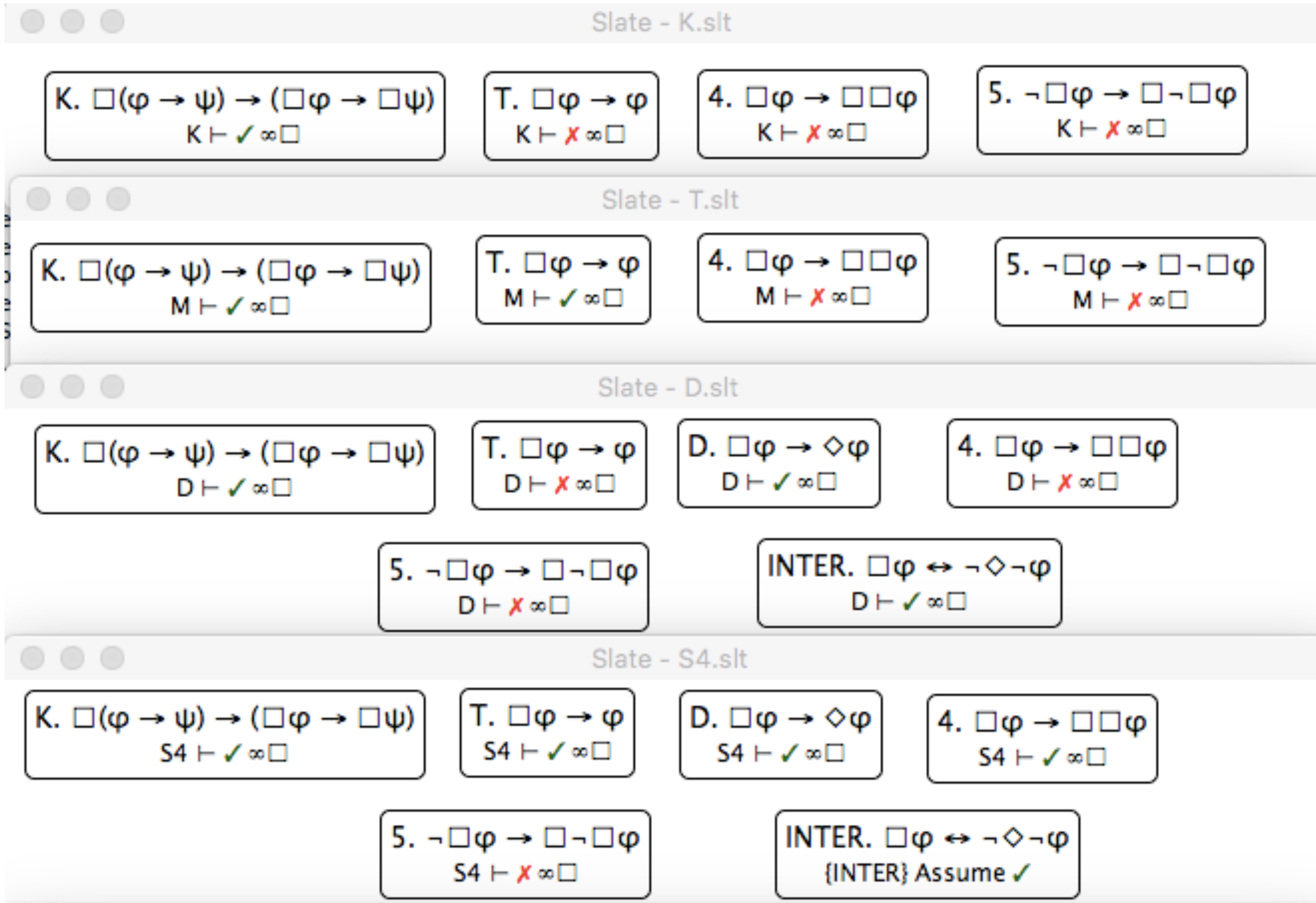

Review: Encapsulation

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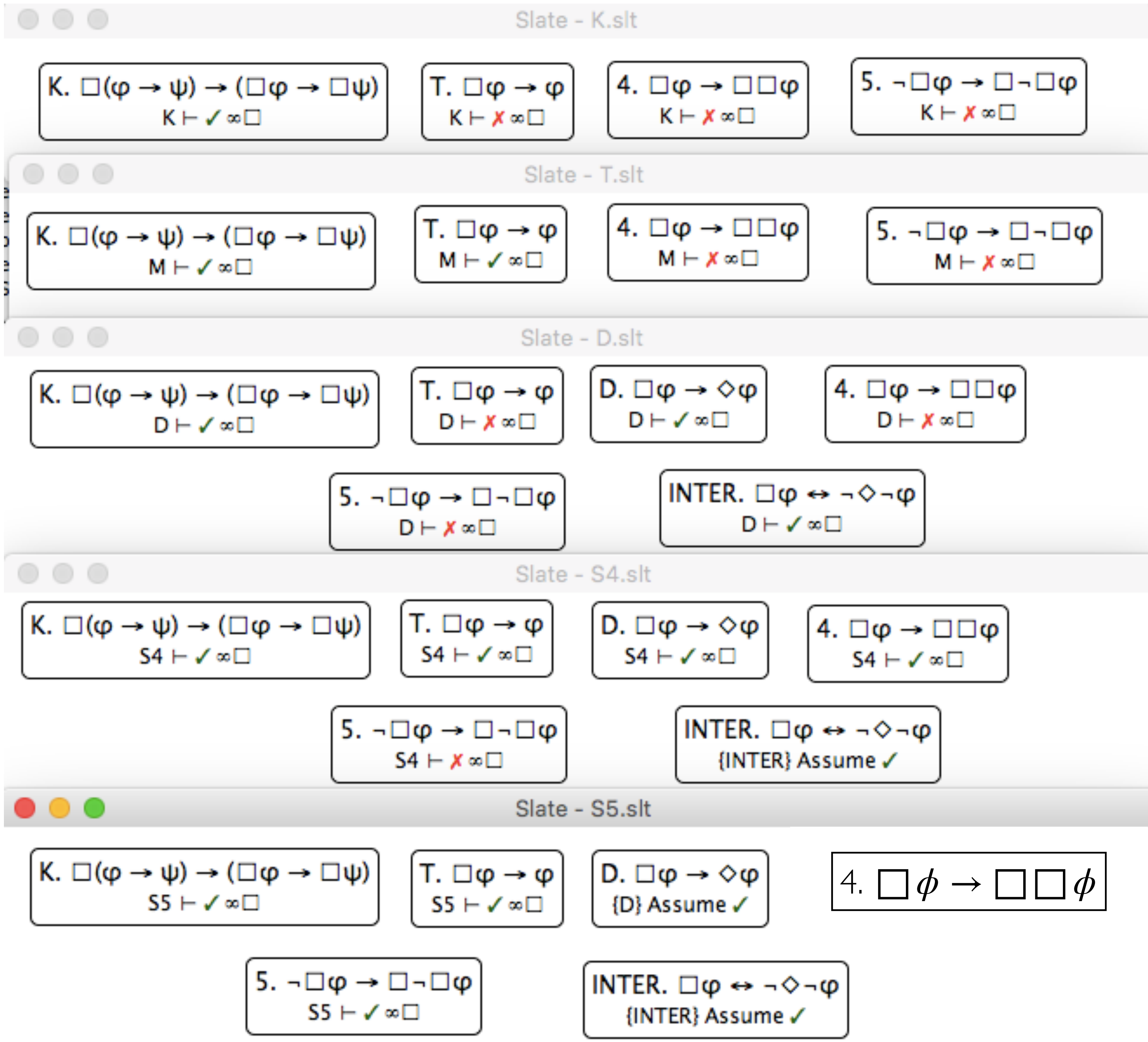
4 = S4

5 = S5



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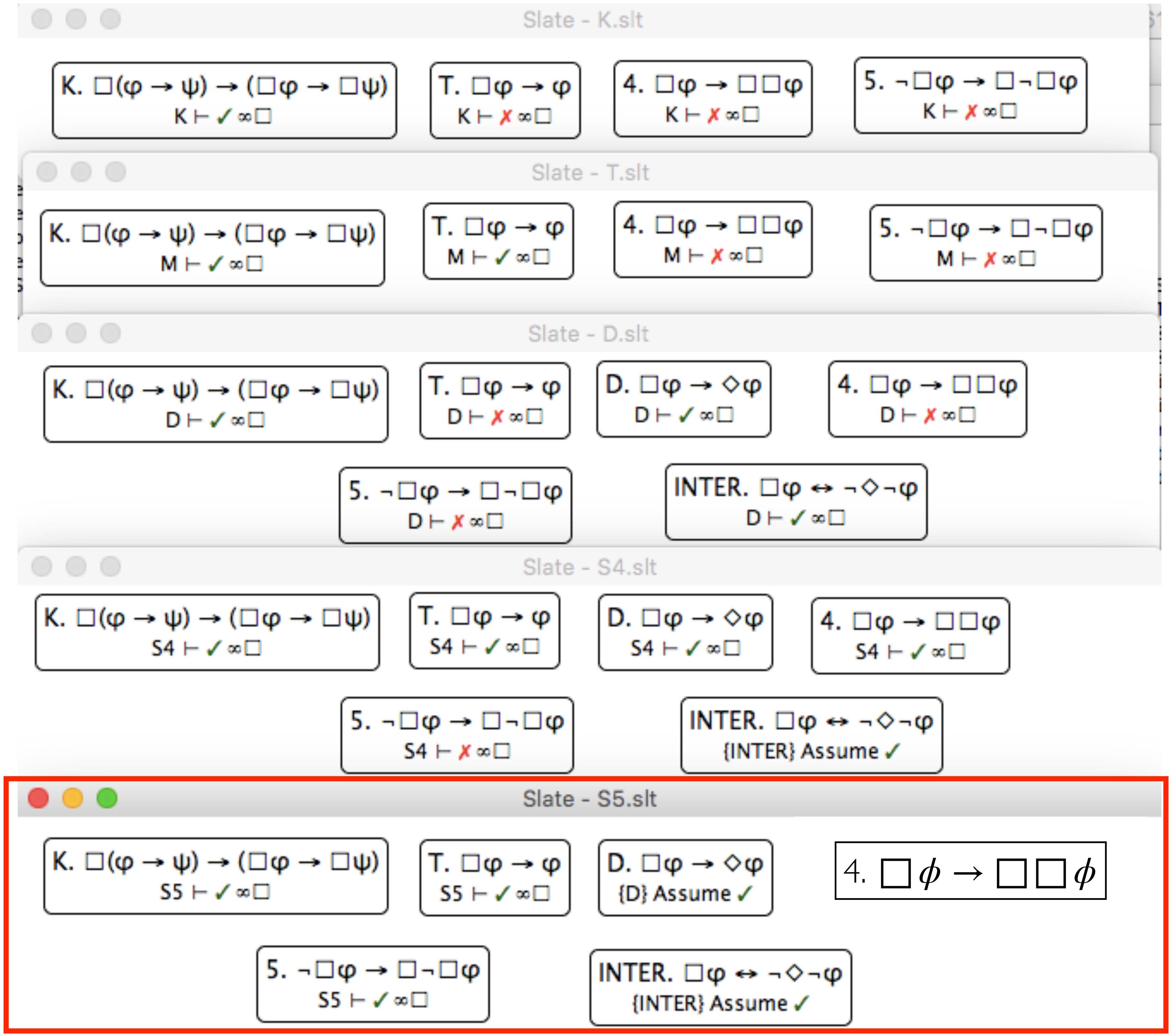

S5 …

The Characteristic Axiom

$$\Diamond\phi \rightarrow \Box\Diamond\phi$$

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$$\Diamond\phi \rightarrow \Box\Diamond\phi$$

$$\neg\Box\psi \rightarrow \Box\neg\Box\psi$$

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$$\Diamond\phi \rightarrow \Box\Diamond\phi$$

$$\neg\Box\psi \rightarrow \Box\neg\Box\psi$$

$$\neg\Box\neg\phi \rightarrow \Box\neg\Box\neg\phi$$

The Characteristic Axiom

$$\Diamond\phi \rightarrow \Box\Diamond\phi$$

$$\neg\Box\psi \rightarrow \Box\neg\Box\psi$$

$$\neg\Box\neg\phi \rightarrow \Box\neg\Box\neg\phi$$

The Characteristic Axiom

$$\Diamond\phi \rightarrow \Box\Diamond\phi$$

$$\neg\Box\psi \rightarrow \Box\neg\Box\psi$$

$$\neg\Box\neg\phi \rightarrow \Box\neg\Box\neg\phi$$

$$\Diamond\phi \rightarrow \Box\Diamond\phi$$

Nice S5 Reduction Lemmas

Nice S5 Reduction Lemmas

The Four Possible Pairs

$\Box\ \Box$
$\Diamond\ \Diamond$
$\Box\ \Diamond$
$\Diamond\ \Box$

Nice S5 Reduction Lemmas

The Four Possible Pairs

$\Box\ \Box$

$\Diamond\ \Diamond$

$\Box\ \Diamond$

$\Diamond\ \Box$

The Four Reduction Principles

$$\Box\phi \leftrightarrow \Box\Box\phi$$

$$\Diamond\phi \leftrightarrow \Diamond\Diamond\phi$$

$$\Box\phi \leftrightarrow \Diamond\Box\phi$$

$$\Diamond\phi \leftrightarrow \Box\Diamond\phi$$

(where $\phi \in \mathscr{L}_{pc}$)

Nice S5 Reduction Lemmas

The Four Possible Pairs

$\Box\,\Box$
$\Diamond\,\Diamond$
$\Box\,\Diamond$
$\Diamond\,\Box$

The Four Reduction Principles

$$\Box\phi \leftrightarrow \Box\Box\phi$$

$$\Diamond\phi \leftrightarrow \Diamond\Diamond\phi$$

$$\Box\phi \leftrightarrow \Diamond\Box\phi$$

$$\Diamond\phi \leftrightarrow \Box\Diamond\phi$$

(where $\phi \in \mathscr{L}_{pc}$)

(verify in HS®)

Quantificational S5$_1$ ...

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Easy peasy: Marry **PS5** + $\mathscr{L}_1$!

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Theorem: $\forall x \Diamond R(x) \rightarrow \forall x \Box \Diamond R(x)$

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Theorem: $\forall x \Diamond R(x) \rightarrow \forall x \Box \Diamond R(x)$

Theorem: $\Diamond \exists x R(x) \leftrightarrow \exists x \Diamond R(x)$

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$\vdots$

Quantificational S5$_1$ . . .

Easy peasy: Marry **PS5** + $\mathscr{L}_1$!

Theorem: $\forall x \Diamond R(x) \rightarrow \forall x \Box \Diamond R(x)$

(verify in HS®)

Theorem: $\Diamond \exists x R(x) \leftrightarrow \exists x \Diamond R(x)$

$\vdots$

The Notorious Barcans

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Barcan Formula: $\vdash_{QS5_1} \Diamond \exists x \phi(x) \rightarrow \exists x \Diamond \phi(x)$

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The Notorious Barcans

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CrossMark

An Argument for P=NP

Selmer Bringsjord[1,2]

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Abstract I articulate a novel modal argument for **P=NP**.

Keywords **P=NP** · Modal logic · Digital physics

The Clay Mathematics Institute offers a $1 million prize for a solution to the **P=?NP** problem.[1] I look forward to receiving my award—but concede that the expected format of a solution is an *object-level* proof, not a meta-level argument like what I provide. On the other hand, certainly the winner needn't provide a *constructive* proof that **P=NP**.[2] Despite Gödel's recently discovered position on the

[1] See http://www.claymath.org/millennium. There are six other "millennium" problems; each of these is also associated with a $1M prize.

[2] As many readers know, the history of the problem is littered with failed attempts to provide non-constructive substantiation of the received view that **P≠NP**.

I'm greatly indebted to Michael Zenzen for many valuable discussions about the **P=?NP** problem and physics (*simpliciter* and digital), and to Jim Fahey for discussions about such physics and mixed-mode dual-diamond operators in modal logic. The presentation of the core arguments herein to editions of Bringsjord's graduate seminar, *Logic & Artificial Intelligence*, and his guest lectures on **P=?NP** in *Formal Foundations of Cognitive Science* graduate seminars, sparked a number of helpful objections and suggestions, for which I'm grateful. I'm indebted as well to two anonymous referees for trenchant comments. Though the two arguments herein (the second of which seems to establish **P=NP**) are for weal or woe Bringsjord's, Joshua Taylor's astute objections catalyzed much thought and crucial refinements.

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Four S5 Bins for Everything …