Review of Extensional (Including Higher-Order) Logics (contrasted with intensional ones)

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Intermediate Formal Logic & AI (IFLAI2) 9/6/2022



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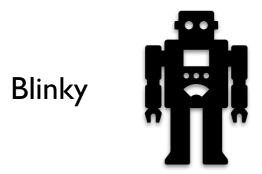
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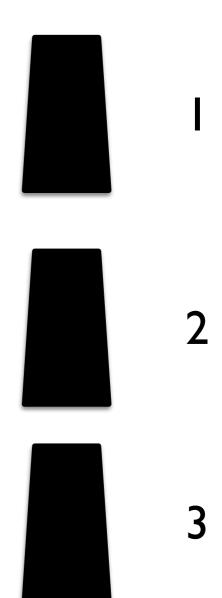
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Be ready for first use of HG® & HS® in class on Sept 12!

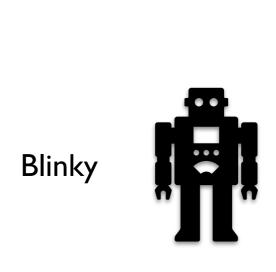
Extensional vs Intensional Logics in a Nutshell Example

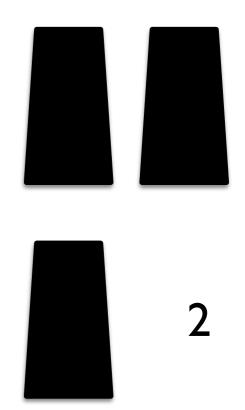
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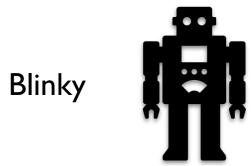


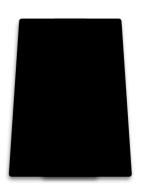


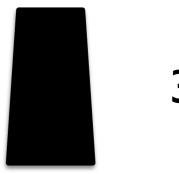


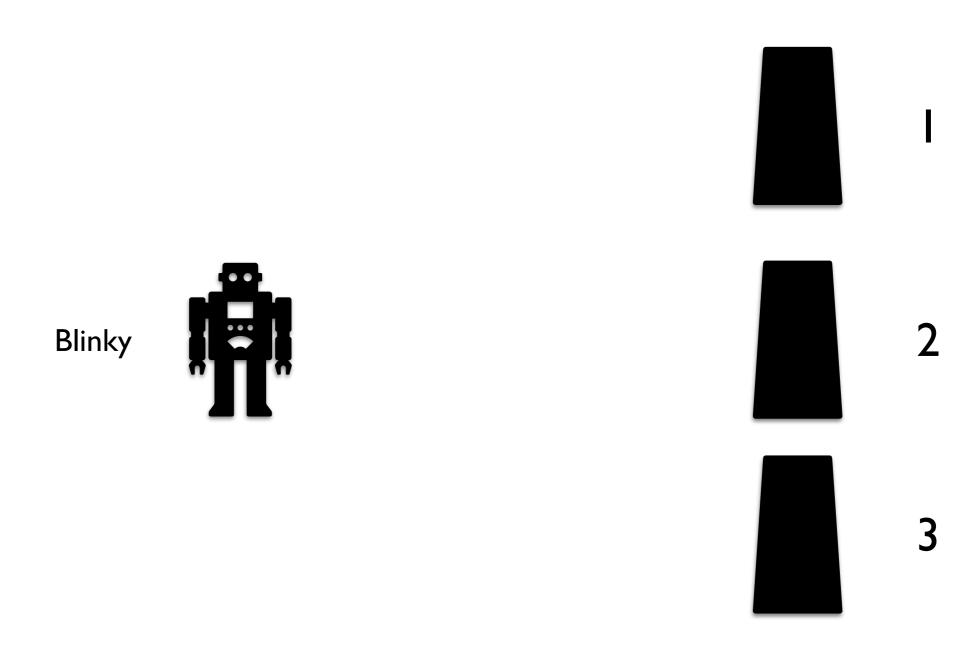




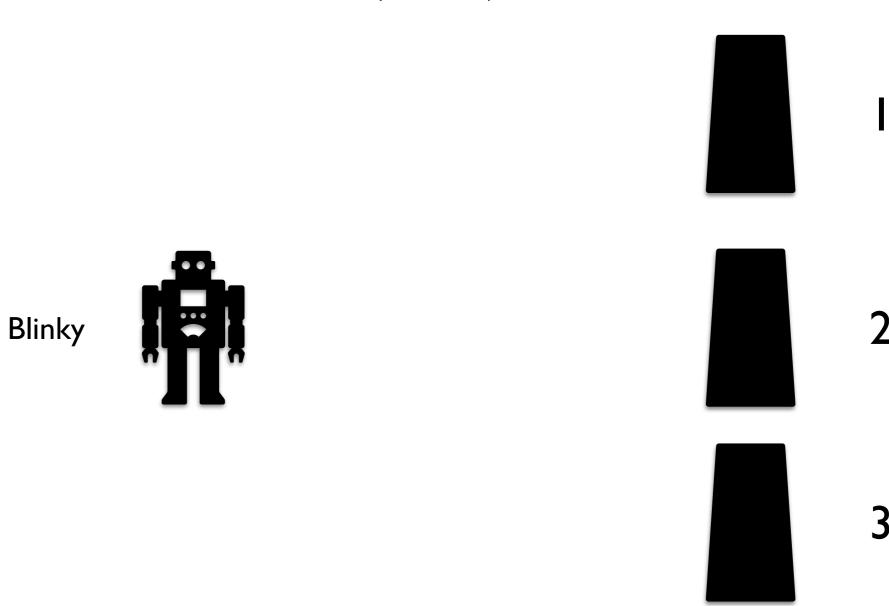


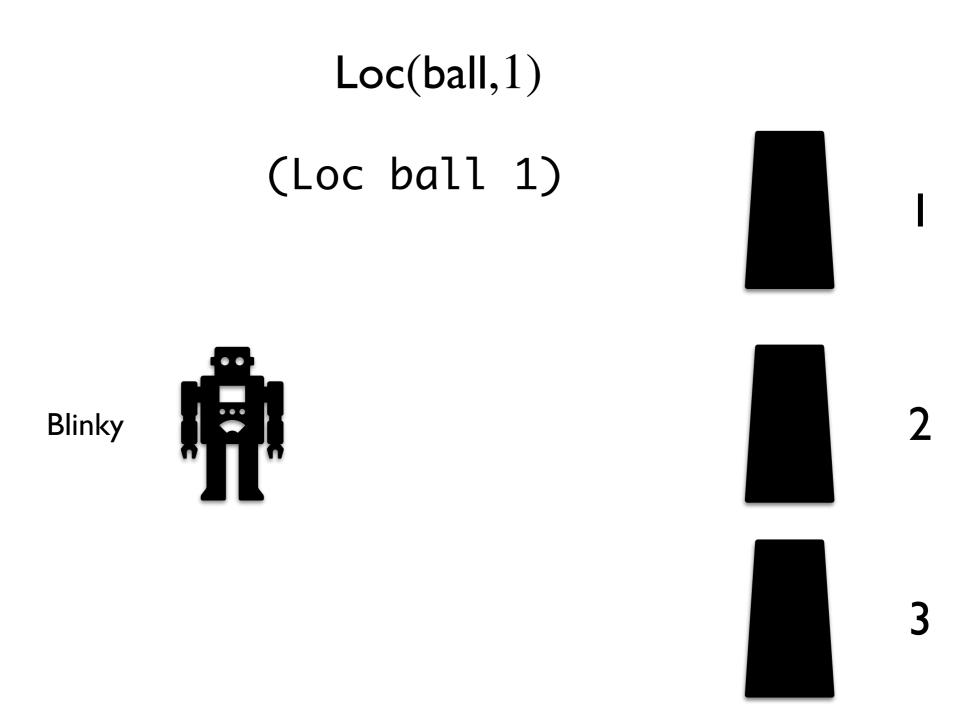


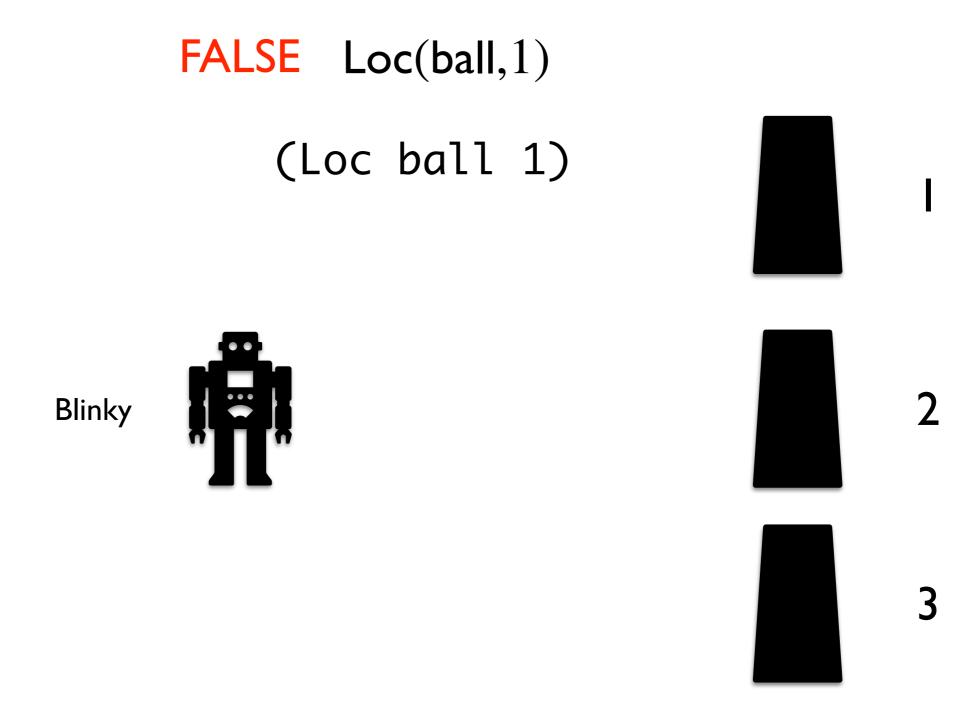




Loc(ball,1)







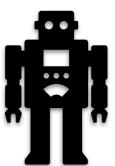
FALSE Loc(ball,1) (Loc ball 1) 2 Blinky

FALSE

(Loc ball 1)



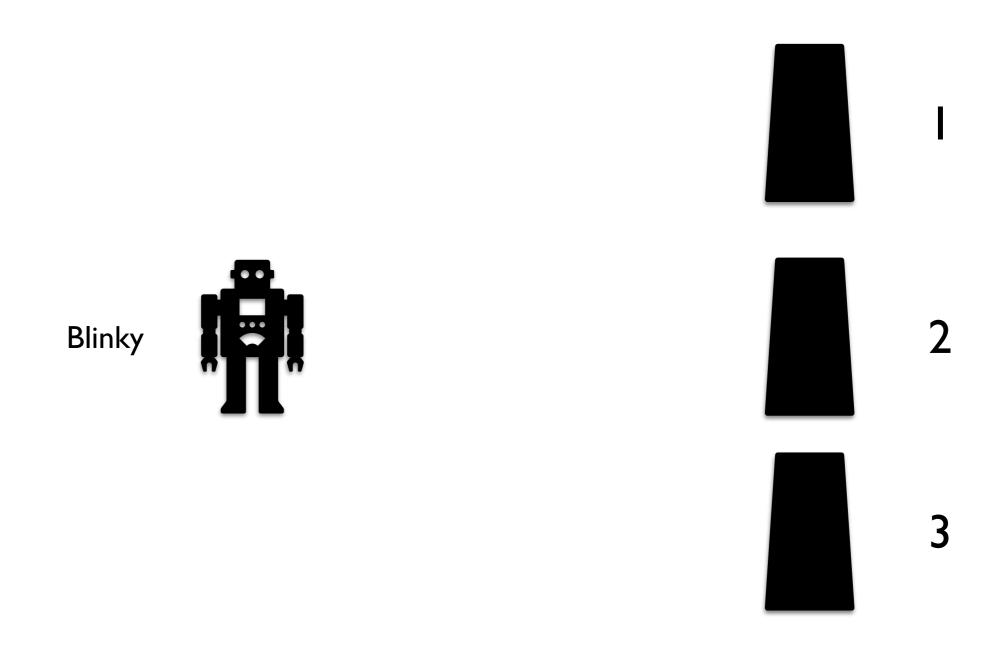




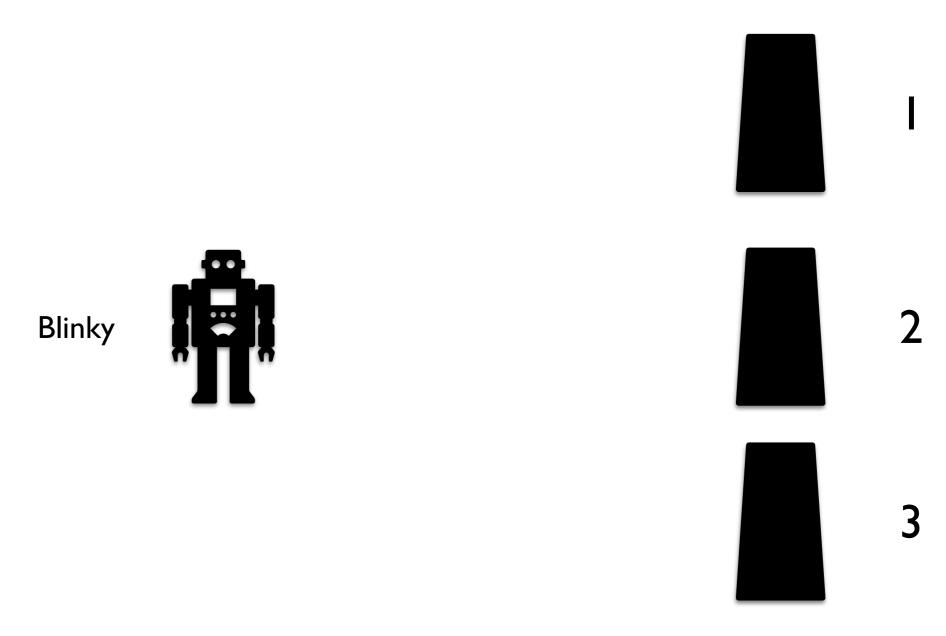
2



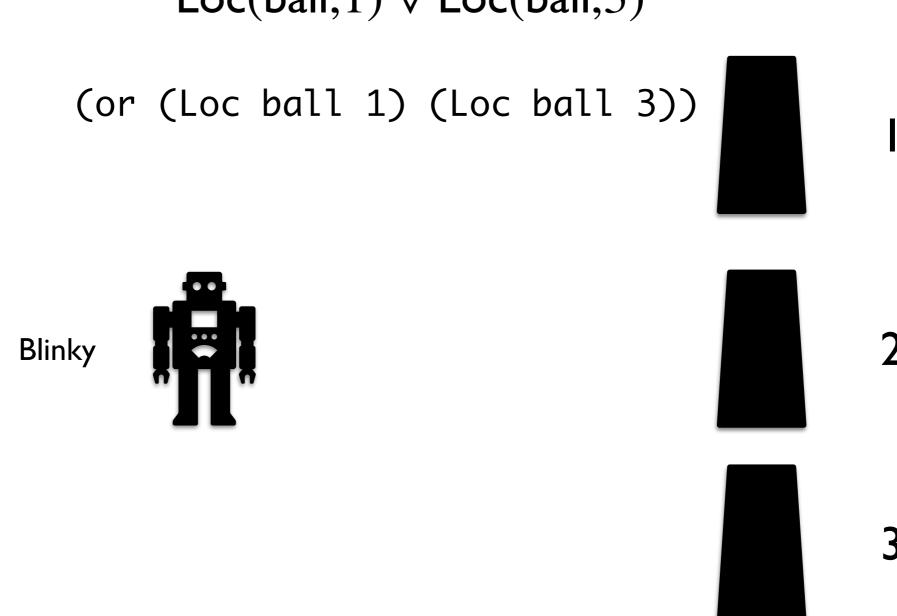
(Loc ball 1) Blinky



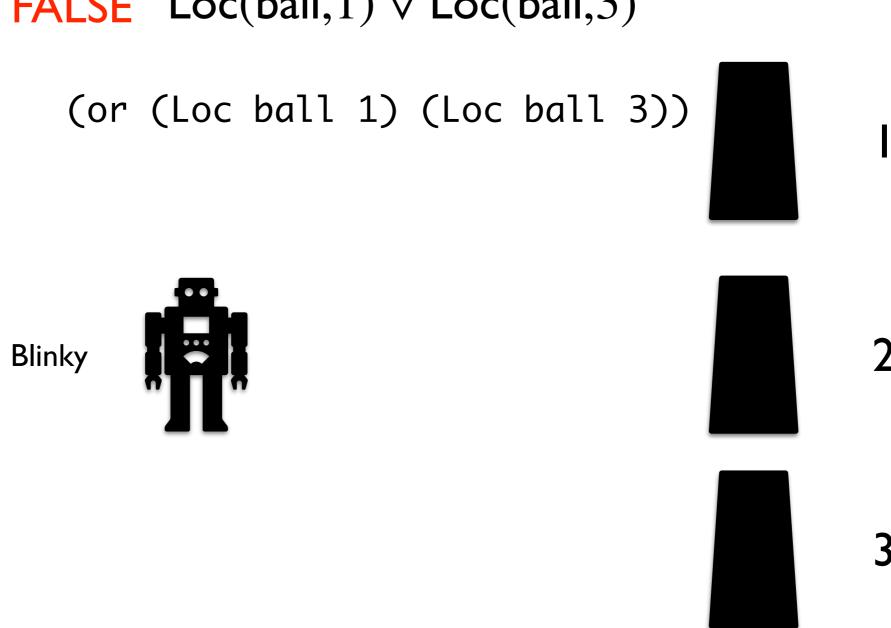
$$Loc(ball,1) \lor Loc(ball,3)$$





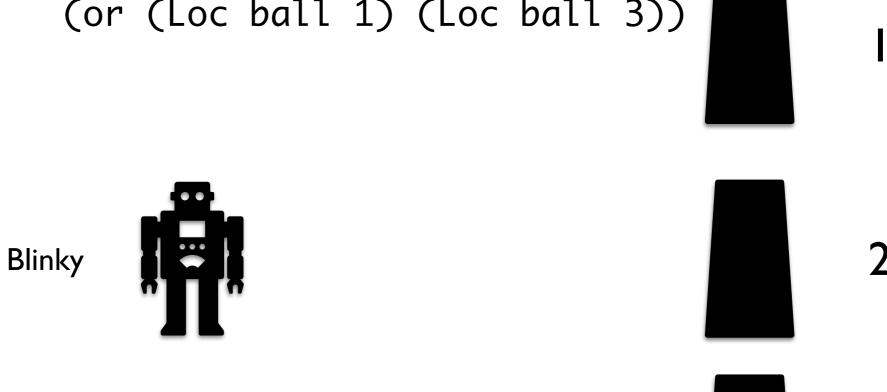




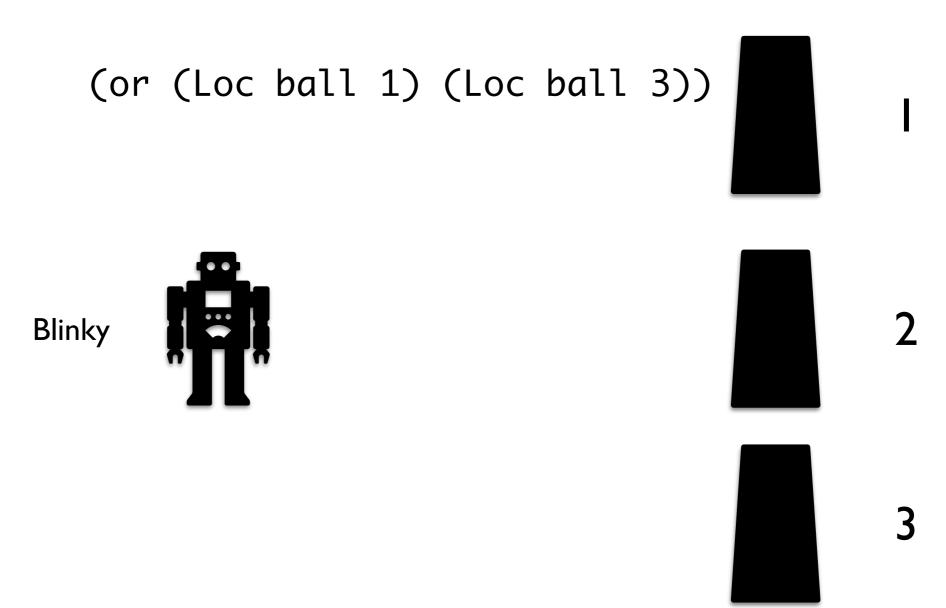


FALSE Loc(ball,1) \times Loc(ball,3)

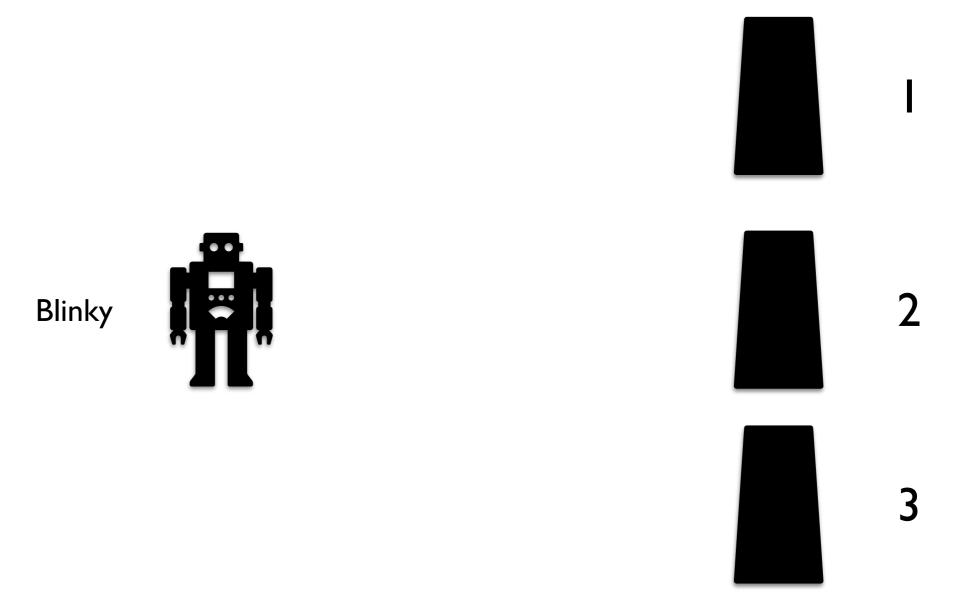
(or (Loc ball 1) (Loc ball 3))



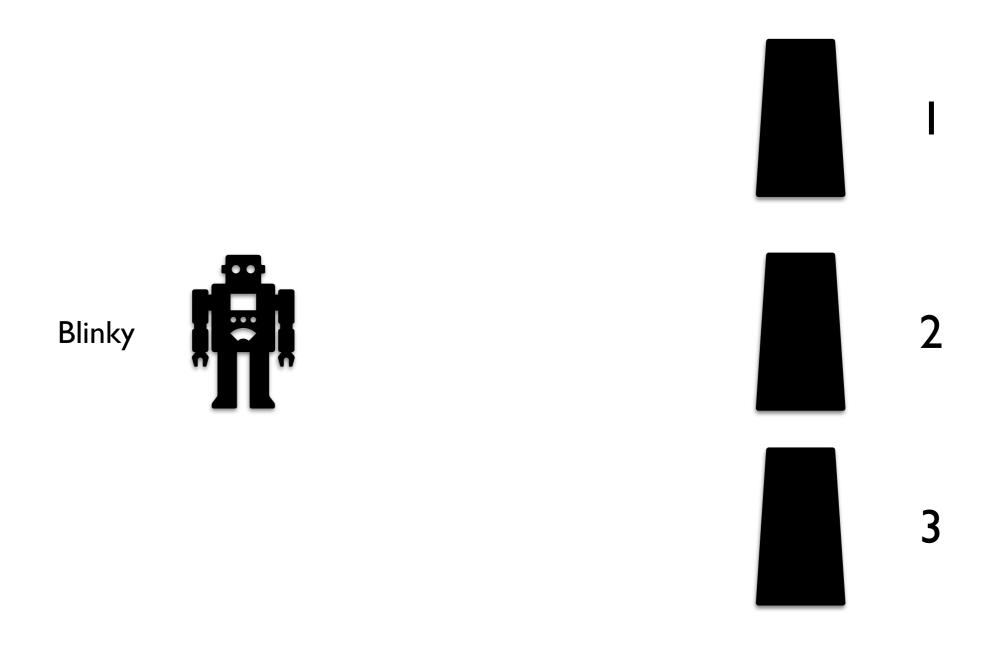
FALSE



FALSE

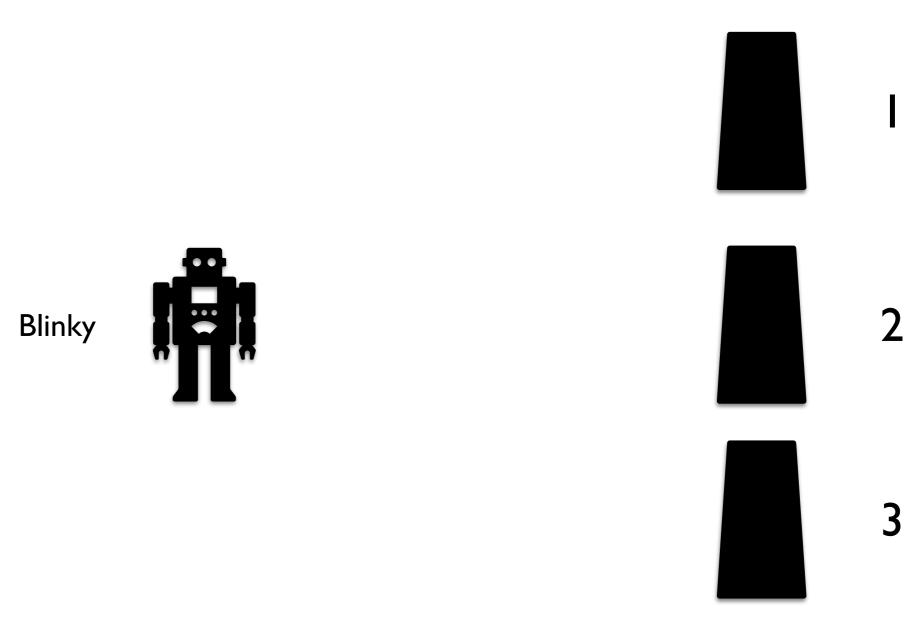


Blinky believes that the ball is in the cup at location #1.



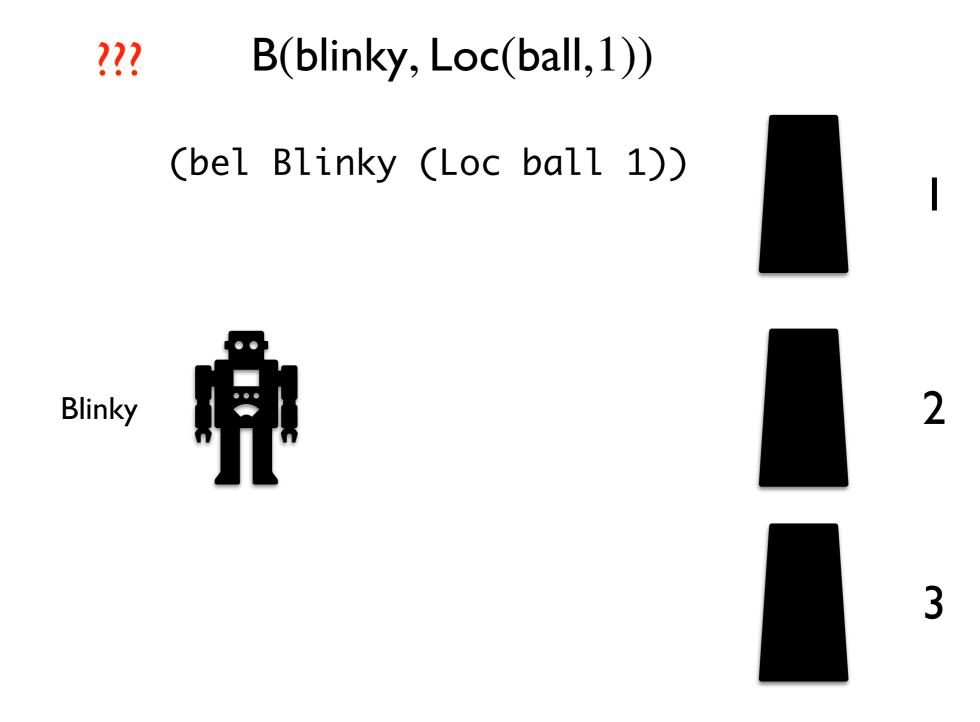
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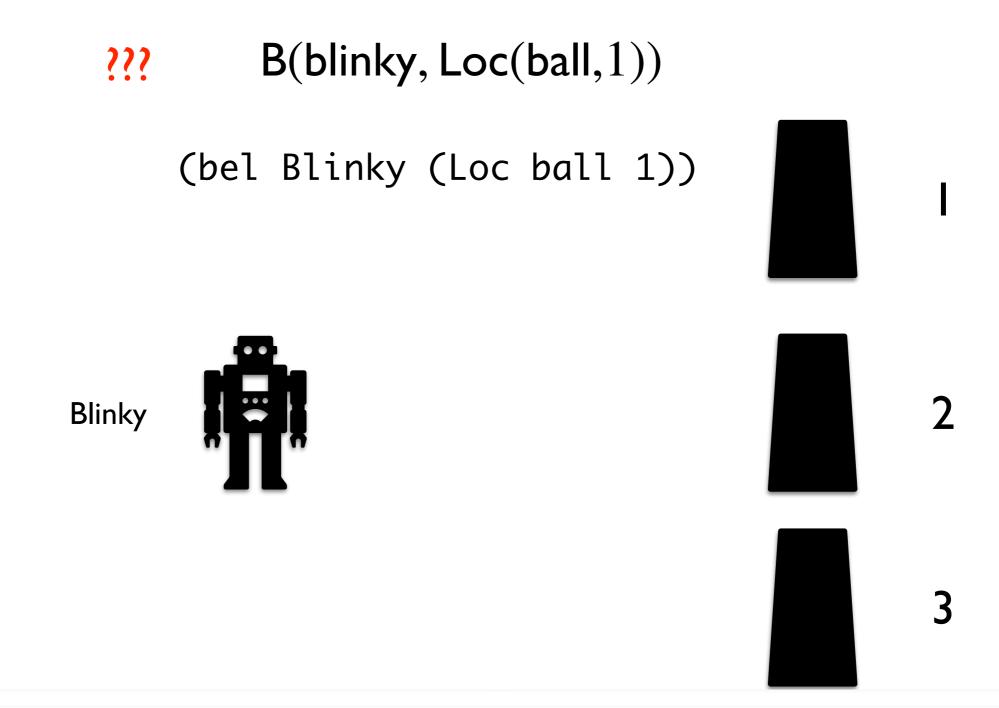
B(blinky, Loc(ball, 1))



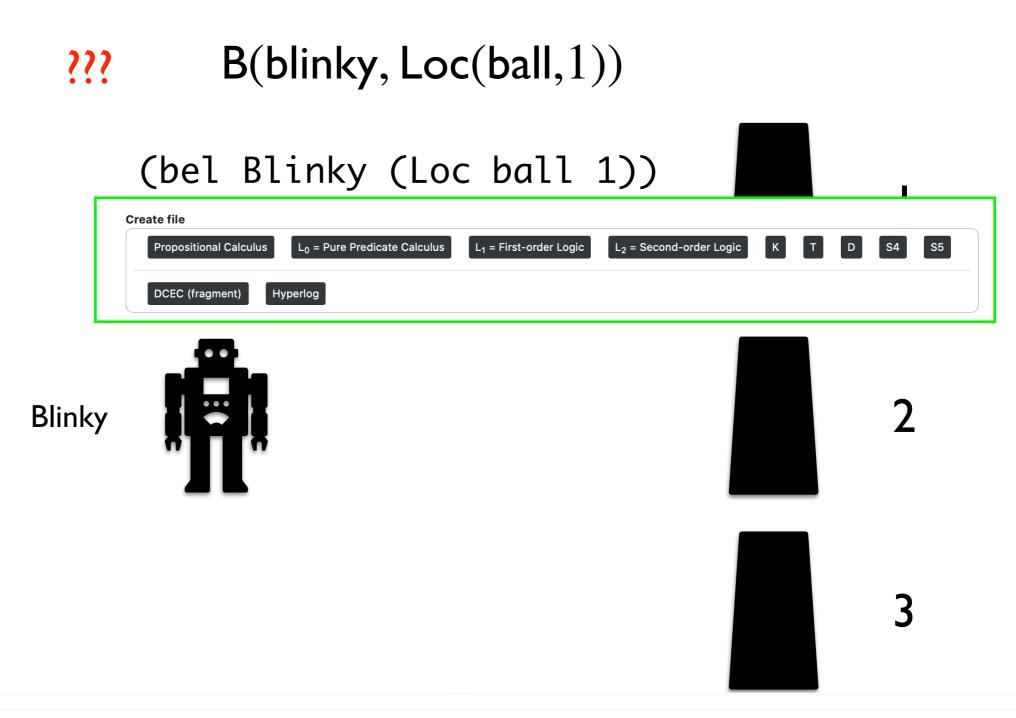


(bel Blinky (Loc ball 1))

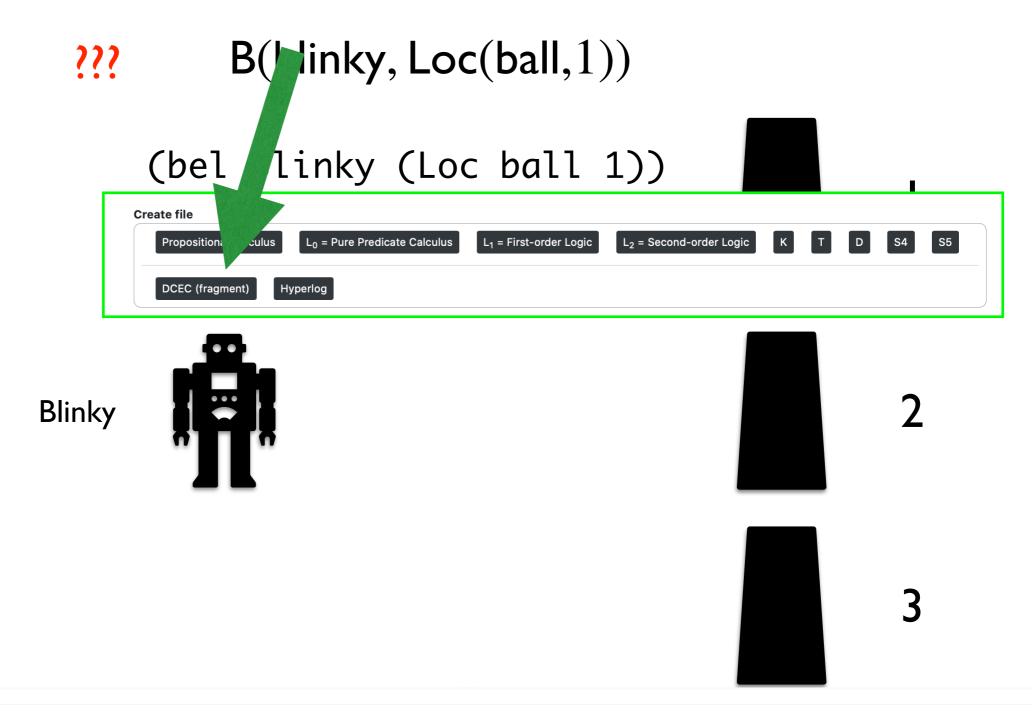




In extensional logics, what is denoted is conflated with meaning (the latter being naïvely compositional), and intensional attitudes like *believes*, *knows*, *hopes*, *fears*, etc cannot be represented and reasoned over smoothly (e.g. without fear of inconsistency rising up).



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Layman Allen Committee Problem ...

"Well, if your Al is so smart, let me see it do my taxes for me, and for a company too."

THE UNIVERSITY OF CHICAGO LAW REVIEW

VOLUME 25

AUTUMN 1957

NUMBER I

TOWARD A MORE SYSTEMATIC DRAFTING AND INTERPRETING OF THE INTERNAL REVENUE CODE: EXPENSES, LOSSES AND BAD DEBTS

LAYMAN E. ALLEN* AND GABRIEL ORECHKOFF†

Suppose that you, as a reasonable man, are asked whether the following two sets of rules mean exactly the same thing. If they do and if you intend to communicate your message as effectively as possible, which of the two sets would you choose to state the organizational rules of your law school?

Set 1

- A. The financial committee shall be chosen from among the general committee.
- B. No one shall be a member of both the general and library committees unless he is also on the financial committee.
- C. No member of the library committee shall be on the financial committee.¹

Set 2

- A. The financial committee shall be chosen from among the general committee.
- B. No member of the general committee shall be on the library committee.

We suspect that most readers will have little difficulty in deciding the second question; but even after careful reading of the two sets of rules, they will remain a little puzzled as to whether both sets say the same thing. We further suspect that lawyers waste a great deal of their mental energy by using inadequate and inappropriate intellectual tools to figure out similar logical problems in their everyday work. A symbolic logician could quickly ascertain that the two sets of rules are equivalent and relieve his mind to focus on the more important problem of deciding which is the more appropriate choice. Since law

^{*} Social Science Research Council Fellow, Yale Law School. Member Connecticut Bar. † Member, New York Bar.

¹ This example is given in Berkeley, The Algebra of States and Events, 78 Scientific Monthly 232 (1954).

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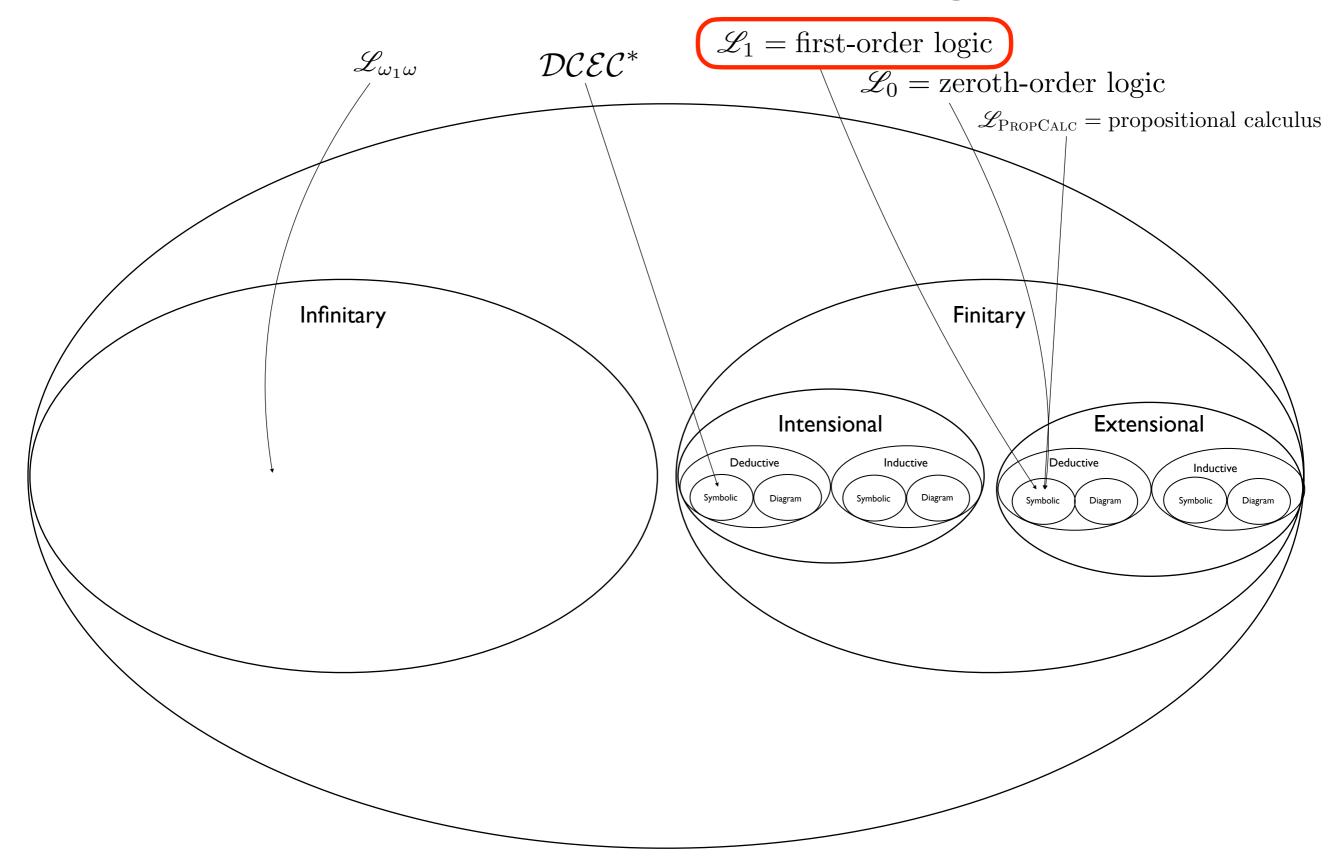
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The Universe of Logics



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There's a thing such that it's both a llama and a non-llama; or there's a thing such that if it's a llama, everything is a llama; or there's a thing such that every llama is a non-llama.

Is this disjunction TRUE, FALSE, or UNKNOWN?

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Can you build a formal, verifying proof in HyperSlate®?

Second-Order Logic and the k-Order Ladder ...

FOL

√ FOL Epistemic + FOL $_{\mathbf{B}_{d}\mathbf{B}_{v}\mathbf{B}_{d}Vv}$

$\begin{array}{c} \text{$\sqrt{\text{FOL}}$} \\ \text{Epistemic + FOL} \text{ (for coverage of "killer" robots, later)} \\ \mathbf{B}_{d}\mathbf{B}_{v}\mathbf{B}_{d}Vv \end{array}$

$\begin{array}{c} \text{$\sqrt{FOL}} \\ \text{$\sqrt{$Epistemic+FOL$}} \text{ (for coverage of "killer" robots, later)} \\ \text{$B_dB_vB_dVv$} \end{array}$

 $\sqrt{\text{Epistemic} + \text{FOL}}$ (for coverage of "killer" robots, later)

TOL

 $\exists X[X(j) \land \neg X(m) \land S(X)]$

√ Epistemic + FOL (for coverage of "killer" robots, later) $\mathbf{B}_d \mathbf{B}_v \mathbf{B}_d V v$

 $\exists X[X(j) \land \neg X(m) \land S(X)]$



 $\sqrt{\text{Epistemic} + \text{FOL}}$ (for coverage of "killer" robots, later)

TOL

 $\exists X[X(j) \land \neg X(m) \land S(X)]$



Double-Minded Man

The Contemporary Craft of Creating Characters Meets Today's Cognitive Architectures: A Case Study in Expressivity*

Selmer Bringsjord • John Licato • Alexander Bringsjord version of 0121161500NY

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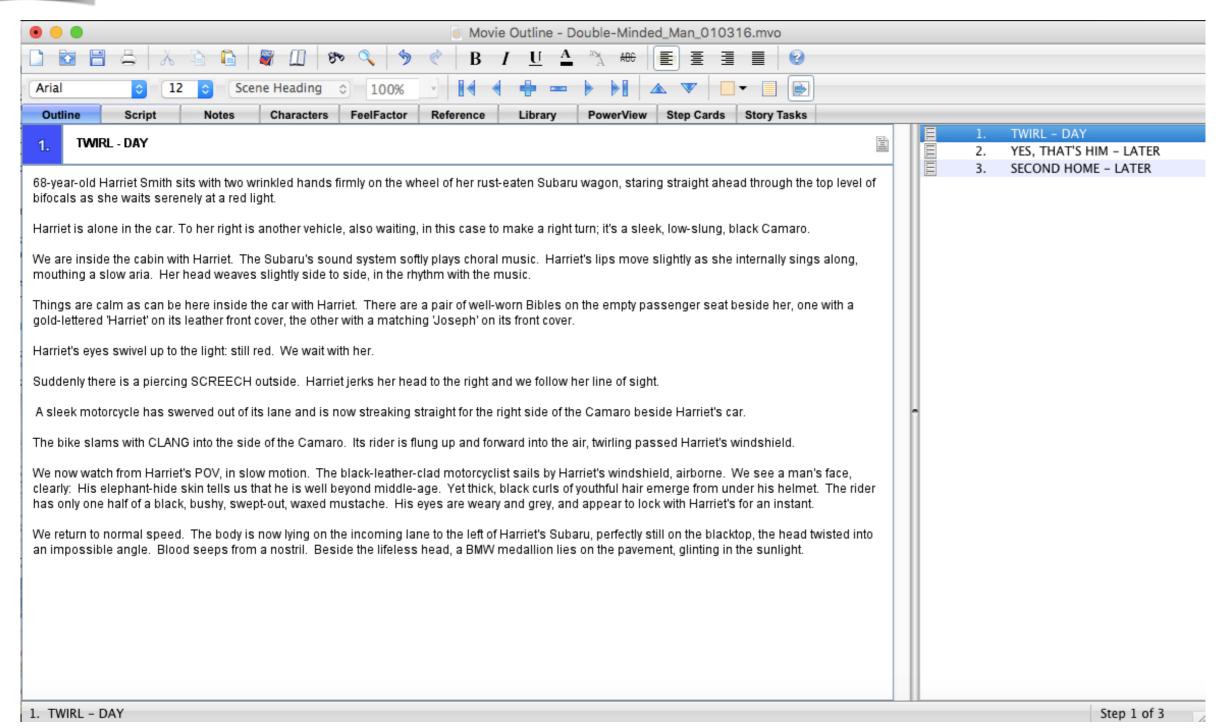
^{*}We are indebted to two anonymous referees, and editor Jeremy Turner, for insightful suggestions and objections. Any remaining deficiencies are due to our own failings.

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Double-Minded Man

Double-Minded Man by

S Bringsjord & A Bringsjord

DRAFT #5 © June 30 2016

Selmer.Bringsjord@gmail.com

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Double-Minded Man

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Double-Minded Man

1. TWIRL - DAY

68-year-old Harriet Smith sits with two wrinkled hands firmly on the wheel of her rust-eaten Subaru wagon, staring straight ahead through the top level of bifocals as she waits serenely at a red light.

Harriet is alone in the car. To her right is another vehicle, also waiting, in this case to make a right turn; it's a sleek, low-slung, black Camaro.

We are inside the cabin with Harriet. The Subaru's sound system softly plays choral music. Harriet's lips move slightly as she internally sings along, mouthing a slow aria. Her head weaves slightly side to side, in the rhythm with the music.

Things are calm as can be here inside the car with Harriet. There are a pair of well-worn Bibles on the empty passenger seat beside her, one with a gold-lettered 'Harriet' on its leather front cover, the other with a matching 'Joseph' on its front cover.

Harriet's eyes swivel up to the light: still red. We wait with her.

Suddenly there is a piercing SCREECH outside. Harriet jerks her head to the right and we follow her line of sight.

A sleek motorcycle has swerved out of its lane and is now streaking straight for the right side of the Camaro beside Harriet's car.

The bike slams with CLANG into the side of the Camaro. Its rider is flung up and forward into the air, twirling passed Harriet's windshield.

We now watch from Harriet's POV, in slow motion. The black-leather-clad motorcyclist sails by Harriet's windshield, airborne. We see a man's face, clearly: His elephant-hide skin tells us that he is well beyond middle-age. Yet thick, black curls of youthful hair emerge from under his helmet. The rider has only one half of a black, bushy, swept-out, waxed mustache. His eyes are weary and grey, and appear to lock with Harriet's for an instant.

We return to normal speed. The body is now lying on the incoming lane to the left of Harriet's Subaru, perfectly still on the blacktop, the head twisted into an impossible angle. Blood seeps from a nostril. Beside the lifeless head, a BMW medallion lies on the pavement, glinting in the sunlight.

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 $Llama(a) \wedge Llama(b) \wedge Likes(a,b) \wedge Llama(fatherOf(a))$

ZOL $Llama(a) \wedge Llama(b) \wedge Likes(a,b) \wedge Llama(fatherOf(a))$

There's some thing which is a llama and likes b (which is also a llama), and whose father is a llama too.

ZOL $Llama(a) \wedge Llama(b) \wedge Likes(a,b) \wedge Llama(fatherOf(a))$

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Things x and y, along with the father of x, share a certain property (and x likes y).

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$$\exists x \exists y \exists R[R(x) \land R(y) \land Likes(x,y) \land R(fatherOf(x))]$$

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Things x and y, along with the father of x, share a certain property; and, x R^2 s y, where R^2 is a positive property.

SOL $\exists x \exists y \exists R[R(x) \land R(y) \land Likes(x,y) \land R(fatherOf(x))]$

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FOL $\exists x[Llama(x) \land Llama(b) \land Likes(x,b) \land Llama(fatherOf(x))]$

There's some thing which is a llama and likes b (which is also a llama), and whose father is a llama too.

ZOL $Llama(a) \wedge Llama(b) \wedge Likes(a,b) \wedge Llama(fatherOf(a))$

$$\exists x, y \; \exists R, R^2[R(x) \land R(y) \land R^2(x, y) \land Positive(R^2) \land R(fatherOf(x))]$$

Things x and y, along with the father of x, share a certain property; and, x R^2 s y, where R^2 is a positive property.

SOL
$$\exists x \exists y \exists R[R(x) \land R(y) \land Likes(x,y) \land R(fatherOf(x))]$$

Things x and y, along with the father of x, share a certain property (and x likes y).

FOL $\exists x[Llama(x) \land Llama(b) \land Likes(x,b) \land Llama(fatherOf(x))]$

There's some thing which is a llama and likes b (which is also a llama), and whose father is a llama too.

ZOL $Llama(a) \wedge Llama(b) \wedge Likes(a,b) \wedge Llama(fatherOf(a))$

TOL $\exists x, y \; \exists R, R^2[R(x) \land R(y) \land R^2(x, y) \land Positive(R^2) \land R(fatherOf(x))]$

Things x and y, along with the father of x, share a certain property; and, x R^2 s y, where R^2 is a positive property.

SOL $\exists x \exists y \exists R[R(x) \land R(y) \land Likes(x,y) \land R(fatherOf(x))]$

Things x and y, along with the father of x, share a certain property (and x likes y).

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ZOL $Llama(a) \wedge Llama(b) \wedge Likes(a,b) \wedge Llama(fatherOf(a))$

TOL $\exists x, y \; \exists R, R^2[R(x) \land R(y) \land R^2(x, y) \land Positive(R^2) \land R(fatherOf(x))]$ \mathscr{L}_3 Things x and y, along with the father of x, share a certain property; and, $x \; R^2$ s y, where R^2 is a positive property.

SOL

 \mathcal{L}_2

 \mathscr{L}_0

 $\exists x \exists y \exists R[R(x) \land R(y) \land Likes(x,y) \land R(fatherOf(x))]$ Things x and y, along with the father of x, share a certain property (and x likes y).

FOL $\exists x[Llama(x) \land Llama(b) \land Likes(x,b) \land Llama(fatherOf(x))]$

There's some thing which is a llama and likes b (which is also a llama), and whose father is a llama too.

ZOL $Llama(a) \wedge Llama(b) \wedge Likes(a,b) \wedge Llama(fatherOf(a))$

•

TOL
$$\exists x, y \; \exists R, R^2[R(x) \land R(y) \land R^2(x, y) \land Positive(R^2) \land R(fatherOf(x))]$$

Things x and y, along with the father of x, share a certain property; and, $x R^2$ s y, where R^2 is a positive property.

SOL
$$\exists x \exists y \exists R[R(x) \land R(y) \land Likes(x,y) \land R(fatherOf(x))]$$

Things x and y, along with the father of x, share a certain property (and x likes y).

 \mathcal{L}_2

 \mathscr{L}_1

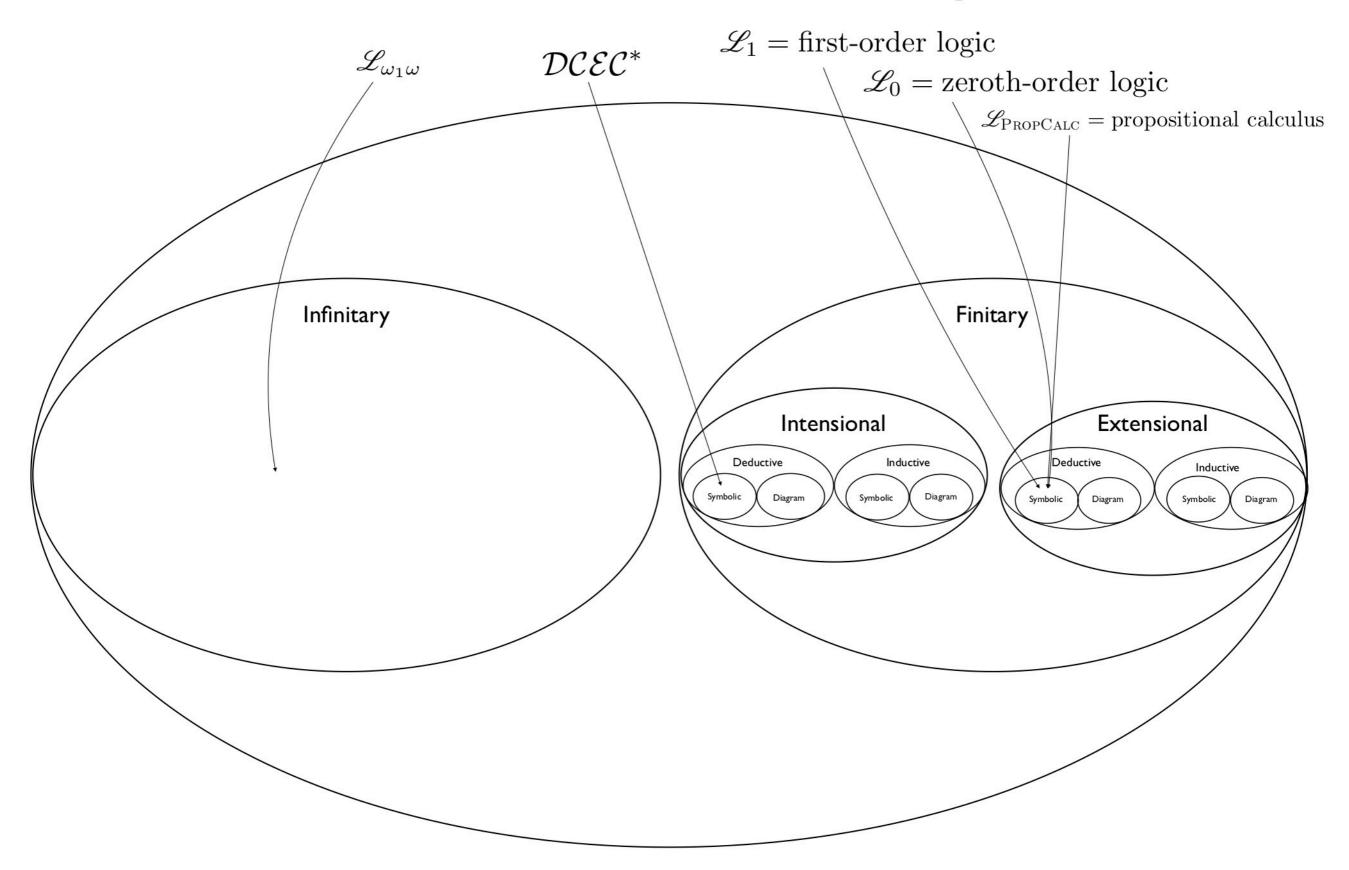
 \mathscr{L}_0

FOL
$$\exists x[Llama(x) \land Llama(b) \land Likes(x,b) \land Llama(fatherOf(x))]$$

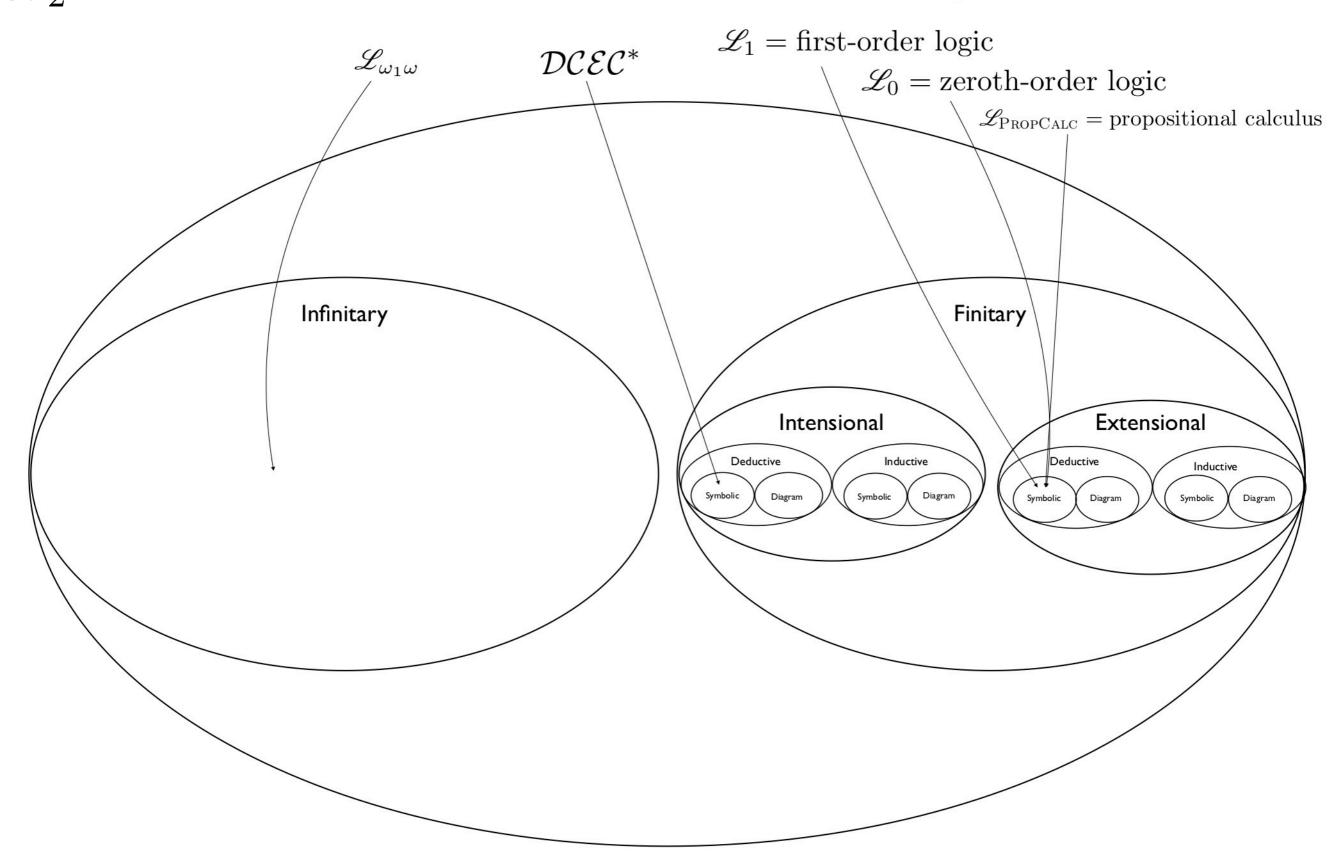
There's some thing which is a llama and likes b (which is also a llama), and whose father is a llama too.

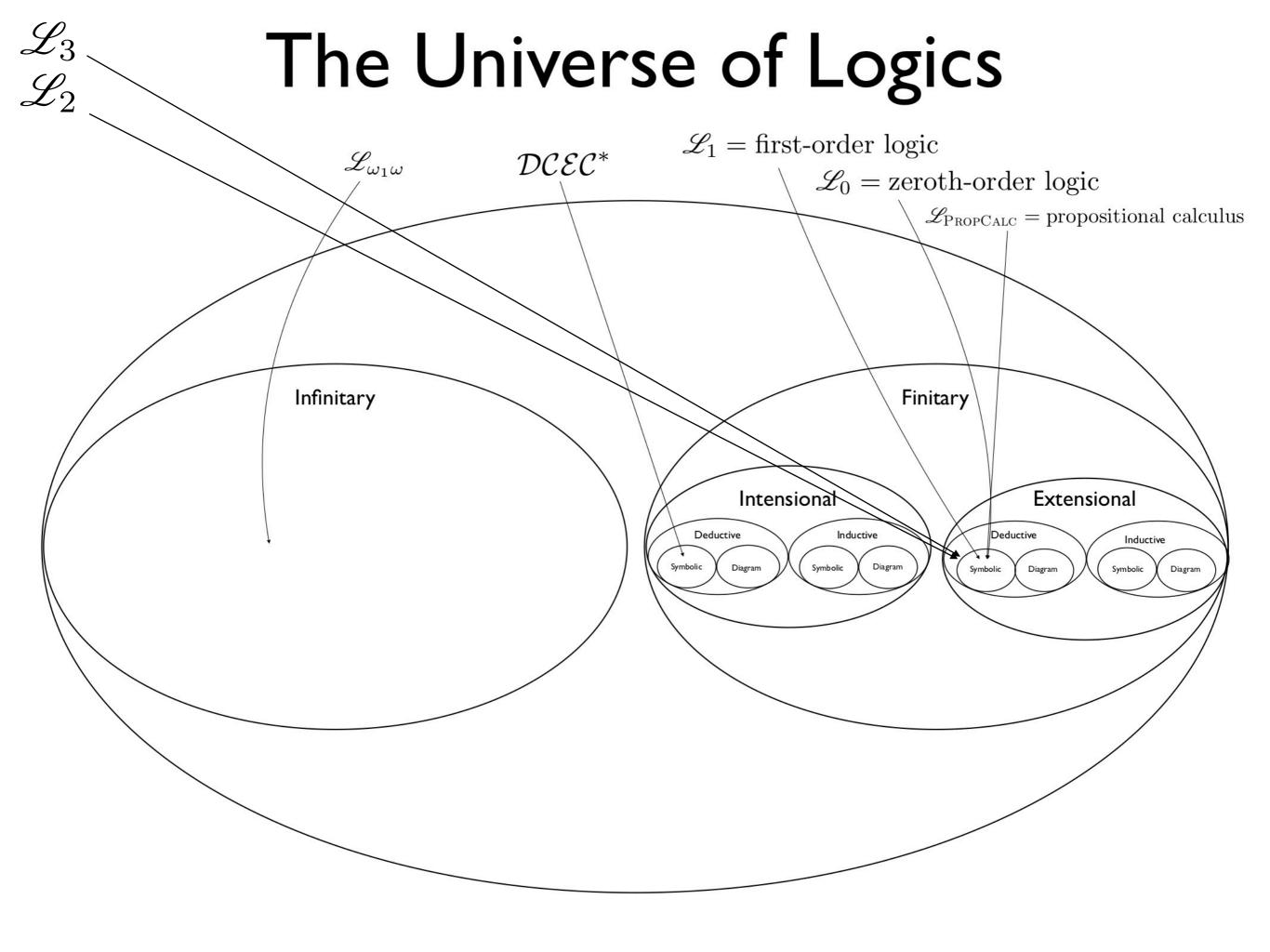
ZOL
$$Llama(a) \wedge Llama(b) \wedge Likes(a,b) \wedge Llama(fatherOf(a))$$

The Universe of Logics



The Universe of Logics





•

TOL
$$\exists x, y \; \exists R, R^2[R(x) \land R(y) \land R^2(x, y) \land Positive(R^2) \land R(fatherOf(x))]$$

Things x and y, along with the father of x, share a certain property; and, $x R^2$ s y, where R^2 is a positive property.

SOL
$$\exists x \exists y \exists R[R(x) \land R(y) \land Likes(x,y) \land R(fatherOf(x))]$$

Things x and y, along with the father of x, share a certain property (and x likes y).

 \mathcal{L}_2

 \mathscr{L}_1

 \mathscr{L}_0

FOL
$$\exists x[Llama(x) \land Llama(b) \land Likes(x,b) \land Llama(fatherOf(x))]$$

There's some thing which is a llama and likes b (which is also a llama), and whose father is a llama too.

ZOL
$$Llama(a) \wedge Llama(b) \wedge Likes(a,b) \wedge Llama(fatherOf(a))$$

•

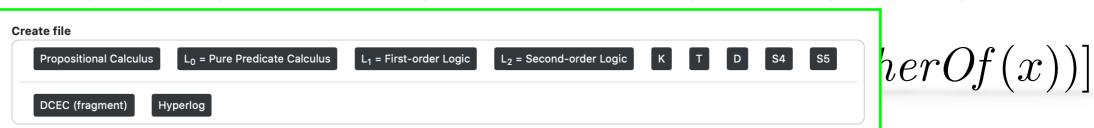
TOL

 $\exists x, y \; \exists R, R^2[R(x) \land R(y) \land R^2(x, y) \land Positive(R^2) \land R(fatherOf(x))]$

 \mathscr{L}_3

Things x and y, along with the father of x, share a certain property; and, x R^2 s y, where R^2 is a positive property.

SOL L



Things x and y, along with the father of x, share a certain property (and x likes y).

FOL

 \mathscr{L}_1

$$\exists x [Llama(x) \land Llama(b) \land Likes(x,b) \land Llama(fatherOf(x))]$$

There's some thing which is a llama and likes b (which is also a llama), and whose father is a llama too.

ZOL

$$Llama(a) \wedge Llama(b) \wedge Likes(a,b) \wedge Llama(fatherOf(a))$$

 \mathscr{L}_0

•

TOL

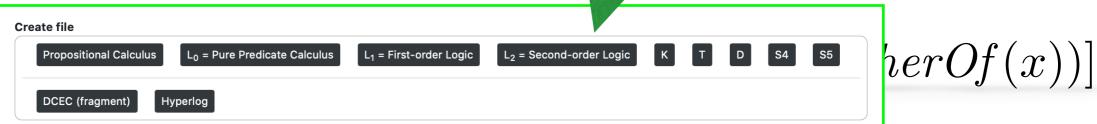
 $\exists x, y \; \exists R, R^2[R(x) \land R(y) \land R^2(x, y) \land Peritive(R^2) \land R(fatherOf(x))]$

 \mathcal{L}_3

Things x and y, along with the father of x, share a certain property; and, $x R^2s y$, where x = x + y + y = 0 a positive property.

SOL

 \mathscr{L}_2



Things x and y, along with the father of x, share a certain property (and x likes y).

FOL

$$\exists x [Llama(x) \land Llama(b) \land Likes(x,b) \land Llama(fatherOf(x))]$$

 \mathscr{L}_1

There's some thing which is a llama and likes b (which is also a llama), and whose father is a llama too.

ZOL

$$Llama(a) \wedge Llama(b) \wedge Likes(a,b) \wedge Llama(fatherOf(a))$$

 \mathscr{L}_0

Og med det .. er vårt klassemøte ferdig!