On Quantificational Modal Logic

(S5-centric)

Selmer Bringsjord

Rensselaer AI & Reasoning (RAIR) Lab
Department of Cognitive Science
Department of Computer Science
Lally School of Management & Technology
Rensselaer Polytechnic Institute (RPI)
Troy, New York 12180 USA

IFLAI2
11/10/2022
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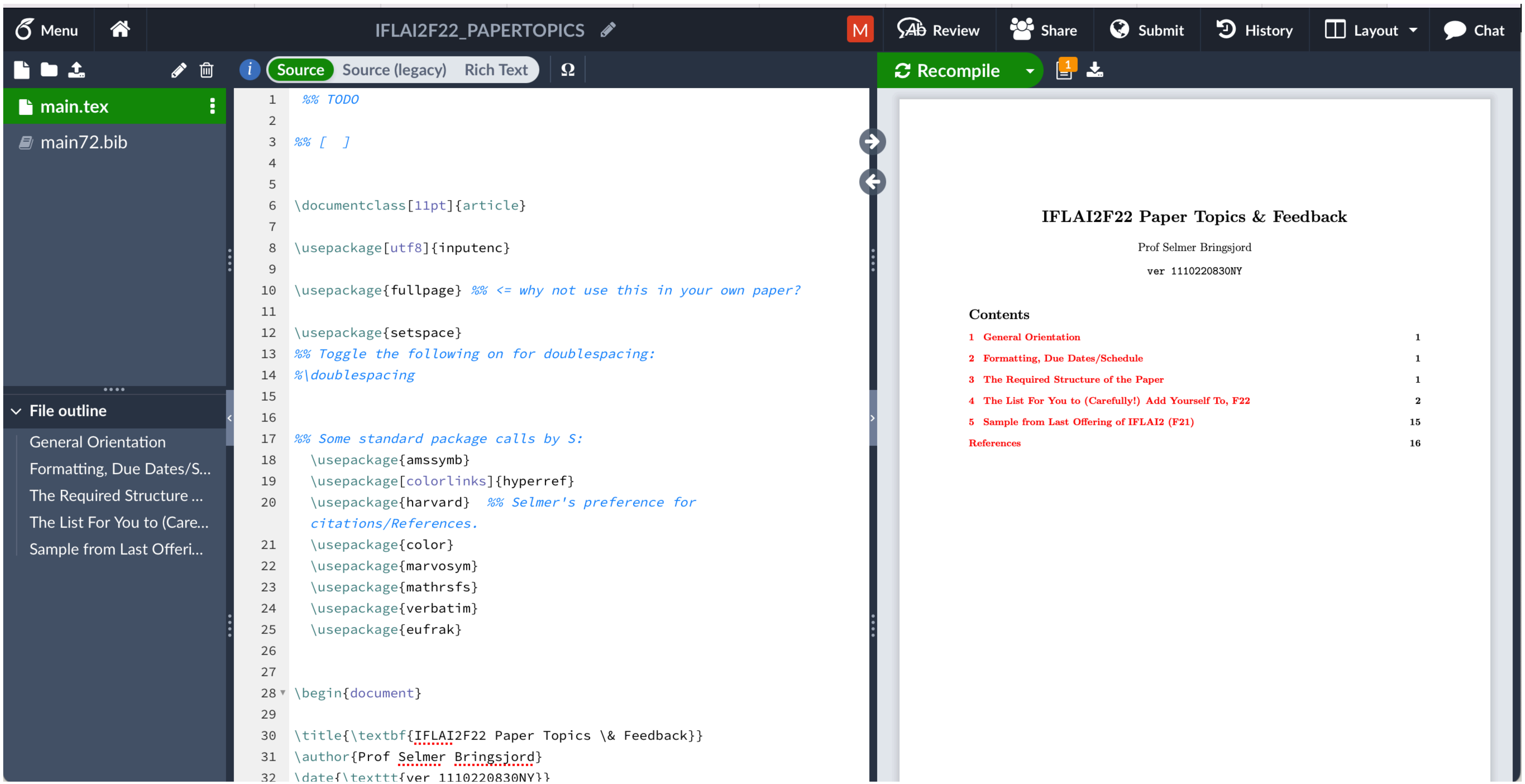
RAIR
Rensselaer AI and Reasoning Lab

Logistics …

Status? Some discussion …



Automatic A on Q1, for all.

Automatic A on Q1, for all.

Q2?

Return to Return-to-Blinky

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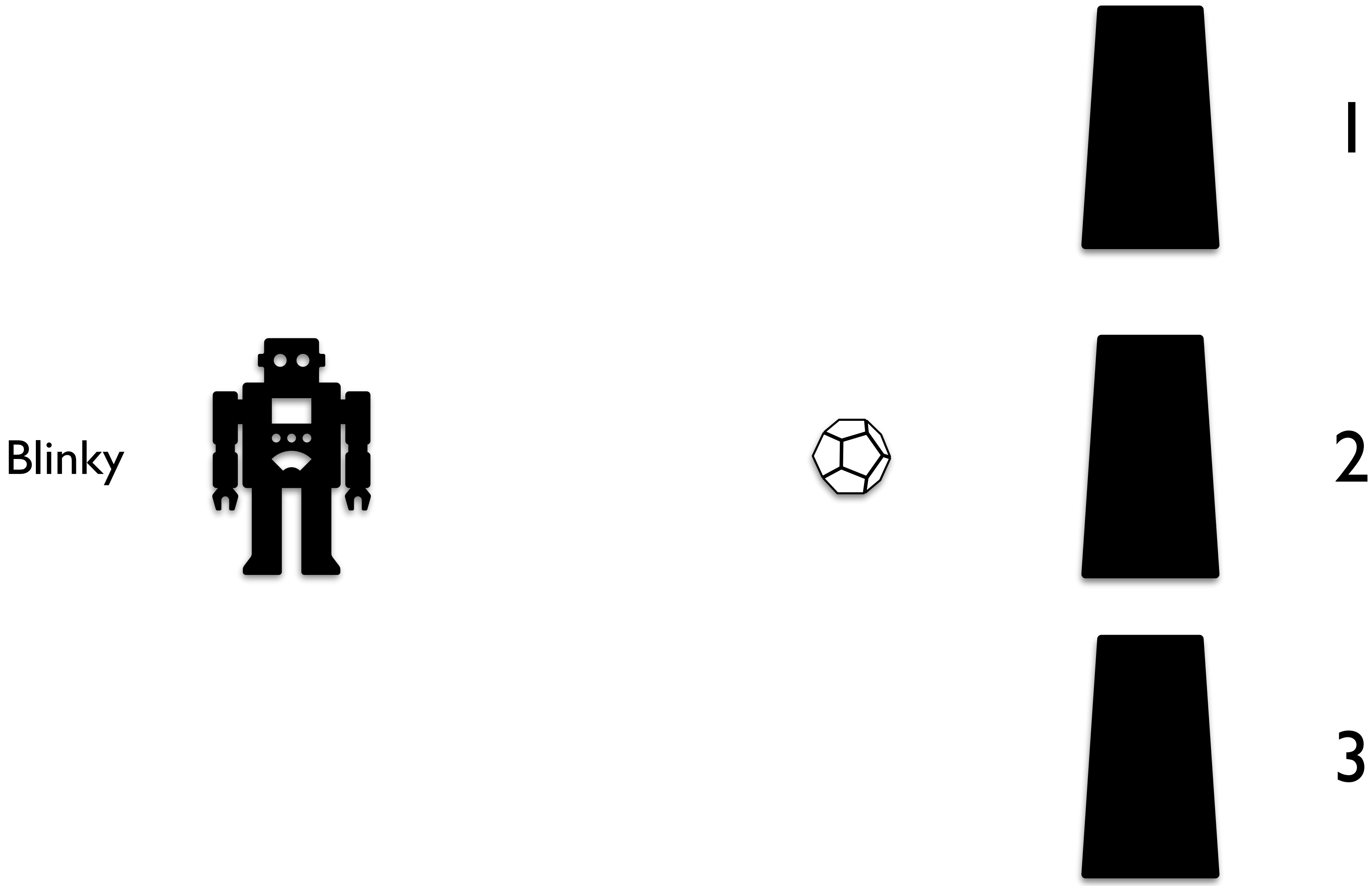

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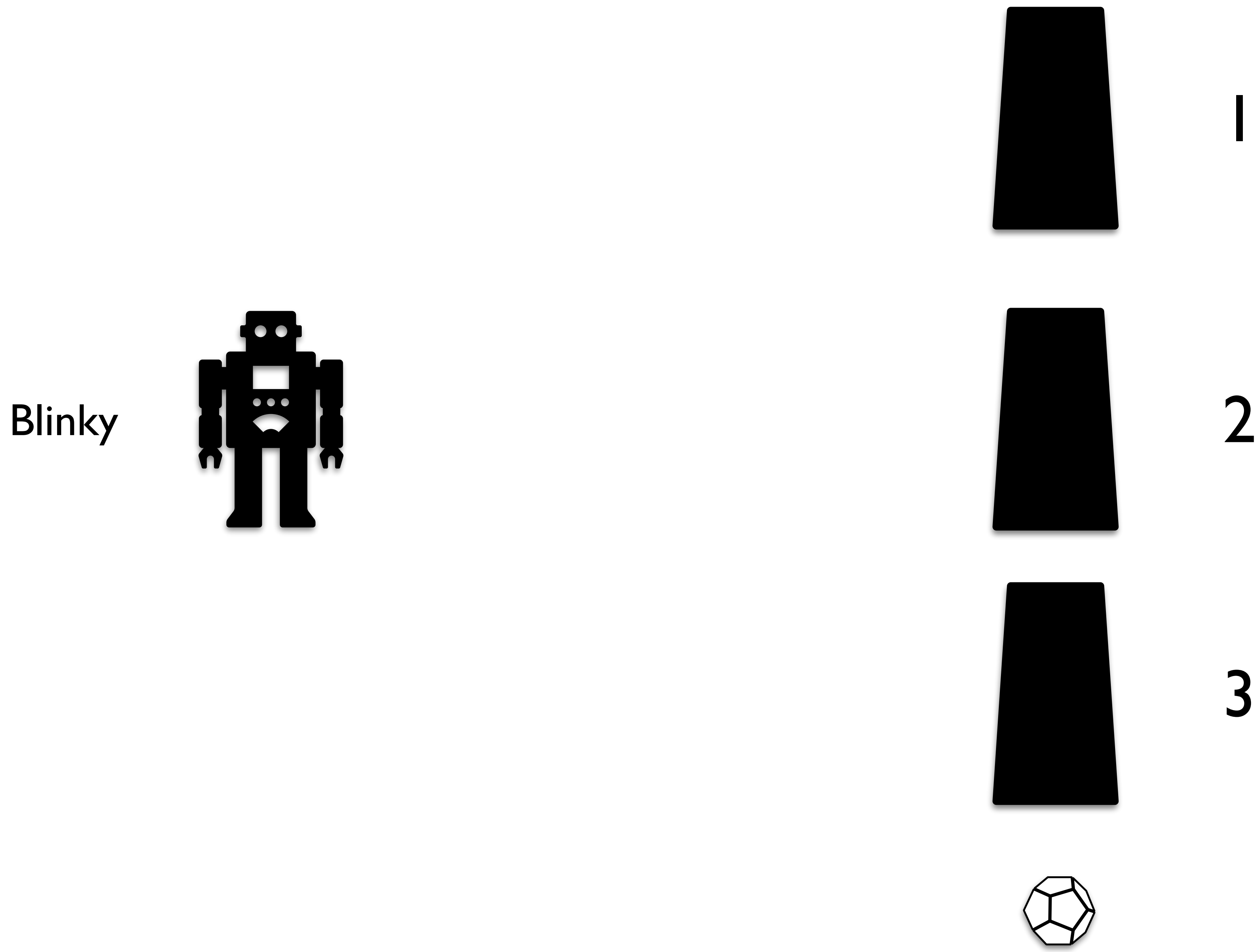

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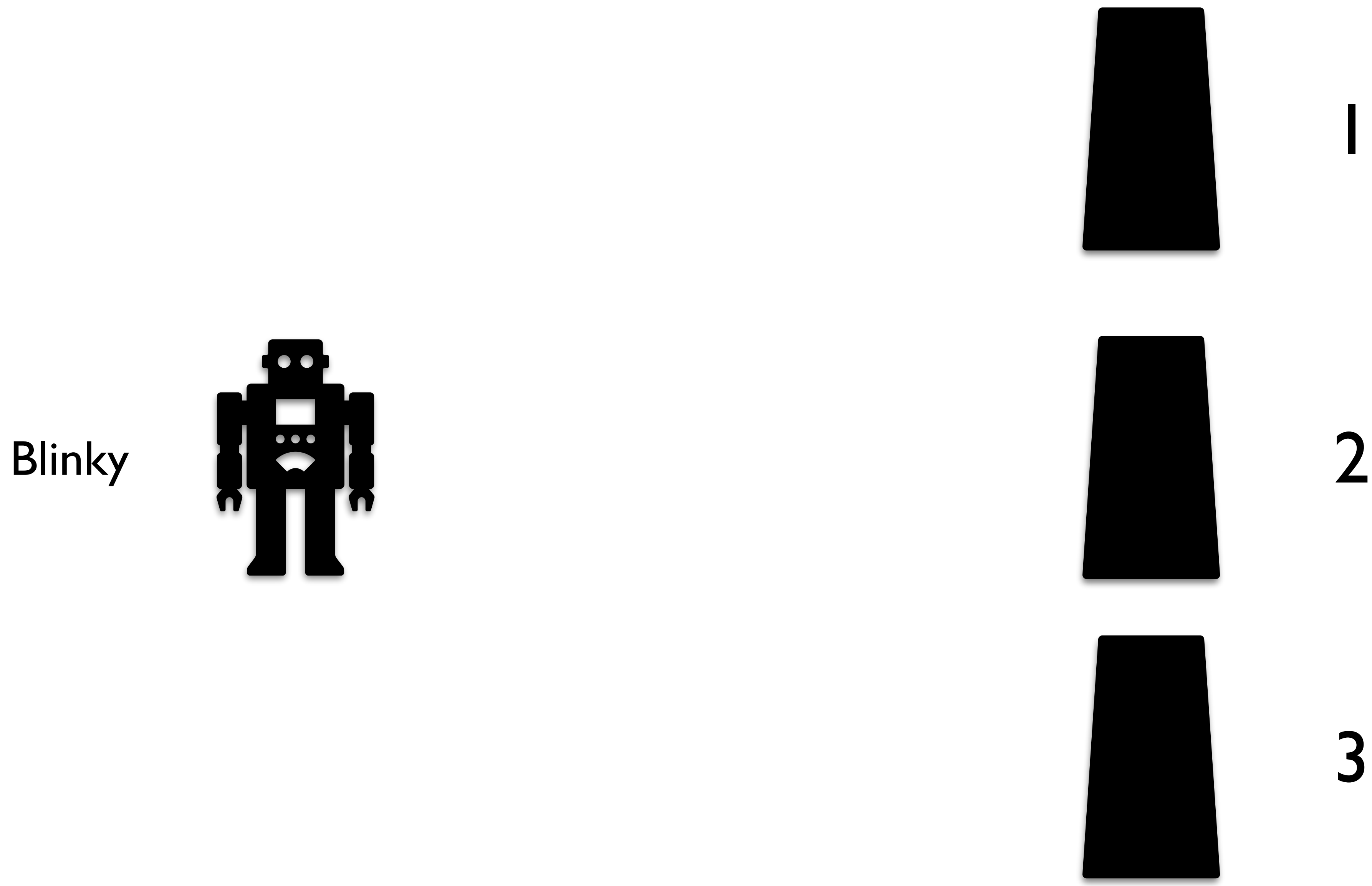
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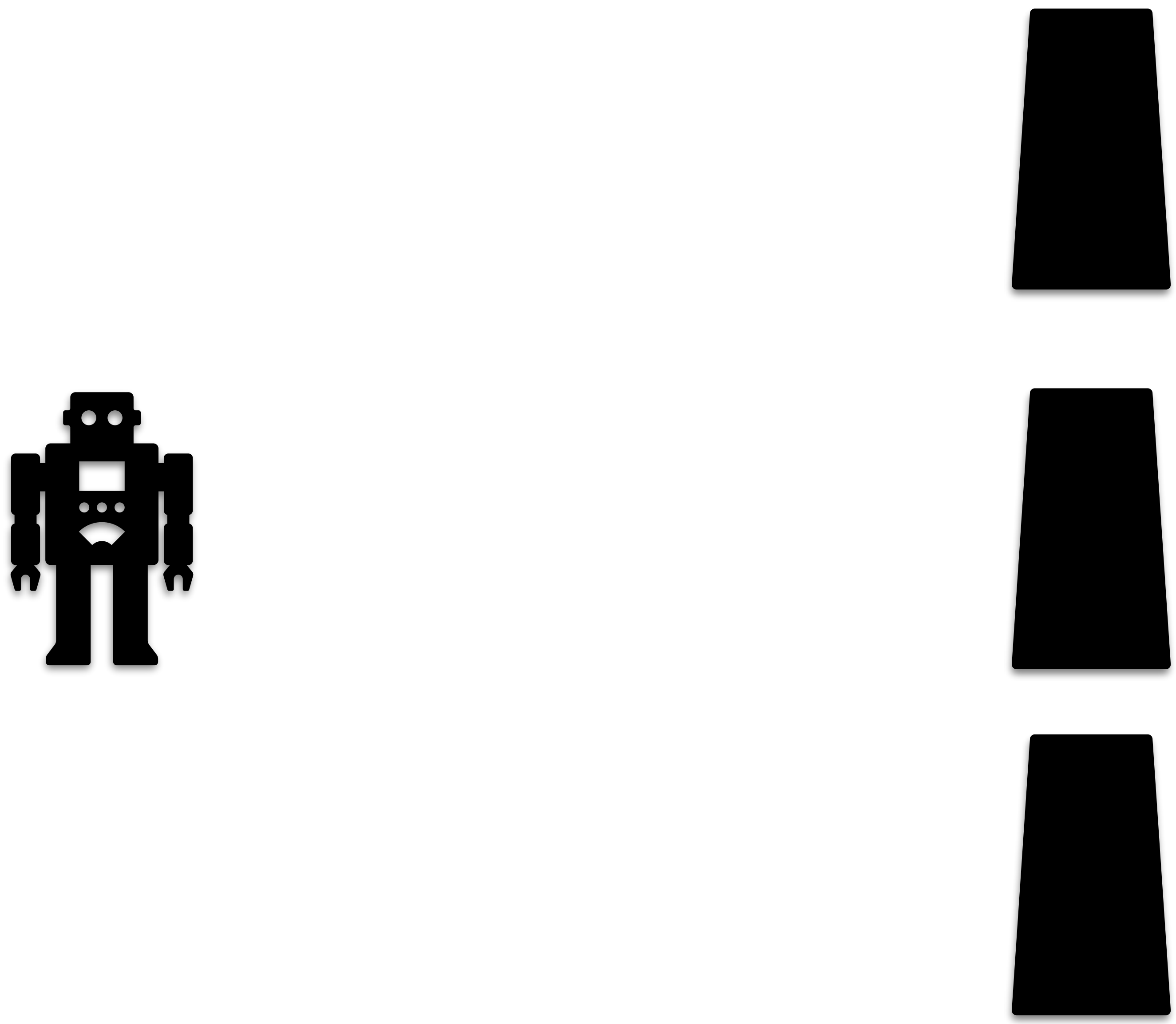

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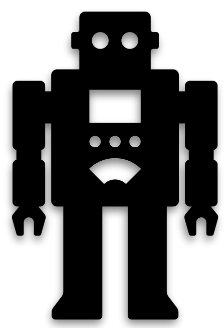

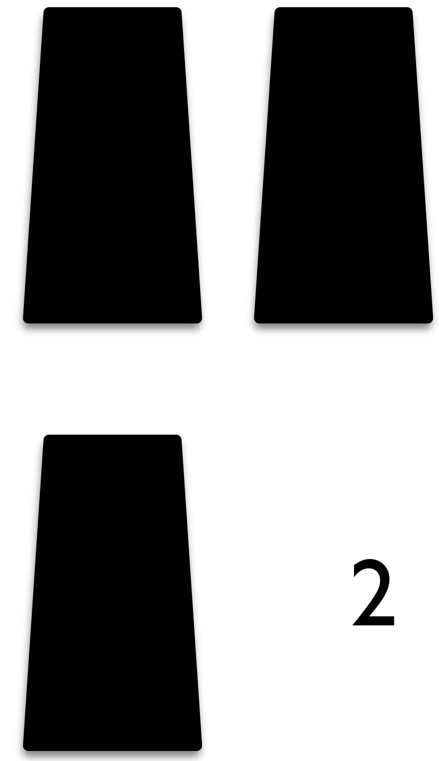

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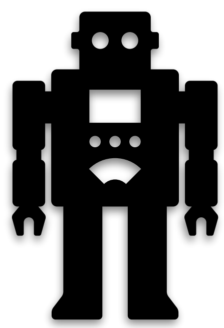

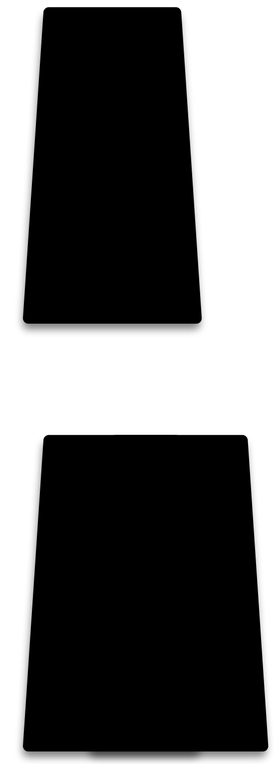

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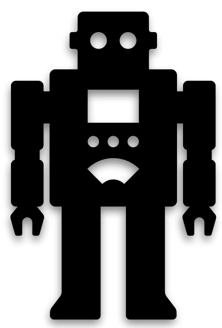

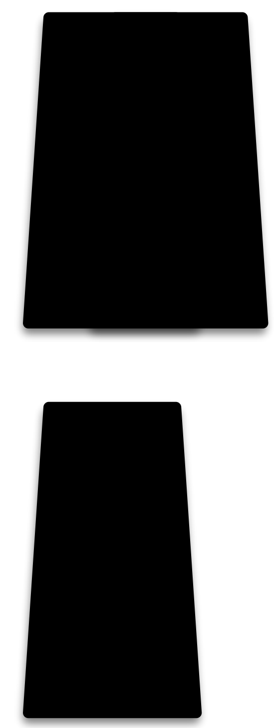

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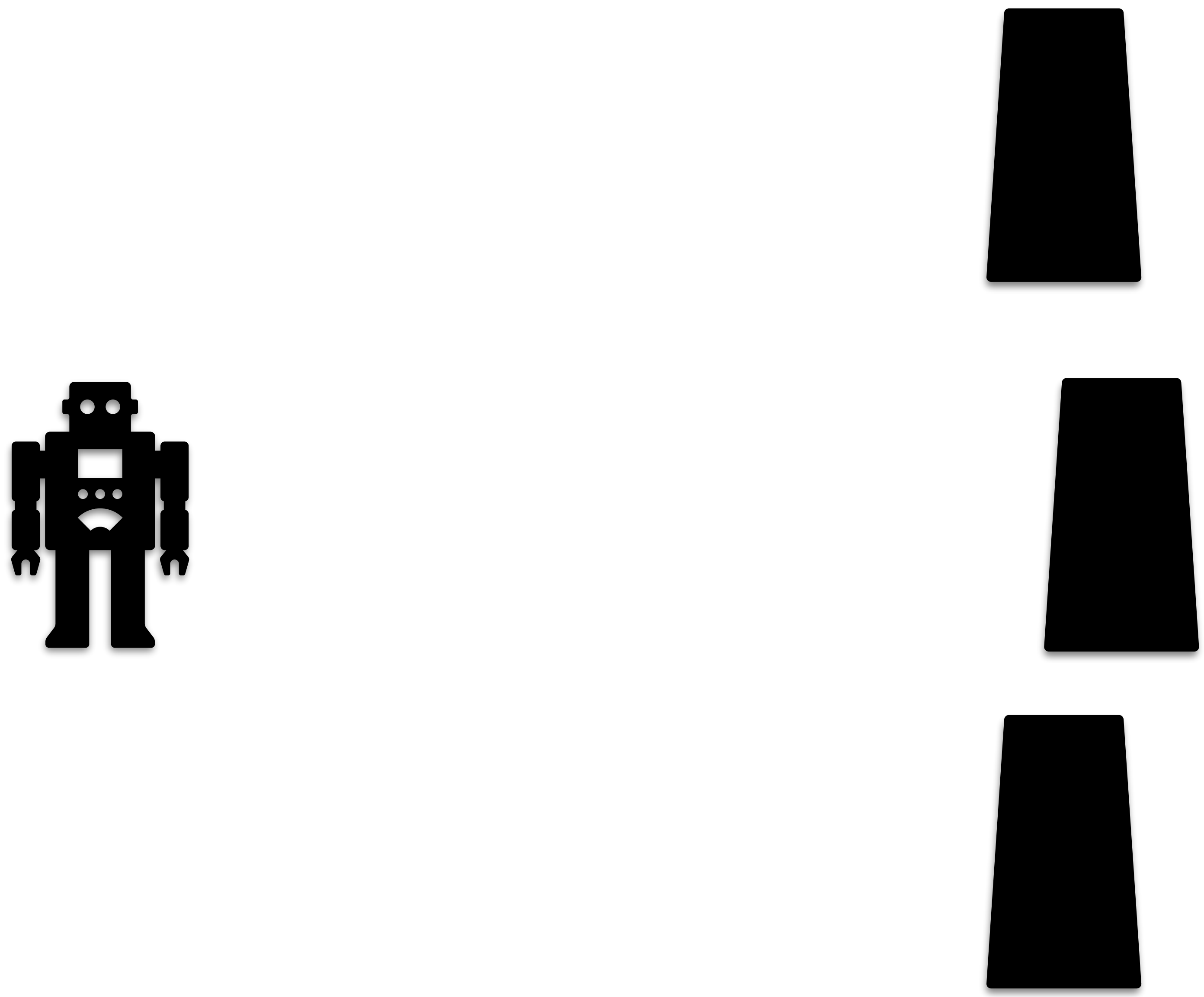

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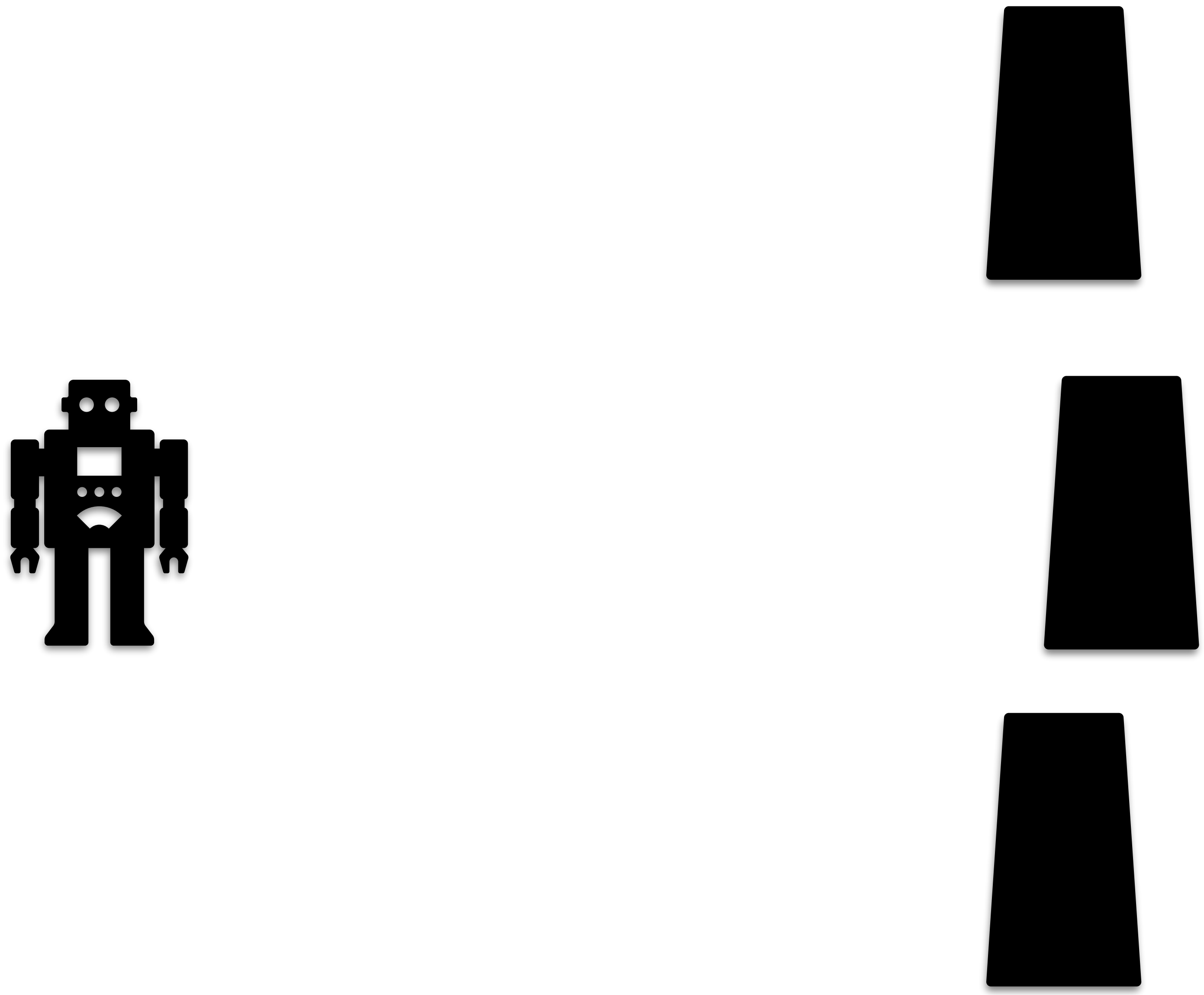

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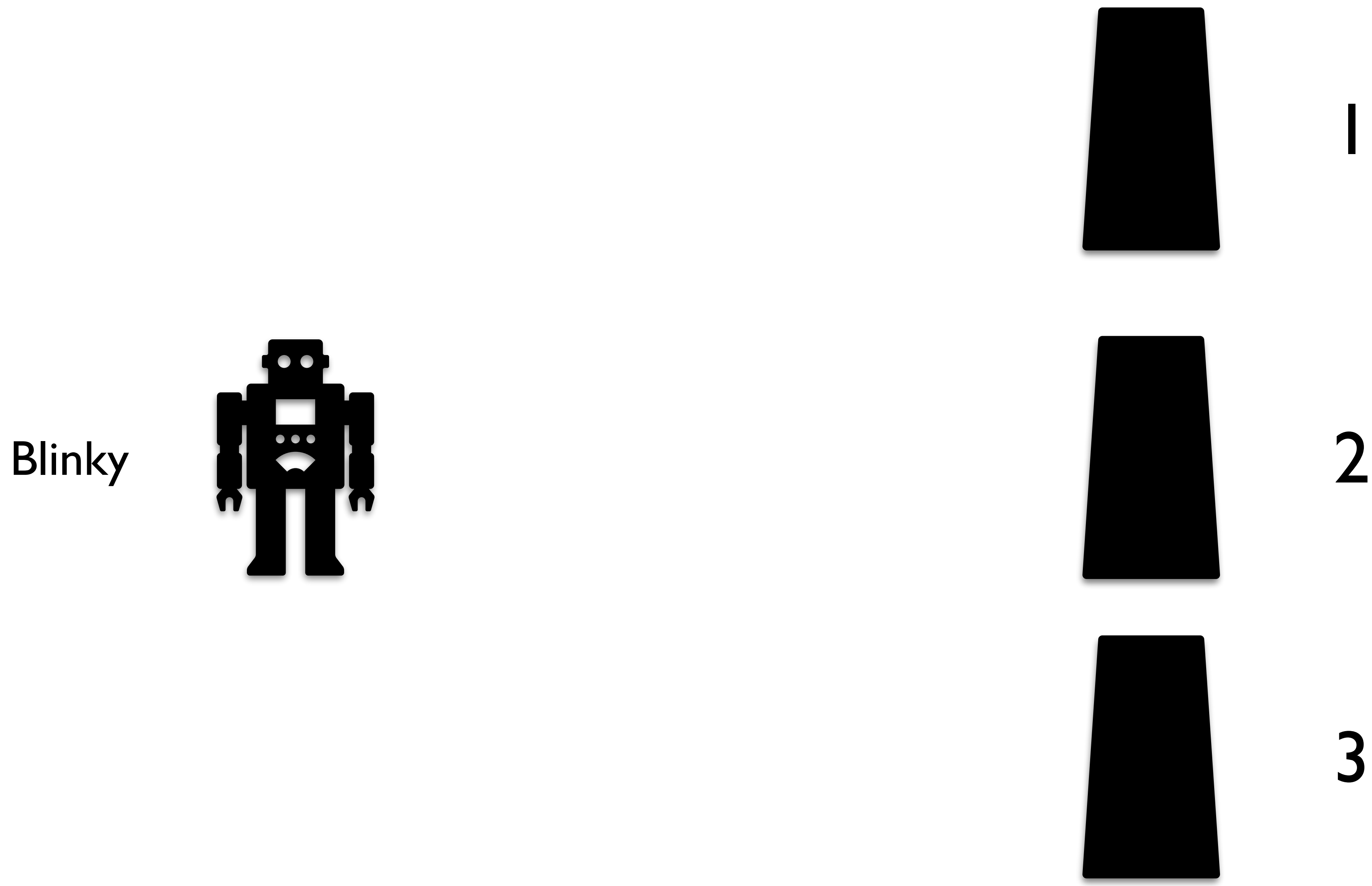
Blinky
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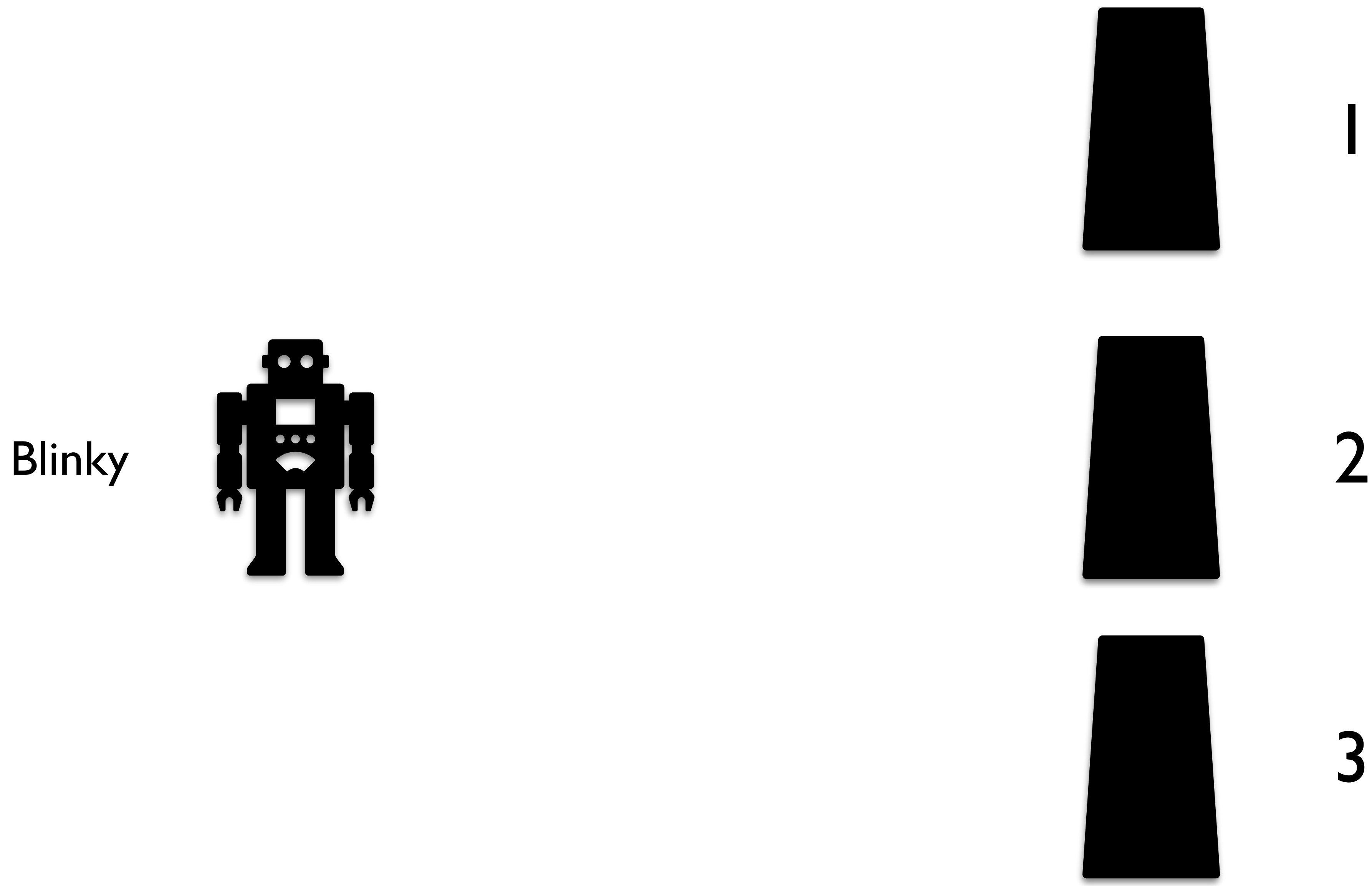
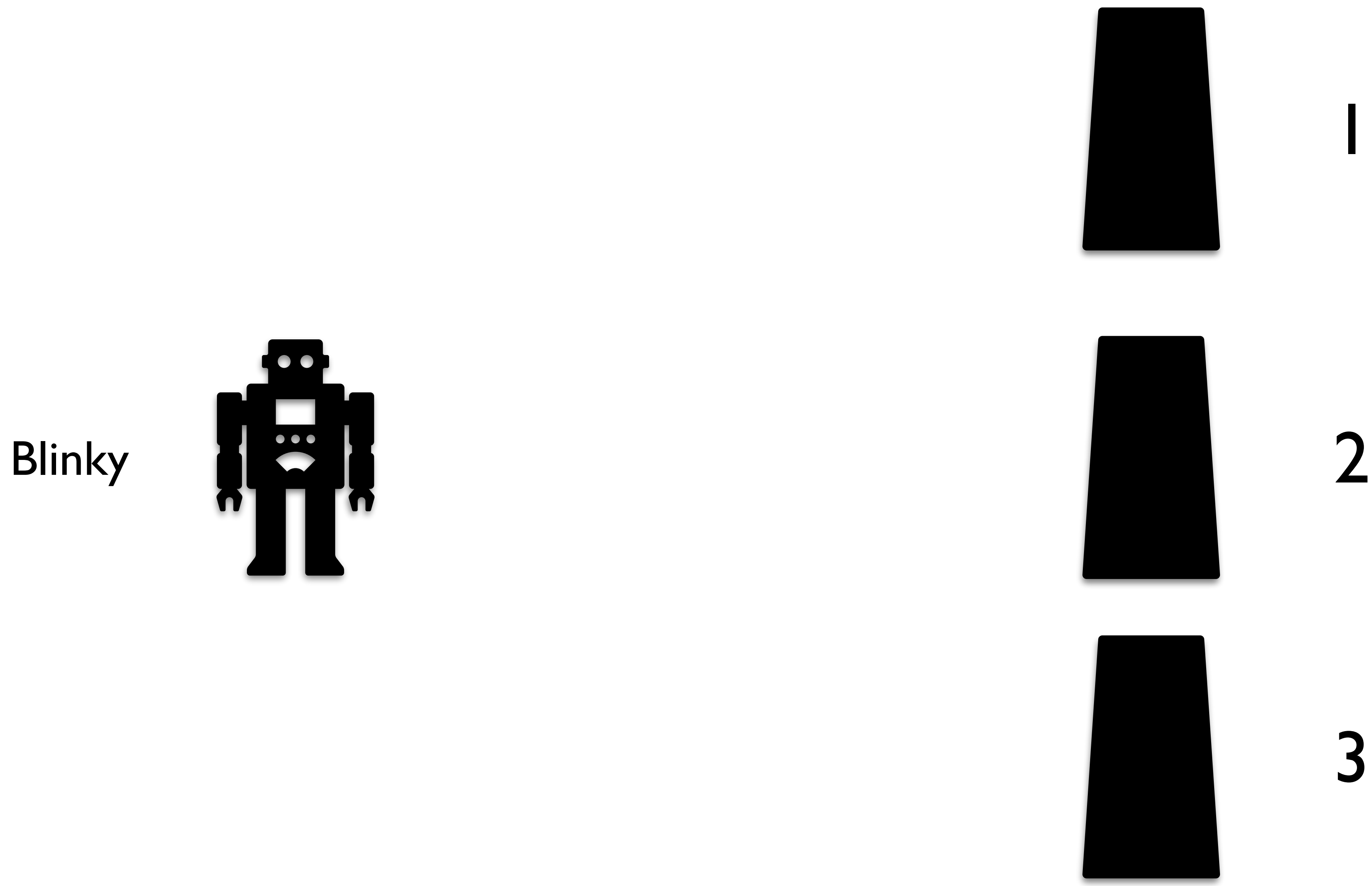
The ball is in the cup at location #1.



The ball is in the cup at location #1.

Loc(ball,1)

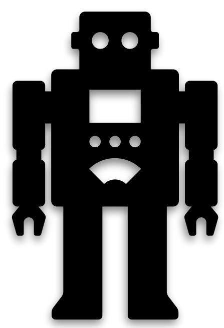


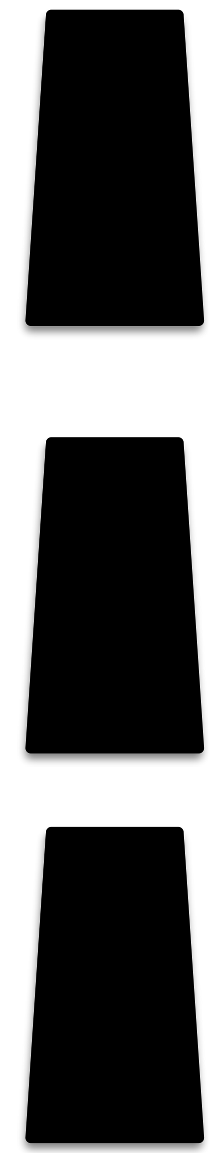
The ball is in the cup at location #1.

Loc(ball,1)

`(Loc ball 1)`

Blinky





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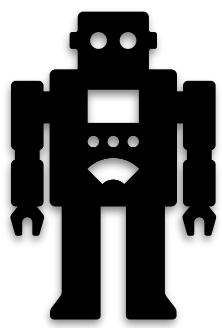
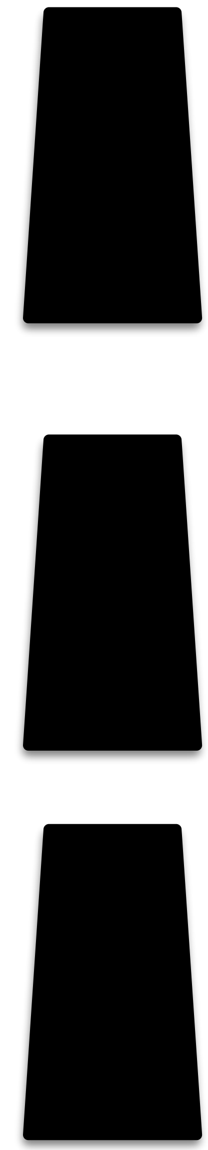
The ball is in the cup at location #1.

FALSE Loc(ball,1)

```
(Loc ball 1)
```

Blinky





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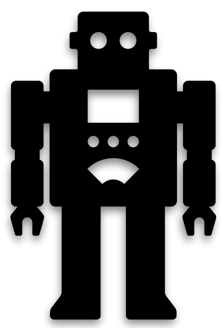
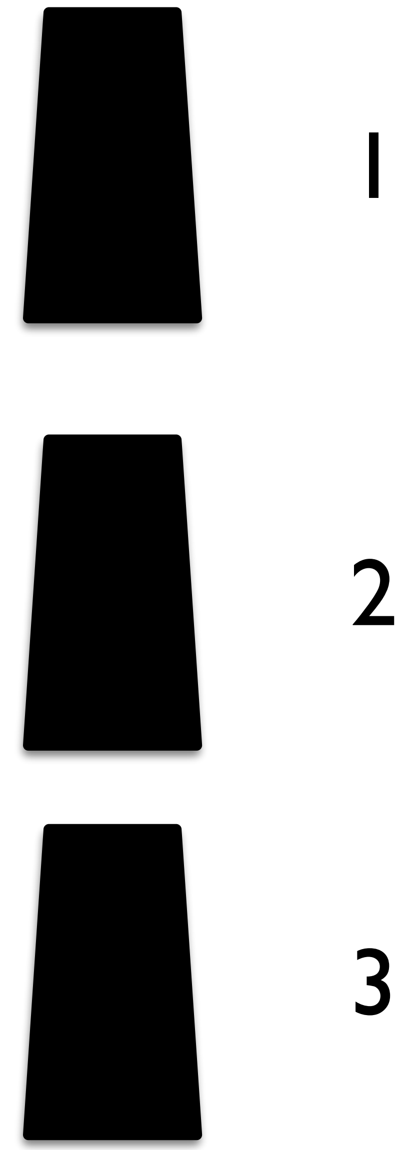
Blinky

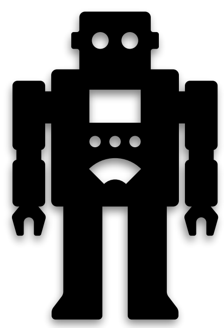


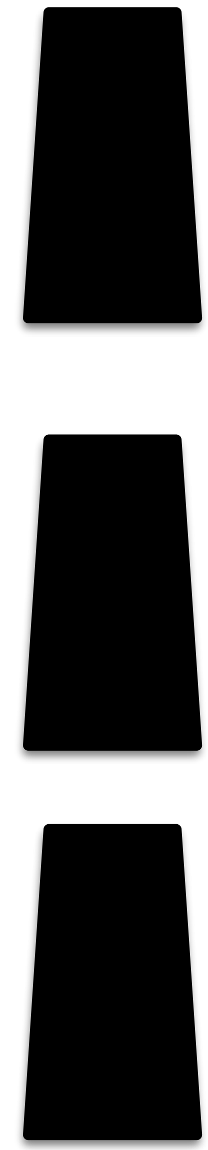


FALSE

`(Loc ball 1)`

Blinky





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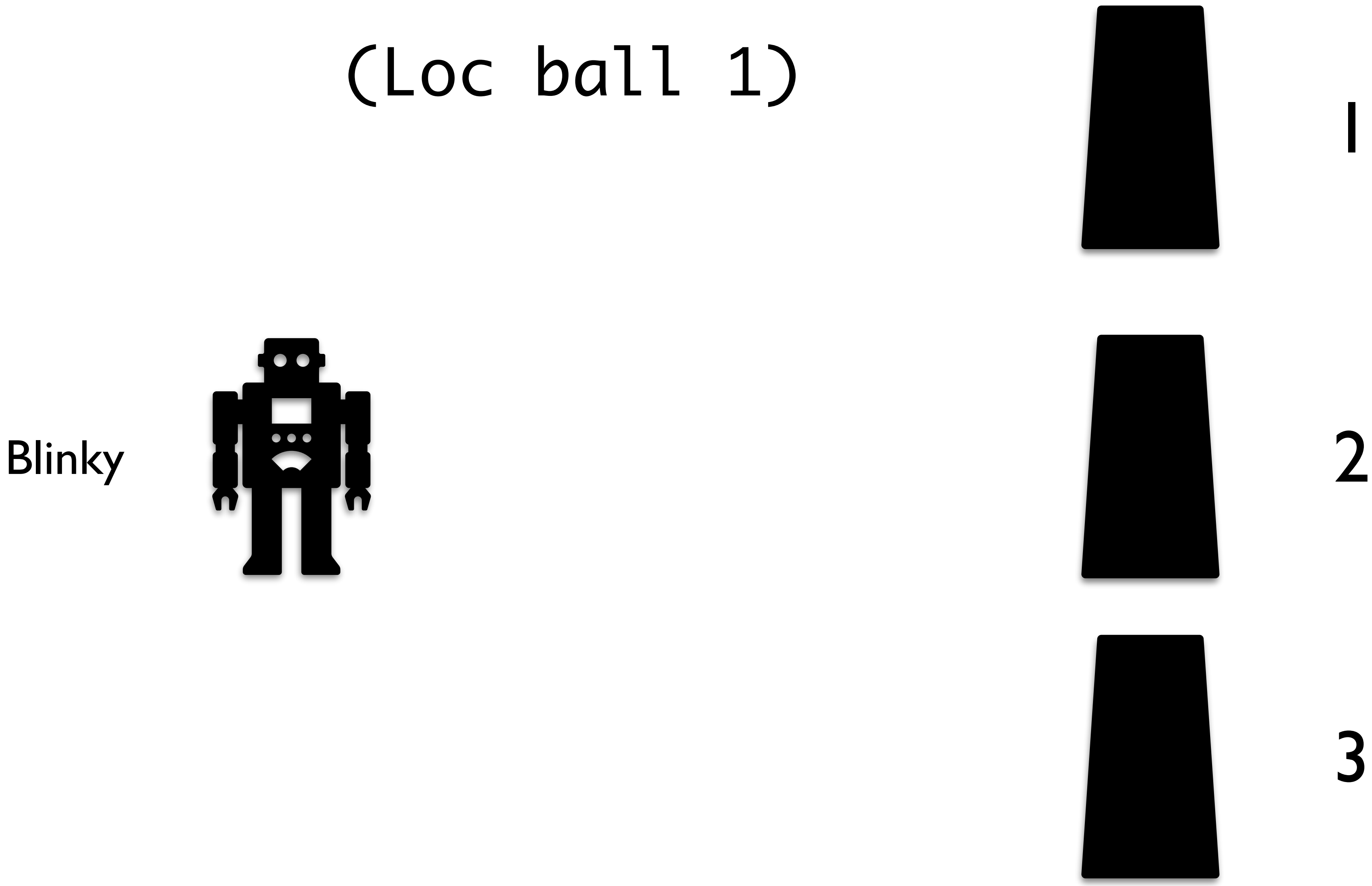
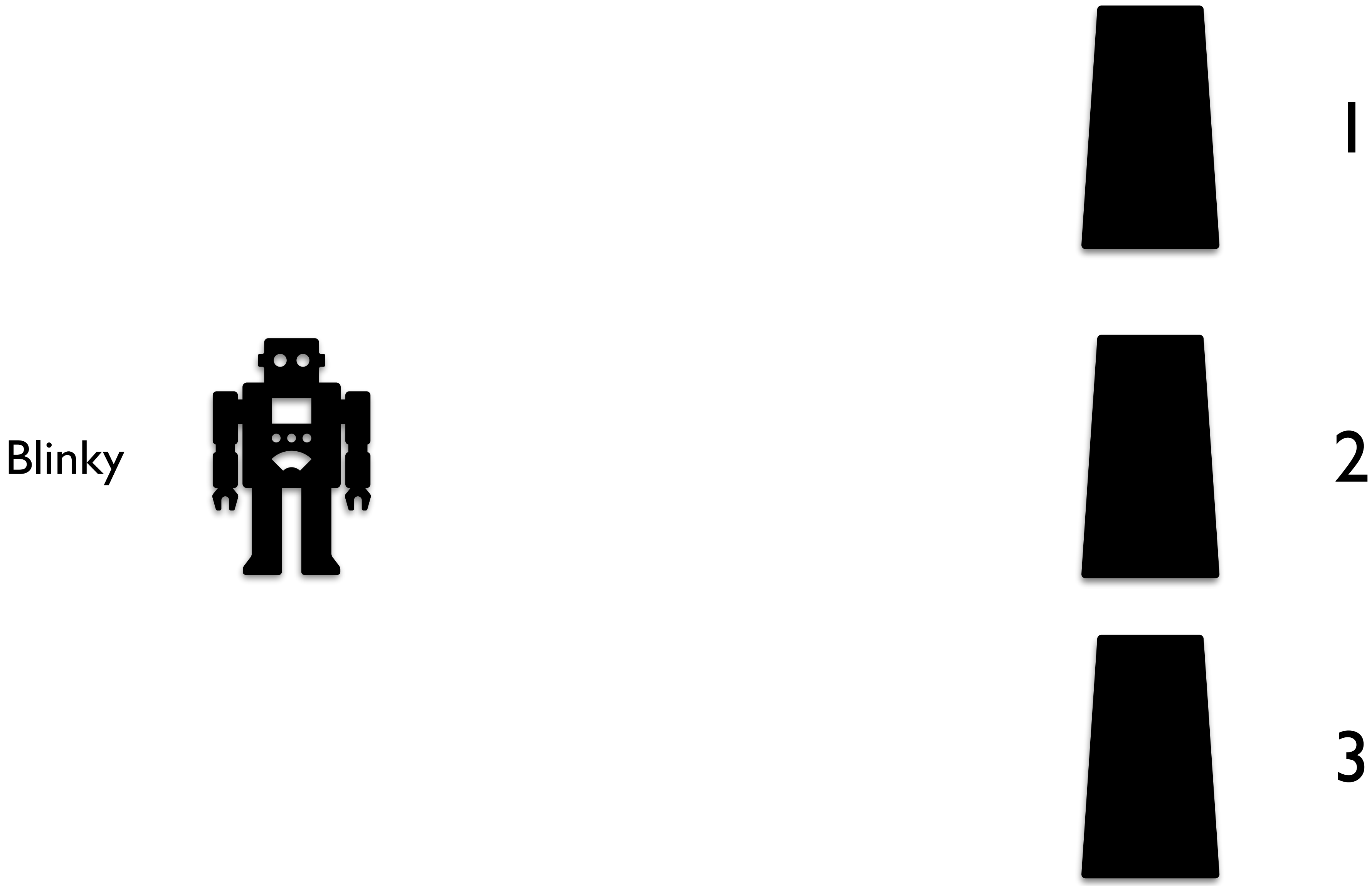
(Loc ball 1)




Blinky
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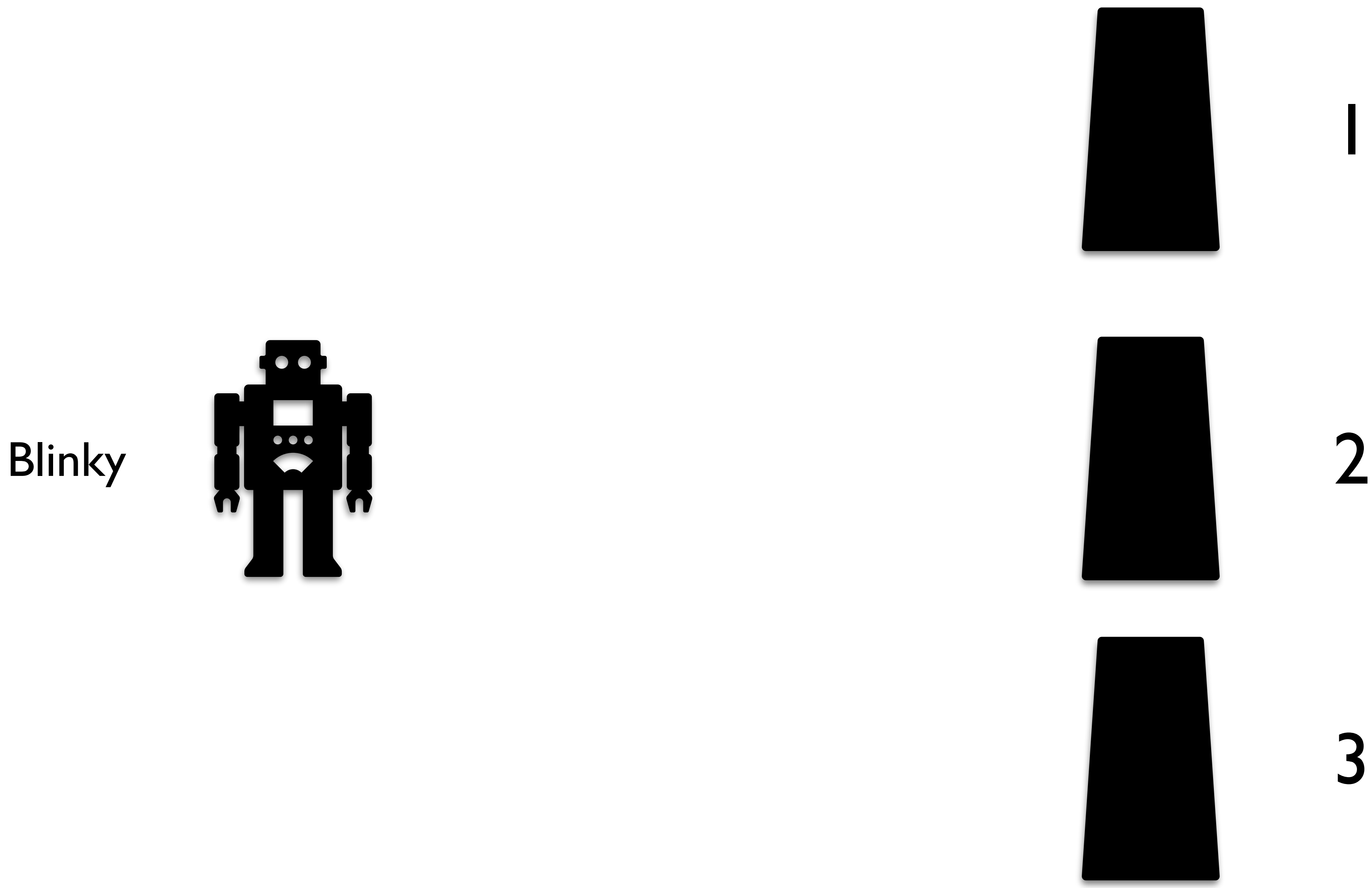
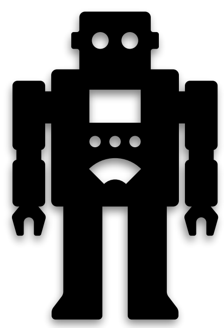
The ball is in the cup at location #1 or the ball is at location #3.

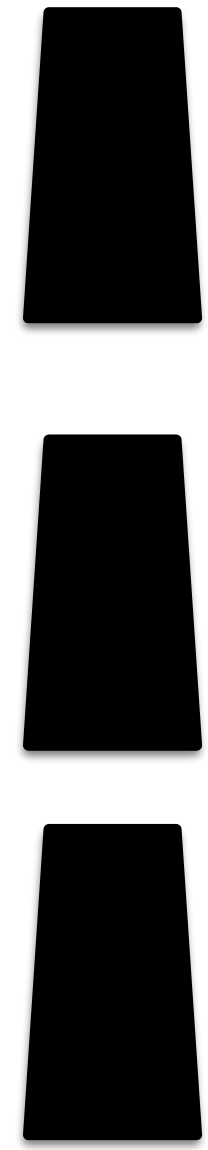


The ball is in the cup at location #1 or the ball is at location #3.

$$\mathrm{Loc}(\mathrm{ball},1) \vee \mathrm{Loc}(\mathrm{ball},3)$$

Blinky





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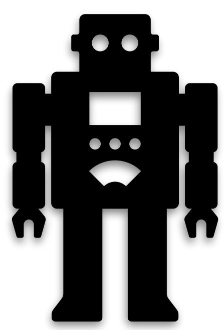
The ball is in the cup at location #1 or the ball is at location #3.

$$\text{Loc}(\text{ball},1) \vee \text{Loc}(\text{ball},3)$$

```
(or (Loc ball 1) (Loc ball 3))
```

1

Blinky 

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The ball is in the cup at location #1 or the ball is at location #3.

FALSE $\mathrm{Loc}(\mathrm{ball}, 1) \lor \mathrm{Loc}(\mathrm{ball}, 3)$

```
(or (Loc ball 1) (Loc ball 3))
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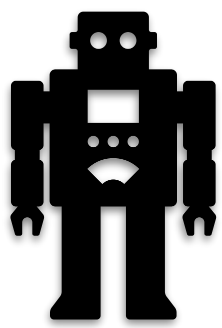
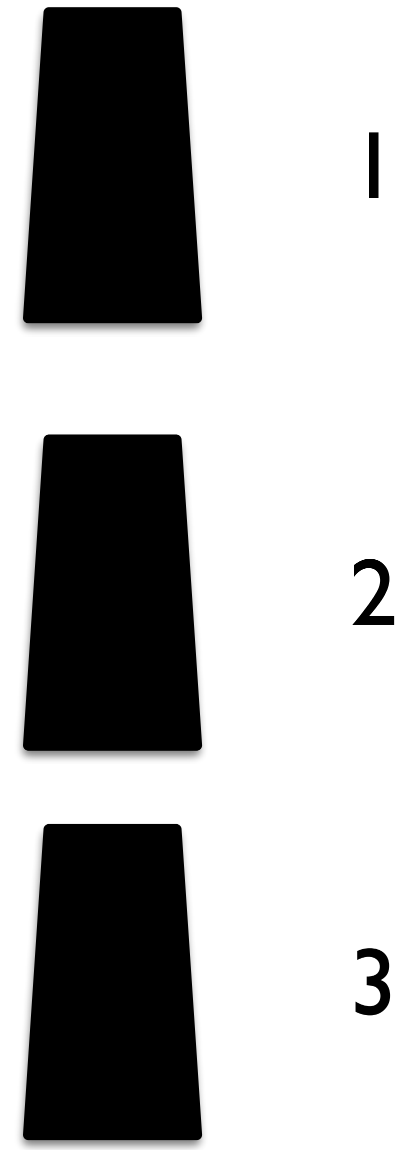
Blinky





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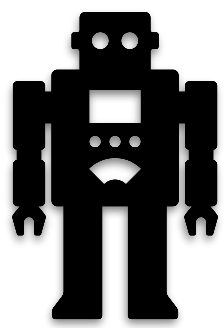

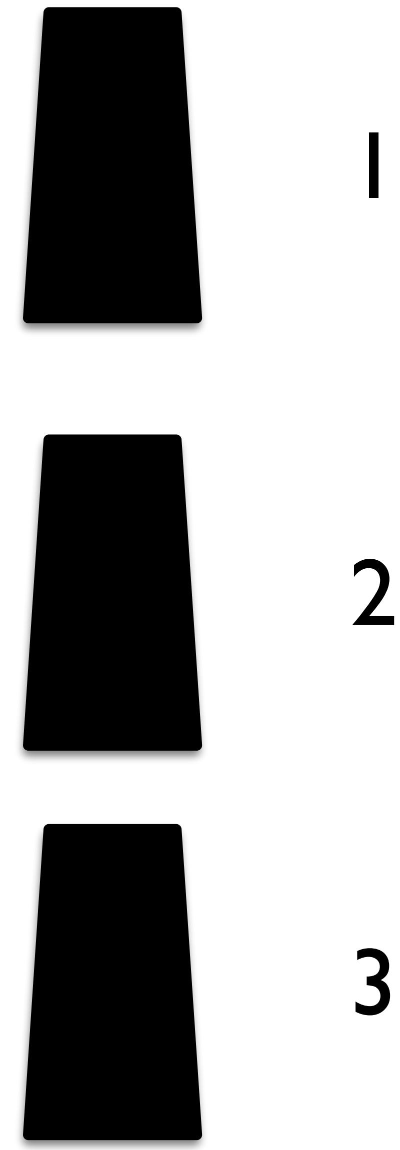

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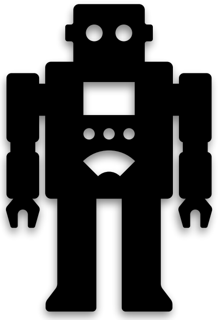

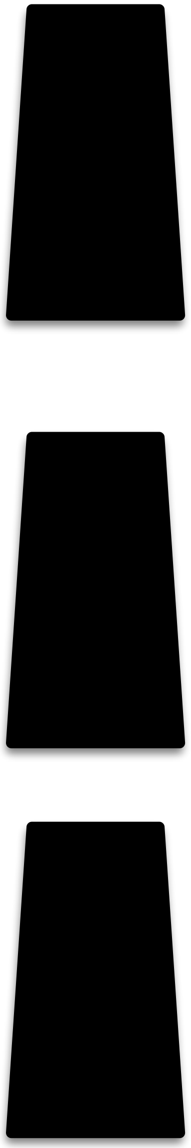

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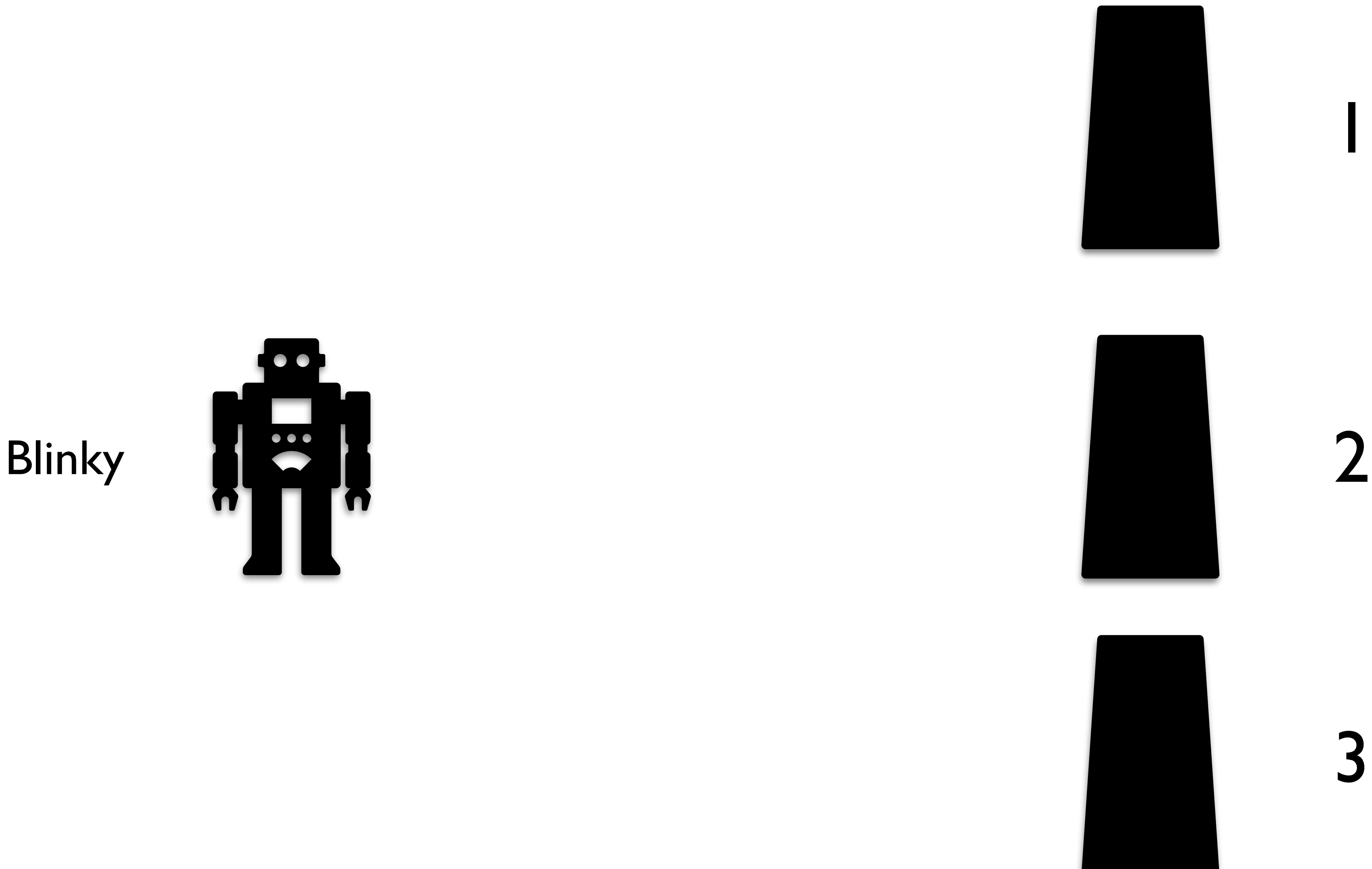
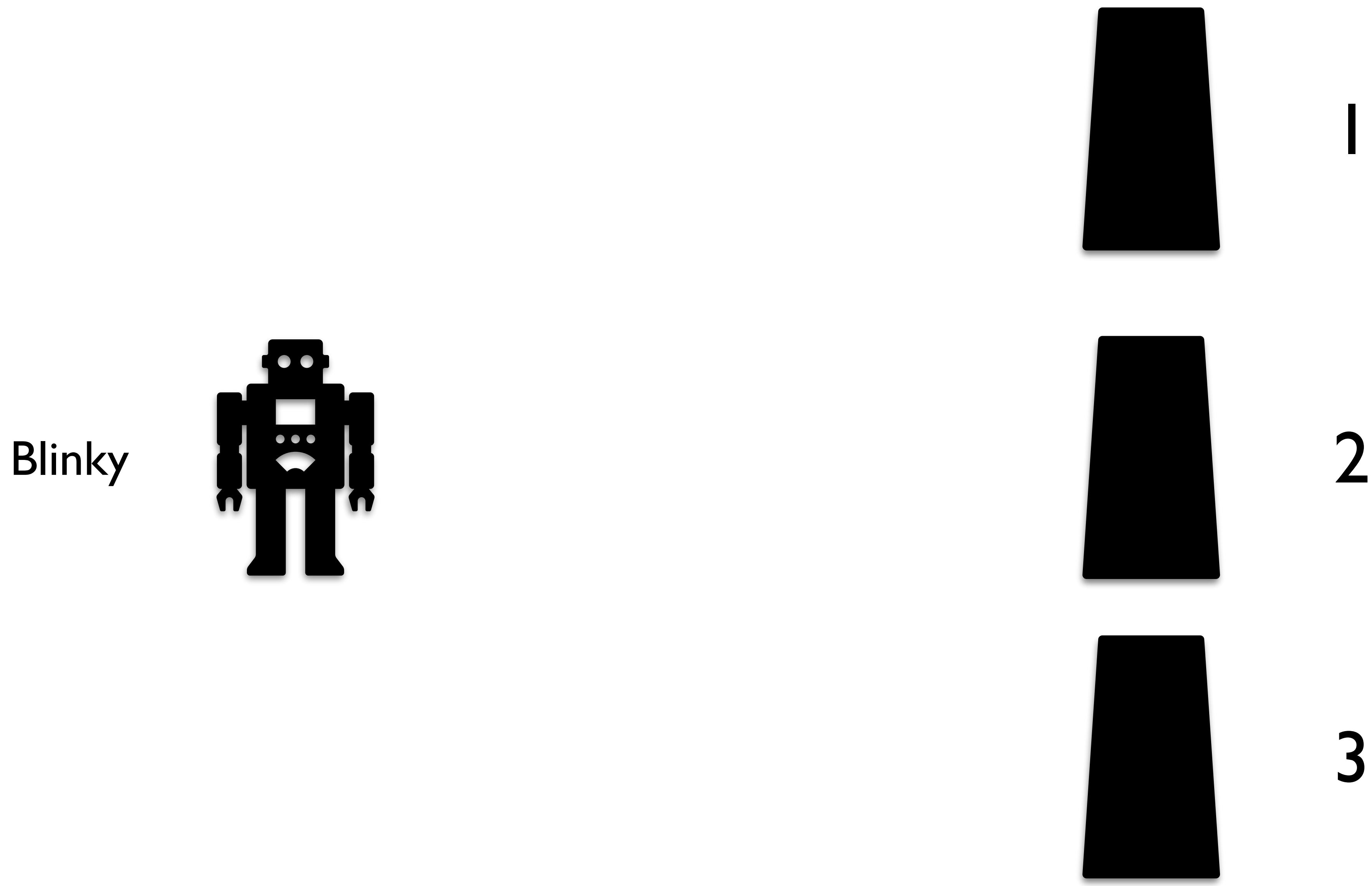
FALSE

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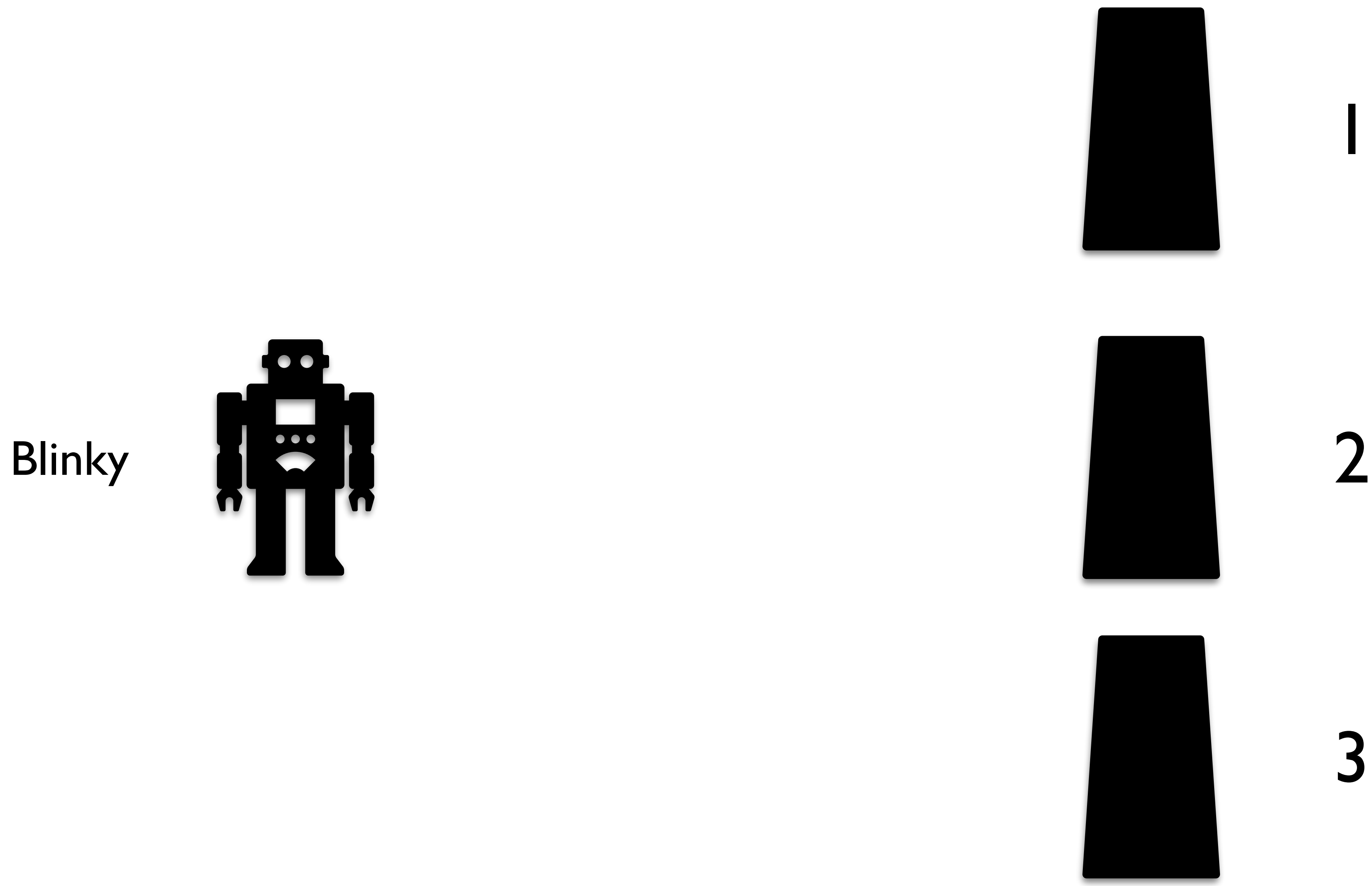
Blinky believes that the ball is in the cup at location #1.



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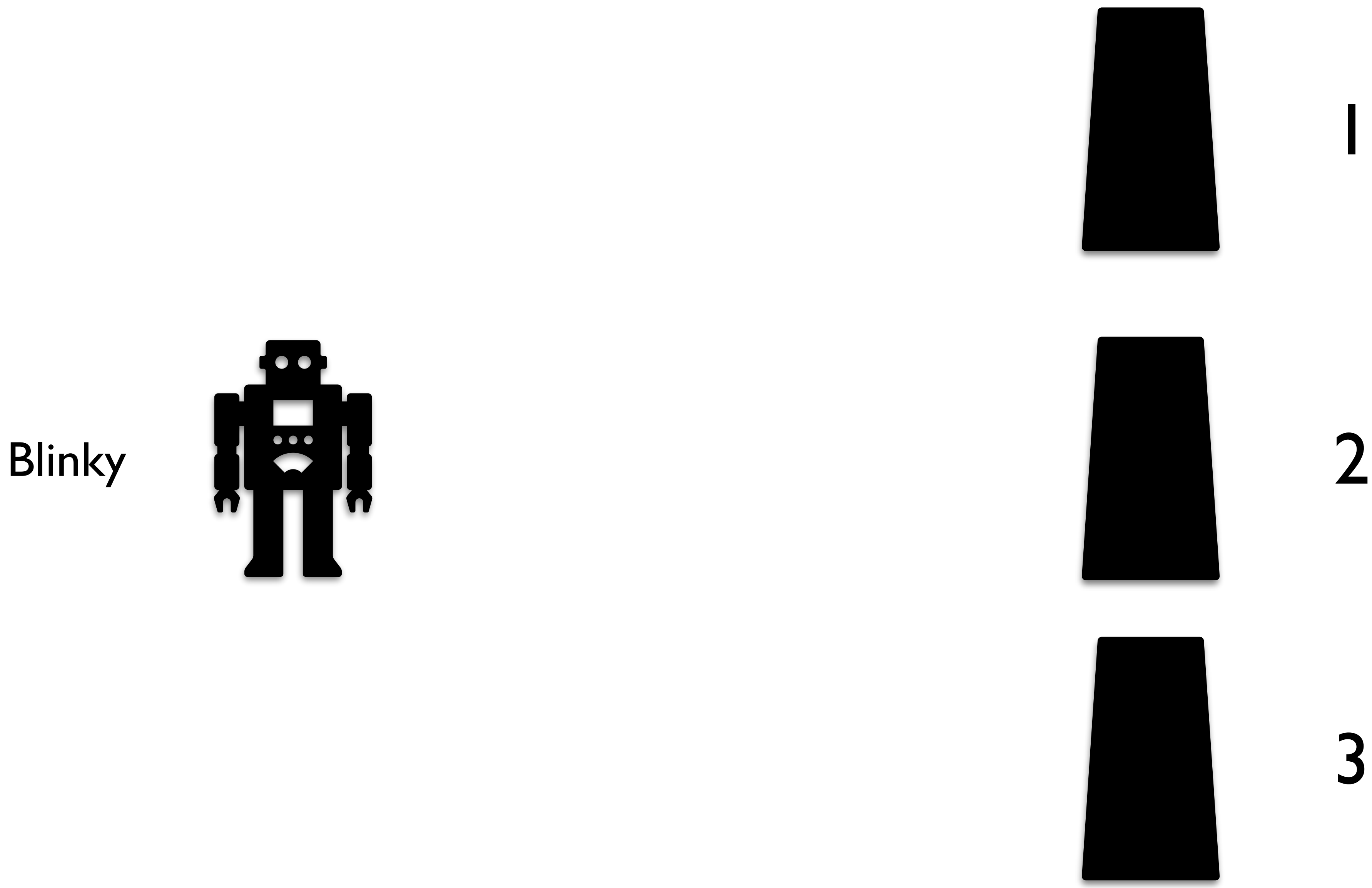
$$B(\text{blinky}, \text{Loc}(\text{ball}, 1))$$



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(Believes! t blinky (Loc ball 1))
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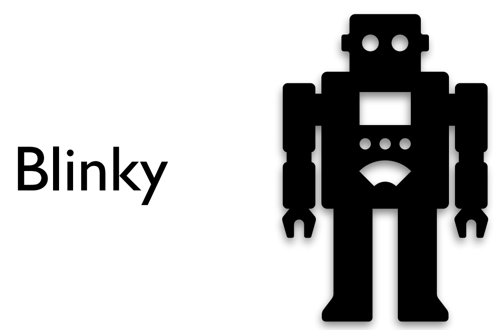

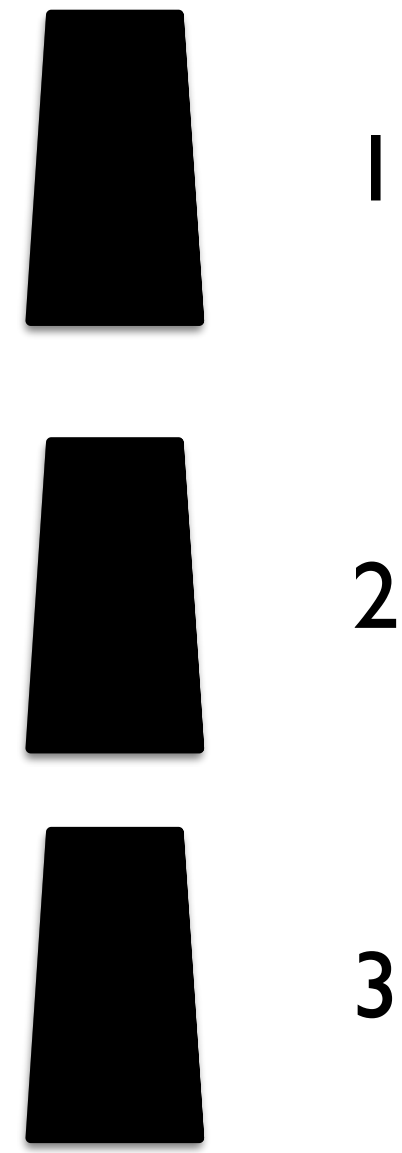

Blinky believes that the ball is in the cup at location #1.

??? B(blinky, Loc(ball,1))

```
(Believes! t blinky (Loc ball 1))
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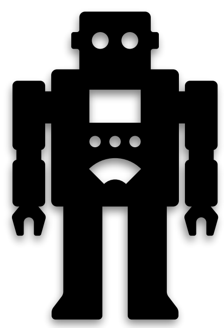
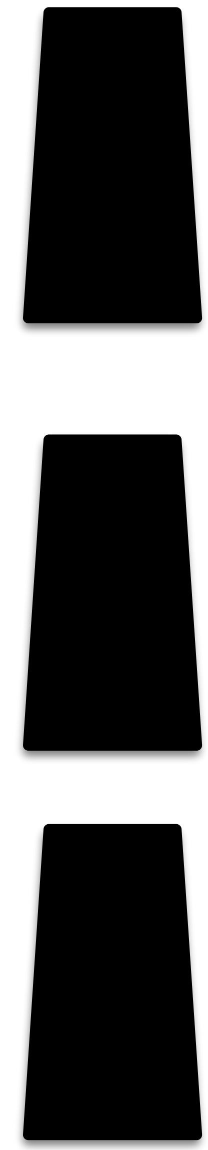
Blinky





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Blinky believes that the ball is in the cup at location #1.

???

$$B(blinky, Loc(ball, 1))$$

```
(Believes! t blinky (Loc ball 1))
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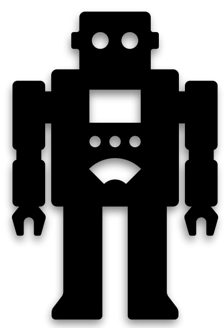
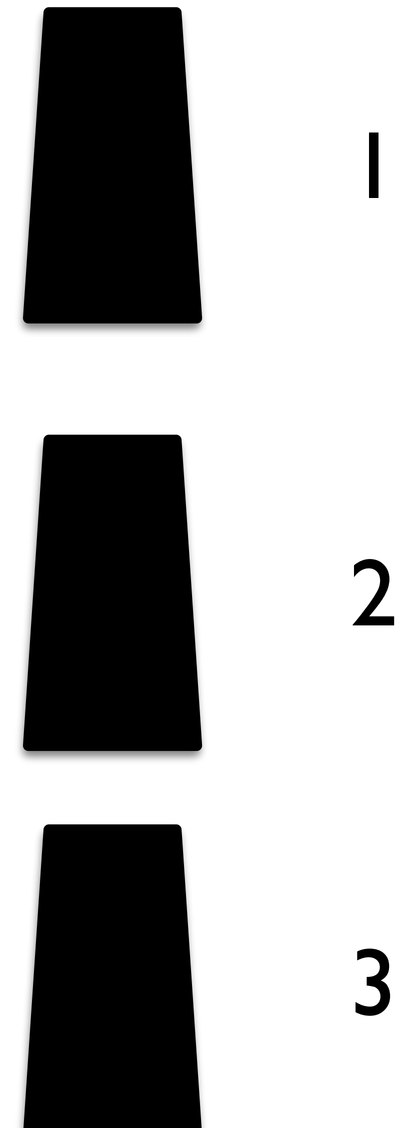
Blinky





In extensional logics, what is denoted is conflated with meaning (the latter being naïvely compositional), and intensional attitudes like *believes*, *knows*, *hopes*, *fears*, etc cannot be represented and reasoned over smoothly (e.g. without fear of inconsistency rising up).

Review: Encapsulation

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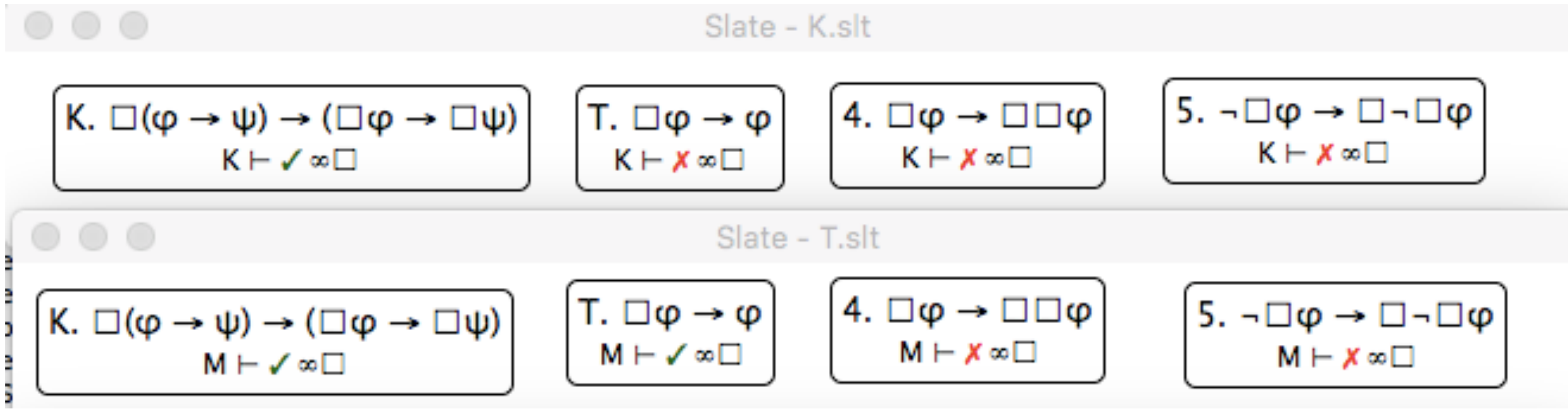

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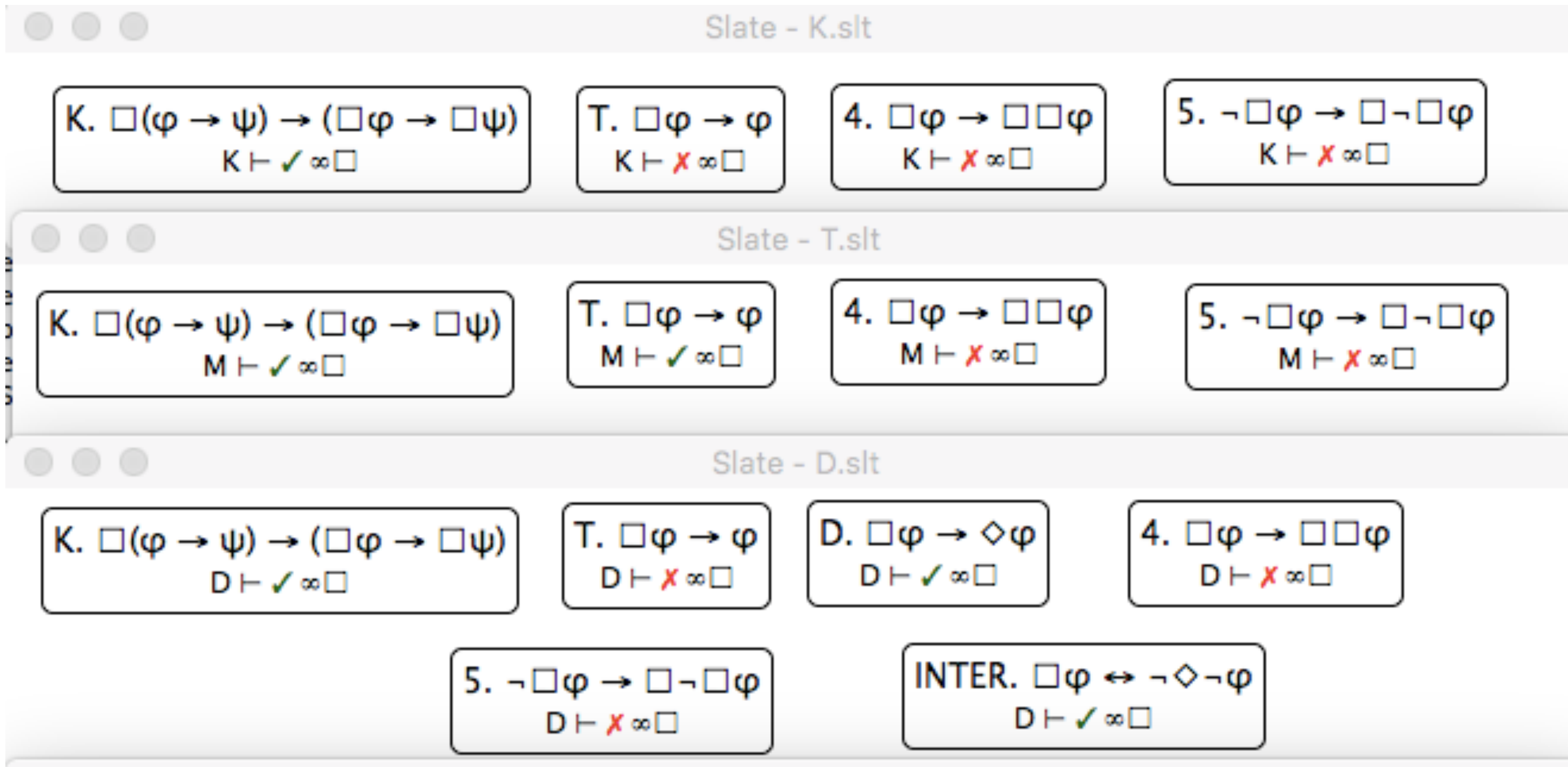

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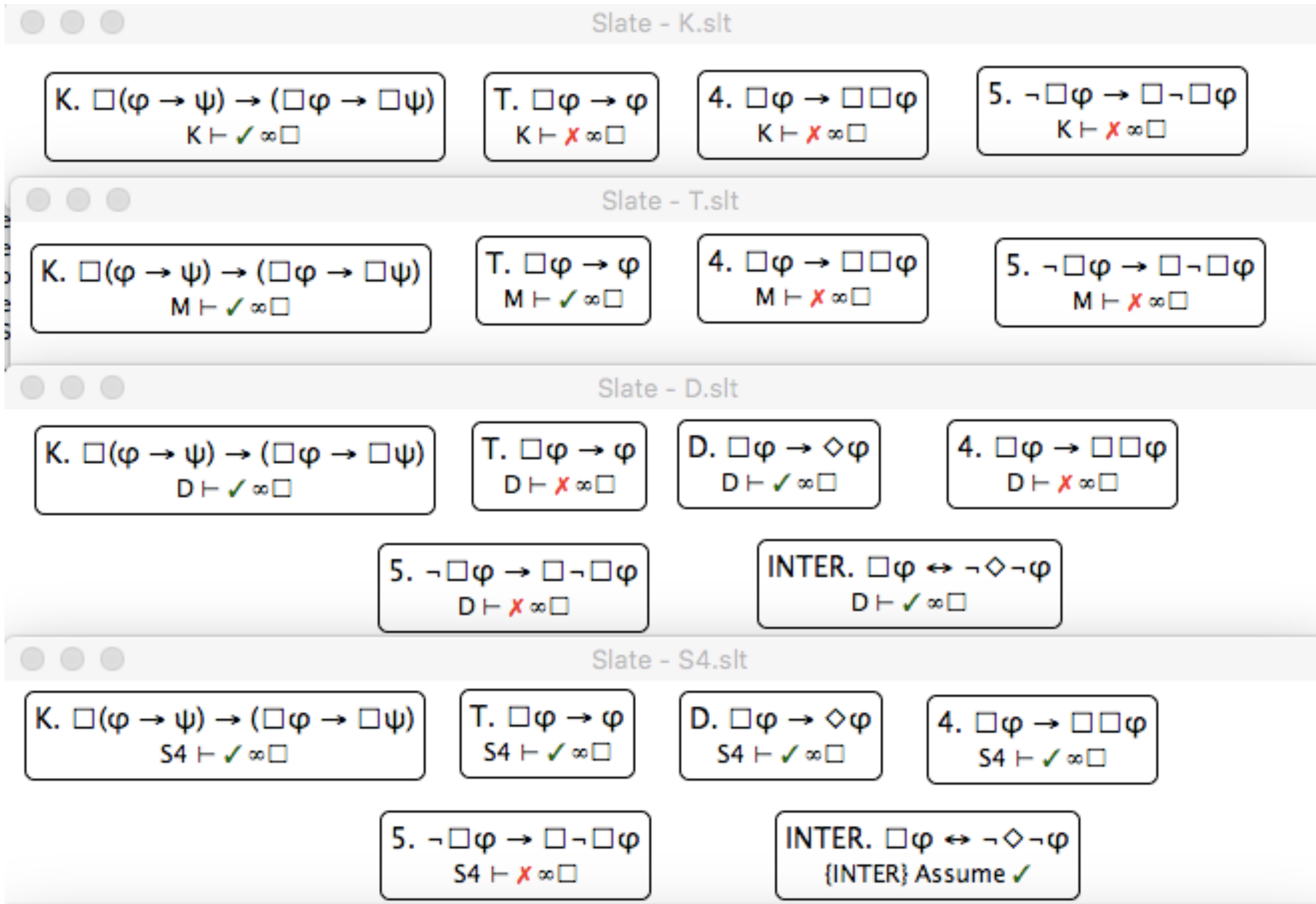

Review: Encapsulation

K

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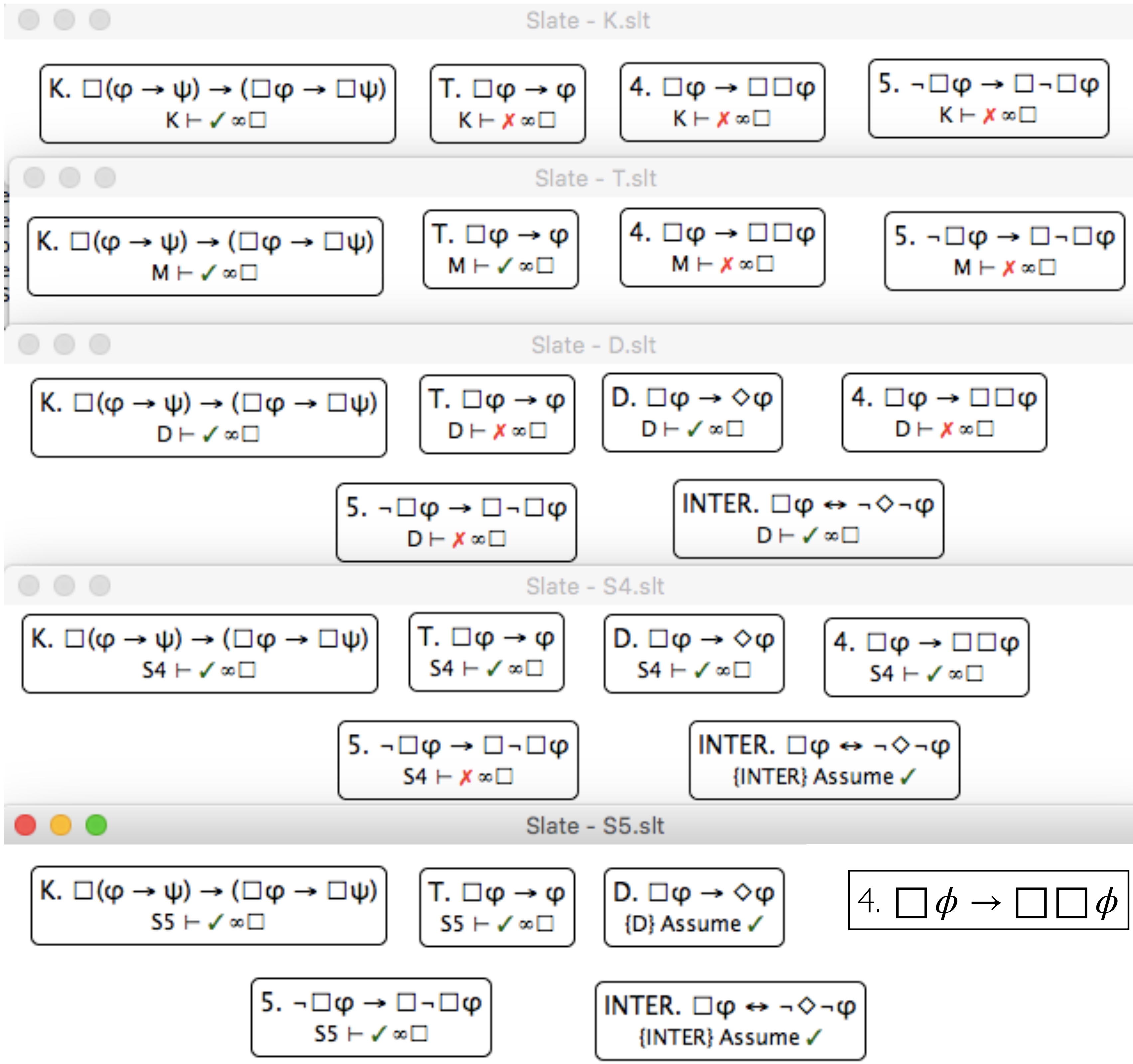
D

4 = S4

5 = S5



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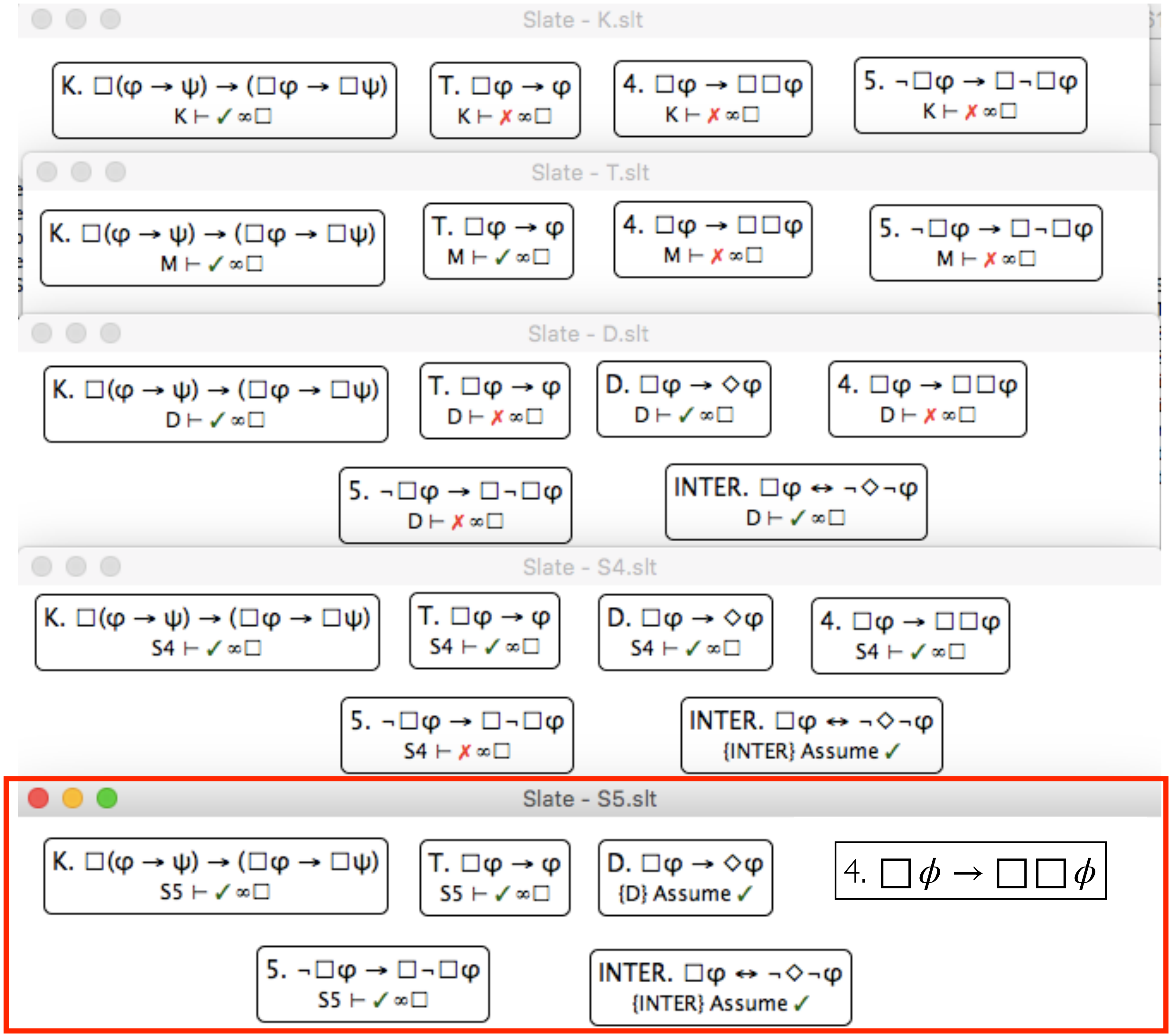

S5 ...

The Characteristic Axiom

$$\Diamond\phi \rightarrow \Box\Diamond\phi$$

The Characteristic Axiom

$$\Diamond\phi \rightarrow \Box\Diamond\phi$$

$$\neg\Box\psi \rightarrow \Box\neg\Box\psi$$

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$$\neg\Box\neg\phi \rightarrow \Box\neg\Box\neg\phi$$

The Characteristic Axiom

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$$\neg\Box\psi \rightarrow \Box\neg\Box\psi$$

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$$\Diamond\phi \rightarrow \Box\Diamond\phi$$

Nice S5 Reduction Lemmas

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<u>The Four Possible Pairs</u>

$\Box\,\Box$

$\Diamond\,\Diamond$

$\Box\,\Diamond$

$\Diamond\,\Box$

Nice S5 Reduction Lemmas

<u>The Four Possible Pairs</u>

$\Box\ \Box$

$\Diamond\ \Diamond$

$\Box\ \Diamond$

$\Diamond\ \Box$

<u>The Four Reduction Principles</u>

$$\Box\phi \leftrightarrow \Box\Box\phi$$

$$\Diamond\phi \leftrightarrow \Diamond\Diamond\phi$$

$$\Box\phi \leftrightarrow \Diamond\Box\phi$$

$$\Diamond\phi \leftrightarrow \Box\Diamond\phi$$

(where $\phi \in \mathscr{L}_{pc}$)

Nice S5 Reduction Lemmas

The Four Possible Pairs

$\Box\,\Box$
$\Diamond\,\Diamond$
$\Box\,\Diamond$
$\Diamond\,\Box$

The Four Reduction Principles

$$\Box\phi \leftrightarrow \Box\Box\phi$$

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$$\Diamond\phi \leftrightarrow \Box\Diamond\phi$$

(where $\phi \in \mathscr{L}_{pc}$)

(verify in HS®)

Quantificational $S5_1$ …

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Quantificational S5 I …

Easy peasy: Marry **PS5** + $\mathscr{L}_1$!

Quantificational S5$_1$ …

Easy peasy: Marry **PS5** + $\mathscr{L}_1$!

Theorem: $\forall x \Diamond R(x) \rightarrow \forall x \Box \Diamond R(x)$

Quantificational S5 I …

Easy peasy: Marry **PS5** + $\mathscr{L}_1$!

Theorem: $\forall x \Diamond R(x) \rightarrow \forall x \Box \Diamond R(x)$

Theorem: $\Diamond \exists x R(x) \leftrightarrow \exists x \Diamond R(x)$

Quantificational S5$_1$ …

Easy peasy: Marry **PS5** + $\mathscr{L}_1$!

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$\vdots$

Quantificational S5$_1$ …

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Theorem: $\Diamond \exists x R(x) \leftrightarrow \exists x \Diamond R(x)$

$\vdots$

The Notorious Barcans

The Notorious Barcans

Barcan Formula: $\vdash_{QS5_1} \Diamond \exists x \phi(x) \rightarrow \exists x \Diamond \phi(x)$

The Notorious Barcans

Barcan Formula: $\vdash_{QS5_1} \Diamond \exists x \phi(x) \rightarrow \exists x \Diamond \phi(x)$

Converse Barcan Formula: $\vdash_{QS5_1} \exists x \Diamond \phi(x) \rightarrow \Diamond \exists x \phi(x)$

The Notorious Barcans

Barcan Formula: $\vdash_{QS5_1} \Diamond \exists x \phi(x) \rightarrow \exists x \Diamond \phi(x)$

Converse Barcan Formula: $\vdash_{QS5_1} \exists x \Diamond \phi(x) \rightarrow \Diamond \exists x \phi(x)$


Minds & Machines (2017) 27:663–672
https://doi.org/10.1007/s11023-017-9454-1

CrossMark

An Argument for P=NP

Selmer Bringsjord[1,2]

Received: 8 November 2017 / Accepted: 9 November 2017 / Published online: 24 November 2017

Abstract I articulate a novel modal argument for **P=NP**.

Keywords **P=NP** · Modal logic · Digital physics

The Clay Mathematics Institute offers a $1 million prize for a solution to the **P=?NP** problem.[1] I look forward to receiving my award—but concede that the expected format of a solution is an *object-level* proof, not a meta-level argument like what I provide. On the other hand, certainly the winner needn't provide a *constructive* proof that **P=NP**.[2] Despite Gödel's recently discovered position on the

[1] See http://www.claymath.org/millennium. There are six other "millennium" problems; each of these is also associated with a $1M prize.

[2] As many readers know, the history of the problem is littered with failed attempts to provide non-constructive substantiation of the received view that **P≠NP**.

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✉ Selmer Bringsjord
Selmer.Bringsjord@gmail.com;
http://www.rpi.edu/~brings

1 Department of Cognitive Science, Rensselaer AI and Reasoning (RAIR) Lab, Rensselaer Polytechnic Institute (RPI), Troy, NY 12180, USA

2 Department of Computer Science, Rensselaer AI and Reasoning (RAIR) Lab, Rensselaer Polytechnic Institute (RPI), Troy, NY 12180, USA

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