

On Quantificational Modal Logic (S5-centric)

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IFLAI2
11/10/2022
ver 1110221040NY



Logistics ...

Status? Some discussion ...

The screenshot shows the Overleaf LaTeX editor interface. The left panel displays the file structure and code for `main.tex`. The right panel shows the rendered document titled "IFLAI2F22 Paper Topics & Feedback".

File Structure:

- `main.tex`
- `main72.bib`

File outline:

- General Orientation
- Formatting, Due Dates/S...
- The Required Structure ...
- The List For You to (Care...
- Sample from Last Offeri...

Code (main.tex):

```
1 %% TODO
2
3 %% [ ]
4
5
6 \documentclass[11pt]{article}
7
8 \usepackage[utf8]{inputenc}
9
10 \usepackage{fullpage} %% <= why not use this in your own paper?
11
12 \usepackage{setspace}
13 %% Toggle the following on for doublespacing:
14 \%doublespacing
15
16
17 %% Some standard package calls by S:
18 \usepackage{amssymb}
19 \usepackage[colorlinks]{hyperref}
20 \usepackage{harvard} %% Selmer's preference for
21   citations/References.
22 \usepackage{color}
23 \usepackage{marvosym}
24 \usepackage{mathrsfs}
25 \usepackage{verbatim}
26 \usepackage{eufrak}
27
28 \begin{document}
29
30 \title{\textbf{IFLAI2F22 Paper Topics \& Feedback}}
31 \author{Prof Selmer Bringsjord}
32 \date{\texttt{ver 1110220830NY}}
```

Document Content:

IFLAI2F22 Paper Topics & Feedback

Prof Selmer Bringsjord
ver 1110220830NY

Contents:

1 General Orientation	1
2 Formatting, Due Dates/Schedule	1
3 The Required Structure of the Paper	1
4 The List For You to (Carefully!) Add Yourself To, F22	2
5 Sample from Last Offering of IFLAI2 (F21)	15
References	16

Automatic A on QI, for all.

Automatic A on Q1, for all.

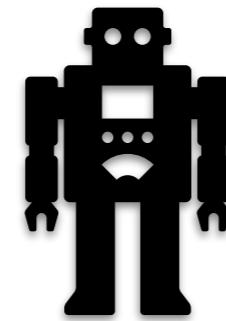
Q2?

Return to Return-to-Blinky

• • •



Blinky



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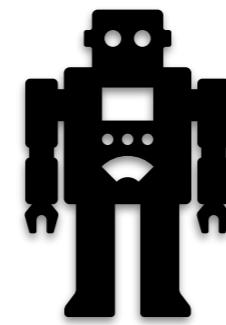
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Blinky



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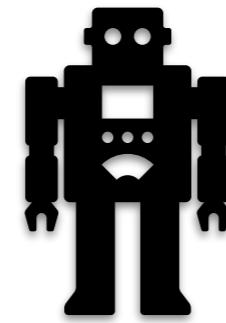


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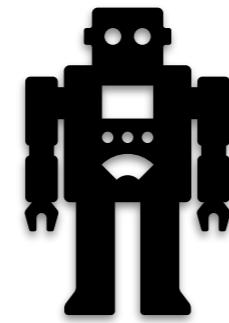
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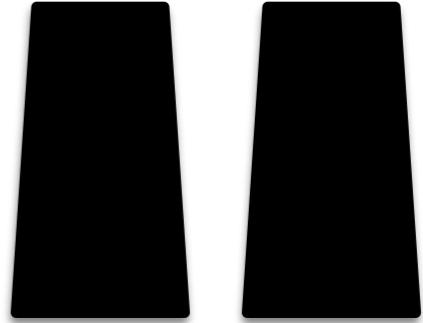
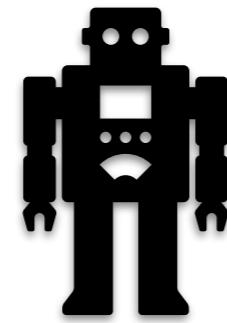
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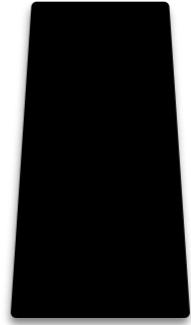
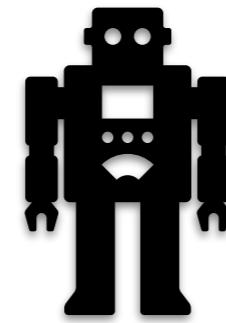


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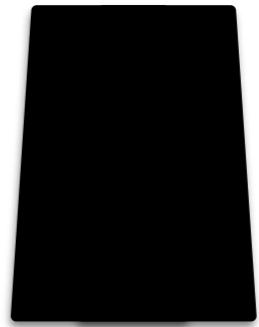
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1

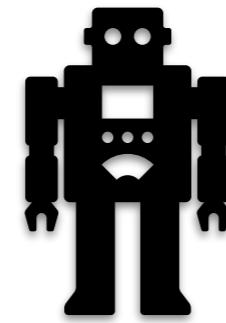


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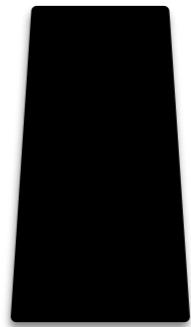
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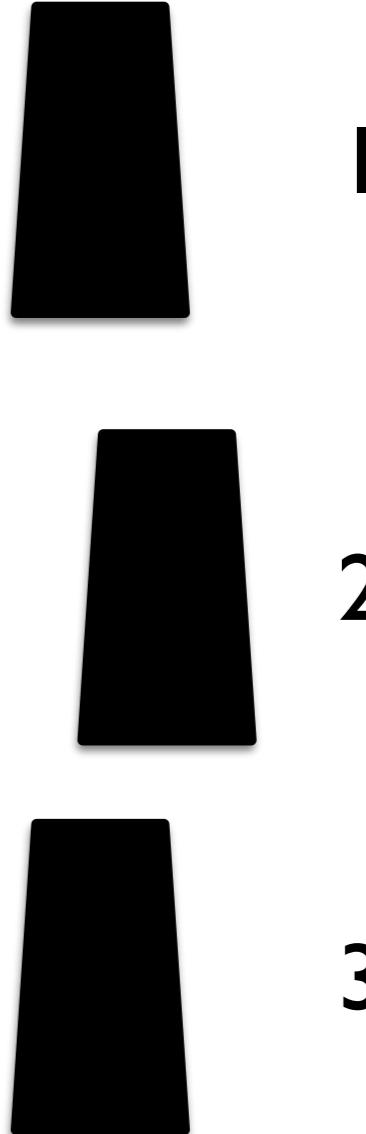
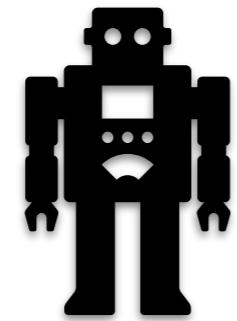
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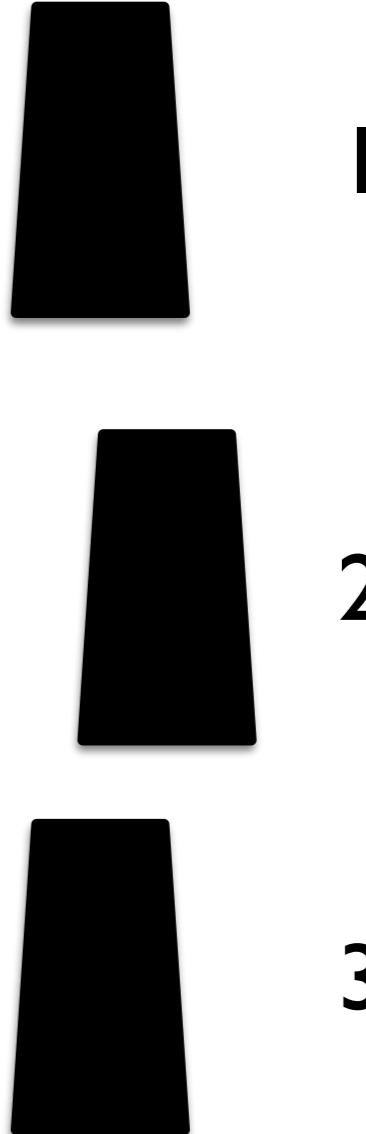
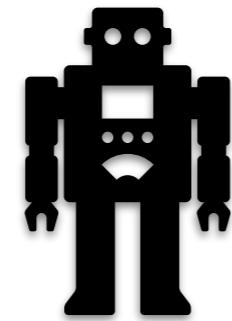


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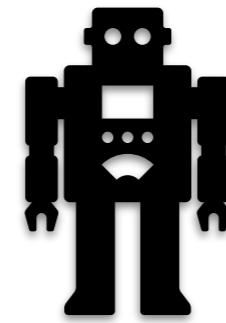


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Blinky



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2

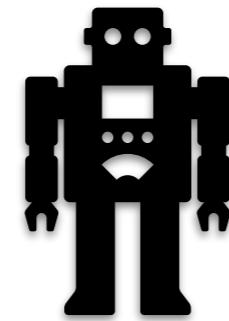


3



The ball is in the cup at location #1.

Blinky



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2

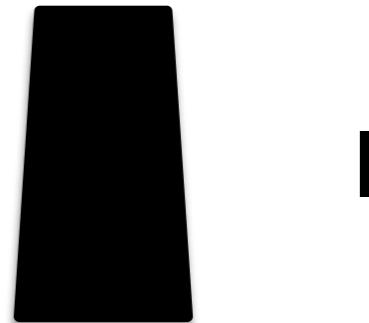


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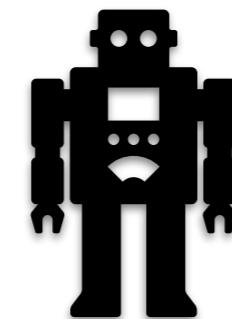


The ball is in the cup at location #1.

Loc(ball,1)



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3





The ball is in the cup at location #1.

Loc(ball,1)

(Loc ball 1)



1

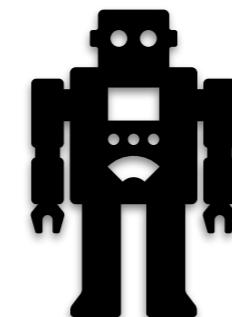


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3

Blinky





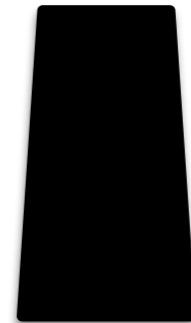
The ball is in the cup at location #1.

FALSE Loc(ball,1)

(Loc ball 1)



1

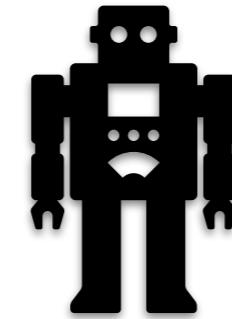


2



3

Blinky





FALSE Loc(ball,1)

(Loc ball 1)



1

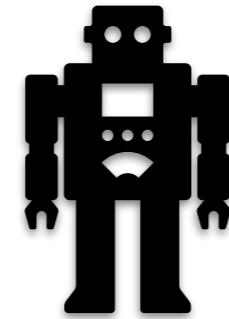


2



3

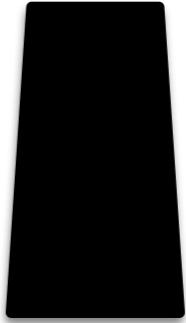
Blinky



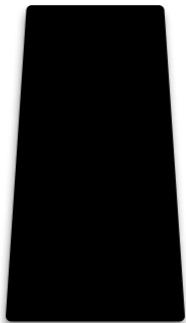


FALSE

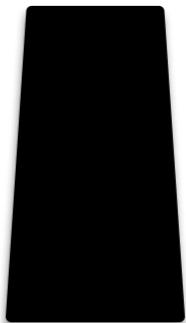
(Loc ball 1)



1

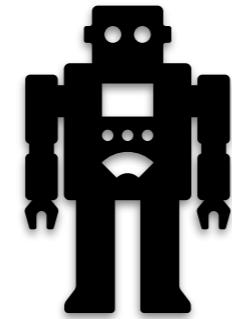


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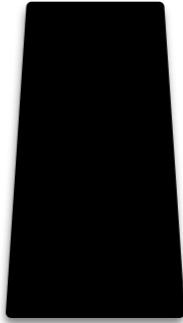
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Blinky



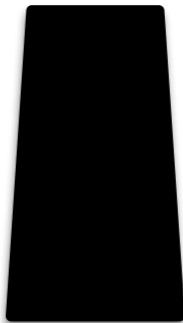
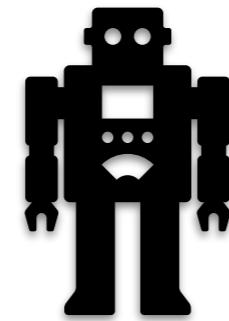


(Loc ball 1)

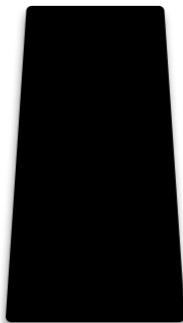


1

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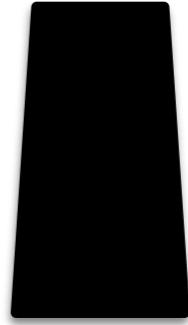
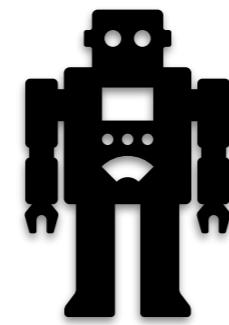
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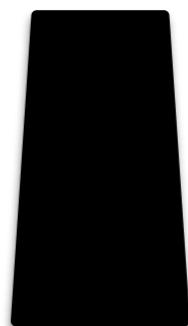
3



Blinky



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2

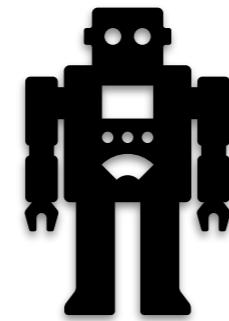


3



The ball is in the cup at location #1 or the ball is at location #3.

Blinky



1



2



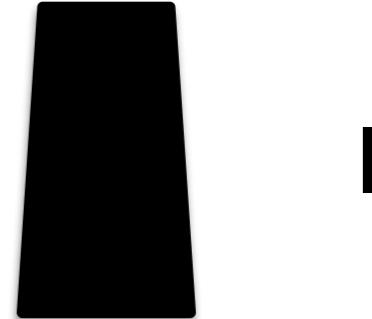
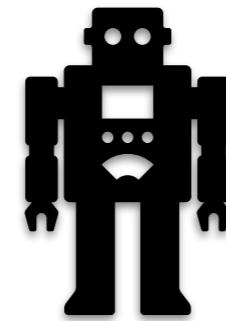
3



The ball is in the cup at location #1 or the ball is at location #3.

$$\text{Loc(ball,1)} \vee \text{Loc(ball,3)}$$

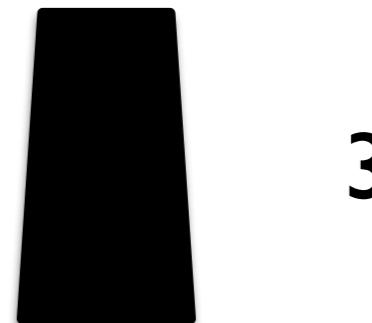
Blinky



1



2



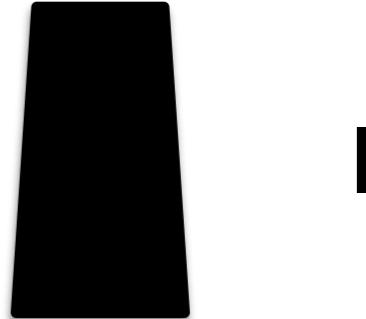
3



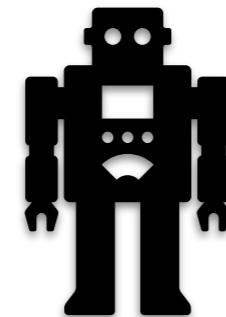
The ball is in the cup at location #1 or the ball is at location #3.

$\text{Loc}(\text{ball}, 1) \vee \text{Loc}(\text{ball}, 3)$

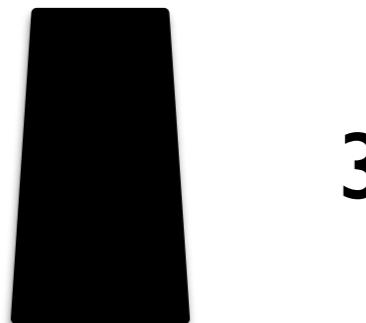
(or (Loc ball 1) (Loc ball 3))



Blinky



2



3

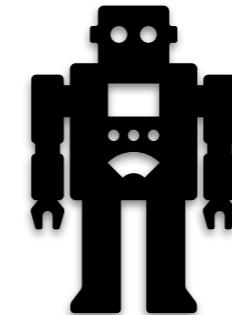


The ball is in the cup at location #1 or the ball is at location #3.

FALSE $\text{Loc}(\text{ball}, 1) \vee \text{Loc}(\text{ball}, 3)$

(or (Loc ball 1) (Loc ball 3))

Blinky



1



2



3



FALSE Loc(ball,1) \vee Loc(ball,3)

(or (Loc ball 1) (Loc ball 3))



1

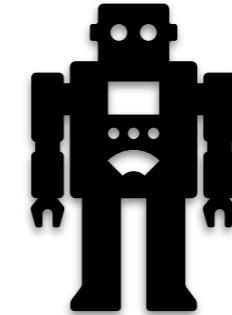


2



3

Blinky





FALSE

(or (Loc ball 1) (Loc ball 3))



1

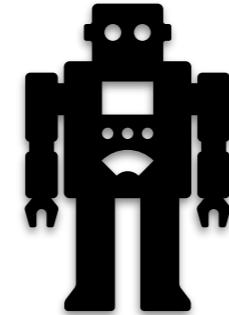


2



3

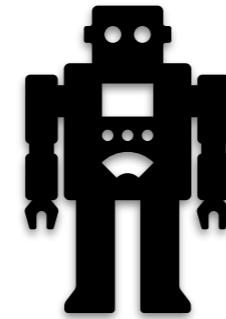
Blinky





FALSE

Blinky



1



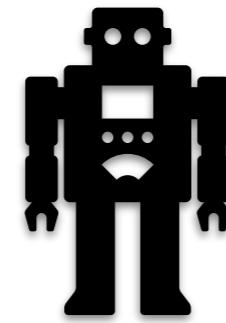
2



3



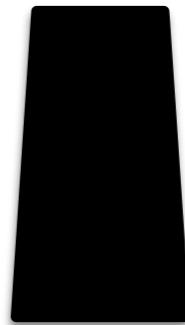
Blinky



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2

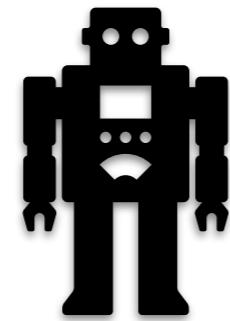


3



Blinky believes that the ball is in the cup at location #1.

Blinky



|



2

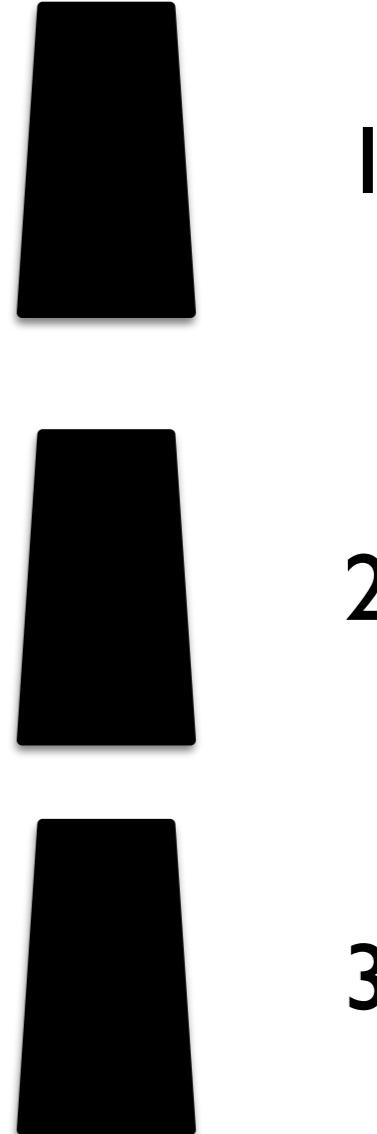
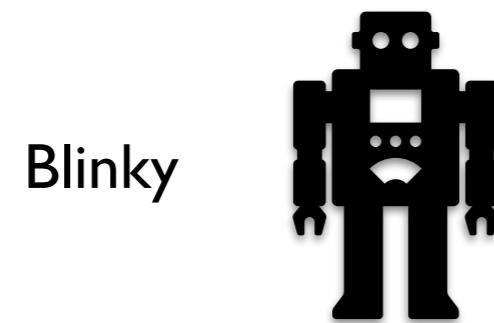


3



Blinky believes that the ball is in the cup at location #1.

$B(\text{blinky}, \text{Loc}(\text{ball}, 1))$





Blinky believes that the ball is in the cup at location #1.

$B(\text{blinky}, \text{Loc}(\text{ball}, 1))$

(Believes! t blinky (Loc ball 1))



1

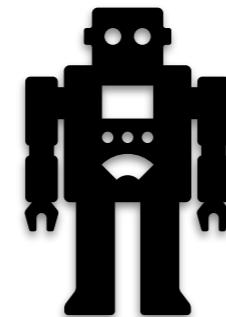


2



3

Blinky





Blinky believes that the ball is in the cup at location #1.

???

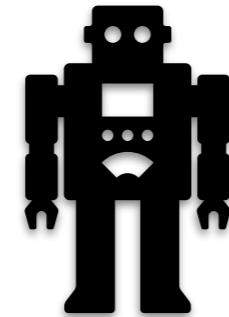
$B(\text{blinky}, \text{Loc}(\text{ball}, 1))$

(Believes! t blinky (Loc ball 1))



|

Blinky



2



3



Blinky believes that the ball is in the cup at location #1.

???

$B(\text{blinky}, \text{Loc}(\text{ball}, 1))$

(Believes! t blinky (Loc ball 1))



1

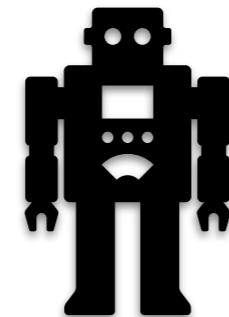


2



3

Blinky



In extensional logics, what is denoted is conflated with meaning (the latter being naively compositional), and intensional attitudes like *believes*, *knows*, *hopes*, *fears*, etc cannot be represented and reasoned over smoothly (e.g. without fear of inconsistency rising up).

Review: Encapsulation

Slate - K.slt

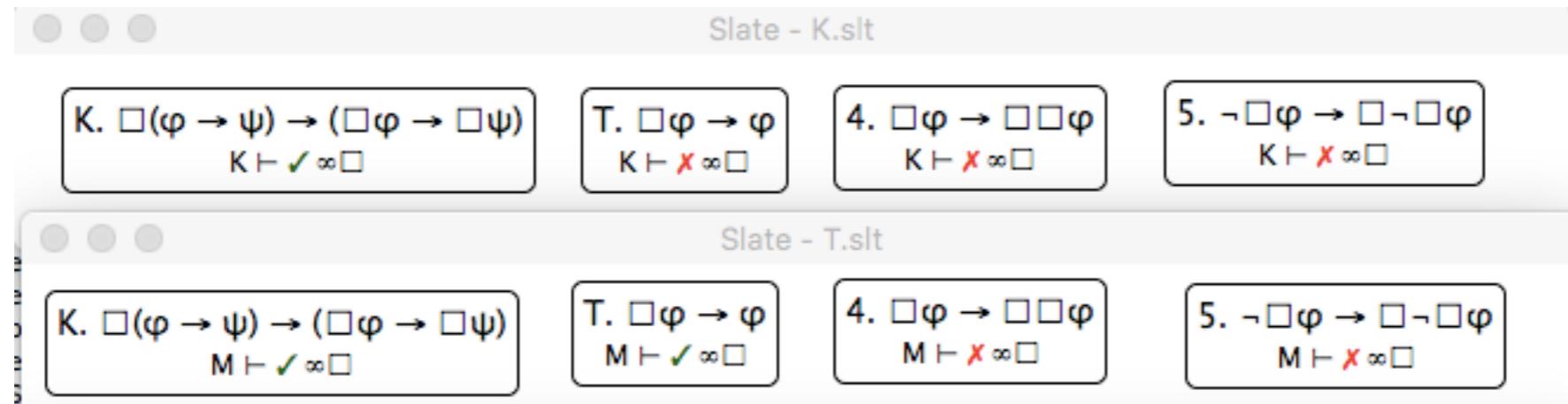
K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$
K $\vdash \checkmark \infty \Box$

T. $\Box\varphi \rightarrow \varphi$
K $\vdash \textcolor{red}{X} \infty \Box$

4. $\Box\varphi \rightarrow \Box\Box\varphi$
K $\vdash \textcolor{red}{X} \infty \Box$

5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$
K $\vdash \textcolor{red}{X} \infty \Box$

Review: Encapsulation



Review: Encapsulation

The screenshot shows the Slate interface with three tabs: **Slate - K.slt**, **Slate - T.slt**, and **Slate - D.slt**. Each tab contains five logical axioms, each enclosed in a rounded rectangle with a thin border:

- K.** $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$
K $\vdash \checkmark \infty \Box$
- T.** $\Box\varphi \rightarrow \varphi$
K $\vdash \text{X} \infty \Box$
- 4.** $\Box\varphi \rightarrow \Box\Box\varphi$
K $\vdash \text{X} \infty \Box$
- 5.** $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$
K $\vdash \text{X} \infty \Box$

The **Slate - T.slt** tab has the same four axioms as K.slt, but the first one is modified:

- K.** $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$
M $\vdash \checkmark \infty \Box$
- T.** $\Box\varphi \rightarrow \varphi$
M $\vdash \checkmark \infty \Box$
- 4.** $\Box\varphi \rightarrow \Box\Box\varphi$
M $\vdash \text{X} \infty \Box$
- 5.** $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$
M $\vdash \text{X} \infty \Box$

The **Slate - D.slt** tab contains different axioms:

- K.** $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$
D $\vdash \checkmark \infty \Box$
- T.** $\Box\varphi \rightarrow \varphi$
D $\vdash \text{X} \infty \Box$
- D.** $\Box\varphi \rightarrow \Diamond\varphi$
D $\vdash \checkmark \infty \Box$
- 4.** $\Box\varphi \rightarrow \Box\Box\varphi$
D $\vdash \text{X} \infty \Box$

Below the D.slt tab are two additional boxes:

- 5.** $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$
D $\vdash \text{X} \infty \Box$
- INTER.** $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$
D $\vdash \checkmark \infty \Box$

Review: Encapsulation

The screenshot shows the Slate interface with four tabs: K.slt, T.slt, D.slt, and S4.slt. Each tab contains five logical laws, each with a status indicator (K, M, or D) and proof details.

- K.slt:**
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$
K $\vdash \checkmark \infty \Box$
 - T. $\Box\varphi \rightarrow \varphi$
K $\vdash \times \infty \Box$
 - 4. $\Box\varphi \rightarrow \Box\Box\varphi$
K $\vdash \times \infty \Box$
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$
K $\vdash \times \infty \Box$
- T.slt:**
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$
M $\vdash \checkmark \infty \Box$
 - T. $\Box\varphi \rightarrow \varphi$
M $\vdash \checkmark \infty \Box$
 - 4. $\Box\varphi \rightarrow \Box\Box\varphi$
M $\vdash \times \infty \Box$
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$
M $\vdash \times \infty \Box$
- D.slt:**
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$
D $\vdash \checkmark \infty \Box$
 - T. $\Box\varphi \rightarrow \varphi$
D $\vdash \times \infty \Box$
 - D. $\Box\varphi \rightarrow \Diamond\varphi$
D $\vdash \checkmark \infty \Box$
 - 4. $\Box\varphi \rightarrow \Box\Box\varphi$
D $\vdash \times \infty \Box$
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$
D $\vdash \times \infty \Box$
 - INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$
D $\vdash \checkmark \infty \Box$
- S4.slt:**
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$
S4 $\vdash \checkmark \infty \Box$
 - T. $\Box\varphi \rightarrow \varphi$
S4 $\vdash \checkmark \infty \Box$
 - D. $\Box\varphi \rightarrow \Diamond\varphi$
S4 $\vdash \checkmark \infty \Box$
 - 4. $\Box\varphi \rightarrow \Box\Box\varphi$
S4 $\vdash \checkmark \infty \Box$
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$
S4 $\vdash \times \infty \Box$
 - INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$
{INTER} Assume \checkmark

Review: Encapsulation

K

T

D

4 = S4

5 = S5

The screenshot shows the Slate application interface with five tabs representing different modal logics:

- Slate - K.slt**: Contains formulas K, T, 4, and 5. Formula 5 is marked as **K ⊢ X ∞□**.
- Slate - T.slt**: Contains formulas K, T, 4, and 5. Formula 5 is marked as **M ⊢ X ∞□**.
- Slate - D.slt**: Contains formulas K, T, D, 4, 5, and INTER. Formulas 5 and INTER are marked as **D ⊢ X ∞□**.
- Slate - S4.slt**: Contains formulas K, T, D, 4, 5, and INTER. Formulas 5 and INTER are marked as **S4 ⊢ X ∞□**. Formula INTER has a note: **{INTER} Assume ✓**.
- Slate - S5.slt**: Contains formulas K, T, D, 4, 5, and INTER. Formulas 5 and INTER are marked as **S5 ⊢ ✓ ∞□**. Formula D has a note: **{D} Assume ✓**.

Review: Encapsulation

K

T

D

4 = S4

5 = S5

The screenshot shows the Slate application interface with five tabs representing different modal logics:

- Slate - K.slt:** Contains formulas K, T, 4, and 5. Formula 5 is marked as $\text{K} \vdash \text{X} \infty \square$.
- Slate - T.slt:** Contains formulas K, T, 4, and 5. Formula 5 is marked as $\text{M} \vdash \text{X} \infty \square$.
- Slate - D.slt:** Contains formulas K, T, D, 4, 5, and INTER. Formulas 5 and INTER are marked as $\text{D} \vdash \text{X} \infty \square$.
- Slate - S4.slt:** Contains formulas K, T, D, 4, 5, and INTER. Formulas 5 and INTER are marked as $\text{S4} \vdash \text{X} \infty \square$. Formula INTER has a note: $\{\text{INTER}\} \text{ Assume } \checkmark$.
- Slate - S5.slt:** Contains formulas K, T, D, 4, 5, and INTER. Formulas 5 and INTER are marked as $\text{S5} \vdash \text{X} \infty \square$. Formula D has a note: $\{\text{D}\} \text{ Assume } \checkmark$. Formula 4 has a note: $\{\text{4}\} \text{ Assume } \checkmark$.

S5 ...

The Characteristic Axiom

$$\Diamond\phi \rightarrow \Box\Diamond\phi$$

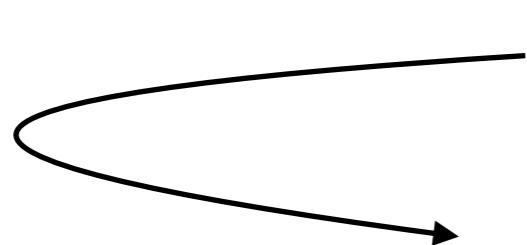
The Characteristic Axiom

$$\Diamond\phi \rightarrow \Box\Diamond\phi$$

$$\neg\Box\psi \rightarrow \Box\neg\Box\psi$$

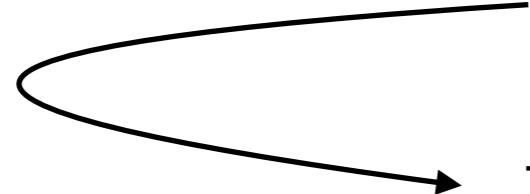
The Characteristic Axiom

$$\Diamond\phi \rightarrow \Box\Diamond\phi$$

$$\neg\Box\psi \rightarrow \Box\neg\Box\psi$$
A hand-drawn style arrow pointing from the first equation to the second.

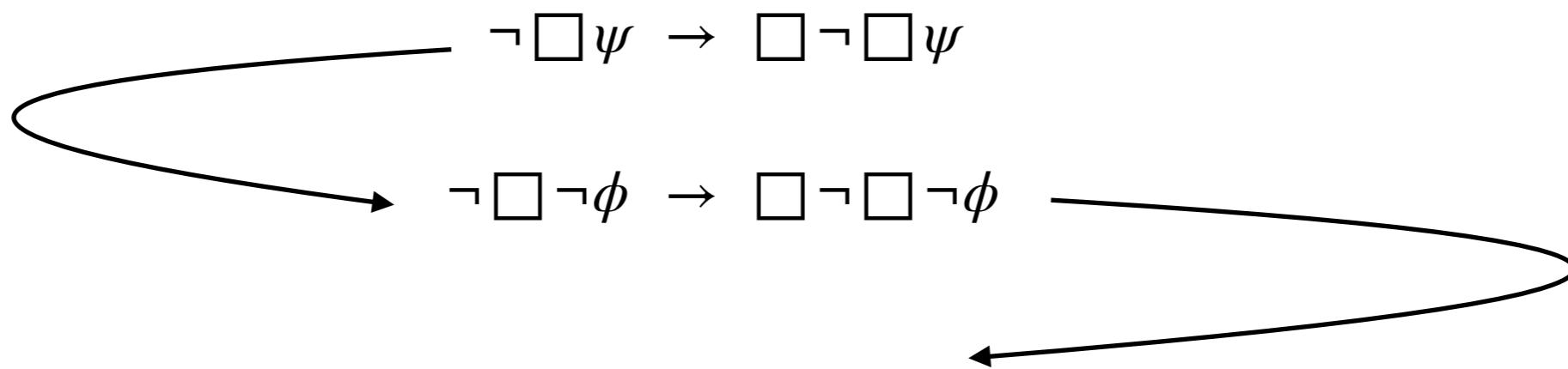
The Characteristic Axiom

$$\Diamond\phi \rightarrow \Box\Diamond\phi$$


$$\neg\Box\psi \rightarrow \Box\neg\Box\psi$$
$$\neg\Box\neg\phi \rightarrow \Box\neg\Box\neg\phi$$

The Characteristic Axiom

$$\Diamond\phi \rightarrow \Box\Diamond\phi$$



The Characteristic Axiom

$$\Diamond\phi \rightarrow \Box\Diamond\phi$$

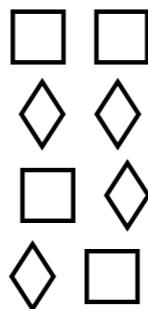
$$\neg\Box\psi \rightarrow \Box\neg\Box\psi$$
$$\neg\Box\neg\phi \rightarrow \Box\neg\Box\neg\phi$$
$$\Diamond\phi \rightarrow \Box\Diamond\phi$$

A diagram illustrating the characteristic axiom. It consists of four formulas arranged in a rectangle: $\Diamond\phi \rightarrow \Box\Diamond\phi$ at the top, $\neg\Box\psi \rightarrow \Box\neg\Box\psi$ on the top edge, $\neg\Box\neg\phi \rightarrow \Box\neg\Box\neg\phi$ on the bottom edge, and $\Diamond\phi \rightarrow \Box\Diamond\phi$ at the bottom. The top and bottom formulas are connected by curved arrows forming a horizontal loop. The left and right formulas are also connected by curved arrows forming a vertical loop.

Nice S5 Reduction Lemmas

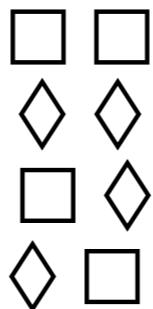
Nice S5 Reduction Lemmas

The Four Possible Pairs



Nice S5 Reduction Lemmas

The Four Possible Pairs



The Four Reduction Principles

$$\square\phi \leftrightarrow \square\square\phi$$

$$\diamond\phi \leftrightarrow \diamond\diamond\phi$$

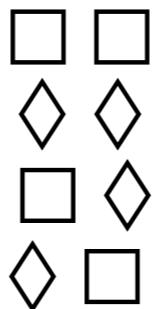
$$\square\phi \leftrightarrow \diamond\square\phi$$

$$\diamond\phi \leftrightarrow \square\diamond\phi$$

(where $\phi \in \mathcal{L}_{pc}$)

Nice S5 Reduction Lemmas

The Four Possible Pairs



The Four Reduction Principles

$$\square\phi \leftrightarrow \square\square\phi$$

$$\diamond\phi \leftrightarrow \diamond\diamond\phi$$

$$\square\phi \leftrightarrow \diamond\square\phi$$

$$\diamond\phi \leftrightarrow \square\diamond\phi$$

(verify in HS[®])

(where $\phi \in \mathcal{L}_{pc}$)

Quantificational S5 | ...

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Easy peasy: Marry **PS5** + \mathcal{L}_1 !

Quantificational S5 | ...

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Theorem: $\forall x \Diamond R(x) \rightarrow \forall x \Box \Diamond R(x)$

Quantificational S5 | ...

Easy peasy: Marry **PS5** + \mathcal{L}_1 !

Theorem: $\forall x \Diamond R(x) \rightarrow \forall x \Box \Diamond R(x)$

Theorem: $\Diamond \exists x R(x) \leftrightarrow \exists x \Diamond R(x)$

Quantificational S5 | ...

Easy peasy: Marry **PS5** + $\mathcal{L}_1^!$

Theorem: $\forall x \Diamond R(x) \rightarrow \forall x \Box \Diamond R(x)$

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Quantificational S5 | ...

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The Notorious Barcans

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Barcan Formula: $\vdash_{QS5_1} \Diamond \exists x\phi(x) \rightarrow \exists x\Diamond\phi(x)$

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Converse Barcan Formula: $\vdash_{QS5_1} \exists x\Diamond\phi(x) \rightarrow \Diamond\exists x\phi(x)$

The Notorious Barcans

Barcan Formula: $\vdash_{QS5_1} \Diamond \exists x \phi(x) \rightarrow \exists x \Diamond \phi(x)$

Converse Barcan Formula: $\vdash_{QS5_1} \exists x \Diamond \phi(x) \rightarrow \Diamond \exists x \phi(x)$

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An Argument for P=NP

Selmer Bringsjord^{1,2}

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Abstract I articulate a novel modal argument for P=NP.

Keywords P=NP · Modal logic · Digital physics

The Clay Mathematics Institute offers a \$1 million prize for a solution to the P=?NP problem.¹ I look forward to receiving my award—but concede that the expected format of a solution is an *object-level* proof, not a meta-level argument like what I provide. On the other hand, certainly the winner needn’t provide a *constructive* proof that P=NP.² Despite Gödel’s recently discovered position on the

¹ See <http://www.claymath.org/millennium>. There are six other “millennium” problems; each of these is also associated with a \$1M prize.

² As many readers know, the history of the problem is littered with failed attempts to provide non-constructive substantiation of the received view that P≠NP.

I’m greatly indebted to Michael Zenzen for many valuable discussions about the P=?NP problem and physics (*simpliciter* and digital), and to Jim Fahey for discussions about such physics and mixed-mode dual-diamond operators in modal logic. The presentation of the core arguments herein to editions of Bringjord’s graduate seminar, *Logic & Artificial Intelligence*, and his guest lectures on P=?NP in *Formal Foundations of Cognitive Science* graduate seminars, sparked a number of helpful objections and suggestions, for which I’m grateful. I’m indebted as well to two anonymous referees for trenchant comments. Though the two arguments herein (the second of which seems to establish P=NP) are for weal or woe Bringjord’s, Joshua Taylor’s astute objections catalyzed much thought and crucial refinements.

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Four S5 Bins for Everything ...

