# Intermediate Formal Logic and AI

(= IFLAI2, rhymes with "eye fly boo")

Crosslistings:
54276-CSCI-4972-01
59729-MATH-4140-01
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Selmer Bringsjord
Rensselaer AI & Reasoning (RAIR) Lab
Department of Cognitive Science
Department of Computer Science
Lally School of Management
Rensselaer Polytechnic Institute (RPI)
Troy NY 12180 USA

Office Hours: Wed 10a–12n (sometime via Zoom); & by appointment

Selmer.Bringsjord@gmail.com

version 0907222000NY

## Contents

# 1 General Orientation

This course is an intermediate class in formal logic, with substantive connections made to logicist (= logic-based) AI, within the LAMA®️ paradigm.[1] (While emphasis is on *deductive* formal logic, there is some coverage of formal *in*ductive and heterogeneous formal logic).[2] AI plays a significant role in advancing learning in the class; and the class includes some coverage of logicist AI and logicist computer programming.[3] While coverage will not be extensive, students are introduced to a new logic-programming language: Hyperlog®️.

We have referred above to "the LAMA®️ paradigm." What is that? This question will be answered in more detail later, but we do volunteer here that while the LAMA®️ paradigm is based upon a number of pedagogical principles, first and foremost among them is what can be labelled the Driving Dictum:[4]

> If you can't prove it, you don't get it.

Turning back to the nature of formal logic, it can accurately be said that it's the science and engineering of reasoning,[5] but even this supremely general slogan fails to convey the flexibility and enormity of the field. For example, all of classical mathematics can be deductively derived from a small set of formulae (e.g., **ZFC** set theory, with which you should already have some familiarity with, and indeed experimenting with in the HyperSlate®️ system) expressed in the formal logic known as 'first-order logic' (= FOL = $\mathscr{L}_1$, with which you are *also* familiar, and, as we shall review and discuss in class, computer science emerged from and is in large part based upon logic (for cogent coverage of this emergence, see Glymour 1992, Halpern, Harper, Immerman, Kolaitis, Vardi & Vianu 2001). Logic is indeed the foundation for *all* at once rational-and-rigorous intellectual

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[1]'LAMA®️' is an acronym for 'Logic: A Modern Approach," and is pronounced to rhyme with 'llama' in contemporary English, the name of the exotic and sure-footed camelid whose binomial name is *Lama glama*, and has in fact been referred to in the past by the single-l 'lama.'

[2]Sometimes 'symbolic' is used in place of 'formal,' but that's a bad practice, since — as students in this class will soon see — formal logic includes the representation of and systematic reasoning over *pictorial* information, and such information is decidedly *not* symbolic. For a discussion of the stark difference between the pictorial vs. the symbolic, and presentation of a formal logic that enables representation of and reasoning over both, see (Arkoudas & Bringsjord 2009), which directly informs Chapter 8 of our LAMA-BDLAHGHS textbook.

[3]We use 'logicist computer programming' to denote a general approach to computer programming based on formal logic; this general approach covers what is called 'logic programming,' which is connected specifically to such languages as Prolog.

[4]It's profitable to ponder a variant of this dictum, applicable in venues [e.g. legal hearings, courtrooms, reports by analysts in various domains that are not exclusively formal (e.g. fundamental investing, intelligence, etc.)] in which reasoning is not only deductive, but inductive, viz. "If you can't show by explicit argument that it's likelihood reaches a sufficient level, you don't get it."

[5]Warning: Increasingly, the term 'reasoning' is used by some who don't *really* do anything related to reasoning, as traditionally understood, to nonetheless label what they do. Fortunately, it's easy to verify that some reasoning is that which is covered by formal logic: *If the reasoning is explicit, links declarative statements or declarative formulae together via explicit, abstract reasoning schemata or rules of inference (giving rise to at least explicit arguments, and often proofs), is surveyable and inspectable, and ultimately machine-checkable, then the reasoning in question is what formal logic is the science and engineering of.* In order to characterize *in*formal logic, one can remove from the previous sentence the requirements that the links must conform to explicit reasoning schemata or rules of inference, and machine-checkability. It follows that so-called informal logic would revolve around arguments, but not proofs. An excellent overview of informal logic, which will be completely ignored in this class and its LAMA-BDLAHSHG textbook, is provided in "Informal Logic" in the Stanford Encyclopedia of Philosophy. In this article, it's made clear that, yes, informal logic concentrates on the nature and uses of argument.

pursuits. (If you can find a counter-example, i.e. such a pursuit that doesn't directly and crucially partake of logic, S Bringsjord would be very interested to see it.)

## 2  Prerequisites

Students are expected to have taken a serious university-level introductory formal-logic course (which hopefully included second-order logic and some propositional modal logic — but given the rather impoverished reach of the vast majority of such courses, this really and truly is a *hope*), and to have a significant degree of logico-mathematical maturity. Phase I of the course will include a review of some introductory formal logic, via problems in HyperSlate®.

## 3  Teaching Assistant

The TA for this course is Shreya Banerjee, a doctoral student in computer science at RPI. Her email address is `shreyabbanerjee@gmail.com`, and her Webex room for meeting is https://rensselaer.webex.com/meet/b Shreya's office hours are Fri 9:30a–11:30a, and will be held by Webex.

## 4  Reading/Videos/Textbook/Courseware

Slide decks and lectures/tutorials (including video versions of such) are part of the crucial content for this course, and will be linked-to from the course web page; in this regard we have a parallel situation to IFLAI1.

Papers that are required reading will be made available to students as we proceed.

Students will purchase a license giving access to the inseparable and symbiotic triadic combination published and maintained by Motalen:

- the e-textbook *Logic: A Modern Approach; Beginning Deductive Logic, Advanced via HyperSlate® and HyperGrader®* (LAMA-BDLAHSHG);

- access to and use of the HyperGrader® AI platform (for, among other things, assessing student work); and

- access to and use of the HyperSlate® AI interactive environment (for, among other things, engineering proofs and logic programs in collaboration with AI).

All three items will be available after purchase in the RPI Bookstore of a barcoded envelope with a personalized starting code/key for registration. Students who previously registered for a version of the online software and ebook will be able to present their email address used in the system and receive a substantial discount. Logistics of the purchase, and the contents of the envelope that purchase will secure, will be encapsulated in the first class meeting, Aug 29 2022, and then gone over in more detail on Sep 1 2022. The first use in earnest of HyperSlate® and HyperGrade® will happen in class on Sep 12 2022, so by the start of class on that day students should have LAMA-BDLAHSHG, and be able to open both HyperSlate® and HyperGrader® on their laptops in class. Updates to LAMA-BDLAHSHG, and additional exercises, will be provided by listing on the course web page (and sometimes by email) through the course of the semester. You will need

to manage many electronic files in the course of this course, and e-housekeeping and e-orderliness are of paramount importance. You will specifically need to assemble a library of completed and partially completed proofs/arguments/truth-trees etc. so that you can use them as building blocks in harder proofs; in other words, building up your own "logical library" will be crucial.

Please note that HyperSlate® and HyperGrader® are trademarked, copyrighted, and Pat. Pend. software: copying and/or reverse-engineering and/or distributing this software to others is strictly prohibited. You will need to submit online a signed version of a License Agreement. This agreement will also reference the textbook, which is copyrighted as well, and since it's an ebook, cannot be copied or distributed or resold in any way.

In addition, occasionally papers may be assigned as reading. Two background ones, indeed, are hereby assigned: (Bringsjord, Taylor, Shilliday, Clark & Arkoudas 2008, Bringsjord 2008).

Finally, slide decks used in class will contain crucial additional content above and beyond LAMA-BDLA and HyperSlate® and HyperGrader® content, and will be available on the web site for the course for study. Along with slide decks, an appreciable number of video and audio tutorials and mini-lectures will be provided as well.

## 5 Schedule

### 5.1 The Four Coverage Areas

This class is divided into four I–IV coverage areas:

I **$Review^+$**. We use HyperSlate® and HyperGrader® to review the logics $\mathscr{L}_{PC}, \mathscr{L}_1,$ $\mathscr{L}_2$, and the propositional modal logics **K**, **T**, **D**. For some students, modal logics may be new; these students will want to pay close attention to, and expend some genuine effort exploring, these logics. Our review will also include brief description of the modal logics **S4** and **S5**; both are still only *propositional* modal logics. Importantly, many students who took "IFLAI1" in the past will not have used a version of HyperSlate® that included **S4** or **S5**. This the reason for the use of '+' after 'Review' in giving a label for Phase I of the class.

II *Metalogic, Including Gödel's Great Theorems.* The standard bulk of intermediate formal logic consists in a series of metatheorems that can be viewed as showing that certain metaproperties hold of certain formal logics and parts thereof. For instance, "completeness" (COMP) is often one of these properties. As we will e.g. see:

– **Theorem**: COMP[$\mathscr{L}_{PC}$]
– **Gödel's Completeness Theorem/GCT**: COMP[$\mathscr{L}_1$]

One of the distinctive aspects of IFLAI2 is that its coverage of metalogic will include nearly all the great theorems of the greatest logician (Kurt Gödel); this coverage will be from Bringsjord's forthcoming *Gödel's Great Theorems*.

III *Advanced Topics in* HyperSlate®. Recall that as said above Phase I includes some new formal logics to be explored in HyperSlate®. In Phase III of the course we take a jump to *quantified* modal logic, and we also explore, to a degree, a more advanced version of HyperSlate® in which the user if allowed to write Clojure

functions that play a role in proofs. (We will only consider relatively simple Clojure functions because of time limitations in our schedule.)

**IV** *Big Questions: e.g. Will AI Match, or Even Exceed, Human Intelligence?.* Our fourth area of coverage includes "big questions" that AI forces us to consider, if we're thoughtful. One example is given in the heading just above, but there are many others, as the student will see. The first big question we'll ponder is whether The Singularity is going to happen or not. Area IV will also include Gödel's "Either-Or" framework for considering whether standard computing (i.e. Turing-machine-level computing) can ever reach human-level intelligence. Here, Bringsjord shall draw from another forthcoming publication, a debate with Rapaport, who holds that such computing will indeed enable AI to reach human-level heights.

## 5.2 Fine-Grained Schedule

A more fine-grained schedule now follows.[6]

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[6]Note that the Rensselaer Academic Calendar is available here.

- **Aug 29**: *General Orientation to the LAMA$^{\circledR}$ Paradigm, Logistics, Mechanics.* The syllabus is reviewed in detail. It's made clear to the students that, in this class, there is a very definite, comprehensive, theoretical position on formal logic and the teaching thereof; this position corresponds to the affirmation of the LAMA$^{\circledR}$ (= Logic: A Modern Approach) paradigm, and that in lockstep with this position the tightly integrated trio of

    1. the LAMA-BDLAHSHG textbook,
    2. HyperSlate$^{\circledR}$ AI-infused proof and program construction environment, and
    3. HyperGrader$^{\circledR}$ AI platform (comprising HyperSlate$^{\circledR}$) for (among other things) automated assessment of proofs,

    are used. Students wishing to learn intermediate formal logic under e.g. the "Stanford" paradigm and with its associated software and not the aforementioned trio from Motalen are encouraged to drop this LAMA$^{\circledR}$-based course and take *Intermediate Logic* at Rensselaer from another instructor.

- **Sep 1**: *Tutorials, Mechanics; Historical and Scientific Context re. Formal Logic, AI, and Logic Machines.* A rapid overview of the relevant history and background of formal logic and AI is provided; this content forms the bulk of the context for our coming investigations and learning.

- **Sep 6** ⌨: *Review of Extensional Logics.* Note, this is a *Tuesday.* The day before is Labor Day: no classes are held that day. We here first explain the core difference between extensional logics such as $\mathscr{L}_1$ and intensional logics, using Blinky the robot and a cup-switching challenge, and the infinitary False Belief Task. We then proceed to explore

    <div align="center">SpecialLlamasDisjunction</div>

    in HyperSlate$^{\circledR}$, and other poblems.

- **Sep 8** ⌨: *Review of Intensional/Modal Logics.* We look at modal-logic **D** in review, and include (for most) new coverage of modal logics **S4** and **S5**. We also look at $\mathcal{DCEC}$ in HyperSlate$^{\circledR}$.

- **Sep 12**: *Church's Theorem, the Halting Problem, and The Singularity.* Students by this point should have HyperSlate$^{\circledR}$ running on their laptops, have their codes registered, have put in their RINs to HyperGrader$^{\circledR}$, and have signed and accepted their LA. Church's Theorem tells us that theoremhood for $\mathscr{L}_1$ is Turing-undecidable (= that the *Entscheidungsproblem* is Turing-unsolvable). How do we prove this? And what implications does Church's Theorem have for the future of AI, and The Singularity in particular?

- **Sep 15**: *Completeness Theorems.* We here cover the first of Gödel's great theorems, which says that $\mathscr{L}_1$ is complete (in the sense that every necessary truth has a proof). We consider Henkin's version of this theorem as well, albeit briefly.

- **Sep 19**: *Gödel's First Incompleteness Theorem (G1).*

- **Sep 22**: *Gödel's Second Incompleteness Theorem (G2).*

- **Sep 26**: *Gödel's Speedup Theorem and Artificial Superintelligence.*

- **Sep 29**: *Formal Logic, AI, Computer Science, and the Immaterial.* Formal logic, AI, and computer science all at least appear to entail some non-physical things (e.g. algorithms, infinite cardinal numbers, etc.) exist. Does the entailment go through? And if it does, does this in turn entail that we are immaterial as well? Affirmative answers to both questions are defended by Bringsjord.

- **Oct 3** ✗: *AI, Consciousness, Cognitive Intelligence, and AGI.* This class is based on work by Bringsjord & Govindarajulu in which a new theory of machine consciousness is set out and associated with a scheme ($\Lambda$) for measuring this consciousness. B&G also here articulate and analyze purported refutations of the Integrated Information Theory of consciousness advanced by Tononi & Koch, and its associated scheme ($\Phi$) for measuring consciousness. In addition, it is explained how the concept of *cognitive intelligence* can be based upon $\Lambda$, and how this has substantive bearing on artificial *general* intelligence = AGI.

- **Oct 6** ✏: *Real Learning (RL).* AI of today has given the world so-called "machine learning," or just 'ML' for short. But do machines doing ML actually learn? A negative answer is given, and defended; and a genuine form of learning (for natural and artificial agents), $\mathcal{RL}$, is introduced and defended. This class is based on (Bringsjord, Govindarajulu, Banerjee & Hummel 2018), among other publications.

- **Oct 10**: *No class: Columbus Day.*

- **Oct 13**: *What is Formal <u>Inductive</u> Logic?* This class includes compressed coverage of so-called "pure inductive logic" (PIL), which has become nearly the sole province of mathematicians and logicians, with AI activity nearly zero. Why? One reason, which we find compelling, is that PIL is devoid of proofs and arguments build on the basis of the formal structures involved. We use the "Grue Paradox" to help explain matters.

- **Oct 17** ▣: *From the Lottery Paradox to Defeasible/Nonmonotonic Logic and AI.* We present and examine The Lottery Paradox as a portal to argument-based defeasible (= nonmonotonic) logic. We also consider the suppression task as a caset study in the applicability of argument-based defeasible logic. Such logic is way to do inductive logic, including automated inductive logic, that is superior to PIL and anything based upon it.

- **Oct 20**: *AI to Surmount Arrow's Impossibility Theorem.* This class is based on (Bringsjord, Govindarajulu & Giancola 2021).

- **Oct 24**: *The Argument for God's Existence from AI.* This class is based on in-*European Journal of Science & Theology* paper, a preprint of which is available here.

- **Oct 27**: *What is the Brain, Computationally and AI Speaking?* We here begin by considering the claim, defended by Richard Granger, that the human brain is fundamentally *less* than a Turing machine (and of course thus its equivalents, e.g. a register machine).

- **Oct 31**: *Logicist Agent-based Economics; AI and Tax Technology.* Can the U.S. federal tax code be captured in formal logic? If so, wouldn't that allow AI to compute minimal tax payments, and certify such payments as minimal?

- **Nov 3** ▣: *Pure General Logic Programming, Functional Programming, Turing-Completeness, and Beyond.* We review the basic paradigms of computer programming. For the imperative case, we use the simple imperative language of (Davis, Sigal & Weyuker 1994), and also discuss register machines, Turing machines (again), KU machines. We also discuss whether programming beyond the Turing Limit makes sense and can be pursued. In this connection we explore the hierarchy $\mathfrak{LM}$.

- **Nov 7**: *Hypergraphical Proof and Programming in HyperLog®.* We here introduce the availability of writing Clojure functions in the context of proofs in HyperLog®.

- **Nov 10**: *Quantified Modal Logic.* We here explore quantified **S5**, including the the infamous Barcan Formula. HyperSlate® is used.

- **Nov 14**: *Killer Robots, **D**, and Beyond in HyperSlate® to DCEC.* We begin here by stating the "PAID Problem," and then the approach to it from Bringsjord et al. advocates. We review that modal logic **D** is painfully inadequate, but now move to some exploration of a version of $\mathcal{DCEC}$ in HyperSlate®.

- **Nov 17**: *The Four Steps (including Logicist AI-ification of the Doctrines of N Effect) to Solve the PAID Problem.*

- **Nov 21**: ***ZFC***. We review and expand our understanding of axiomatic set theory, and of the relative size of infinite sets. **ZFC** in HyperSlate® is visited and explored.

- **Nov 24**: **No Class** (Thanksgiving).

- **Nov 28**: *Gödel's Greatest Theorem: The Continuum Hypothesis.* We turn to Sherlock Holmes for help in understanding Gödel's result that CH cannot be proved false on the basis of **ZFC**.

- **Dec 1**: *Gödel's Time-Travel Theorem.* We here visit the world known as "Flatland," and use it to articulate a visual Gödelian proof that backwards time travel is possible. We also consider the the Paradox of Proust/Looping Painter Paradox, and Bringsjord provides his analysis and solution.

- **Dec 5**: *Gödel's "God Theorem".* Did Gödel prove that God exists? We discuss this question, and look in some detail at his attempt to do so, which has become an active area in AI of today.

- **Dec 8**: *The "Games" of Gödel and His "Diophantine Disjunction".* We here assess the theorems of Gödel by considering them in connection with games measured computationally and logically, so as to answer the question (Q) as to whether an AI could ever match Gödel. We also consider Gödel's view on this question, which he connected to a certain disjunction involving Diophantine problems. Bringsjord answers Q in the negative, and provide supporting argument for this position. His position is contrasted with Bill Rapaport's contrary position.

# 6 Grading

Grades are based on four factors:

1. All required problems in HyperSlate®, when completed and certified correct by HyperGrader®, earns the student an A/4 for 50% of her/his final grade. Students cannot pass the course unless all these required problems are solved and certified correct. It is not expected that passing all of these problems will be onerous; in this regard, IFLAI2 is perhaps a bit different than IFLAI1.

2. Answers to questions regarding metalogic/metatheorems covered. These questions will go out be email; answers will need to come back as pdfs. This will constitute 20% of one's grade. Usually the questions will ask for (informal) proofs or proof-sketches, or — a concept to be explained — sub-proofs of steps that are/seem mysterious in metaproofs that you are presented with. At most there will be three of these questions for the course.

3. A 3-page paper written as a critique of a position on formal logic, AI, and the mind advanced by Bringsjord. (It will be easy to find a position that you vehemently disagree with. The topic must be pre-accepted by Bringsjord.) This paper will be submitted in two versions, a first version on which feedback is given, and then a final version submitted after that that takes account of this feedback. Overleaf will be used for this process (for proposing topics, clearing topics, for Selmer to write feedback, and for writing papers (thus they must be written in LaTex); space is courtesy of Motalen, and off campus/separate from any RPI technology/infrastructure. This paper is 20% of one's grade. Bringsjord's positions are expressed as declarative propositions, and will often have a philosophical dimension. As an example, here is a position that will be advanced:

   $\overline{\text{AI}^{=\text{HI}}}$ It is logically/mathematically impossible for AI (as defined today in the textbooks and primary literature of the field of AI) to match (let alone exceed) human intelligence.

4. Finally, the remaining 10% of one's grade is based on participation through discussion and email, etc. Cogent critique from students of Bringsjord's positions on "big questions" re. AI and the mind.

# 7 Some Learning Outcomes

There are three desired outcomes. *One*: Students will be able to/refresh their ability to carry out/execute formal proofs and disproofs, and simple pure logic programs, in collaboration with AI, within the HyperSlate® system and its workspaces, at the level of the propositional and predicate calculi, and propositional modal logic (the aforementioned systems **T**, **S4**, **D**, and **S5**). *Two*: Students will understand the main metatheorems of intermediate formal logic, and all of those achieved by Gödel (as enumerated above). *Three*, students will be able to debate, verbally and in cogent prose, some of the profound questions raised by AI (questions enumerated above).

# 8 Academic Honesty

Student-teacher relationships are built on mutual respect and trust. Students must be able to trust that their teachers have made responsible decisions about the structure and content of the course, and that they're conscientiously making their best effort to help students learn. Teachers must be able to trust that students do their work conscientiously and honestly, making their best effort to learn. Acts that violate this mutual respect and trust undermine the educational process; they counteract and contradict our very reason for being at Rensselaer and will not be tolerated. Any student who engages in any form of academic dishonesty will receive an F in this course and will be reported to the Dean of Students for further disciplinary action. (The *Rensselaer Handbook* defines various forms of Academic Dishonesty and procedures for responding to them. All of these forms are violations of trust between students and teachers. Please familiarize yourself with this portion of the handbook.) In particular, all solutions submitted to HyperGrader$^{\textregistered}$ for course credit under a student id are to be the work of the student associated with that id alone, and not in any way copied or based on the work of anyone else.

# References

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