

Could AI Ever Match Gödelian Greatness?

(Essays on Artificial vs. Personal Intelligence)

Selmer Bringsjord

Intro to Formal Logic (& AI) (IFLAI)

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No.

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<https://cse.buffalo.edu/~rapaport/Papers/aidebate.pdf>

Will AI Succeed? The “Yes” Position

William J. Rapaport

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July 30, 2025

Abstract

This is a draft of the “Yes” side of a proposed debate book, *Will AI Succeed?*. The “No” position will be taken by Selmer Bringsjord, and will be followed by rejoinders on each side.

AI should be considered as the branch of computer science that investigates whether, and to what extent, cognition is computable. Computability is a logical or mathematical notion. So, the only way to prove that something—including (some aspect of) cognition—is *not* computable is via a logical or mathematical argument. Because no such argument has met with general acceptance (in the way that other proofs of non-computability—such as the Halting Problem—have been generally accepted), there is no logical reason to think that AI *won't* eventually match human intelligence. Along the way, I discuss the Turing Test as a measure of AI's success at showing the computability of various aspects of cognition, and I consider the potential roadblocks set by consciousness, qualia, and mathematical intuition.

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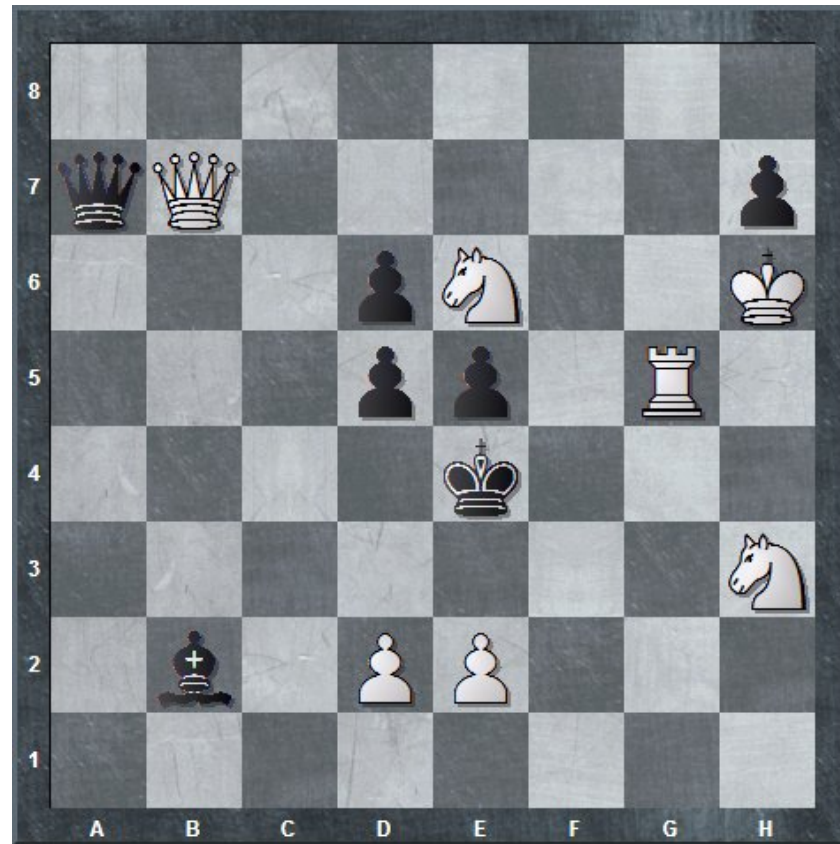
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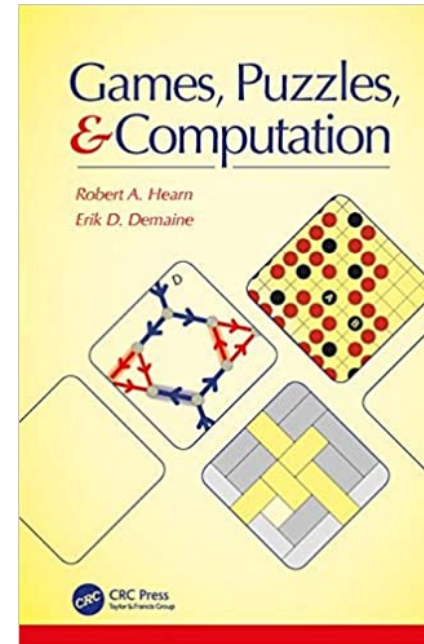
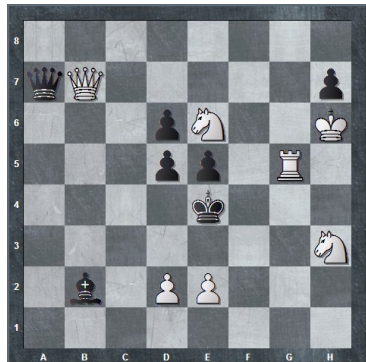
Gödel's Greatness & Games

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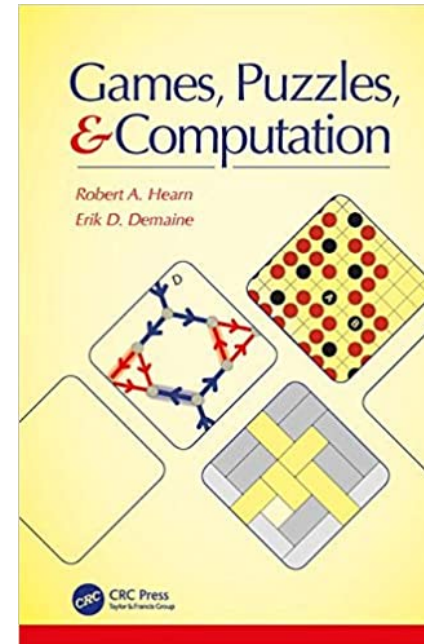
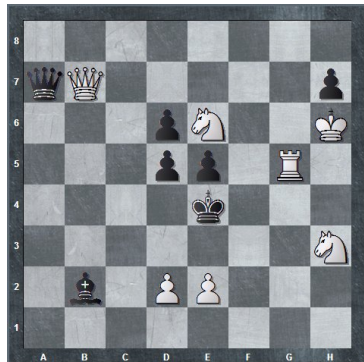
Mate in 2 Problem



Mate in 2 Problem



Mate in 2 Problem



The Constraint-Logic Formalism

The general model of games we will develop is based on the idea of a *constraint graph*; by adding rules defining legal moves on such graphs we get *constraint logic*. In later chapters the graphs and the rules will be specialized to produce games with different numbers of players: zero, one, two, etc. A game played on a constraint graph is a computation of a sort, and simultaneously serves as a useful problem to reduce to other games to show their hardness.

In the game complexity literature, the standard problem used to show games hard is some kind of game played with a Boolean formula. The Satisfiability problem (SAT), for example, can be interpreted as a puzzle: the player must existentially make a series of variable selections, so that the formula is true. The corresponding model of computation is nondeterminism, and the natural complexity class is NP. Adding alternating existential and universal quantifiers creates the Quantified Boolean Formula problem (QBF), which has a natural interpretation as a two-player game [158].

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on



Logico-Mathematical Landscape that Has Gödel Turning in His Grave

Super-Serious Human Cognitive Power

Serious Human Cognitive Power

Mere Calculative Cognitive Power

Entscheidungsproblem

Logico-Mathematical Landscape that Has Gödel Turning in His Grave

Super-Serious Human Cognitive Power

Serious Human Cognitive Power



Gödel

Entscheidungsproblem

Mere Calculative Cognitive Power

Logico-Mathematical Landscape that Has Gödel Turning in His Grave

Super-Serious Human Cognitive Power

Serious Human Cognitive Power



Gödel



Turing

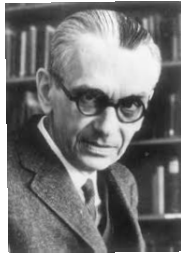
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Turing

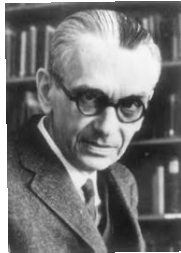
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Entscheidungsproblem

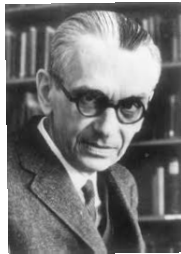
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Serious Human Cognitive Power

Podcast: The Turing Test is Dead.
Long Live the Lovelace Test.



Gödel



Entscheidungsproblem

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Entscheidungsproblem

Mere Calculative Cognitive Power

Logico-Mathematical Landscape that Has Gödel Turning in His Grave

Analytical Hierarchy

Serious Human Cognitive Power



Gödel



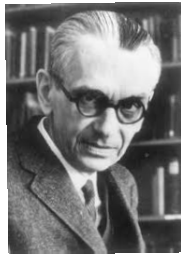
Mere Calculative Cognitive Power

Entscheidungsproblem

Logico-Mathematical Landscape that Has Gödel Turning in His Grave

Analytical Hierarchy

Arithmetical Hierarchy



Gödel



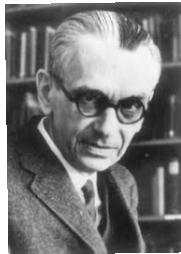
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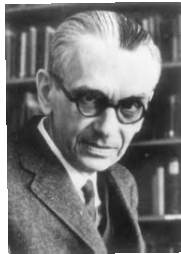
Entscheidungsproblem

Polynomial Hierarchy

Logico-Mathematical Landscape that Has Gödel Turning in His Grave

Analytical Hierarchy

Arithmetical Hierarchy



Gödel



Entscheidungsproblem

Polynomial Hierarchy

$$\mathbf{P} \subseteq \mathbf{NP} \subseteq \mathbf{PSPACE} = \mathbf{NPSPACE} \subseteq \mathbf{EXPTIME} \subseteq \mathbf{NEXPTIME} \subseteq \mathbf{EXPSPACE}$$

Logico-Mathematical Landscape that Has Gödel Turning in His Grave

Analytical Hierarchy

Arithmetical Hierarchy



Gödel



\vdots
 Π_2
 Σ_2
 Π_1
 Σ_1
 Σ_0

Entscheidungsproblem

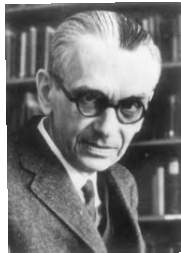
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Analytical Hierarchy

Arithmetical Hierarchy



Gödel



Go:AlphaGo



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Jeopardy!:



Go:AlphaGo



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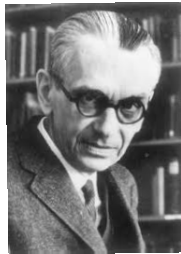
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Jeopardy!:

Chess: Deep Blue

Go: AlphaGo

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Jeopardy!:

Checkers: Chinook



Chess: Deep Blue



Go: AlphaGo



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Chess: Deep Blue



Checkers: Chinook

Go: AlphaGo



$P \subseteq NP \subseteq PSPACE = NPSPACE \subseteq EXPTIME \subseteq NEXPTIME \subseteq EXPSPACE$

1994

Checkers: Tinsley vs. Chinook



Name: Marion Tinsley
Profession: Teach mathematics
Hobby: Checkers
Record: Over 42 years
loss only 2 games
of checkers
World champion for over 40
years

Mr. Tinsley suffered his 4th and 5th losses against Chinook

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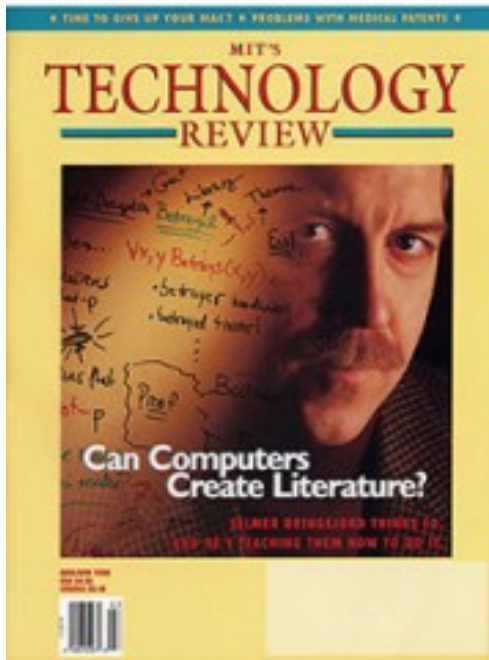
2011





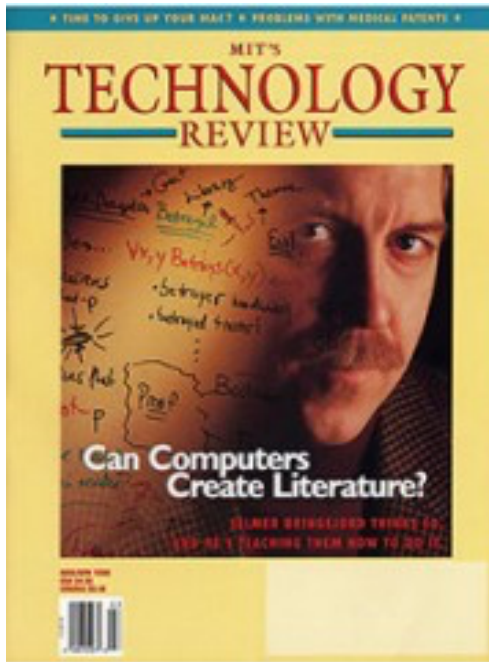
1998

“Chess is Too Easy”



1998

“Chess is Too Easy”





1998

Some of Gödel's great work is at the level of chess.

But to *fully* “gamify” Gödel,
we need a harder game! ...

Rengo Kriegspiel



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American Go E-Journal

US Go Congress Goes a Little Crazy

Wednesday August 13, 2014



"White plays capturing black, putting herself and black into atari," calls Crazy Go TD Terry Benson. He officiated several games of Rengo Kriegspiel on Tuesday evening – a pair go game in which all four players face away from the main board and play their stones on their own empty board in front of them; the only clues about where their opponents — and even their partner — have played comes when they make an illegal move, or play where their own team or their opponents already have stones. Rengo Kriegspiel is only one of dozens of variants on the game of go that were played by an enthusiastic crowd of around 100 players. Familiar games include Magnetic Go, 4 Color Go, Tessellation Go, 3D Go, Spiral Go, and Blind Go. "After all these years, it's still crazy," said TD and Crazy Go founder Terry Benson. New Crazy Go games, never before played at a Go Congress, were even invented on the spot. Four players donned sleeping masks to block their vision and transformed Blind Go into Rengo Blind Go, and a few other players added the fundamentals of Tiddlywinks to their go game. Spectators and players alike are enthusiastic about the creativity of the games and the fun of adding a little Crazy to Go; "Crazy Go is my favorite part of the Congress!" said Bob Crites.
- report/photos by Karoline Li



CATEGORIES:

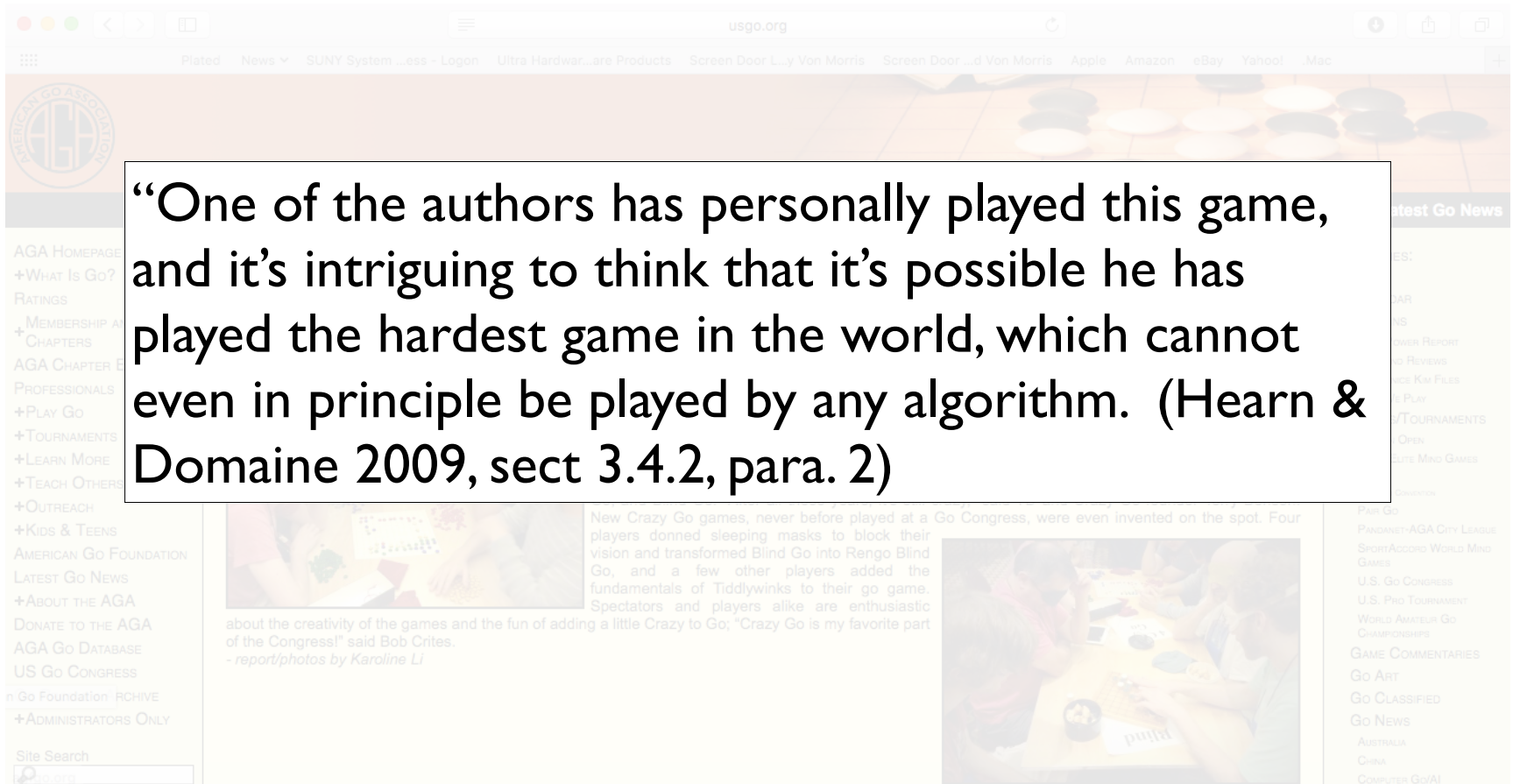
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JOHN POWER REPORT
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WHY WE PLAY

EVENTS/TOURNAMENTS
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IMSA ELITE MIND GAMES
OTHER
N.A. GO CONVENTION
PAIR GO
PANDANET-AGA CITY LEAGUE
SPORTACCORD WORLD MIND GAMES
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WORLD AMATEUR GO CHAMPIONSHIPS

GAME COMMENTARIES
GO ART
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GO NEWS
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Rengo Kriegspiel

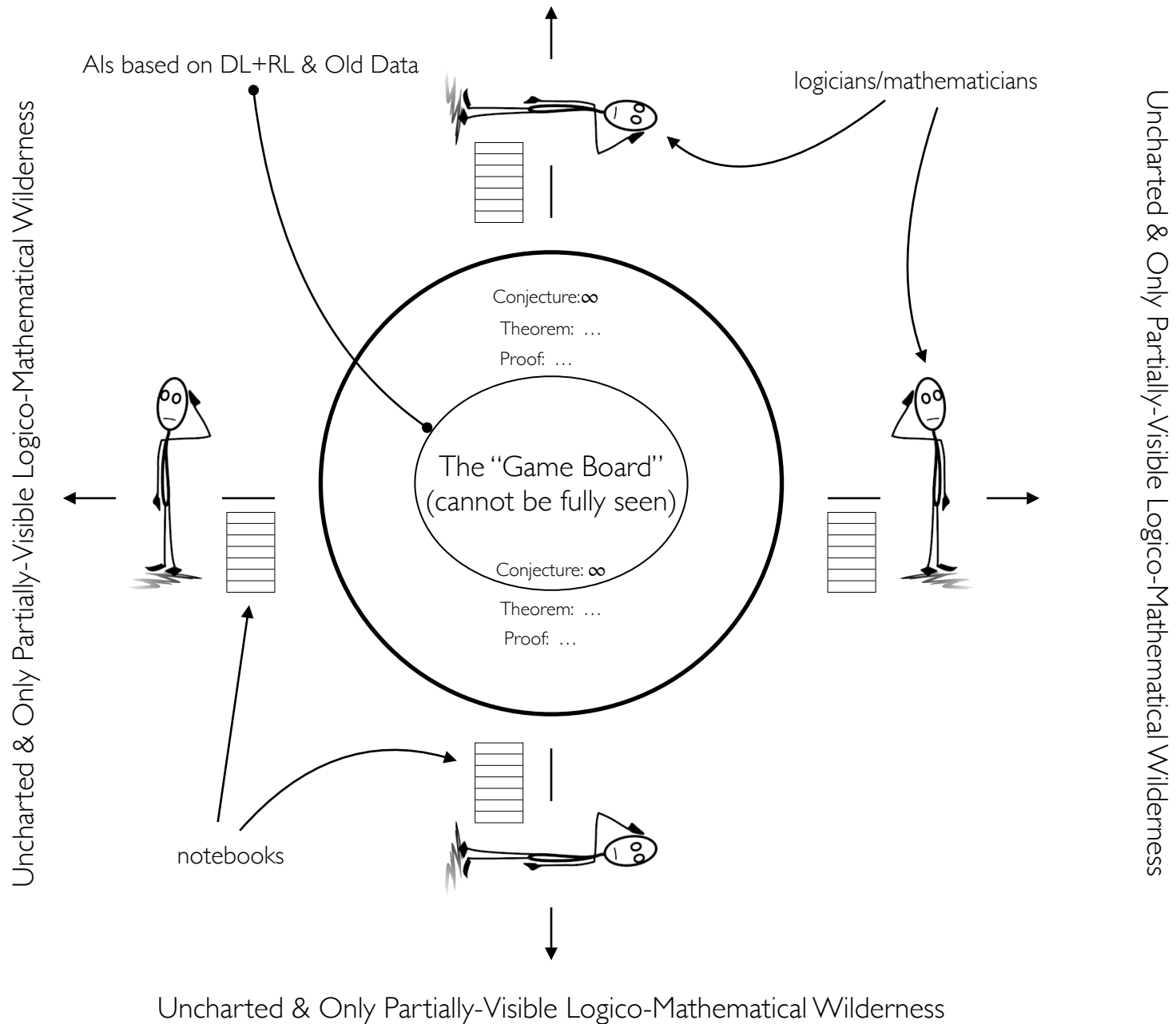
“One of the authors has personally played this game, and it’s intriguing to think that it’s possible he has played the hardest game in the world, which cannot even in principle be played by any algorithm. (Hearn & Domaine 2009, sect 3.4.2, para. 2)



The Gödel Game

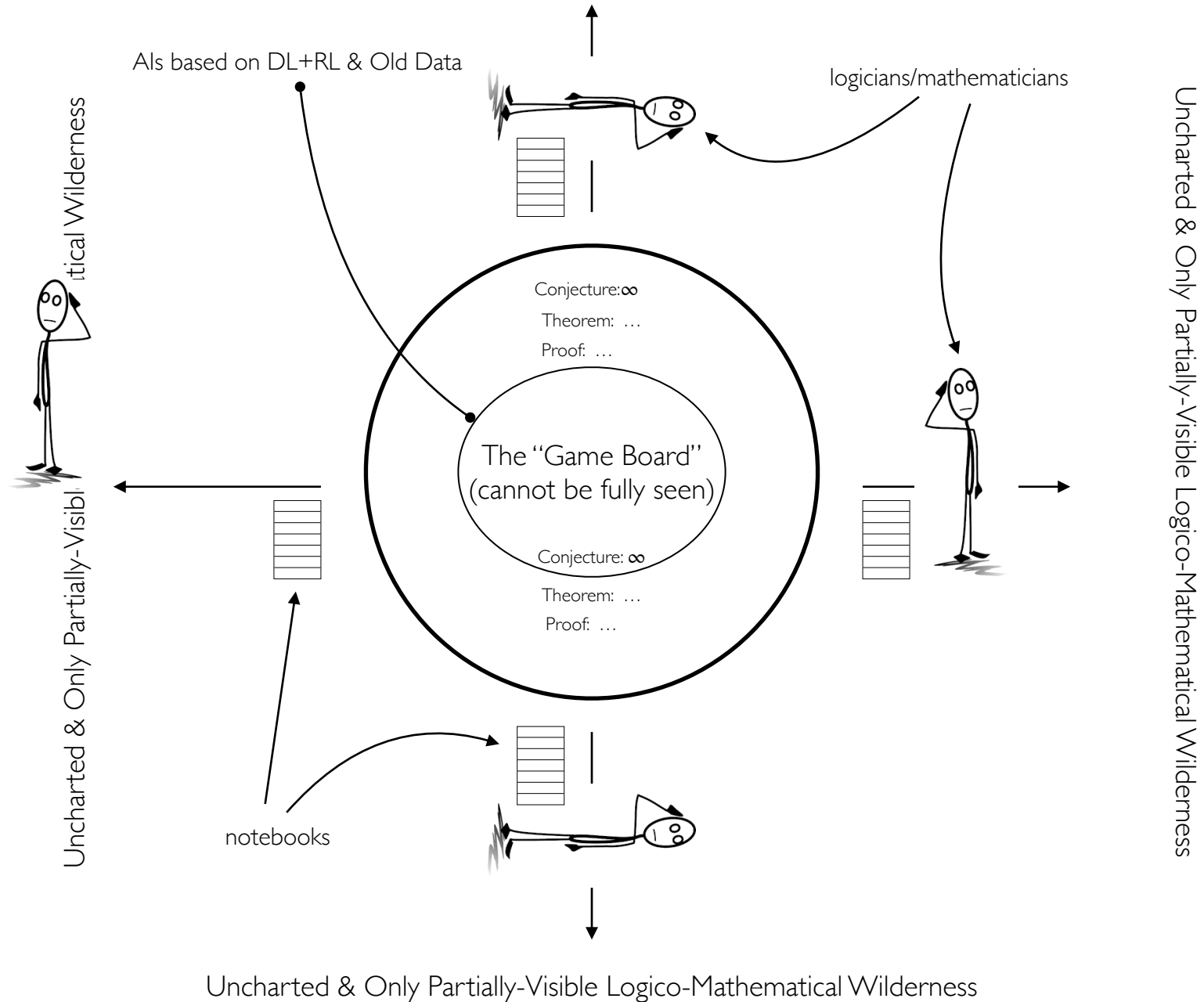
The Gödel Game

Uncharted & Only Partially-Visible Logico-Mathematical Wilderness



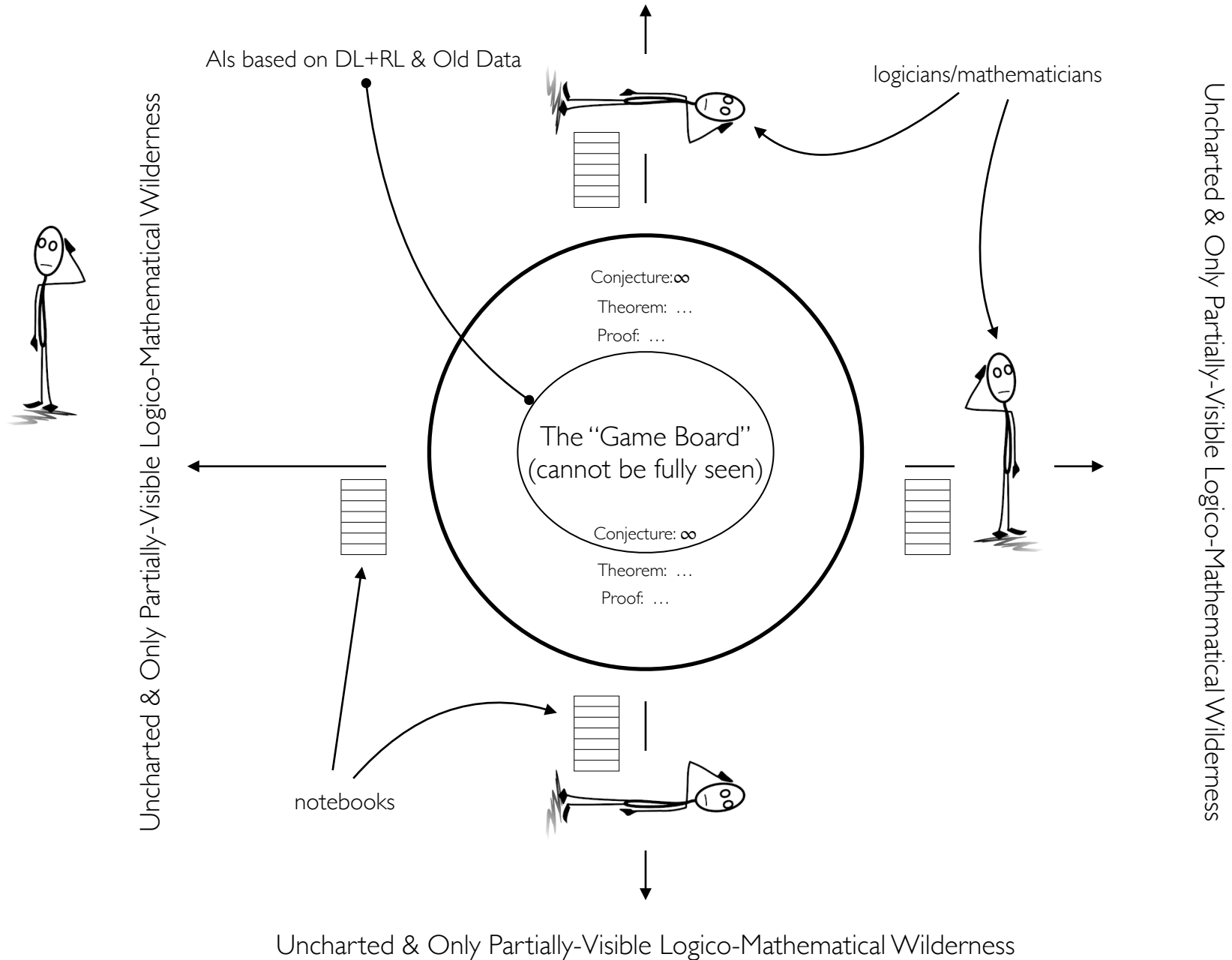
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The Gödel Game

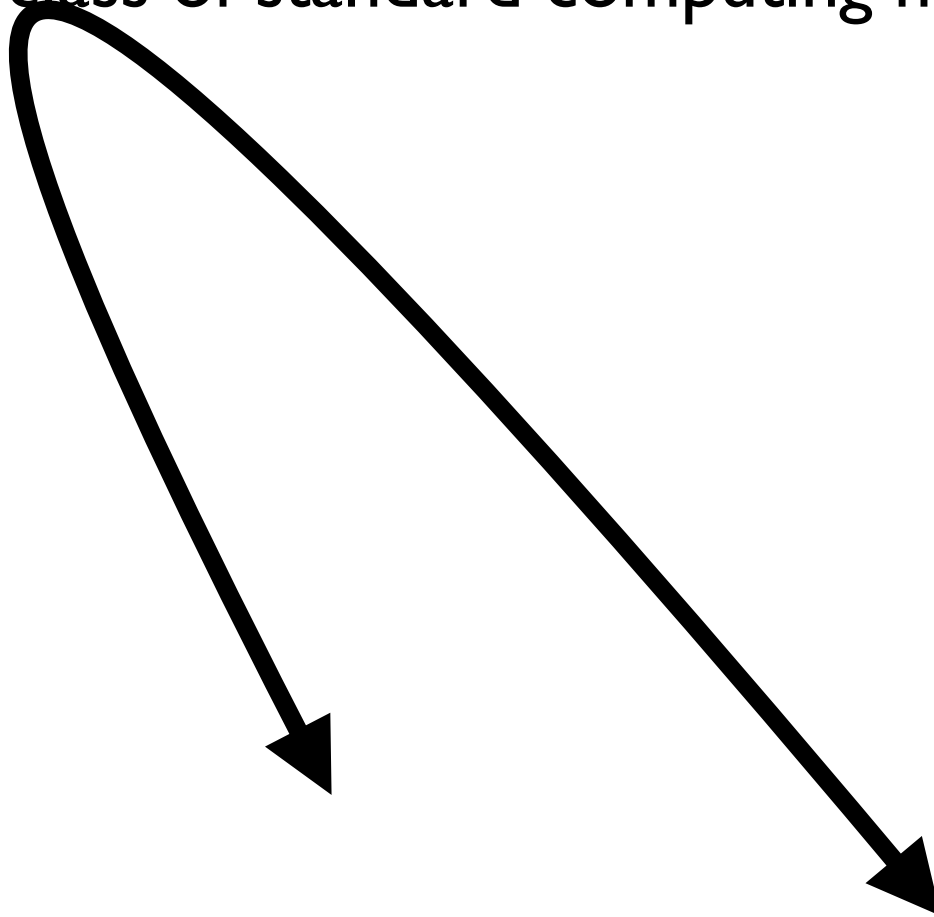
Uncharted & Only Partially-Visible Logico-Mathematical Wilderness



Gödel's Either/Or ...

The Question

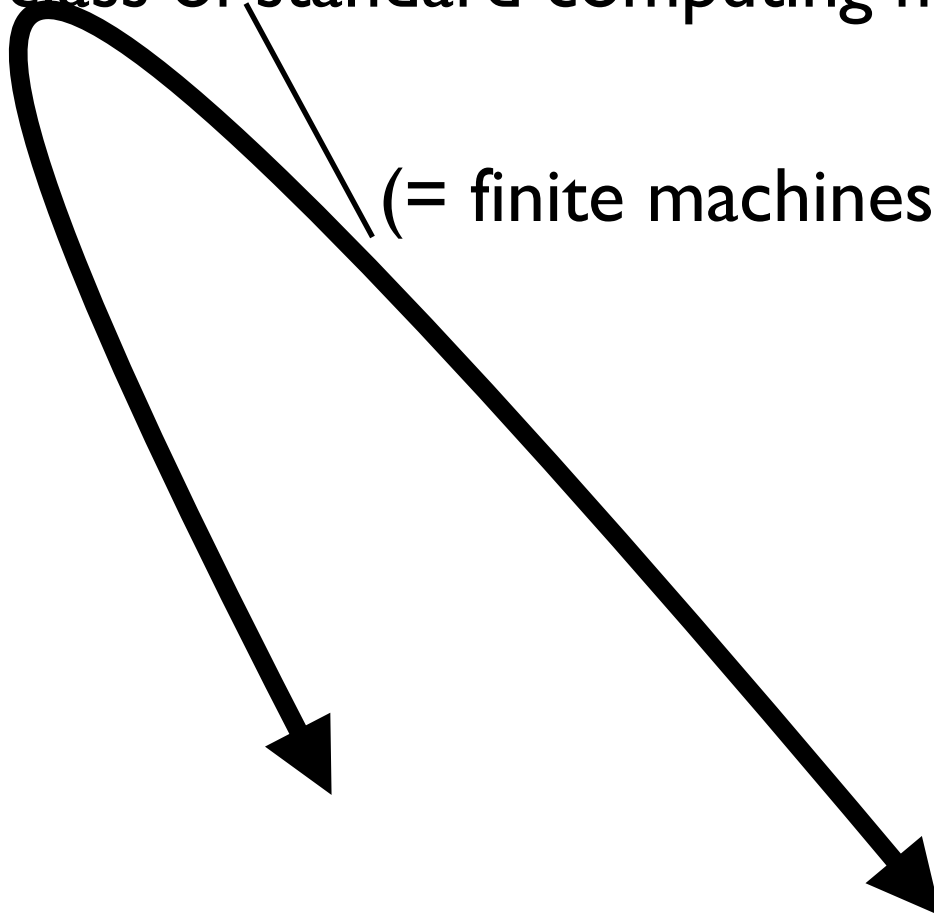
Q* Is the human mind more powerful than the class of standard computing machines?



The Question

Q* Is the human mind more powerful than the class of standard computing machines?

(= finite machines)



The Question

Q* Is the human mind more powerful than the class of standard computing machines?

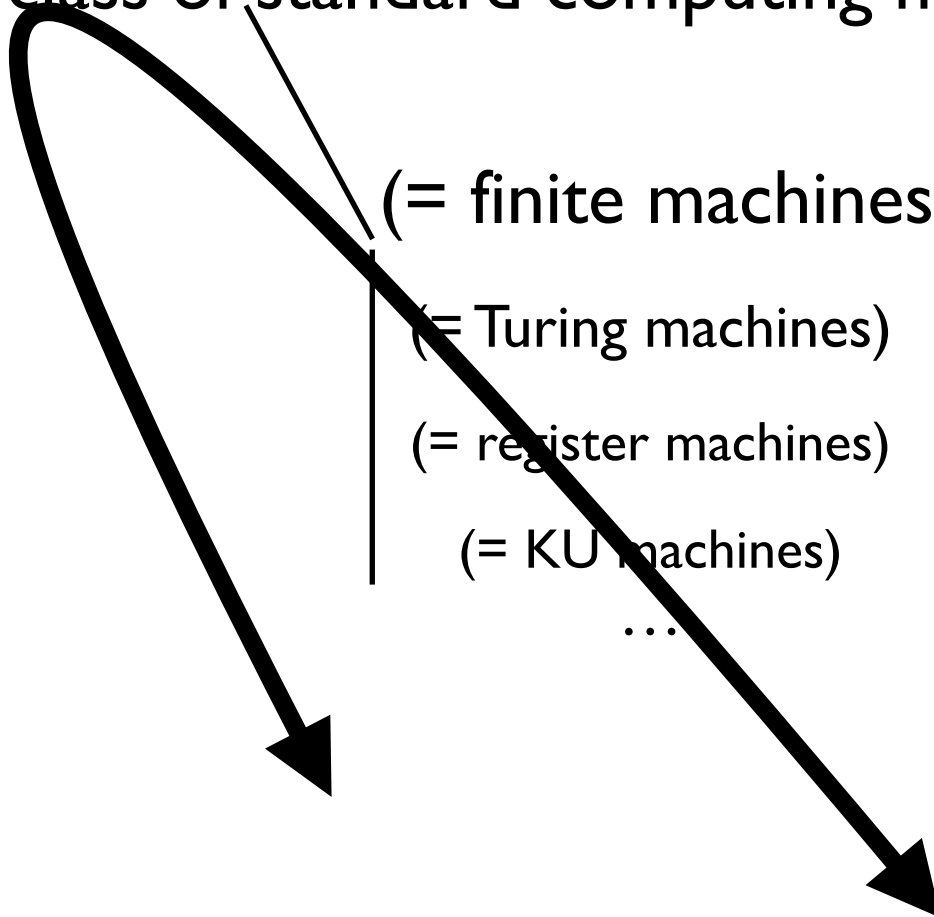
(= finite machines)

(= Turing machines)

(= register machines)

(= KU machines)

...



The Question

Q* Is the human mind more powerful than the class of standard computing machines?

(= finite machines)

(= Turing machines)

(= register machines)

(= KU machines)

...



No.

Yes.

Gödel's Either/Or

“[E]ither ... the human mind (even within the realm of pure mathematics) infinitely surpasses the power of any finite machine, or else there exist absolutely [humanly?] unsolvable diophantine problems.”

— Gödel, 1951, Providence RI

Gödel's Either/Or

“[E]ither ... the human mind (even within the realm of pure mathematics) infinitely surpasses the power of any finite machine, or else there exist absolutely [humanly?] unsolvable diophantine problems.”
— Gödel, 1951, Providence RI

More precisely, what does this mean?

PT as a Diophantine Equation

Equations of this sort were introduced to you in middle-school, when you were asked to find the hypotenuse of a right triangle when you knew its sides; the familiar equation, the famous Pythagorean Theorem that most adults will remember at least echoes of into their old age, is:

$$(PT) \quad a^2 + b^2 = c^2,$$

and this is of course equivalent to

$$(PT') \quad a^2 + b^2 - c^2 = 0,$$

which is a Diophantine equation. Such equations have at least two unknowns (here, we of course have three: a, b, c), and the equation is solved when positive integers for the unknowns are found that render the equation true. Three positive integers that render (PT') true are

$$a = 4, b = 3, c = 5.$$

It is *mathematically impossible* that there is a finite computing machine capable of solving any Diophantine equation given to it as a challenge (!).

... which means that the 10th of Hilbert's Problems is settled:

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Hilbert's problems

From Wikipedia, the free encyclopedia

Hilbert's problems are twenty-three problems in [mathematics](#) published by German mathematician [David Hilbert](#) in 1900. The problems were all unsolved at the time, and several of them were very influential for 20th-century mathematics. Hilbert presented ten of the problems (1, 2, 6, 7, 8, 13, 16, 19, 21, and 22) at the [Paris](#) conference of the [International Congress of Mathematicians](#), speaking on August 8 in the [Sorbonne](#). The complete list of 23 problems was published later, most notably in English translation in 1902 by [Mary Frances Winston Newson](#) in the *[Bulletin of the American Mathematical Society](#)*.^[1]

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David Hilbert

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10th	Find an algorithm to determine whether a given polynomial Diophantine equation with integer coefficients has an integer solution.	Resolved. Result: Impossible; Matiyasevich's theorem implies that there is no such algorithm.	1970
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It was a *team* effort, actually; it's not due solely to Matiyasevich, and is often denoted as the 'MRDP Theorem.'

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Julia **R**obinson

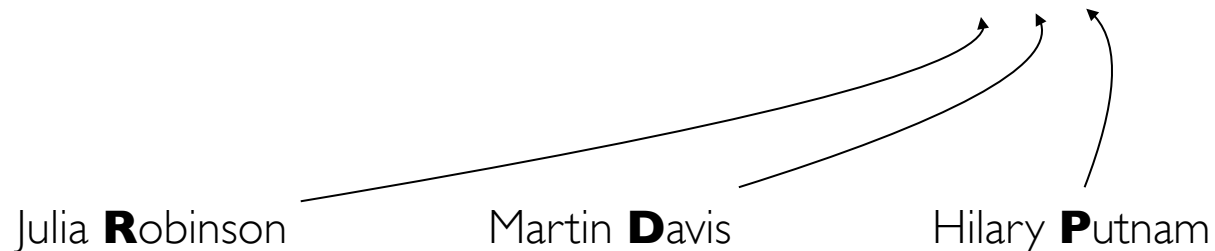
Martin **D**avis

Hilary **P**utnam

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Background

problem?⁷ In his lecture, Gödel precisely defines diophantine problems, but we don't need to bother with all of the details here; we only need to appreciate the general structure of such a problem, and that can be achieved quickly as follows, given what was introduced in Chapter 2.

Each diophantine problem has at its core a polynomial \mathcal{P} whose variables are comprised by two lists, x_1, x_2, \dots, x_n and y_1, y_2, \dots, y_m ; all variables must be integers, and the same for subscripts n and m . To represent a polynomial in a manner that announces its variables, we can write

$$\mathcal{P}(x_1, x_2, \dots, x_k, y_1, y_2, \dots, y_j).$$

But Gödel was specifically interested in whether, for all integers that can be set to the variables x_i , there are integers that can be set to the y_j , such that the polynomial equals 0. To make this clearer, first, here are two particular, simple equations that employ polynomials that are both instances of the needed form:

$$\text{E1} \quad 3x - 2y = 0$$

$$\text{E2} \quad 2x^2 - y = 0$$

All we need to do now is prefix these equations with quantifiers in the pattern Gödel gave. This pattern is quite simple: universally quantify over each x_i variable (using the now-familiar \forall), after which we existentially quantify over each y_i variable (using the also-now-familiar \exists). Thus, here are the two diophantine problems that correspond to the pair E1 and E2 from just above:

$$\text{P1} \quad \text{Is it true that } \forall x \exists y (3x - 2y = 0)?$$

$$\text{P2} \quad \text{Is it true that } \forall x \exists y (2x^2 - y = 0)?$$

Great Paper!



Hilbert's Tenth Problem is Unsolvable

Author(s): Martin Davis

Source: *The American Mathematical Monthly*, Vol. 80, No. 3 (Mar., 1973), pp. 233-269

Published by: [Mathematical Association of America](#)

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Hilbert's Tenth Problem is Unsolvable

Author(s): Martin Davis

Source: *The American Mathematical Monthly*, Vol. 80, No. 3 (Mar., 1973), pp. 233-269

Published by: [Mathematical Association of America](#)

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DEFINITION. A set S of ordered n -tuples of positive integers is called **Diophantine** if there is a polynomial $P(x_1, \dots, x_n, y_1, \dots, y_m)$, where $m \geq 0$, with integer coefficients such that a given n -tuple $\langle x_1, \dots, x_n \rangle$ belongs to S if and only if there exist positive integers y_1, \dots, y_m for which

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1973]

HILBERT'S TENTH PROBLEM IS UNSOLVABLE

235

$$P(x_1, \dots, x_n, y_1, \dots, y_m) = 0.$$

Borrowing from logic the symbols " \exists " for "there exists" and " \Leftrightarrow " for "if and only if", the relation between the set S and the polynomial P can be written succinctly as:

$$\langle x_1, \dots, x_n \rangle \in S \Leftrightarrow (\exists y_1, \dots, y_m) [P(x_1, \dots, x_n, y_1, \dots, y_m) = 0],$$

or equivalently:

$$S = \{ \langle x_1, \dots, x_n \rangle \mid (\exists y_1, \dots, y_m) [P(x_1, \dots, x_n, y_1, \dots, y_m) = 0] \}.$$

Note that P may (and in non-trivial cases always will) have negative coefficients. The word "**polynomial**" should always be so construed in the article except where the contrary is explicitly stated. Also all numbers in this article are positive integers unless the contrary is stated.

133-269

on a wide range of
 facilitate new forms

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Great Paper!

Notice that this is a perfect fit with how we used formal logic to present and understand the Polynomial Hierarchy and the Arithmetic Hierarchy.



Unsolvability

Mathematical Monthly, Vol. 81, No. 1, January 1973
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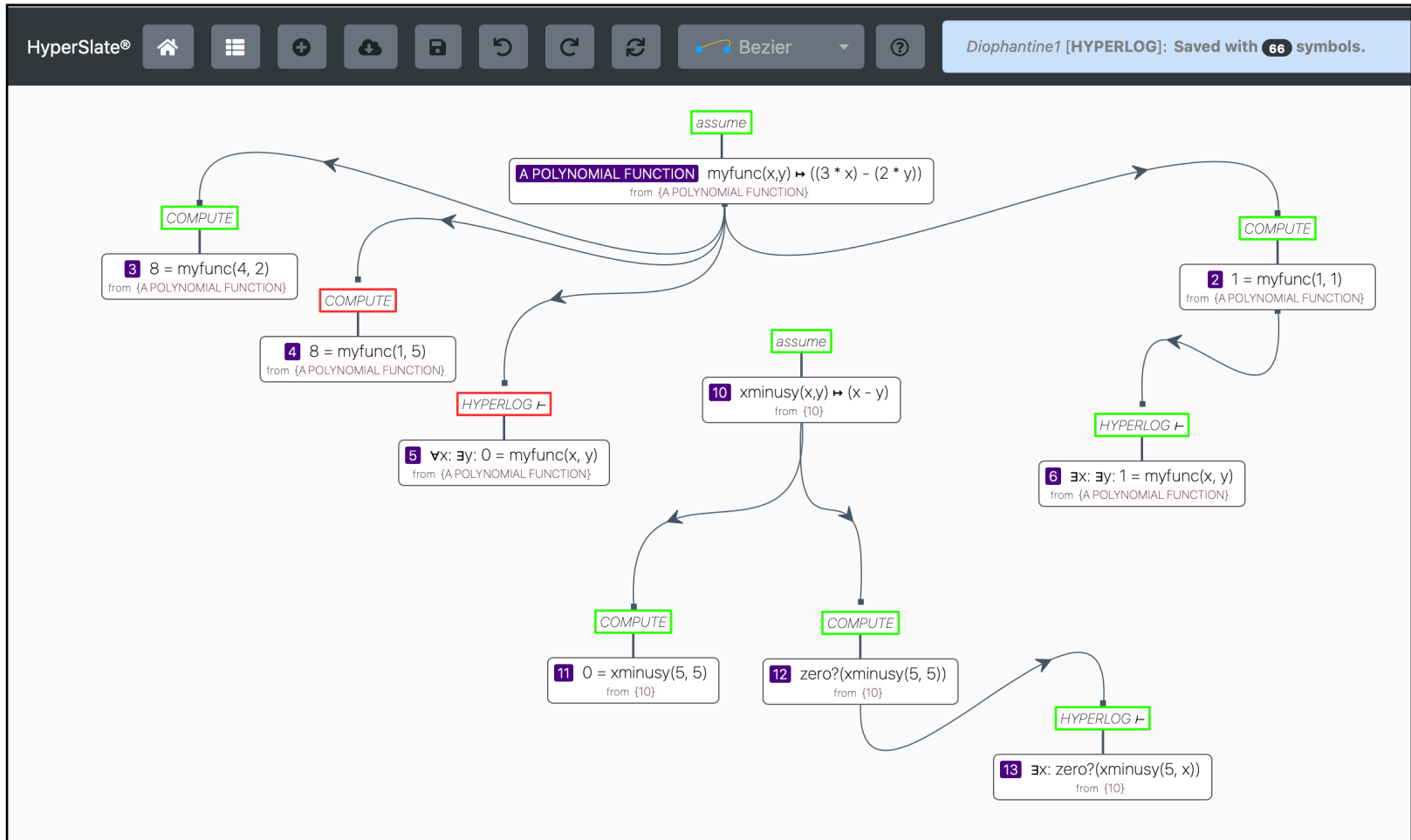
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Diophantine “Threat” in the Programming Language Hyperlog[®]



The Crux

$\exists \mathcal{P}$ s.t. no human mind could ever decide $\forall x_1 \forall x_2 \cdots \forall x_k \exists y_1 \exists y_2 \cdots \exists y_j (\mathcal{P}(x_1, x_2, \dots, x_k, y_1, y_2, \dots, y_j))$?

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The human mind is *not* infinitely more powerful than any standard computing machine.

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No.



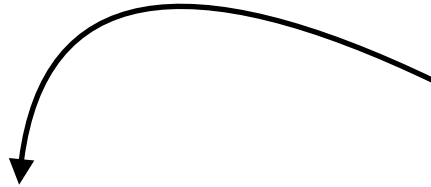
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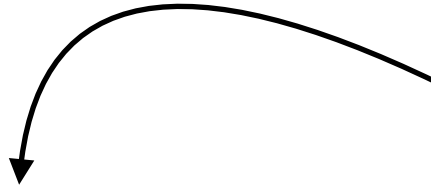
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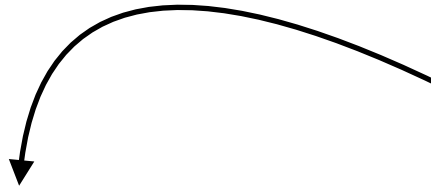
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Where else early in his career does Dr Gödel use this form? \$20

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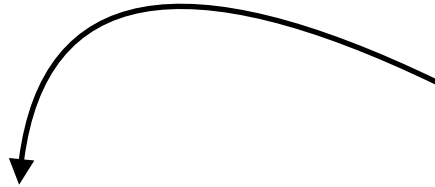
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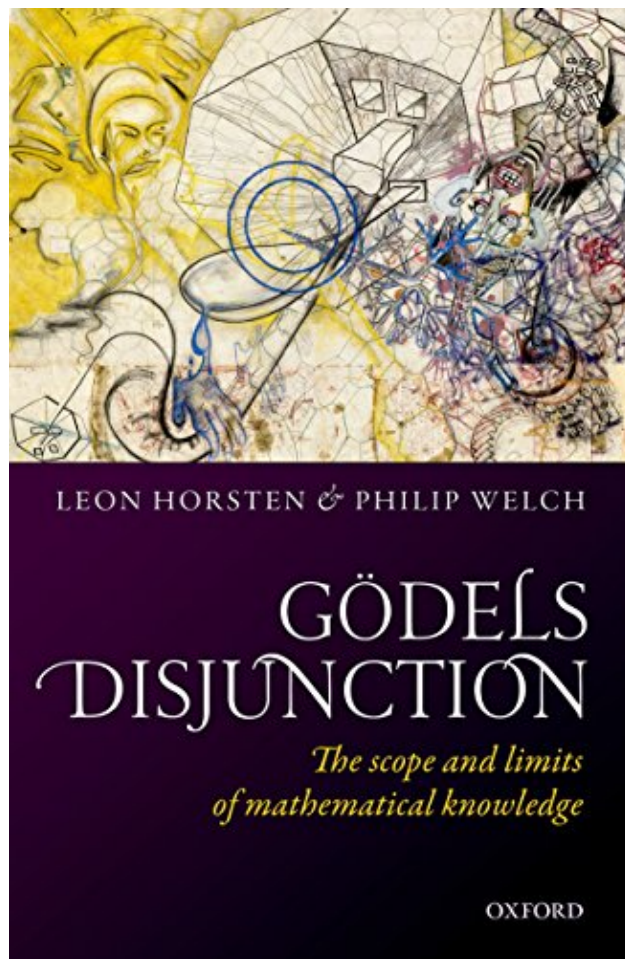
No.



The human mind *is* infinitely more powerful than any standard computing machine.

Entire book on Gödel's Either-Or ...

Entire book on Gödel's Either-Or ...



Earlier Gödelian Argument for the “No.”

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Outline

Abstract

1. Introduction
2. Clarifying computationalism, the view to be overthro...
3. The essence of hypercomputation: harnessing the in...
4. Gödel on minds exceeding (Turing) machines by “co...
5. Setting the context: the busy beaver problem
6. The new Gödelian argument
7. Objections
8. Conclusion

References

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Figures (1)



Tables (1)

Table 1



Applied Mathematics and Computation

Volume 176, Issue 2, 15 May 2006, Pages 516–530



A new Gödelian argument for hypercomputing minds based on the busy beaver problem ☆

Selmer Bringsjord , Owen Kellett, Andrew Shilliday, Joshua Taylor, Bram van Heuveln, Yingrui Yang, Jeffrey Baumes, Kyle Ross

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<https://doi.org/10.1016/j.amc.2005.09.071>

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Abstract

Do human persons hypercompute? Or, as the doctrine of *computationalism* holds, are they information processors at or below the Turing Limit? If the former, given the essence of hypercomputation, persons must in some real way be capable of infinitary information processing. Using as a springboard Gödel's little-known assertion that the human mind has a power “converging to infinity”, and as an anchoring problem Rado's [T. Rado, On non-computable functions, Bell System Technical Journal 41 (1963) 877–884] Turing-uncomputable “busy beaver” (or Σ) function, we present in this short paper a new argument that, in fact, human persons can hypercompute. The argument is intended to be formidable, not conclusive: it brings Gödel's intuition to a greater level of precision, and places it within a sensible case against computationalism.

Bringsjord vs. Rapaport ...

Will AI Match (Or Even Exceed) Human Intelligence?



No.



Yes.

Will AI Match (Or Even Exceed) Human Intelligence?



No.



Yes.



?

Will AI Succeed? The “Yes” Position

William J. Rapaport

**Department of Computer Science and Engineering,
Department of Philosophy, Department of Linguistics,
and Center for Cognitive Science
University at Buffalo, The State University of New York,
Buffalo, NY 14260-2500
rapaport@buffalo.edu
<http://www.cse.buffalo.edu/~rapaport/>**

July 30, 2025

Abstract

This is a draft of the “Yes” side of a proposed debate book, *Will AI Succeed?*. The “No” position will be taken by Selmer Bringsjord, and will be followed by rejoinders on each side.

AI should be considered as the branch of computer science that investigates whether, and to what extent, cognition is computable. Computability is a logical or mathematical notion. So, the only way to prove that something—including (some aspect of) cognition—is *not* computable is via a logical or mathematical argument. Because no such argument has met with general acceptance (in the way that other proofs of non-computability—such as the Halting Problem—have been generally accepted), there is no logical reason to think that AI *won't* eventually match human intelligence. Along the way, I discuss the Turing Test as a measure of AI's success at showing the computability of various aspects of cognition, and I consider the potential roadblocks set by consciousness, qualia, and mathematical intuition.

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Will AI Match (Or Even Exceed) Human Intelligence?



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4: And finally, the sledgehammer is used: *phenomenal consciousness*.

Logistics; Submission Info (Final Projects)

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- Grades: Everyone has an A for 40% (Required) — but finish if it's open for you. 20% A if you came/come :). Gift of 20% for everyone. = 80% A = 4.0.

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*Med nok penger, kan
logikk løse alle problemer.*