Re. Hyperlog® (& PGLP)

NSG • SB
Intro to Logic-Based AI

Flavor: Simplified
Version: Nov 10 17:17 NY 2025

COMPUTE

Inference Schema

$(\mathbf{defn}\ f\ [\mathsf{x}\ \mathsf{y}\ \mathsf{z}]\ (\ldots))$

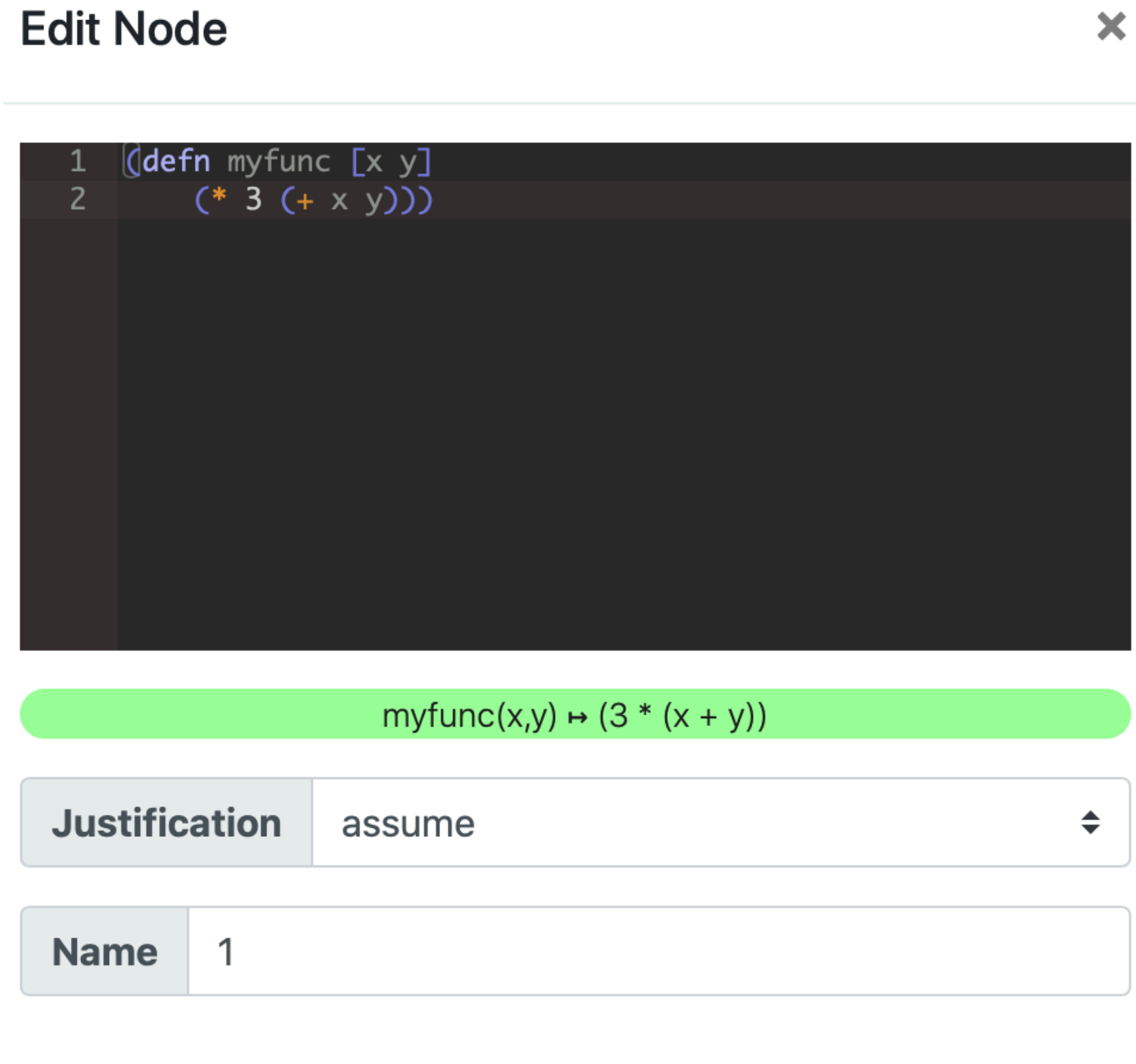
$(\mathbf{defn}\ h\ [\mathsf{x}\ \mathsf{y}\ \mathsf{z}]\ (\ldots))$

$(\mathbf{defn}\ g\ [\mathsf{x}\ \mathsf{y}\ \mathsf{z}]\ (\ldots))$

$(f\ a\ b\ c) \rightsquigarrow \sigma$

$$(f\ a\ b\ c) = \sigma$$

Example


Edit Node
1 (defn myfunc [x y]
2 (* 3 (+ x y)))
myfunc(x,y) ↦ (3 * (x + y))
Justification
assume
Name
1

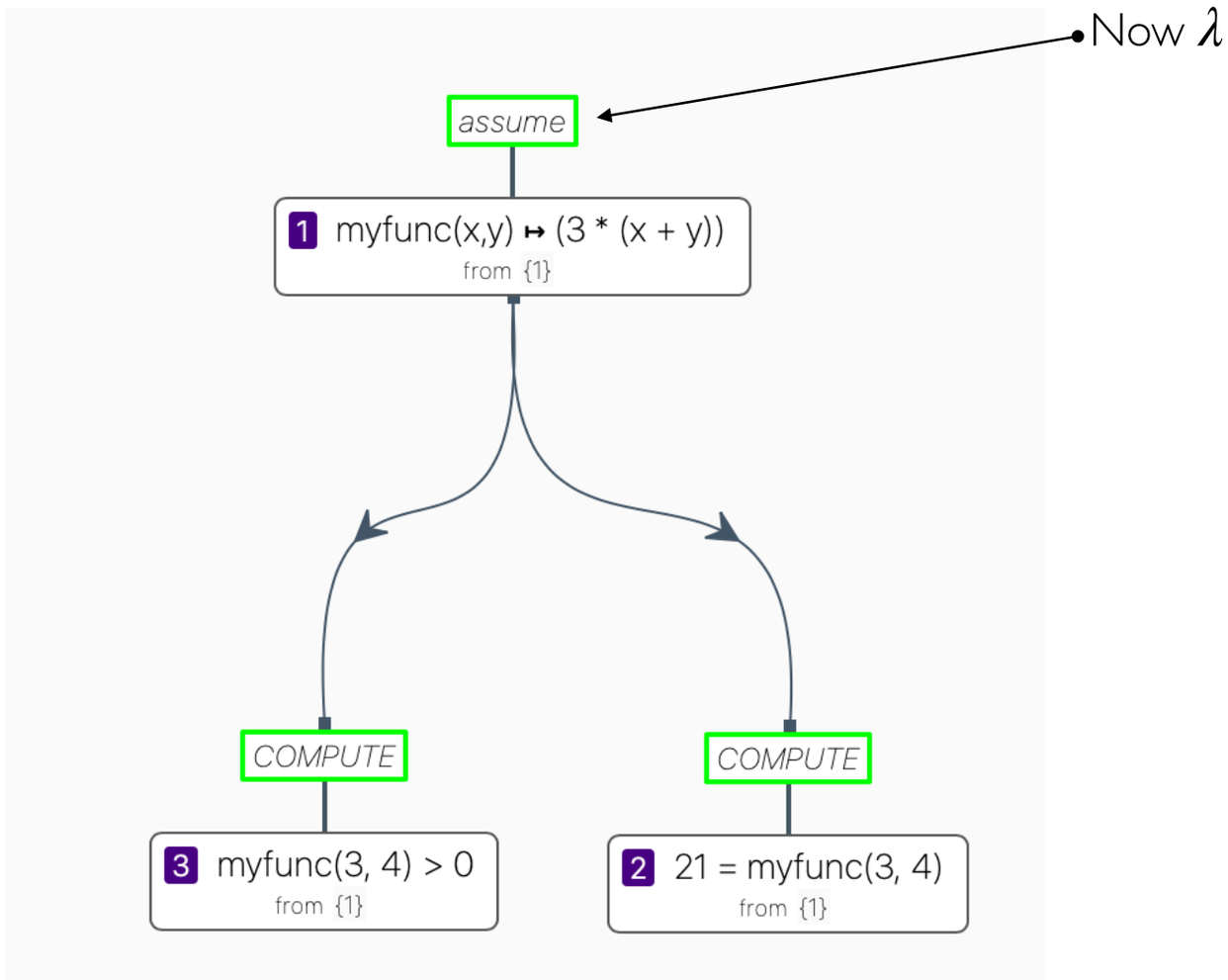
ILBAI (F24) Restrictions

- Only non-recursive functions.
- Pure functions built from {+, *, -, /}.
- No other operators or functions allowed — but
  - now conditional code permitted and "live."

Announcement!

- The justification of defining a function in Clojure in order to make it live is the λ justification.

```clojure
1  (defn myfunc [x y]
2      (* 3 (+ x y)))
```



Implicit Axioms — From Which **You** Can Select and Put Into Nodes in Your Workspace

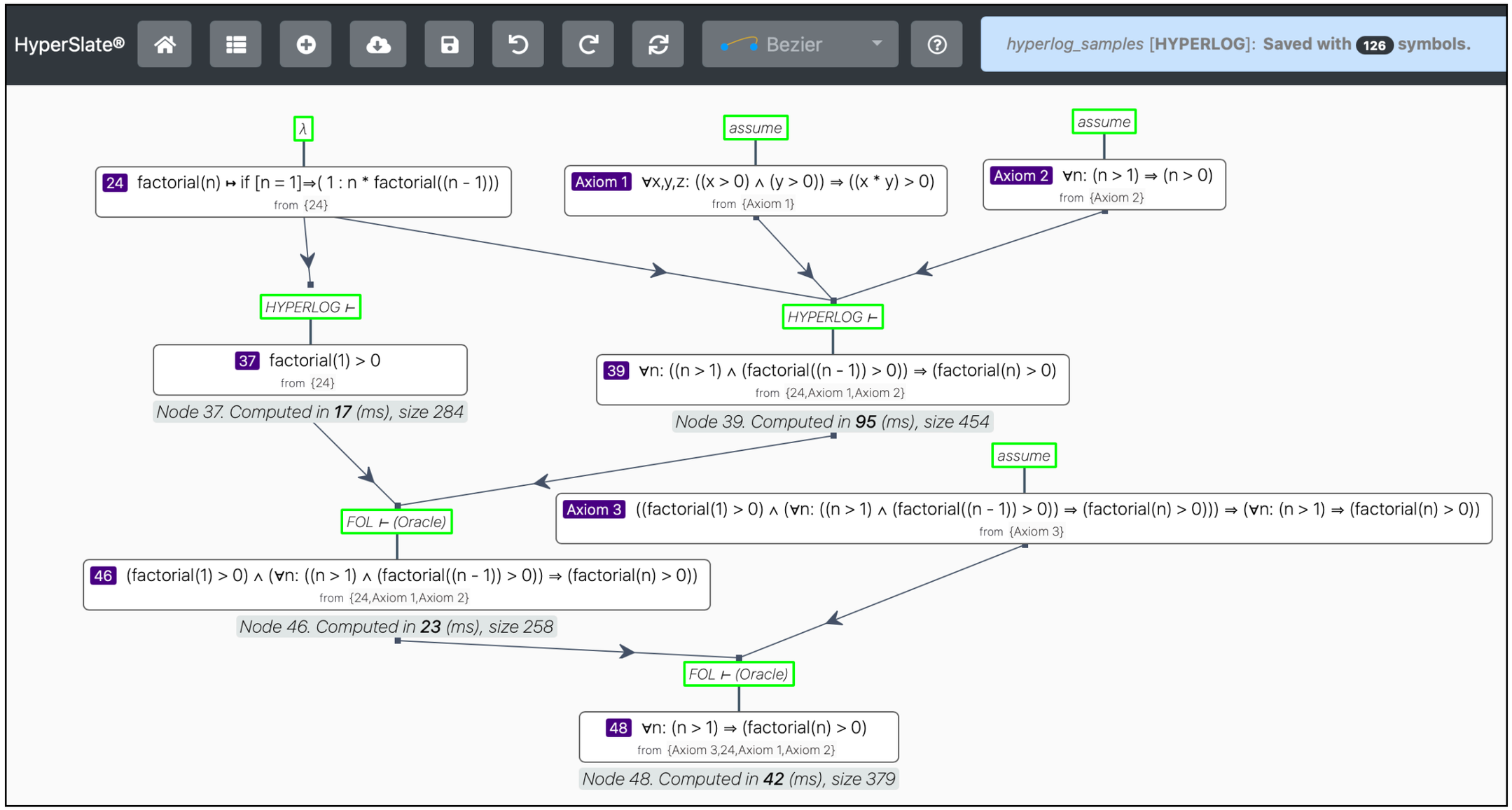
(redundancy allowed)

```
"(forall (x y) (= (+ x y) (+ y x)))", // Commutativity of addition
"(forall (x y) (= (* x y) (* y x)))", // Commutativity of multiplication
"(forall (x y z) (= (+ x (+ y z)) (+ (+ x y) z)))", // Associativity of addition
"(forall (x y z) (= (* x (* y z)) (* (* x y) z)))", // Associativity of multiplication
"(forall (x y z) (= (* z (+ x y)) (+ (* z x) (* z y))))", // Distributivity of multiplication

"(forall (x) (= (+ x 0) x))", // Identity for addition
"(forall (x) (= (* x 1) x))", // Identity for multiplication
"(forall (x) (= (* x 0) 0))",    // Multiplication by zero
"(forall (x) (exists (y) (= (+ x y) 0)))", // Existence of additive inverse
"(forall (x) (exists (y) (if (not (= x 0)) (= (* x y) 1))))", // Existence of multiplicative inverse
"(forall (x y) (= (- x y) (+ x (* -1 y))))", // Definition of subtraction
"(forall (x y) (= (/ x y) (* x (inv y))))", // Definition of division
"(forall (x) (if (not (= x 0)) (= (* x (/ 1 x)) 1)))", // Division property for non-zero x


// Axioms for < and >
"(forall (x y) (if (< x y) (not (= x y))))",  // If x < y, then x != y
"(forall (x y) (if (> x y) (not (= x y))))",  // If x > y, then x != y
"(forall (x y) (if (< x y) (> y x)))",  // If x < y, then y > x
"(forall (x y) (if (> x y) (< y x)))",  // If x > y, then y < x
"(forall (x y z) (if (and (< x y) (< y z)) (< x z)))",  // Transitivity of <
"(forall (x y z) (if (and (> x y) (> y z)) (> x z)))",  // Transitivity of >
"(forall (x y z) (if (< x y) (< (+ x z) (+ y z))))",  // Addition preserves <
"(forall (x y z) (if (< x y) (< (* x z) (* y z))))",  // Multiplication preserves < for positive z
 "(forall (x y) (if (< x y) (> y x)))",  // If x < y, then y > x
 "(forall (x y) (if (> x 0) (> (+ x y) y )))", // If x > 0 then x + y > y
 "(forall (x y) (if (> x 0) (<  y (+ x y))))" // If x > 0 then x + y > y
```

Sample File
(circulation prohibited)



https://world.logicamodernapproach.com/@/hyperlog_samples

Next, Zoombinis Exploration …

Zoombinis Exploration

Can we prove that Mairy is a bridge I-Passer?

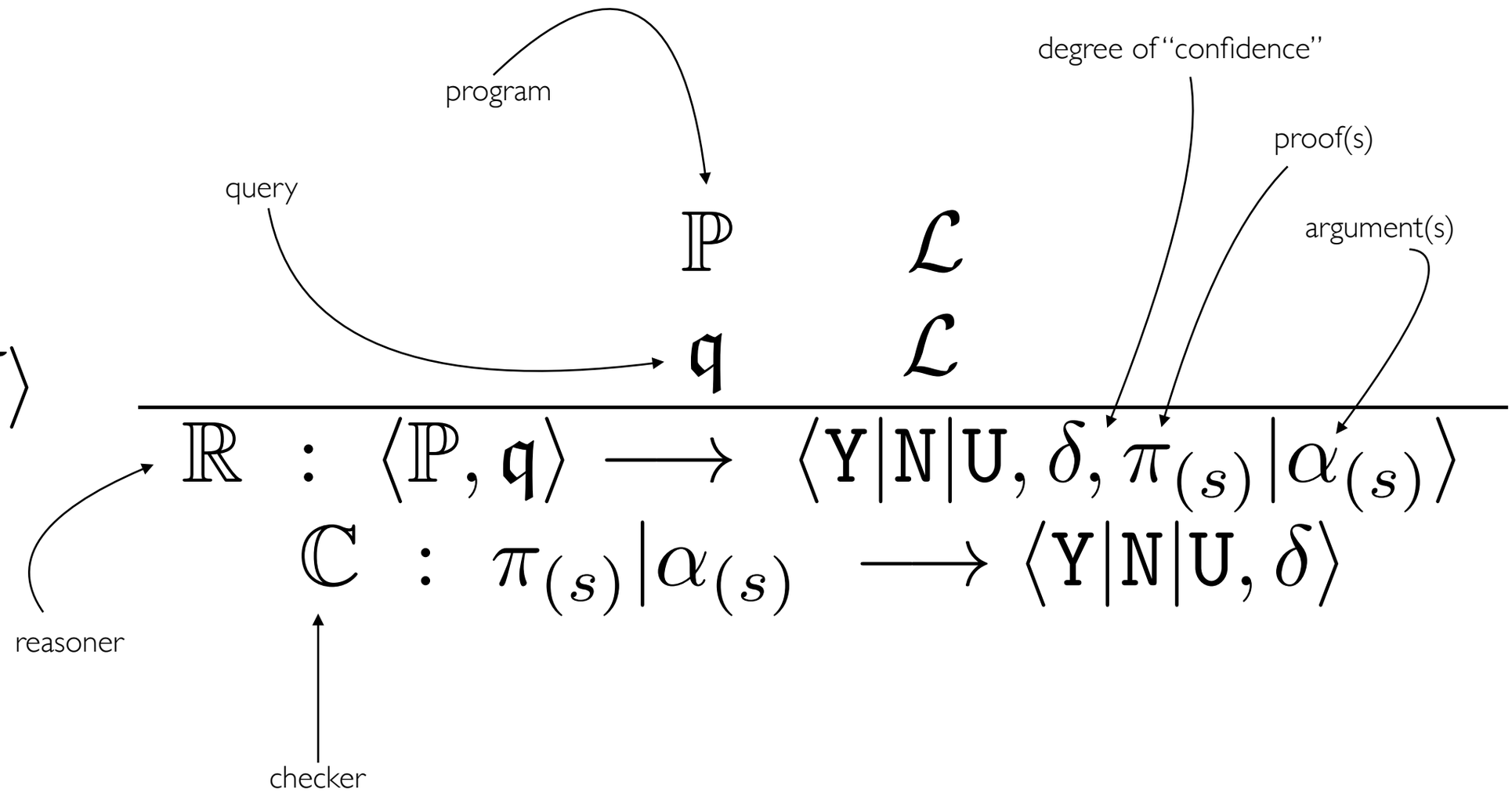
https://world.logicamodernapproach.com/hyperslate/public/Selmer.Bringsjord@gmail.com/TsetlinesqueFourZoombinis

(Pure General) Logic Programming …

$$\mathcal{L} := \langle L, \mathcal{I} \rangle$$

$$\frac{\begin{array}{cc} \mathbb{P} & L \\ \mathfrak{q} & L \end{array}}{\begin{array}{c} \mathbb{R} \ : \ \langle \mathbb{P}, \mathfrak{q} \rangle \longrightarrow \langle \mathtt{Y} | \mathtt{N} | \mathtt{U}, \delta, \pi_{(s)} | \alpha_{(s)} \rangle \\ \mathbb{C} \ : \ \pi_{(s)} | \alpha_{(s)} \longrightarrow \langle \mathtt{Y} | \mathtt{N} | \mathtt{U}, \delta \rangle \end{array}}$$

$$\mathscr{L} = \langle \mathcal{L}, \mathcal{I} \rangle$$



For just "logic programming," and a vintage approach that goes back to circa 1970, restrict this to a FOL or a fragment thereof, and use resolution as the only inference schema.

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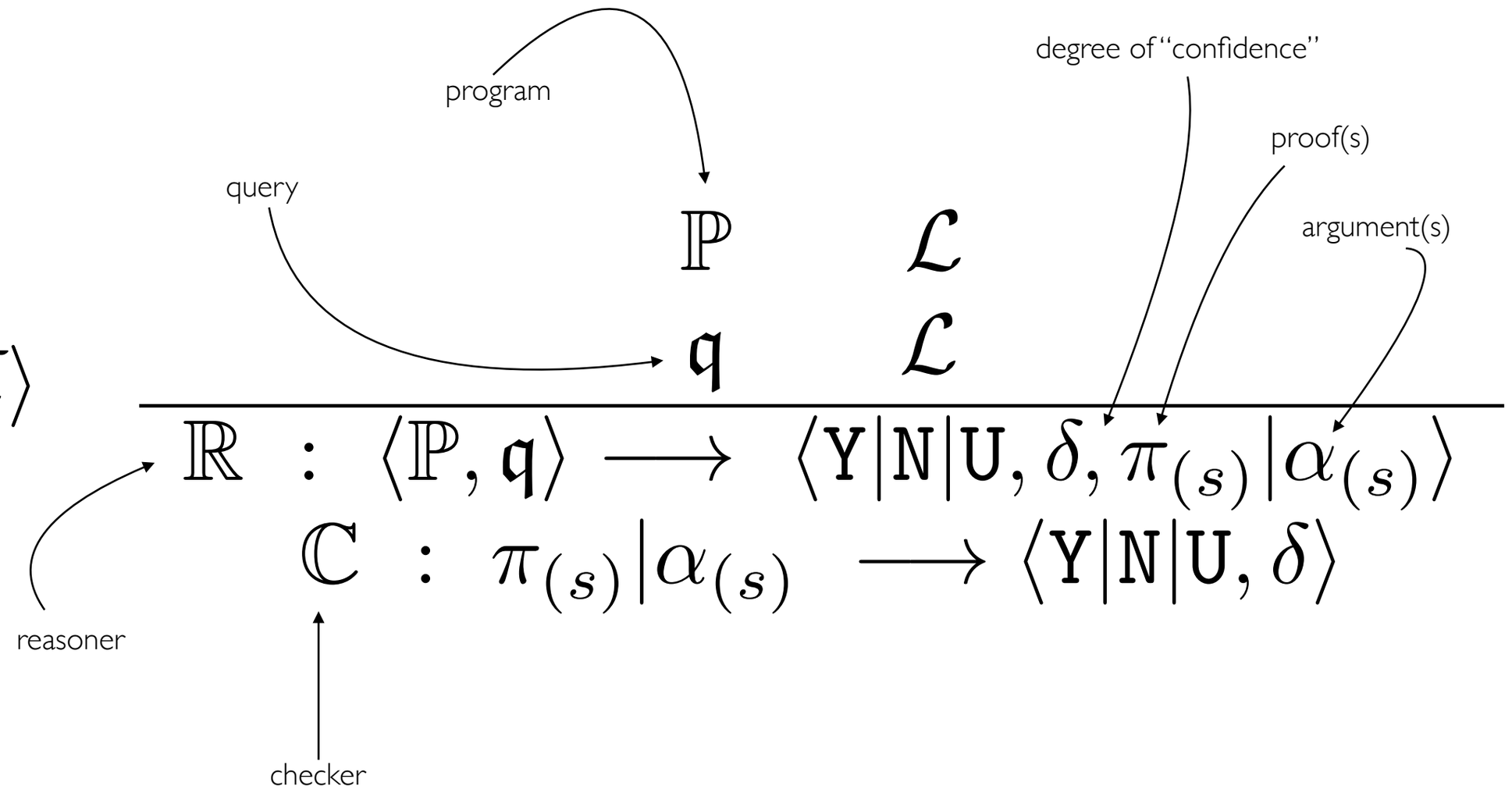

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$$\mathscr{L} = \langle \mathcal{L}, \mathcal{I} \rangle$$

program

query

$\mathbb{P}$ $\mathcal{L}$

$\mathfrak{q}$ $\mathcal{L}$

degree of "confidence"

proof(s)

argument(s)

$$\mathbb{R} \ : \ \langle \mathbb{P}, \mathfrak{q} \rangle \longrightarrow \langle \mathtt{Y}|\mathtt{N}|\mathtt{U}, \delta, \pi_{(s)}|\alpha_{(s)} \rangle$$

$$\mathbb{C} \ : \ \pi_{(s)}|\alpha_{(s)} \longrightarrow \langle \mathtt{Y}|\mathtt{N}|\mathtt{U}, \delta \rangle$$

reasoner

checker

On the Anatomy of a PGLP Program

On the Anatomy of a PGLP Program

Linguistics

$$
\begin{array}{lll}
\vdots & \vdots & \vdots \\
L_2^{\mu} & \text{meta-level}_2 \text{ language} & (\{\phi\} \vdash \psi \wedge \{\psi\} \vdash \delta) \vdash_{\mu_2} \{\phi\} \vdash \delta \\
L_1^{\mu} & \text{meta-level}_1 \text{ language} & \exists x \ \mathrm{rank}(\phi) = x \quad \{\phi\} \vdash \psi \quad \mathfrak{U} \models \phi \\
L & \text{object-level language} & \phi \quad \psi \quad \delta
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A collection of (deductive and/or inductive) inference schemata.

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Selection of language, inference schemata, plus formulae/meta-formulae $= \mathbb{P}_{\mathscr{L}}$

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Selection of language, inference schemata, plus formulae/meta-formulae $= \mathbb{P}_{\mathscr{L}}$ + ShadowReasoner

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UI

$\mathfrak{LM}$

$\mathcal{A}^n\mathcal{H}$ (Analytic Hierarchy)

Δ^1_2

Π^1_1

Infinite Time Turing Machines (ITTMs)

$\mathcal{A}^r\mathcal{H}$ (Arithmetic Hierarchy)

∞ ∞ ∞

Σ^0_3 Π^0_3

Δ^0_3

Σ^0_2 Π^0_2

Δ^0_2

Σ^0_1 Π^0_1

Δ^0_1

(decidable formulae)

Human Persons
(according to Bringsjord)

Human Brains
(according to Granger)

$\mathcal{CH}$ (Chomsky Hierarchy)

Turing Machines (TMs)
Linear Bounded Automata (LBAs)
Push Down Automata (PDAs)
Finite State Automata (FSAs)

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Higher-Order Logic/s

Cognitive Calculi

(add intentional operators)

Infinitary Logics

Analytic Hierarchy $\mathbb{P}\ \mathfrak{q}\ \mathscr{L}_2\ \mathbb{R}\ \mathbb{C}$

Arithmetic Hierarchy $\mathbb{P}\ \mathfrak{q}\ \mathscr{L}_1\ \mathbb{R}\ \mathbb{C}$

Turing Machines

Polynomial Hierarchy $\mathbb{P}\ \mathfrak{q}\ \mathscr{L}_0\ \mathbb{R}\ \mathbb{C}$

$\mathbb{P}\ \mathfrak{q}\ \mathscr{L}_{\mathrm{PC}}\ \mathbb{R}\ \mathbb{C}$

program

query

Extensional Logics
(prop calc to start)

automated reasoner

proof/argument checker

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Cognitive Calculi

(add intentional operators)

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Slutten