# Brief Remarks on Heterogeneous Logic

**Selmer Bringsjord**

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Troy, New York 12180 USA

*Intro to Formal Logic*
preliminary version of 4/15/2019

R A I R
Rensselaer AI and Reasoning Lab

Substrate made possible by:

# Logistics/Review

# Logistics/Review

- New Required Problems w/ Apr 30 deadline, out.

# Logistics/Review

- New Required Problems w/ Apr 30 deadline, out.
- Questions, comments, objections re solution to The PAID Problem from the RAIR Lab? re Robotic Jungle Jim?

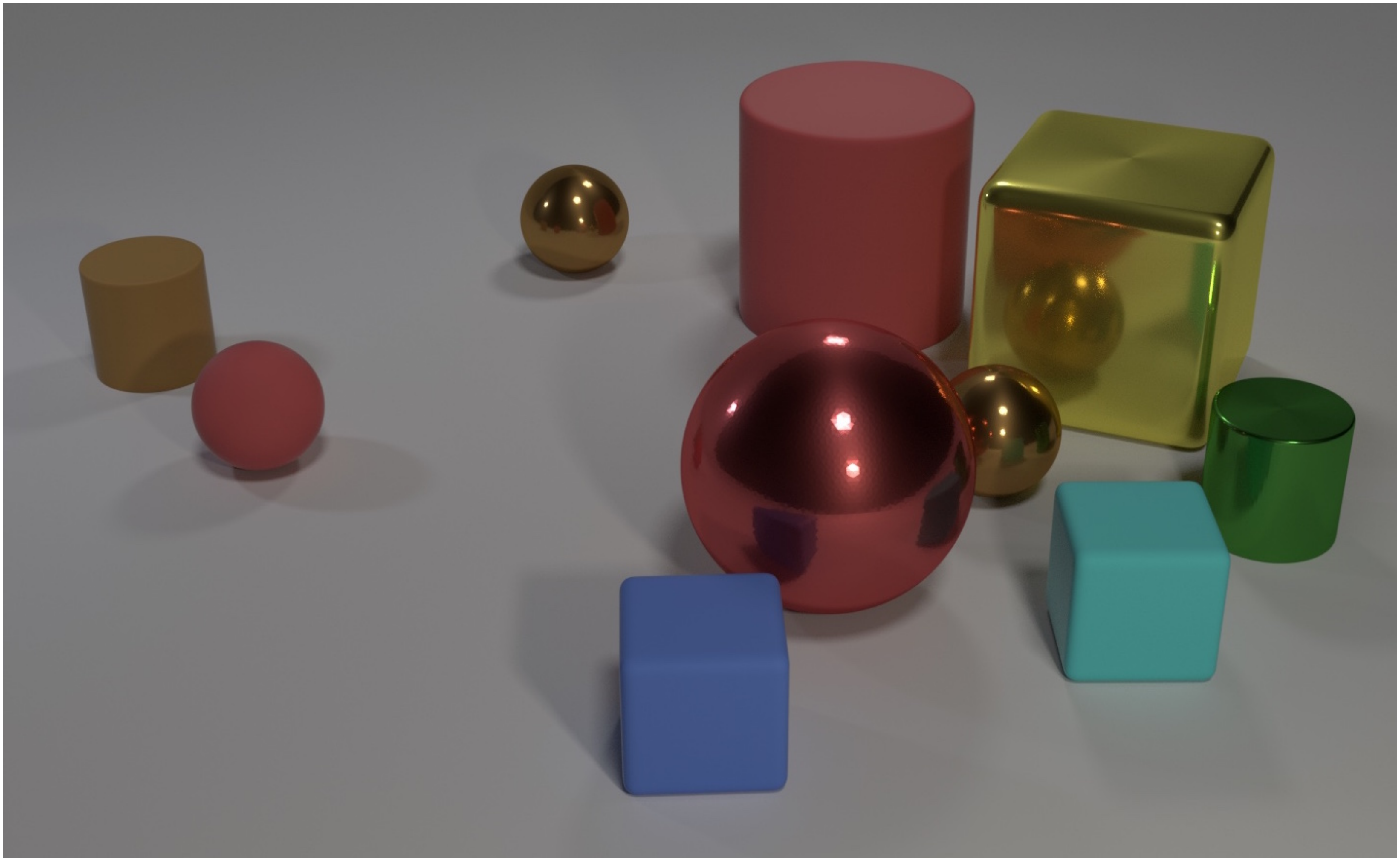
# Why pursue visual/ heterogeneous logic?

*Visual*QA (= VQA)

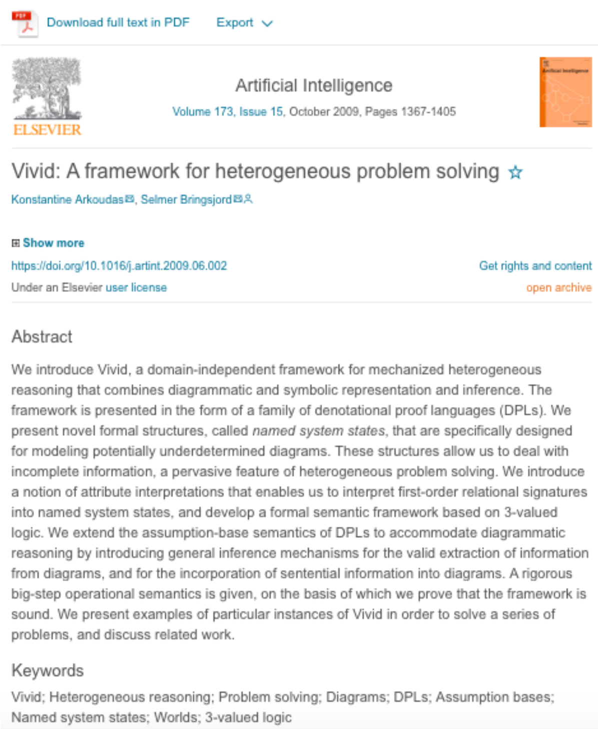
https://cs.stanford.edu/people/jcjohns/clevr/

http://www.visualqa.org

# E.g. a CLEVR Scene:



# Evolution of Visual Logic in RAIR Lab



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Artificial Intelligence
Volume 173, Issue 15, October 2009, Pages 1367-1405

## Vivid: A framework for heterogeneous problem solving

Konstantine Arkoudas, Selmer Bringsjord

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### Abstract

We introduce Vivid, a domain-independent framework for mechanized heterogeneous reasoning that combines diagrammatic and symbolic representation and inference. The framework is presented in the form of a family of denotational proof languages (DPLs). We present novel formal structures, called *named system states*, that are specifically designed for modeling potentially underdetermined diagrams. These structures allow us to deal with incomplete information, a pervasive feature of heterogeneous problem solving. We introduce a notion of attribute interpretations that enables us to interpret first-order relational signatures into named system states, and develop a formal semantic framework based on 3-valued logic. We extend the assumption-base semantics of DPLs to accommodate diagrammatic reasoning by introducing general inference mechanisms for the valid extraction of information from diagrams, and for the incorporation of sentential information into diagrams. A rigorous big-step operational semantics is given, on the basis of which we prove that the framework is sound. We present examples of particular instances of Vivid in order to solve a series of problems, and discuss related work.

Keywords
Vivid; Heterogeneous reasoning; Problem solving; Diagrams; DPLs; Assumption bases; Named system states; Worlds; 3-valued logic

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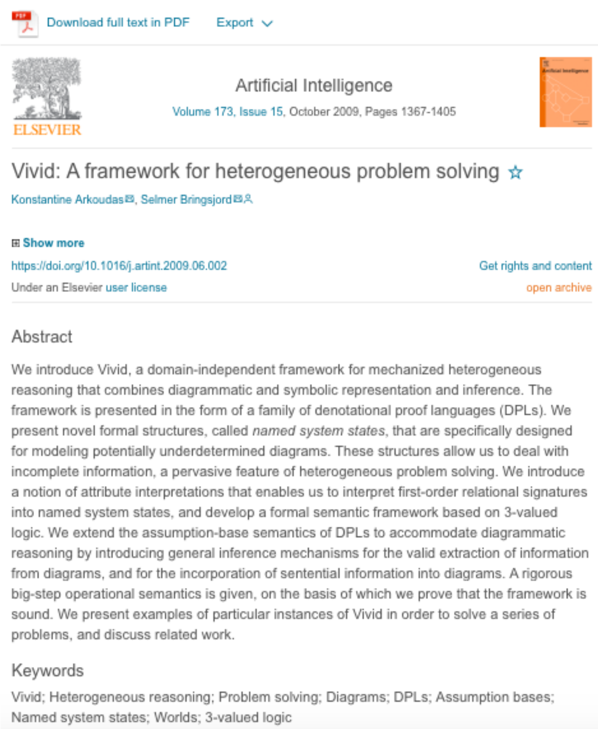
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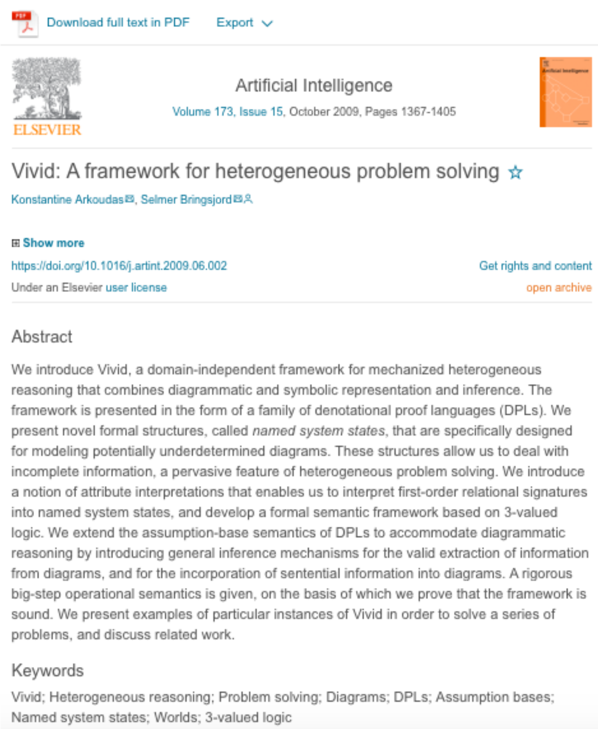
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Vivid in Slate
(hypergraphical natural deduction)

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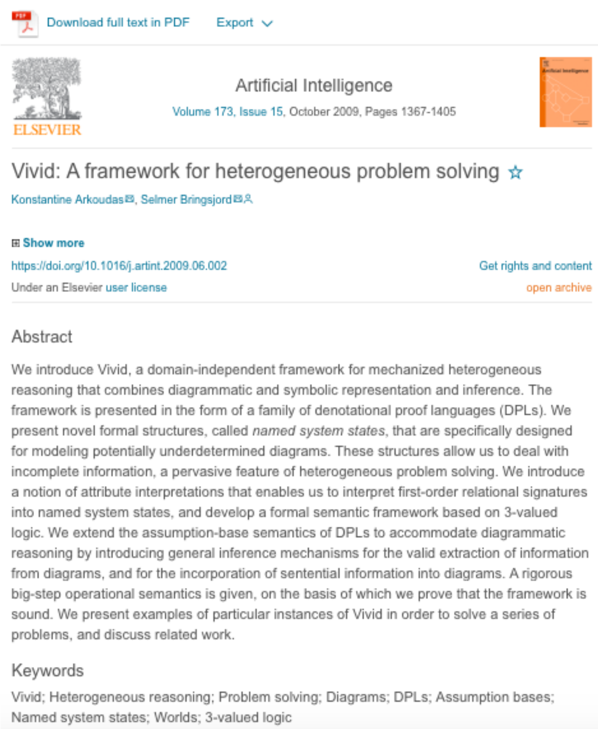
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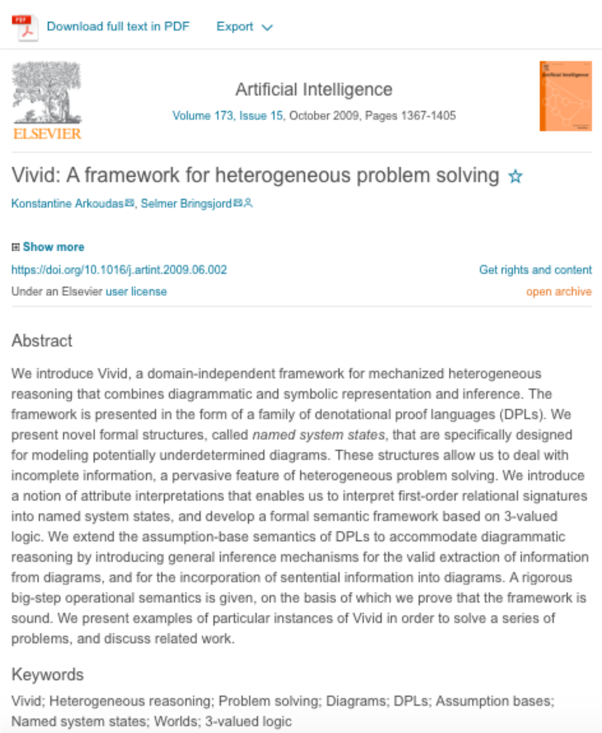
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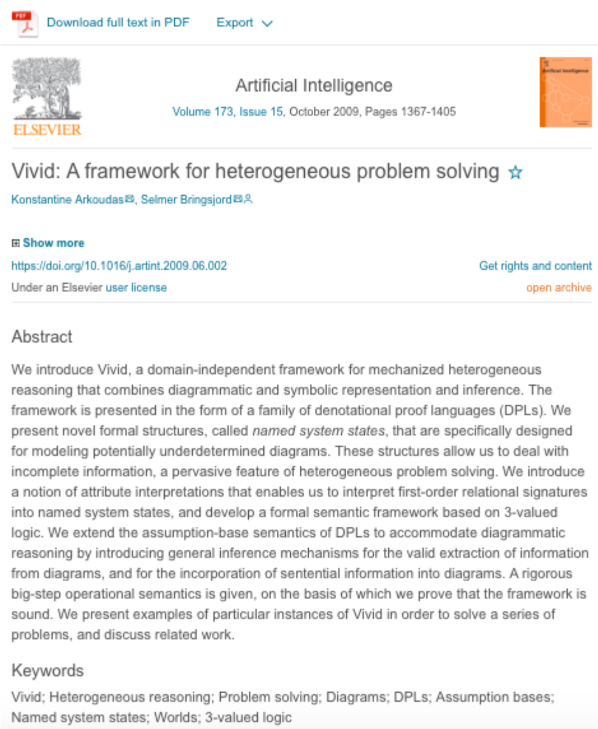

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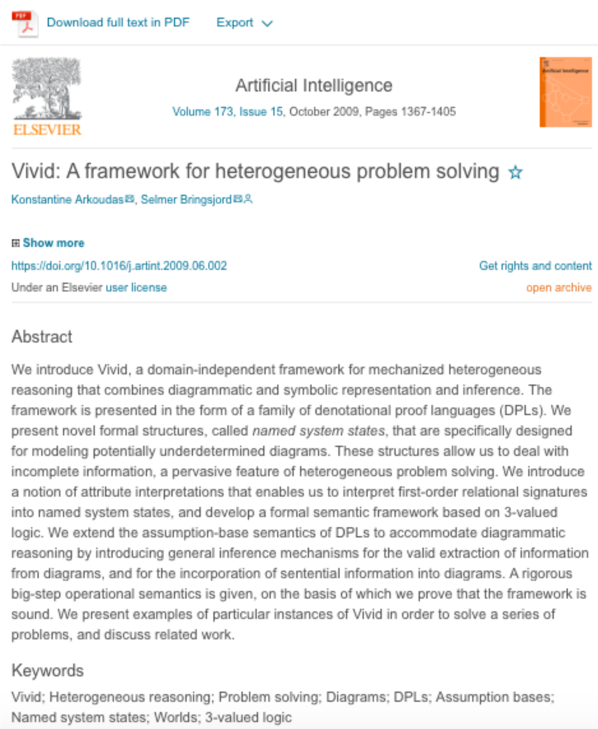
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## Vivid Proof (Atriya Sen)

What color is the object on the left side of the small rubber thing?

1. We define a *vocabulary* $\Sigma_{image} = \{C_{image}, R_{image}, V_{image}\}$, where the set of *constant symbols* $C_{image} = \{o_1, o_2, o_3\}$ for an image containing three objects, the set of *relation symbols* $R_{image} = \{left, is\_small, is\_rubber, is\_grey\}$ for this particular test question, and the set of *variables* $V_{clock} = \emptyset$.

2. We define an *attribute structure*

$\mathcal{A}_{image} = \{color : \{red, blue, \ldots\}, shape : \{cube, cylinder, \ldots\}, material : \{rubber, metal \ldots\}, size : \{small, large \ldots\}, posx, posy; \{R_1, R_2, R_3, R_4\}\}$

where

$R_1(shape_1, size_1, material_1, color_1, posx_1, posy_1, shape_2, size_2, material_2, color_2, posx_2, posy_2) \iff posx_1 < posx_2$

, $R_2(shape_1, size_1, material_1, color_1, posx_1, posy_1) \iff color_1 = grey$, and so on.

3. We define an *interpretation* $I_{image}$ of $\Sigma_{image}$ onto $\mathcal{A}_{image}$ such that $left^{I_{image}} = R_1$ and the *profile*

$Prof(left) = [(shape,1), (size,1), (material,1), (color,1), (posx,1), (posy,1), (shape,2), (size,2), (material,2), (color,2), (posx,2), (posy,2)]$

, $is\_grey^{I_{image}} = R_2$,

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4. We define a *system* of three observed objects $ob_1$, $ob_2$, and $ob_3$, and the attribute structure given by $\mathcal{A}_{image}$, a *constant assignment* $\rho$ mapping observed objects $ob_1$, $ob_2$, and $ob_3$ into $o_1$, $o_2$, and $o_3$ respectively, and a *state* $\sigma$ encoding the observed properties of the objects as presented.

Now, the Vivid proof which suffices as an answer to the test problem is:

Observe $is\_small(ob3) \wedge is\_rubber(ob3)$

Observe $left(ob1, ob3)$

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Observe $is\_grey(ob1)$ QED

## 8.2 Jumping In With Seating Puzzles

Specifically, we now get started in earnest with a class of diagrams that denote seating arrangements. Here's a simple diagram intended to denote five chairs in a row, each of which is either empty or filled with one of four possible people:

$$\underline{?}\quad \underline{?}\quad \underline{?}\quad \underline{?}\quad \underline{?}$$

To make it easier on ourselves, we label the seats from left to right, starting with 1:

$$\begin{array}{ccccc} \underline{?} & \underline{?} & \underline{?} & \underline{?} & \underline{?} \\ s1 & s2 & s3 & s4 & s5 \end{array}$$

In addition, assume that the only people who can occupy seats are Alvin ($a$), Billy ($b$), Cindy ($c$), and Dora ($d$). A seat can also be empty; we denote this condition by the constant $e$. (Here, obviously, these lower-case Roman letters are symbolic

Our four people are to be seated in a row of five seats. This seating arrangement must satisfy the following three conditions:

- C1 *a* and *c* should flank the empty seat.
- C2 *c* should be closer to the middle seat than *b*.
- C3 *b* and *d* should be seated next to each other.

Given this information, reach the following three goals:

- G1 prove that the empty seat can't be in the middle and can't be on either end;
- G2 settle whether it can be determined who must be seated in the middle seat;
- G3 settle whether it can be determined who is to be seated on the two ends.

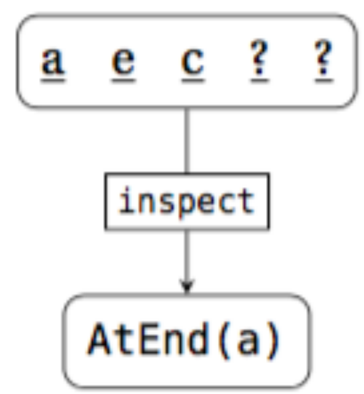
**Proof**: From all the permutations in which our four characters are seated (with a remaining empty seat), which totals 5! = 120), we can eliminate as inconsistent with condition C1 all but these six possibilities:

| | | | | | |
|---|---|---|---|---|---|
| P1 | a | e | c | ? | ? |
| P2 | ? | a | e | c | ? |
| P3 | ? | ? | a | e | c |
| P4 | c | e | a | ? | ? |
| P5 | ? | c | e | a | ? |
| P6 | ? | ? | c | e | a |

But possibilities P2–P5 each lead to absurdity when combined with the conjunction of C2 and C3. Hence by `disjunctive syllogism` over the disjunction of all six possibilities P1–P6 we deduce P1 $\vee$ P6, from which we in turn can deduce (i) G1, (ii) that $c$ is in the middle seat and hence an affirmative answer settles G2, and (iii) that any of $a$, $b$, $d$ can occupy an end seat, and from this a negative answer to the query presented in G3. **QED** proof given above is a diagram. We set P$i$ to $\partial_i$. Now consider specifically $\partial_1$, that is:

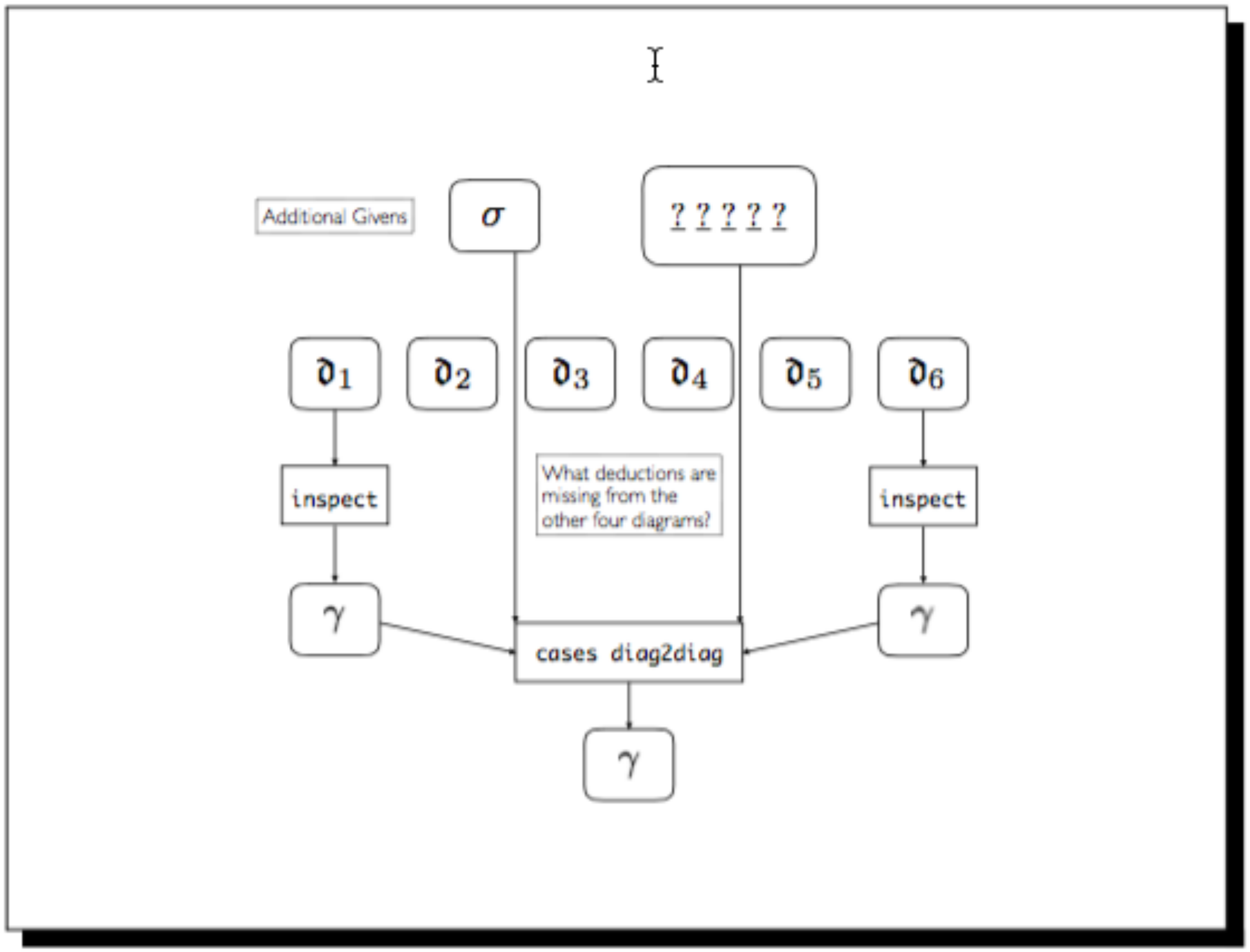
a e c ? ?

We can deduce by `inspect`[5] that `AtEnd(a)`. We can also deduce by `inspect` that `At(a,s1)`. Here's the first of these inferences in Slate style:




(8.1)

Figure 8.3: Hetergeneous Slate Proof that Solves the Seating Puzzle




Features original art & gameplay
The Classic Mystery Face Game
GUESS WHO ?
Is your person wearing a hat?
Does your person have blue eyes?
WARNING:


FREE
akinator

# The Basic Intuition of What Diagrams Are

# The Basic Intuition of What Diagrams Are

## 4. Attribute structures and systems

Very broadly, a diagram depicts a finite number of objects, and conveys (though perhaps only partially) the values of certain *observable attributes* of these objects. Of course what counts as an object and what counts as an attribute depends on our underlying theory and on our purposes, i.e., on *how* we choose to look at a diagram. That is, how we parse the raw pictorial information of a diagram varies widely, depending on the application at hand. But, assuming that we have fixed what constitutes an object and which observable object attributes we are interested in, the upshot of diagram parsing can be thought of as a mapping that assigns one or more values to every observable attribute of every object represented in the diagram. If exactly one value is assigned to every attribute and object, then the diagram is completely determined; there is no ambiguity or imprecision about it. Incomplete diagrammatic information is captured by assigning multiple values to a single object and attribute; e.g., if a diagram does not completely determine the color of an object $o$, we might assign a *set* of several color values to $o$, say, green, red, and blue. What follows is a formal development of these intuitions.

# The Basic Intuition of What Diagrams Are

We see objects in diagrams, a finite number of them, and these objects are depicted in diagrams as having the *values* of *attributes*. But these values may be only partially given. If *o* is an object, say an object intended to depict a street light, and color is an *attribute*, *values* eg may be *red*, *green*, or *yellow*.

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# Peek @ Underlying Math

$$\mathcal{A} \coloneqq \langle\{A_1, A_2, \ldots, A_k\}; \mathcal{R} \coloneqq \langle R_1, \ldots, R_j\rangle\rangle$$

For every $R \in \mathcal{R}, D(R) \subseteq \{A_1, \ldots, A_k\}$

$$\mathcal{S} \coloneqq \langle\{o_1, \ldots, o_n\}; \mathcal{A}\rangle$$

# Peek @ Underlying Math

attribute structure

attributes (sets)

computable relations

$$\mathcal{A} := \langle \{A_1, A_2, \ldots, A_k\}; \mathcal{R} := \langle R_1, \ldots, R_j \rangle \rangle$$

For every $R \in \mathcal{R}, D(R) \subseteq \{A_1, \ldots, A_k\}$

$$\mathcal{S} := \langle \{o_1, \ldots, o_n\}; \mathcal{A} \rangle$$

# Peek @ Underlying Math

attribute structure — attributes (sets) — computable relations

$$\mathcal{A} := \langle \{A_1, A_2, \ldots, A_k\}; \mathcal{R} := \langle R_1, \ldots, R_j \rangle \rangle$$

For every $R \in \mathcal{R}, D(R) \subseteq \{A_1, \ldots, A_k\}$

system structure

$$\mathcal{S} := \langle \{o_1, \ldots, o_n\}; \mathcal{A} \rangle$$

# E.g.,

# E.g.,

# E.g.,

$$\langle \{c\}; hours : \{0, \ldots, 23\}, minutes : \{0, \ldots, 59\} \rangle$$

# E.g.,

attributes (sets)

$$\langle \{c\}; hours : \{0, \ldots, 23\}, minutes : \{0, \ldots, 59\} \rangle$$

# E.g.,

attributes (sets)

$$\langle \{c\}; hours : \{0, \ldots, 23\}, minutes : \{0, \ldots, 59\} \rangle$$

labels

# E.g.,

$$\langle \{f\}; emotion : \{happy, sad, angry, distrust, fearful\} \rangle$$

# E.g.,

attributes (sets)

$$\langle \{f\}; emotion : \{happy, sad, angry, distrust, fearful\} \rangle$$

# E.g.,

attributes (sets)

$$\langle \{f\}; emotion : \{happy, sad, angry, distrust, fearful\} \rangle$$

label

# Peek @ Underlying Math

A **state** of a system $\mathcal{S} := \langle \{o_1, \ldots, o_n\}, \{A_1, \ldots, A_k\} \rangle$

is a set of functions $\sigma := \{\delta_1, \ldots, \delta_k\}$ where each $\delta_i$ is s.t.

$$\delta_i : \{o_1, \ldots, o_n\} \longrightarrow [\mathcal{P}(A_i) - \emptyset]$$

$$\sigma_1 : \quad hours(c) = 10, minutes(c) = 0$$

Figure 8.4: Portrait of an "Emotionally Mysterious" Larry (diagram $\mathfrak{d}_?$; by KB Foushée)


Now, suppose that the following conjunction, $\phi$, holds:

$$\neg\mathsf{Happy}(\mathsf{larry},\mathsf{t}) \land \neg\mathsf{Angry}(\mathsf{larry},\mathsf{t}) \land \neg\mathsf{Distrustful}(\mathsf{larry},\mathsf{t}) \land \neg\mathsf{Fearful}(\mathsf{larry},\mathsf{t})$$

Figure 8.4: Portrait of an "Emotionally Mysterious" Larry (diagram $\partial_?$; by KB Foushée)


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Figure 8.5: Result of Deduction by `thinning` Applied to Diagram $\partial_?$ and $\phi$ (by KB Foushée)

Figure 8.5: Result of Deduction by `thinning` Applied to Diagram $\mathfrak{d}_?$ and $\phi$ (by KB Foushée)

## 8.3 Exercises

1. Examine Figure 8.3 carefully, if you haven't done so already. Complete the proof, by specifying and deploying the missing givens, and by adding the deductions that are missing below each of the diagrams $\mathfrak{d}_2$–$\mathfrak{d}_4$.
2. What, exactly, is the general form of `cases diag2diag`? Write down the inference schema in graphical form. (We suggest that you generalize from the proof produced by an answer to the previous exercise.)
3. What, exactly, is the general form of `thinning`? Write down the inference schema in graphical form.
4. As you know, we have (informally) introduced the inference schema `thinning`. What do you think the inference schema called `widening` is, given that it is accurately said to be the inverse of `thinning`? Write down a specification of it, as your best hypothesis.