Motivating Paradoxes, Puzzles, and $\mathcal{R}$,

Part I

Selmer Bringsjord

Intro to (<u>Formal</u>) Logic (via, and also to, AI)

1/14/19

<u>Selmer.Bringsjord@gmail.com</u>

Logistics …

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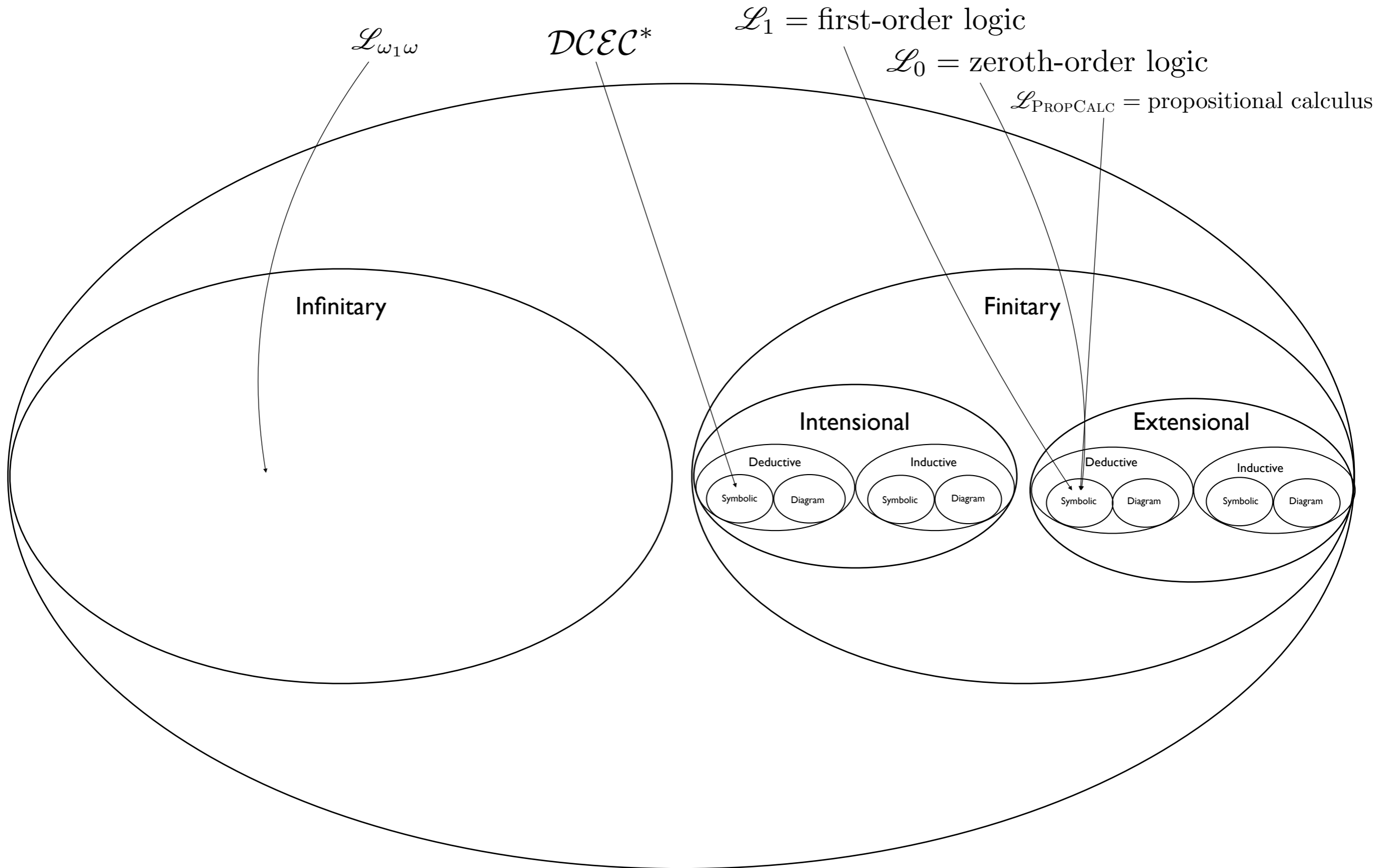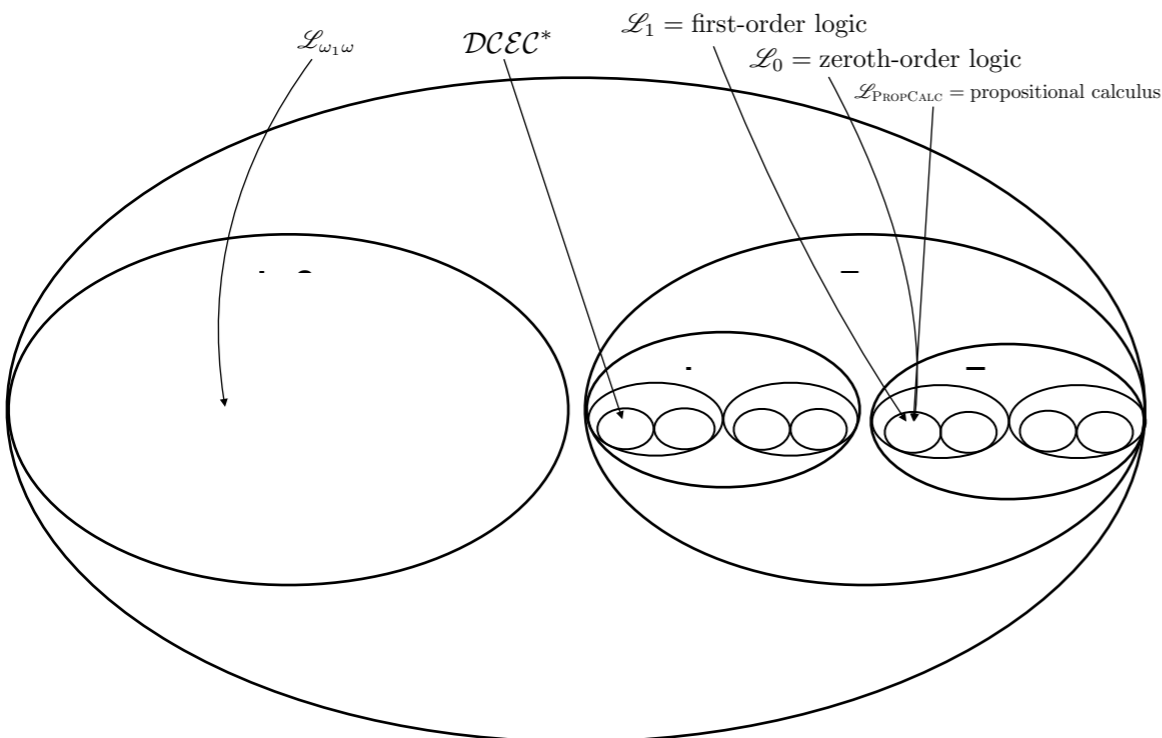
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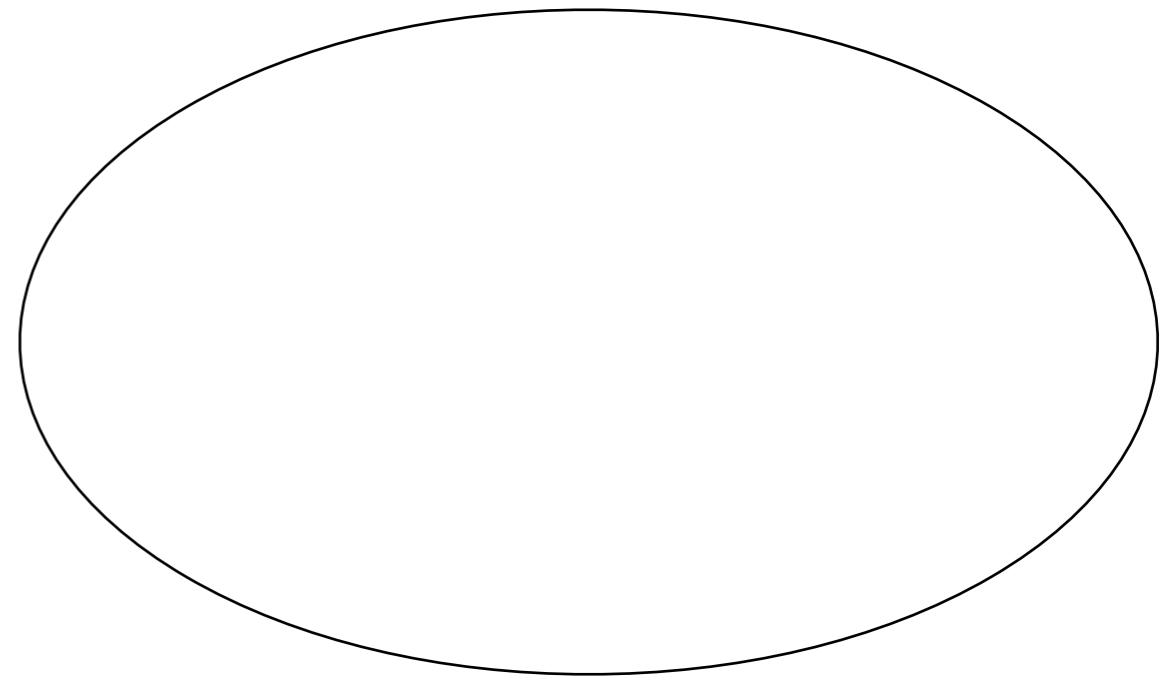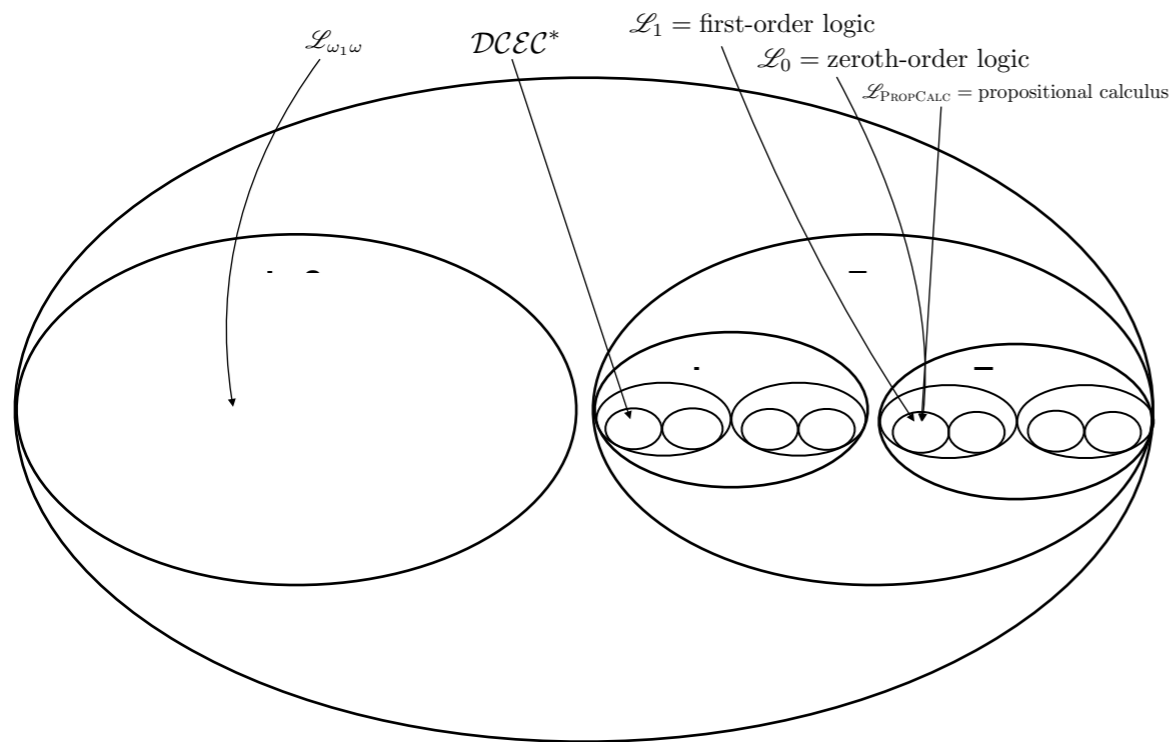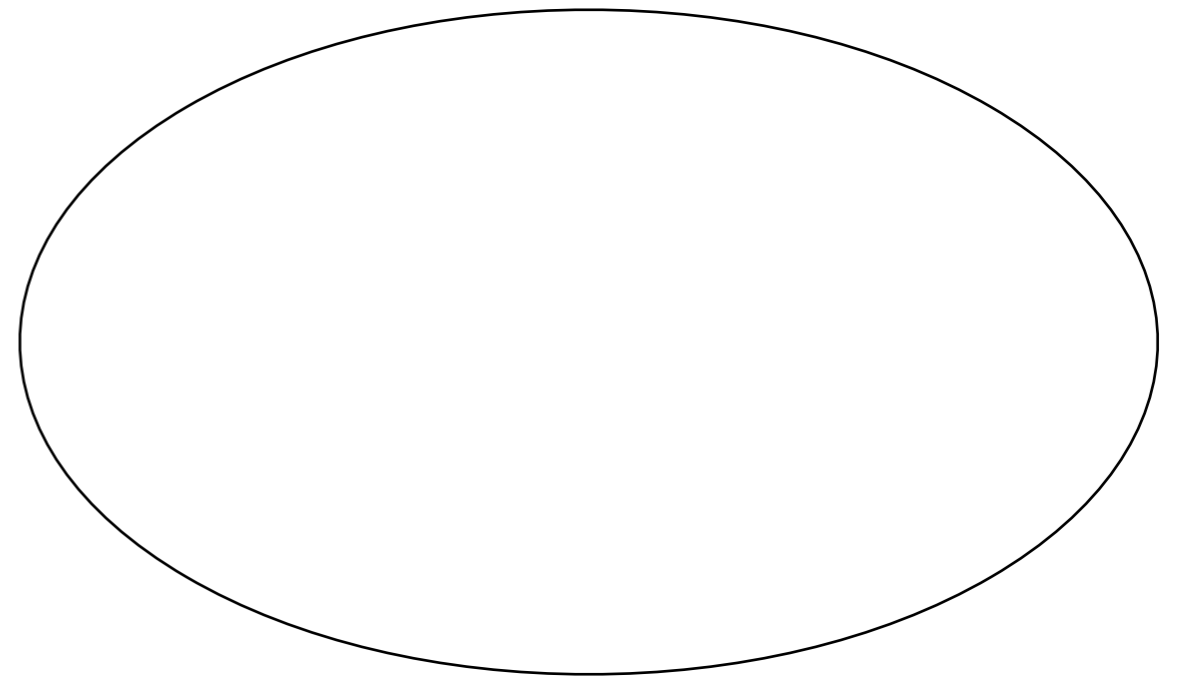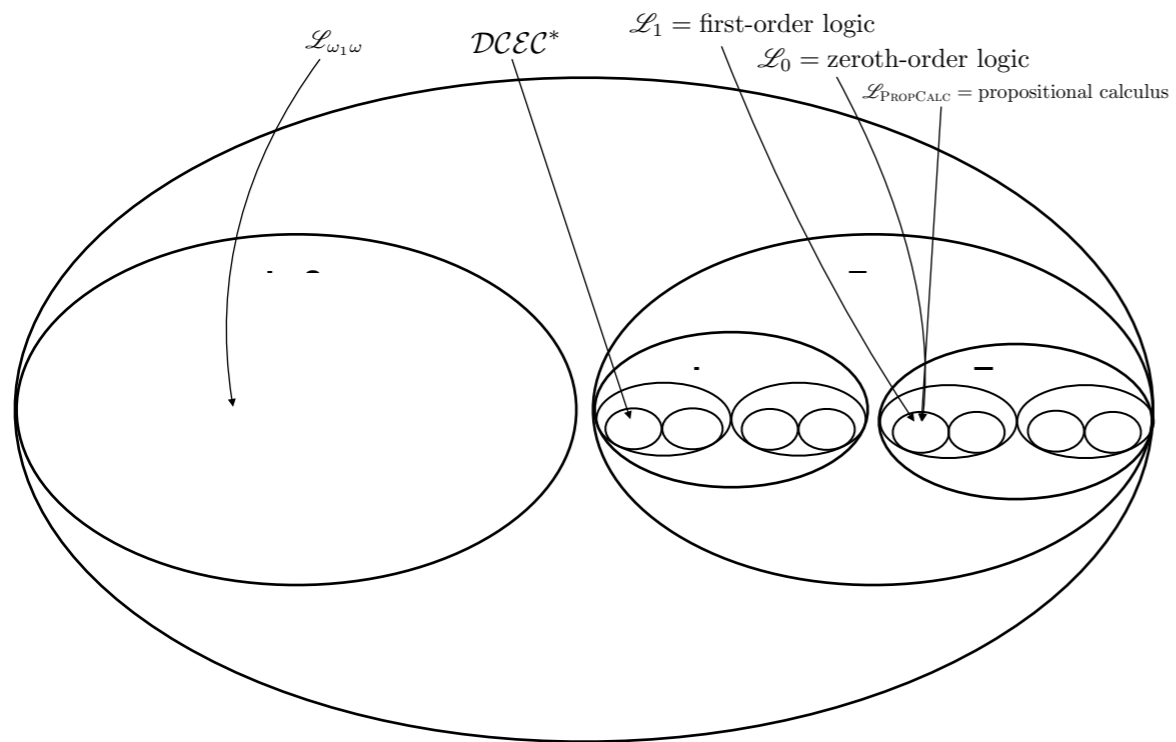
The Universe of Logics

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$\mathcal{DCEC}^*$

$\mathscr{L}_1$ = first-order logic

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"The Oracle of AI"

https://www.cbsnews.com/news/60-minutes-ai-facial-and-emotional-recognition-how-one-man-is-advancing-artificial-intelligence/

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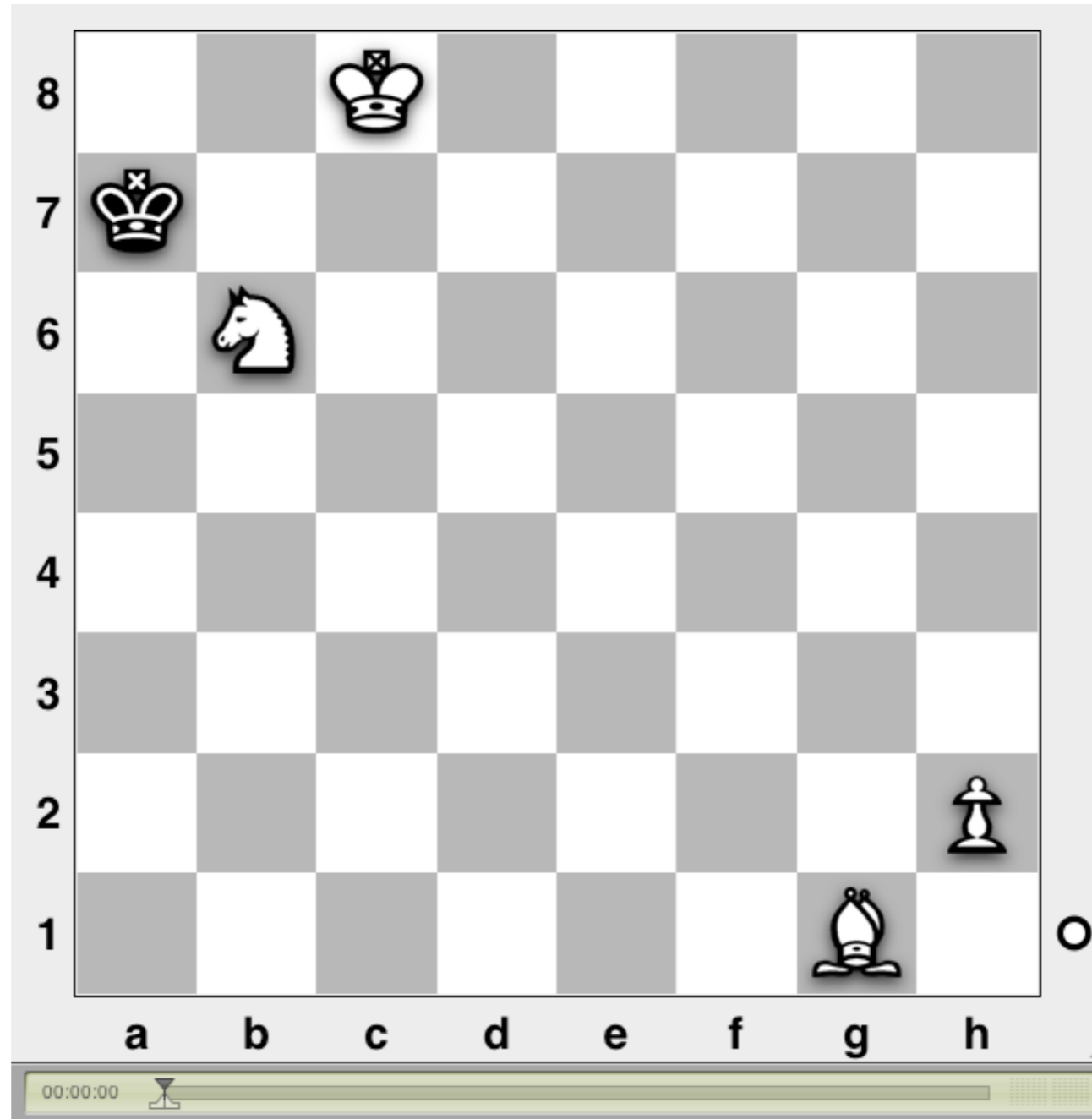- The key to becoming rational. Or are you *already* rational? …

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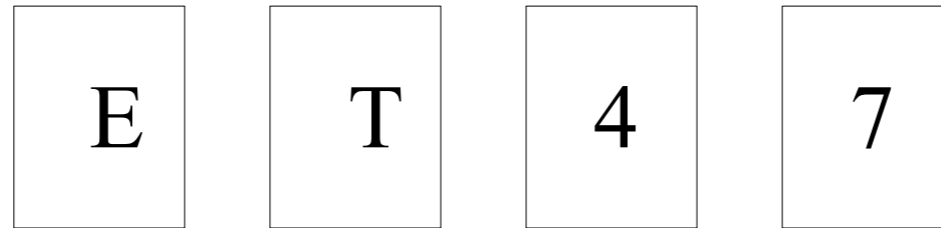
- …

It's White's turn. What move did Black just make?
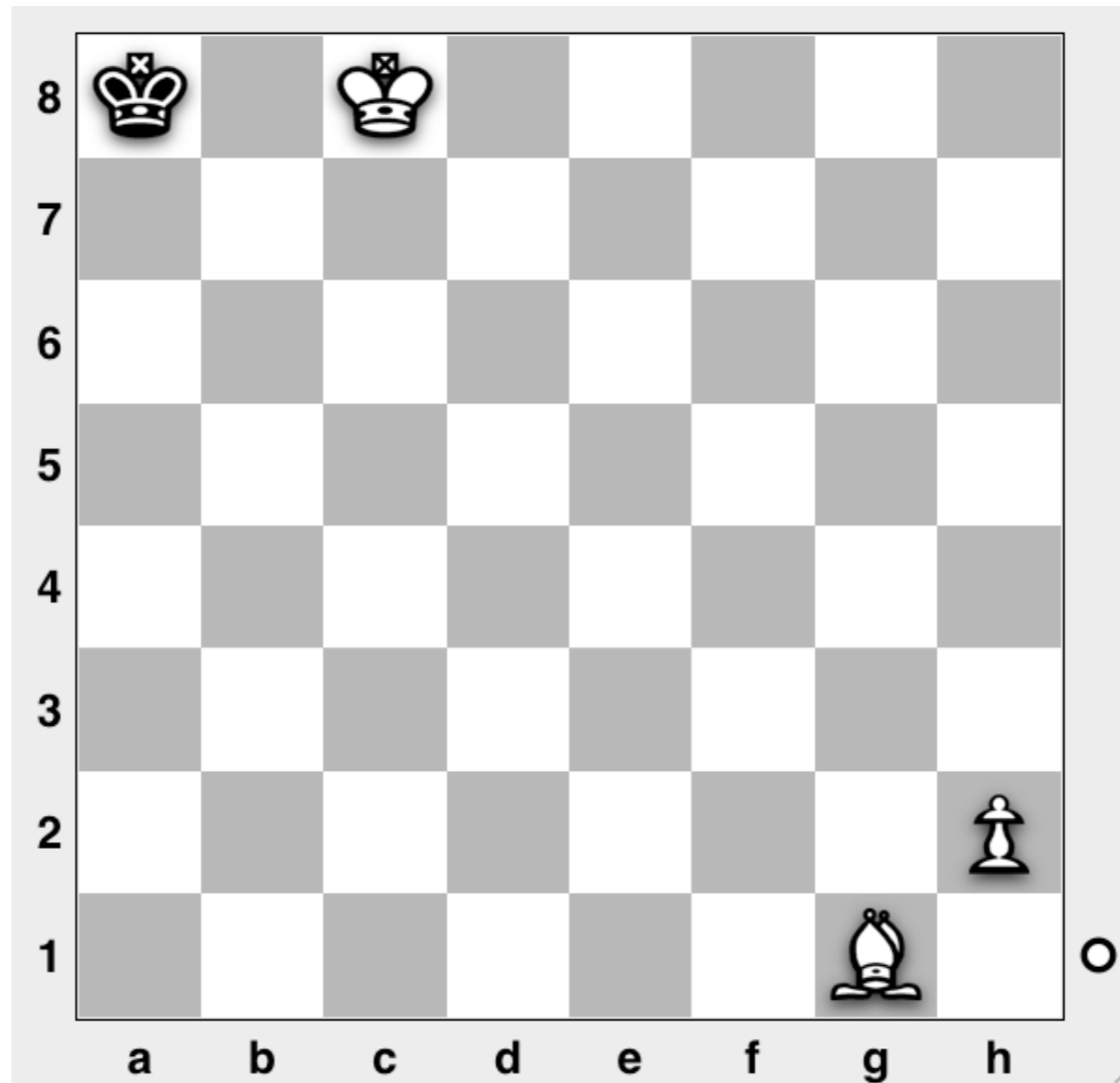
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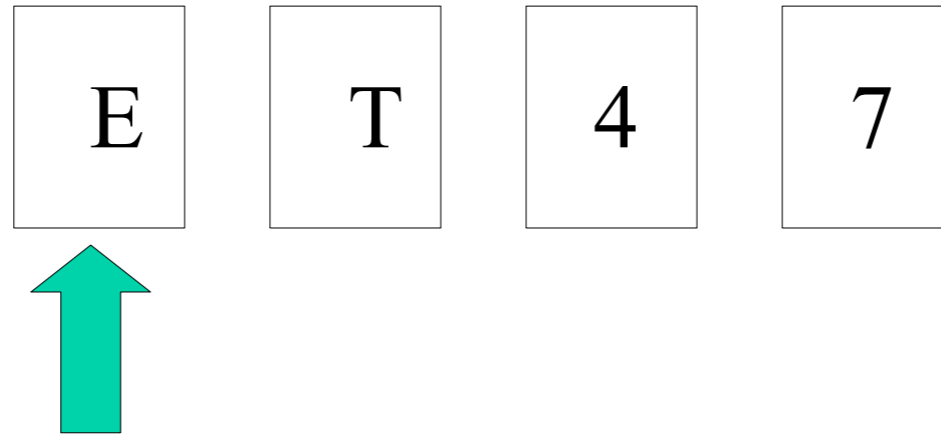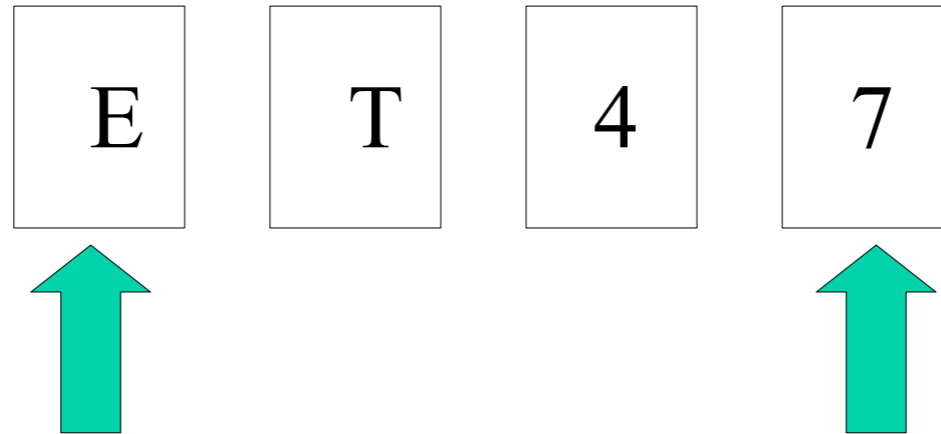
Simple Selection Task

| E | T | 4 | 7 |

Suppose I claim that the following rule is true.

    If a card has a vowel on one side, it has an even number on the other side.

Which card or cards, if any, should you turn over in order to try to efficiently decide whether the rule is true or false?
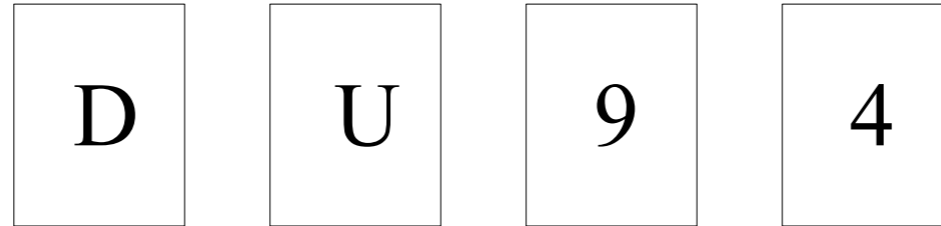
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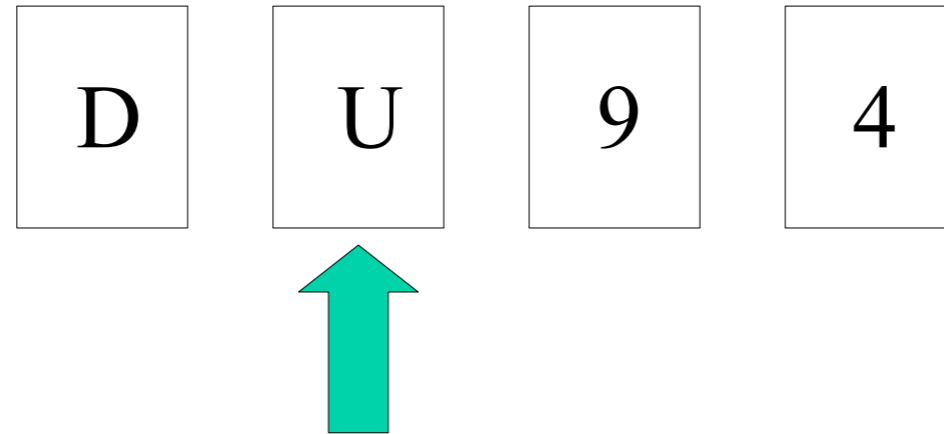
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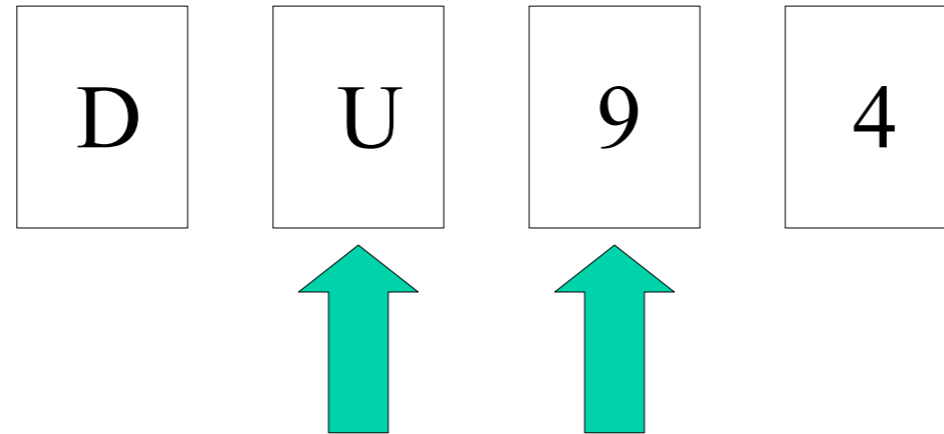
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Given the statements

¬a ∨ ¬b
b
c → a

which one of the following statements must also be true?

c
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h
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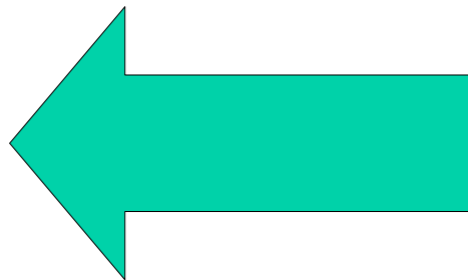¬c
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Which one of the following statements is logically equivalent to the following statement: "If you are not part of the solution, then you are part of the problem."

If you are part of the solution, then you are not part of the problem.

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Given the statements

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¬(d ∨ e)

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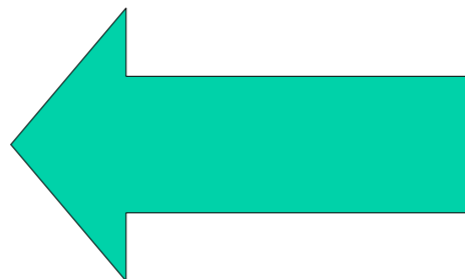¬(d ∨ e)

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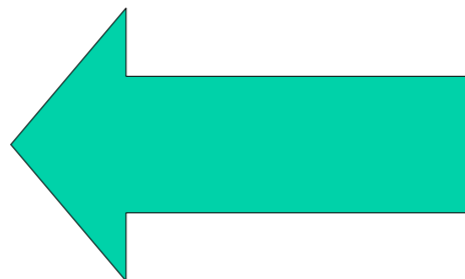
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The Original King-Ace

Suppose that the following premise is true:

If there is a king in the hand, then there is an ace in the hand, or else if there isn't a king in the hand, then there is an ace.

What can you infer from this premise?

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NO! ~~There is an ace in the hand.~~ NO!

In fact, what you *can* infer is that there *isn't* an ace in the hand!

King-Ace 2

Suppose that the following premise is true:

If there is a king in the hand, then there is an ace in the hand; or if there isn't a king in the hand, then there is an ace; but not both of these if-then statements are true.

What can you infer from this premise?

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Train-to-Princeton Problem

Everyone loves anyone who loves someone.

Larry loves Lucy.

Can you infer that everyone loves Lucy?

ANSWER:

PROOF:

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ANSWER: Yup.

PROOF: ??

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(1) The following three assertions are either all true or all false:

> If Billy helped, Doreen helped.
>
> If Doreen helped, Frank did as well.
>
> If Frank helped, so did Emma.

(2) The following assertion is definitely true: Billy helped.

Can it be inferred from (1) and (2) that Emma helped?

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(2) The following assertion is definitely true: Billy helped.

Can it be inferred from (1) and (2) that Emma helped?

YUP! — & now prove it!