FOL II: Universal Intro

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Intro to Logic
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R A I R
Rensselaer AI and Reasoning Lab

Please asap add your RIN in your Profile; thank you.

Re Test 1 Grades

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- 1 problem correct (including resurrection problem): B

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- 2 problems correct: A

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- 3 problems correct: A+

Re Test 1 Grades

- 1 problem correct (including resurrection problem): B
- 2 problems correct: A
- 3 problems correct: A+
- 4 or 5 problems correct: should be in clear contention for winning it all

Test 1 Solutions …

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HyperGrader Required Problems:

Self-paced, yes, and deadline now in countdown — but interconnected!

$\boxed{\text{BogusBiconditional}}$

tertium_non_datur

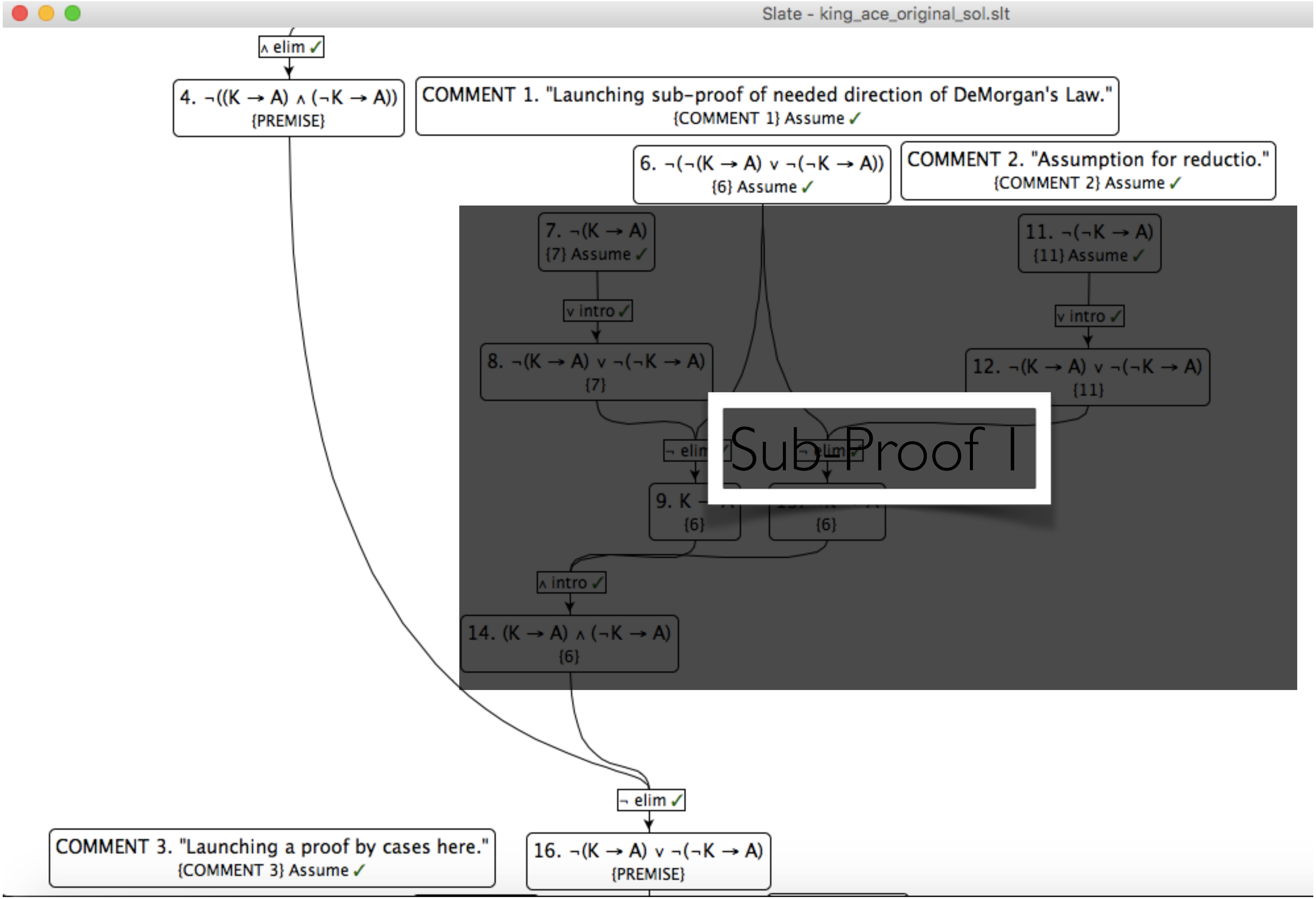
Disj_Elim

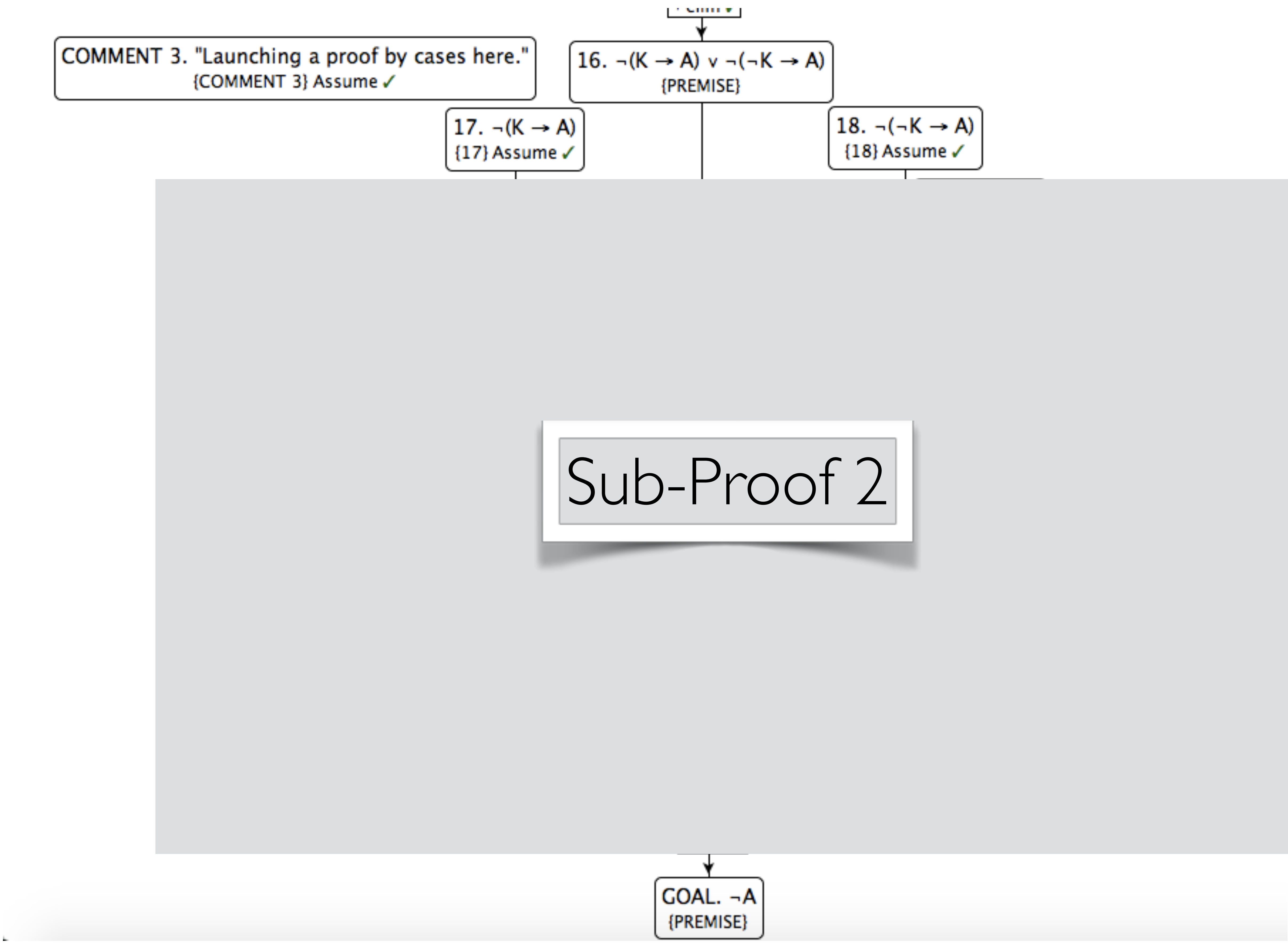
BogusBiconditional

BogusBiconditional

tertium_non_datur   Disj_Elim

Further Comments on Proof Plan for KingAce2


Slate - king_ace_original_sol.slt
∧ elim ✓
4. ¬((K → A) ∧ (¬K → A))
{PREMISE}
COMMENT 1. "Launching sub-proof of needed direction of DeMorgan's Law."
{COMMENT 1} Assume ✓
6. ¬(¬(K → A) ∨ ¬(¬K → A))
{6} Assume ✓
COMMENT 2. "Assumption for reductio."
{COMMENT 2} Assume ✓
7. ¬(K → A)
{7} Assume ✓
∨ intro ✓
8. ¬(K → A) ∨ ¬(¬K → A)
{7}
11. ¬(¬K → A)
{11} Assume ✓
∨ intro ✓
12. ¬(K → A) ∨ ¬(¬K → A)
{11}
Sub-Proof I
{6}
{6}
∧ intro ✓
14. (K → A) ∧ (¬K → A)
{6}
¬ elim ✓
COMMENT 3. "Launching a proof by cases here."
{COMMENT 3} Assume ✓
16. ¬(K → A) ∨ ¬(¬K → A)
{PREMISE}


COMMENT 3. "Launching a proof by cases here."
{COMMENT 3} Assume ✓
16. ¬(K → A) ∨ ¬(¬K → A)
{PREMISE}
17. ¬(K → A)
{17} Assume ✓
18. ¬(¬K → A)
{18} Assume ✓
Sub-Proof 2
GOAL. ¬A
{PREMISE}

Next New (*Not*-So-Easy!) Inference Rule in FOL

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- `universal introduction`

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- `universal introduction`
  - If something *a* is an *R*, and the constant/name *a* is *genuinely arbitrary*, then we can deduce that everything is an *R*.

The Inference Schema

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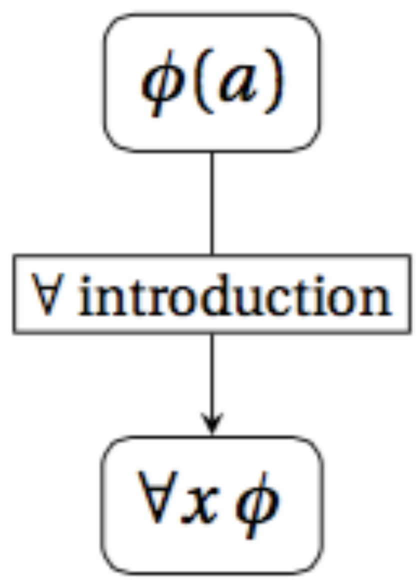


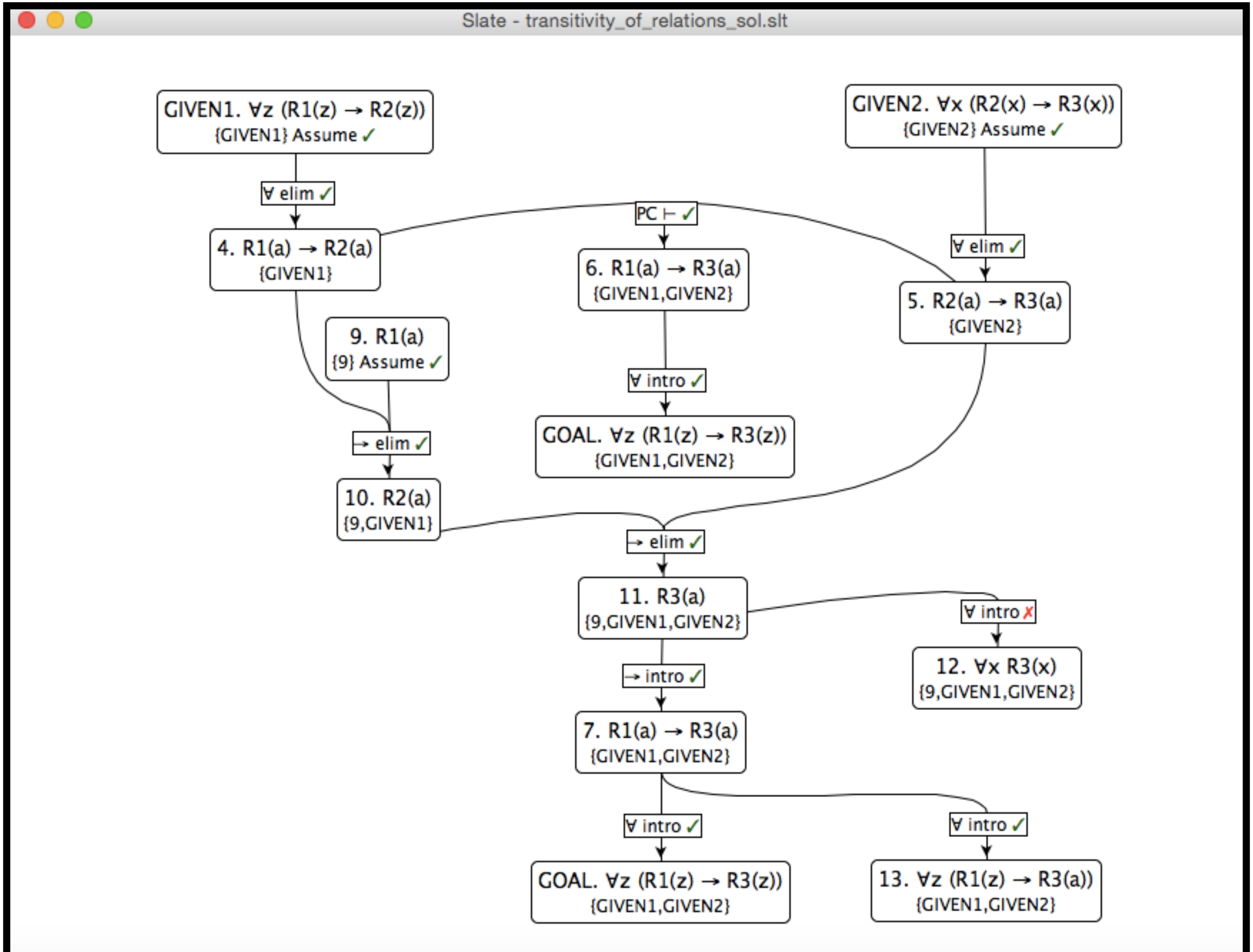
provided that a does not appear free in any in-scope assumption of ϕ, and that no occurrence of a appear in the inferred $\forall x\, \phi$ (3.16)

The Inference Schema

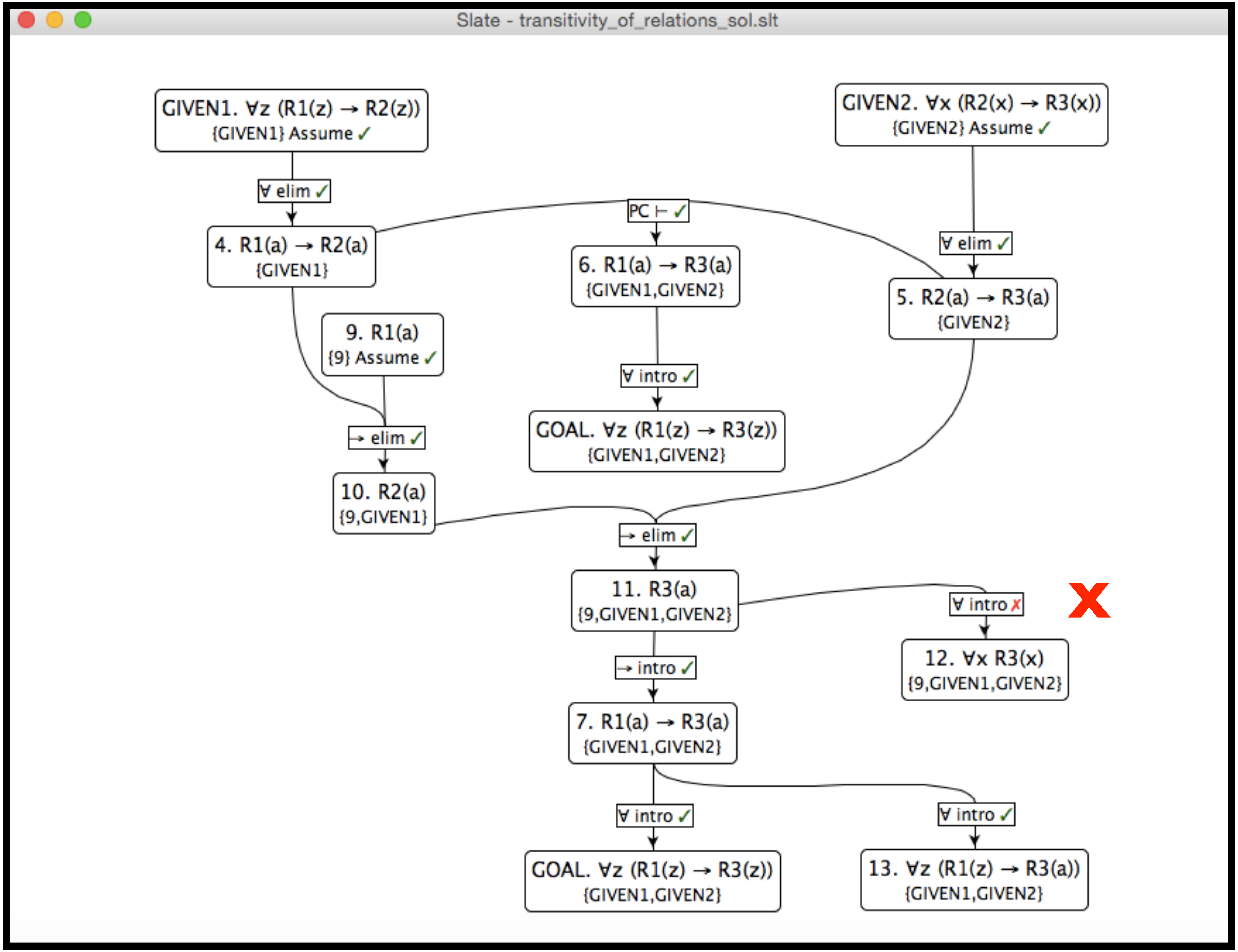
$$\begin{array}{c} \boxed{\phi(a)} \\ \Big\downarrow \boxed{\forall \text{ introduction}} \\ \boxed{\forall x\, \phi} \end{array} \qquad \text{provided that } a \text{ does not appear free in any in-scope assumption of } \phi\text{, and that no occurrence of } a \text{ appear in the inferred } \forall x\, \phi \qquad (3.16)$$

(Why the provisos?)

Example


Slate - transitivity_of_relations_sol.slt
GIVEN1. ∀z (R1(z) → R2(z))
{GIVEN1} Assume ✓
GIVEN2. ∀x (R2(x) → R3(x))
{GIVEN2} Assume ✓
∀ elim ✓
PC ⊢ ✓
4. R1(a) → R2(a)
{GIVEN1}
6. R1(a) → R3(a)
{GIVEN1,GIVEN2}
∀ elim ✓
5. R2(a) → R3(a)
{GIVEN2}
9. R1(a)
{9} Assume ✓
∀ intro ✓
→ elim ✓
GOAL. ∀z (R1(z) → R3(z))
{GIVEN1,GIVEN2}
10. R2(a)
{9,GIVEN1}
→ elim ✓
11. R3(a)
{9,GIVEN1,GIVEN2}
∀ intro ✗
12. ∀x R3(x)
{9,GIVEN1,GIVEN2}
→ intro ✓
7. R1(a) → R3(a)
{GIVEN1,GIVEN2}
∀ intro ✓
∀ intro ✓
GOAL. ∀z (R1(z) → R3(z))
{GIVEN1,GIVEN2}
13. ∀z (R1(z) → R3(a))
{GIVEN1,GIVEN2}

Example



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PC ⊢ ✓
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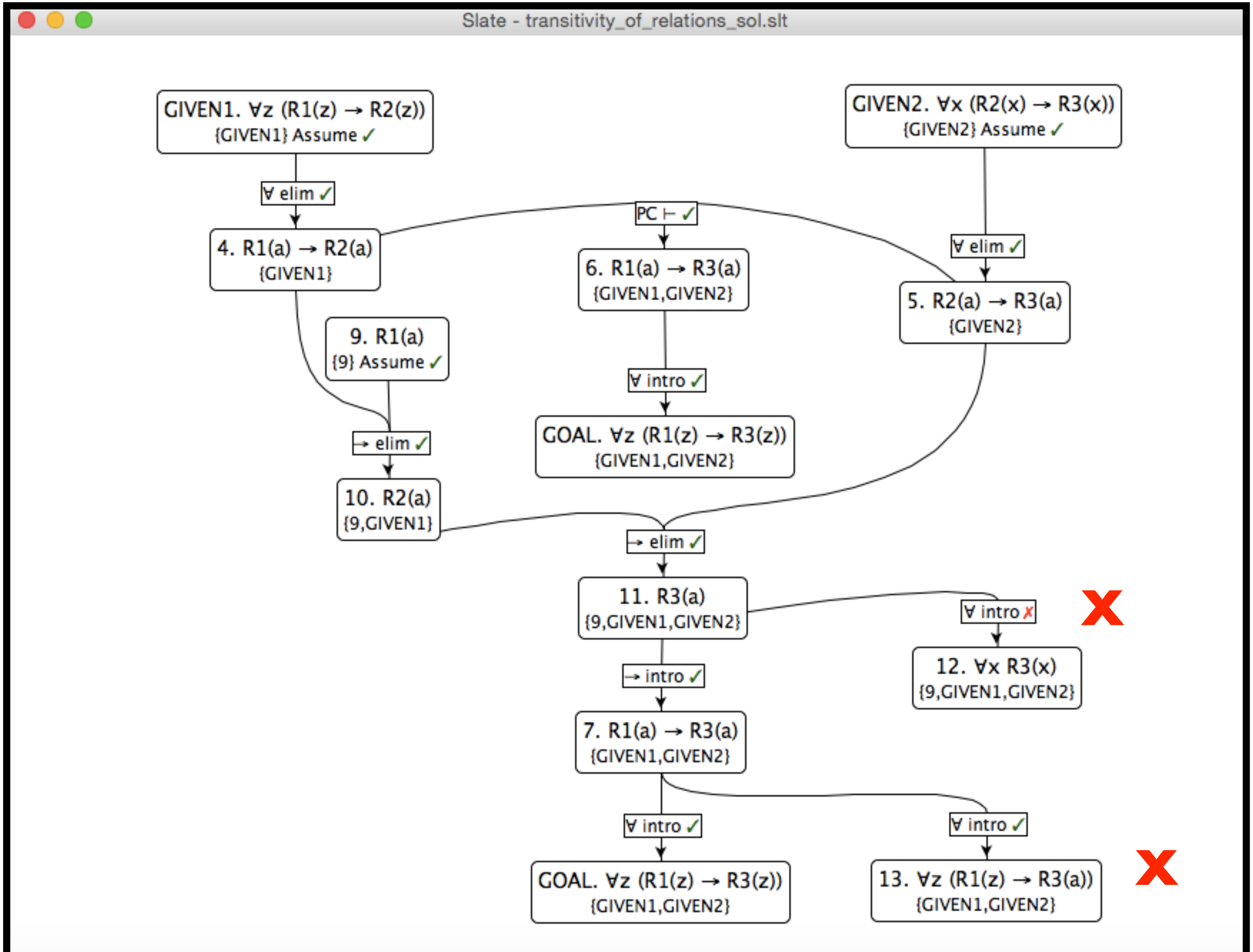
6. R1(a) → R3(a)
{GIVEN1,GIVEN2}
5. R2(a) → R3(a)
{GIVEN2}
9. R1(a)
{9} Assume ✓
∀ intro ✓
→ elim ✓
GOAL. ∀z (R1(z) → R3(z))
{GIVEN1,GIVEN2}
10. R2(a)
{9,GIVEN1}
→ elim ✓
11. R3(a)
{9,GIVEN1,GIVEN2}
∀ intro ✗
x
12. ∀x R3(x)
{9,GIVEN1,GIVEN2}
→ intro ✓
7. R1(a) → R3(a)
{GIVEN1,GIVEN2}
∀ intro ✓
∀ intro ✓
GOAL. ∀z (R1(z) → R3(z))
{GIVEN1,GIVEN2}
13. ∀z (R1(z) → R3(a))
{GIVEN1,GIVEN2}

Example



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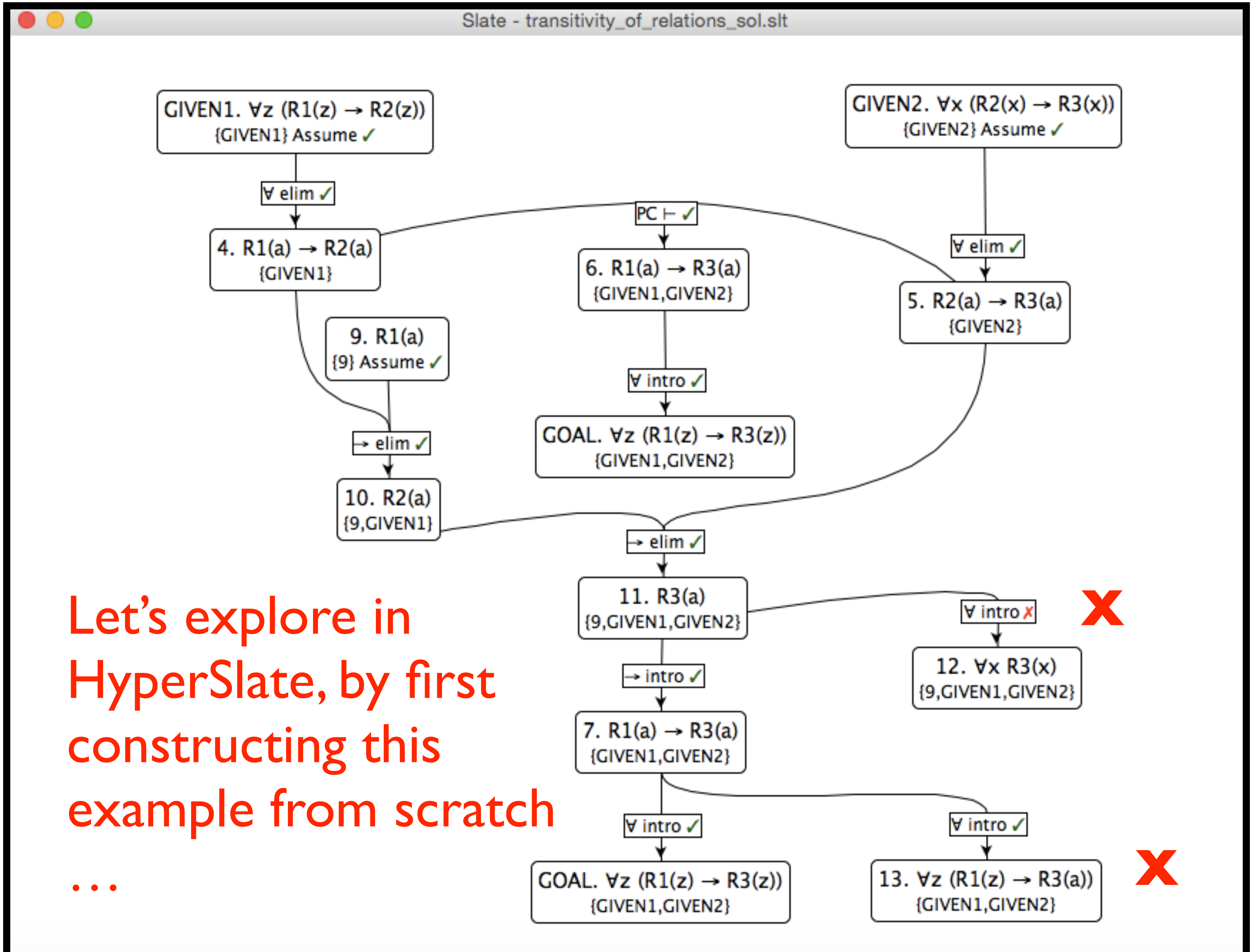
9. R1(a)
{9} Assume ✓
∀ intro ✓
→ elim ✓
GOAL. ∀z (R1(z) → R3(z))
{GIVEN1,GIVEN2}
10. R2(a)
{9,GIVEN1}
→ elim ✓
11. R3(a)
{9,GIVEN1,GIVEN2}
∀ intro ✗
X
12. ∀x R3(x)
{9,GIVEN1,GIVEN2}
→ intro ✓
7. R1(a) → R3(a)
{GIVEN1,GIVEN2}
∀ intro ✓
∀ intro ✓
GOAL. ∀z (R1(z) → R3(z))
{GIVEN1,GIVEN2}
13. ∀z (R1(z) → R3(a))
{GIVEN1,GIVEN2}
X

Example



Let's explore in HyperSlate, by first constructing this example from scratch

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Suggested Practice Problems in HyperSlate!

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$$\{\forall \mathtt{x}(\mathtt{R}(\mathtt{x}) \leftrightarrow \mathtt{S}(\mathtt{x})), \forall \mathtt{x}\mathtt{R}(\mathtt{x})\} \vdash \forall \mathtt{x}\mathtt{S}(\mathtt{x})\} \ \textbf{?}$$

Suggested Practice Problems in HyperSlate!

$$\{\forall \mathtt{x}(\mathtt{R}(\mathtt{x}) \leftrightarrow \mathtt{S}(\mathtt{x})), \forall \mathtt{x}\mathtt{R}(\mathtt{x})\} \vdash \forall \mathtt{x}\mathtt{S}(\mathtt{x})\} \ \textbf{?}$$

$$\{\forall \mathtt{x}[\mathtt{Norsk}(\mathtt{x}) \rightarrow \forall \mathtt{y}(\mathtt{Svensk}(\mathtt{y}) \rightarrow \mathtt{Smarter}(\mathtt{x},\mathtt{y}))]\} \vdash \forall \mathtt{x},\mathtt{y}[(\mathtt{Norsk}(\mathtt{x}) \land \mathtt{Svensk}(\mathtt{y})) \rightarrow \mathtt{Smarter}(\mathtt{x},\mathtt{y})] \ \textbf{?}$$

Suggested Practice Problems in HyperSlate!

$$\{\forall x(R(x) \leftrightarrow S(x)), \forall x R(x)\} \vdash \forall x S(x)\} \ ?$$

$$\{\forall x[\mathtt{Norsk}(x) \rightarrow \forall y(\mathtt{Svensk}(y) \rightarrow \mathtt{Smarter}(x,y))]\} \vdash \forall x,y[(\mathtt{Norsk}(x) \land \mathtt{Svensk}(y)) \rightarrow \mathtt{Smarter}(x,y)] \ ?$$

$$\begin{aligned}&\{\forall x,y[(\mathtt{Norsk}(x) \land (\mathtt{Svensk}(y)) \rightarrow \mathtt{Smarter}(x,y)],\\ &\ \forall x,y[(\mathtt{Svensk}(x) \land (\mathtt{Dansk}(y)) \rightarrow \mathtt{Smarter}(x,y)]\} \vdash\\ &\qquad \forall x,y[(\mathtt{Norsk}(x) \land (\mathtt{Dansk}(y)) \rightarrow \mathtt{Smarter}(x,y)] \ ?\end{aligned}$$