FOL III

Selmer Bringsjord

Rensselaer AI & Reasoning (RAIR) Lab
Department of Cognitive Science
Department of Computer Science
Lally School of Management & Technology
Rensselaer Polytechnic Institute (RPI)
Troy, New York 12180 USA

Intro to Logic
3/11/2019

$\mathscr{L}_1$ FOL III

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Live-action on HyperGrader …

ThxForThePCOracle

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(under Exercises)

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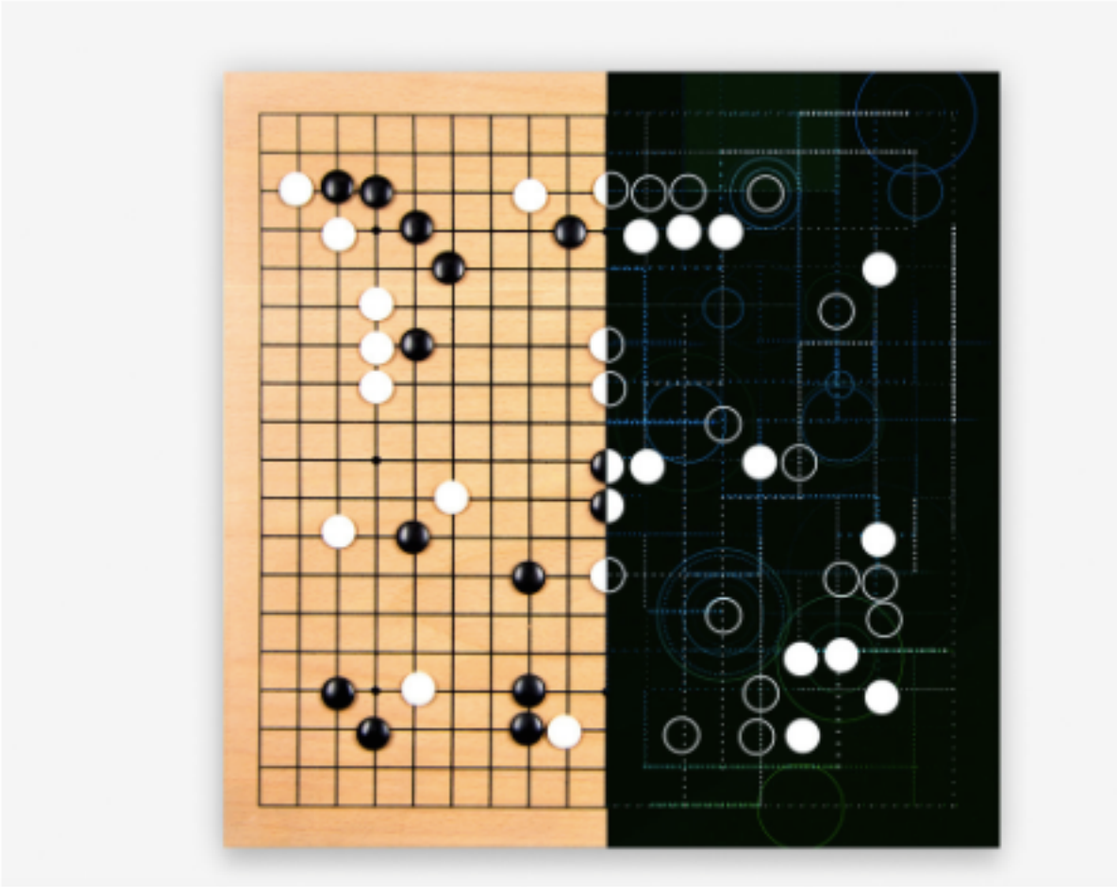
(under Exercises)

Please attempt that now; thx.

Interlude re Formal Logic & Games …

IN A HUGE BREAKTHROUGH, GOOGLE'S AI BEATS A TOP PLAYER AT THE GAME OF GO



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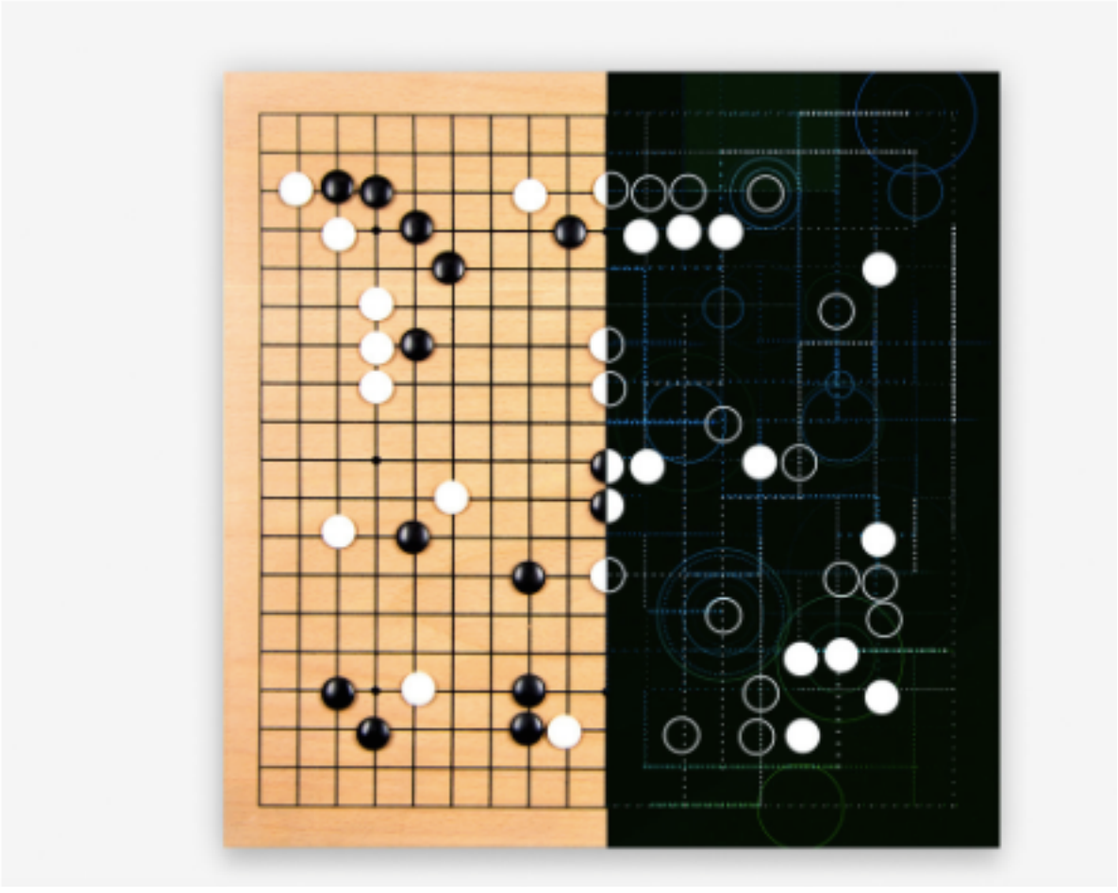

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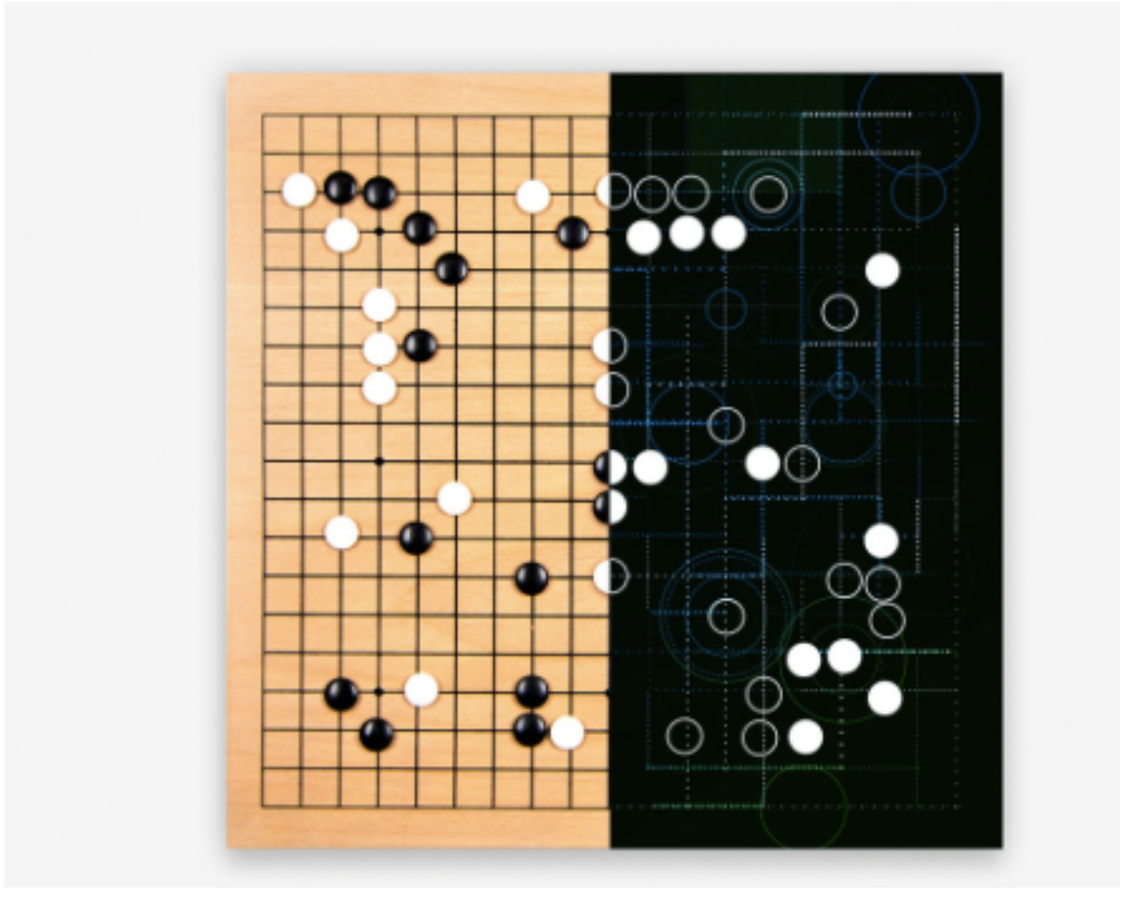

The *Entscheidungsproblem*

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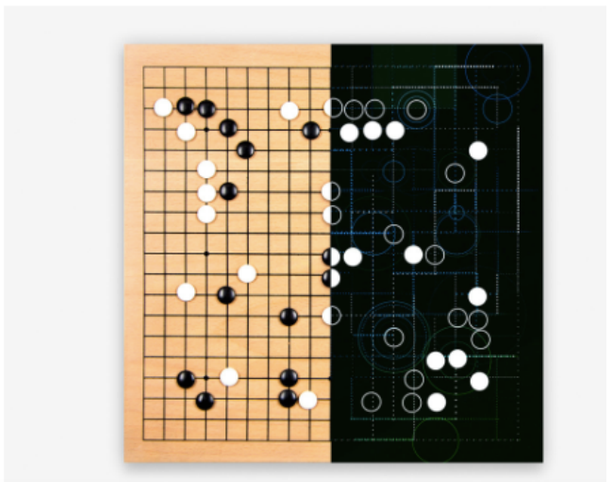

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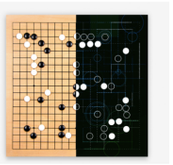

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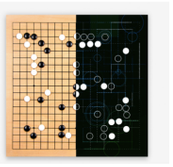

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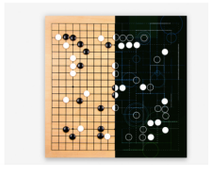
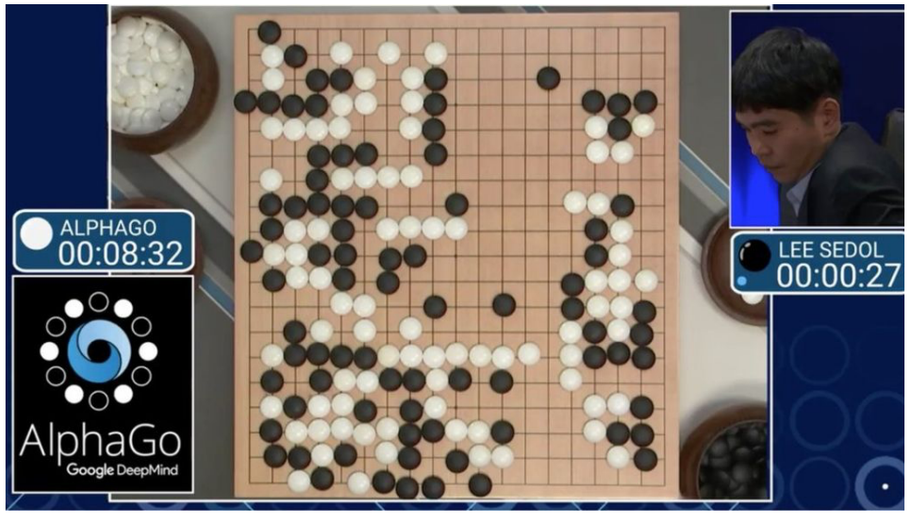
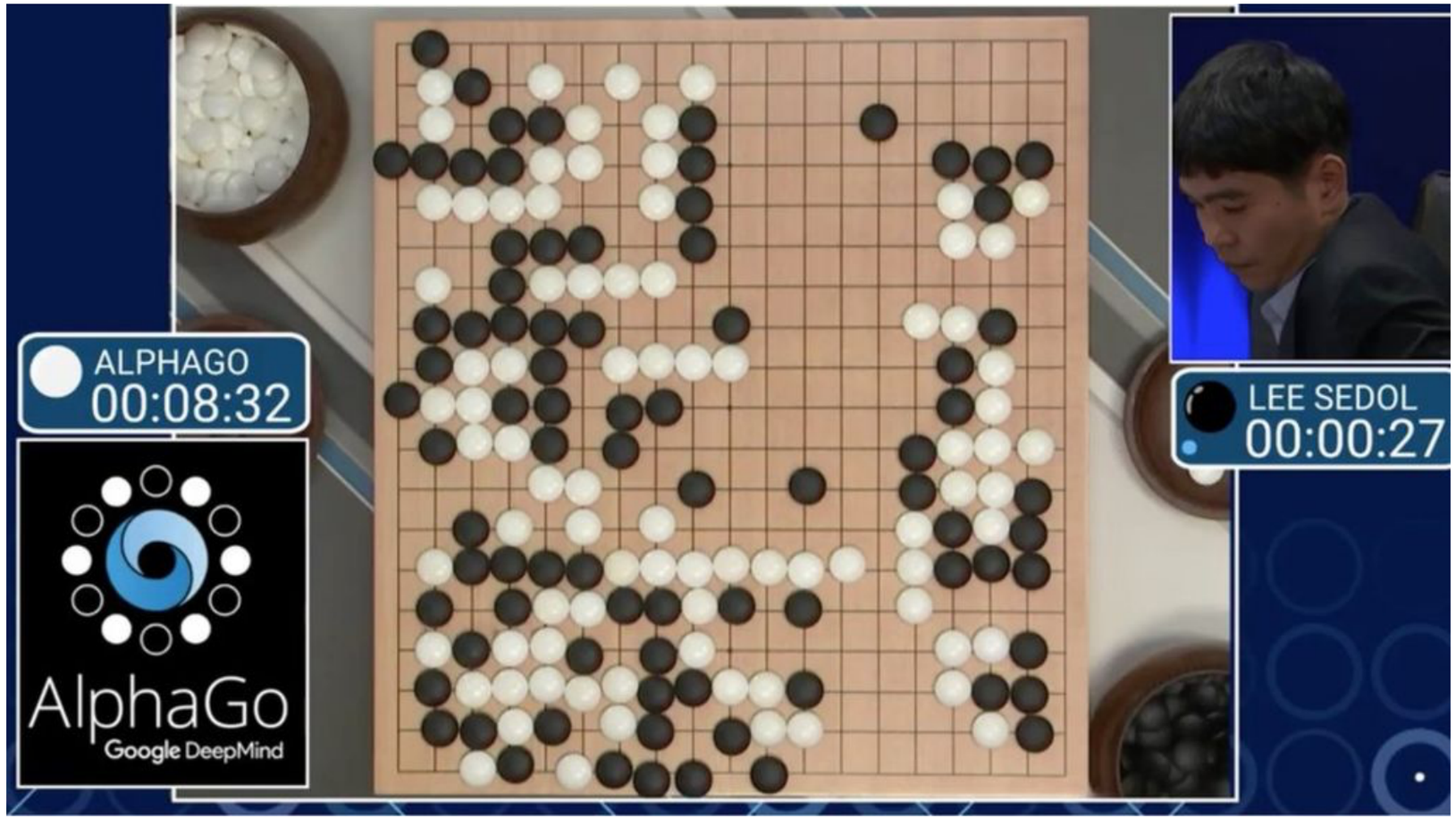



ALPHAGO
00:08:32
AlphaGo
Google DeepMind
LEE SEDOL
00:00:27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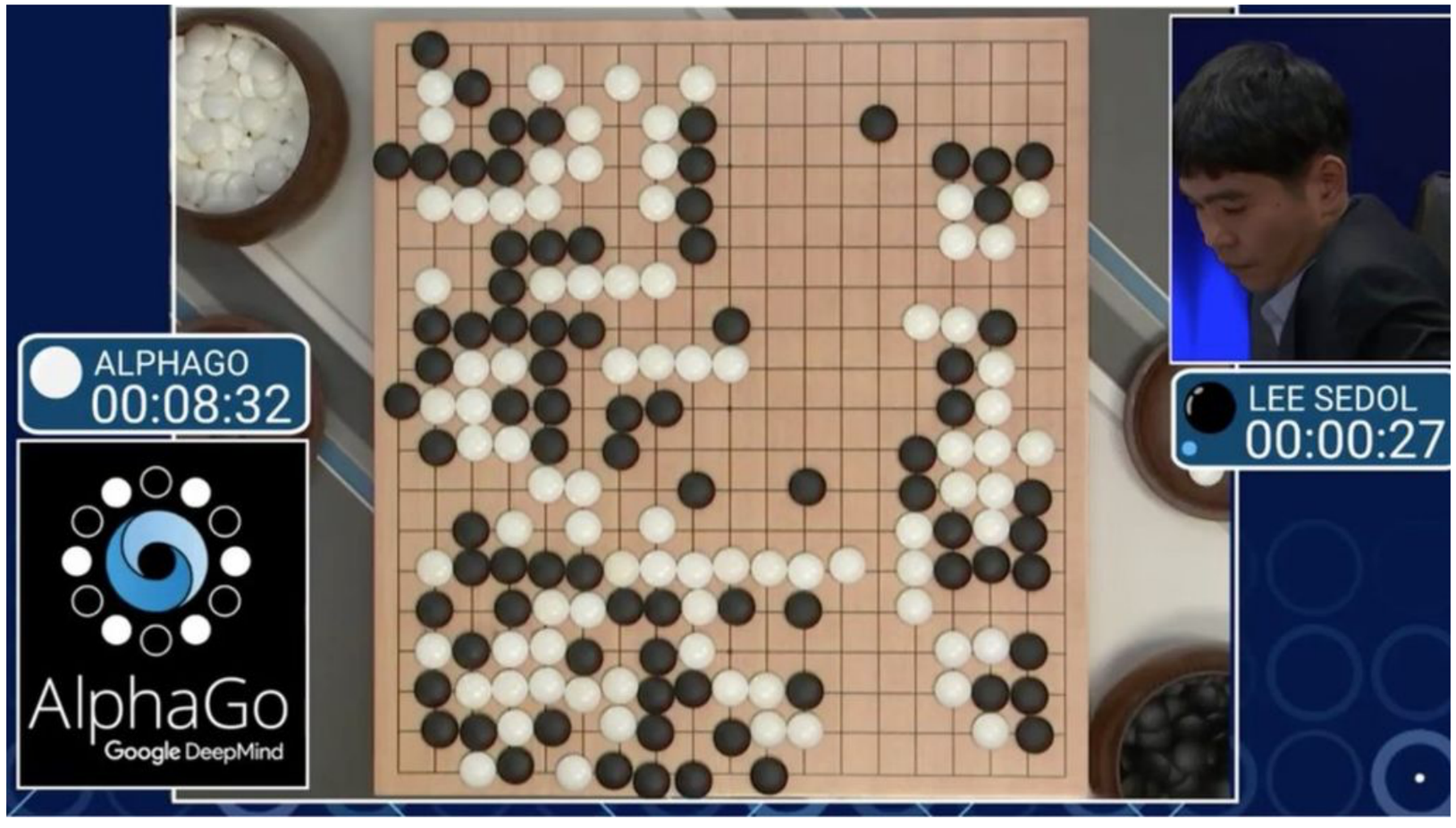
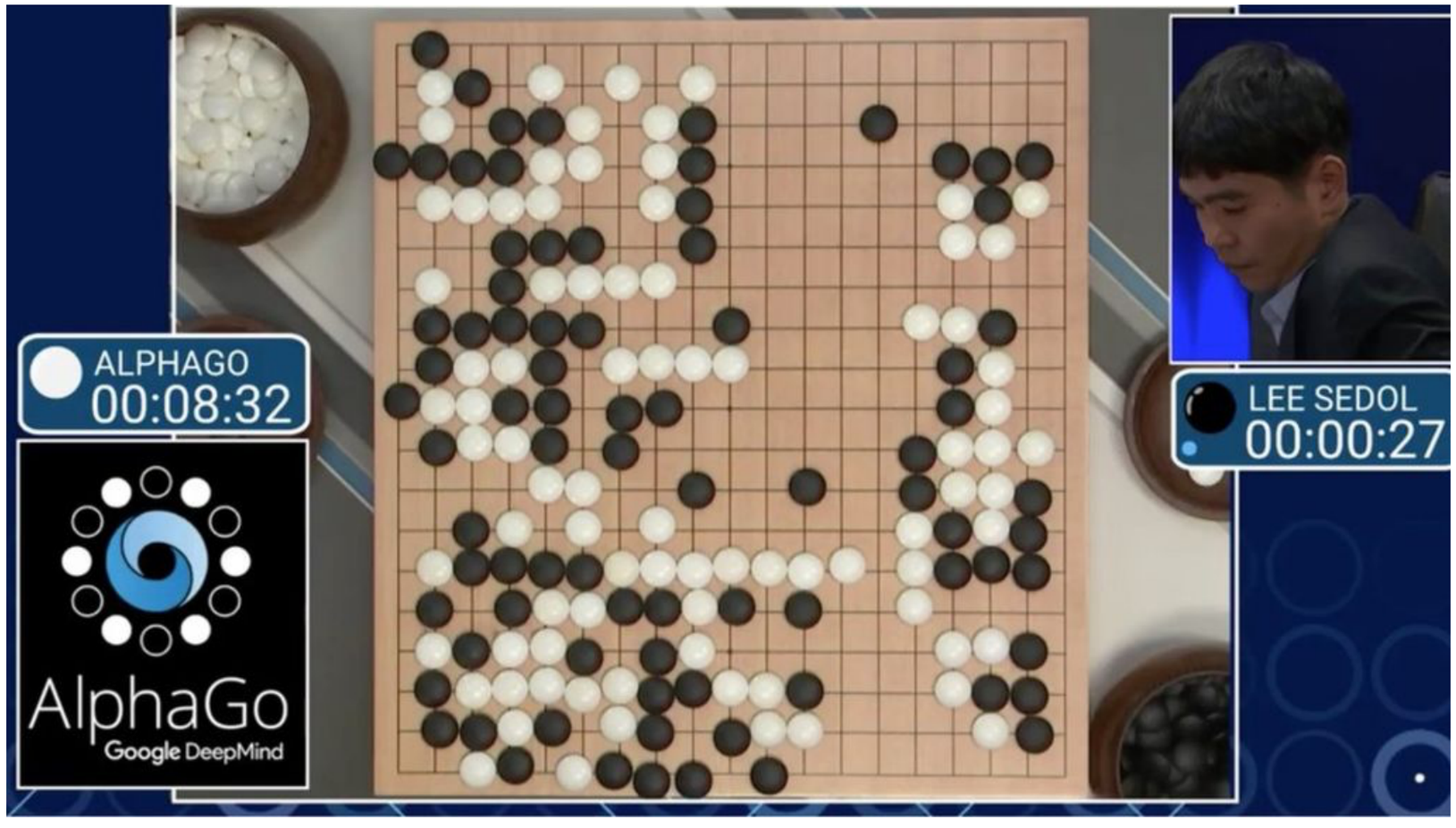
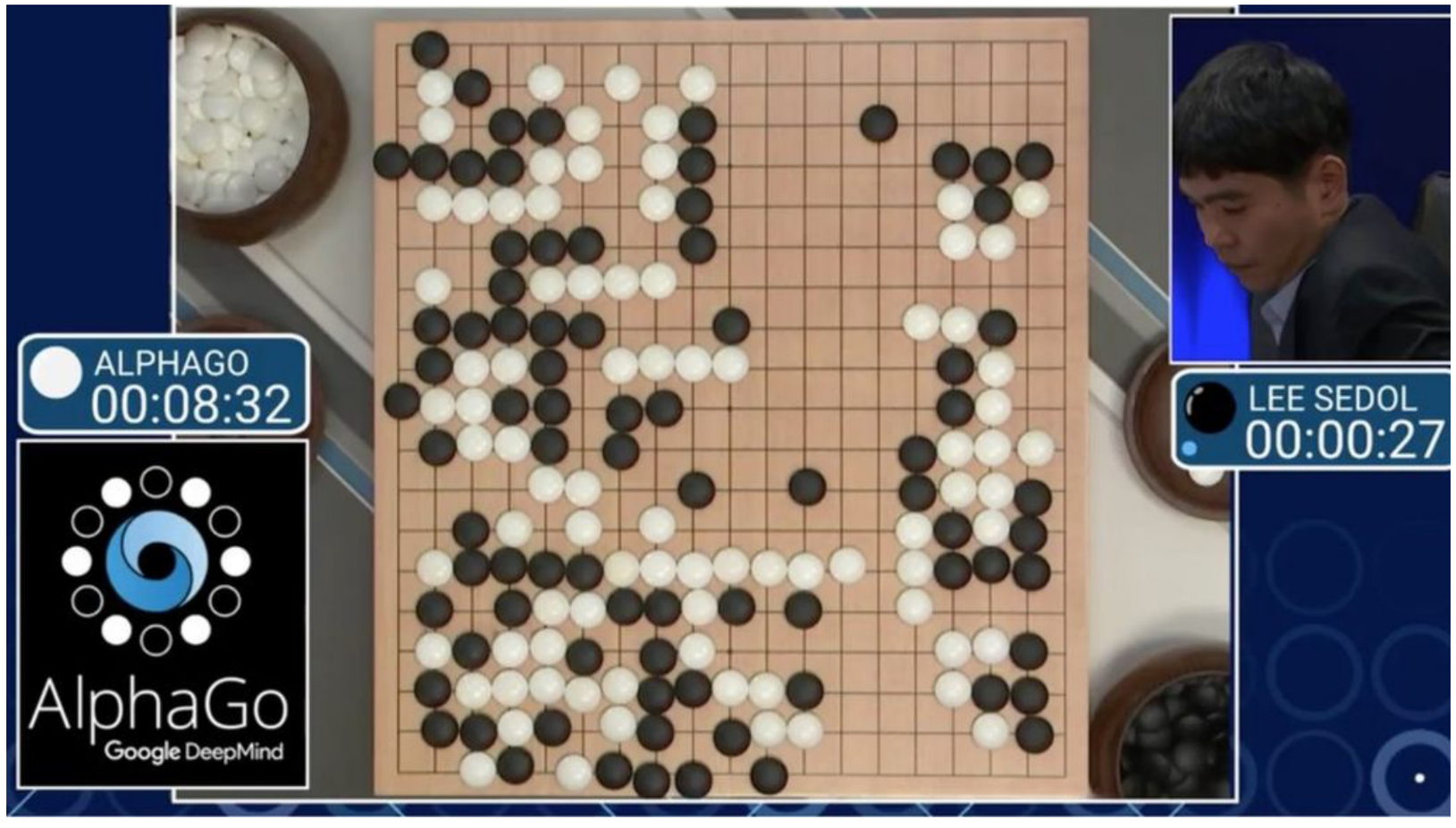


Praiseworthy AI *simplicter*, perhaps.



Praiseworthy AI *simplicter*, perhaps.

But certainly *not* AI = HI!



Praiseworthy AI *simplicter*, perhaps.

But certainly *not* AI = HI!

"AlphaGo, from the perspective of South, how many majuscule Roman letters are in black? Why do you say that?"

Logico-Mathematical Landscape that Has Them Turning in Their Graves

Super-Serious Human Cognitive Power

Serious Human Cognitive Power

Entscheidungsproblem

Mere Calculative Cognitive Power

Logico-Mathematical Landscape that Has Them Turning in Their Graves

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Polynomial Hierarchy

$$\mathbf{P} \subseteq \mathbf{NP} \subseteq \mathbf{PSPACE} = \mathbf{NPSPACE} \subseteq \mathbf{EXPTIME} \subseteq \mathbf{NEXPTIME} \subseteq \mathbf{EXPSPACE}$$

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$$\vdots$$
$$\Pi_2$$
$$\Sigma_2$$
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Rengo Kriegspiel

usgo.org

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US Go Congress Goes a Little Crazy

Wednesday August 13, 2014

"White plays capturing black, putting herself and black into atari," calls Crazy Go TD Terry Benson. He officiated several games of Rengo Kriegspiel on Tuesday evening – a pair go game in which all four players face away from the main board and play their stones on their own empty board in front of them; the only clues about where their opponents — and even their partner — have played comes when they make an illegal move, or play where their own team or their opponents already have stones. Rengo Kriegspiel is only one of dozens of variants on the game of go that were played by an enthusiastic crowd of around 100 players. Familiar games include Magnetic Go, 4 Color Go, Tessellation Go, 3D Go, Spiral Go, and Blind Go. "After all these years, it's still crazy," said TD and Crazy Go founder Terry Benson. New Crazy Go games, never before played at a Go Congress, were even invented on the spot. Four players donned sleeping masks to block their vision and transformed Blind Go into Rengo Blind Go, and a few other players added the fundamentals of Tiddlywinks to their go game. Spectators and players alike are enthusiastic about the creativity of the games and the fun of adding a little Crazy to Go; "Crazy Go is my favorite part of the Congress!" said Bob Crites.

- report/photos by Karoline Li

Categories:
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Redmond Reviews
The Janice Kim Files
Why We Play
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Cotsen Open
IMSA Elite Mind Games
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Pair Go
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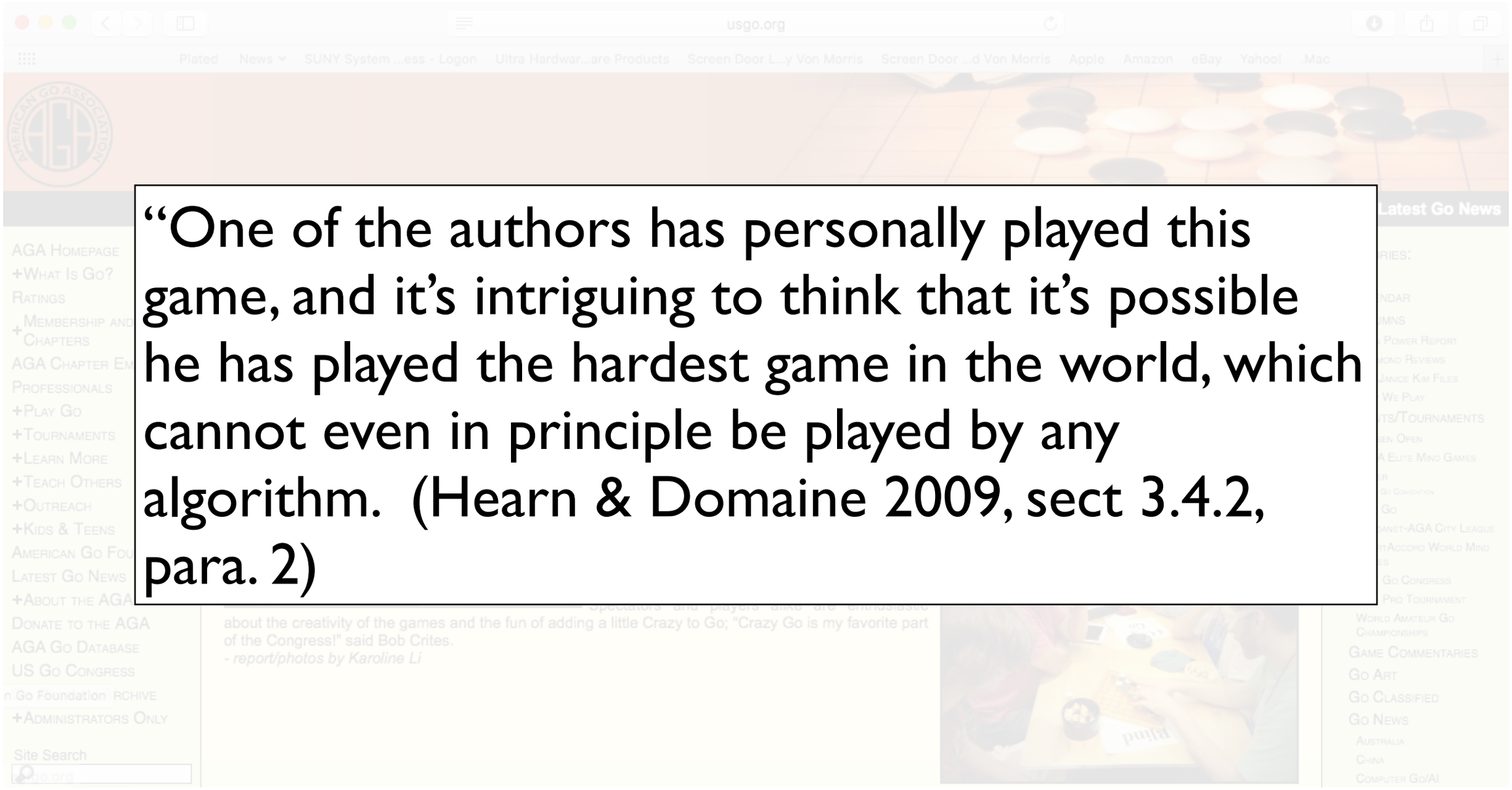
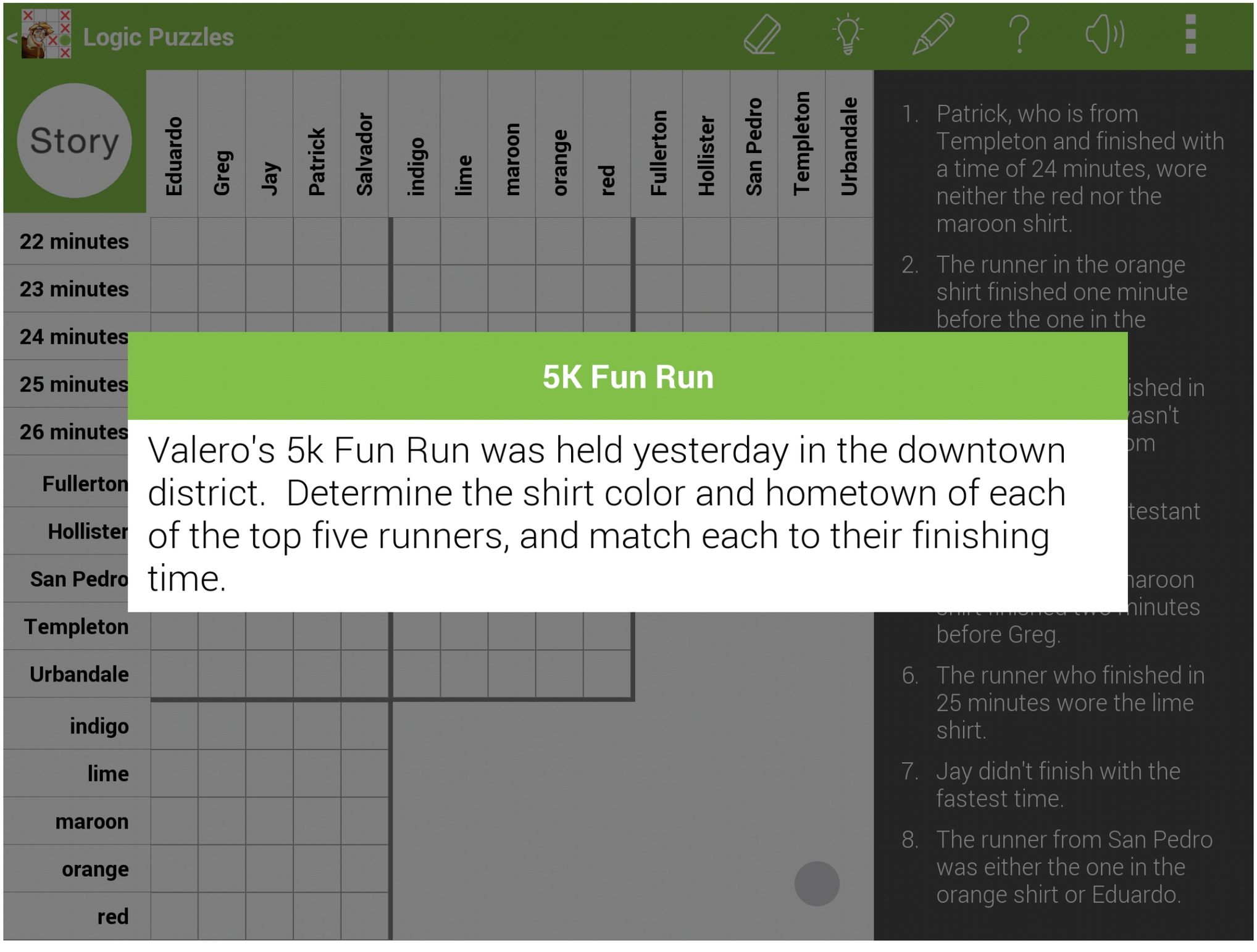
Rengo Kriegspiel



"One of the authors has personally played this game, and it's intriguing to think that it's possible he has played the hardest game in the world, which cannot even in principle be played by any algorithm. (Hearn & Domaine 2009, sect 3.4.2, para. 2)

But starting simpler …

Tabular “Deduction” (Example)



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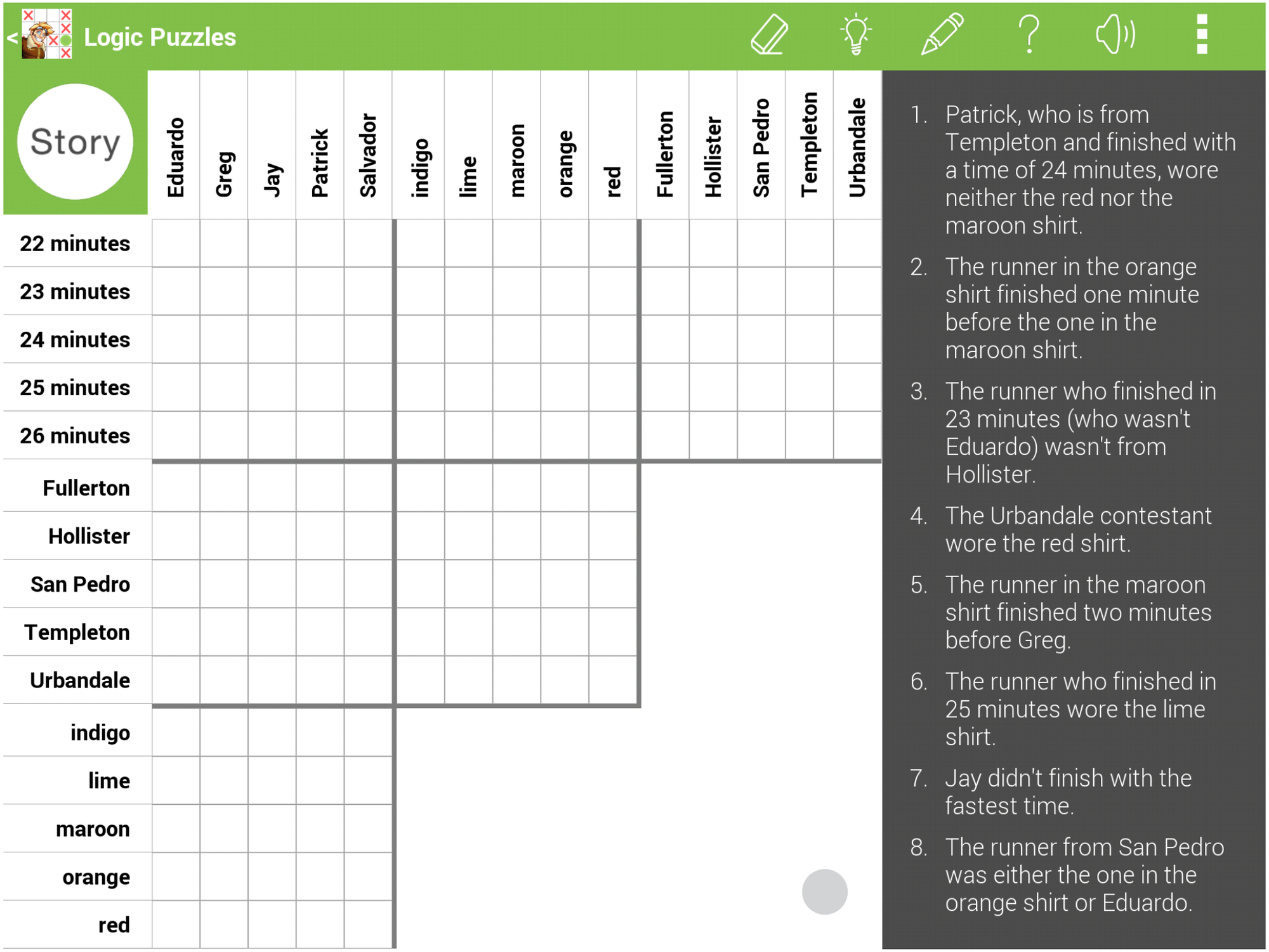

Tabular "Deduction" (Example)

Logic Puzzles

Story	Eduardo	Greg	Jay	Patrick	Salvador	indigo	lime	maroon	orange	red	Fullerton	Hollister	San Pedro	Templeton	Urbandale
22 minutes															
23 minutes															
24 minutes															
25 minutes															
26 minutes															
Fullerton				X											
Hollister				X											
San Pedro				X											
Templeton	X	X	X	●	X										
Urbandale				X											
indigo															
lime															
maroon															
orange															
red															

1. Patrick, who is from Templeton and finished with a time of 24 minutes, wore neither the red nor the maroon shirt.
2. The runner in the orange shirt finished one minute before the one in the maroon shirt.
3. The runner who finished in 23 minutes (who wasn't Eduardo) wasn't from Hollister.
4. The Urbandale contestant wore the red shirt.
5. The runner in the maroon shirt finished two minutes before Greg.
6. The runner who finished in 25 minutes wore the lime shirt.
7. Jay didn't finish with the fastest time.
8. The runner from San Pedro was either the one in the orange shirt or Eduardo.

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In HyperSlate …

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Templeton	✕	✕	✕	●	✕					✕					
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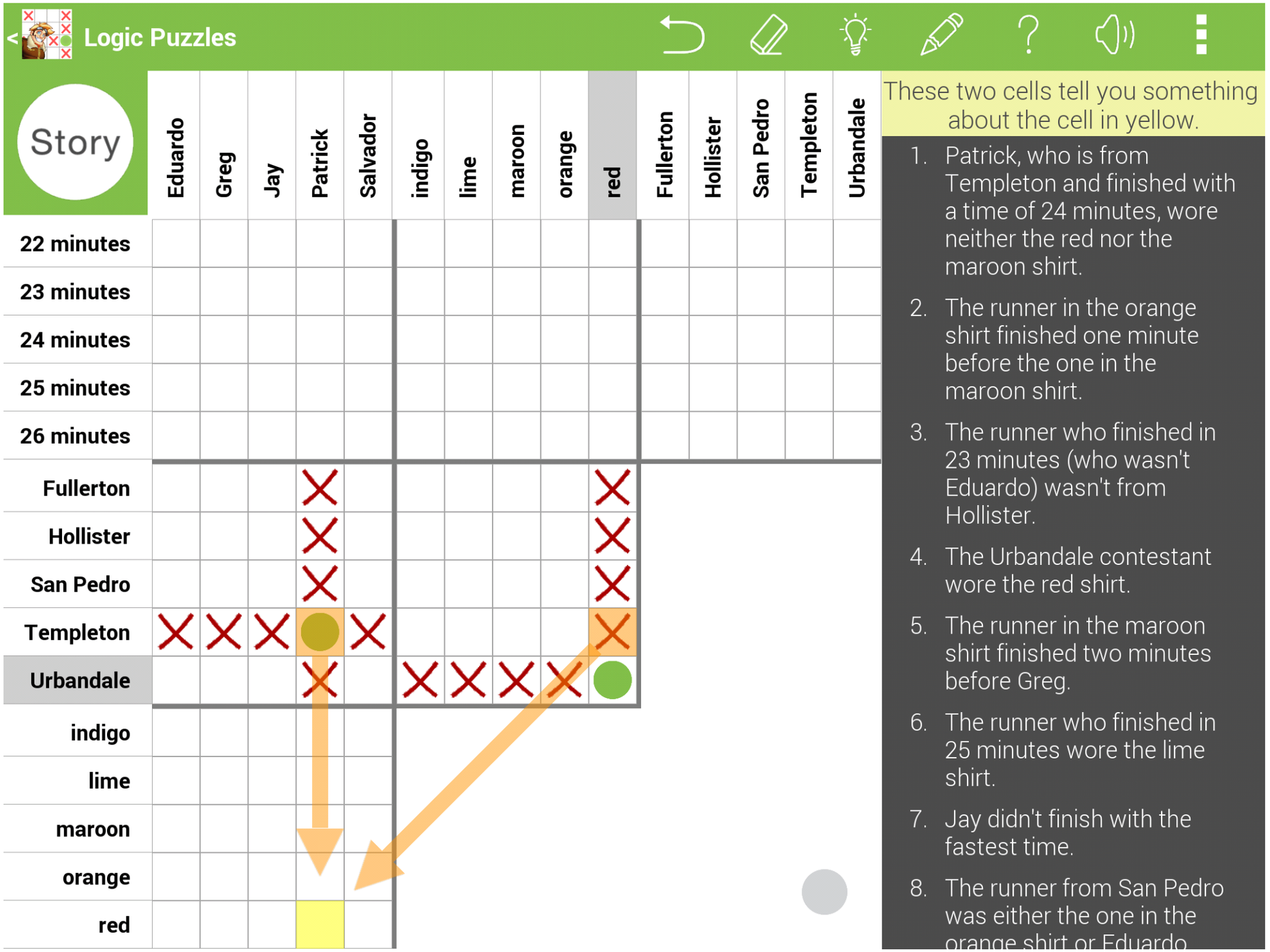


Tabular "Deduction": It's Taught!

Example

Grace, Dylan, Kira, and Diego are each wearing different colored shirts. Grace's shirt is red. Dylan's shirt is not white. Kira's shirt is not green. Diego's shirt is not yellow or white. What color shirt is each person wearing?

First, make a chart to show what you know.

- Each shirt is a different color.
- Grace's shirt is red.
- Dylan's shirt is not white.
- Kira's shirt is not green.
- Diego's shirt is not yellow or white.

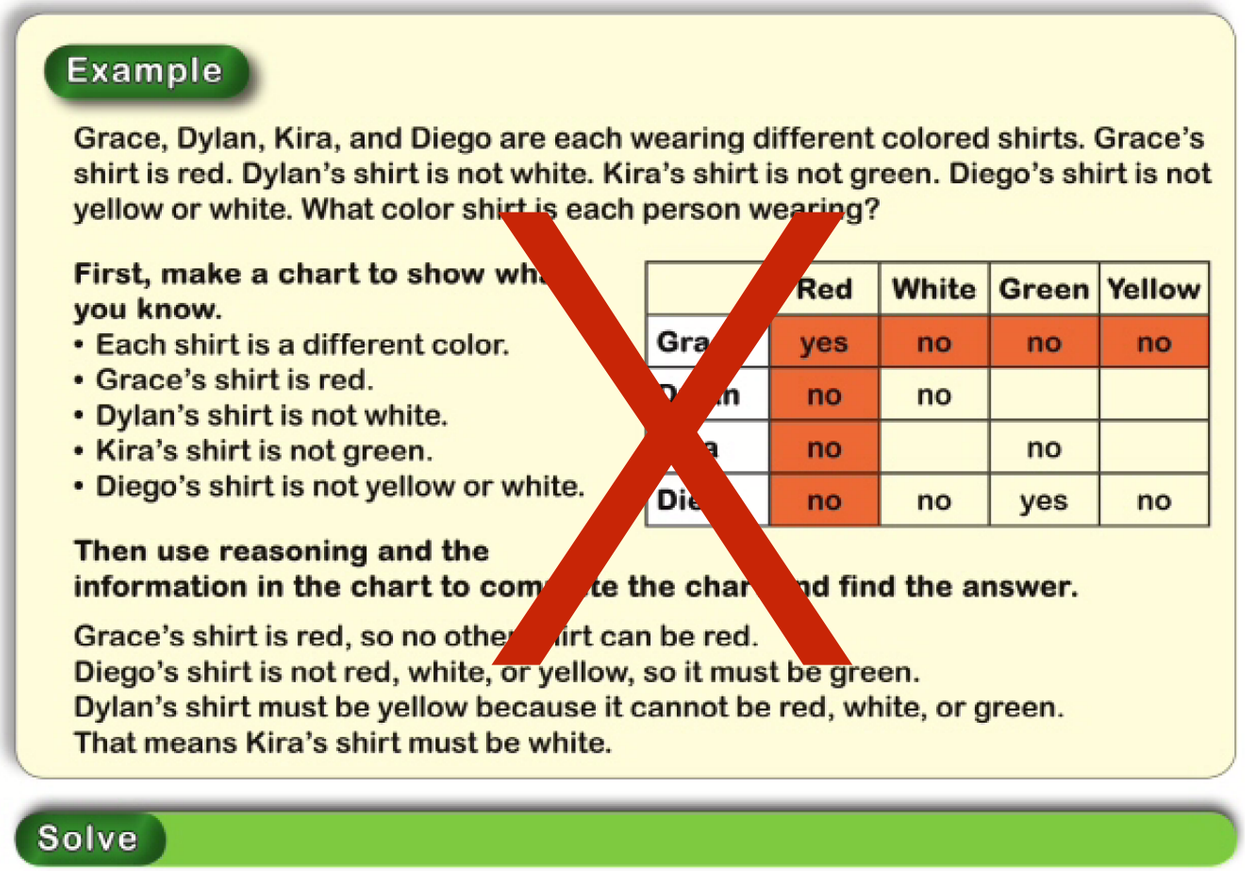
	Red	White	Green	Yellow
Grace	yes	no	no	no
Dylan	no	no		
Kira	no		no	
Diego	no	no	yes	no

Then use reasoning and the information in the chart to complete the chart and find the answer.

Grace's shirt is red, so no other shirt can be red.
Diego's shirt is not red, white, or yellow, so it must be green.
Dylan's shirt must be yellow because it cannot be red, white, or green.
That means Kira's shirt must be white.

Solve

Tabular "Deduction": It's Taught!



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- Dylan's shirt is not white.
- Kira's shirt is not green.
- Diego's shirt is not yellow or white.

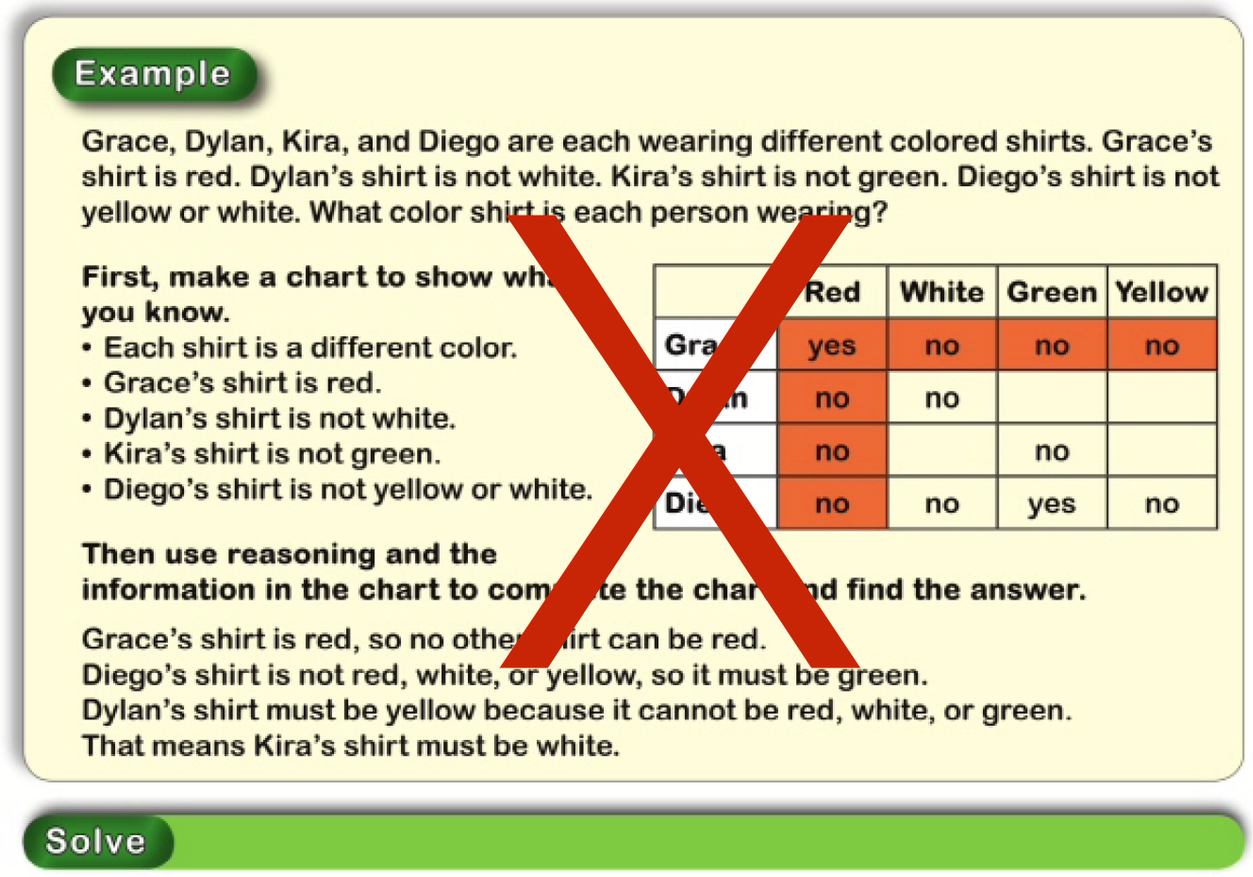
	Red	White	Green	Yellow
Gra	yes	no	no	no
n	no	no		
a	no		no	
Die	no	no	yes	no

Then use reasoning and the information in the chart to com te the char nd find the answer.

Grace's shirt is red, so no othe irt can be red.
Diego's shirt is not red, white, or yellow, so it must be green.
Dylan's shirt must be yellow because it cannot be red, white, or green.
That means Kira's shirt must be white.

Solve

Tabular "Deduction": It's Taught!



Example

Grace, Dylan, Kira, and Diego are each wearing different colored shirts. Grace's shirt is red. Dylan's shirt is not white. Kira's shirt is not green. Diego's shirt is not yellow or white. What color shirt is each person wearing?

First, make a chart to show wh… you know.

- Each shirt is a different color.
- Grace's shirt is red.
- Dylan's shirt is not white.
- Kira's shirt is not green.
- Diego's shirt is not yellow or white.

	Red	White	Green	Yellow
Gra…	yes	no	no	no
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That means Kira's shirt must be white.

Solve

IMHO very bad idea—if *before* real learning of deduction to answer "Why, exactly? Prove it!"

Tabular "deduction" not the skill that's needed.

Recall from Lesson 4-8 that the complex numbers $a + bi$ and $a - bi$ are conjugates. Similarly, the irrational numbers $a + \sqrt{b}$ and $a - \sqrt{b}$ are conjugates. If a complex number or an irrational number is a root of a polynomial equation with rational coefficients, so is its conjugate.

TAKE NOTE Theorem

Conjugate Root Theorem

If $P(x)$ is a polynomial with *rational* coefficients, then irrational roots of $P(x) = 0$ that have the form $a + \sqrt{b}$ occur in conjugate pairs. That is, if $a + \sqrt{b}$ is an irrational root with a and b rational, then $a - \sqrt{b}$ is also a root.

If $P(x)$ is a polynomial with *real* coefficients, then the complex roots of $P(x) = 0$ occur in conjugate pairs. That is, if $a + bi$ is a complex root with a and b real, then $a - bi$ is also a root.

Every quadratic polynomial equation has two roots, every cubic polynomial equation has three roots, and so on.

This result is related to the *Fundamental Theorem of Algebra.* The German mathematician Carl Friedrich Gauss (1777–1855) is credited with proving this theorem.

TAKE NOTE Theorem

The Fundamental Theorem of Algebra

If $P(x)$ is a polynomial of degree $n \geq 1$, then $P(x) = 0$ has exactly n roots, including multiple and complex roots.

Practice and Problem-Solving Exercises - *Continued*

Determine whether each of the following statements is *always*, *sometimes*, or *never* true.

41. A polynomial function with real coefficients has real zeros.

42. Polynomial functions with complex coefficients have one complex zero.

43. A polynomial function that does not intercept the x-axis has complex roots only.

44. Reasoning A 4th-degree polynomial function has zeros at 3 and $5 - i$. Can $4 + i$ also be a zero of the function? Explain your reasoning.

45. Open-Ended Write a polynomial function that has four possible rational zeros but no actual rational zeros.

46. Reasoning Show that the Fundamental Theorem of Algebra must be true for all quadratic polynomial functions.

C • Challenge

47. Use the Fundamental Theorem of Algebra and the Conjugate Root Theorem to show that any odd degree polynomial equation with real coefficients has at least one real root.

48. Reasoning What is the maximum number of points of intersection between the graphs of a quartic and a quintic polynomial function?

49. Reasoning What is the least possible degree of a polynomial with rational coefficients, leading coefficient 1, constant term 5, and zeros at $\sqrt{2}$ and $\sqrt{3}$? Show that such a polynomial has a rational zero and indicate this zero.

Tabular "deduction" not the skill that's needed.

8:29 AM iPad 8:29 AM 91

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Theorems About Roots of Polynomial Equations

Practice and Problem-Solving Exercises - *Continued*

Determine whether each of the following statements is *always*, *sometimes*, or *never* true.

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Back to FOL …

Our Final New Inference Rule in FOL

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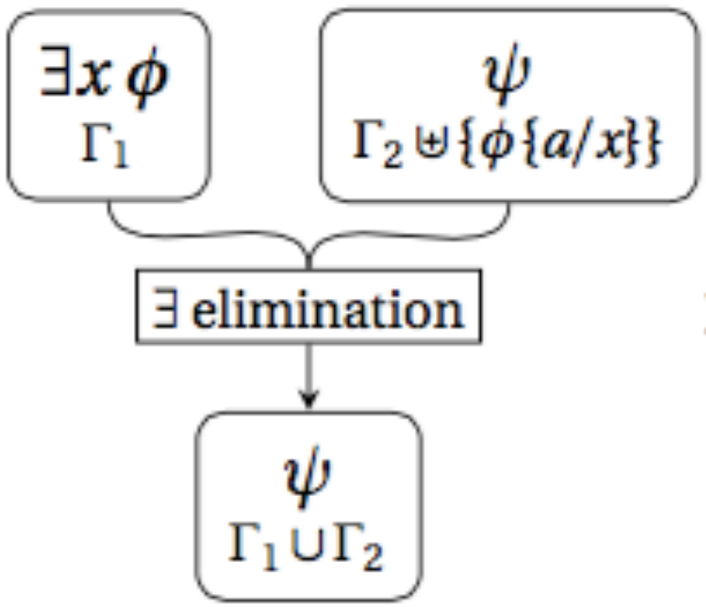
- `existential elimination` (intuitively put):

Our Final New Inference Rule in FOL

- `existential elimination` (intuitively put):
  - If we know that (i) there's something *x* which is an *R*, and (ii) on the supposition that *a* is an arbitrary representative (a "witness") of such an *x* we can prove *P*, then we are permitted to deduce *P* from (i) alone.

existential elimination, precise version:



provided that a does not appear free in Γ_1, Γ_2, or ψ

Starting State of Workspace

(Assumes a domain of e.g. players on a March-madness basketball court.)

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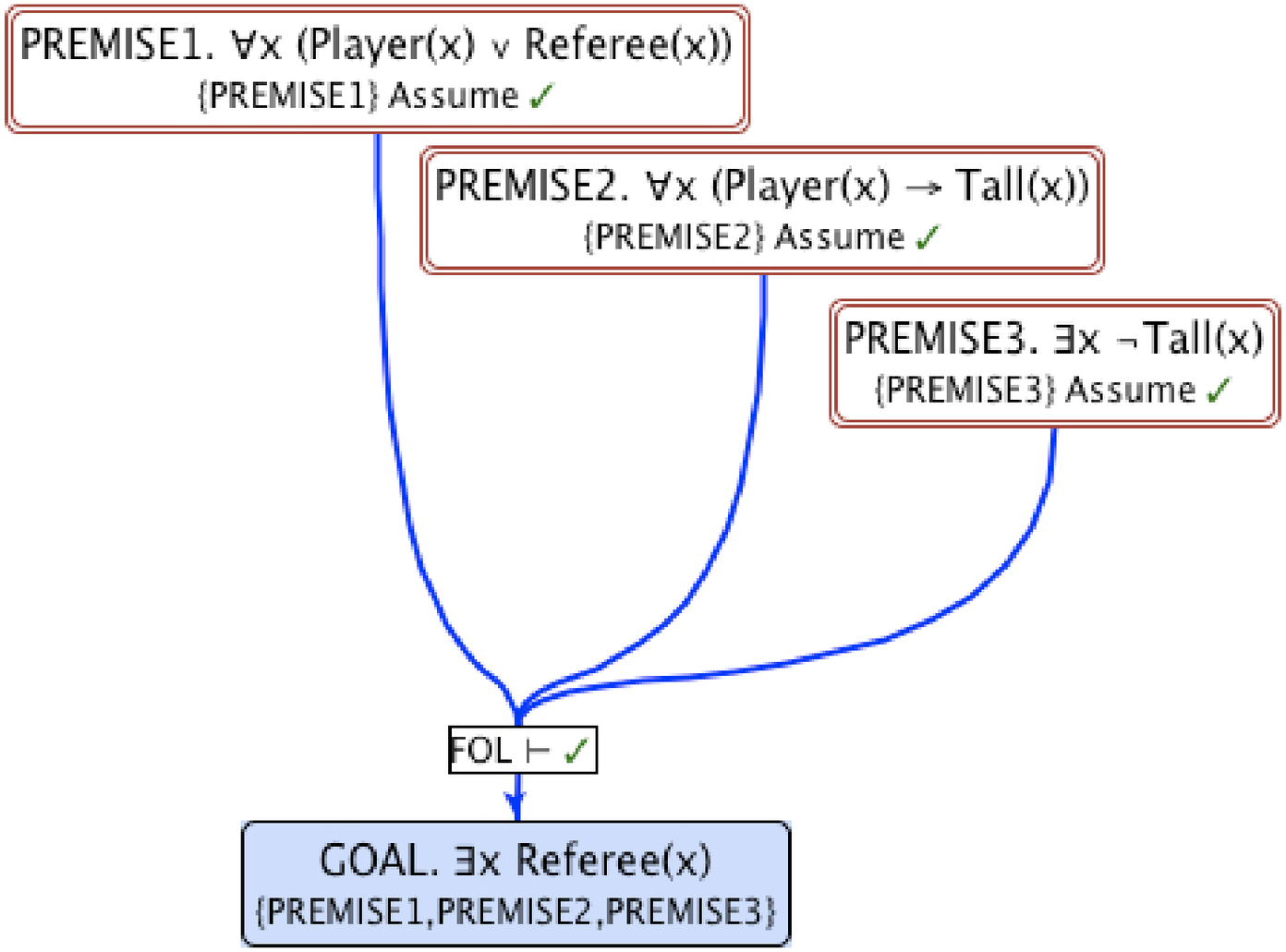
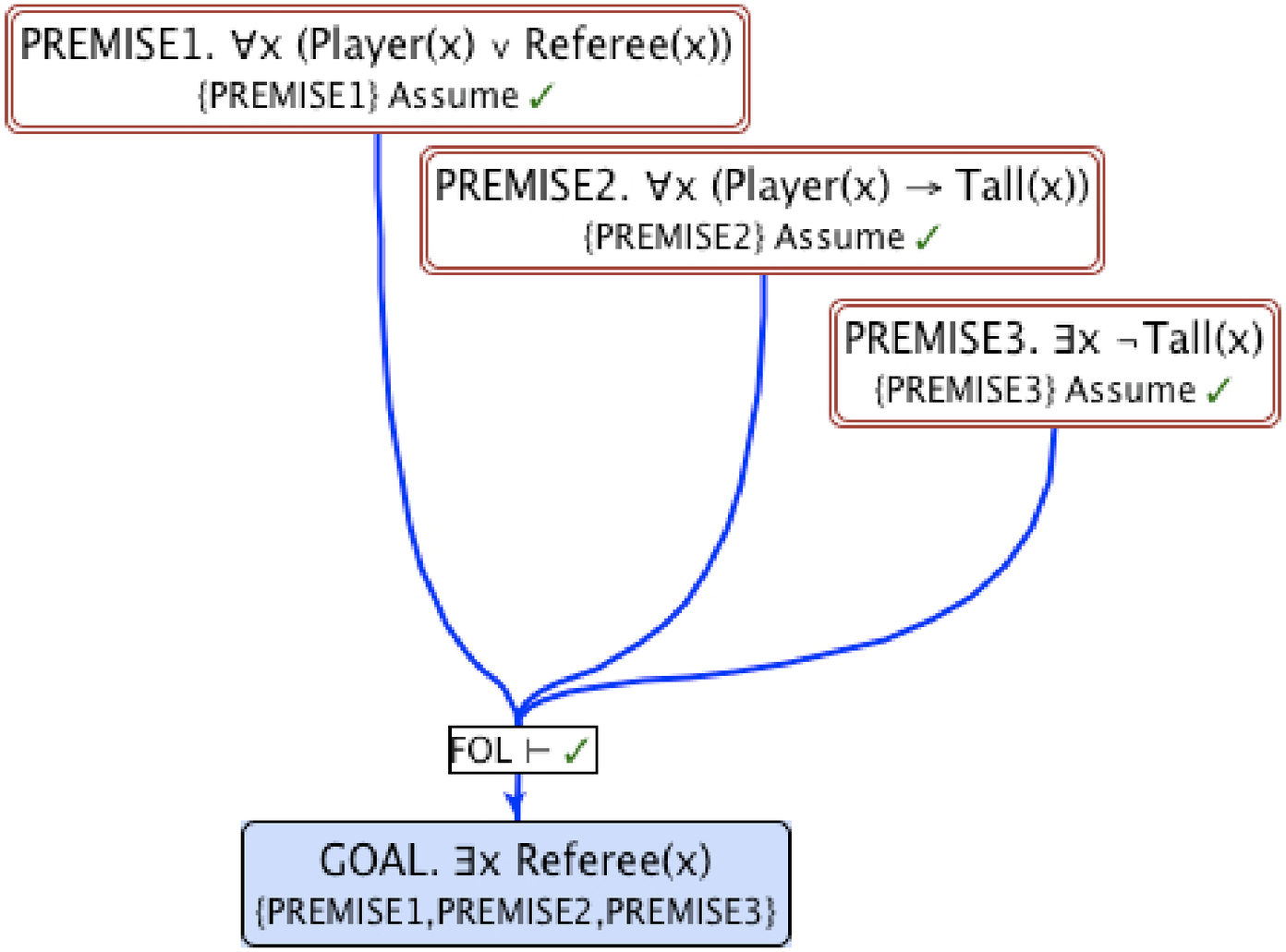


Starting State of Workspace

(Assumes a domain of e.g. players on a March-madness basketball court.)

"Each and every thing is
either a player or a referee."



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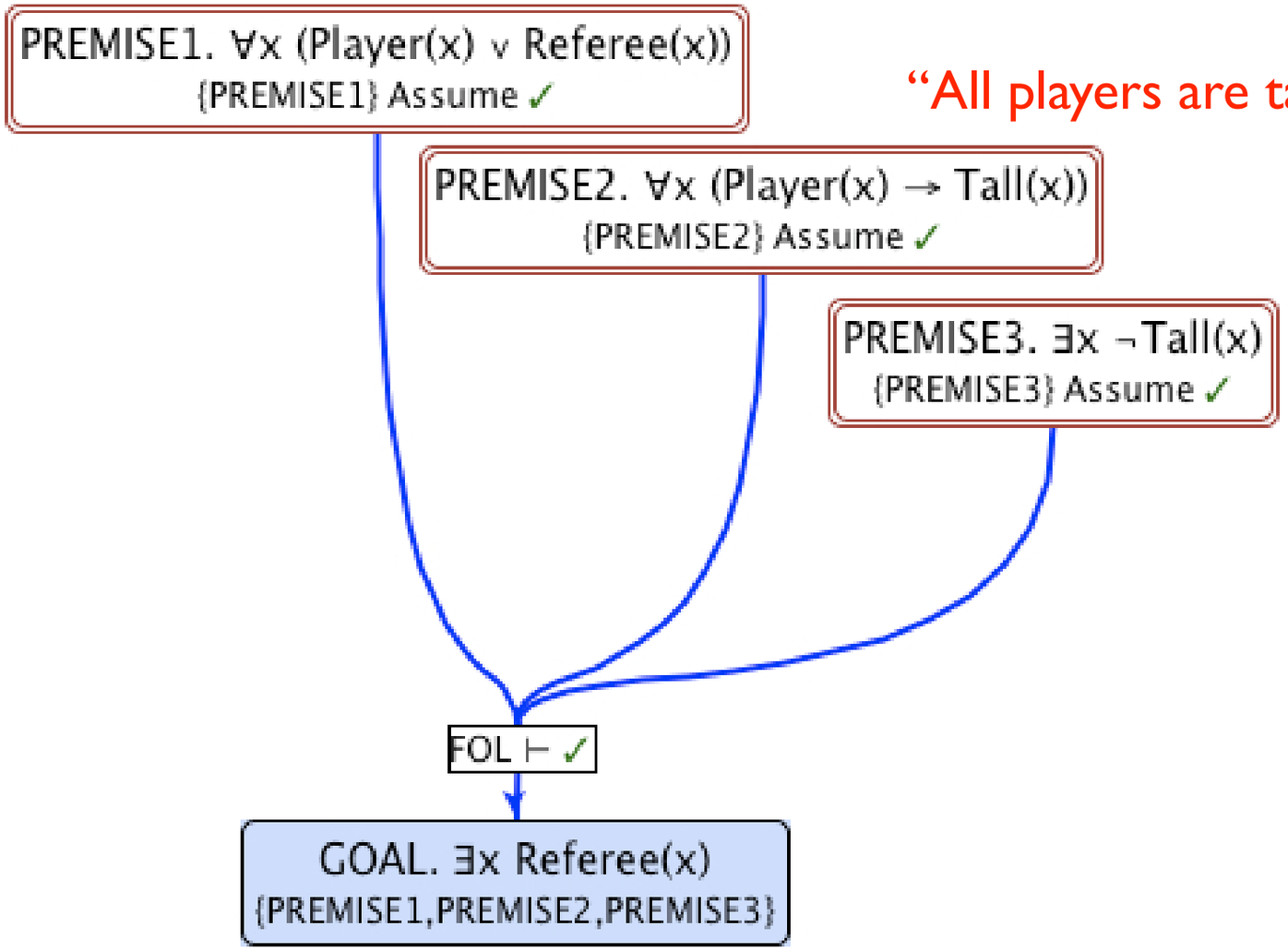

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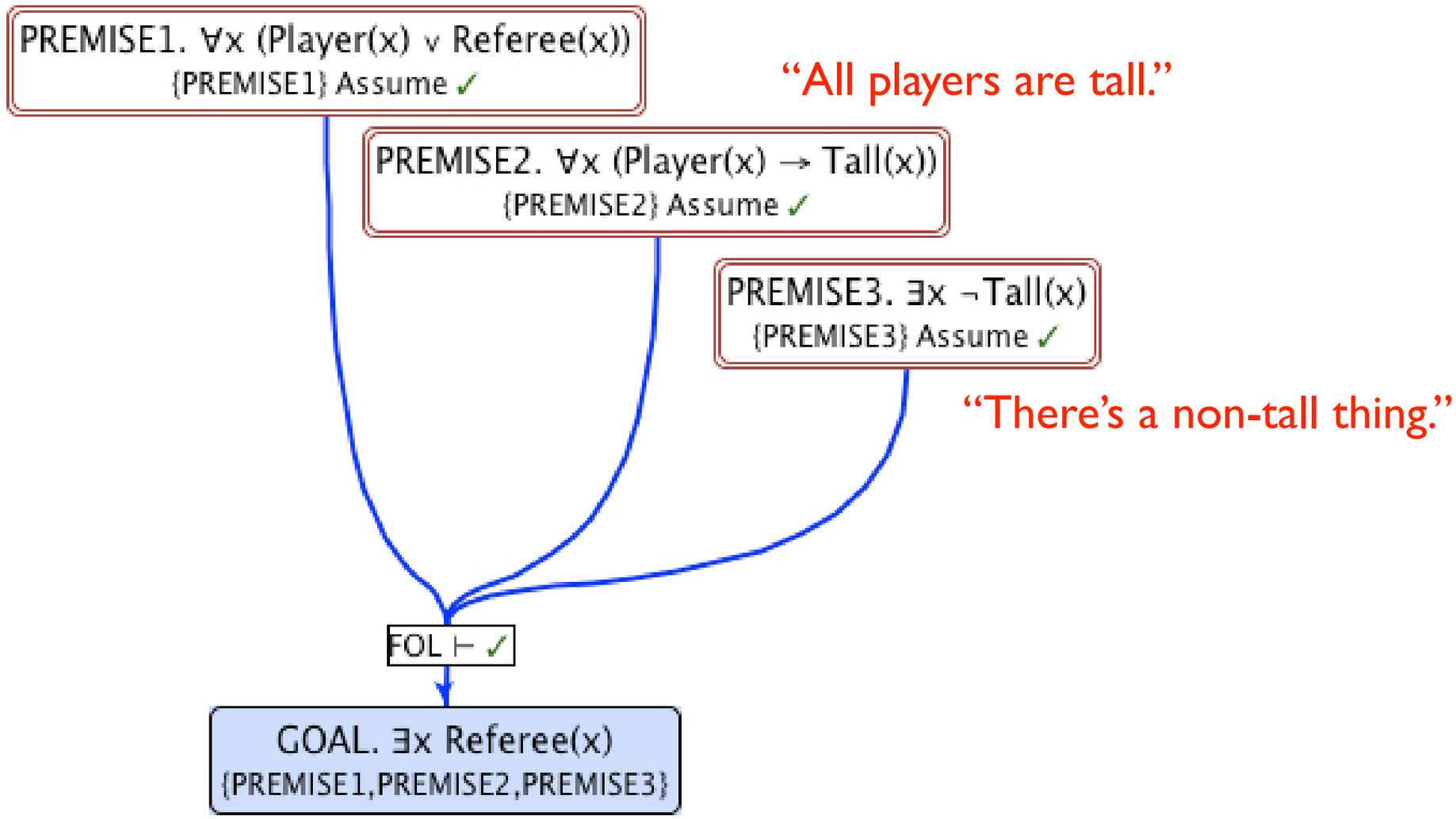


"All players are tall."

"There's a non-tall thing."

Step 1

PREMISE1. ∀x (Player(x) ∨ Referee(x))
{PREMISE1} Assume ✓

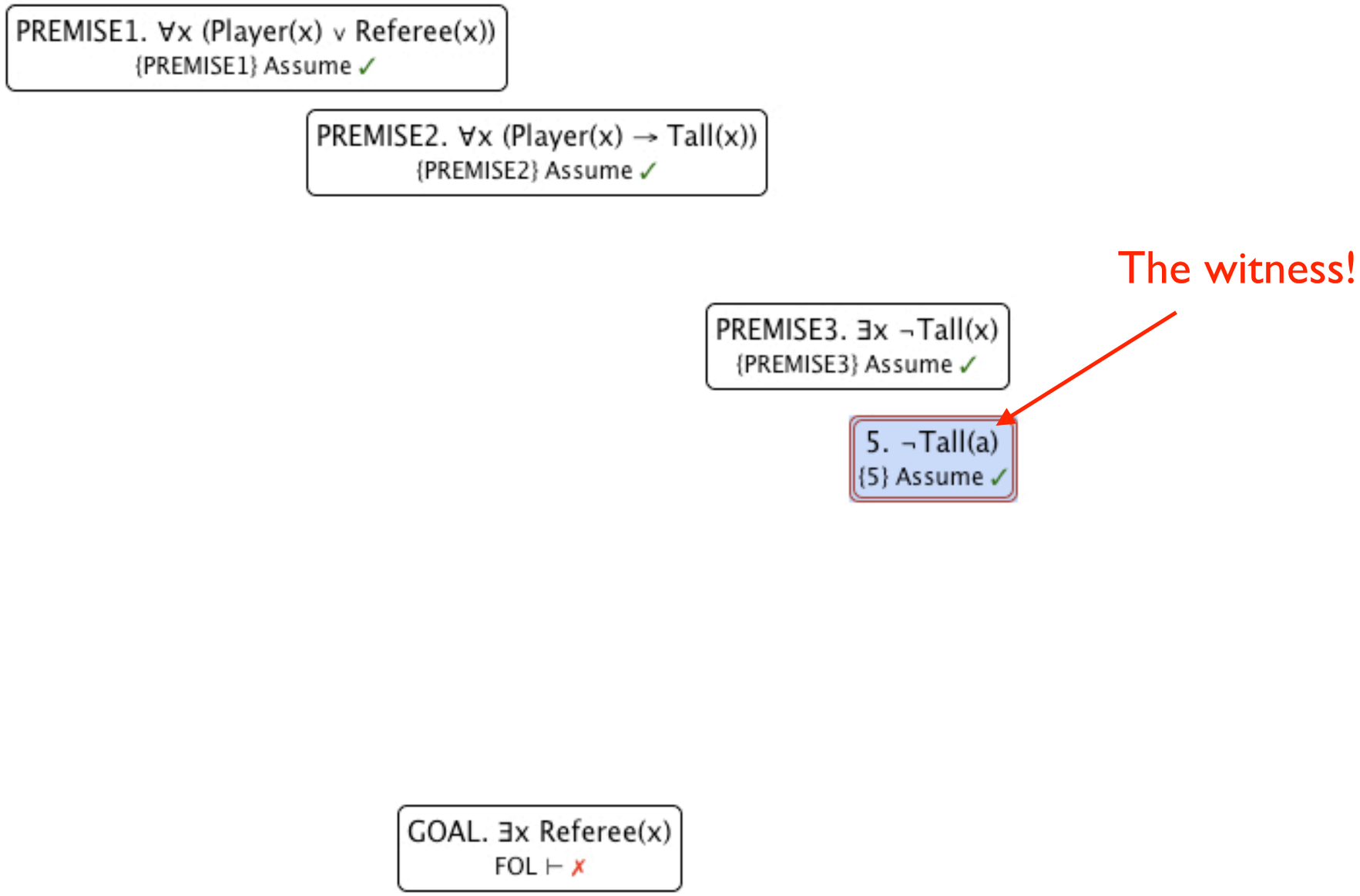
PREMISE2. ∀x (Player(x) → Tall(x))
{PREMISE2} Assume ✓

PREMISE3. ∃x ¬Tall(x)
{PREMISE3} Assume ✓

5. ¬Tall(a)
{5} Assume ✓

GOAL. ∃x Referee(x)
FOL ⊢ ✗

Step 1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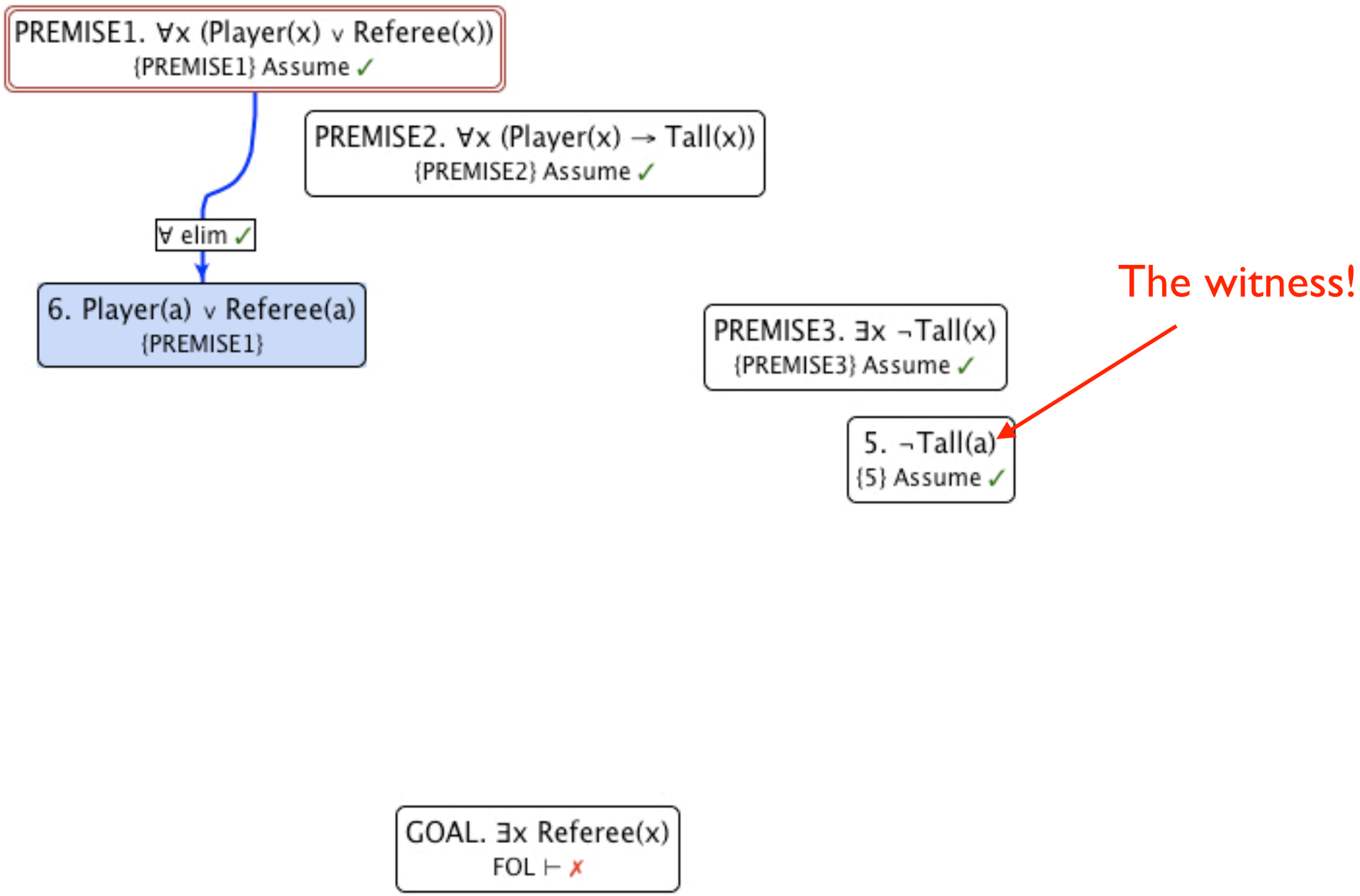
{PREMISE2} Assume ✓
PREMISE3. ∃x ¬Tall(x)
{PREMISE3} Assume ✓
The witness!
5. ¬Tall(a)
{5} Assume ✓
GOAL. ∃x Referee(x)
FOL ⊢ ✗

Step 2


PREMISE1. ∀x (Player(x) ∨ Referee(x))
{PREMISE1} Assume ✓
PREMISE2. ∀x (Player(x) → Tall(x))
{PREMISE2} Assume ✓
∀ elim ✓
6. Player(a) ∨ Referee(a)
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PREMISE3. ∃x ¬Tall(x)
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The witness!
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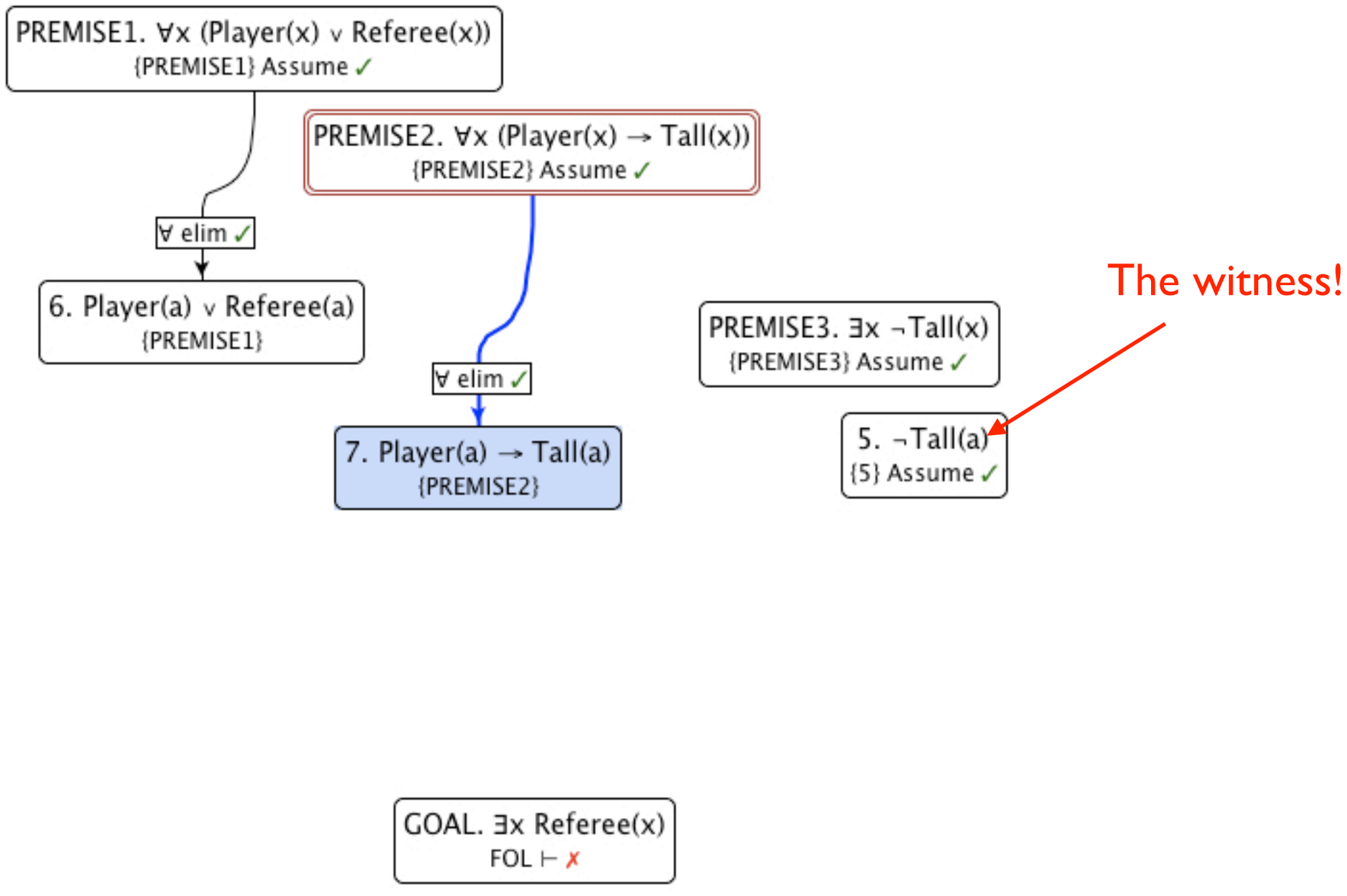
GOAL. ∃x Referee(x)
FOL ⊢ ✗

Step 3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{PREMISE1}
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5. ¬Tall(a)
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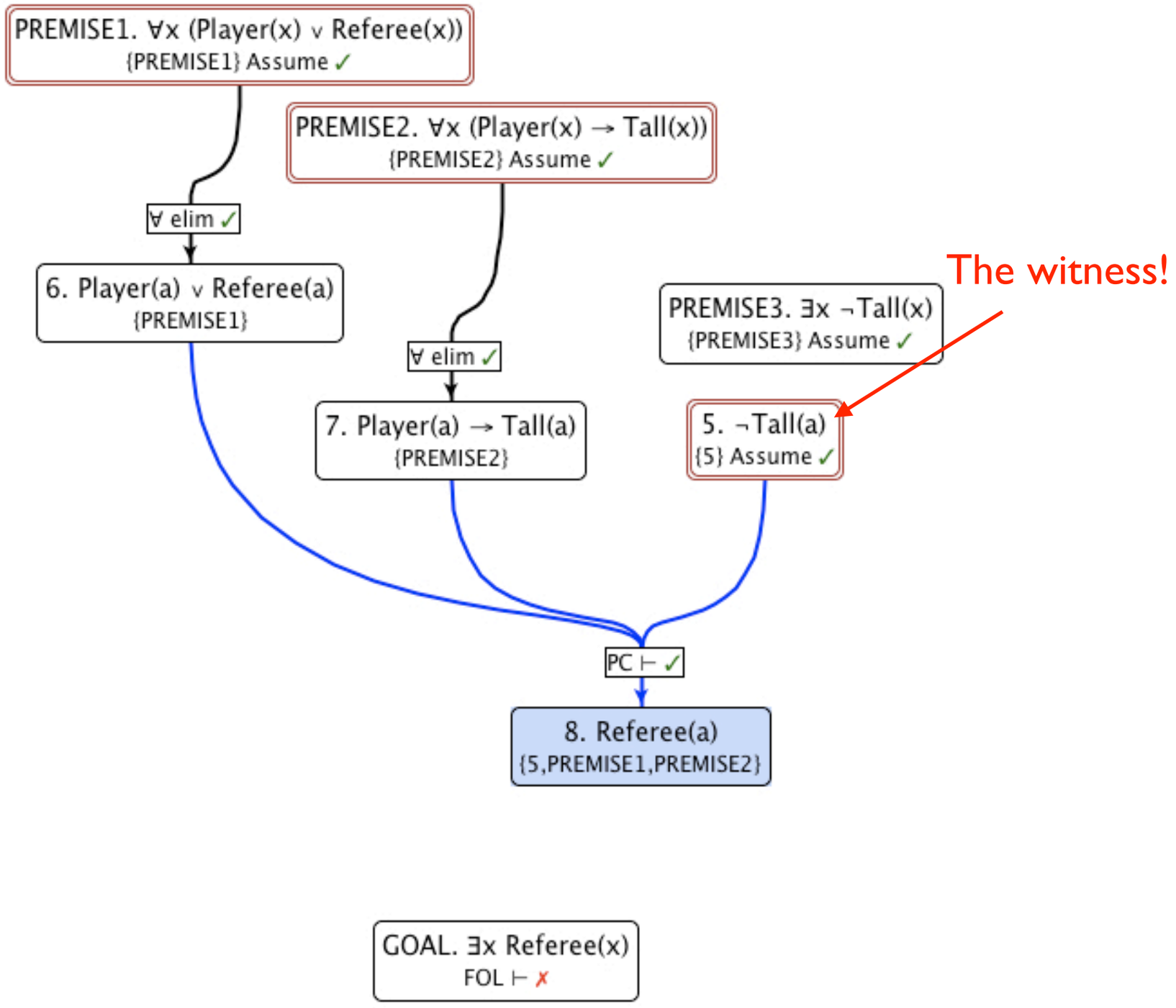
The witness!
GOAL. ∃x Referee(x)
FOL ⊢ ✗

Step 4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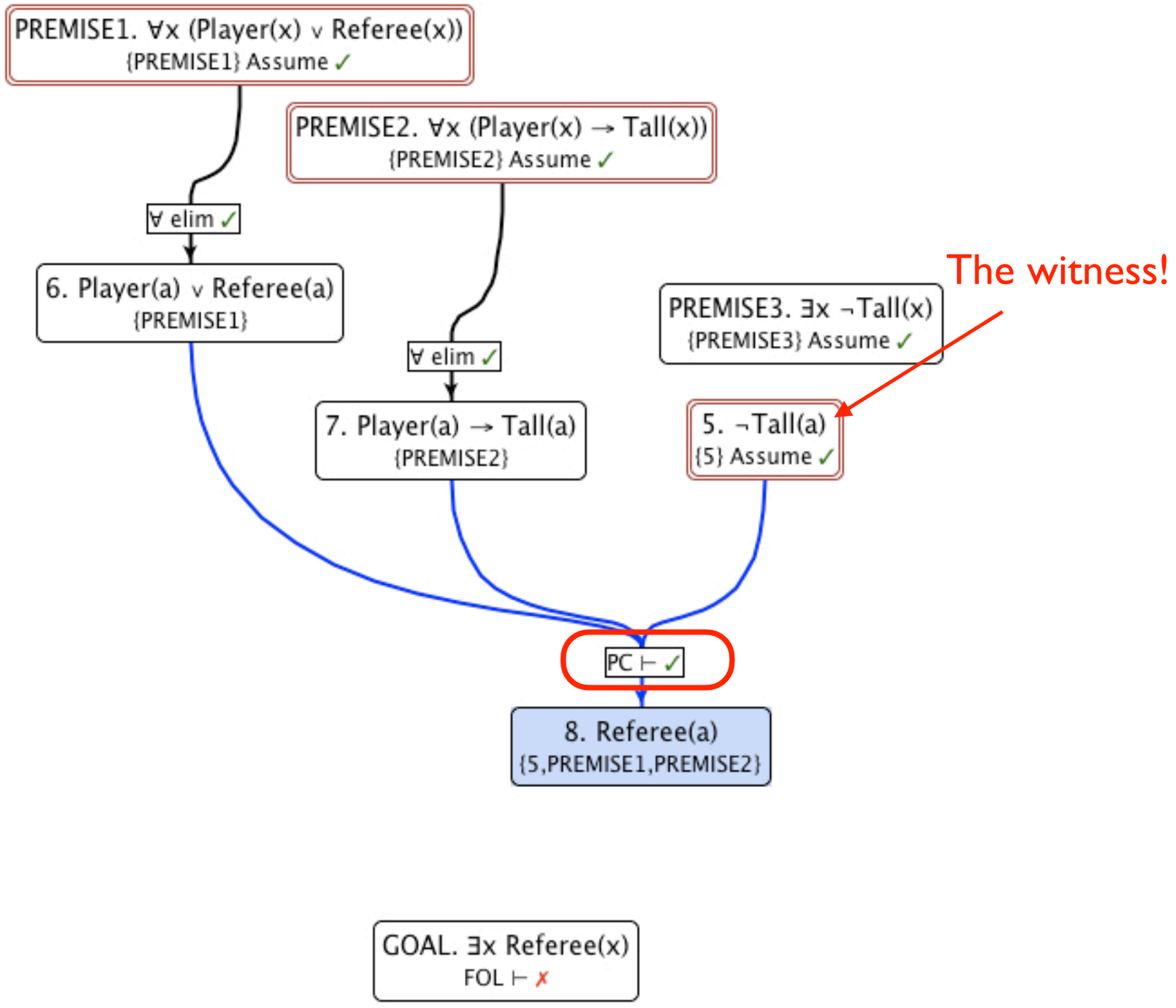
∀ elim ✓
7. Player(a) → Tall(a)
{PREMISE2}
PREMISE3. ∃x ¬Tall(x)
{PREMISE3} Assume ✓
The witness!
5. ¬Tall(a)
{5} Assume ✓
PC ⊢ ✓
8. Referee(a)
{5,PREMISE1,PREMISE2}
GOAL. ∃x Referee(x)
FOL ⊢ ✗

Step 4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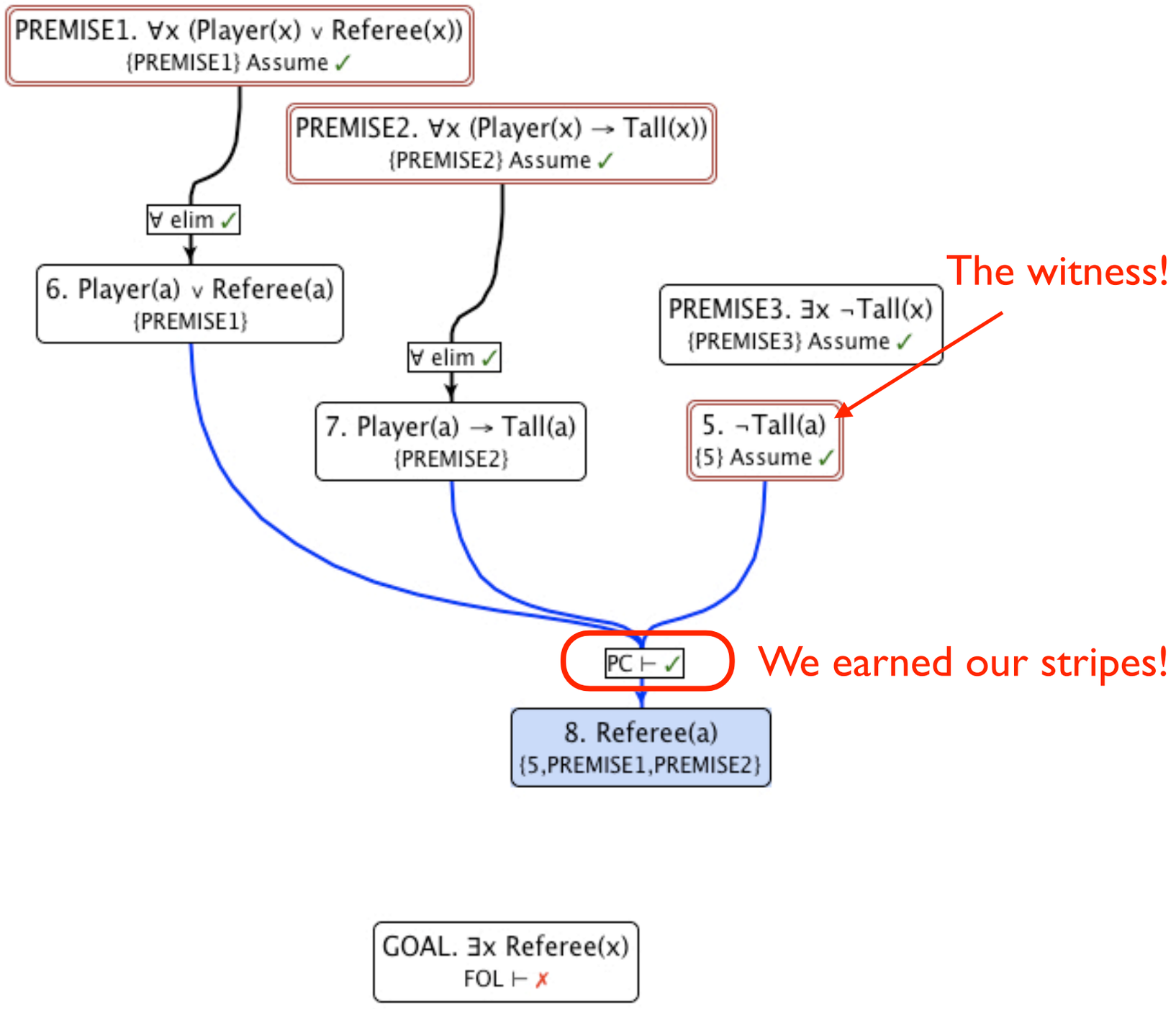
FOL ⊢ ✗

Step 4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{5,PREMISE1,PREMISE2}
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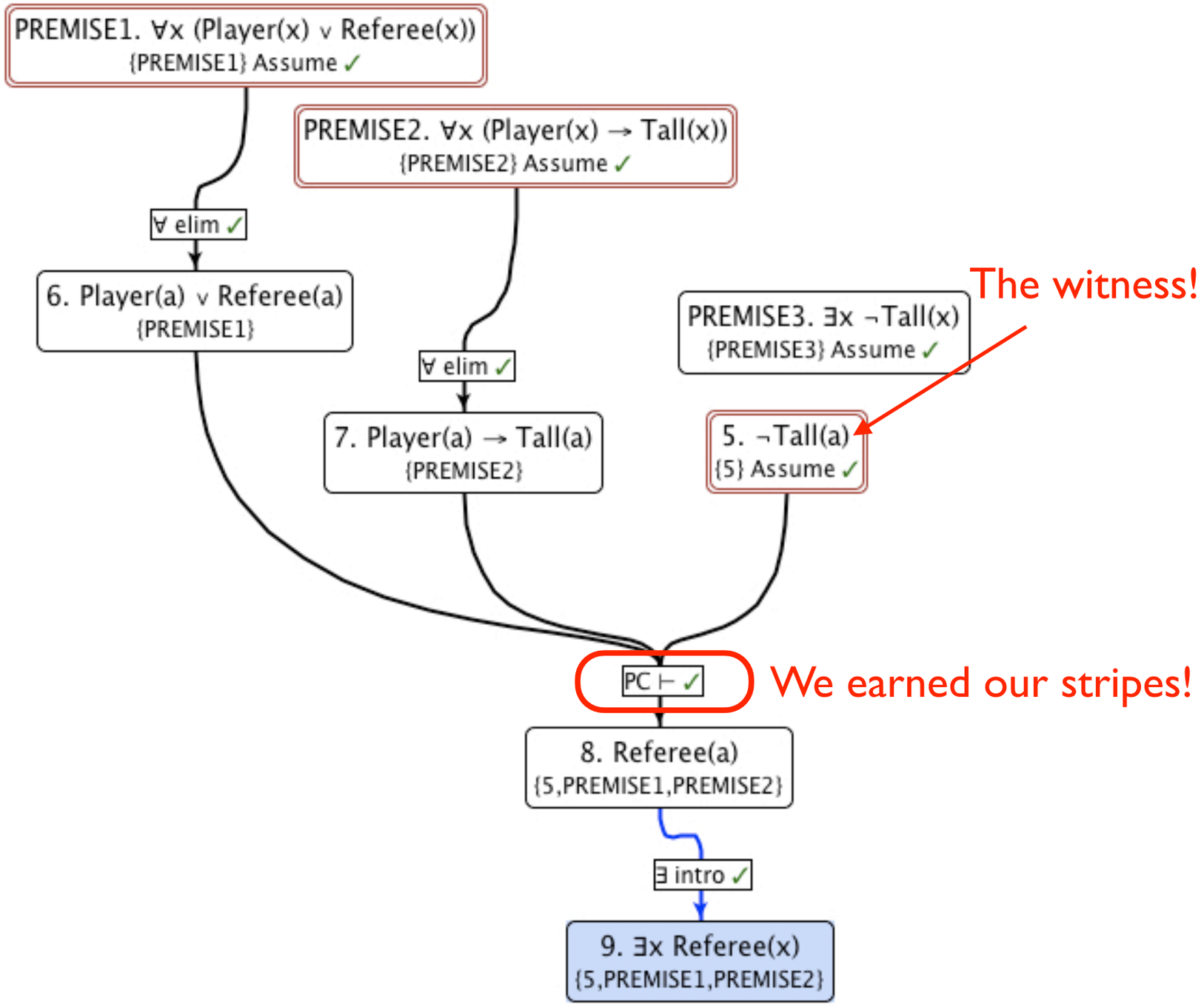
FOL ⊢ ✗

Step 5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PC ⊢ ✓
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8. Referee(a)
{5,PREMISE1,PREMISE2}
∃ intro ✓
9. ∃x Referee(x)
{5,PREMISE1,PREMISE2}
GOAL. ∃x Referee(x)
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Step 6


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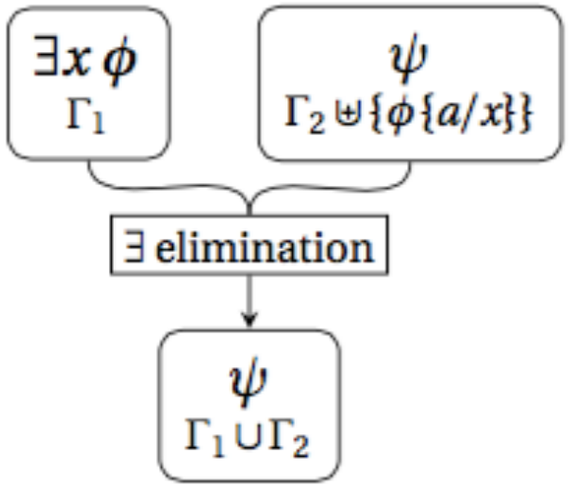
$\exists x\,\phi$
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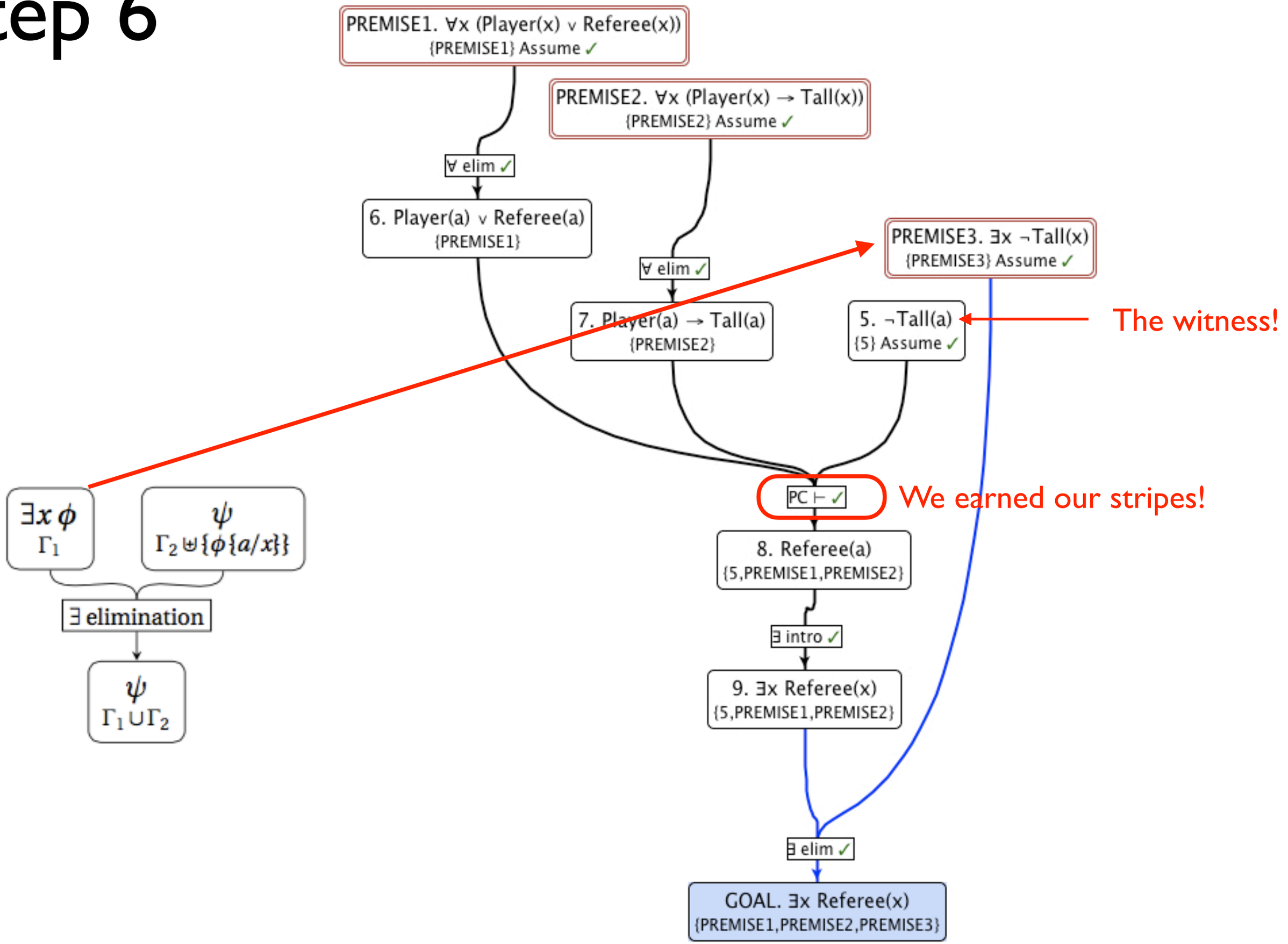
Step 6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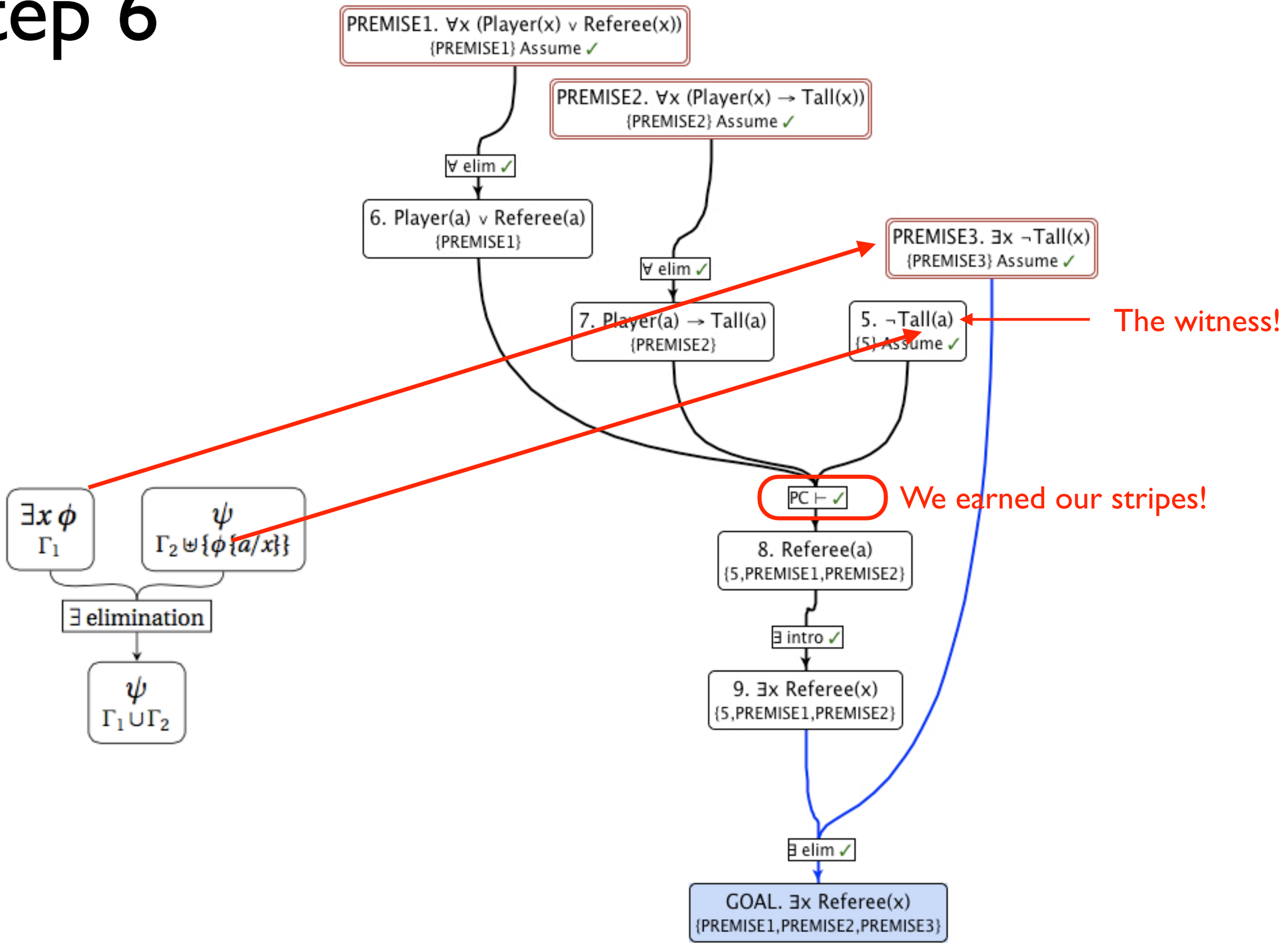
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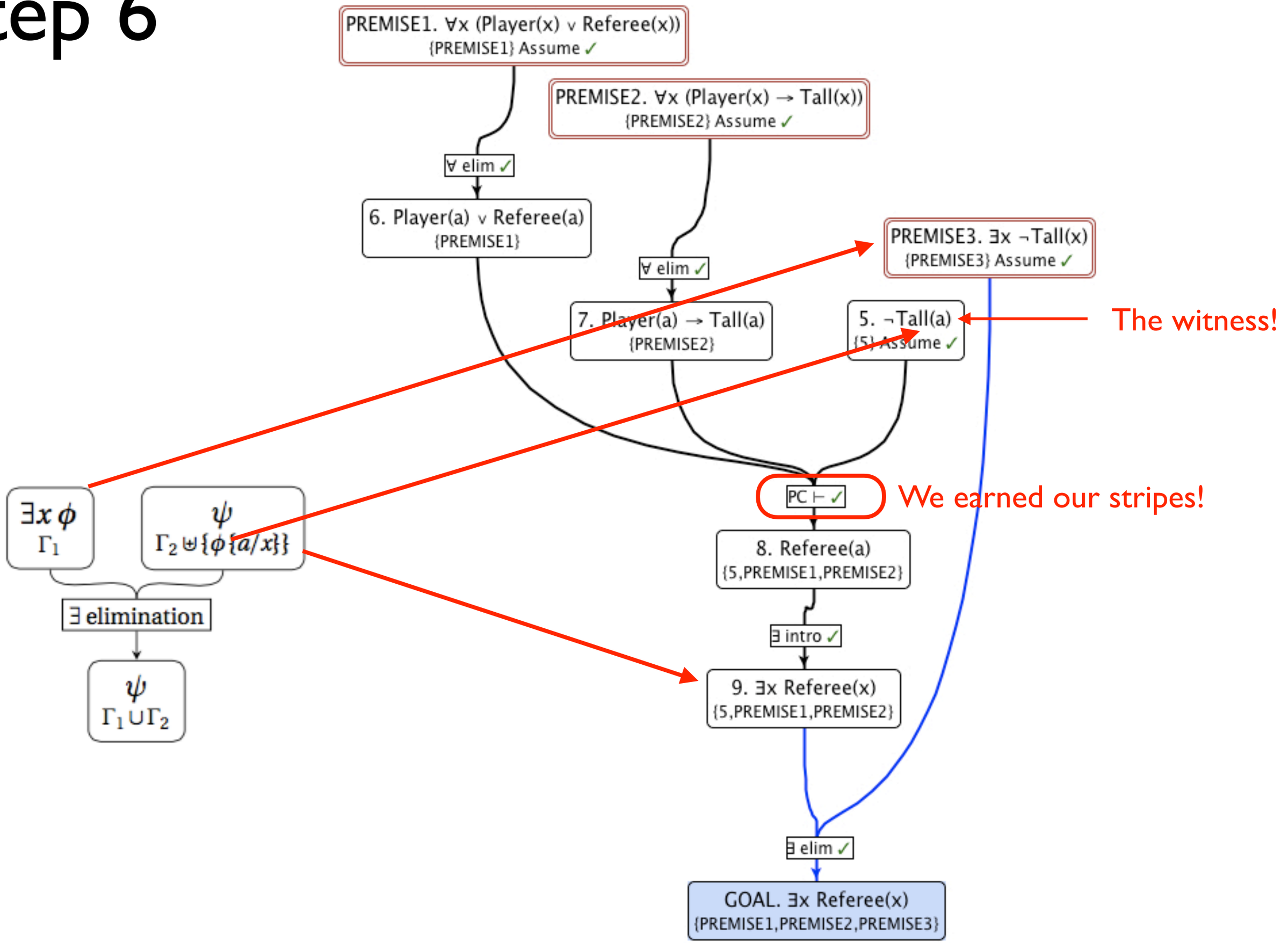
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{5,PREMISE1,PREMISE2}
∃ elim ✓
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{PREMISE1,PREMISE2,PREMISE3}
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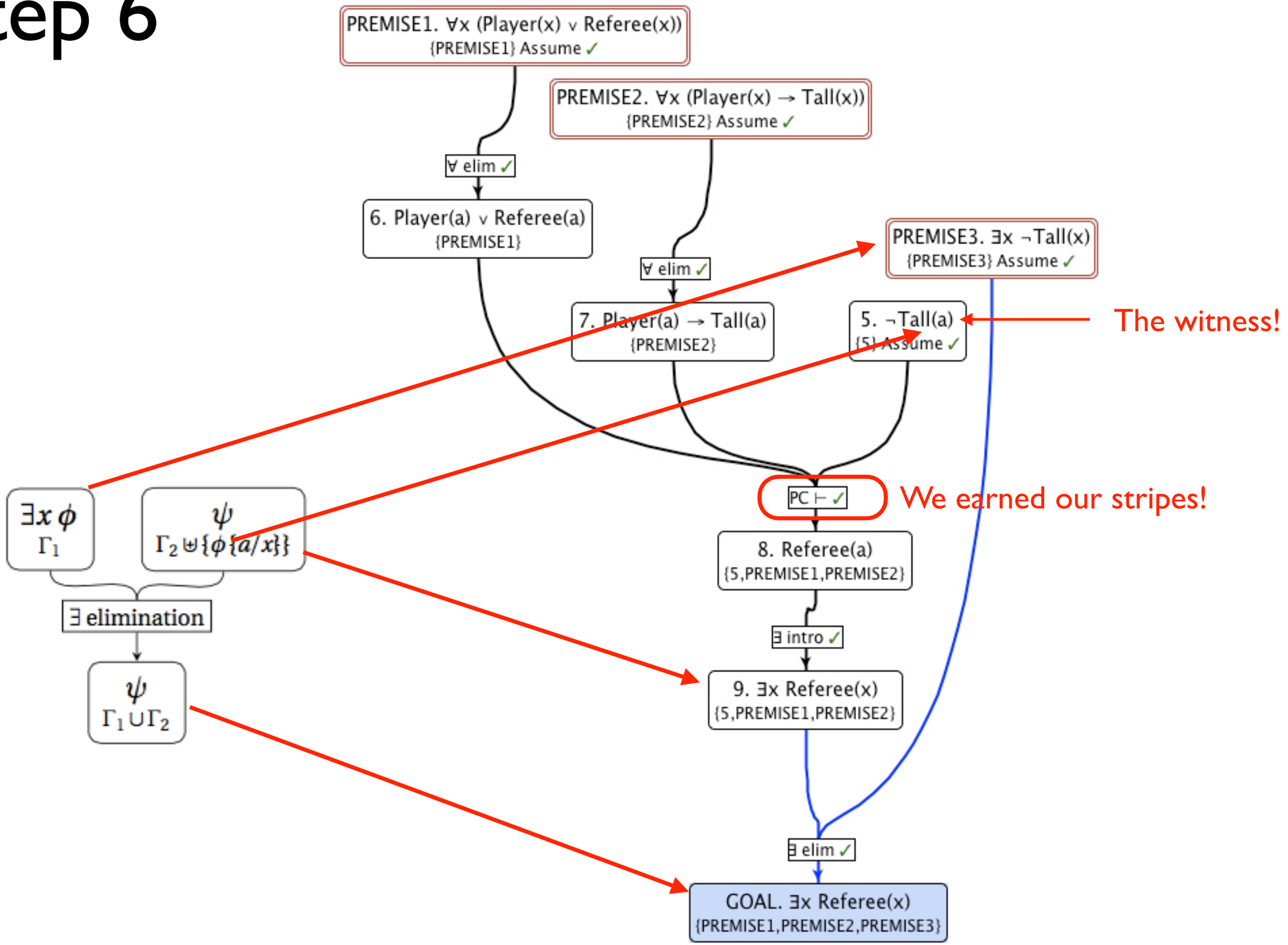
Step 6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{PREMISE1,PREMISE2,PREMISE3}
$\exists x\,\phi$
Γ_1
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ψ
$\Gamma_1 \cup \Gamma_2$

$$\{\forall \mathtt{x}(\mathtt{Scared}(\mathtt{x}) \leftrightarrow \mathtt{Small}(\mathtt{x})), \exists \mathtt{x} \neg \mathtt{Scared}(x)\} \vdash \exists \mathtt{x} \neg \mathtt{Small}(x)$$

$$\{\exists \mathtt{x}, \mathtt{y}\mathtt{Contiguous}(\mathtt{x}, \mathtt{y}), \forall \mathtt{x}, \mathtt{y}(\mathtt{Contiguous}(\mathtt{x}, \mathtt{y}) \rightarrow \neg \mathtt{SameCountry}(\mathtt{x}, \mathtt{y}))\} \vdash \exists \mathtt{x}, \mathtt{y} \neg \mathtt{SameCountry}(\mathtt{x}, \mathtt{y})$$