Exhortation; A Pop Problem; Truth Trees; FOL IV & Measuring Intelligence

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Intro to (Formal) Logic
3/14/2019

R A I R
Rensselaer AI and Reasoning Lab

Exhortation …

Make sure you're up-to-date today-ish, fully, on HyperGrader's "Required Problems."

New Pop Problem `FregTHEN2` now …

Truth Trees vs. Truth Tables

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Violent breakage between tabular calculation and proof construction.

Truth Trees vs. ~~Truth Tables~~

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LAMA's hypergraphs achieve seamless unification.

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First very simple: truth-tree for *modus ponens* …

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Violent breakage between tabular calculation and proof construction.

LAMA’s hypergraphs achieve seamless unification.

First very simple: truth-tree for *modus ponens* …

$\{P \rightarrow Q, P\} \vdash Q$

GIVEN1. P → Q
PC ⊢ ✗

GIVEN2. P
PC ⊢ ✗

$\{P \rightarrow Q, P\} \vdash Q$

GIVEN1. $P \rightarrow Q$

PC $\vdash$ ✗

GIVEN2. P

PC $\vdash$ ✗

NEGATED GOAL. $\neg Q$

PC $\vdash$ ✗

$\{P \rightarrow Q, P\} \vdash Q$

GIVEN1. $P \rightarrow Q$

PC $\vdash$ ✗

GIVEN2. P

PC $\vdash$ ✗

NEGATED GOAL. $\neg Q$

PC $\vdash$ ✗

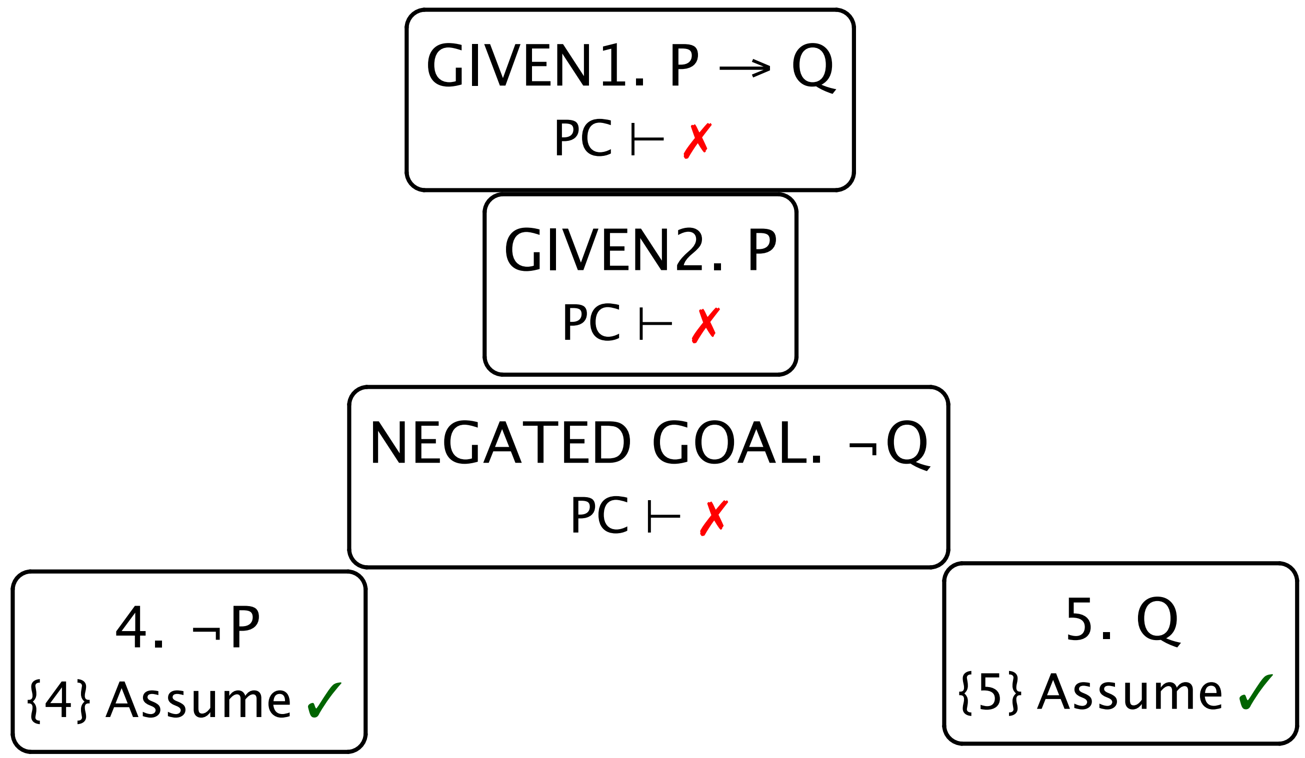
4. $\neg P$

{4} Assume ✓

5. Q

{5} Assume ✓

$\{P \rightarrow Q, P\} \vdash Q$



Either way, a contradiction!

Slightly Harder Truth Tree

Slightly Harder Truth Tree

$$\vdash (\mathtt{P} \rightarrow (\mathtt{Q} \rightarrow \mathtt{R})) \rightarrow ((\mathtt{P} \rightarrow \mathtt{Q}) \rightarrow (\mathtt{P} \rightarrow \mathtt{R}))$$

Slightly Harder Truth Tree

$$\vdash (\mathtt{P} \to (\mathtt{Q} \to \mathtt{R})) \to ((\mathtt{P} \to \mathtt{Q}) \to (\mathtt{P} \to \mathtt{R}))$$

(This is the axiom THEN-2 in Frege's (brutal) axiomatization of the propositional calculus.)

Frege

Slightly Harder Truth Tree

$$\vdash (\mathtt{P} \to (\mathtt{Q} \to \mathtt{R})) \to ((\mathtt{P} \to \mathtt{Q}) \to (\mathtt{P} \to \mathtt{R}))$$

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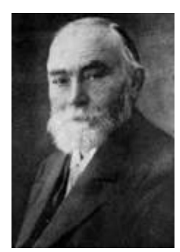

Frege

https://en.wikipedia.org/wiki/Frege%27s_propositional_calculus

Slightly Harder Truth Tree

$$\vdash (\mathtt{P} \to (\mathtt{Q} \to \mathtt{R})) \to ((\mathtt{P} \to \mathtt{Q}) \to (\mathtt{P} \to \mathtt{R}))$$

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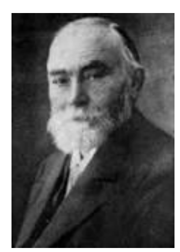

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The Rules of the Game!

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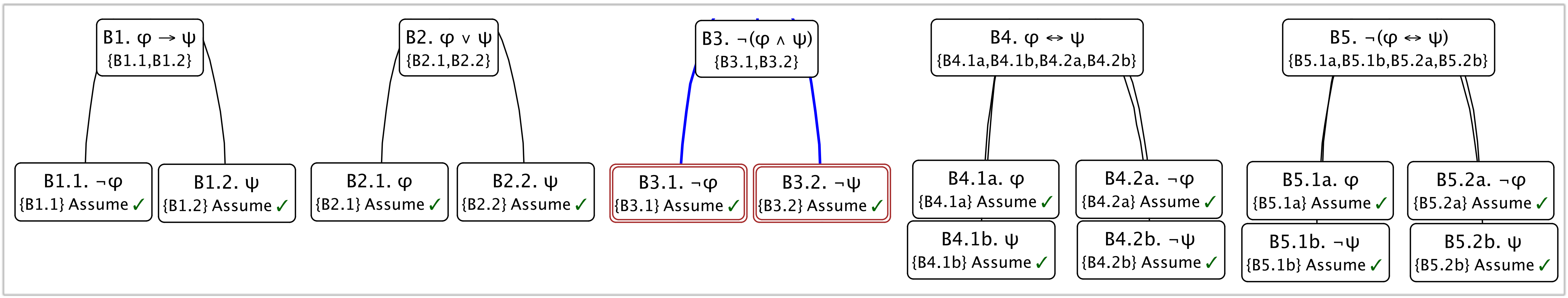

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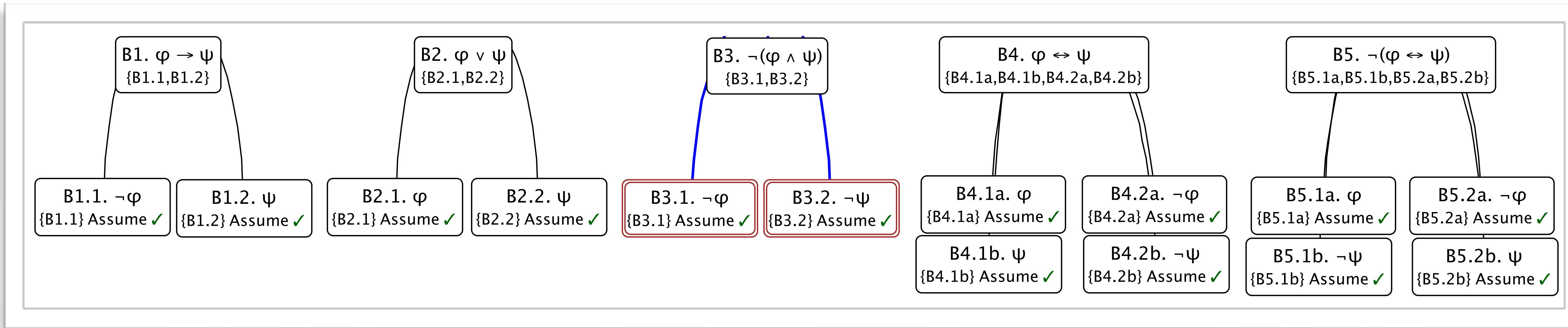

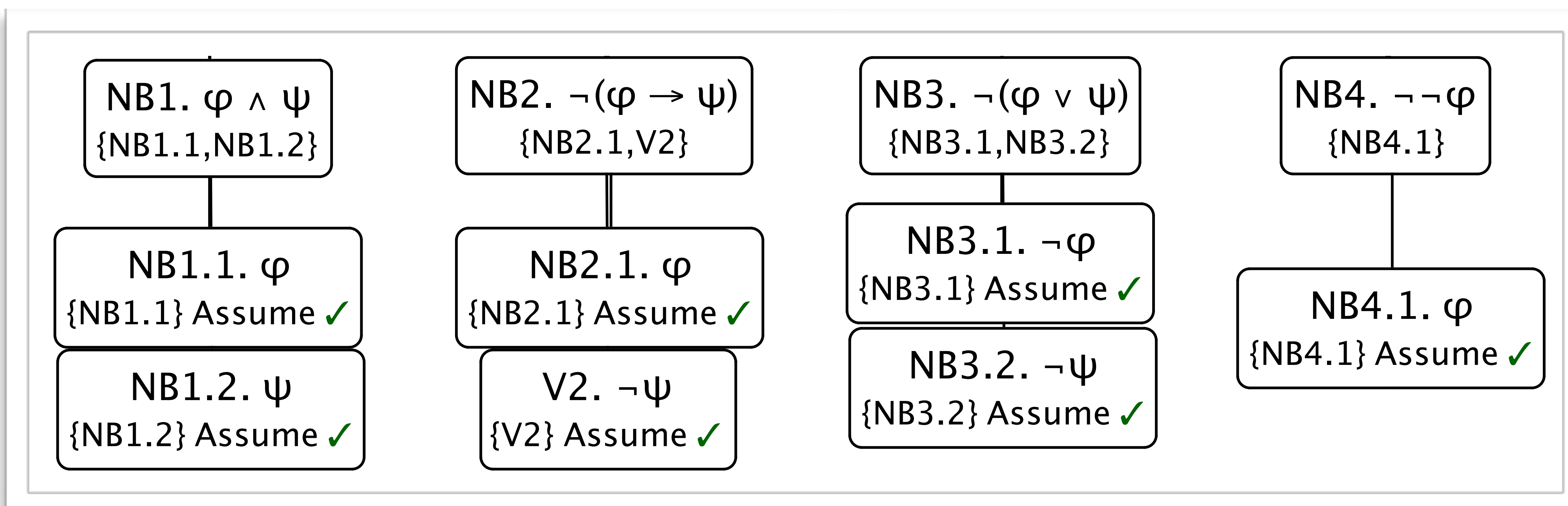

The Rules of the Game!

The Rules of the Game!

Questions?

Theorem?

Theorem?

Let ϕ be a theorem in the proposition calculus $= \mathcal{L}_{\mathrm{PC}}$.
Then the truth-tree algorithm will lead to no open branches.

Measuring Intelligence & AI/The Singularity

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The Singularity (superhuman machine intelligence) is near!!

Measuring Intelligence & AI/The Singularity

The Singularity (superhuman machine intelligence) is near!!

Is that so? And how are you measuring intelligence, pray tell?

Measuring Intelligence & AI/The Singularity

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Polynomial Hierarchy

Measuring Intelligence & AI/The Singularity

Polynomial Hierarchy

$$\mathbf{P} \subseteq \mathbf{NP} \subseteq \mathbf{PSPACE} = \mathbf{NPSPACE} \subseteq \mathbf{EXPTIME} \subseteq \mathbf{NEXPTIME} \subseteq \mathbf{EXPSPACE}$$

Measuring Intelligence & AI/The Singularity

Checkers:Chinook

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Measuring Intelligence & AI/The Singularity

Go:AlphaGo

Polynomial Hierarchy

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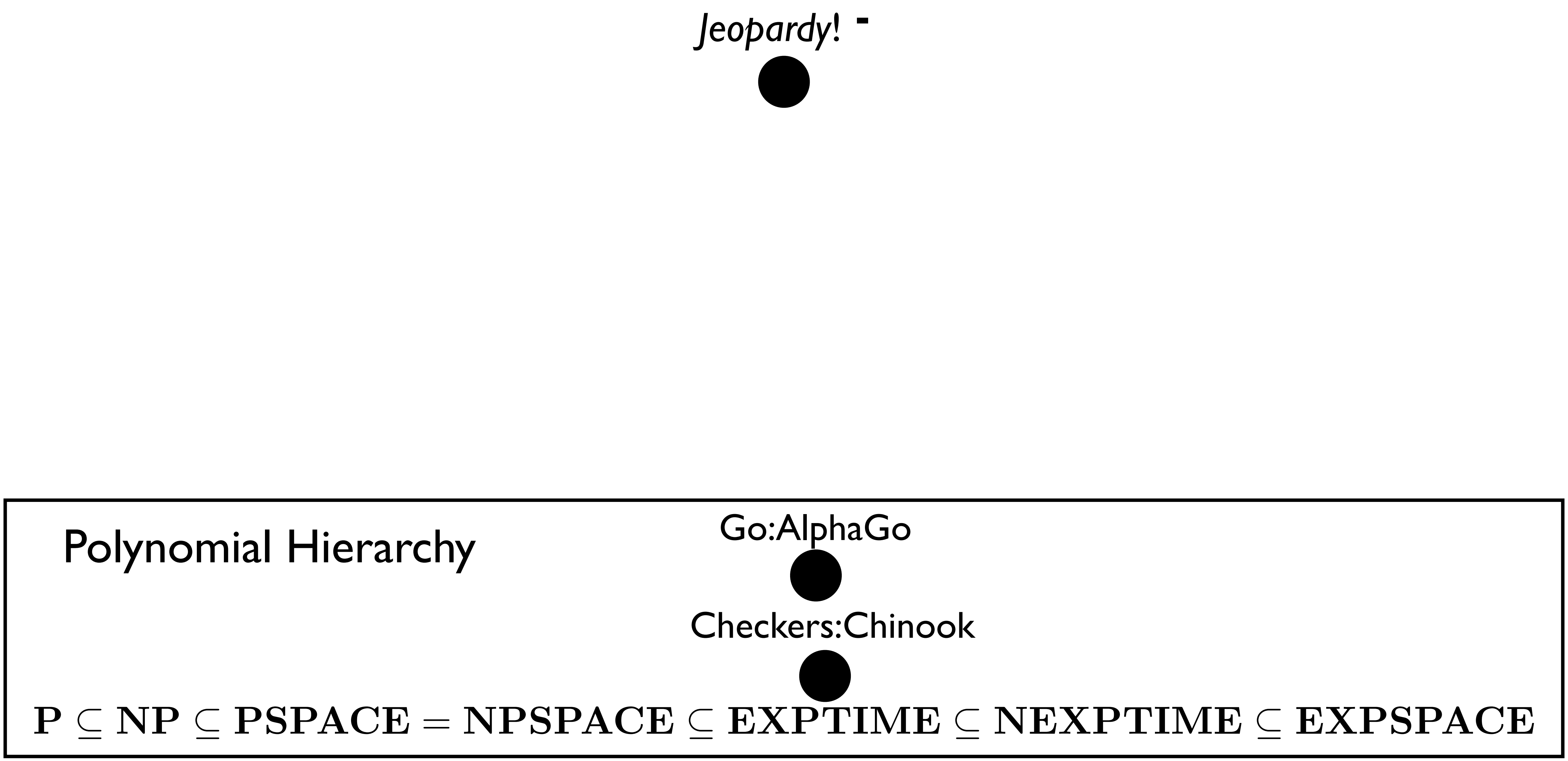
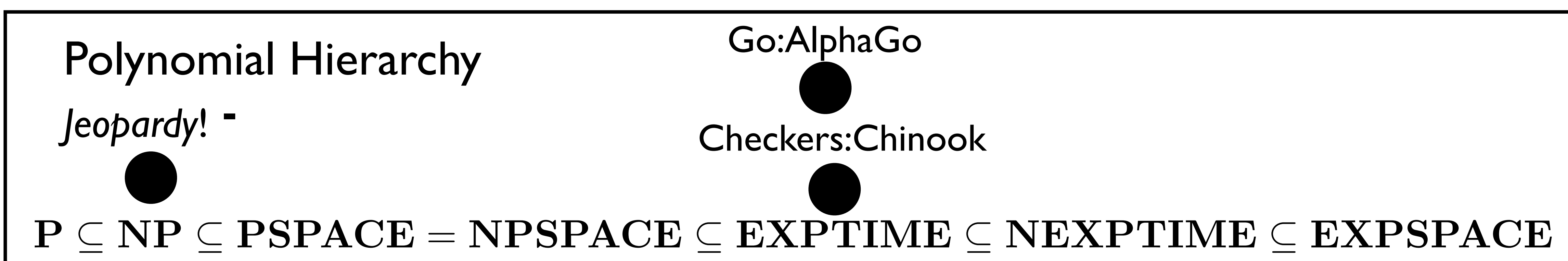


Measuring Intelligence & AI/The Singularity



Measuring Intelligence & AI/The Singularity

Go:AlphaGo

Polynomial Hierarchy

Jeopardy!

Checkers:Chinook

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Measuring Intelligence & AI/The Singularity

Arithmetical Hierarchy


Polynomial Hierarchy

Go:AlphaGo

Jeopardy! -

Checkers:Chinook

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Measuring Intelligence & AI/The Singularity

Arithmetical Hierarchy

$\vdots$

Π_2

Σ_2

Π_1

Σ_1

Σ_0

Polynomial Hierarchy

Go:AlphaGo

Jeopardy!

Checkers:Chinook

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Measuring Intelligence & AI/The Singularity

Arithmetical Hierarchy

$\vdots$

"Hey, do these two Java programs compute the very same function?" Π_2

Σ_2

Π_1

  Σ_1

Σ_0

Polynomial Hierarchy

Jeopardy!

Go:AlphaGo

Checkers:Chinook

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Measuring Intelligence & AI/The Singularity

Analytical Hierarchy

Arithmetical Hierarchy

$\vdots$

"Hey, do these two Java programs compute the very same function?" Π_2

Σ_2

Π_1

 

Σ_1

Σ_0

Polynomial Hierarchy

Go:AlphaGo

Jeopardy! -

Checkers:Chinook

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Measuring Intelligence & AI/The Singularity

Analytical Hierarchy

Arithmetical Hierarchy

This, all of this, is derived from consideration of first-order logic and second-order logic, with an emphasis on *quantification* and proof.

$\vdots$

“Hey, do these two Java programs compute the very same function?” Π_2

Σ_2

Π_1

 

Σ_1

Σ_0

Polynomial Hierarchy

Go:AlphaGo

Jeopardy! -

Checkers:Chinook

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An “Advanced” Topic for Measuring Intelligence …

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- FOL formulae that (only) enforce domain size:

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$$\exists x \exists y (x \neq y)$$

An "Advanced" Topic for Measuring Intelligence …

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$\exists x \exists y (x \neq y)$ at least two things

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$\exists x \exists y \exists z (x \neq y \wedge y \neq z \wedge x \neq z)$

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$$\exists x \exists y \exists z (x \neq y \land y \neq z \land x \neq z)$$ at least three things

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- FOL formulae that (only) enforce domain size:

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$$\vdots$$

$$\underline{\phi_n}$$

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$$\underline{\phi_n}$$ domain of at least *n* things

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$$\exists x \forall y (y = x)$$

An "Advanced" Topic for Measuring Intelligence …

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$$\exists x \exists y \exists z (x \neq y \wedge y \neq z \wedge x \neq z) \quad \text{at least three things}$$

$$\vdots$$

$$\underline{\phi_n} \qquad \text{domain of at least } n \text{ things}$$

$$\exists x \forall y (y = x) \quad \text{at most one thing}$$

An "Advanced" Topic for Measuring Intelligence …

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$\vdots$

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$\exists x \forall y (y = x)$ at most one thing

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An "Advanced" Topic for Measuring Intelligence …

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$$\vdots$$

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$\exists x_1 \exists x_2 \exists x_3 \forall y (y = x_1 \vee y = x_2 \vee y = x_3)$

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$\vdots$

$\underline{\phi_n}$ domain of at least *n* things

$\exists x \forall y (y = x)$ at most one thing

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$\exists x_1 \exists x_2 \exists x_3 \forall y (y = x_1 \lor y = x_2 \lor y = x_3)$ at most three things

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$$\exists x_1 \exists x_2 \exists x_3 \forall y (y = x_1 \lor y = x_2 \lor y = x_3)$$ at most three things

$$\vdots$$

$$\overline{\phi_n}$$

CLEVR

https://cs.stanford.edu/people/jcjohns/clevr/

Addition to RAIR-Lab Interoperability for AI …

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U.S. NAVAL RESEARCH LABORATORY

$\mathcal{CC}$

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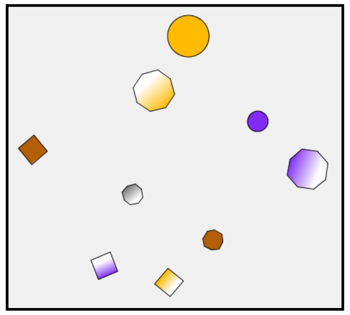
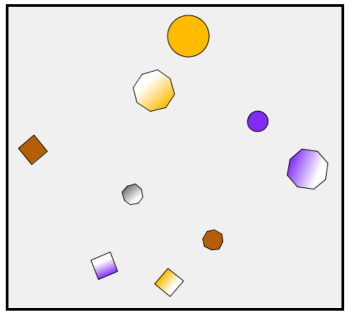


“AI, are there more than two spheres? Answer & justify.”

$\mathscr{C}\mathscr{C}$

. . .



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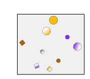

...

“AI, are there more than two spheres? Answer & justify.”

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$\mathcal{CC}$

Φ

$\cdots$

U.S. NAVAL RESEARCH LABORATORY

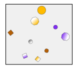
"AI, are there more than two spheres? Answer & justify."

RAIR
Rensselaer AI and Reasoning Lab

U.S. NAVAL RESEARCH LABORATORY

Φ

$\mathcal{CC}$



• • •

“AI, are there more than two spheres? Answer & justify.”

Rensselaer AI and Reasoning Lab

$\mathcal{C}\mathcal{C}$

Φ

$\ldots$

“AI, are there more than two spheres? Answer & justify.”

$$\mathcal{C}\mathcal{C}$$

$$\Phi'$$

$$\cdots$$

"AI, are there more than two spheres? Answer & justify."

Φ'

$\mathcal{CC}$

Φ

. . .

"AI, are there more than two spheres? Answer & justify."

AI: "Yes! And here's the proof."

Φ'

$\mathcal{CC}$

Φ

$\ldots$

"AI, are there more than two spheres? Answer & justify."

AI: "Yes! And here's the proof."

So we go from VQA to VQAJV!

$\mathcal{CC}$

Φ

. . .

Measuring AI Intelligence via (in part) Logic:Quantification

Toby Walsh: “The Singularity May Never Be Near”
(http://arxiv.org/pdf/1602.06462v1.pdf)

Measuring AI Intelligence via (in part) Logic:Quantification

Toby Walsh: “The Singularity May Never Be Near” (http://arxiv.org/pdf/1602.06462v1.pdf)

“I will not tackle here head on what we mean by measuring the intelligence of machines (or of humans). I will simply suppose there is such a property as intelligence, that it can be measured and compared, and that the technological singularity is when this measure increases exponentially fast in an appropriate and reasonable scale.” (p. 1)

But logico-mathematical definitions of intelligence for animals, humans, machines, aliens, gods … *are* possible; recall our consideration of the *Entscheidungsproblem*. We can specifically challenge today's AI on the basis of simple quantification and simple deduction …

First, need some numerical quantifiers:

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$$\exists x \forall y (y = x \land \phi(x)) \text{ will be } \exists^{=1} x \phi(x)$$

First, need some numerical quantifiers:

$$\exists x \forall y (y = x \wedge \phi(x)) \text{ will be } \exists^{=1} x \phi(x)$$

$$\exists x \exists y \exists z (x \neq y \wedge y \neq z \wedge x \neq z \wedge \phi(x, y, z)) \text{ will be } \exists^{\geq 3} x \phi(x)$$

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How do we define formulae of this type: $\exists^{=k} x \psi(x)$

First, need some numerical quantifiers:

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How do we define formulae of this type: $\exists^{=k} x \psi(x)$

How do we define formulae of this type: $\exists^{\leq n} x \psi(x)$

First, need some numerical quantifiers:

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$$\text{How do we define formulae of this type: } \exists^{=k} x \psi(x)$$

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$$\vdots$$

First, need some numerical quantifiers:

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$$\vdots$$

Okay, now AI:

First, need some numerical quantifiers:

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$$\text{How do we define formulae of this type: } \exists^{=k} x \psi(x)$$

$$\text{How do we define formulae of this type: } \exists^{\leq n} x \psi(x)$$

$$\vdots$$

Okay, now AI:

At least seven kenspeckle blookers are red.

First, need some numerical quantifiers:

$$\exists x \forall y (y = x \wedge \phi(x)) \text{ will be } \exists^{=1} x \phi(x)$$

$$\exists x \exists y \exists z (x \neq y \wedge y \neq z \wedge x \neq z \wedge \phi(x, y, z)) \text{ will be } \exists^{\geq 3} x \phi(x)$$

$$\text{How do we define formulae of this type: } \exists^{=k} x \psi(x)$$

$$\text{How do we define formulae of this type: } \exists^{\leq n} x \psi(x)$$

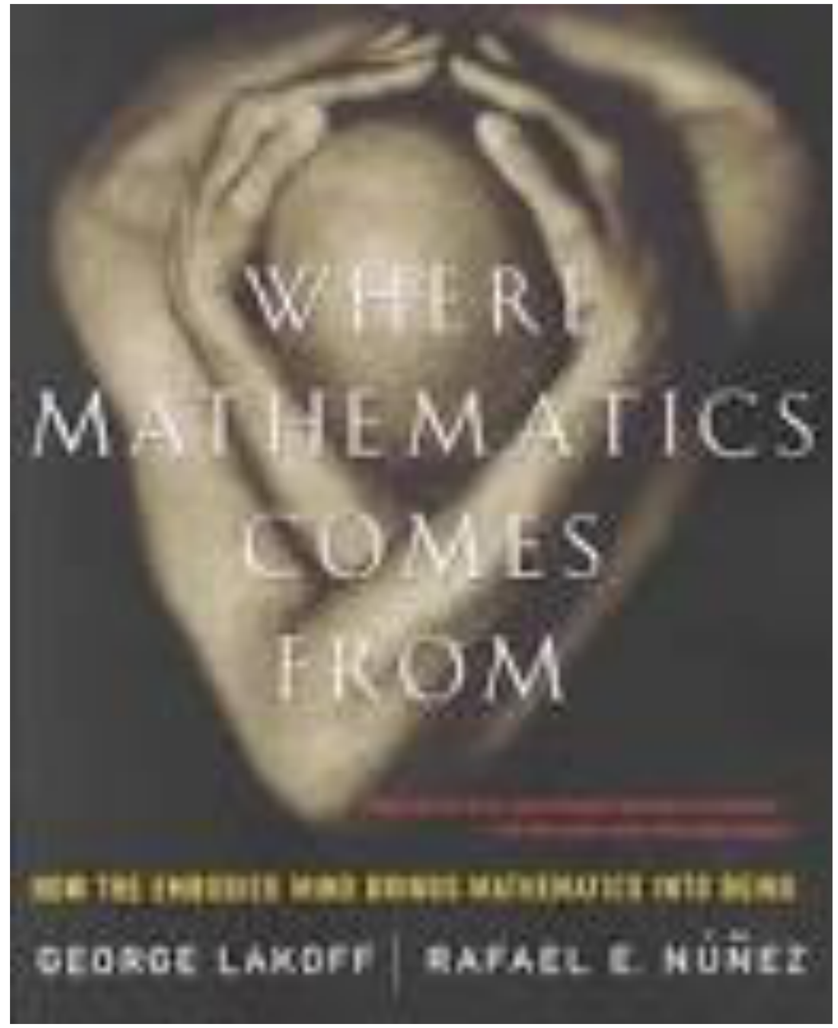
$$\vdots$$

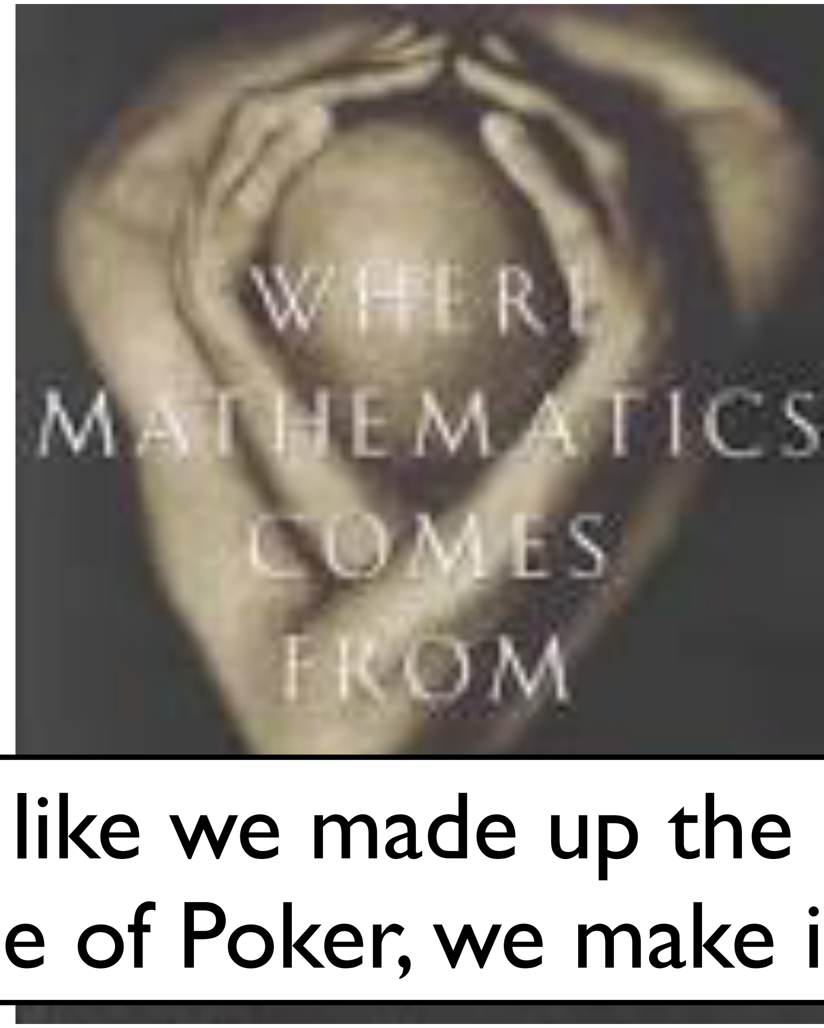
Okay, now AI:

At least seven kenspeckle blookers are red.

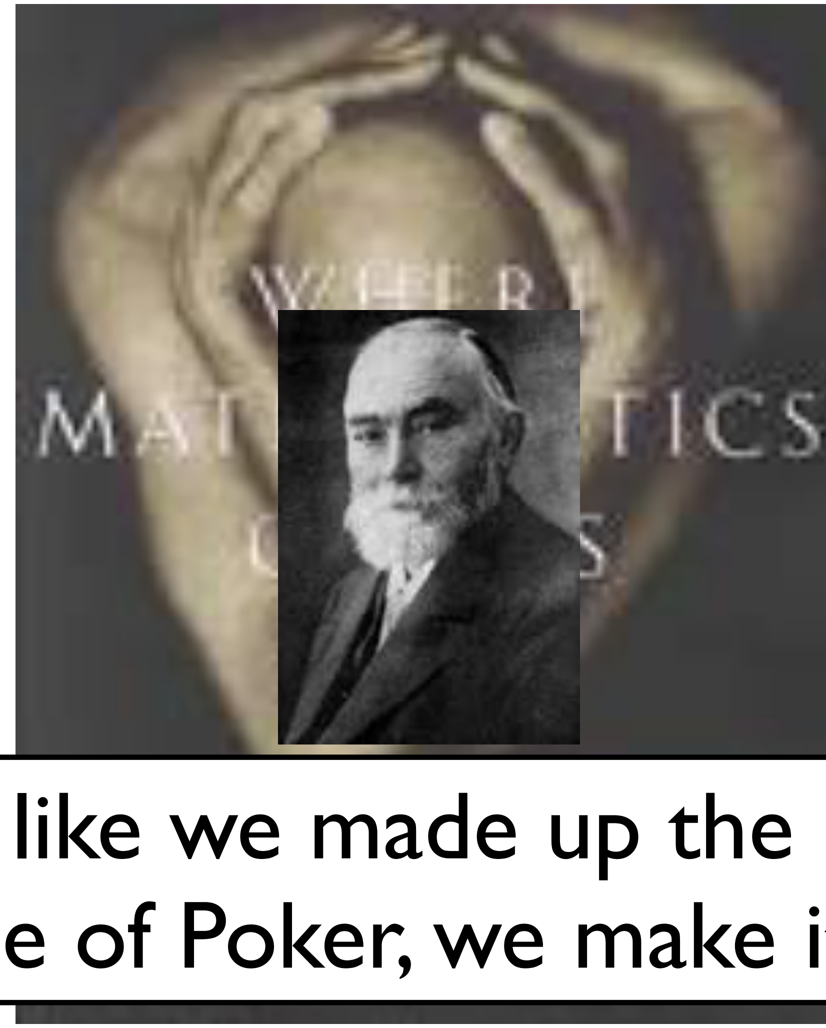
Given this, is it true that there are two red blookers? Why, exactly?

Coming: Astrologic …


WHERE
MATHEMATICS
COMES
FROM
HOW THE EMBODIED MIND BRINGS MATHEMATICS INTO BEING
GEORGE LAKOFF
RAFAEL E. NÚÑEZ



Just like we made up the game of Poker, we make it up!



Just like we made up the
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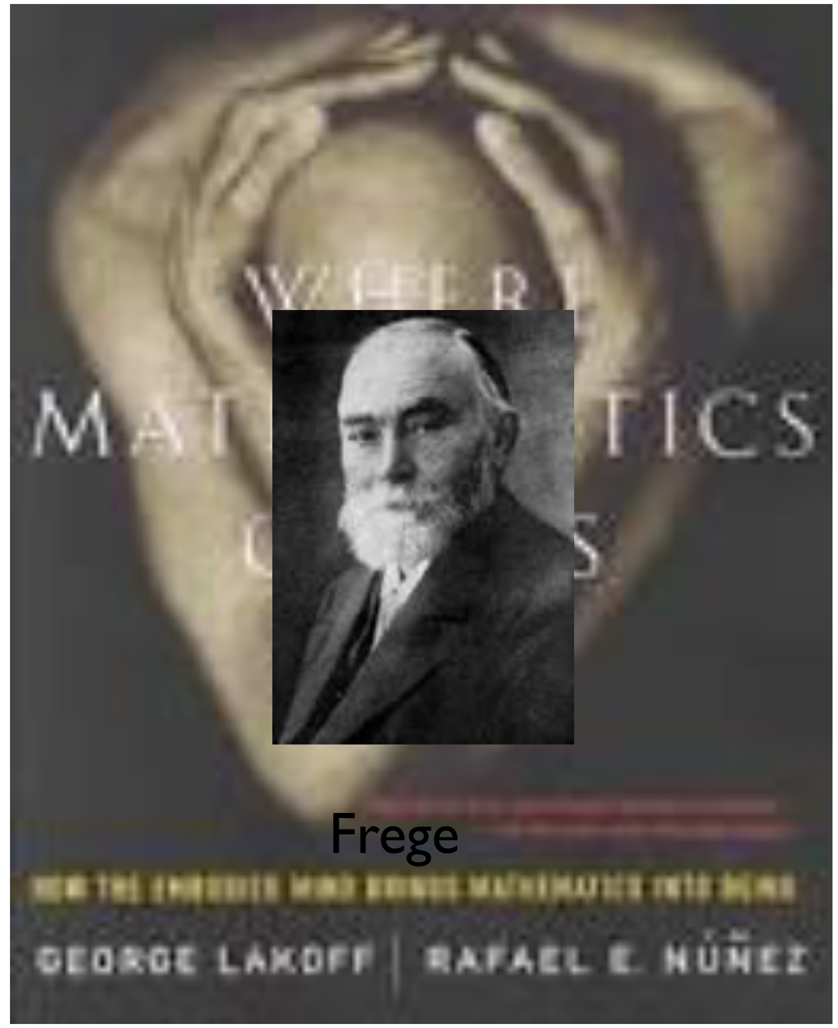
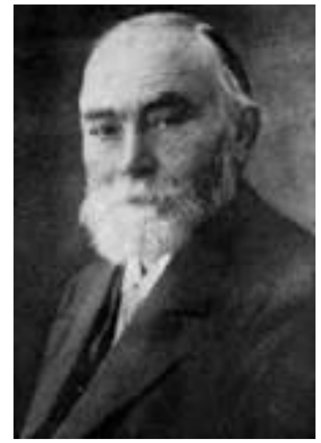

HOW THE EMBODIED MIND BRINGS MATHEMATICS INTO BEING
GEORGE LAKOFF
RAFAEL E. NÚÑEZ

Frege



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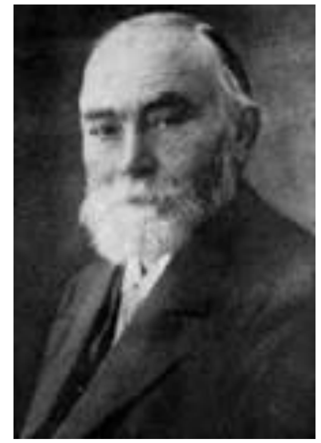

Frege

Don’t be silly. It comes from using logic for discovery, specifically from using …

FOL (which prominently includes *quantification*) + finding arithmetic (which models reality that any aliens —indeed, any sentient-and-truly-intelligent minds — must grasp) + …