Exhortation; A Pop Problem; Truth Trees; FOL IV & Measuring Intelligence

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Intro to (Formal) Logic 3/14/2019



Exhortation ...

Make sure you're up-to-date today-ish, fully, on HyperGrader's "Required Problems."

New Pop Problem FregTHEN2 now ...

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Violent breakage between tabular calculation and proof construction.

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Violent breakage between tabular calculation and proof construction.

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Violent breakage between tabular calculation and proof construction.

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First very simple: truth-tree for modus ponens ...

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LAMA's hypergraphs achieve seamless unification.

First very simple: truth-tree for modus ponens ...

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$$GIVEN1. P \rightarrow Q$$
$$PC \vdash \mathbf{X}$$
$$GIVEN2. P$$
$$PC \vdash \mathbf{X}$$

GIVEN1b. $P \rightarrow Q$ {GIVEN1b} Assume \checkmark







 $\{\mathtt{P} \to \mathtt{Q}, \mathtt{P} \models \mathtt{Q}$

-ζ



GIVEN1b. $P \rightarrow Q$
{GIVEN1b} Assume 🗸



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Either way, a contradiction!

GIVEN1b. $P \rightarrow Q$ {GIVEN1b} Assume \checkmark

 $\vdash (\mathtt{P} \to (\mathtt{Q} \to \mathtt{R})) \to ((\mathtt{P} \to \mathtt{Q}) \to (\mathtt{P} \to \mathtt{R}))$

$$\vdash (\mathtt{P} \to (\mathtt{Q} \to \mathtt{R})) \to ((\mathtt{P} \to \mathtt{Q}) \to (\mathtt{P} \to \mathtt{R}))$$

(This is the axiom THEN-2 in Frege's (brutal) axiomatization of the propositional calculus.)



Frege

$$\vdash (\mathtt{P} \to (\mathtt{Q} \to \mathtt{R})) \to ((\mathtt{P} \to \mathtt{Q}) \to (\mathtt{P} \to \mathtt{R}))$$

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Frege <u>https://en.wikipedia.org/wiki/Frege%27s_propositional_calculus</u>

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Frege <u>https://en.wikipedia.org/wiki/Frege%27s_propositional_calculus</u>













Questions?



Theorem?

Theorem?

Let ϕ be a theorem in the proposition calculus = \mathcal{L}_{PC} . Then the truth-tree algorithm will lead to no open branches.

The Singularity (superhuman machine intelligence) is near!!

The Singularity (superhuman machine intelligence) is near!!

Is that so? And how are you measuring intelligence, pray tell?

Is that so? And how are you measuring intelligence, pray tell?

Polynomial Hierarchy

Polynomial Hierarchy

 $\mathbf{P} \subseteq \mathbf{NP} \subseteq \mathbf{PSPACE} = \mathbf{NPSPACE} \subseteq \mathbf{EXPTIME} \subseteq \mathbf{NEXPTIME} \subseteq \mathbf{EXPSPACE}$

Checkers: Chinook

Polynomial Hierarchy

 $\mathbf{P} \subseteq \mathbf{NP} \subseteq \mathbf{PSPACE} = \mathbf{NPSPACE} \subseteq \mathbf{EXPTIME} \subseteq \mathbf{NEXPTIME} \subseteq \mathbf{EXPSPACE}$




























Analytical Hierarchy



Analytical Hierarchy

Arithmetical Hierarchy

This, all of this, is derived from consideration of first-order logic and second-order logic, with an emphasis on *quantification* and proof.

"Hey, do these two Java programs compute the very same function?"



Polynomial Hierarchy

Jeopardy! -

Checkers:Chinook

Go:AlphaGo

 \bullet $P \subseteq NP \subseteq PSPACE = NPSPACE \subseteq EXPTIME \subseteq NEXPTIME \subseteq EXPSPACE$

• FOL formulae that (only) enforce domain size:

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$$\exists x \exists y (x \neq y)$$

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CLEVR

https://cs.stanford.edu/people/jcjohns/clevr/

Addition to RAIR-Lab Interoperability for AI ...









"Al, are there more than two spheres? Answer & justify."









"Al, are there more than two spheres? Answer & justify."









"Al, are there more than two spheres? Answer & justify."








































Al: "Yes! And here's the proof."







Al: "Yes! And here's the proof."

So we go from VQA to VQAJV!





Rensselaer Al and Reasoning Lab

Measuring AI Intelligence via (in part) Logic:Quantification

Toby Walsh: "The Singularity May Never Be Near" (<u>http://arxiv.org/pdf/1602.06462v1.pdf</u>)

Measuring AI Intelligence via (in part) Logic:Quantification

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"I will not tackle here head on what we mean by measuring the intelligence of machines (or of humans). I will simply suppose there is such a property as intelligence, that it can be measured and compared, and that the technological singularity is when this measure increases exponentially fast in an appropriate and reasonable scale." (p. 1)

But logico-mathematical definitions of intelligence for animals, humans, machines, aliens, gods ... are possible; recall our consideration of the Entscheidungsproblem. We can specifically challenge today's AI on the basis of simple quantification and simple deduction ...

 $\exists x \forall y (y = x \land \phi(x)) \text{ will be } \exists^{=1} x \phi(x)$

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 $\exists x \exists y \exists z (x \neq y \land y \neq z \land x \neq z \land \phi(x, y, z)) \text{ will be } \exists^{\geq 3} x \phi(x)$

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How do we define formulae of this type: $\exists^{=k} x \psi(x)$

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Okay, now AI:

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> Okay, now AI: At least seven kenspeckle blookers are red.

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Okay, now AI:

At least seven kenspeckle blookers are red.

Given this, is it true that there are two red blookers? Why, exactly?

Coming: Astrologic ...





Just like we made up the game of Poker, we make it up!



Just like we made up the game of Poker, we make it up!





Frege



Frege

Don't be silly. It comes from using logic for discovery, specifically from using ...

FOL (which prominently includes *quantification*) + finding arithmetic (which models reality that any aliens —indeed, any sentient-and-trulyintelligent minds — must grasp) + ...