Propositional Calculus I: The Formal Language, Rules of Inference (initial), Application to Some Motivating Problems

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Intro to (Formal) Logic
1/28/2019
Re-re-re...orientation w.r.t. web pages ...
The Starting Code Purchased in Bookstore Should By Now’ve Been Used to Register & Subsequently Sign In

Your code for starting the registration process is:

To access HyperGrader, HyperSlate, the license agreement, and to obtain the textbook LAMA-BDLA, go to::

https://rpi.logicamodernapproach.com
How’d We Arrive Here?
(Selmer’s Leibnizian Whirlwind History of Logic)

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Intro to Logic
1/24/2019
How’d We Arrive Here?
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Intro to Logic
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Questions/comments/objections ...?
Micro-homily:
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skipping to ~ p. 34!
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M. Chi: Self-testers end up being self-made.
Micro-homily:

 skipping to ~ p. 34!

M. Chi: Self-testers end up being self-made.
Micro-homily:

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M. Chi: Self-testers end up being self-made.

“What category of English sentences does logic focus on?”
## Chapter 2. Propositional Calculus

<table>
<thead>
<tr>
<th>Syntax</th>
<th>Formula Type</th>
<th>Sample Representation</th>
</tr>
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<tbody>
<tr>
<td>$P, P_1, P_2, Q, Q_1, \ldots$</td>
<td>Atomic Formulas</td>
<td>“Larry is lucky.” as $L_l$</td>
</tr>
<tr>
<td>$\neg \phi$</td>
<td>Negation</td>
<td>“Gary isn’t lucky.” as $\neg L_g$</td>
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<tr>
<td>$\phi_1 \wedge \ldots \wedge \phi_n$</td>
<td>Conjunction</td>
<td>“Both Larry and Carl are lucky.” as $L_l \wedge L_c$</td>
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<td>$\phi_1 \vee \ldots \vee \phi_n$</td>
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<td>“Either Billy is lucky or Alvin is.” as $L_b \vee L_a$</td>
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<td>$\phi \rightarrow \psi$</td>
<td>Conditional (Implication)</td>
<td>“If Ron is lucky, so is Frank.” as $L_r \rightarrow L_f$</td>
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<tr>
<td>$\phi \leftrightarrow \psi$</td>
<td>Biconditional (Coimplication)</td>
<td>“Tim is lucky if and only if Kim is.” as $L_t \leftrightarrow L_k$</td>
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Table 2.1: Syntax of the Propositional Calculus. Note that $\phi$, $\psi$, and $\phi_i$ stand for arbitrary formulas.
The Formal Language

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Table 2.1: Syntax of the Propositional Calculus. Note that $\phi$, $\psi$, and $\phi_i$ stand for arbitrary formulas.

Exercise: Is this language Roger-decidable? Prove it!
“NYS I” Revisited

Given the statements

\[ \neg a \lor \neg b \]
\[ b \]
\[ c \rightarrow a \]

which one of the following statements must also be true?

\[ c \]
\[ \neg b \]
\[ \neg c \]
\[ h \]
\[ a \]
\[ \text{none of the above} \]
"NYS I" Revisited

Given the statements

\( \neg a \lor \neg b \)

b

c \rightarrow a

which one of the following statements must also be true?

c
\( \neg b \)
\( \neg c \)
h
a

none of the above
Our First Rule of Inference: PC (Entailment) Oracle
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“NYS 3” Revisited

Given the statements
¬¬c
¬a → a
¬(d ∨ e)

which one of the following statements must also be true?

¬c
e
h
¬a
all of the above
“NYS 3” Revisited

Given the statements
\( \neg c \)
\( c \rightarrow a \)
\( \neg a \lor b \)
\( b \rightarrow d \)
\( \neg (d \lor e) \)

which one of the following statements must also be true?

\( \neg c \)
\( e \)
\( h \)
\( \neg a \)
all of the above
“NYS 3” Revisited

Given the statements
\neg \neg c
\neg a \lor b
b \rightarrow d
\neg (d \lor e)

Show in HyperSlate that each of the first four options can be proved using the PC entailment oracle.

which one of the following statements must also be true?

\neg c
e
h
\neg a
all of the above