

Propositional Calculus I: The Formal Language, Rules of Inference (initial), Application to Some Motivating Problems

Selmer Bringsjord

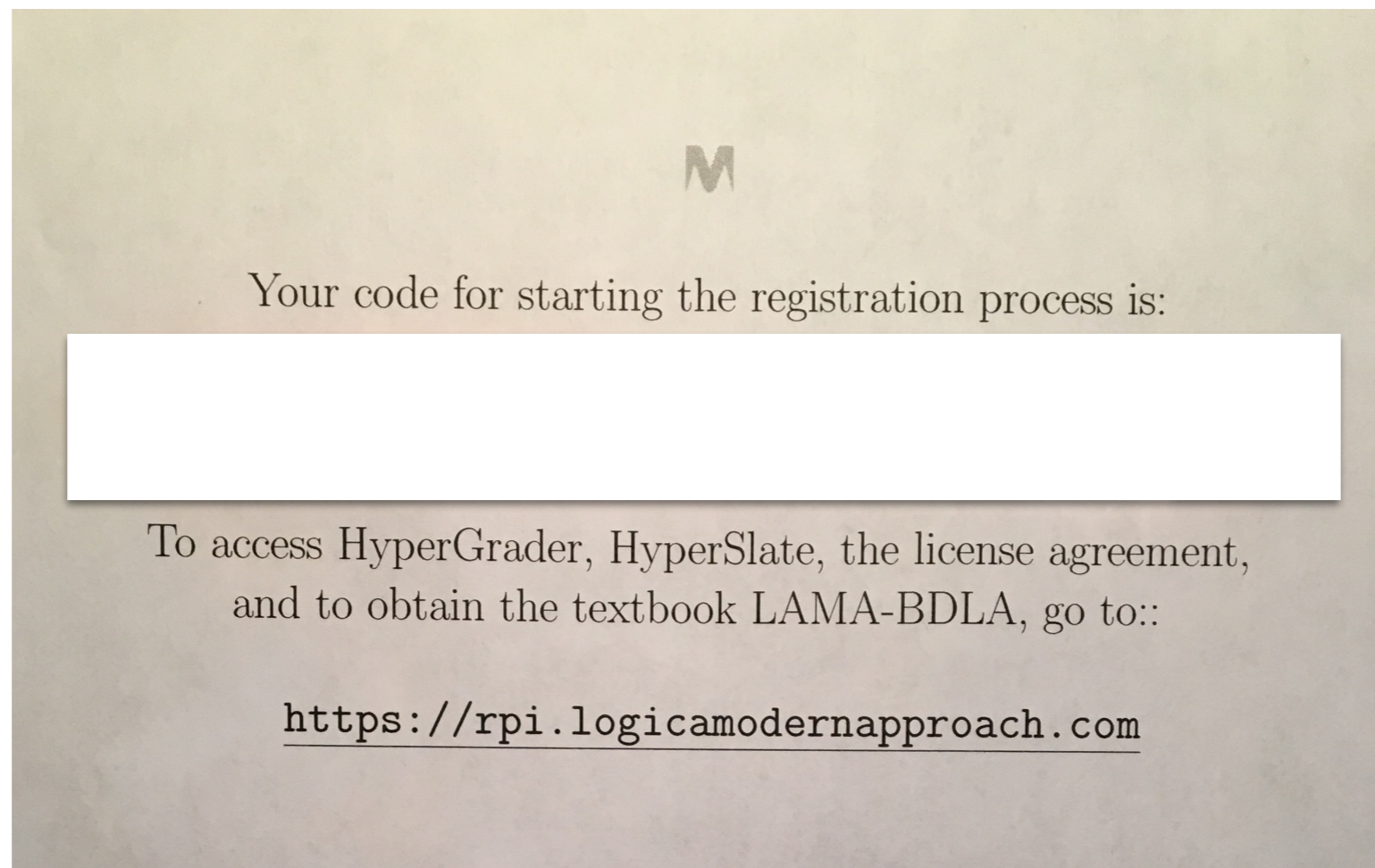
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Intro to (Formal) Logic
1/28/2019



Re-re-re...orientation w.r.t. web pages ...

The Starting Code Purchased in Bookstore Should By Now've Been Used to Register & Subsequently Sign In



How'd We Arrive Here?

(Selmer's Leibnizian Whirlwind History of Logic)

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Questions/comments/objections ...?

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Micro-homily:

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skipping to ~ p. 34!

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M. Chi: Self-testers end up being self-made.

Micro-homily:

skipping to ~ p. 34!



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Micro-homily:

skipping to ~ p. 34!



M. Chi: Self-testers end up being self-made.

“What category of English sentences does logic focus on?”

The Formal Language

CHAPTER 2. PROPOSITIONAL CALCULUS

Syntax	Formula Type	Sample Representation
$P, P_1, P_2, Q, Q_1, \dots$	Atomic Formulas	"Larry is lucky." as L_l
$\neg\phi$	Negation	"Gary isn't lucky." as $\neg L_g$
$\phi_1 \wedge \dots \wedge \phi_n$	Conjunction	"Both Larry and Carl are lucky." as $L_l \wedge L_c$
$\phi_1 \vee \dots \vee \phi_n$	Disjunction	"Either Billy is lucky or Alvin is." as $L_b \vee L_a$
$\phi \rightarrow \psi$	Conditional (Implication)	"If Ron is lucky, so is Frank." as $L_r \rightarrow L_f$
$\phi \leftrightarrow \psi$	Biconditional (Coimplication)	"Tim is lucky if and only if Kim is." as $L_t \leftrightarrow L_k$

Table 2.1: Syntax of the Propositional Calculus. Note that ϕ , ψ , and ϕ_i stand for arbitrary formulas.

The Formal Language

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Exercise: Is this language Roger-decidable? Prove it!

“NYS I” Revisited

Given the statements

$$\neg a \vee \neg b$$

b

$$c \rightarrow a$$

which one of the following statements must also be true?

c

$\neg b$

$\neg c$

h

a

none of the above

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Given the statements

$$\neg a \vee \neg b$$

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$$c \rightarrow a$$

which one of the following statements must also be true?

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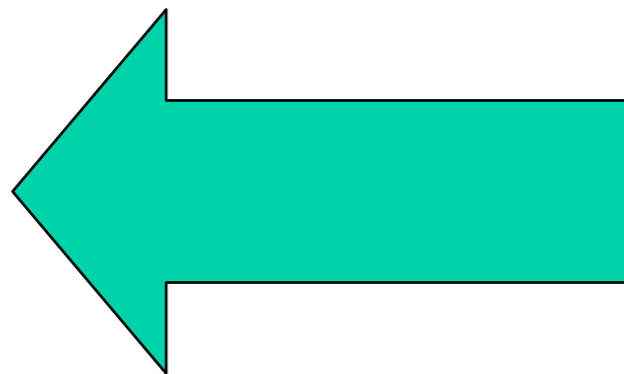
$\neg b$

$\neg c$

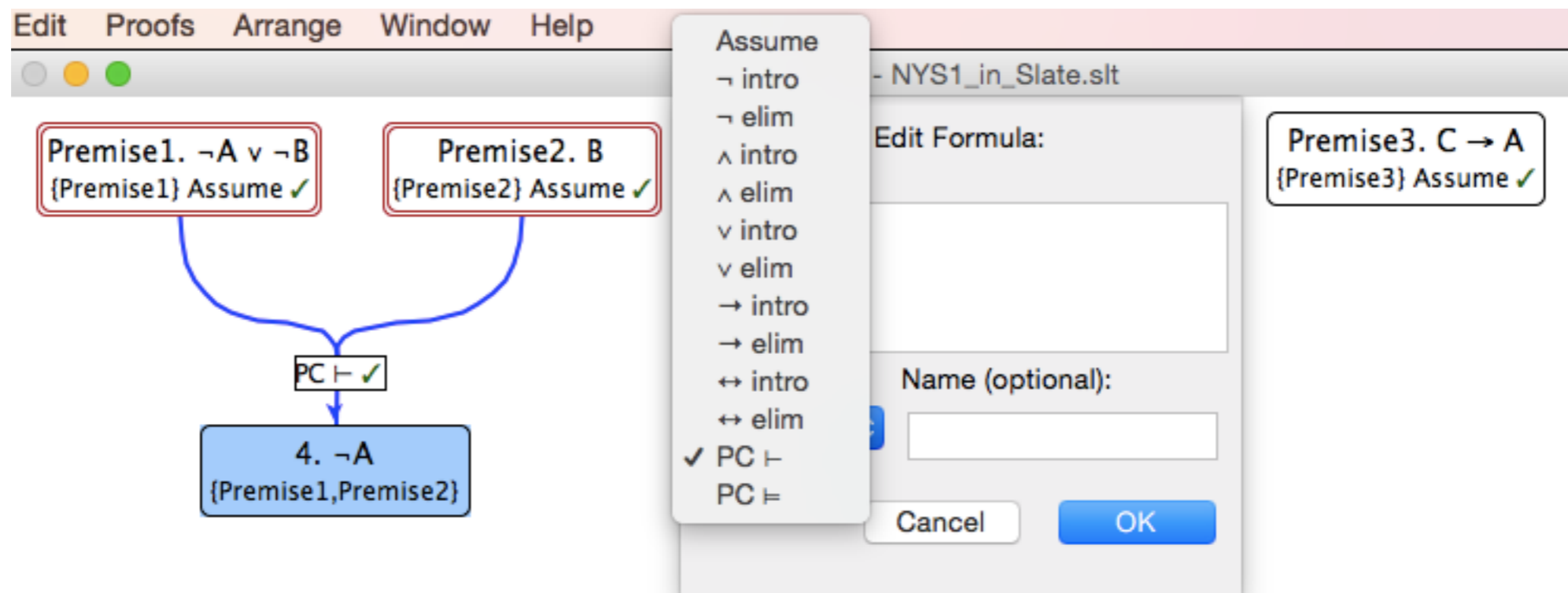
h

a

none of the above



Our First Rule of Inference: PC (Entailment) Oracle



Our First Rule of Inference: PC (Entailment) Oracle

The screenshot shows a proof editor window titled "NYS1_in_Slate.slt" with a menu bar containing "Edit", "Proofs", "Arrange", "Window", and "Help". The main workspace displays a proof tree with three nodes:

- Premise1.** $\neg A \vee \neg B$
{Premise1} Assume ✓
- Premise2.** B
{Premise2} Assume ✓
- 4.** $\neg A$
{Premise1, Premise2}

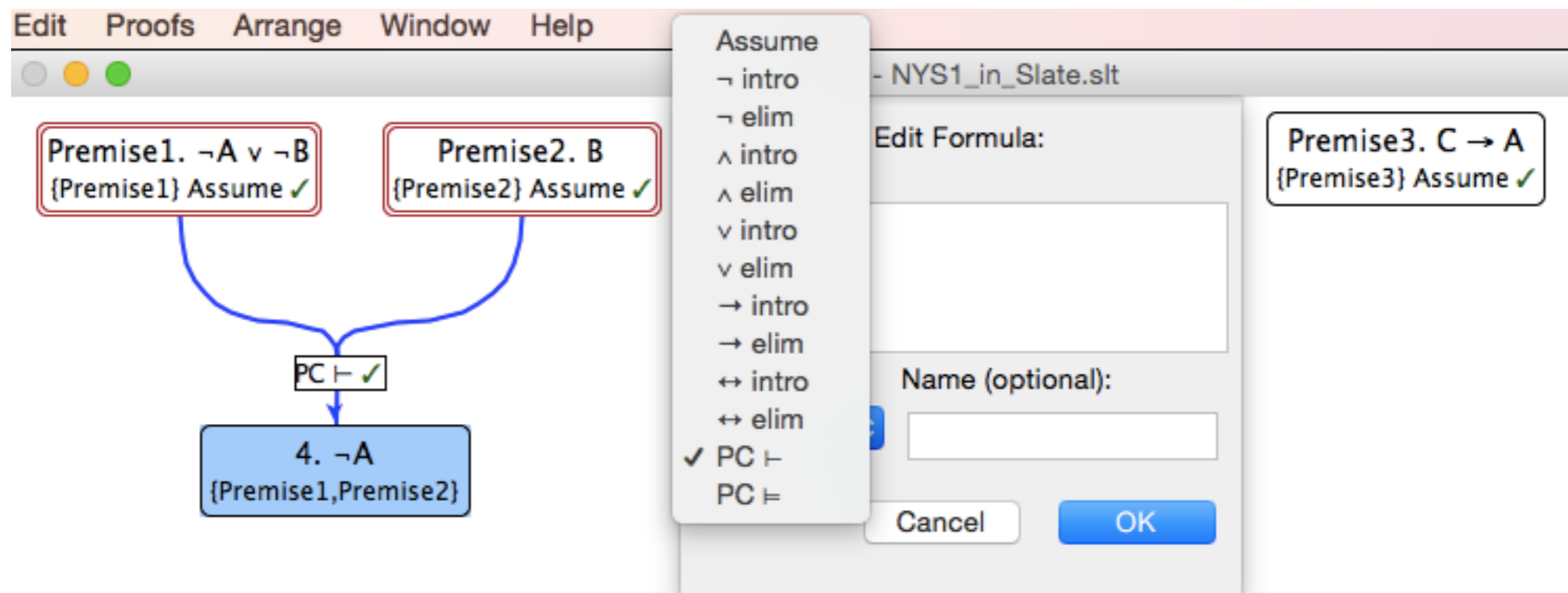
Blue arrows indicate that Premise1 and Premise2 are used to derive the conclusion 4. A small box labeled "PC ⊢ ✓" is positioned between the premises and the conclusion, indicating the rule used for the inference.

An "Edit Formula:" dialog box is open, showing a list of inference rules. The rule "PC ⊢" is highlighted with a red circle. Other visible rules include:

- Assume
- \neg intro
- \neg elim
- \wedge intro
- \wedge elim
- \vee intro
- \vee elim
- \rightarrow intro
- \rightarrow elim
- \leftrightarrow intro
- \leftrightarrow elim

The dialog box also includes a "Name (optional):" field and "Cancel" and "OK" buttons. To the right of the dialog, another node is visible: "Premise3. $C \rightarrow A$ " with "{Premise3} Assume ✓".

Our First Rule of Inference: PC (Entailment) Oracle



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Our First Rule of Inference: PC (Entailment) Oracle

“NYS 3” Revisited

Given the statements

$\neg\neg c$

$c \rightarrow a$

$\neg a \vee b$

$b \rightarrow d$

$\neg(d \vee e)$

which one of the following statements must also be true?

$\neg c$

e

h

$\neg a$

all of the above

“NYS 3” Revisited

Given the statements

$\neg\neg c$

$c \rightarrow a$

$\neg a \vee b$

$b \rightarrow d$

$\neg(d \vee e)$

which one of the following statements must also be true?

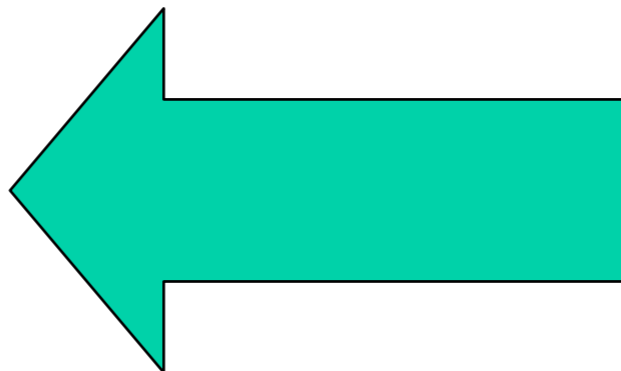
$\neg c$

e

h

$\neg a$

all of the above



“NYS 3” Revisited

Given the statements

$\neg\neg c$

$c \rightarrow a$

$\neg a \vee b$

$b \rightarrow d$

$\neg(d \vee e)$

Show in HyperSlate that each of the first four options can be proved using the PC entailment oracle.

which one of the following statements must also be true?

$\neg c$

e

h

$\neg a$

all of the above

