Propositional Calculus II:

More Rules of Inference, Application to Additional Motivating Problems

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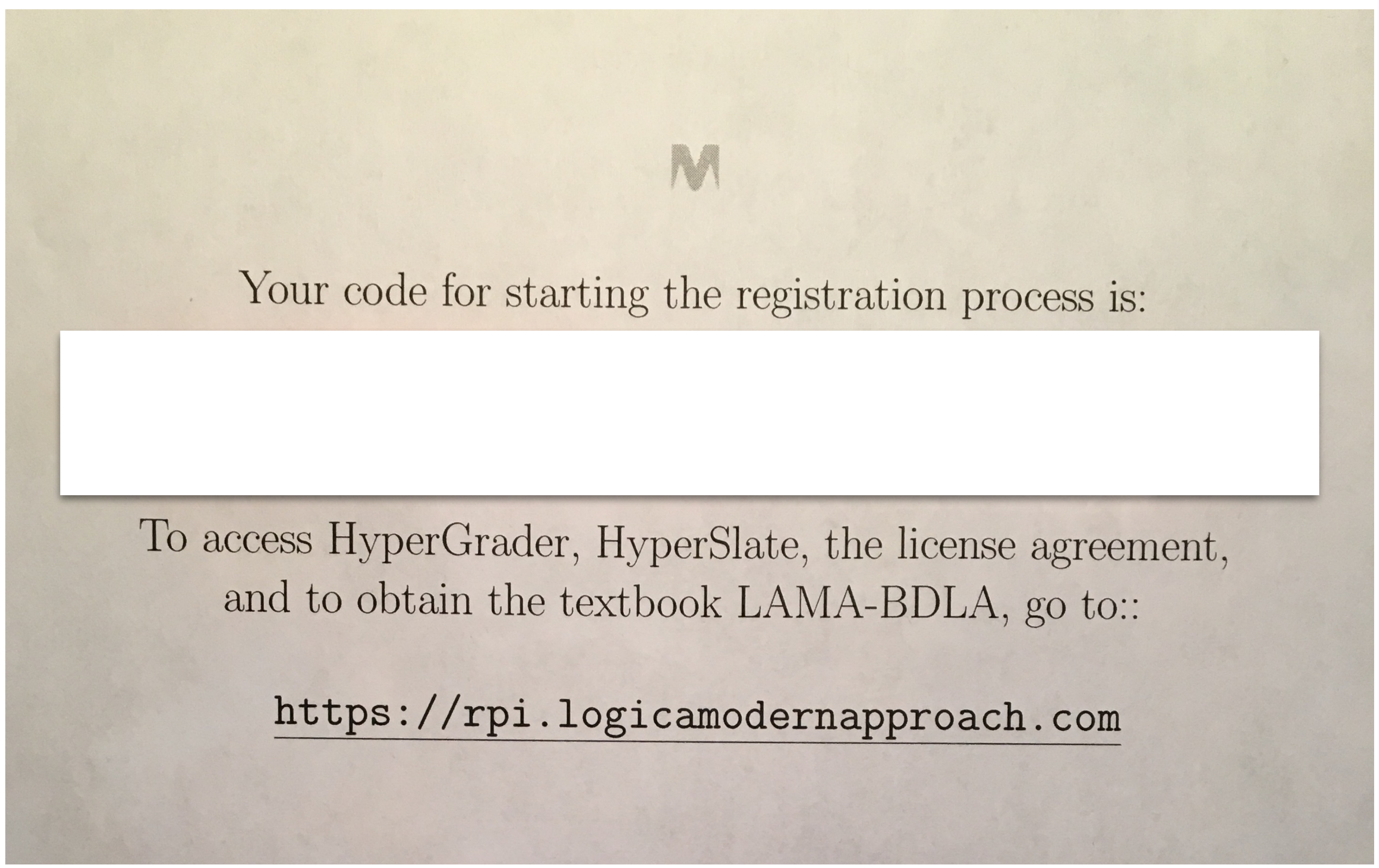
Intro to Logic
1/31/2019

R A I R
Rensselaer AI and Reasoning Lab

Re-re-re…orientation w.r.t. web pages …

The Starting Code Purchased in Bookstore Should By Now've Been Used to Register & Subsequently Sign In

Assume everyone has done switching_conjuncts_fine!



Your code for starting the registration process is:

To access HyperGrader, HyperSlate, the license agreement, and to obtain the textbook LAMA-BDLA, go to::

https://rpi.logicamodernapproach.com

E-Housekeeping Pts

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- Make sure OS fully up-to-date.

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- Chrome best (but I use Safari).

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- Always work in the same browser window with multiple tabs; must do this with email and HyperGrader & HyperSlate.

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Intro to Logic
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Last time we introduced and and lauded the power of **oracles**, and questions … and now … picking up where we left off …

“NYS 3” Revisited

Given the statements

$\neg\neg c$

$c \rightarrow a$

$\neg a \vee b$

$b \rightarrow d$

$\neg(d \vee e)$

which one of the following statements must also be true?

$\neg c$

e

h

$\neg a$

all of the above

“NYS 3” Revisited

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“NYS 3” Revisited

Given the statements

$\neg\neg c$

$c \rightarrow a$

$\neg a \vee b$

$b \rightarrow d$

$\neg(d \vee e)$

After last class, should have done …
Exercise: Show in HyperSlate™ that each of the first four options can be proved using the PC entailment oracle.

which one of the following statements must also be true?

$\neg c$

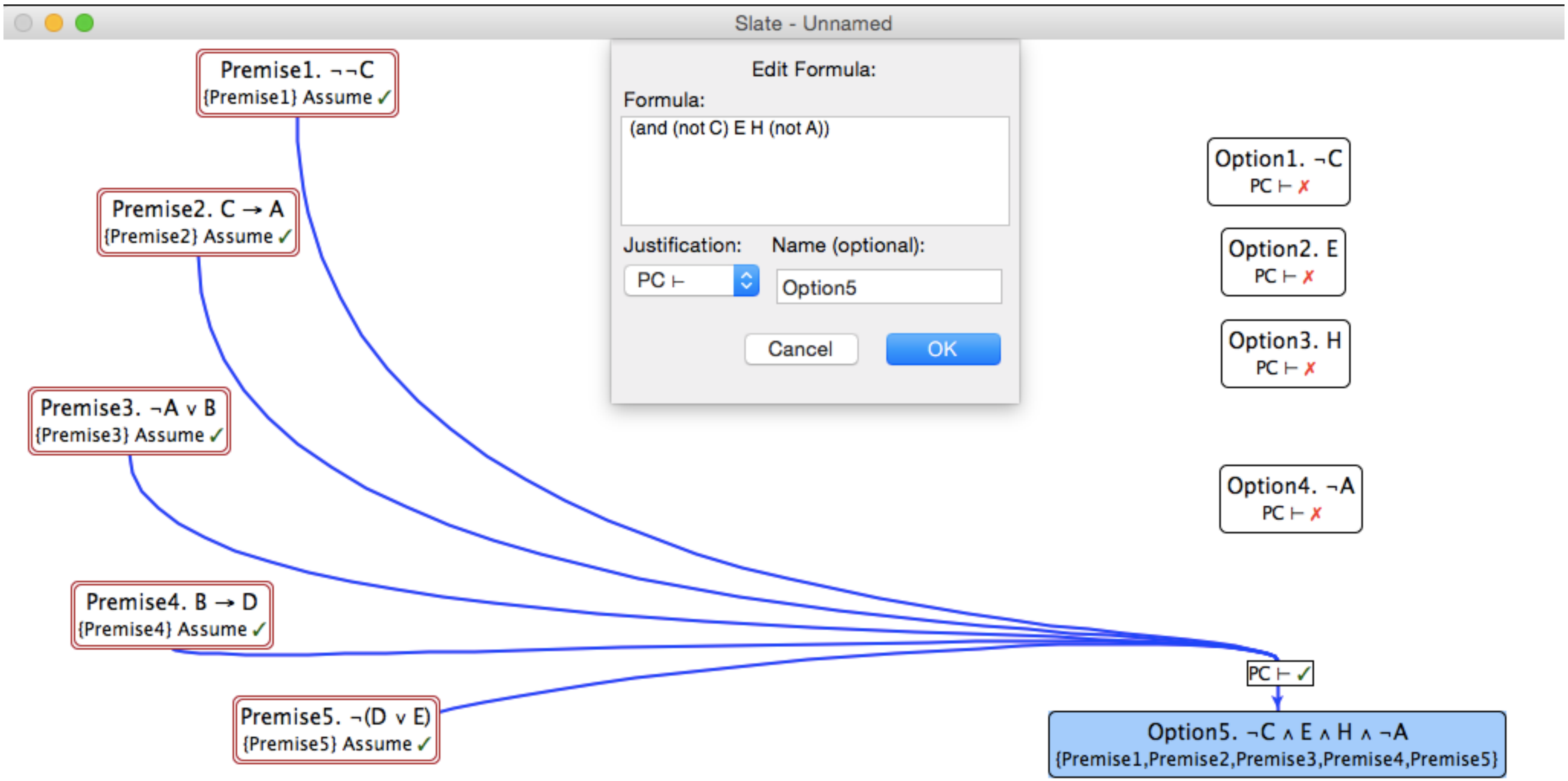
e

h

$\neg a$

all of the above

(and (not C) E H (not A))



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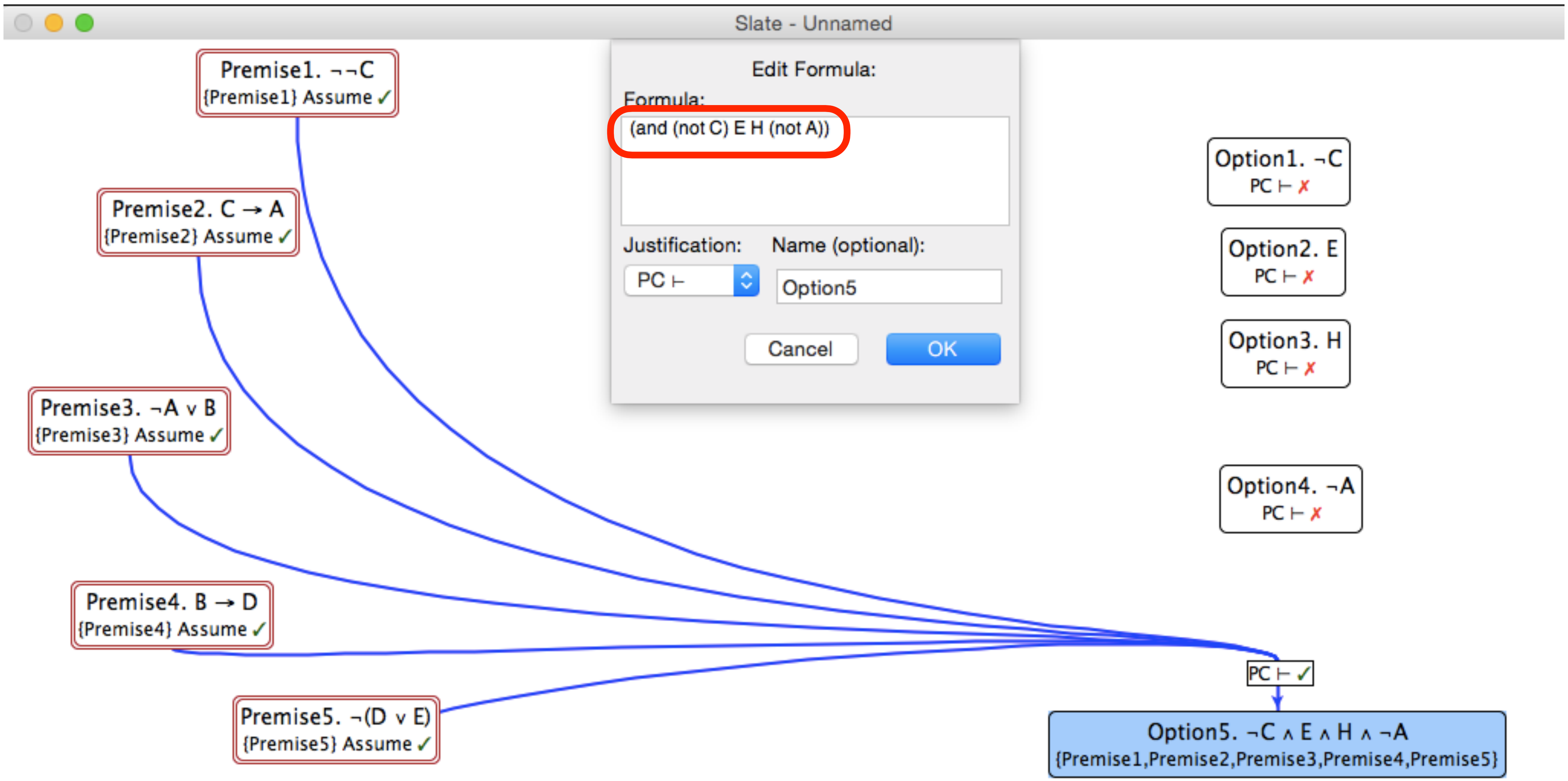

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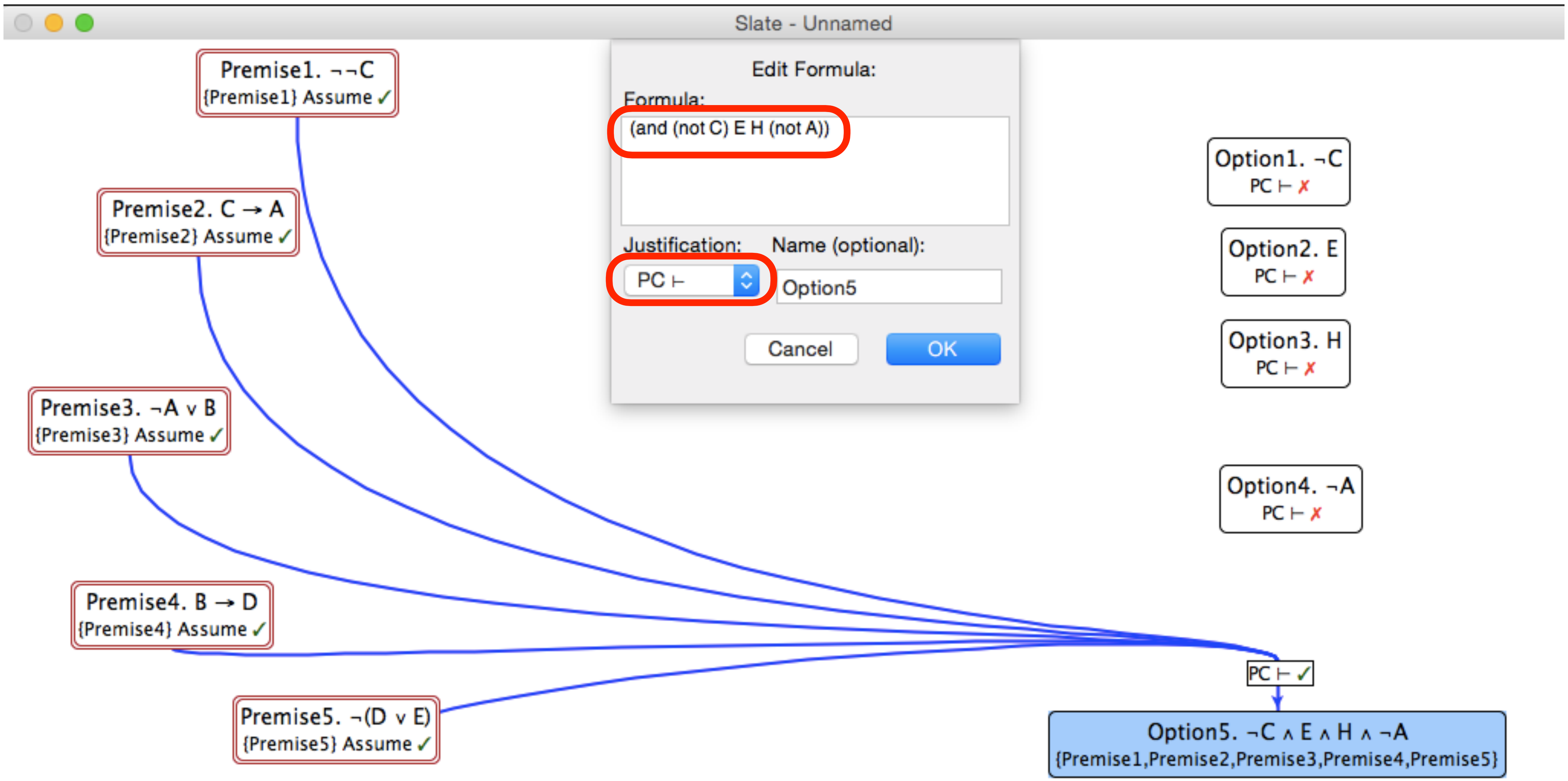

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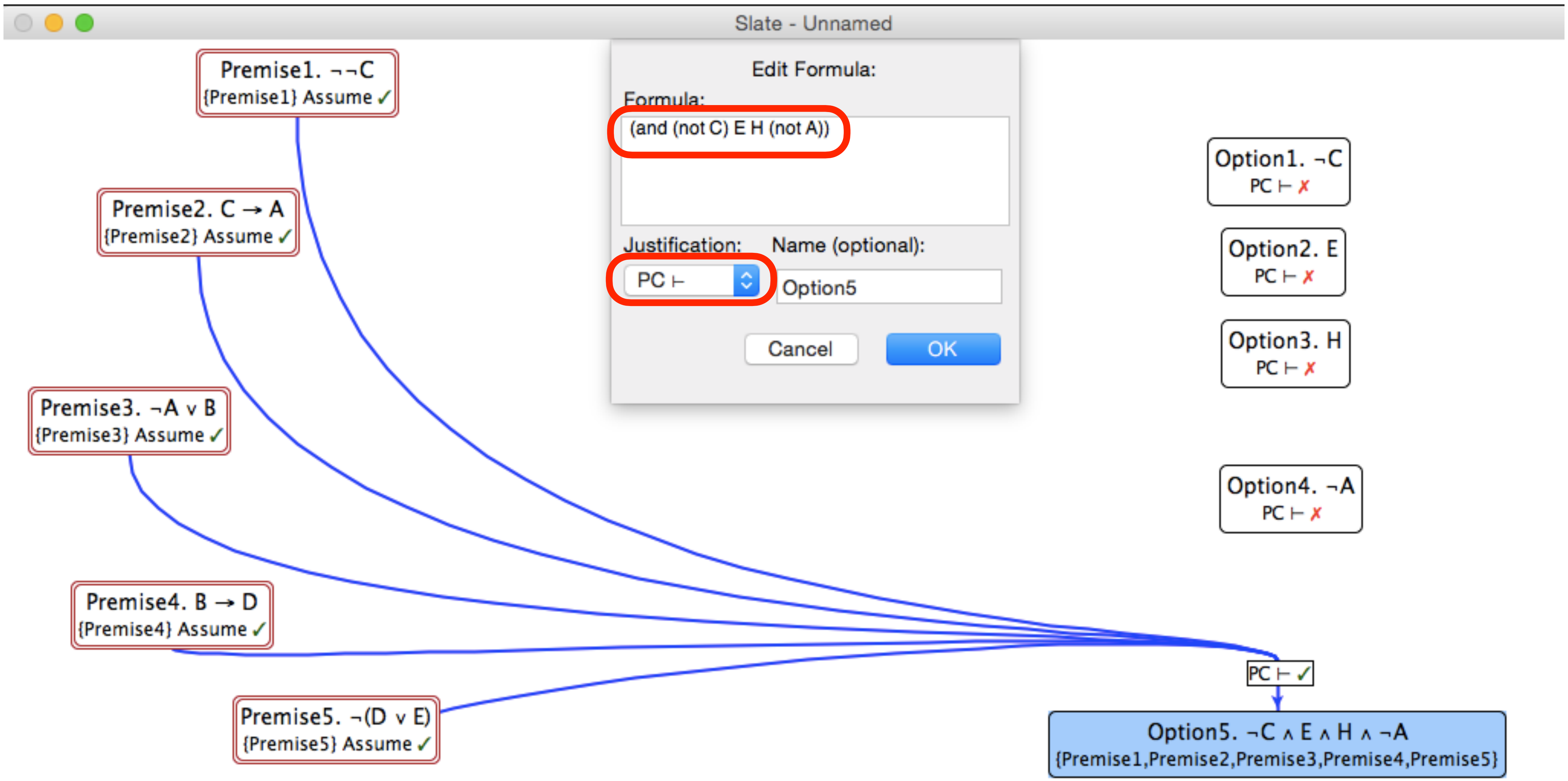

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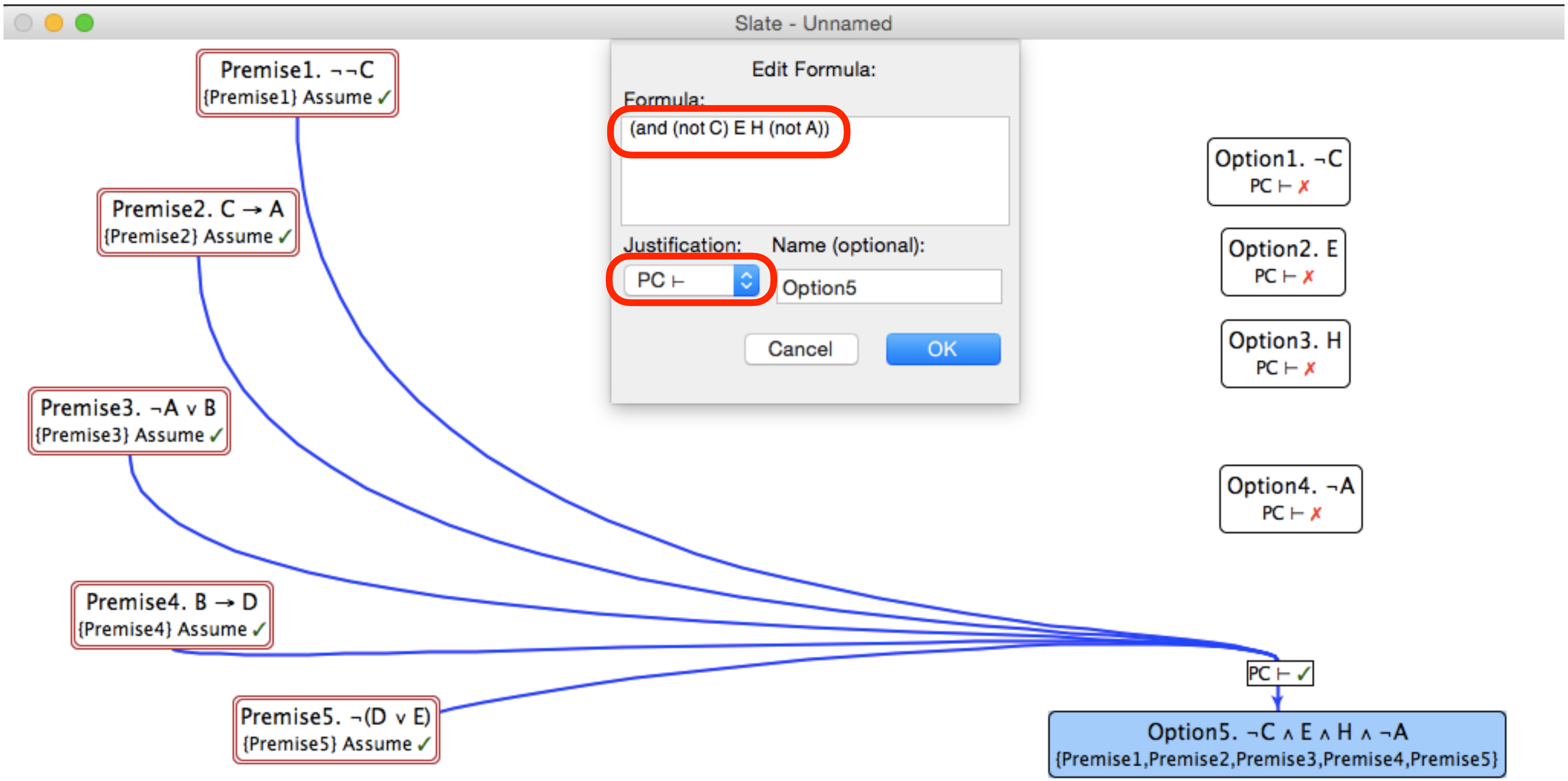

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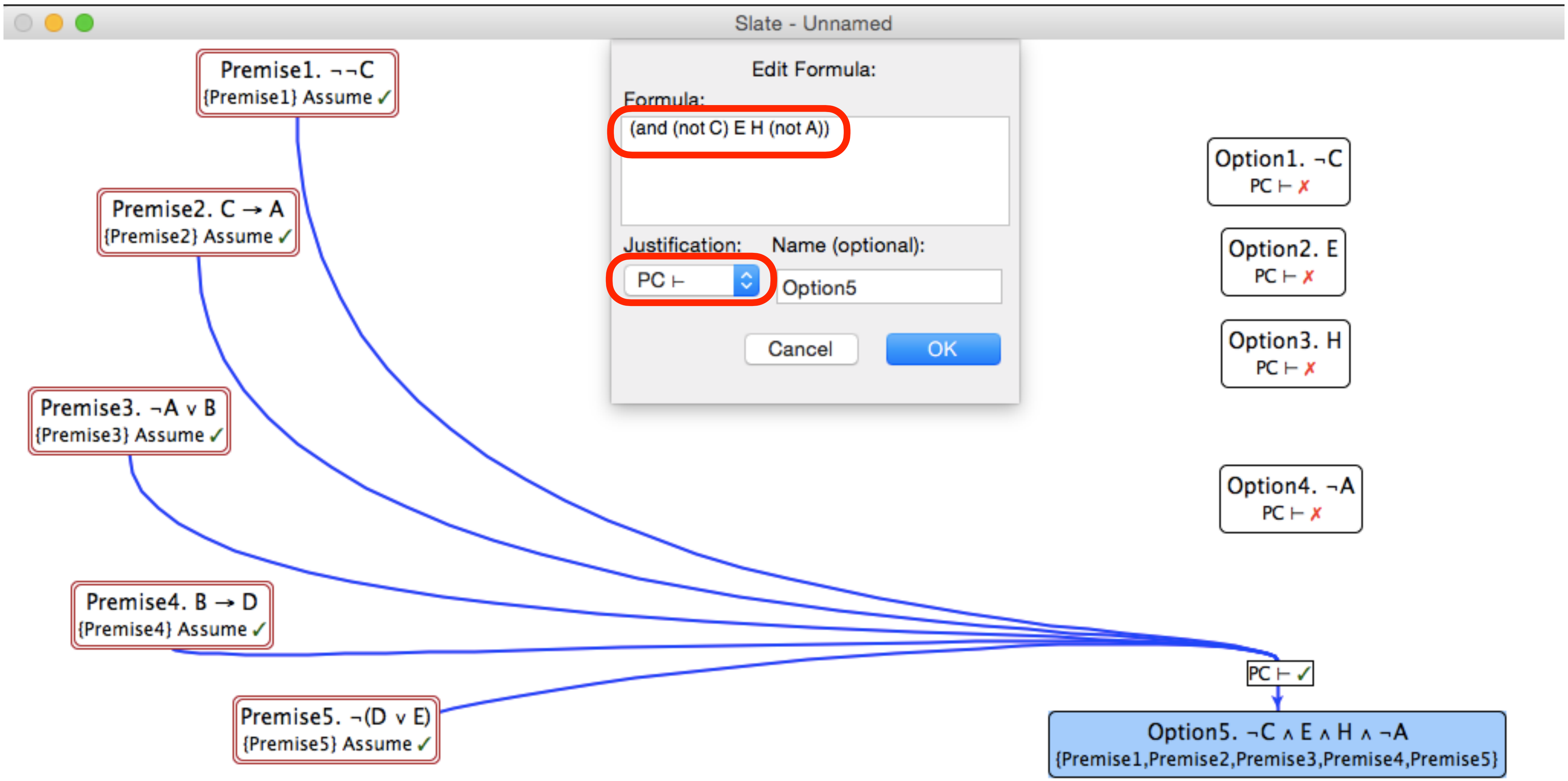

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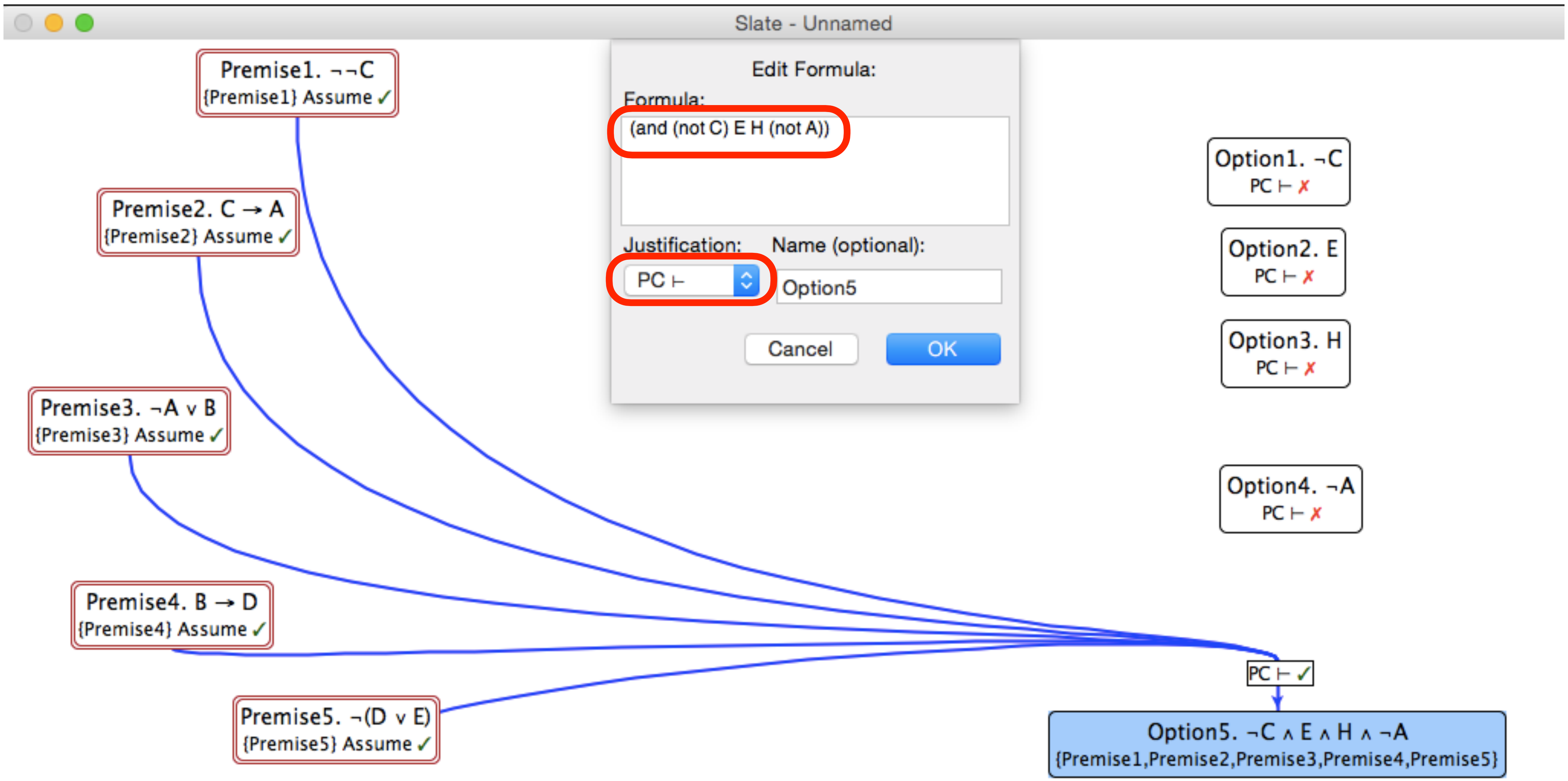
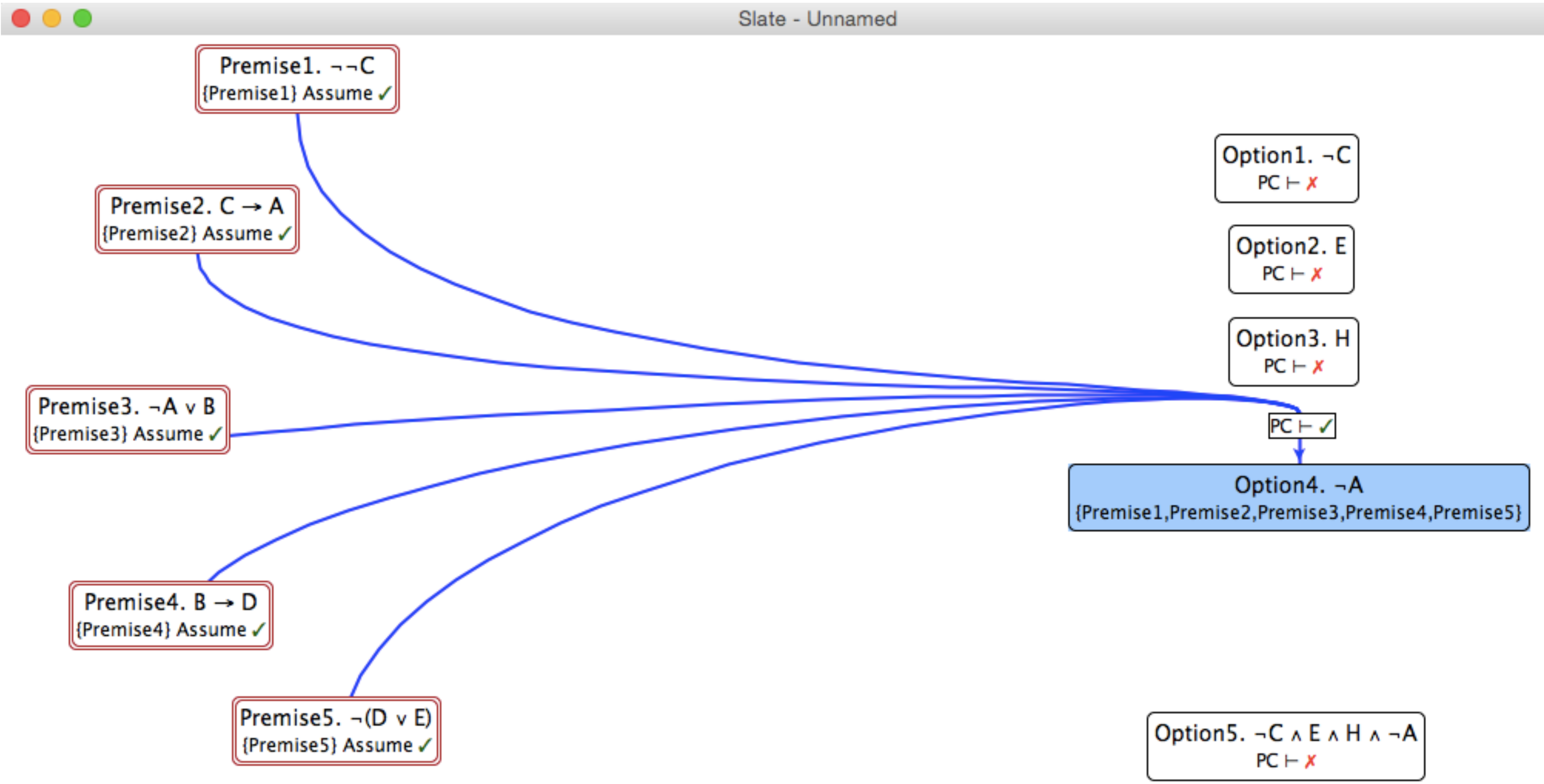


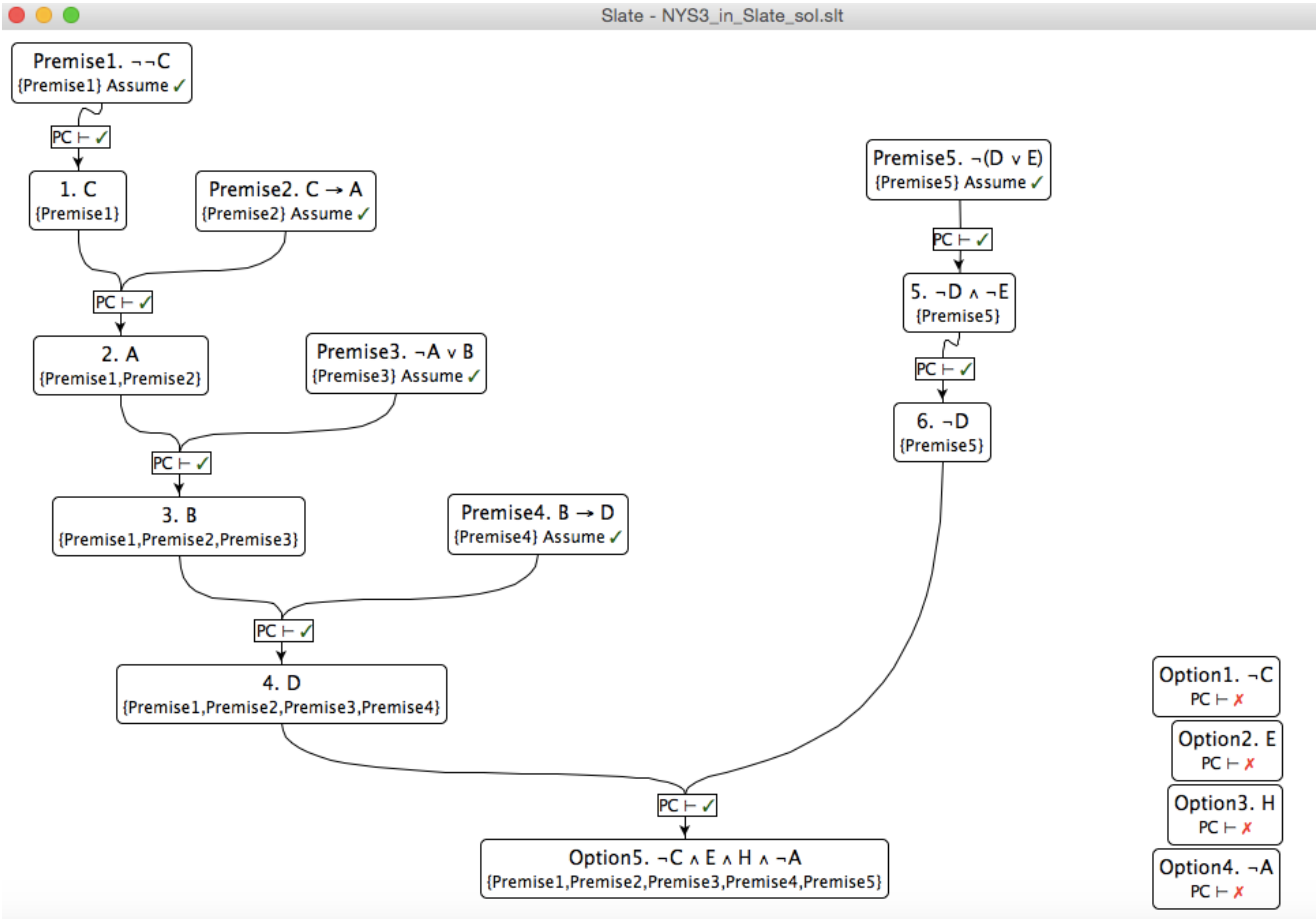

Slate - Unnamed
Premise1. ¬¬C
{Premise1} Assume ✓
Premise2. C → A
{Premise2} Assume ✓
Premise3. ¬A ∨ B
{Premise3} Assume ✓
Premise4. B → D
{Premise4} Assume ✓
Premise5. ¬(D ∨ E)
{Premise5} Assume ✓
Option1. ¬C
PC ⊢ ✗
Option2. E
PC ⊢ ✗
Option3. H
PC ⊢ ✗
PC ⊢ ✓
Option4. ¬A
{Premise1,Premise2,Premise3,Premise4,Premise5}
Option5. ¬C ∧ E ∧ H ∧ ¬A
PC ⊢ ✗

Proof Plan …

Proof Plan …

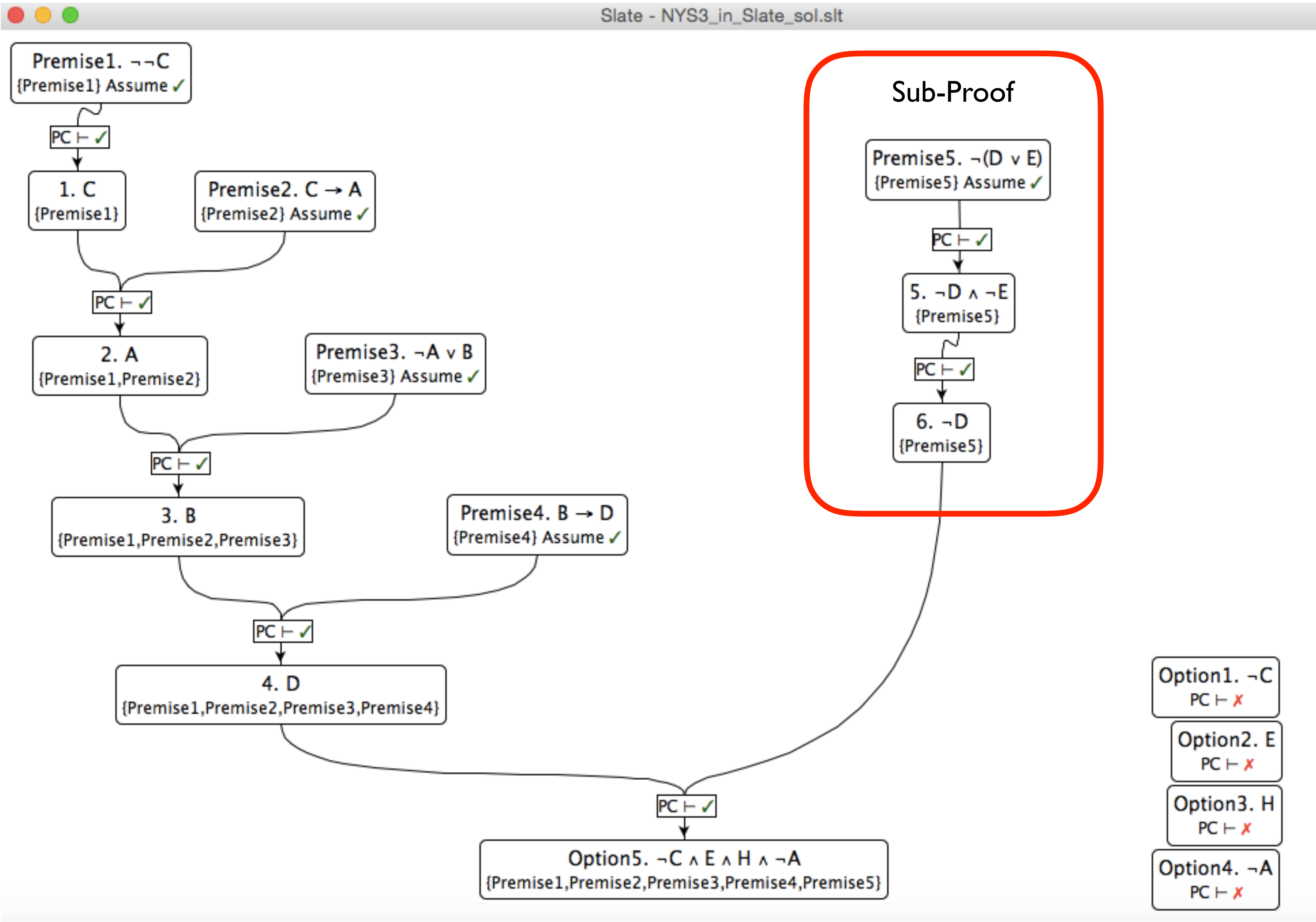
Proof Plan …

Proof Plan …



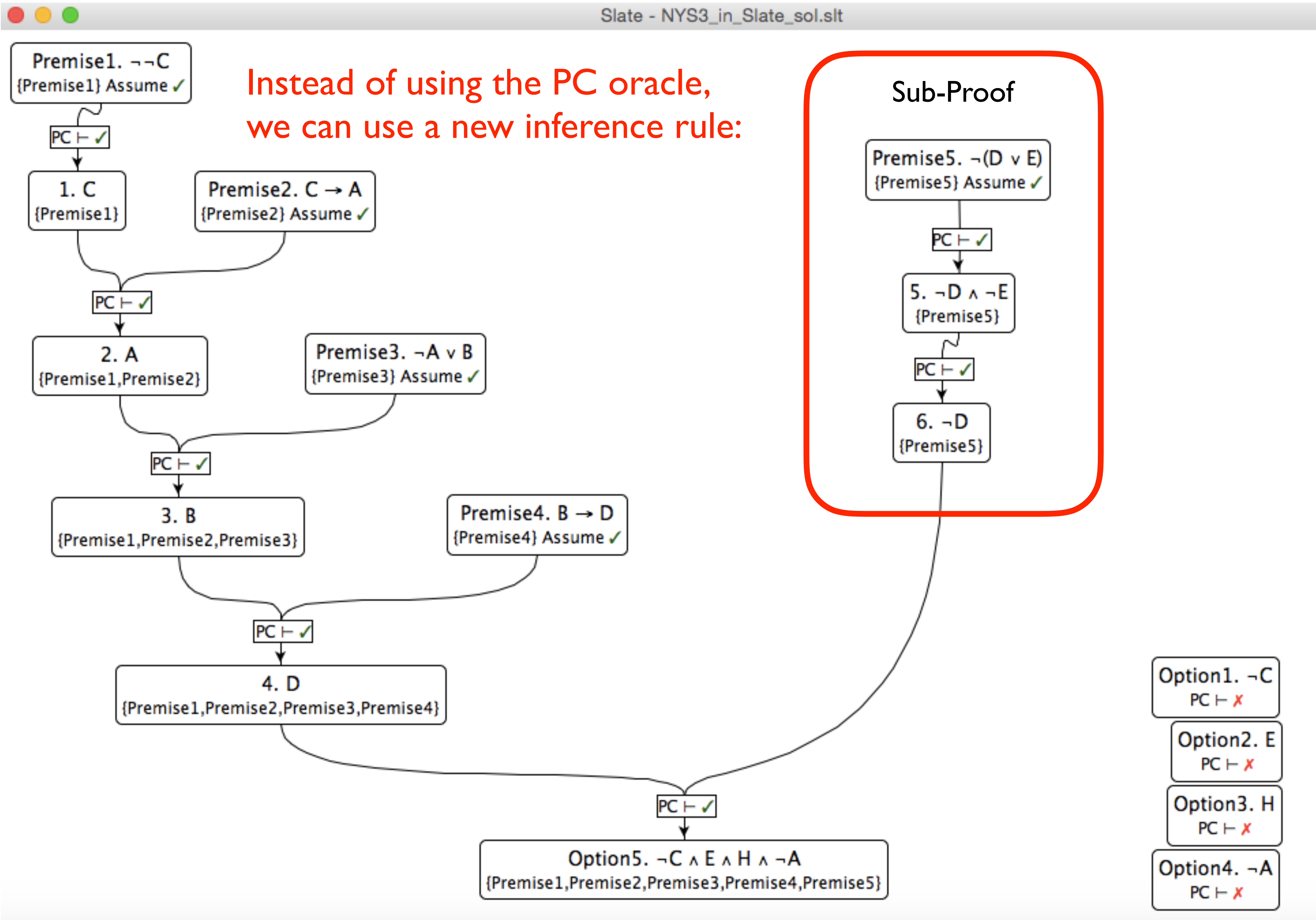
Slate - NYS3_in_Slate_sol.slt
Premise1. ¬¬C
{Premise1} Assume ✓
PC ⊢ ✓
1. C
{Premise1}
Premise2. C → A
{Premise2} Assume ✓
PC ⊢ ✓
2. A
{Premise1,Premise2}
Premise3. ¬A ∨ B
{Premise3} Assume ✓
PC ⊢ ✓
3. B
{Premise1,Premise2,Premise3}
Premise4. B → D
{Premise4} Assume ✓
PC ⊢ ✓
4. D
{Premise1,Premise2,Premise3,Premise4}
Premise5. ¬(D ∨ E)
{Premise5} Assume ✓
PC ⊢ ✓
5. ¬D ∧ ¬E
{Premise5}
PC ⊢ ✓
6. ¬D
{Premise5}
PC ⊢ ✓
Option5. ¬C ∧ E ∧ H ∧ ¬A
{Premise1,Premise2,Premise3,Premise4,Premise5}
Option1. ¬C
PC ⊢ ✗
Option2. E
PC ⊢ ✗
Option3. H
PC ⊢ ✗
Option4. ¬A
PC ⊢ ✗

Proof Plan …


Slate - NYS3_in_Slate_sol.slt
Premise1. ¬¬C
{Premise1} Assume ✓
PC ⊢ ✓
1. C
{Premise1}
Premise2. C → A
{Premise2} Assume ✓
PC ⊢ ✓
2. A
{Premise1,Premise2}
Premise3. ¬A ∨ B
{Premise3} Assume ✓
PC ⊢ ✓
3. B
{Premise1,Premise2,Premise3}
Premise4. B → D
{Premise4} Assume ✓
PC ⊢ ✓
4. D
{Premise1,Premise2,Premise3,Premise4}
Sub-Proof
Premise5. ¬(D ∨ E)
{Premise5} Assume ✓
PC ⊢ ✓
5. ¬D ∧ ¬E
{Premise5}
PC ⊢ ✓
6. ¬D
{Premise5}
PC ⊢ ✓
Option5. ¬C ∧ E ∧ H ∧ ¬A
{Premise1,Premise2,Premise3,Premise4,Premise5}
Option1. ¬C
PC ⊢ ✗
Option2. E
PC ⊢ ✗
Option3. H
PC ⊢ ✗
Option4. ¬A
PC ⊢ ✗

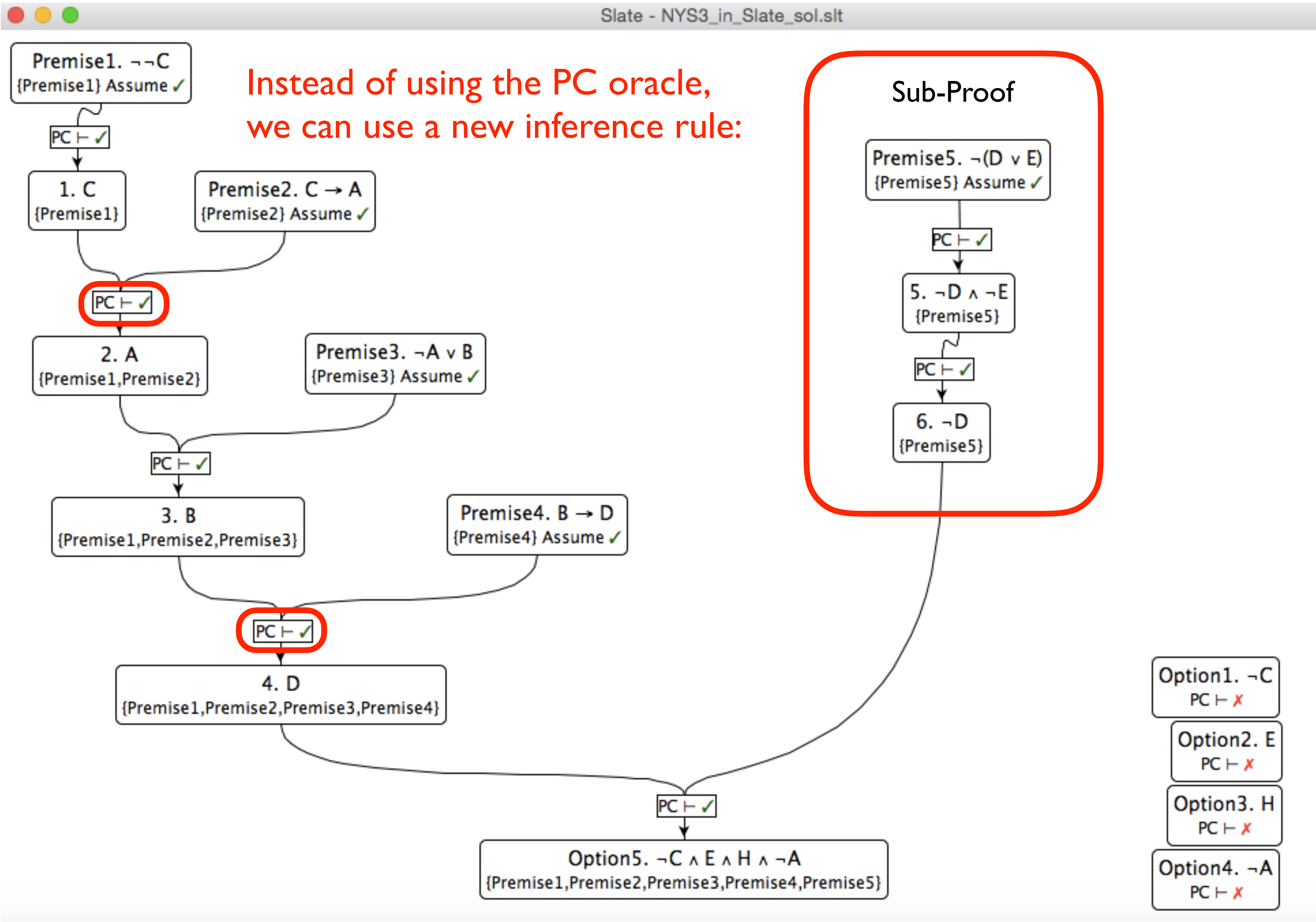
Proof Plan …

Instead of using the PC oracle,
we can use a new inference rule:



Proof Plan …

Instead of using the PC oracle, we can use a new inference rule:

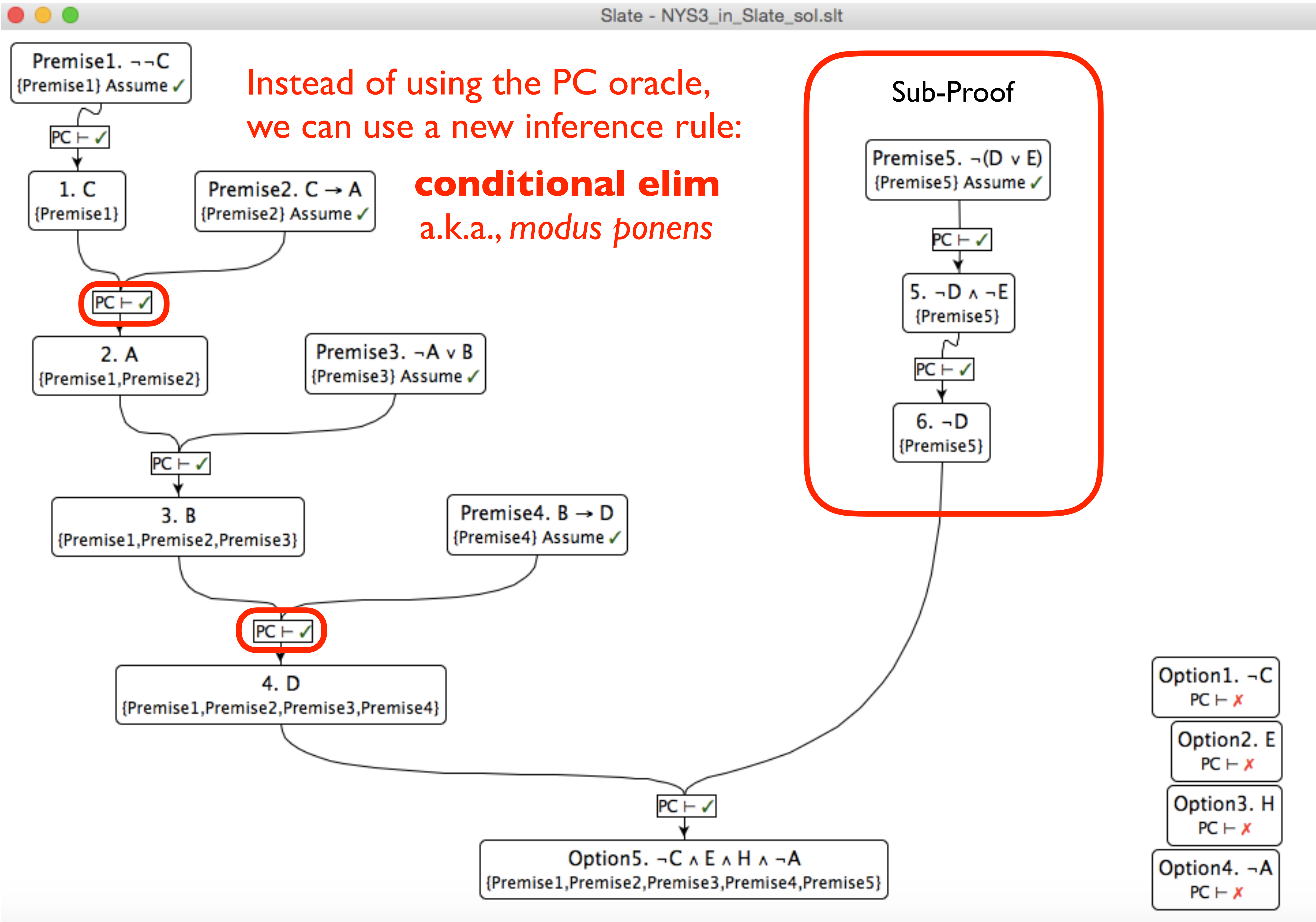


Proof Plan …

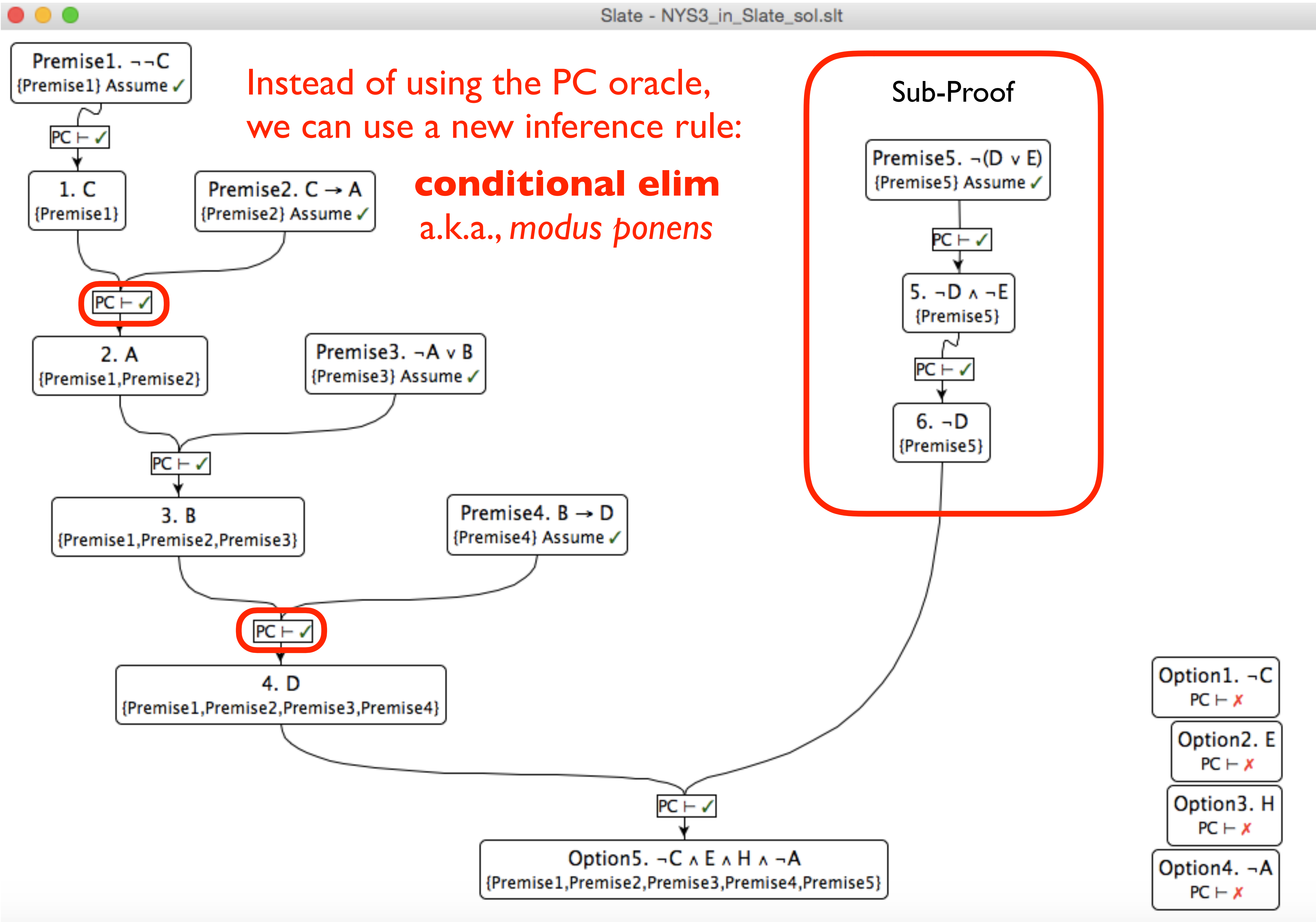
Instead of using the PC oracle, we can use a new inference rule:

conditional elim

a.k.a., *modus ponens*

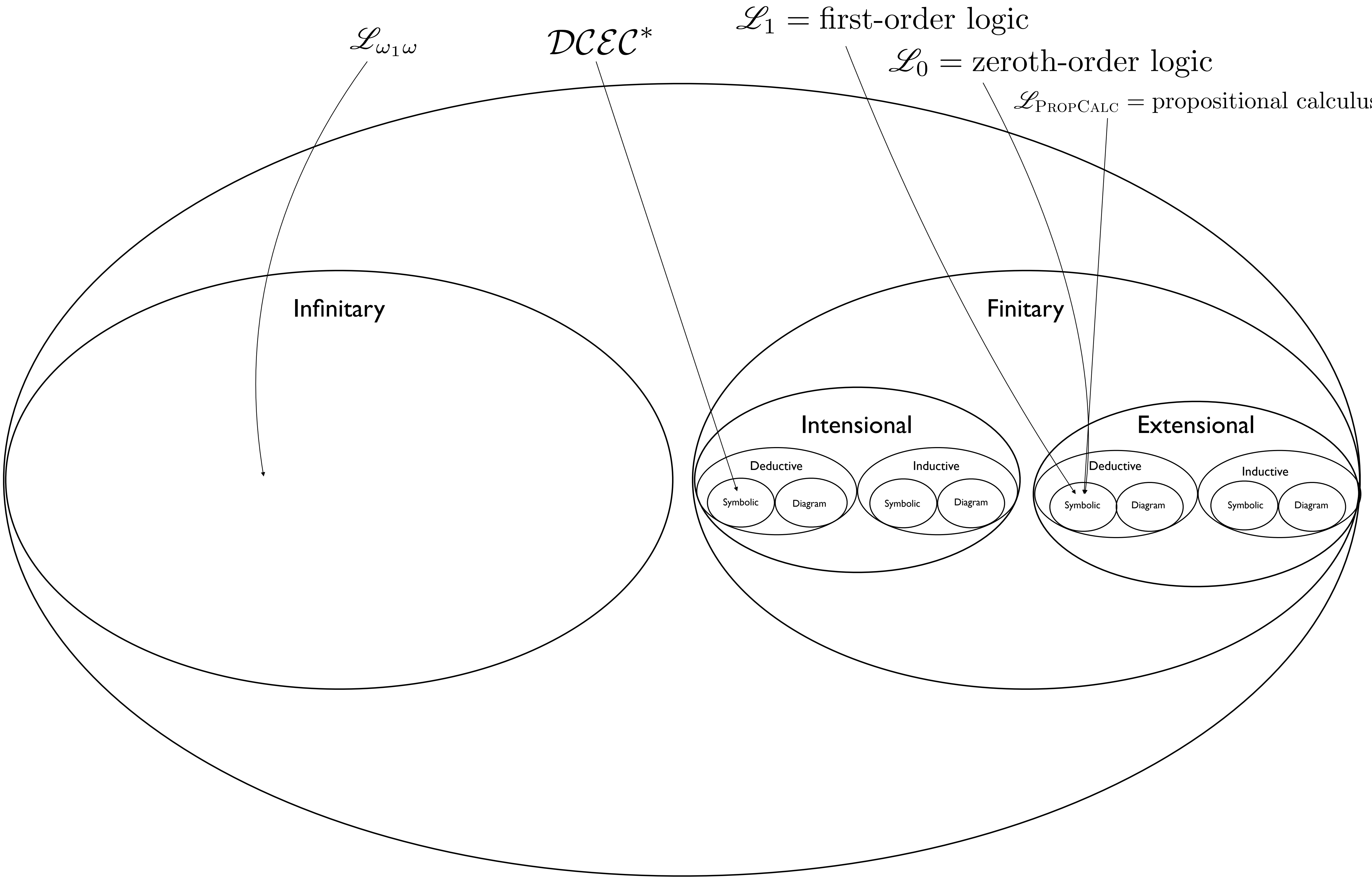


Proof Plan …



Slate - NYS3_in_Slate_sol.slt
Premise1. ¬¬C
{Premise1} Assume ✓
PC ⊢ ✓
1. C
{Premise1}
Premise2. C → A
{Premise2} Assume ✓
PC ⊢ ✓
2. A
{Premise1,Premise2}
Premise3. ¬A ∨ B
{Premise3} Assume ✓
PC ⊢ ✓
3. B
{Premise1,Premise2,Premise3}
Premise4. B → D
{Premise4} Assume ✓
PC ⊢ ✓
4. D
{Premise1,Premise2,Premise3,Premise4}
Instead of using the PC oracle,
we can use a new inference rule:
conditional elim
a.k.a., modus ponens
Sub-Proof
Premise5. ¬(D ∨ E)
{Premise5} Assume ✓
PC ⊢ ✓
5. ¬D ∧ ¬E
{Premise5}
PC ⊢ ✓
6. ¬D
{Premise5}
PC ⊢ ✓
Option5. ¬C ∧ E ∧ H ∧ ¬A
{Premise1,Premise2,Premise3,Premise4,Premise5}
Option1. ¬C
PC ⊢ ✗
Option2. E
PC ⊢ ✗
Option3. H
PC ⊢ ✗
Option4. ¬A
PC ⊢ ✗

The Universe of Logics



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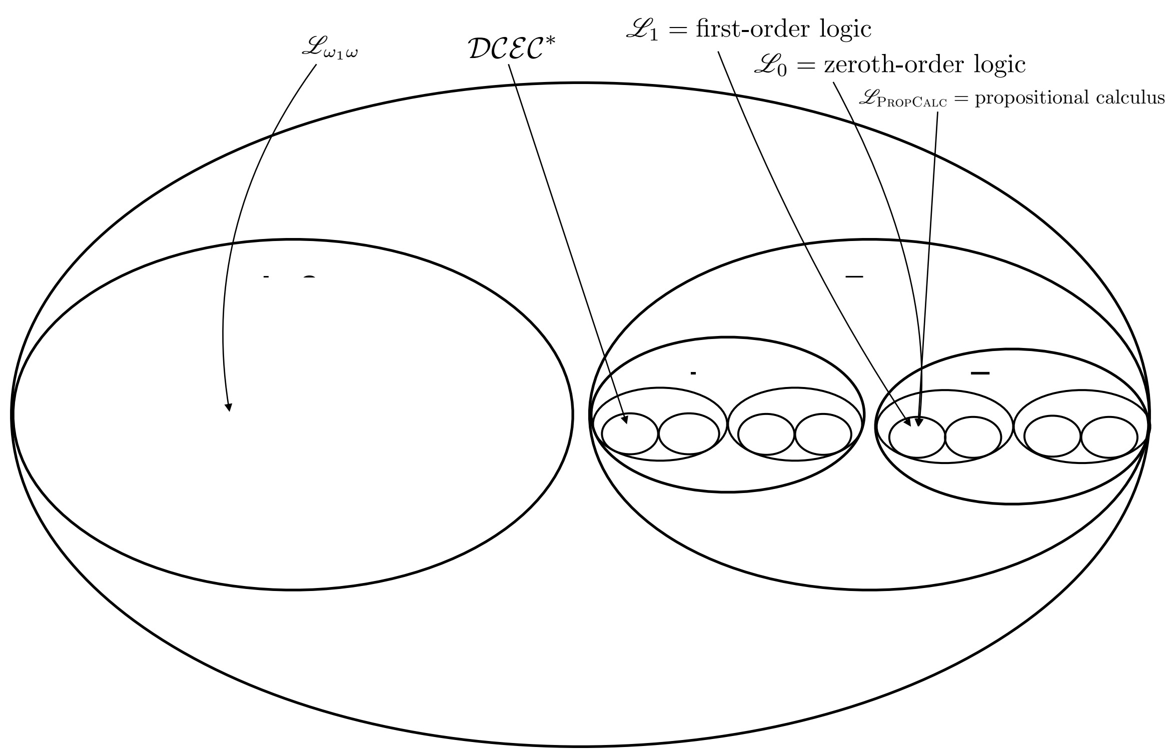

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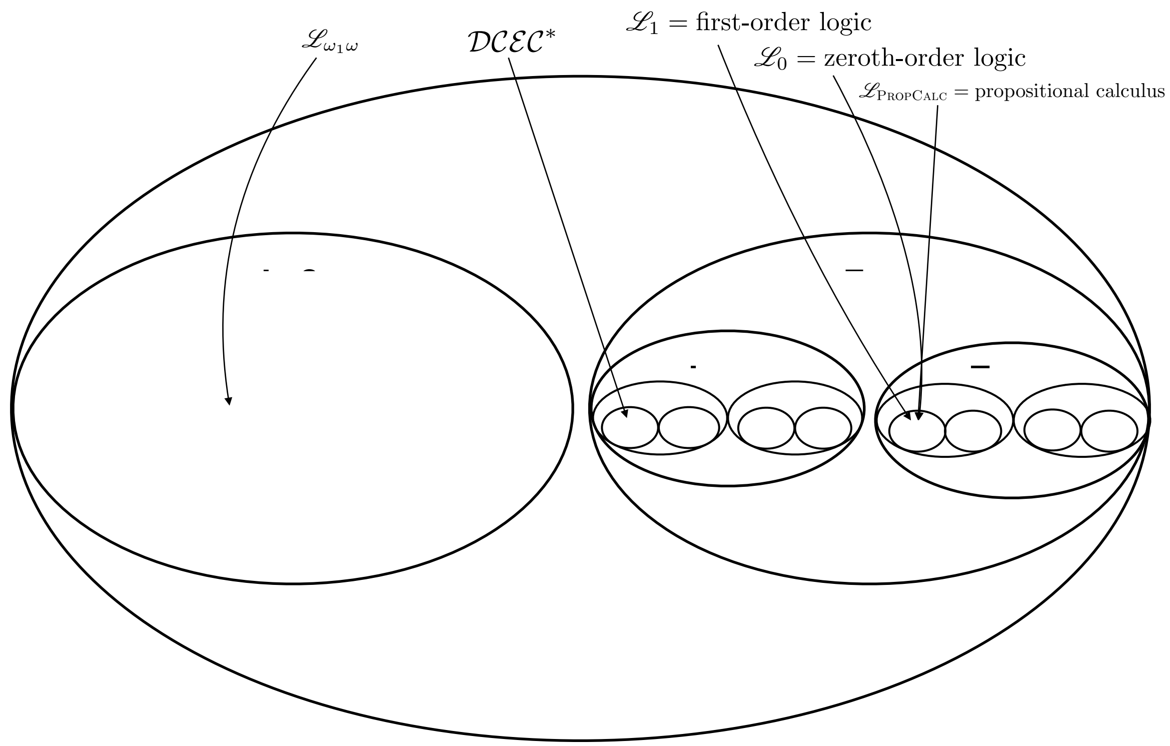
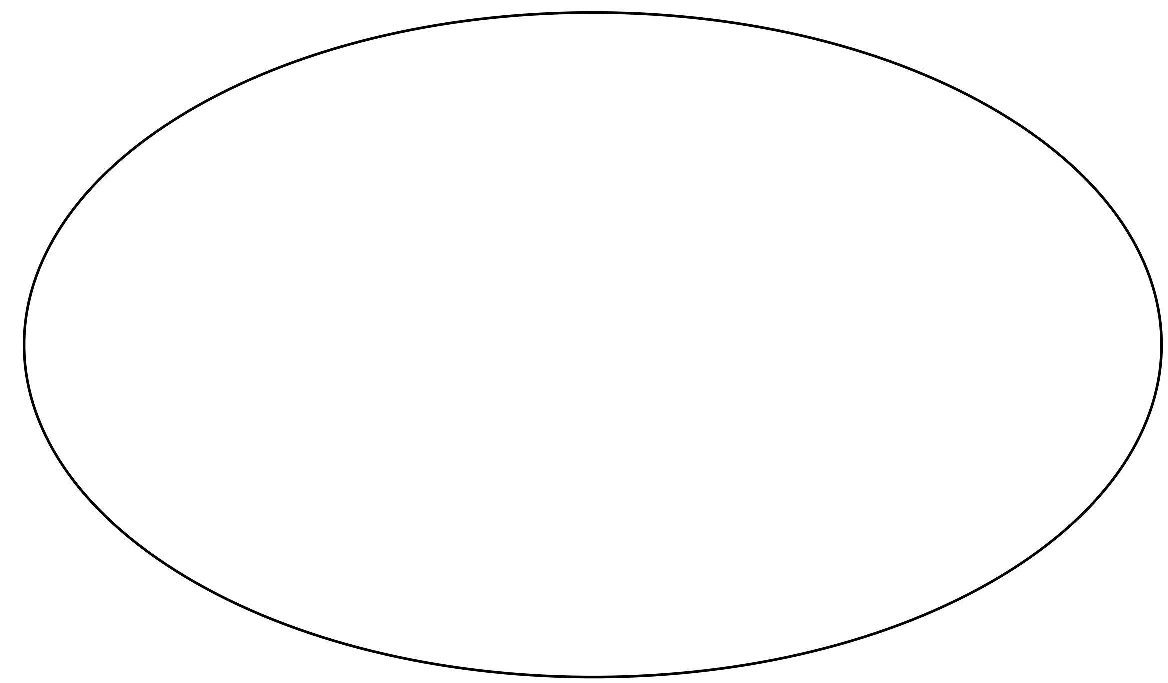


The Physical Universe



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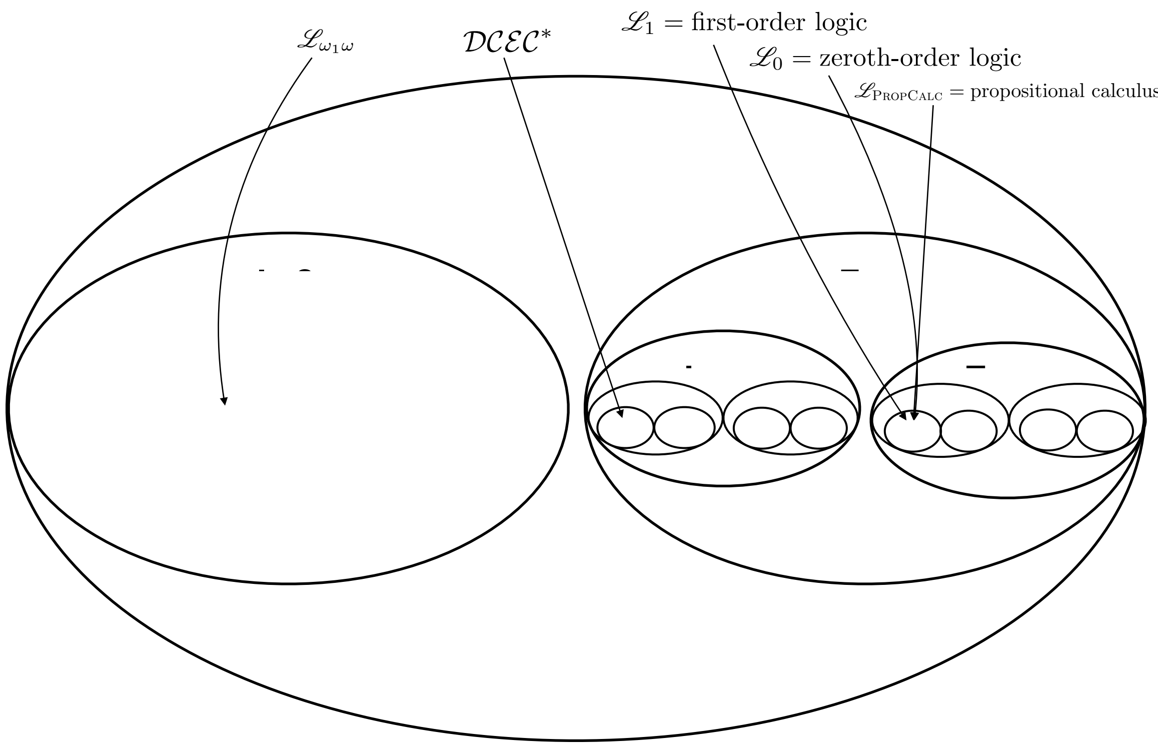

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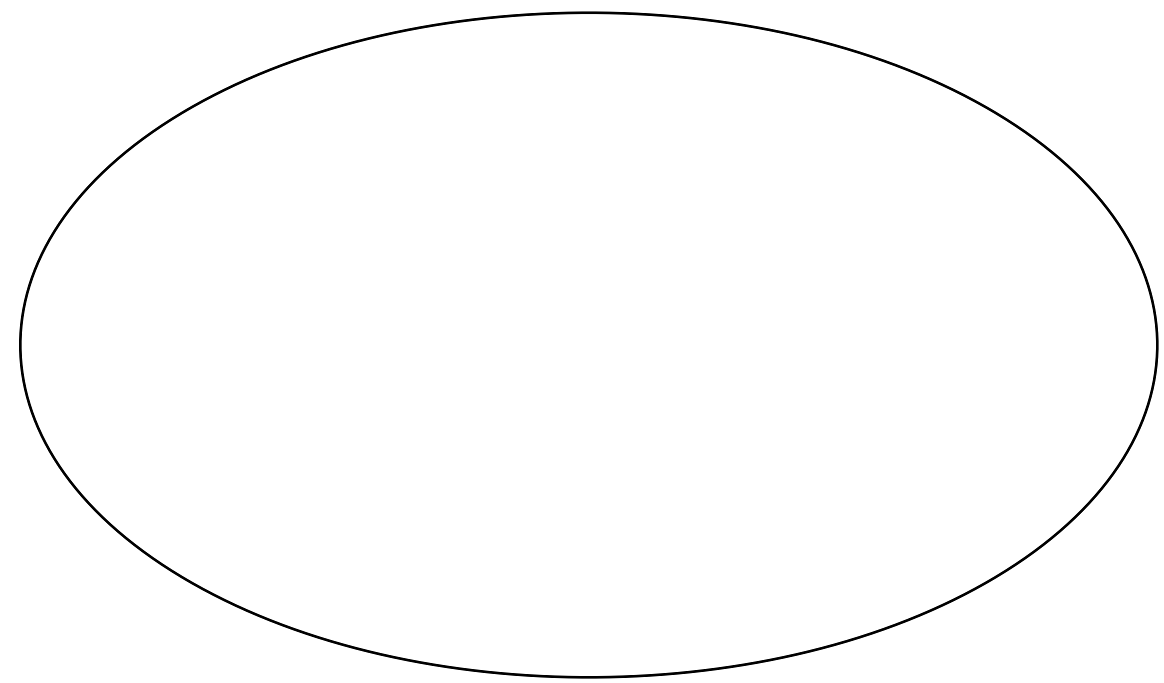
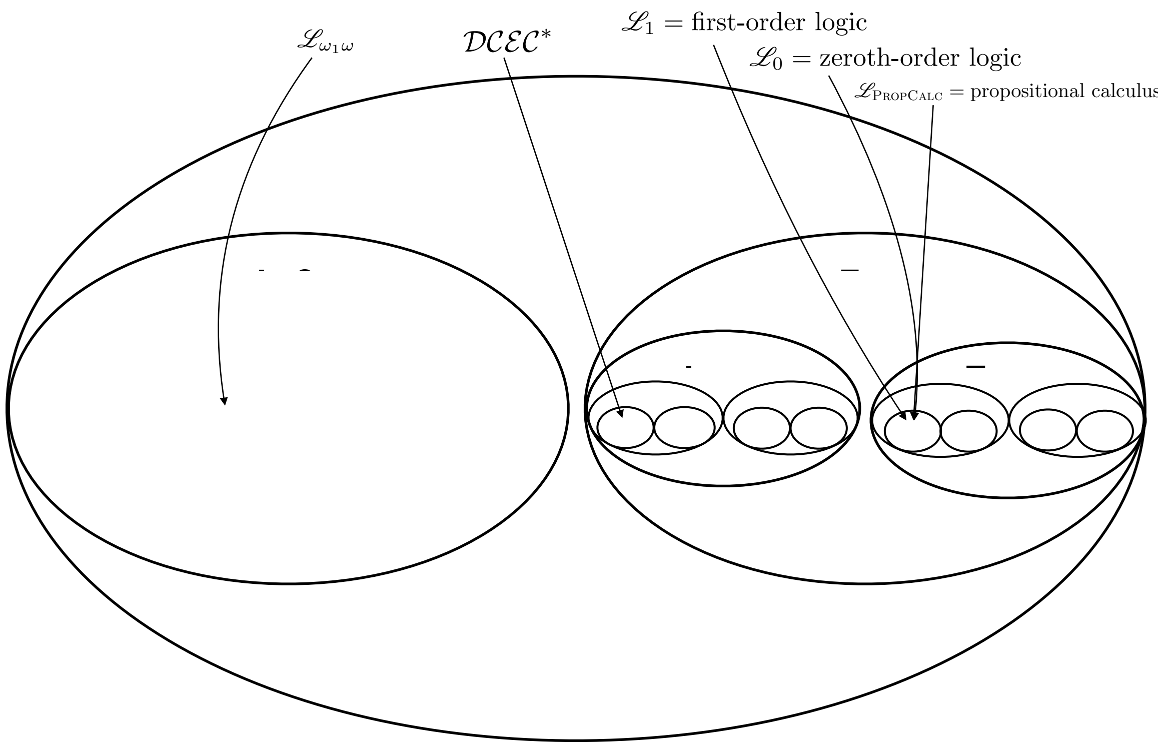
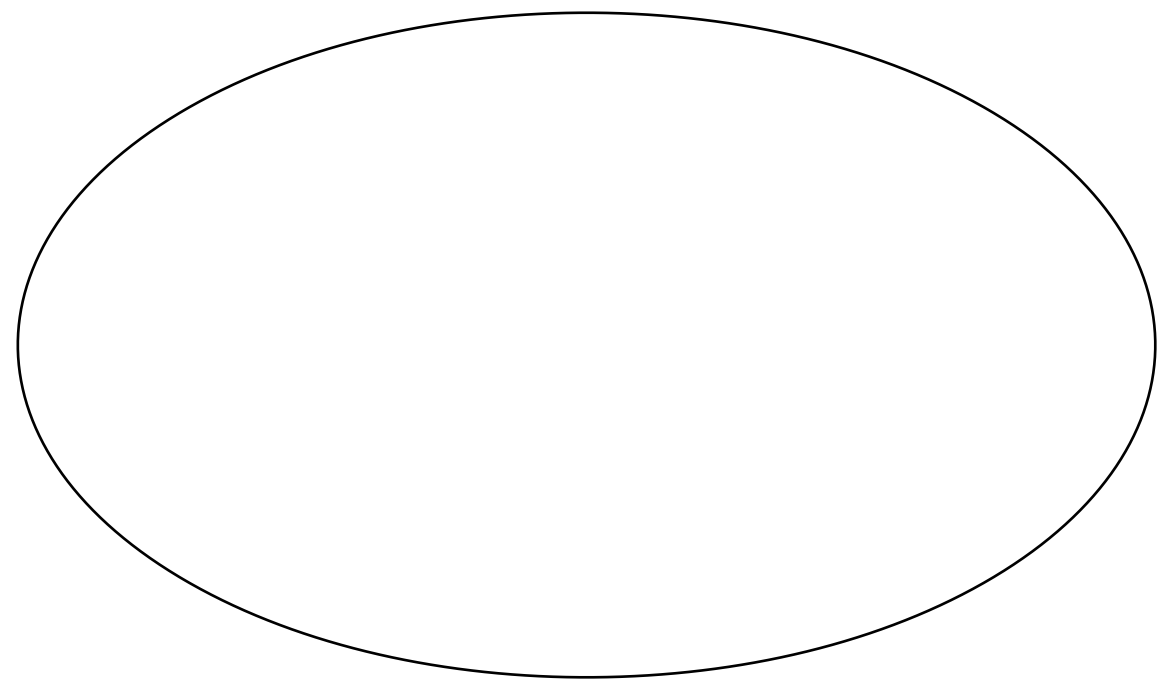


Non-Physical

The Universe of Logics

The Physical Universe





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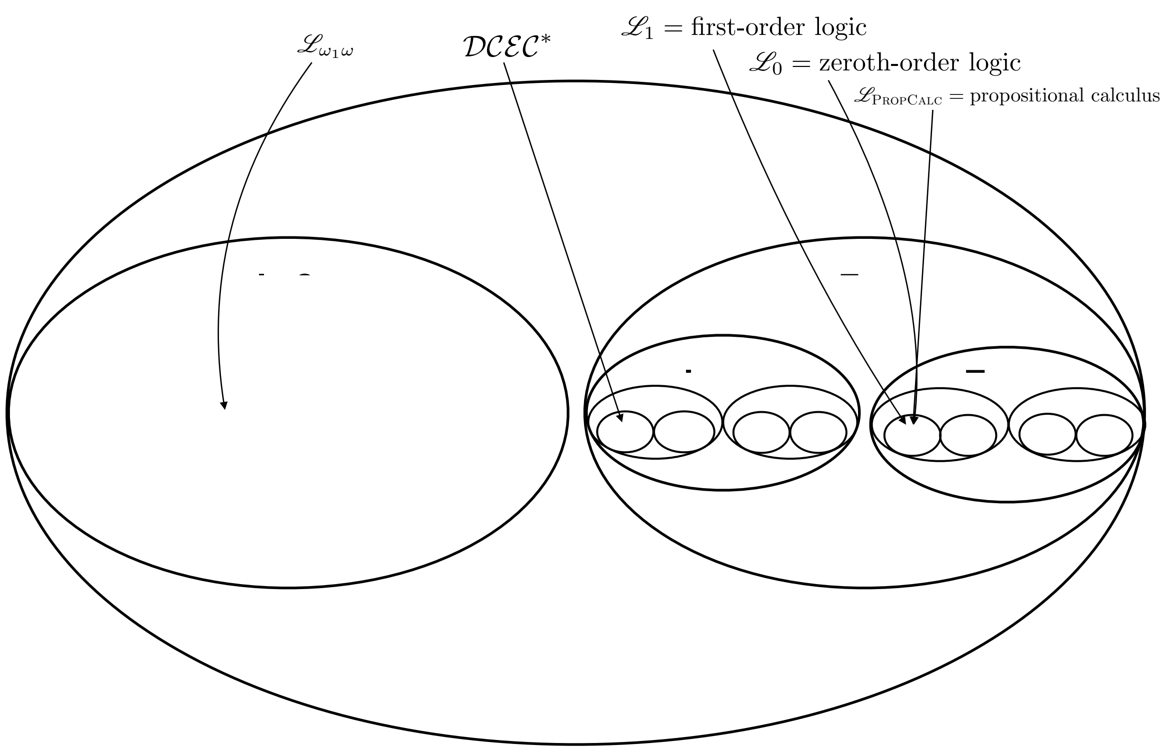

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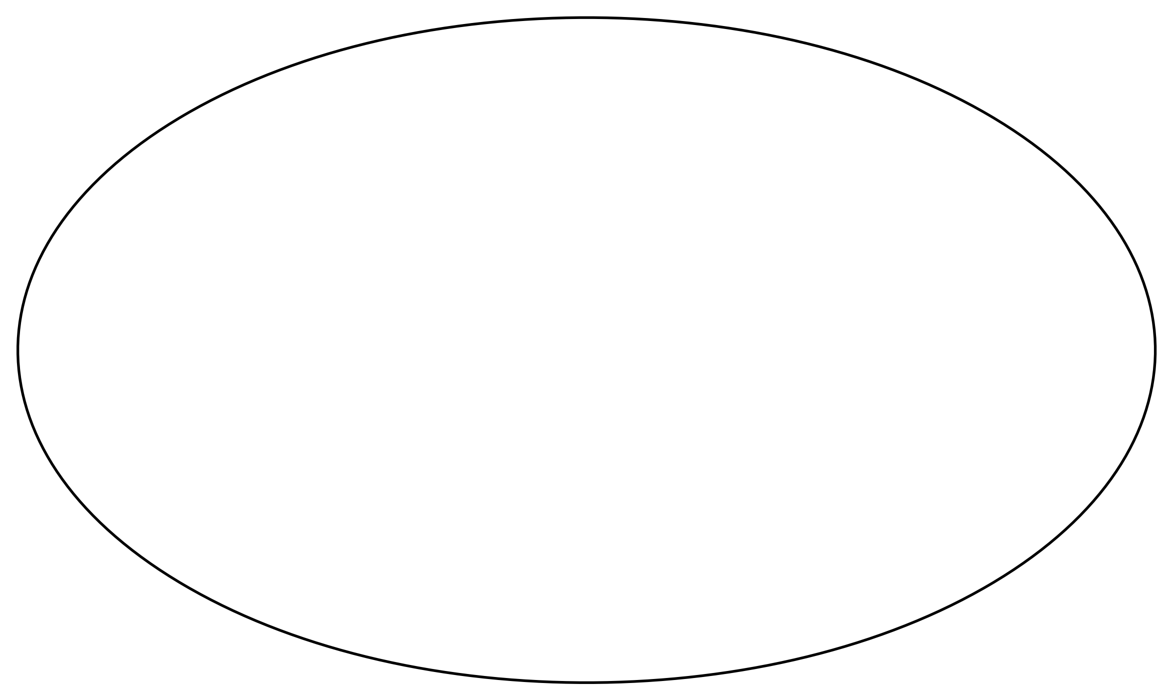

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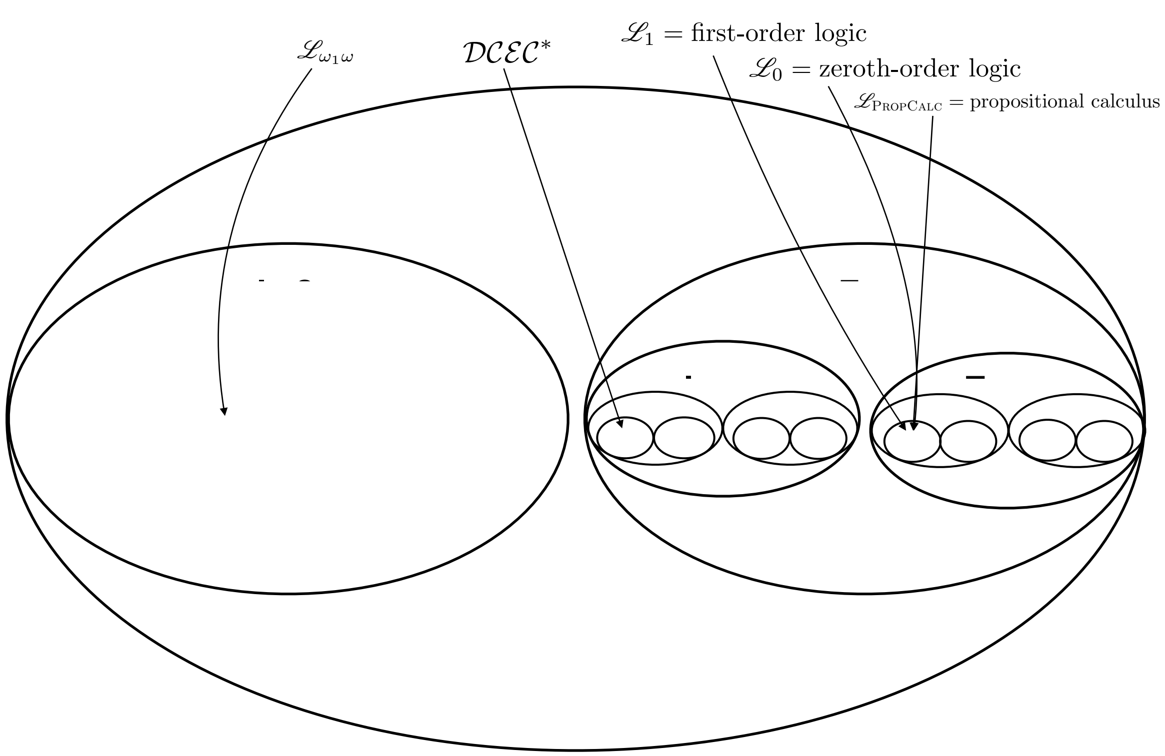

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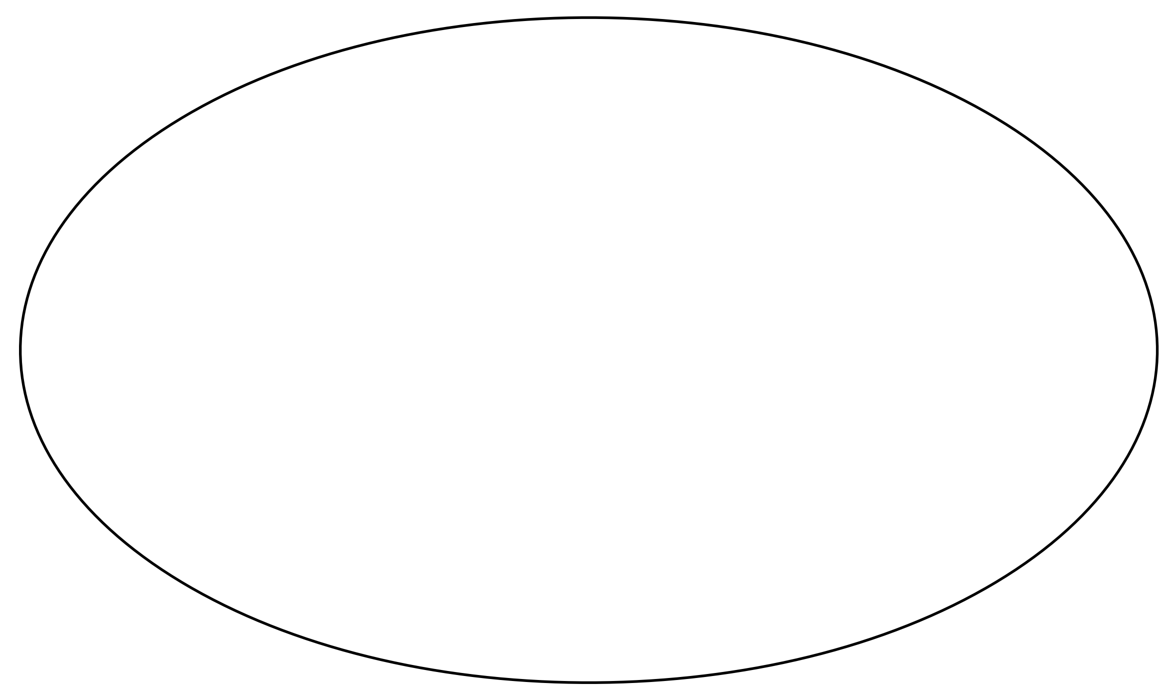

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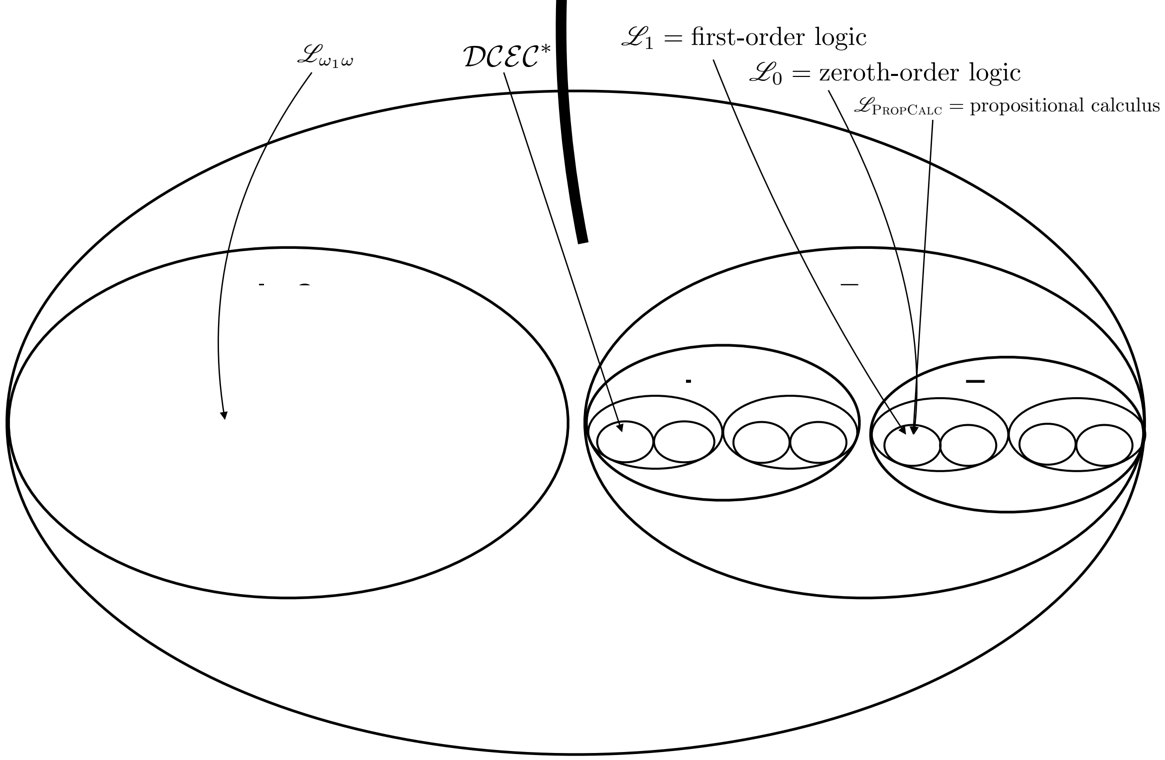

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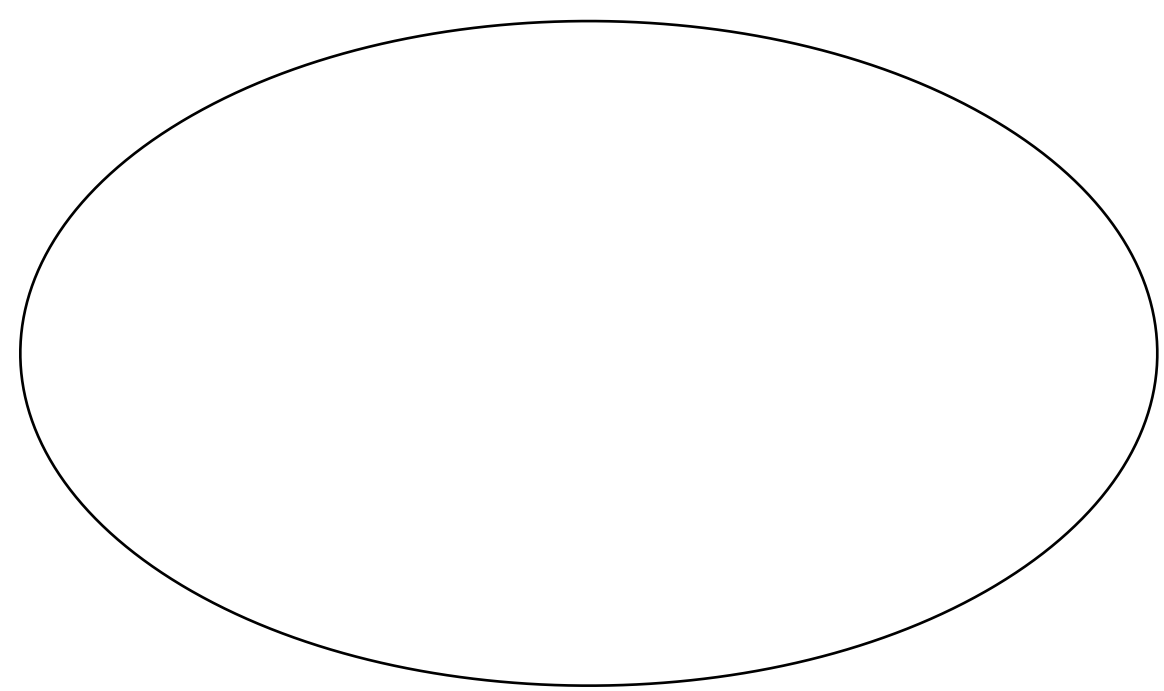

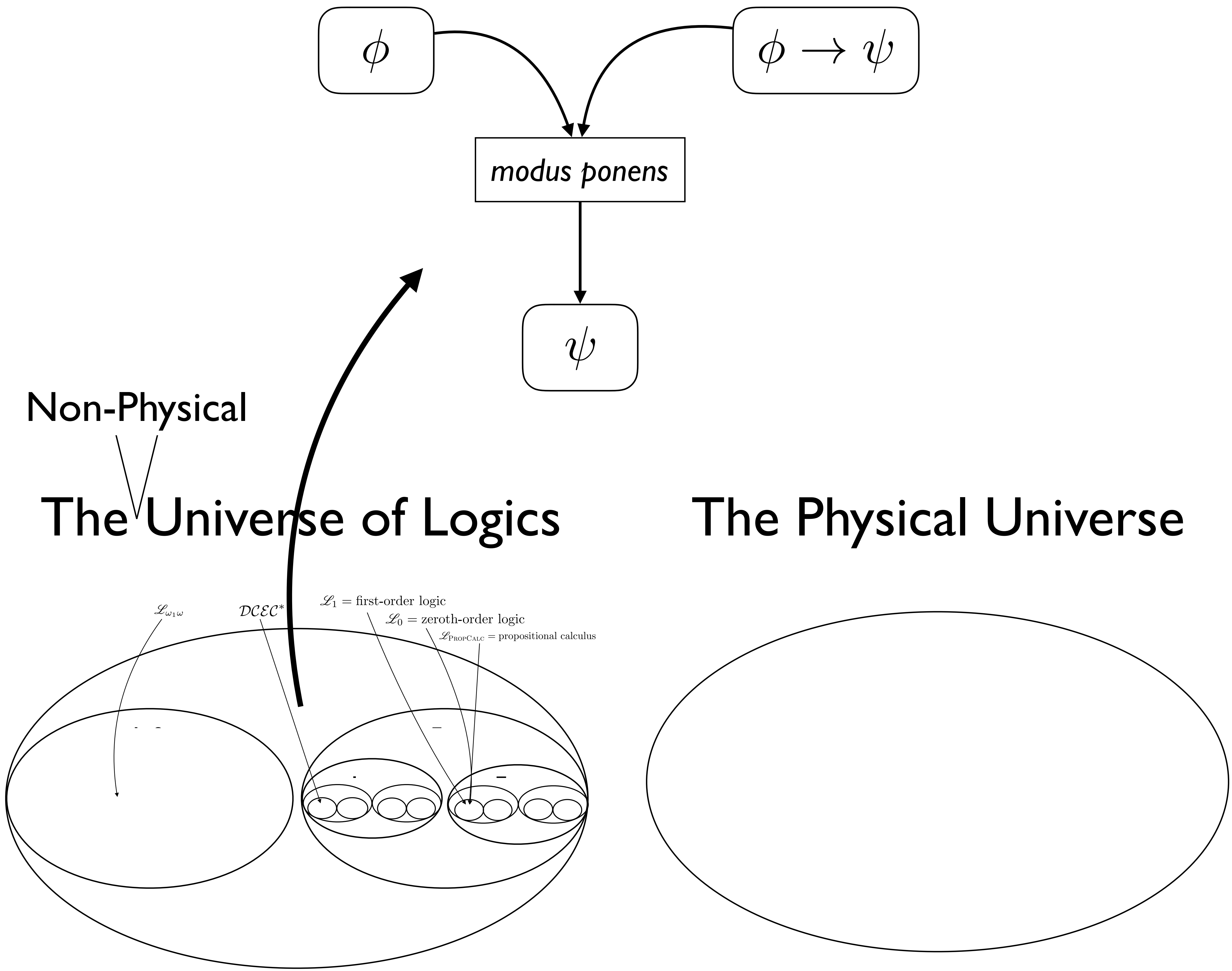
ϕ
$\phi \to \psi$
modus ponens
ψ
Non-Physical
The Universe of Logics
The Physical Universe
$\mathscr{L}_{\omega_1\omega}$
$\mathcal{DCEC}^*$
$\mathscr{L}_1$ = first-order logic
$\mathscr{L}_0$ = zeroth-order logic
$\mathscr{L}_{\text{PropCalc}}$ = propositional calculus

The Original King-Ace

Suppose that the following premise is true:

> If there is a king in the hand, then there is an ace in the hand, or else if there isn't a king in the hand, then there is an ace.

What can you infer from this premise?

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There is an ace in the hand.

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NO! ~~There is an ace in the hand.~~ NO!

In fact, what you *can* infer is that there *isn't* an ace in the hand!

King-Ace Solved

Proposition: There is *not* an ace in the hand.

Proof: We know that at least one of the if-thens (i.e., at least one of the **conditionals**) is false. So we have two cases to consider, viz., that $K \Rightarrow A$ is false, and that $\neg K \Rightarrow A$ is false. Take first the first case; accordingly, suppose that $K \Rightarrow A$ is false. Then it follows that K is true (since when a conditional is false, its antecedent holds but its consequent doesn't), and A is false. Now consider the second case, which consists in $\neg K \Rightarrow A$ being false. Here, in a direct parallel, we know $\neg K$ and, once again, $\neg A$. In both of our two cases, which are exhaustive, there is no ace in the hand. The proposition is established. **QED**

King-Ace 2

Suppose that the following premise is true:

If there is a king in the hand, then there is an ace in the hand; or if there isn't a king in the hand, then there is an ace; but not both of these if-then statements are true.

What can you infer from this premise?

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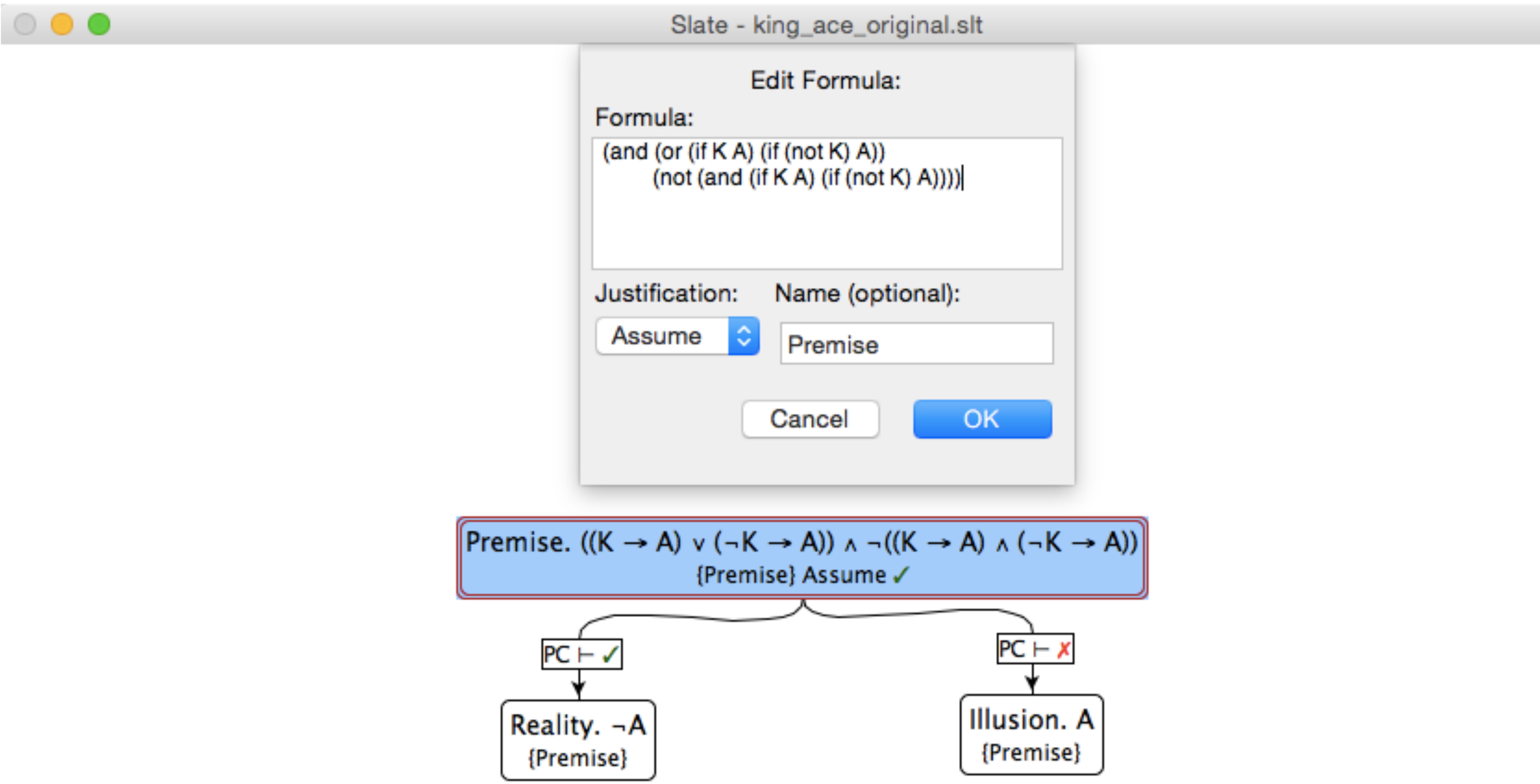
Study the S-expression



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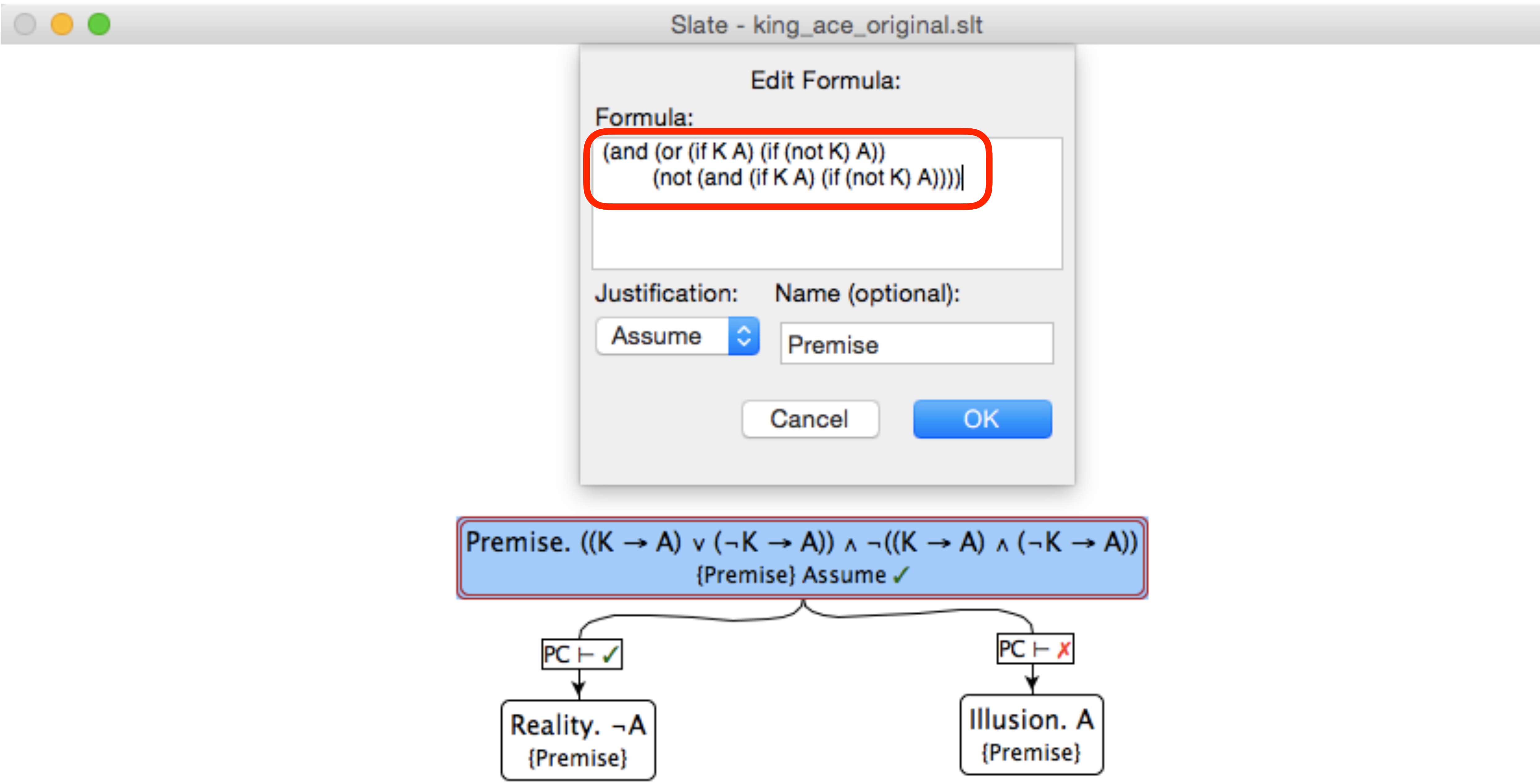

We need another rule of inference to crack this problem … …

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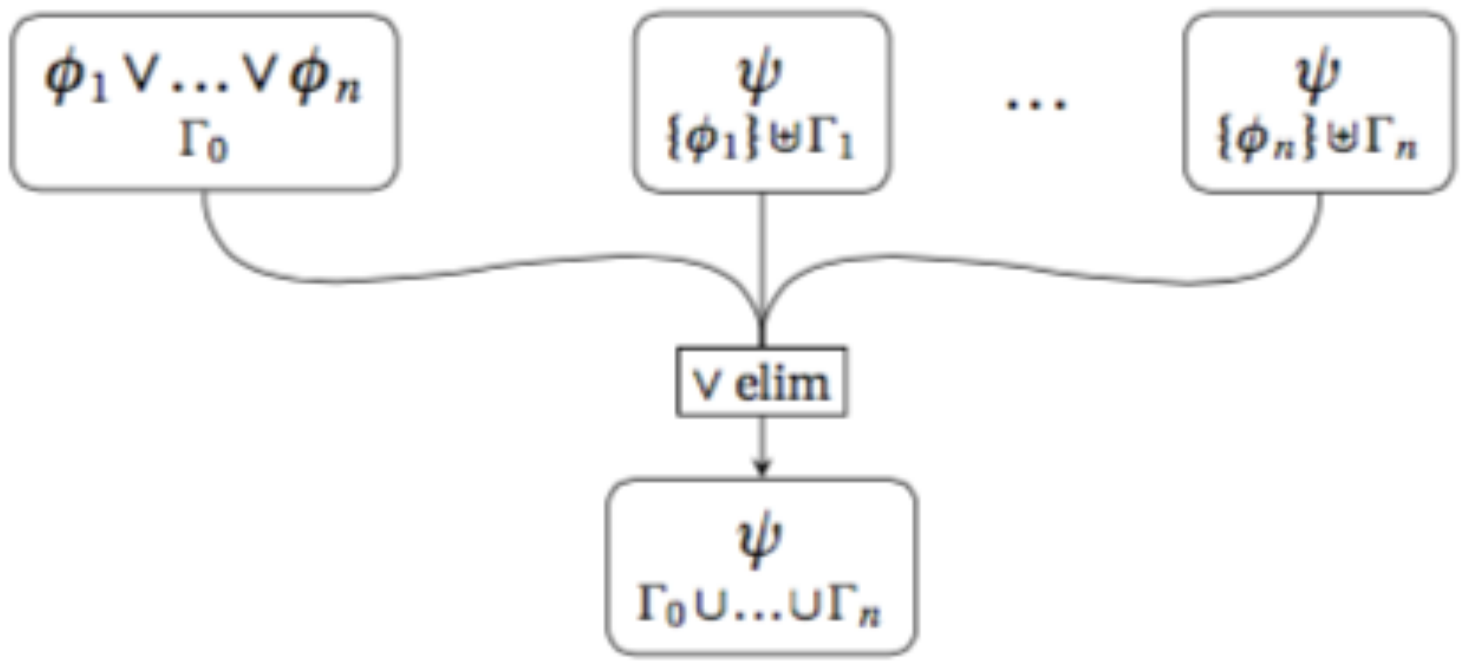
`disjunction elimination`

From ~ p. 54 in LAMA-BDLA

from each ϕ_i, then we may conclude ψ. That is, if we can, for each ϕ_i, assume ϕ_i and show that ψ follows, then we may conclude ψ from the disjunction $\phi_1 \vee \ldots \vee \phi_n$ and the derivations of ψ. There is one more subtle point, however. In the days-of-the-week example above, the conclusion that Susan has class on a weekday should not be in the scope of both the assumptions that she has class on Monday and that she has class on Tuesday; these assumptions are *discharged*. Disjunction elimination discharges each assumption ϕ_i from the line of reasoning that corresponds to that case.



(2.25)

The various Γ_i on the premises of disjunction elimination might make this rule seem more complicated than it really is. Their presence makes it clear that the only assumptions discharged from each line of reasoning is the assumption corresponding to that particular case.

From ~ p. 54 in LAMA-BDLA

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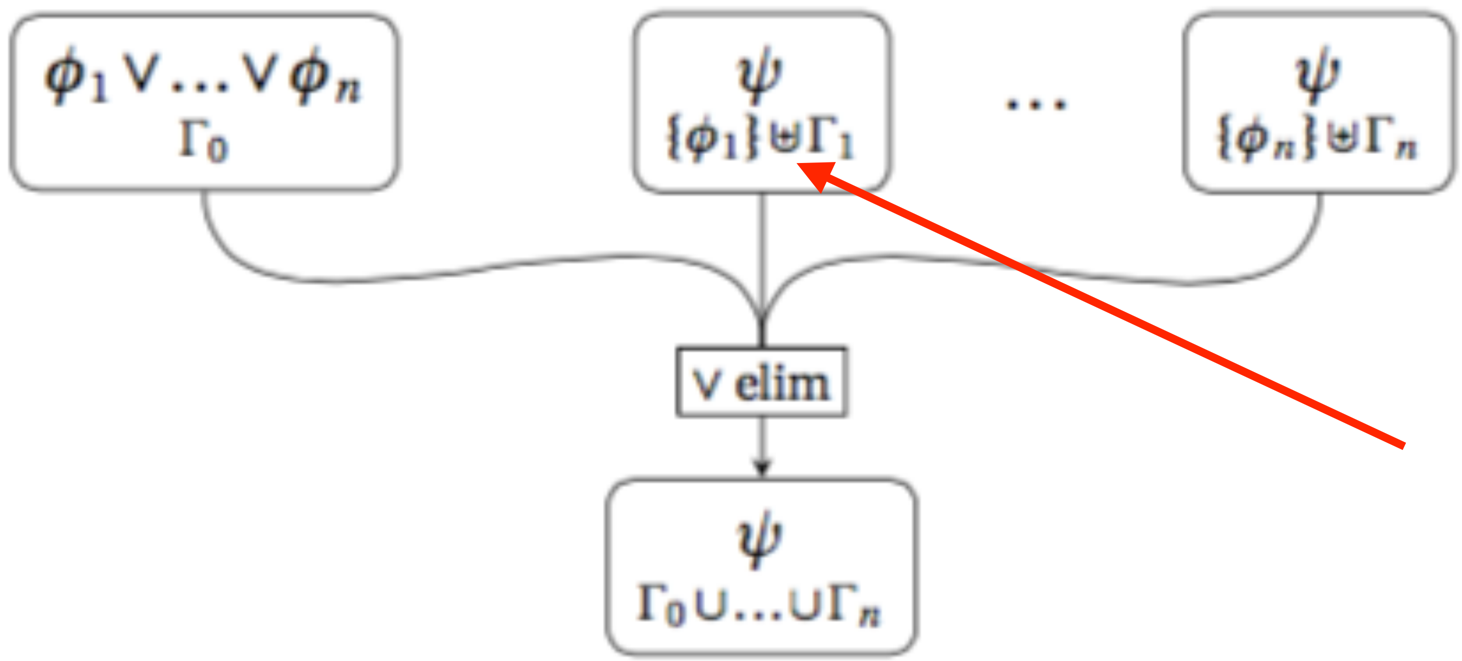

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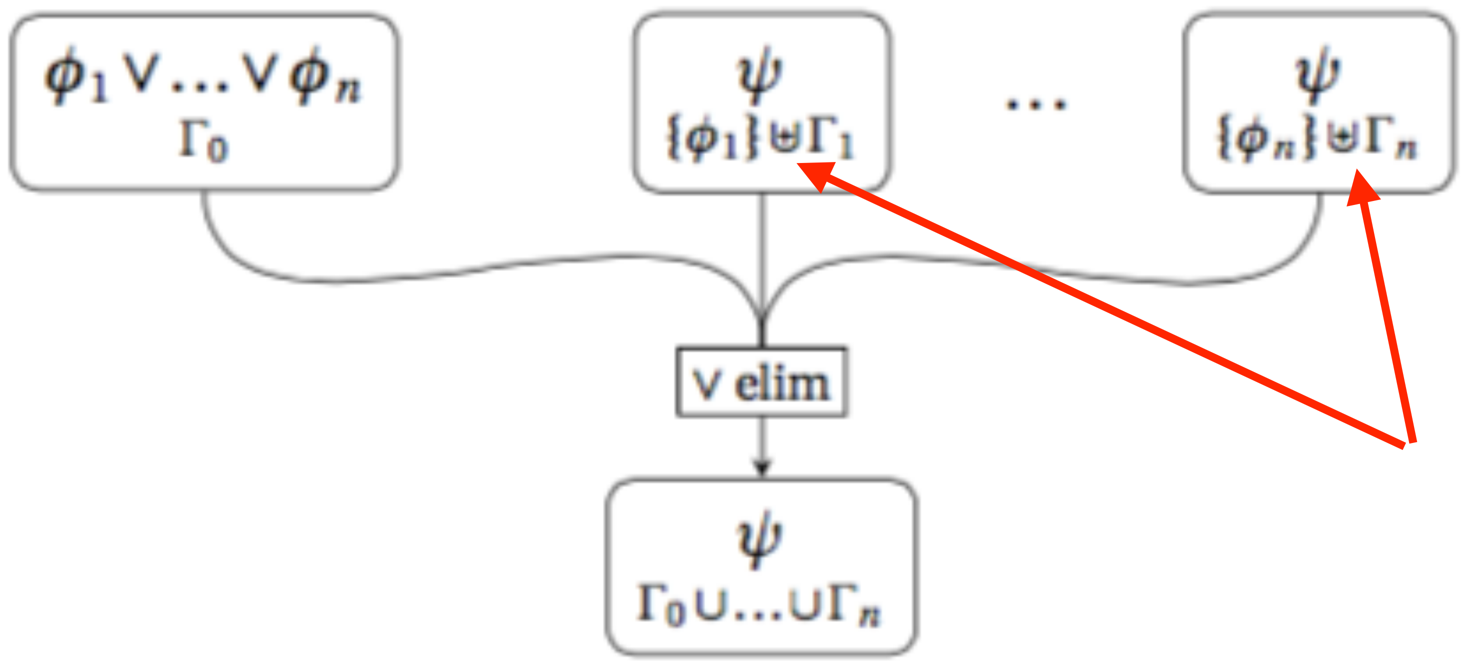
From ~ p. 54 in LAMA-BDLA

from each ϕ_i, then we may conclude ψ. That is, if we can, for each ϕ_i, assume ϕ_i and show that ψ follows, then we may conclude ψ from the disjunction $\phi_1 \vee \ldots \vee \phi_n$ and the derivations of ψ. There is one more subtle point, however. In the days-of-the-week example above, the conclusion that Susan has class on a weekday should not be in the scope of both the assumptions that she has class on Monday and that she has class on Tuesday; these assumptions are *discharged*. Disjunction elimination discharges each assumption ϕ_i from the line of reasoning that corresponds to that case.



(2.25)

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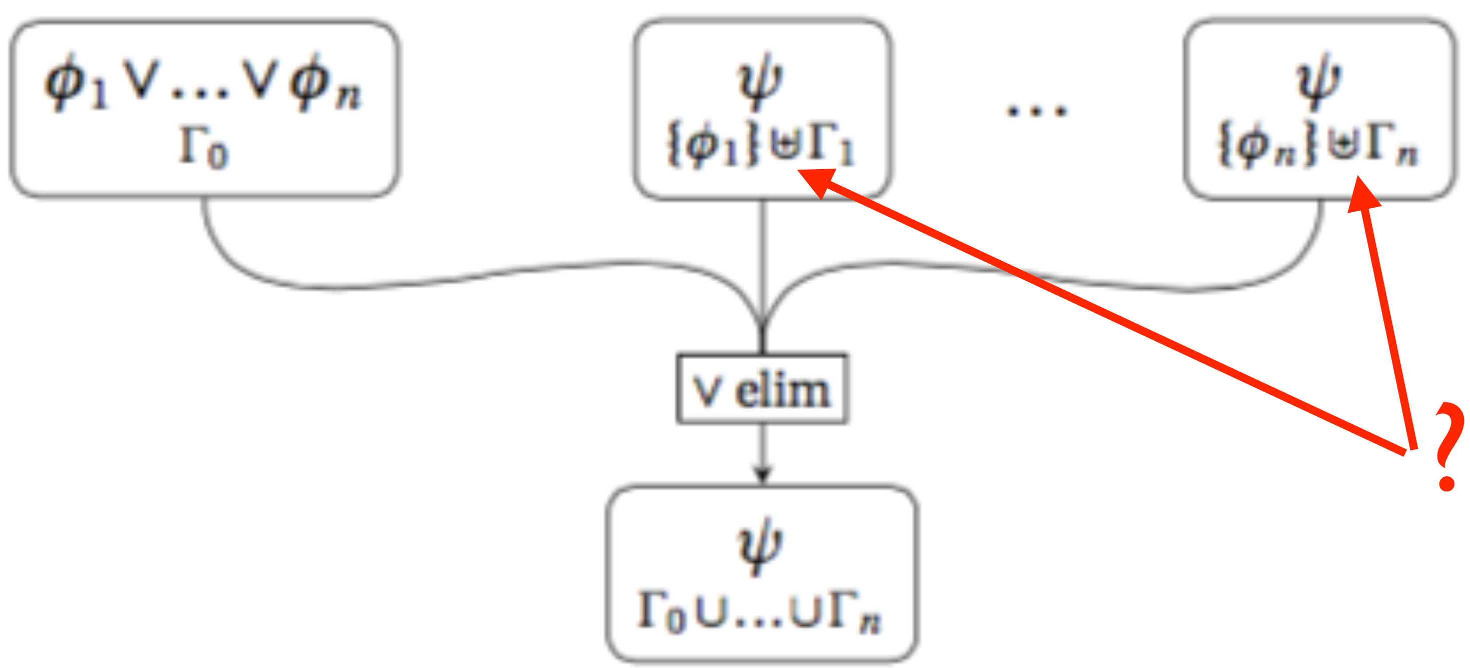

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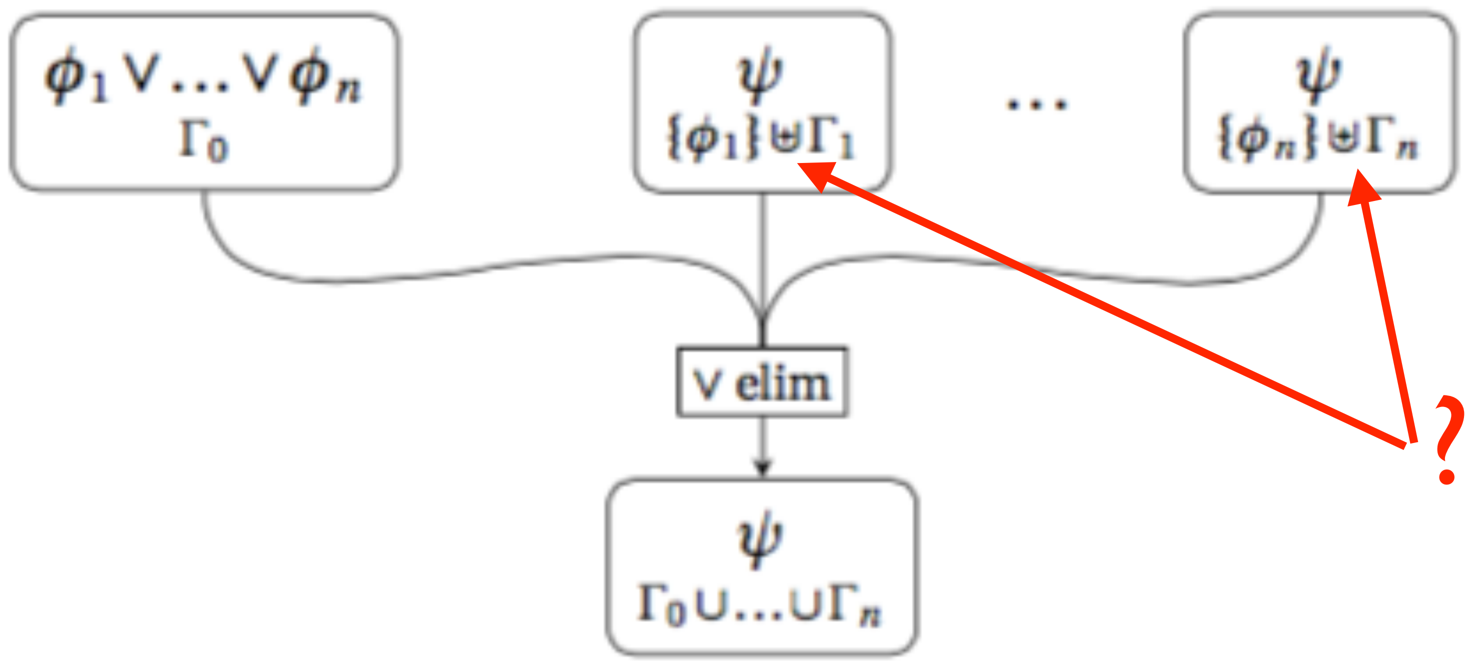

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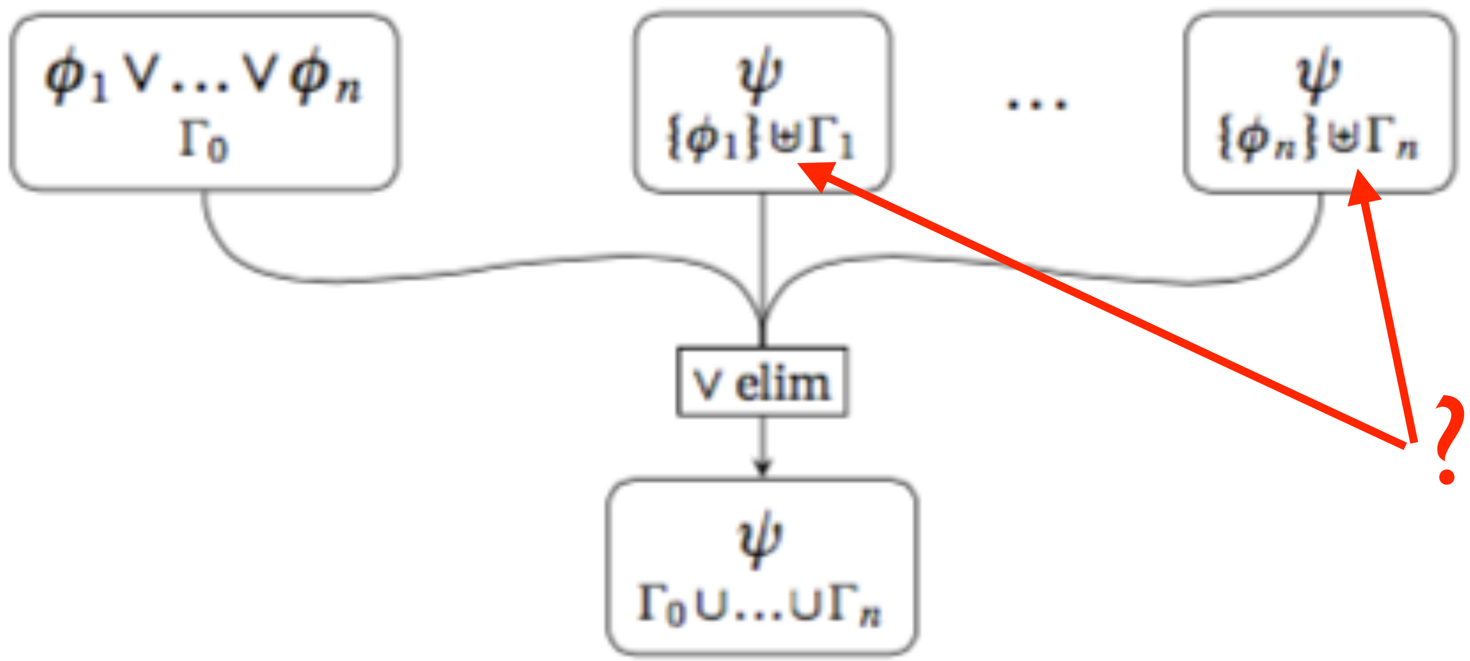

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King-Ace 2

Suppose that the following premise is true:

If there is a king in the hand, then there is an ace in the hand; or if there isn't a king in the hand, then there is an ace; but not both of these if-then statements are true.

What can you infer from this premise?

NO! ~~There is an ace in the hand.~~ NO!

In fact, what you *can* infer is that there *isn't* an ace in the hand!

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If there is a king in the hand, then there is an ace in the han[…] *hen there is* […] *stateme*[…]

Exercise (on HyperGrader™): Finish the proof in HyperSlate™ — with no remaining use of an oracle.

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In fact, what you *can* infer is that there *isn't* an ace in the hand!