

# How'd We Arrive Here?

(Selmer's Leibnizian Whirlwind History of Logic)

## Selmer Bringsjord

Rensselaer AI & Reasoning (RAIR) Lab  
Department of Cognitive Science  
Department of Computer Science  
Lally School of Management & Technology  
Rensselaer Polytechnic Institute (RPI)  
Troy, New York 12180 USA

Intro to Logic  
1/24/2019



# The Starting Code to Purchase in Bookstore

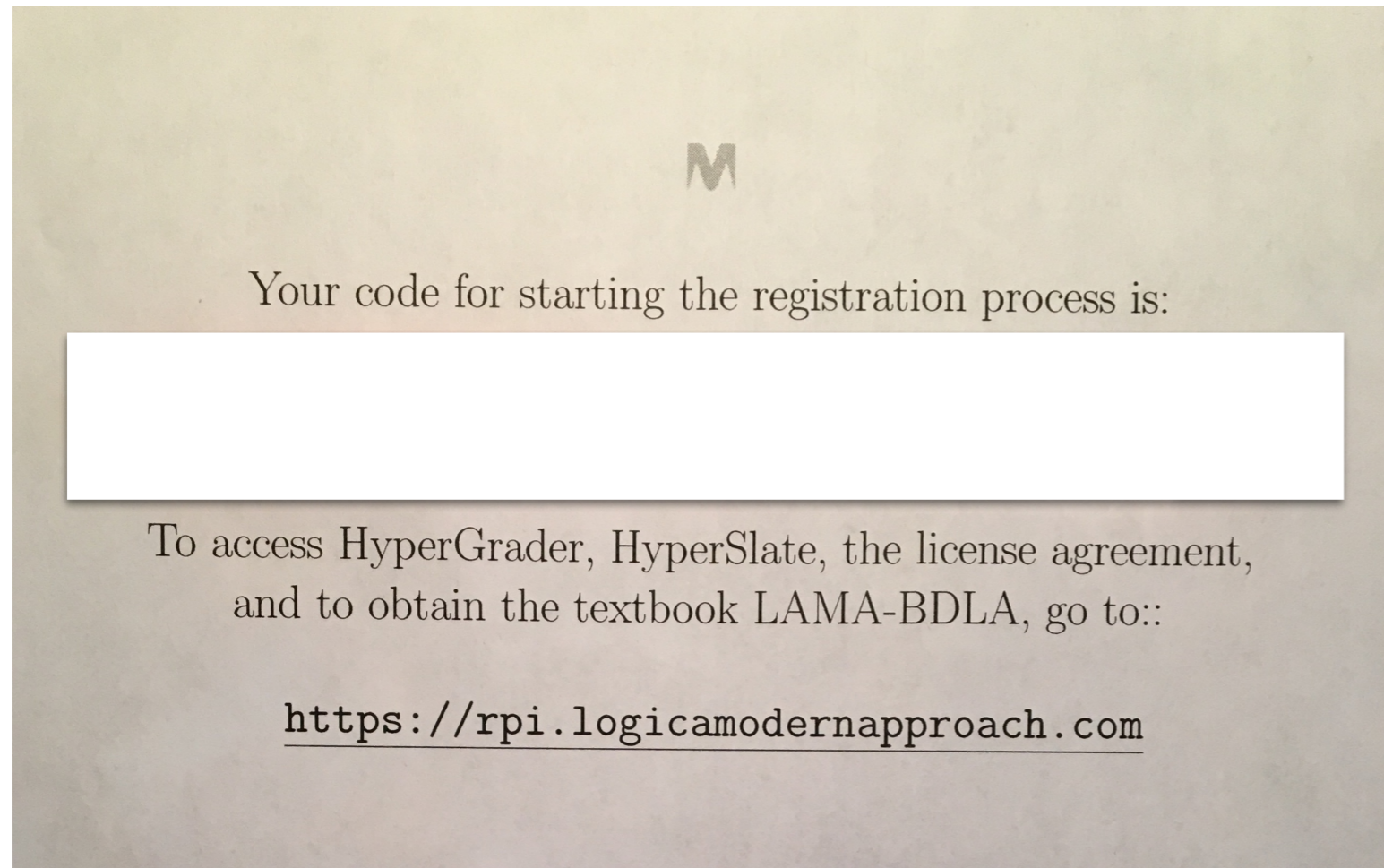
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Your code for starting the registration process is:

To access HyperGrader, HyperSlate, the license agreement,  
and to obtain the textbook LAMA-BDLA, go to::

<https://rpi.logicamodernapproach.com>

# The Starting Code to Purchase in Bookstore



Once seal broken on envelope, no return. Remember from first class, any reservations, opt for "Stanford" paradigm, with its software instead of LAMA™ paradigm!

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**Caveat; Apology; Rain Check**

# Caveat; Apology; Rain Check

LAMA-BDLA

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LAMA-BDLA

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LAMA-BDLA





# Caveat; Apology; Rain Check

LAMA-BDLA



LAMA-BIL, a bit.



# The Monty Hall Problem



\$1M





# The Monty Hall Problem



\$1M





# The Monty Hall Problem



\$1M





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\$1M





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# The Monty Hall Problem



\$1M



# MHP Defined

Jones has come to a game show, and finds himself thereon selected to play a game on national TV with the show's suave host, Full Monty. Jones is told correctly by Full that hidden behind one of three closed, opaque doors facing the two of them is \$1,000,000, while behind each of the other two is a feculent, obstreperous llama whose value on the open market is charitably pegged at \$1. Full reminds Jones that this is a game, and a fair one, and that if Jones ends up selecting the door with \$1M behind it, all that money will indeed be his. (Jones' net worth has nearly been exhausted by his expenditures in traveling to the show.) Full also reminds Jones that he (= Full) knows what's behind each door, fixed in place until the game ends.

Full asks Jones to select which door he wants the contents of. Jones says, "Door 1." Full then says: "Hm. Okay. Part of this game is my revealing at this point what's behind one of the doors you didn't choose. So ... let me show you what's behind Door 3." Door 3 opens to reveal a very unsavory llama. Full now to Jones: "Do you want to switch to Door 2, or stay with Door 1? You'll get what's behind the door of your choice, and our game will end." Full looks briefly into the camera, directly.

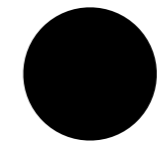
(PI.1) What should Jones do if he's rational?

(PI.2) Prove that your answer is correct. (Diagrammatic proofs are allowed.)

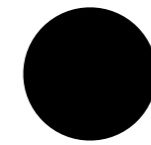
(PI.3) A quantitative hedge fund manager with a PhD in finance from Harvard zipped this email off to Full before Jones made his decision re. switching or not: "Switching would be a royal waste of time (and time is money!). Jones hasn't a doggone clue what's behind Door 1 or Door 2, and it's obviously a 50/50 chance to win whether he stands firm or switches. So the chap shouldn't switch!" Is the fund manager right? Prove that your diagnosis is correct.

(PI.4) Can these answers and proofs be exclusively Bayesian in nature?





2019



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# DCEC\*

## Syntax

$S ::=$  Object | Agent | Self  $\square$  Agent | ActionType | Action  $\sqsubseteq$  Event |  
Moment | Boolean | Fluent | Numeric

$action$  : Agent  $\times$  ActionType  $\rightarrow$  Action

$initially$  : Fluent  $\rightarrow$  Boolean

$holds$  : Fluent  $\times$  Moment  $\rightarrow$  Boolean

$happens$  : Event  $\times$  Moment  $\rightarrow$  Boolean

$clipped$  : Moment  $\times$  Fluent  $\times$  Moment  $\rightarrow$  Boolean

$f ::=$   $initiates$  : Event  $\times$  Fluent  $\times$  Moment  $\rightarrow$  Boolean

$terminates$  : Event  $\times$  Fluent  $\times$  Moment  $\rightarrow$  Boolean

$prior$  : Moment  $\times$  Moment  $\rightarrow$  Boolean

$interval$  : Moment  $\times$  Boolean

$*$  : Agent  $\rightarrow$  Self

$payoff$  : Agent  $\times$  ActionType  $\times$  Moment  $\rightarrow$  Numeric

$t ::= x : S \mid c : S \mid f(t_1, \dots, t_n)$

$t$  : Boolean  $\mid \neg\phi \mid \phi \wedge \psi \mid \phi \vee \psi \mid$

$\mathbf{P}(a, t, \phi) \mid \mathbf{K}(a, t, \phi) \mid \mathbf{C}(t, \phi) \mid \mathbf{S}(a, b, t, \phi) \mid \mathbf{S}(a, t, \phi)$

$\phi ::= \mathbf{B}(a, t, \phi) \mid \mathbf{D}(a, t, holds(f, t')) \mid \mathbf{I}(a, t, happens(action(a^*, \alpha), t'))$

$\mathbf{O}(a, t, \phi, happens(action(a^*, \alpha), t'))$

## Rules of Inference

$\frac{}{\mathbf{C}(t, \mathbf{P}(a, t, \phi) \rightarrow \mathbf{K}(a, t, \phi))} [R_1] \quad \frac{}{\mathbf{C}(t, \mathbf{K}(a, t, \phi) \rightarrow \mathbf{B}(a, t, \phi))} [R_2]$

$\frac{\mathbf{C}(t, \phi) \ t \leq t_1 \dots t \leq t_n}{\mathbf{K}(a_1, t_1, \dots, \mathbf{K}(a_n, t_n, \phi) \dots)} [R_3] \quad \frac{\mathbf{K}(a, t, \phi)}{\phi} [R_4]$

$\frac{}{\mathbf{C}(t, \mathbf{K}(a, t_1, \phi_1 \rightarrow \phi_2)) \rightarrow \mathbf{K}(a, t_2, \phi_1) \rightarrow \mathbf{K}(a, t_3, \phi_2)} [R_5]$

$\frac{}{\mathbf{C}(t, \mathbf{B}(a, t_1, \phi_1 \rightarrow \phi_2)) \rightarrow \mathbf{B}(a, t_2, \phi_1) \rightarrow \mathbf{B}(a, t_3, \phi_2)} [R_6]$

$\frac{}{\mathbf{C}(t, \mathbf{C}(t_1, \phi_1 \rightarrow \phi_2)) \rightarrow \mathbf{C}(t_2, \phi_1) \rightarrow \mathbf{C}(t_3, \phi_2)} [R_7]$

$\frac{}{\mathbf{C}(t, \forall x. \phi \rightarrow \phi[x \rightarrow t])} [R_8] \quad \frac{}{\mathbf{C}(t, \phi_1 \leftrightarrow \phi_2 \rightarrow \neg\phi_2 \rightarrow \neg\phi_1)} [R_9]$

$\frac{}{\mathbf{C}(t, [\phi_1 \wedge \dots \wedge \phi_n \rightarrow \phi] \rightarrow [\phi_1 \rightarrow \dots \rightarrow \phi_n \rightarrow \psi])} [R_{10}]$

$\frac{\mathbf{B}(a, t, \phi) \ \phi \rightarrow \psi}{\mathbf{B}(a, t, \psi)} [R_{11a}] \quad \frac{\mathbf{B}(a, t, \phi) \ \mathbf{B}(a, t, \psi)}{\mathbf{B}(a, t, \psi \wedge \phi)} [R_{11b}]$

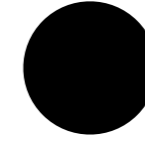
$\frac{\mathbf{S}(s, h, t, \phi)}{\mathbf{B}(h, t, \mathbf{B}(s, t, \phi))} [R_{12}]$

$\frac{\mathbf{I}(a, t, happens(action(a^*, \alpha), t'))}{\mathbf{P}(a, t, happens(action(a^*, \alpha), t))} [R_{13}]$

$\mathbf{B}(a, t, \phi) \ \mathbf{B}(a, t, \mathbf{O}(a^*, t, \phi, happens(action(a^*, \alpha), t')))$

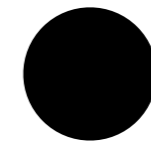
$\frac{\mathbf{O}(a, t, \phi, happens(action(a^*, \alpha), t'))}{\mathbf{K}(a, t, \mathbf{I}(a^*, t, happens(action(a^*, \alpha), t')))} [R_{14}]$

$\frac{\phi \leftrightarrow \psi}{\mathbf{O}(a, t, \phi, \gamma) \leftrightarrow \mathbf{O}(a, t, \psi, \gamma)} [R_{15}]$



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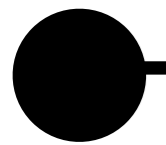
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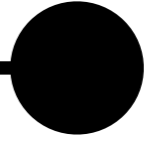


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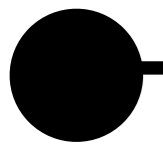


350 BC



2019

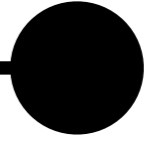
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350 BC



Euclid



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# Euclidean “Magic”

**Theorem:** There are infinitely many primes.

**Proof:** We take an indirect route. Let  $\Pi = p_1 = 2, p_2 = 3, p_3 = 5, \dots, p_k$  be a finite, exhaustive consecutive sequence of prime numbers. Next, let  $\mathbf{M}_\Pi$  be  $p_1 \times p_2 \times \dots \times p_k$ , and set  $\mathbf{M}'_\Pi$  to  $\mathbf{M}_\Pi + 1$ . Either  $\mathbf{M}'_\Pi$  is prime, or not; we thus have two (exhaustive) cases to consider.

- C1 Suppose  $\mathbf{M}'_\Pi$  is prime. In this case we immediately have a prime number beyond any in  $\Pi$  — contradiction!
- C2 Suppose on the other hand that  $\mathbf{M}'_\Pi$  is *not* prime. Then some prime  $p$  divides  $\mathbf{M}'_\Pi$ . (Why?) Now,  $p$  itself is either in  $\Pi$ , or not; we hence have two sub-cases. Supposing that  $p$  is in  $\Pi$  entails that  $p$  divides  $\mathbf{M}_\Pi$ . But we are operating under the supposition that  $p$  divides  $\mathbf{M}'_\Pi$  as well. This implies that  $p$  divides 1, which is absurd (a contradiction). Hence the prime  $p$  is outside  $\Pi$ .

Hence for *any* such list  $\Pi$ , there is a prime outside the list. That is, there are infinitely many primes. **QED**

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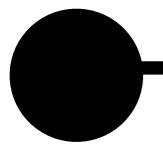
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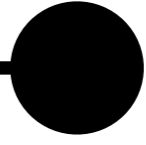
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350 BC

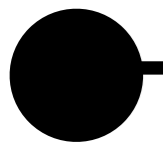


Euclid



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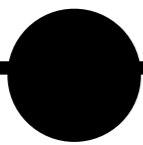
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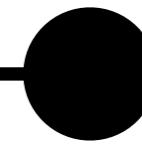
350 BC



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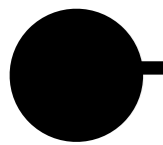
300 BC



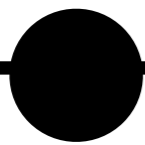
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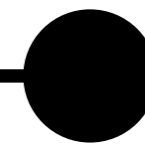




350 BC



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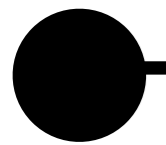
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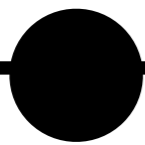
I don't believe in magic! Why exactly is that so convincing? What exactly is he doing?!?



350 BC



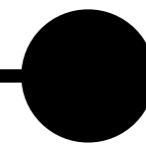
Euclid



300 BC



*Organon*



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He's using syllogisms!

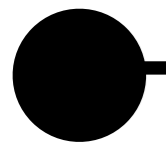
E.g.,

All As are Bs.

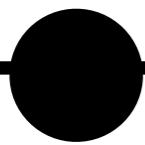
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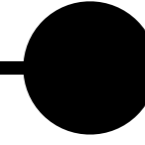
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350 BC



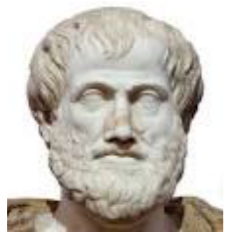
300 BC



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Euclid



*Organon*

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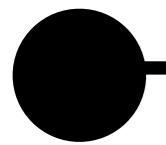
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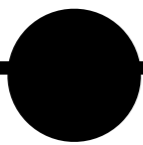
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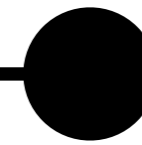


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Balderdash!

He's using syllogisms!



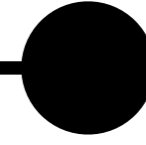
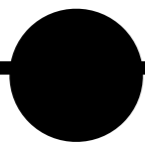
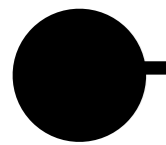
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350 BC

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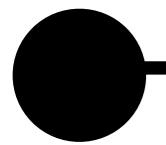


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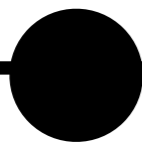
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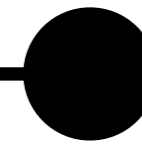
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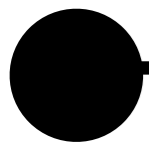
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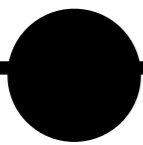
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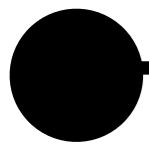


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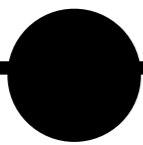
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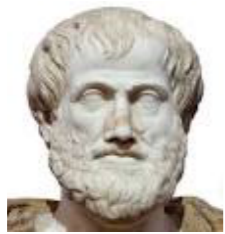
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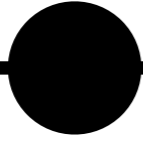
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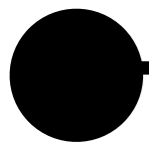
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1666

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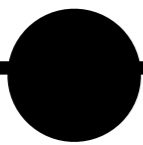




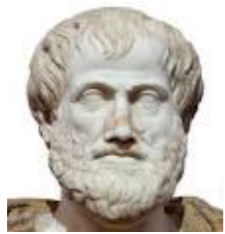
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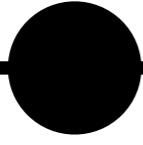
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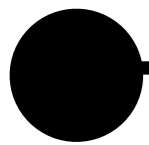


1666



Leibniz

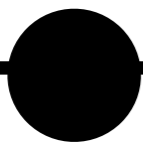
*Intro to (Formal) Logic @ RPI*



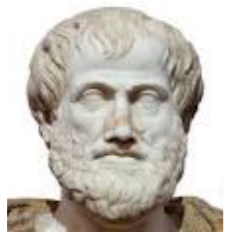
350 BC



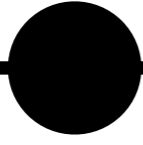
Euclid



300 BC



*Organon*



1666

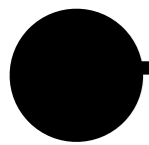


Leibniz



*Intro to (Formal) Logic @ RPI*

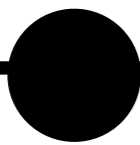
“Universal  
Computational  
Logic”



350 BC



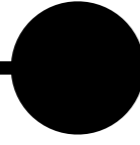
Euclid



300 BC



*Organon*



1666

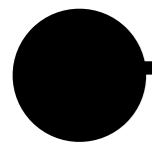


Leibniz



*Intro to (Formal) Logic @ RPI*

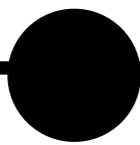
# “Universal Computational Logic”



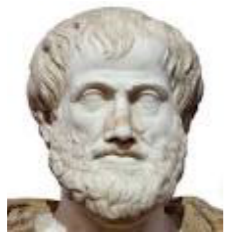
350 BC



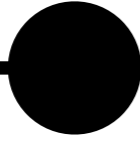
Euclid



300 BC



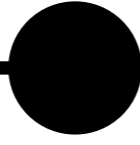
*Organon*



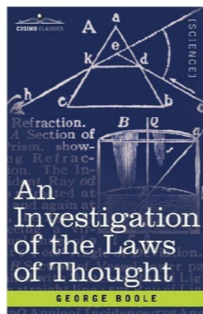
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Leibniz

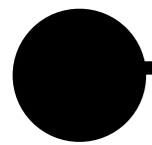


1854



*Intro to (Formal) Logic @ RPI*

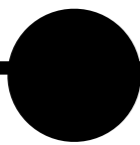
# “Universal Computational Logic”



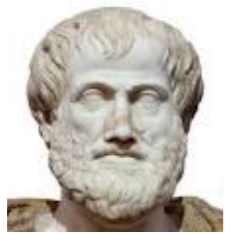
350 BC



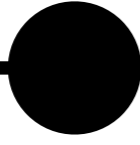
Euclid



300 BC



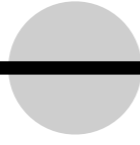
*Organon*



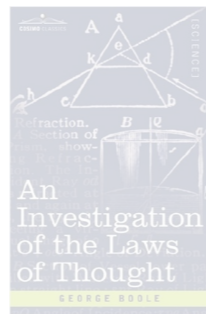
1666



Leibniz



1854

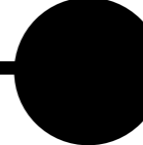
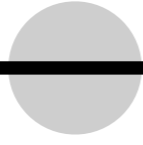
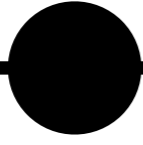
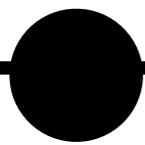
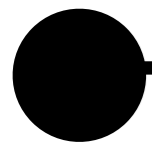


*Intro to (Formal) Logic @ RPI*

“Universal Computational Logic”



Logic Theorist  
(birth of modern logicist AI)



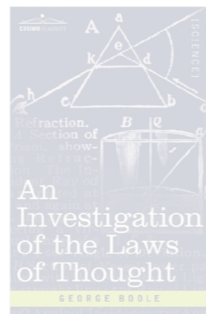
350 BC

300 BC

1666

1854

1956



Euclid

*Organon*

Leibniz

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Simon

*Intro to (Formal) Logic @ RPI*

# “Astonishing” Logic Theorist Proof @ Dawn of AI

# “Astonishing” Logic Theorist Proof @ Dawn of AI

1	$(\phi \vee \phi) \rightarrow \phi$	axiom
2	$(\neg\phi \vee \neg\phi) \rightarrow \neg\phi$	substitution
3	$(\phi \rightarrow \neg\phi) \rightarrow \neg\phi$	a “replacement rule”
4	$(A \rightarrow \neg A) \rightarrow \neg A$	substitution



# “Astonishing” Logic Theorist Proof @ Dawn of AI

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At dawn of AI: 10 seconds.

# “Astonishing” Logic Theorist Proof @ Dawn of AI

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2	$(\neg\phi \vee \neg\phi) \rightarrow \neg\phi$	substitution
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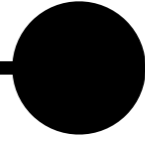
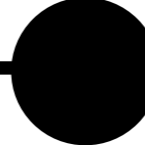
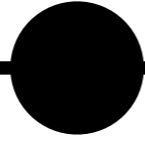
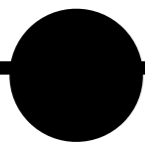
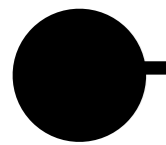
At dawn of AI: 10 seconds.

AI of today: vanishingly small amount of time.

# “Universal Computational Logic”



## Logic Theorist (birth of modern logicist AI)



350 BC

300 BC

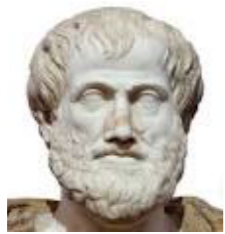
1854

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2019



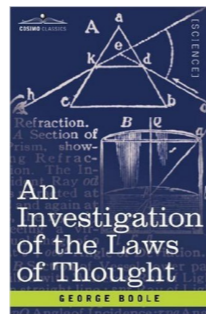
Euclid



*Organon*



Leibniz



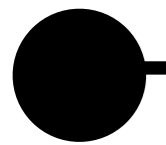
Simon

*Intro to (Formal) Logic @ RPI*

“Universal  
Computational  
Logic”



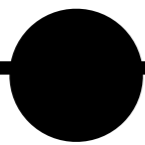
Logic Theorist  
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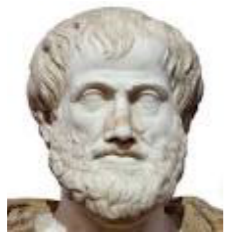
350 BC



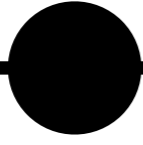
Euclid



300 BC



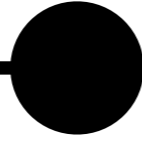
*Organon*



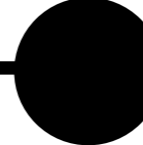
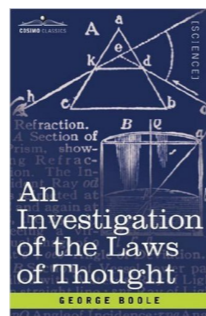
1666



Leibniz



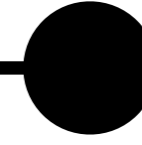
1854



1956



Simon



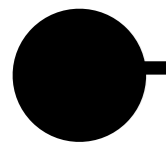
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*Intro to (Formal) Logic @ RPI*

“Universal  
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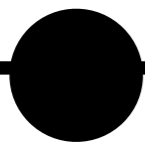
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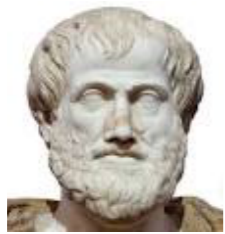
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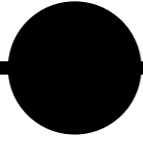
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300 BC



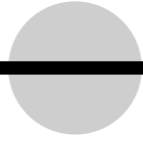
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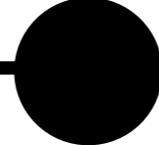
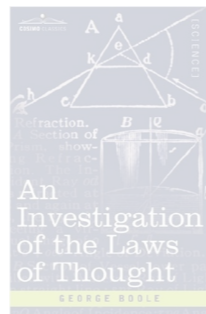
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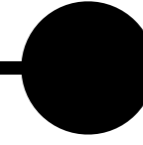
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Simon



2019

*Intro to (Formal) Logic @ RPI*

“Universal  
Computational  
Logic”



Logic Theorist  
(birth of modern logicist AI)



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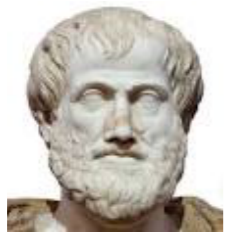
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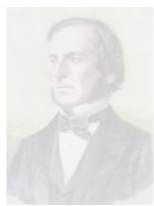
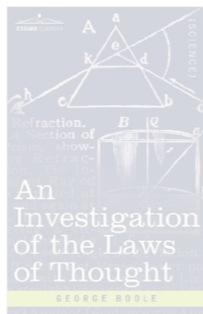
Euclid



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Leibniz



Simon

*Intro to (Formal) Logic @ RPI*

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“Universal  
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Logic Theorist  
(birth of modern logicist AI)



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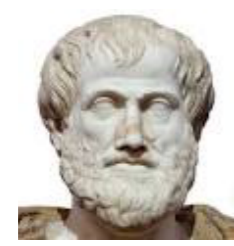
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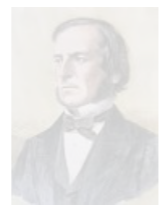
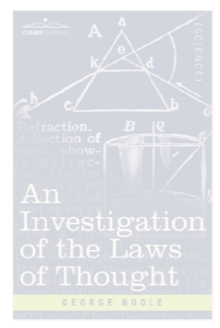
Euclid



*Organon*



Leibniz



Simon

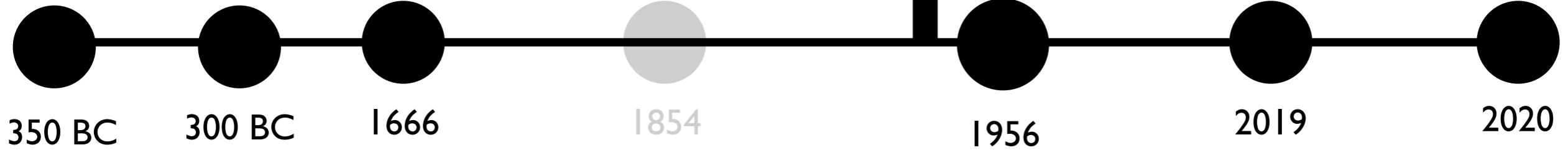
*Intro to (Formal) Logic @ RPI*

# Entscheidungsproblem

“Universal Computational Logic”



Logic Theorist  
(birth of modern logicist AI)



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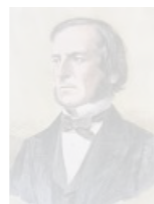
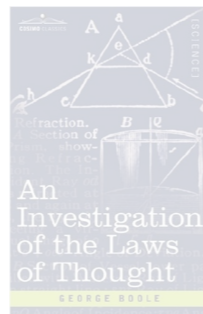
Euclid



Organon



Leibniz



Simon

Intro to (Formal) Logic @ RPI

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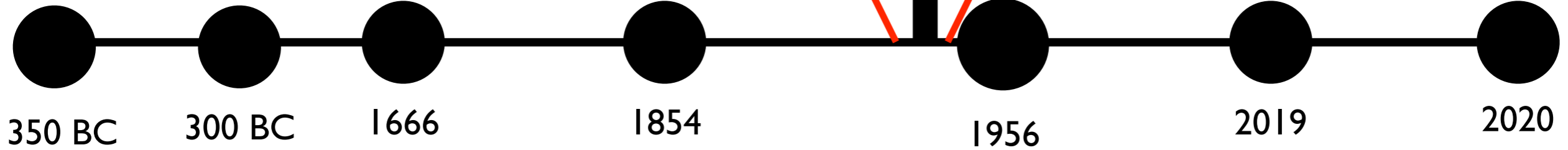
# Entscheidungsproblem



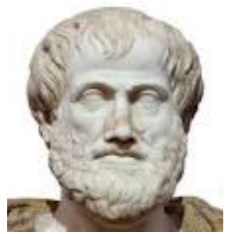
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Logic Theorist  
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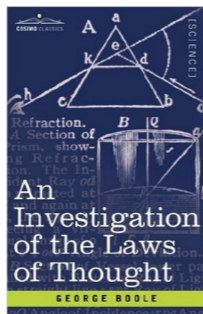
Euclid



Organon



Leibniz



Simon

Intro to Logic @ RPI

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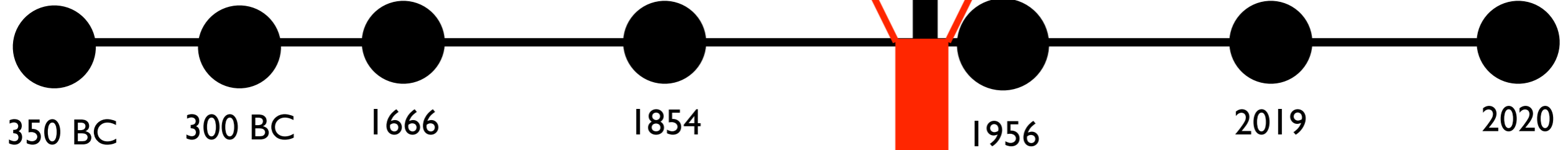
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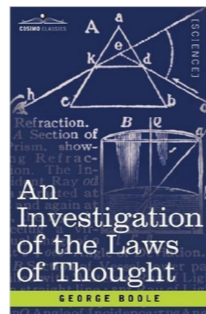
Euclid



Organon



Leibniz



Simon

Intro to Logic @ RPI

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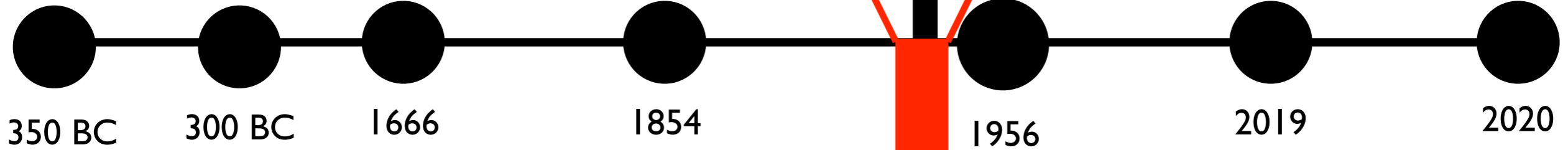
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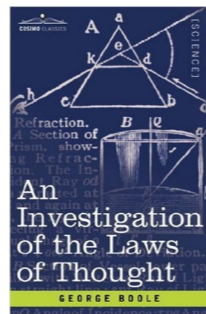
Euclid



Organon



Leibniz



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Simon

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Frege

Intro to Logic @ RPI

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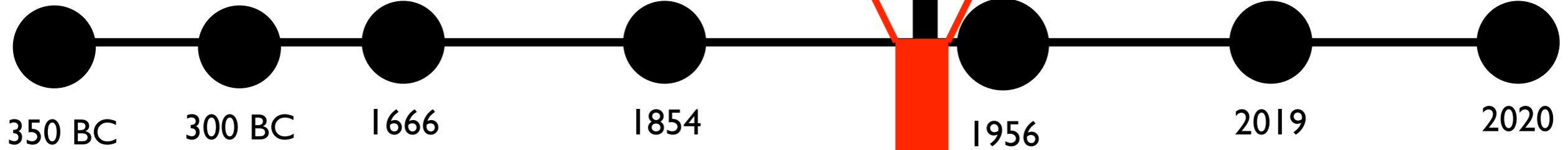
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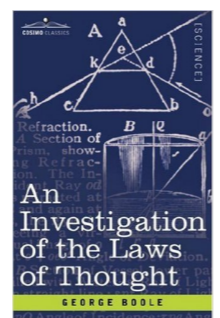
Euclid



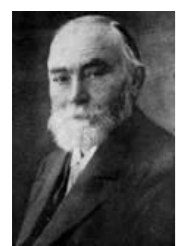
Organon



Leibniz



Simon



Frege

Exceeds Leibniz & de-mystifies Euclid: the “compellingness” of these proofs consists in their being, at bottom, formal proofs in first-order logic (FOL).

Intro to Logic @ RPI

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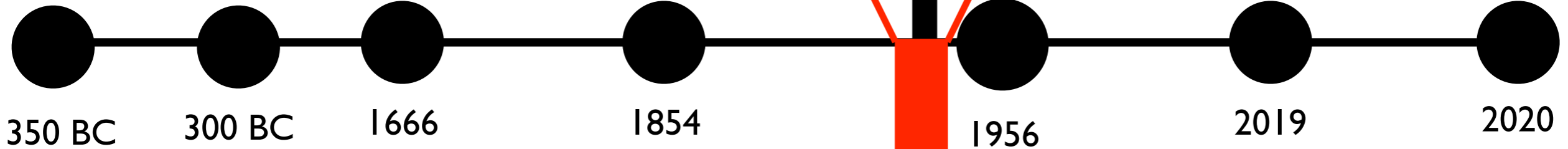
# Entscheidungsproblem



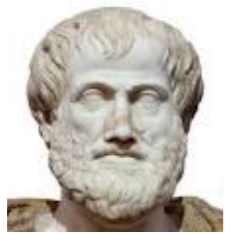
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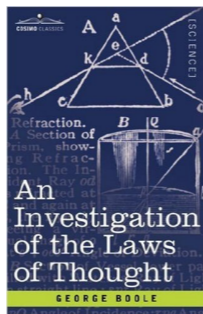
Euclid



Organon



Leibniz



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Church



Simon

Intro to Logic @ RPI

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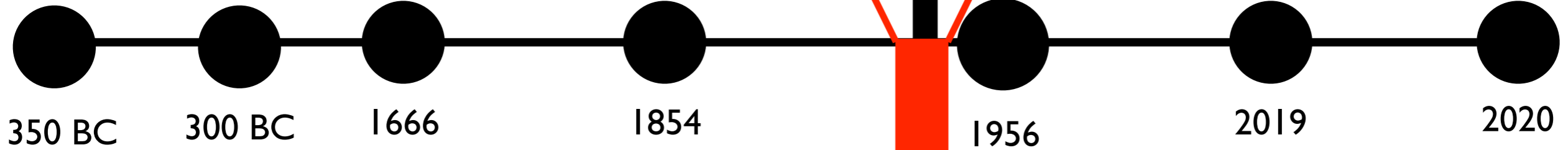
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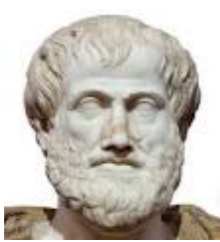
“Universal Computational Logic”



Logic Theorist  
(birth of modern logicist AI)



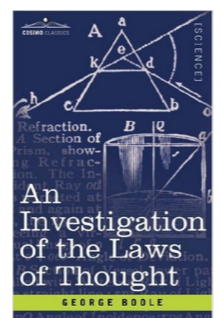
Euclid



Organon



Leibniz



Boole



Simon



Church



Turing



Frege

Exceeds Leibniz & de-mystifies Euclid: the “compellingness” of these proofs consists in their being, at bottom, formal proofs in first-order logic (FOL).

Intro to Logic @ RPI

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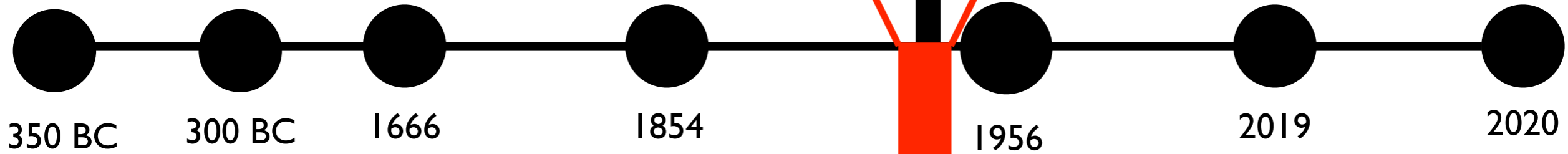
# Entscheidungsproblem



“Universal Computational Logic”



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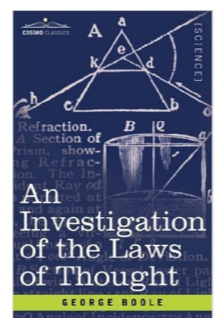
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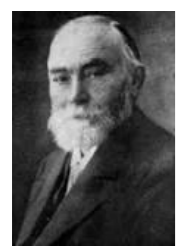


1854



Simon

1956



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Church



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Post

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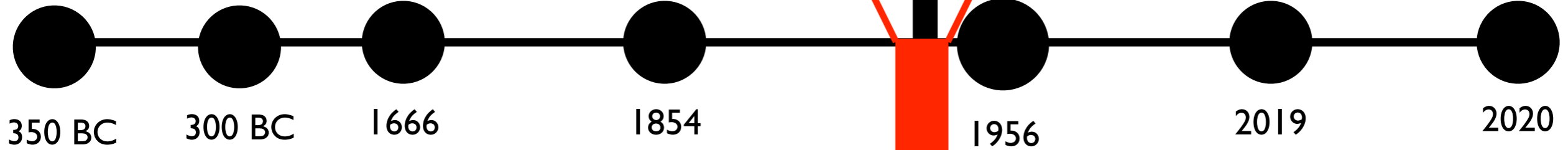
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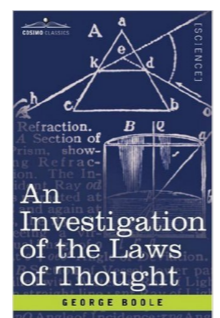
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Church



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Post

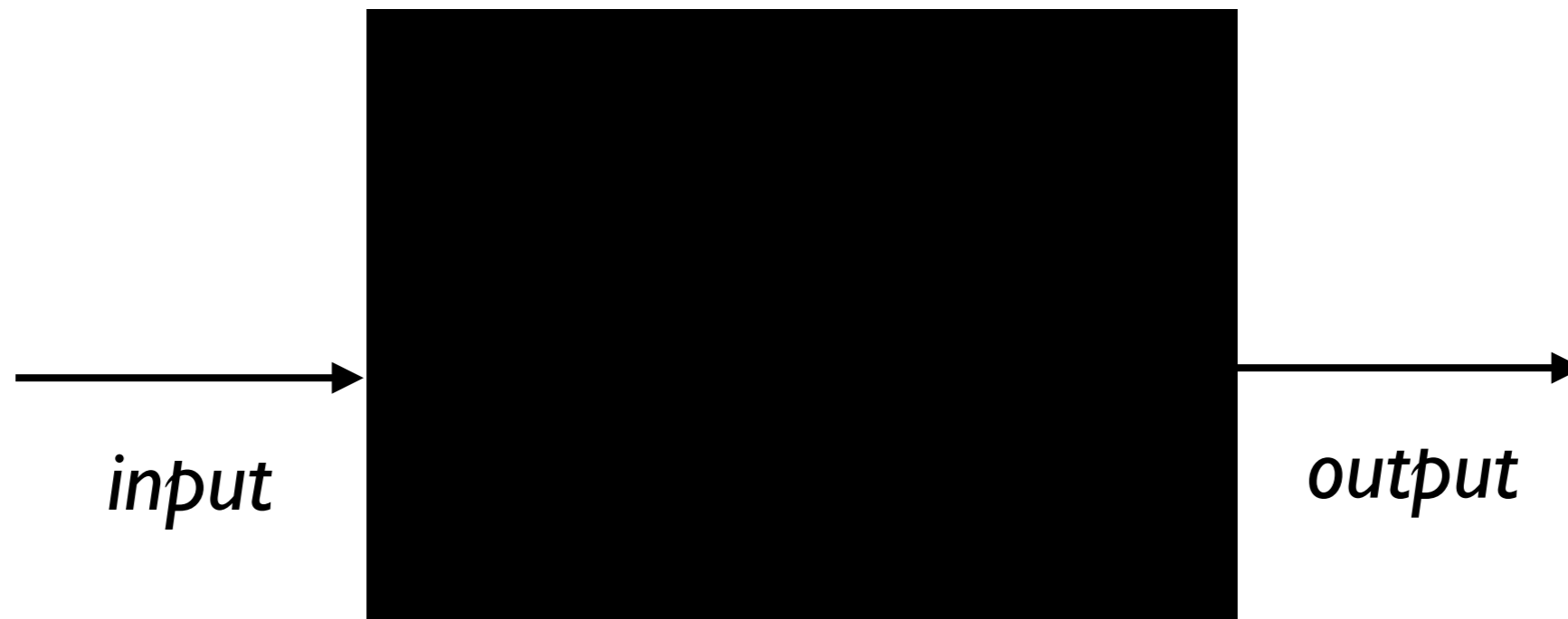
Intro to Logic @ RPI

Here’s what a computer is, and given that, sorry, the Entscheidungsproblem can’t be solved by such a machine!

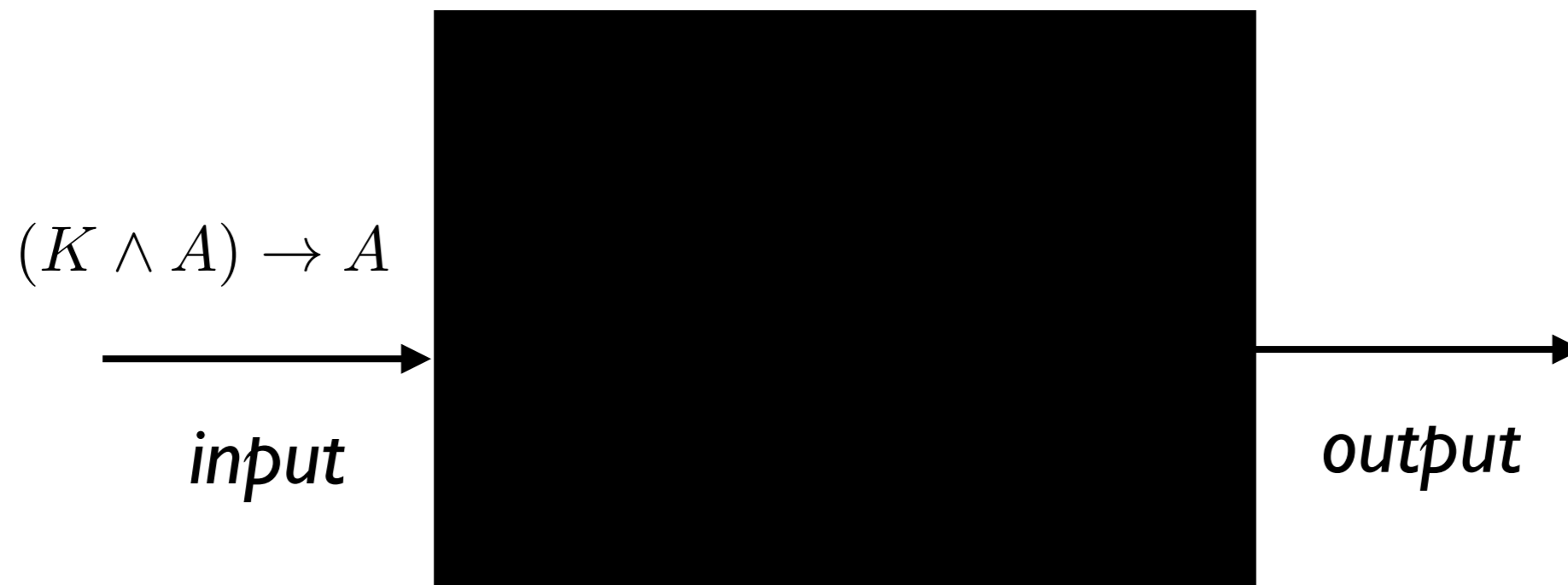
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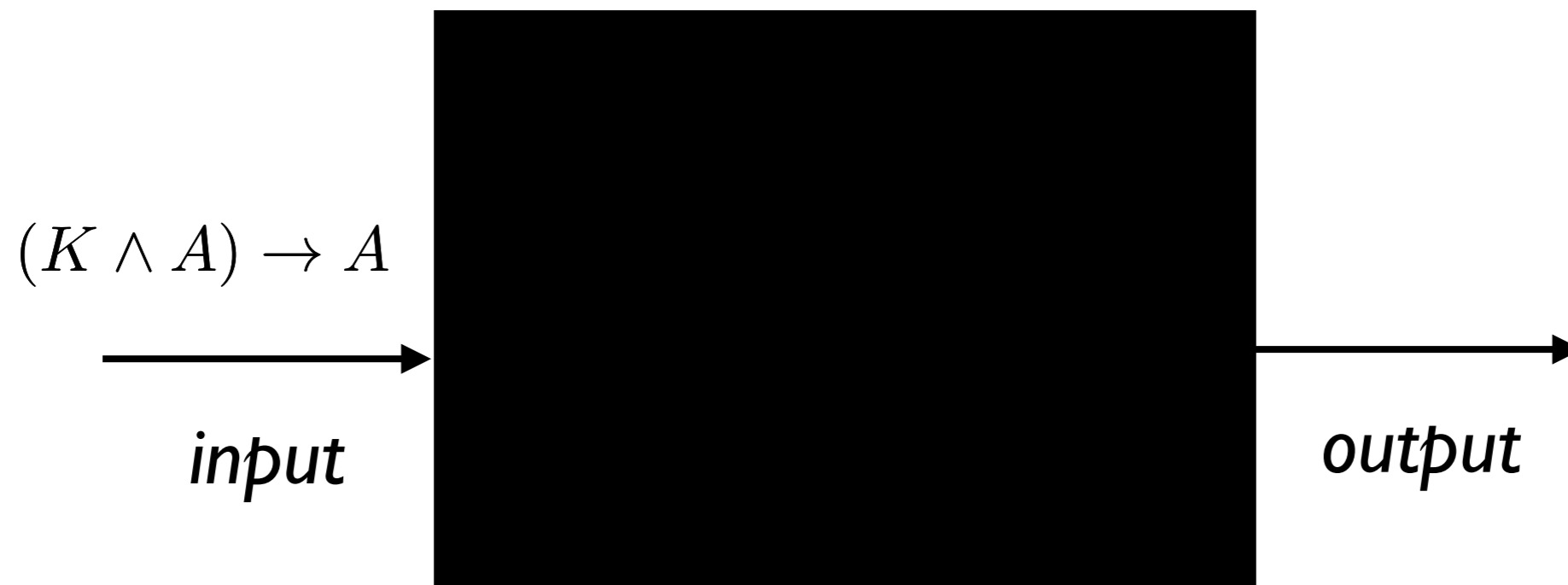
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( $\text{THEOREM}_{\text{PC}}$ )  
for the Propositional Calculus



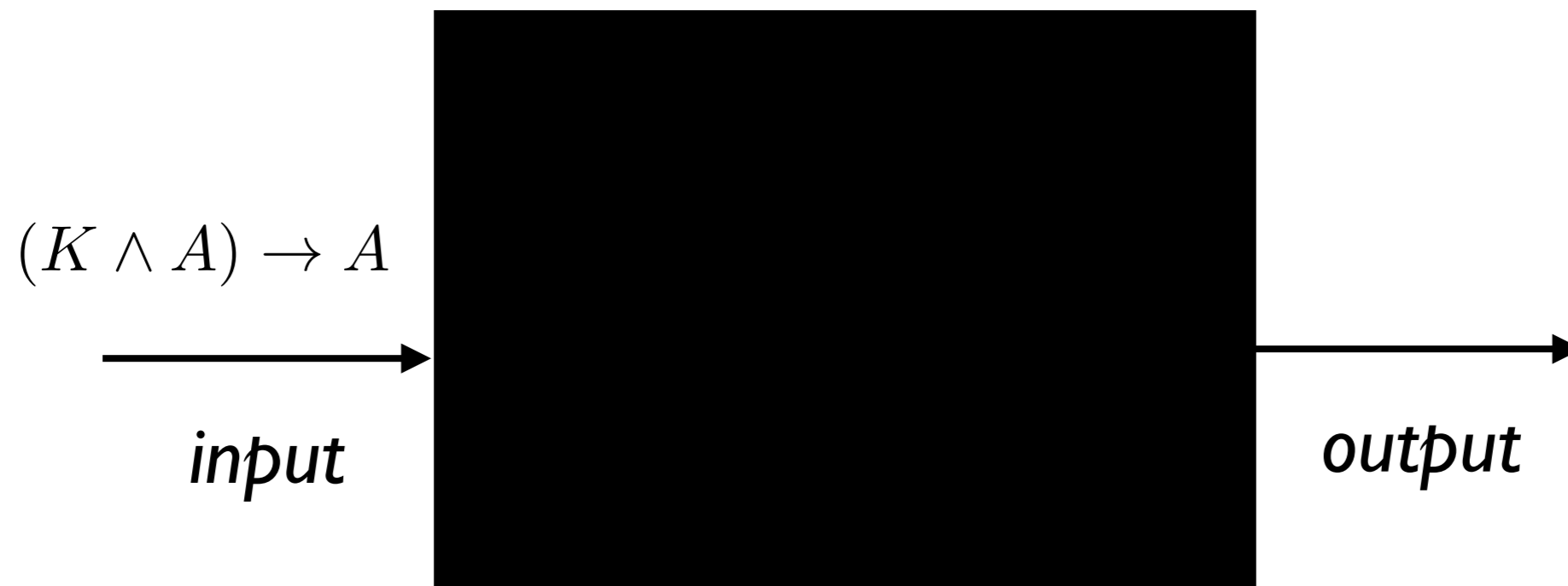
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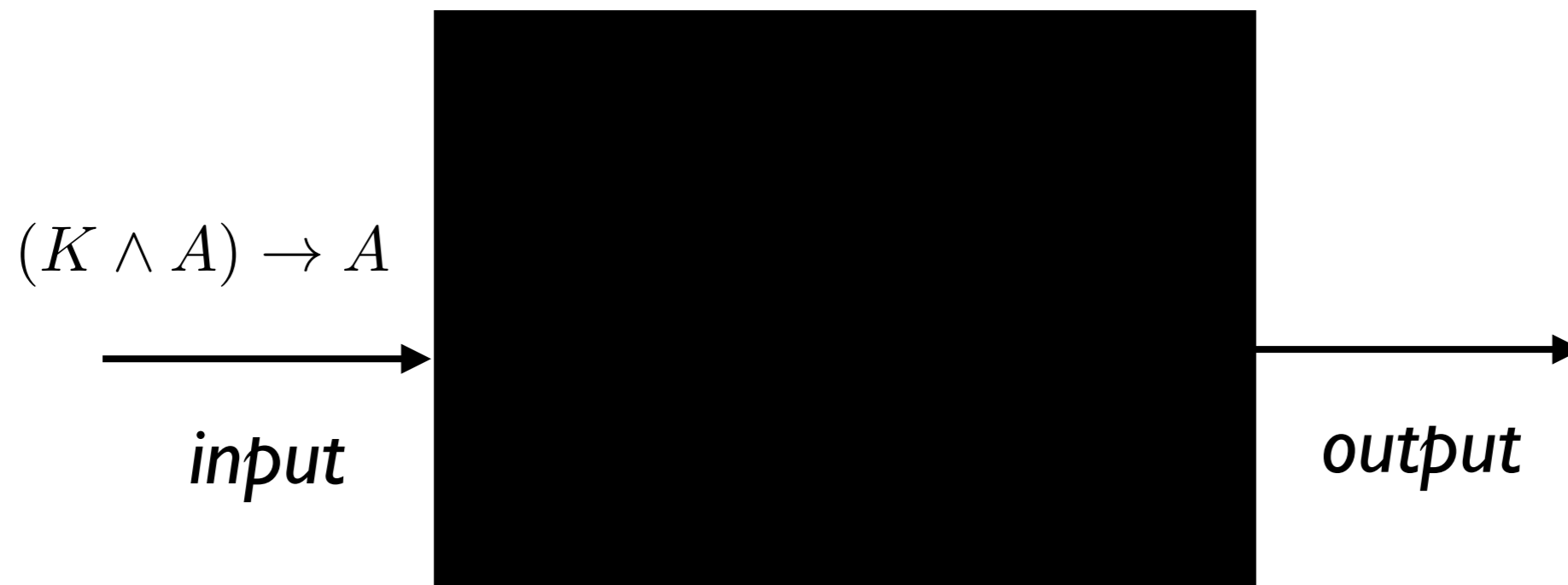
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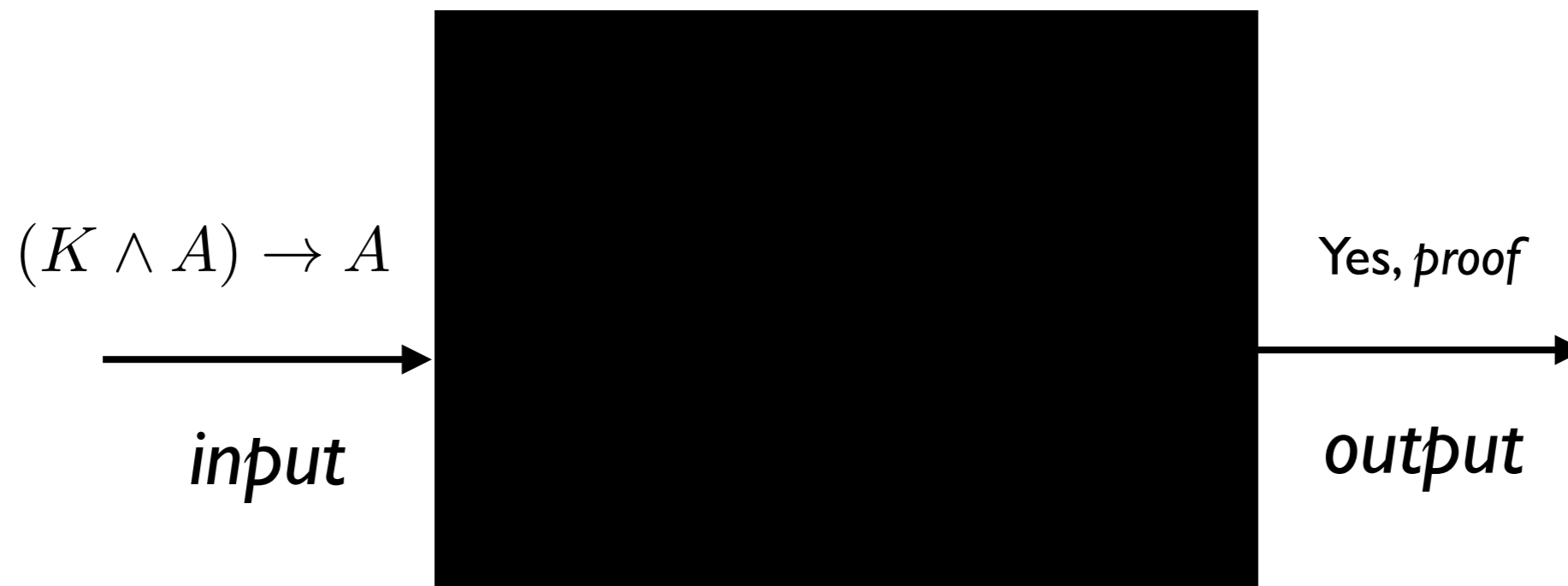
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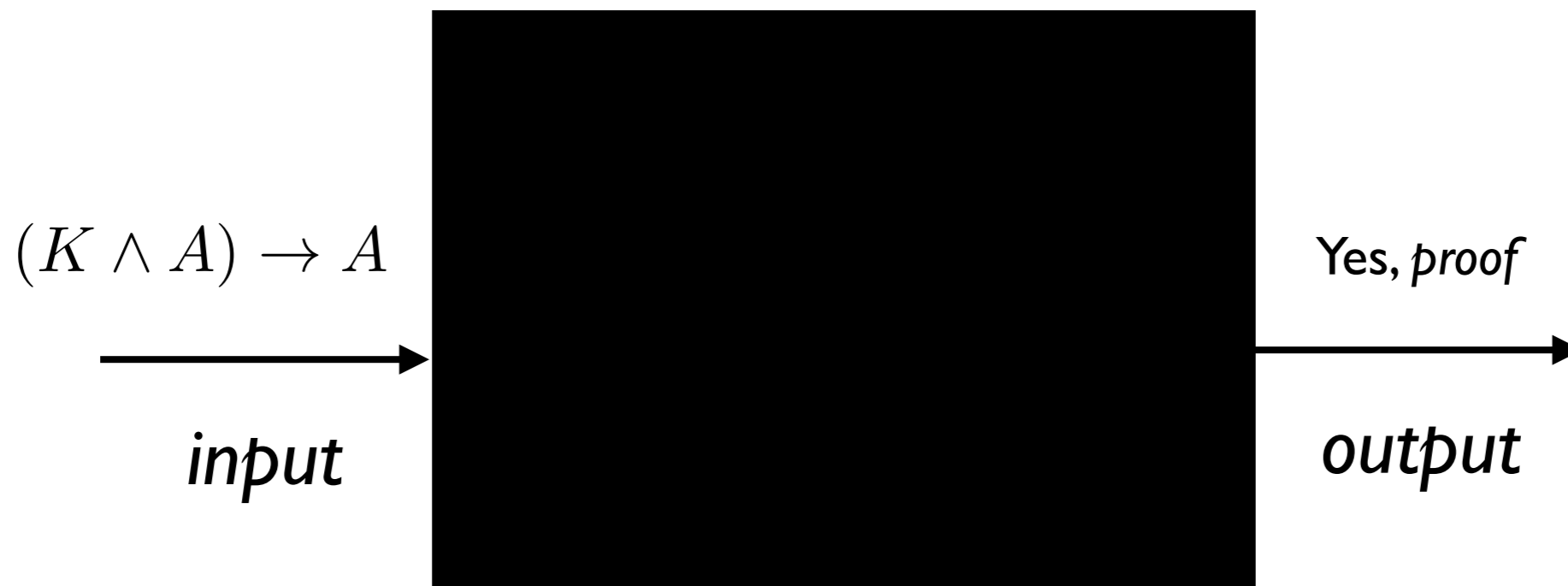
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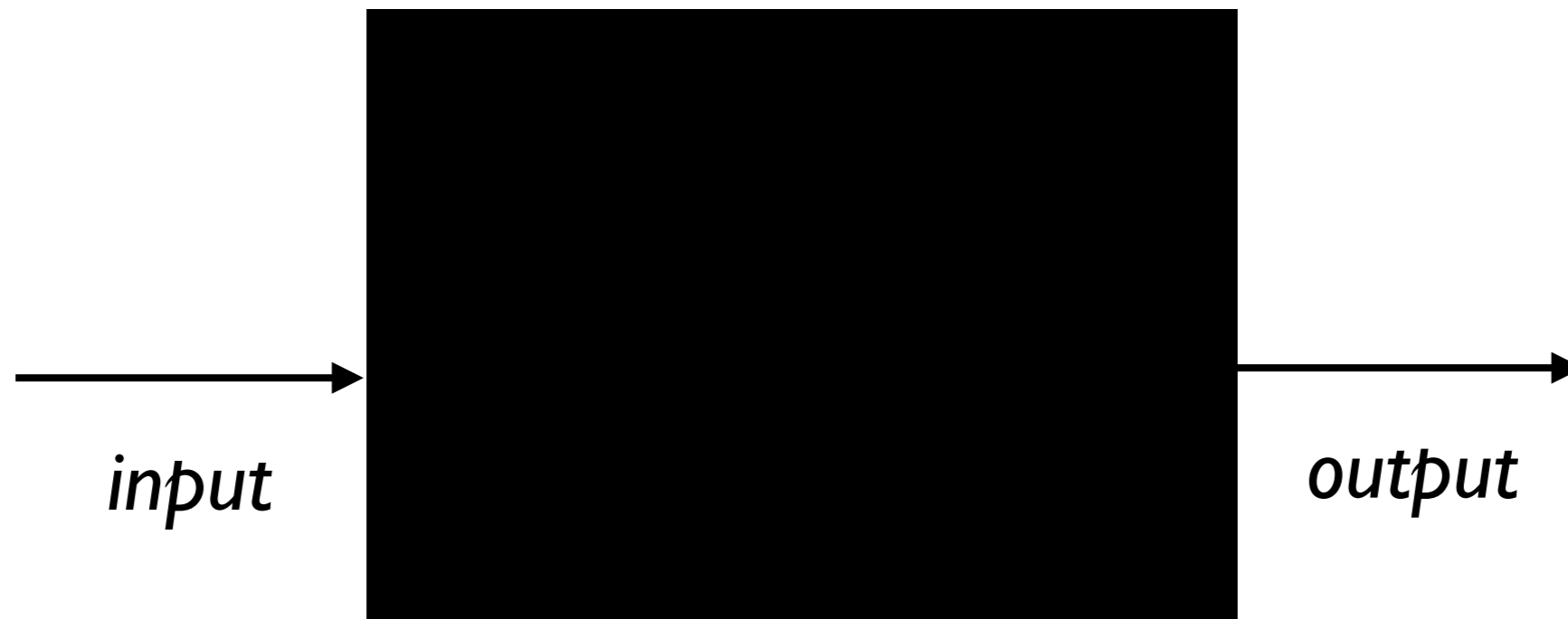


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**Hard!! — for apparently no polynomial-time algorithm for this!**

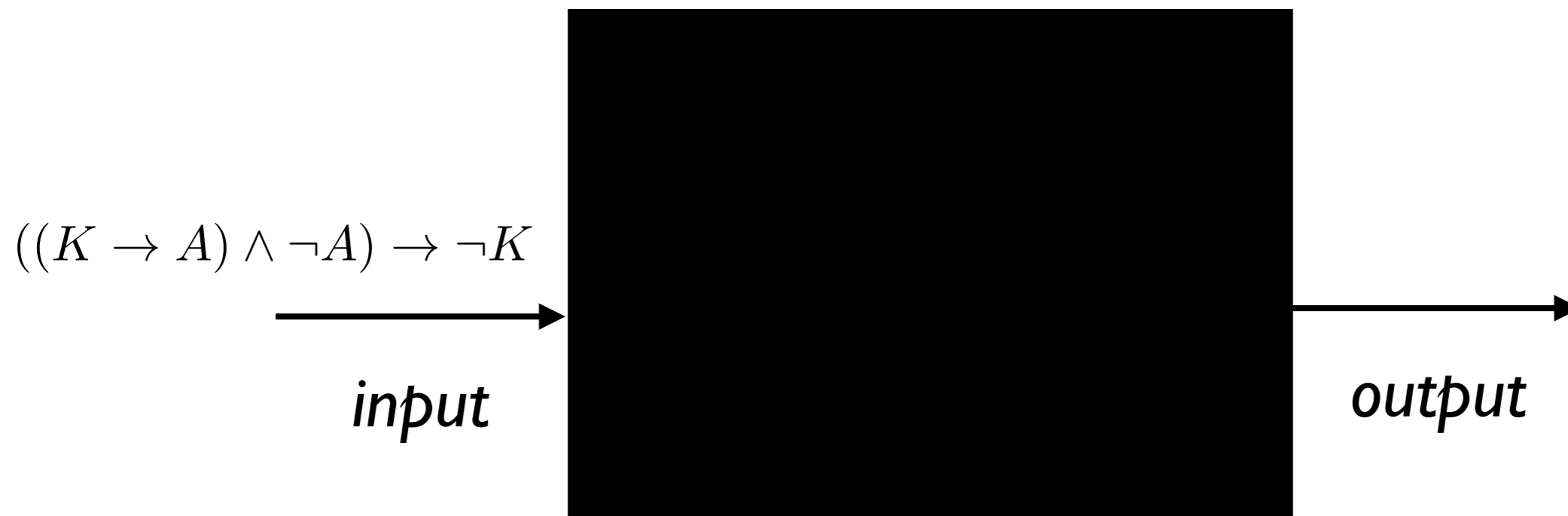
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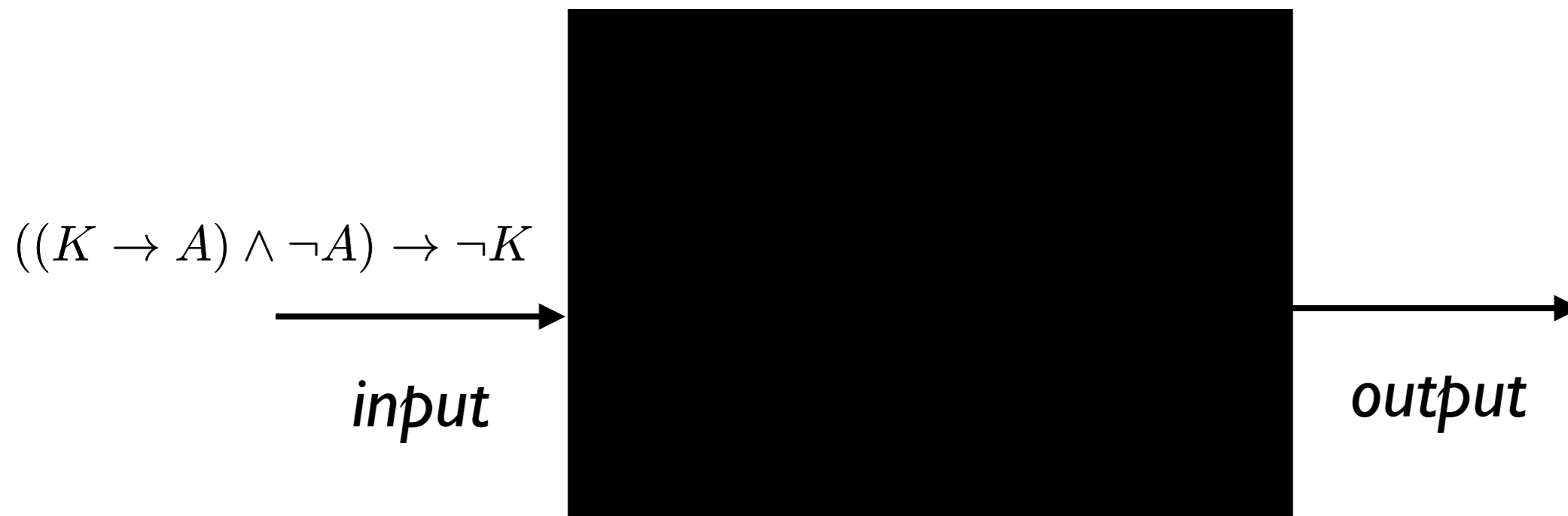


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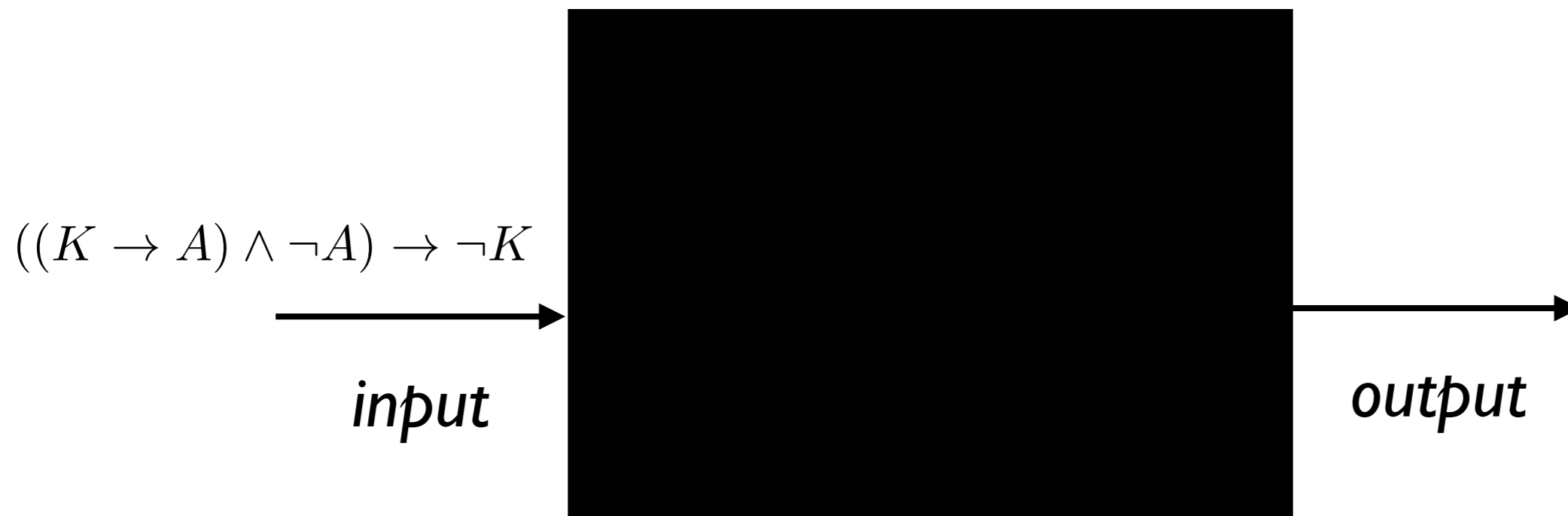
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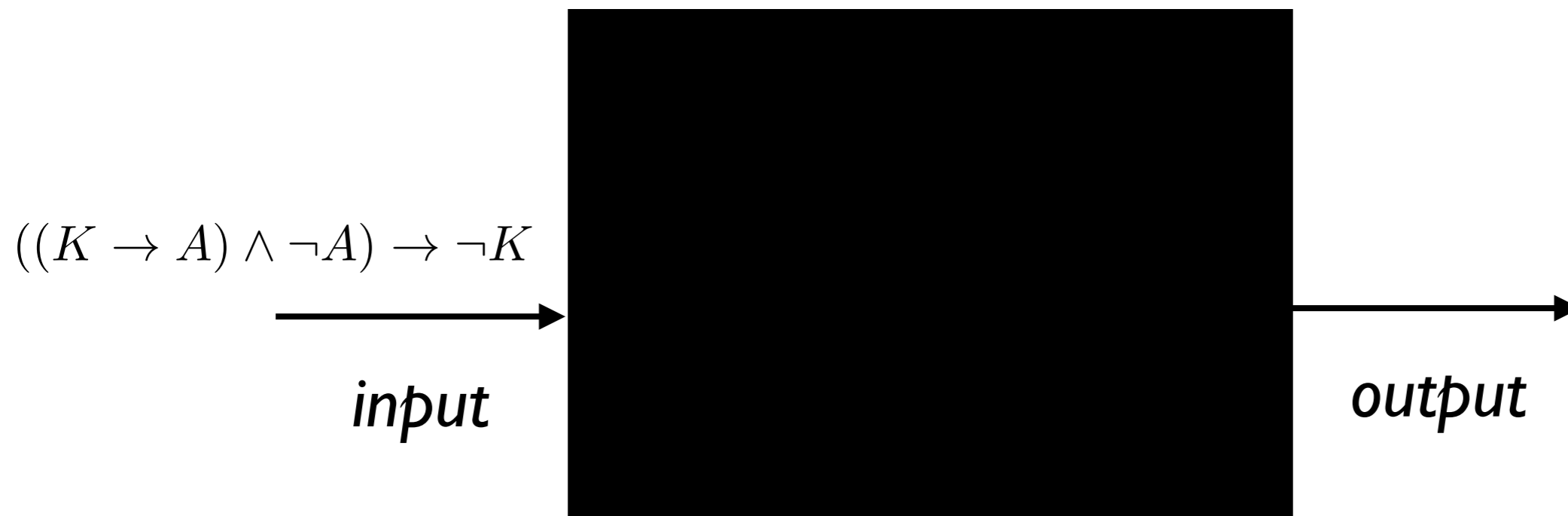
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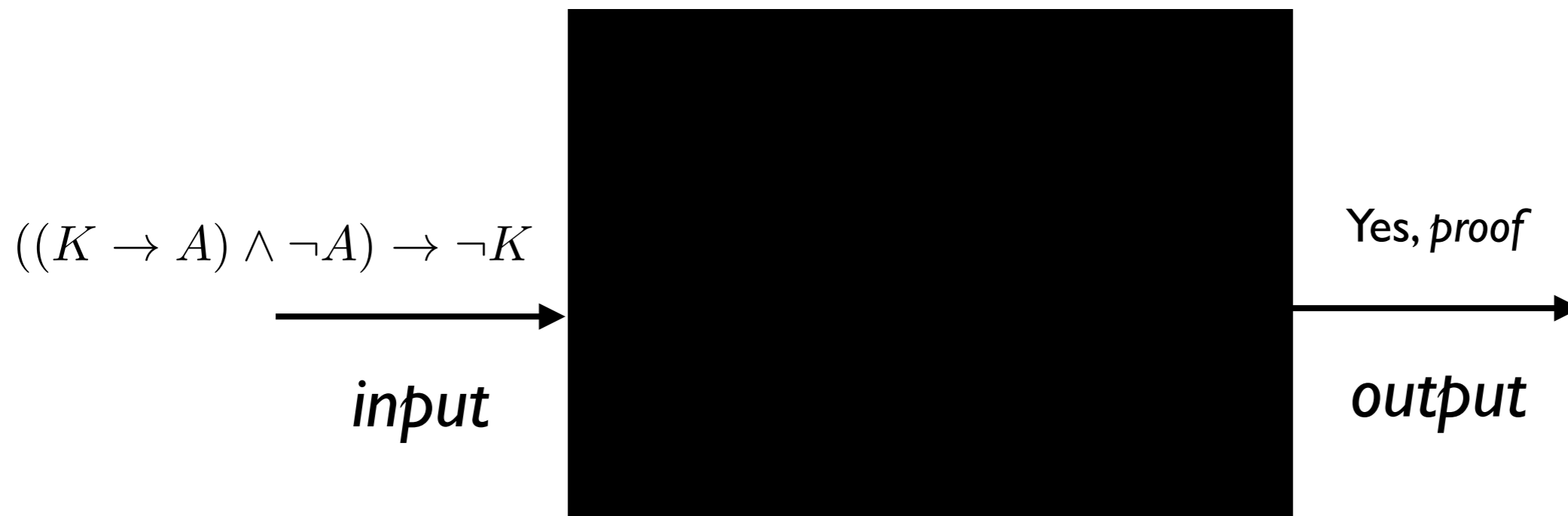
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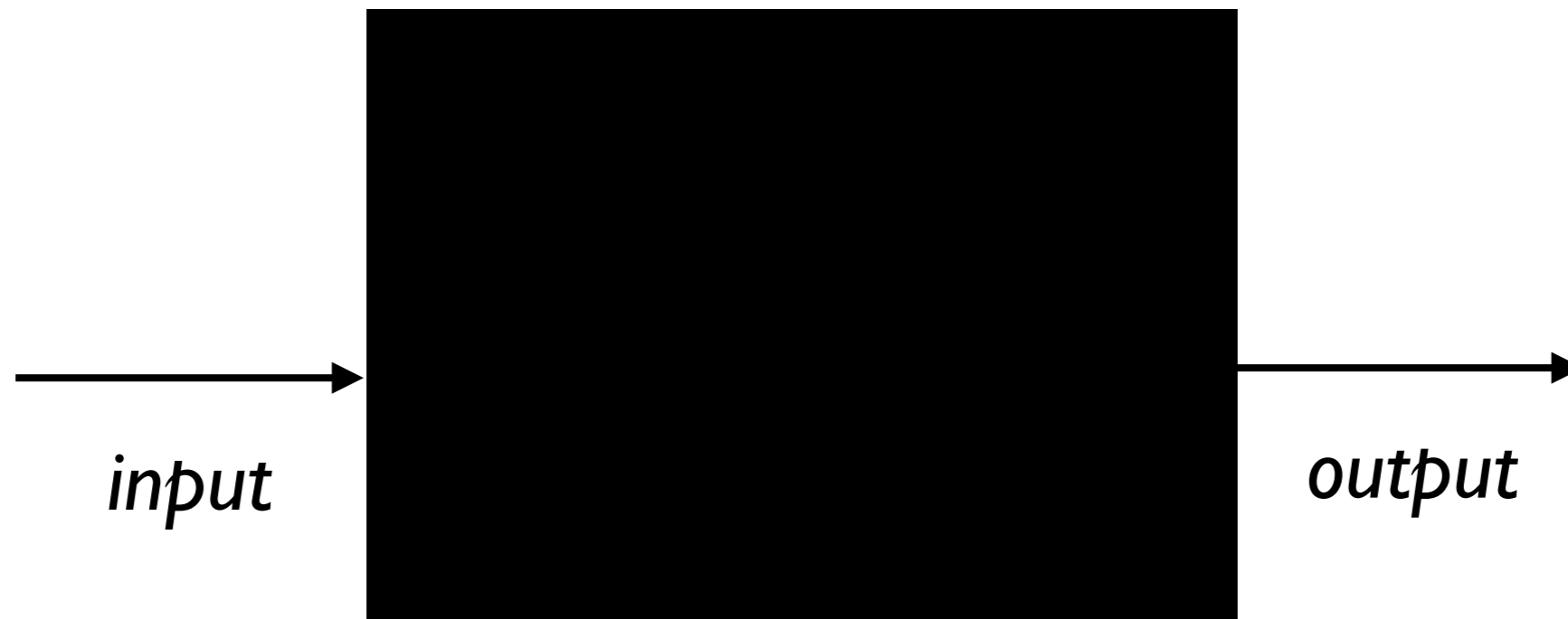
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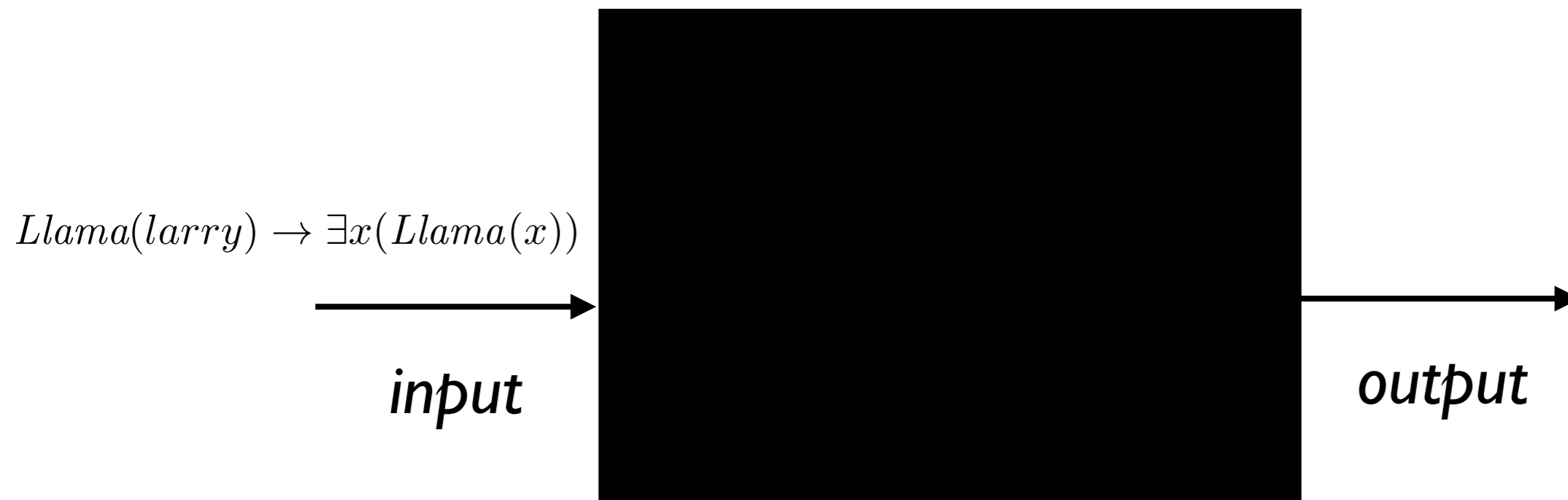


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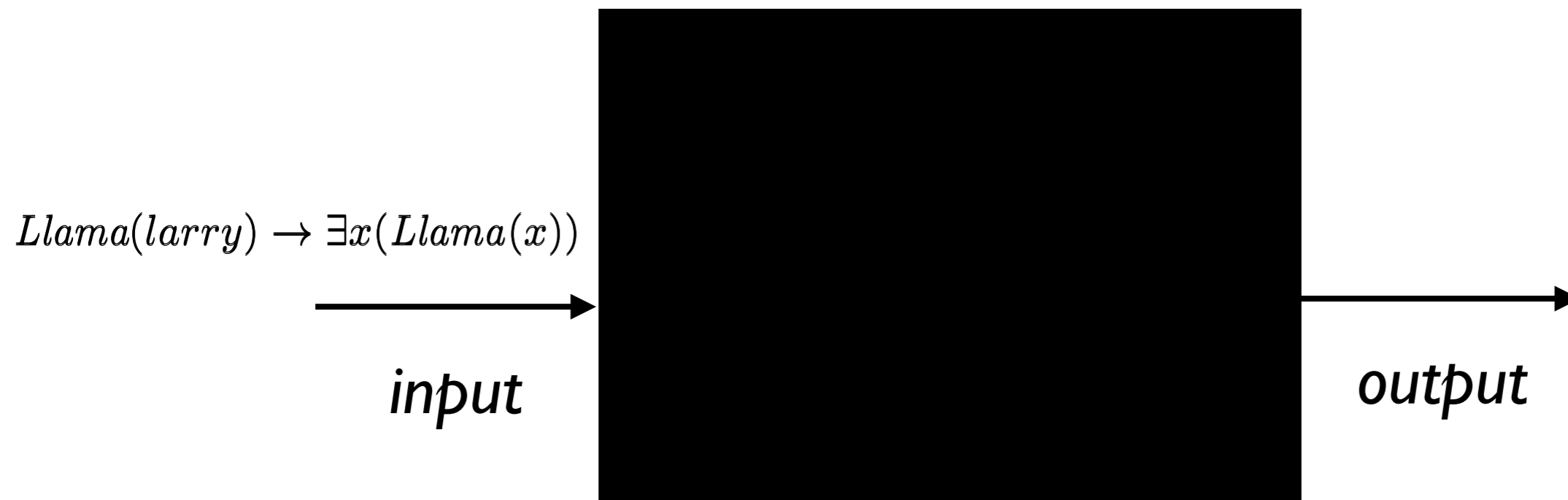
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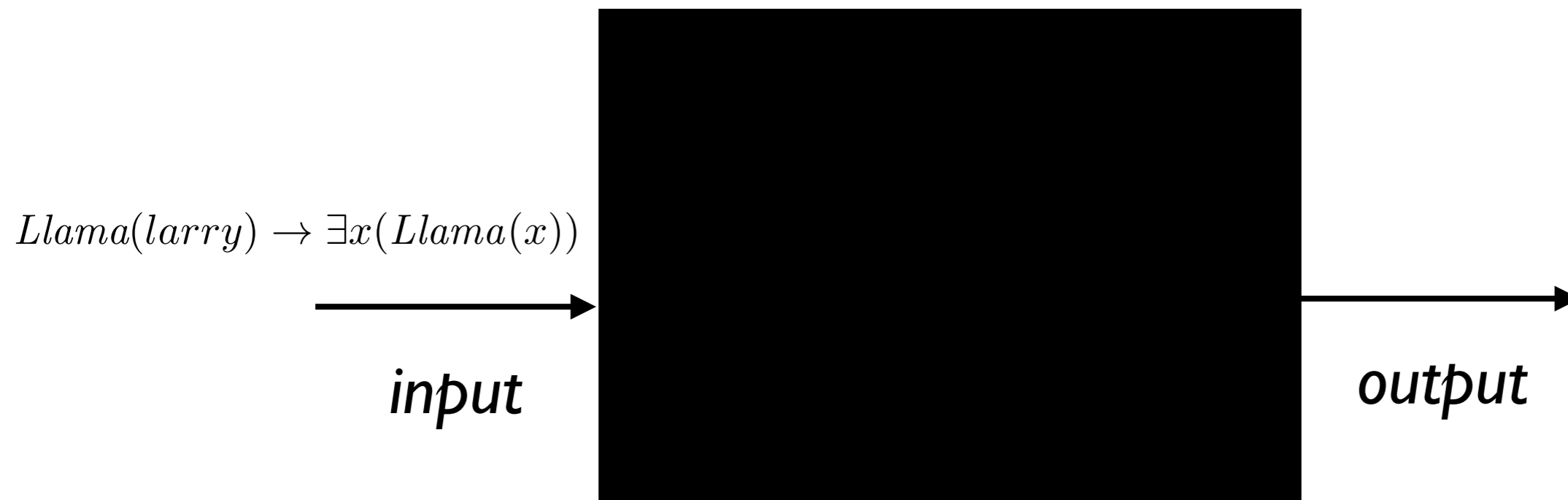


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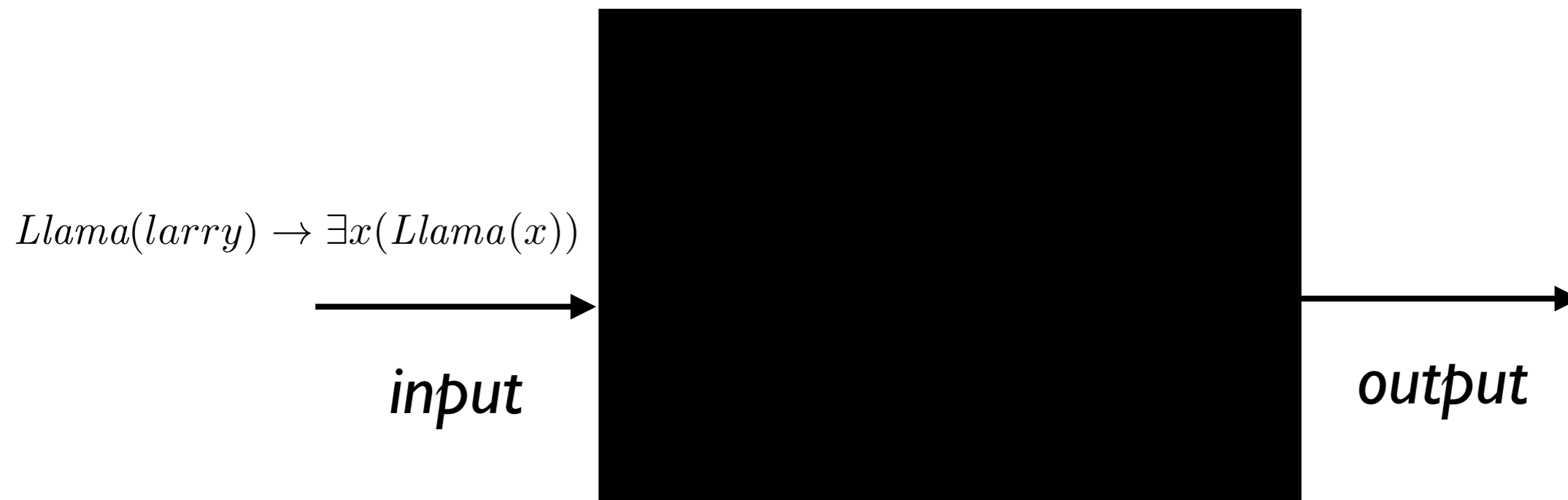




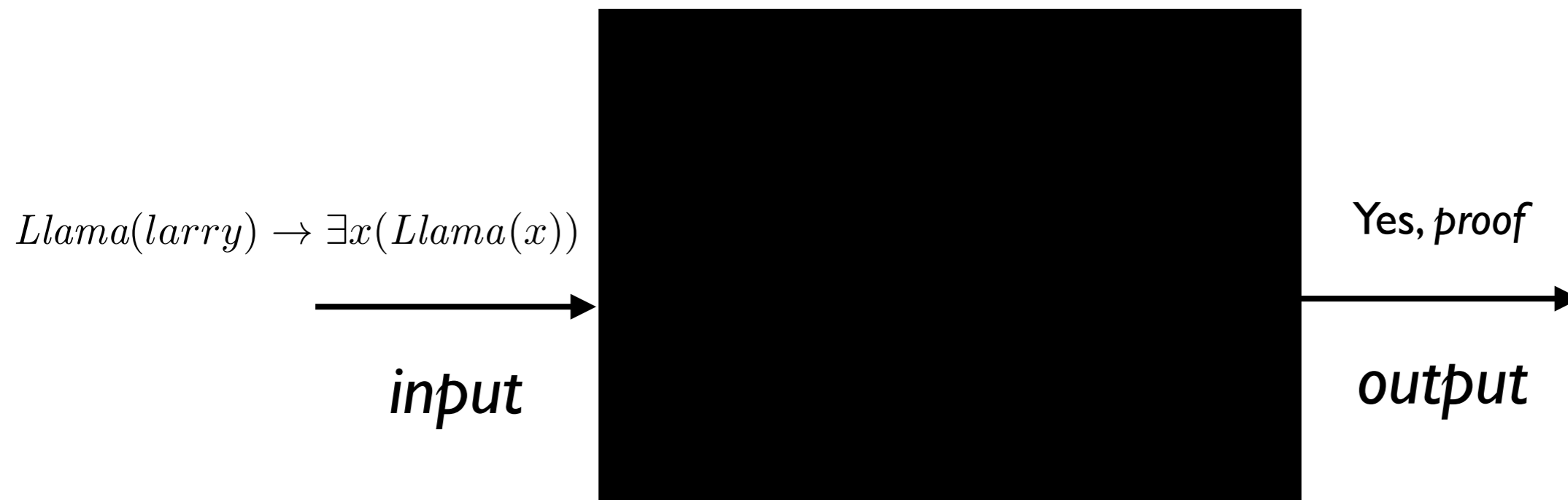
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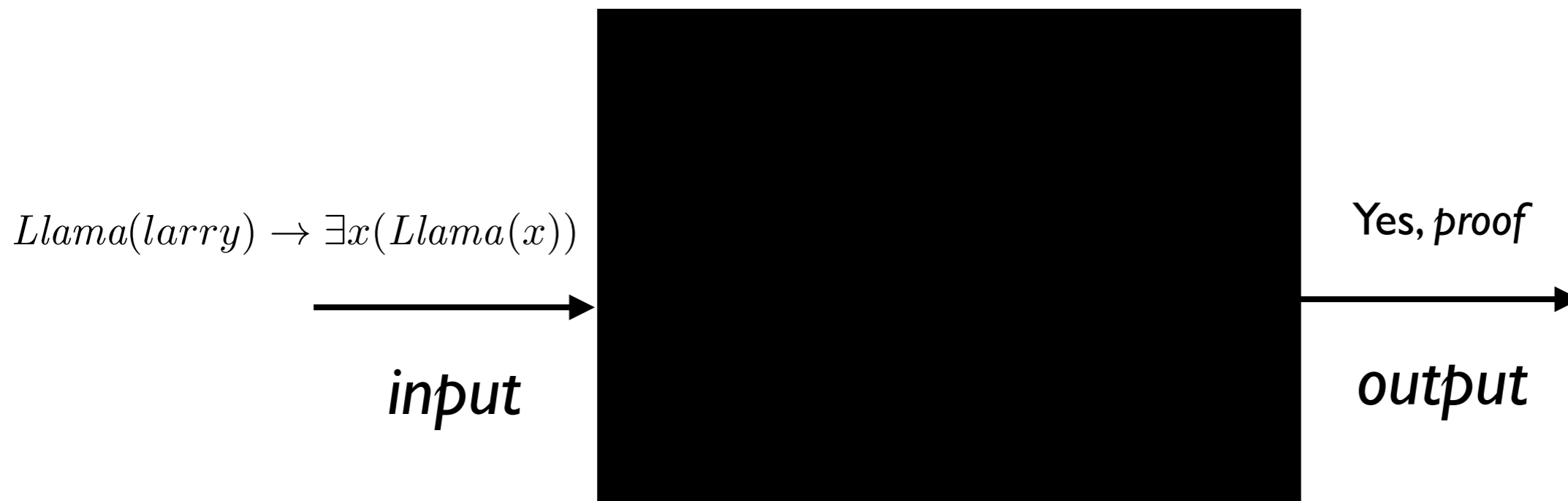
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**Not just hard: *impossible* for a (and this needed to be *invented* in the course of clarifying and solving the problem) standard computing machine.**

Applying this to ...  
The Singularity Question

# Applying this to ...

## The Singularity Question

*A:*

**Premise 1** There will be AI (created by HI and such that  $AI = HI$ ).

**Premise 2** If there is AI, there will be  $AI^+$  (created by AI).

**Premise 3** If there is  $AI^+$ , there will be  $AI^{++}$  (created by  $AI^+$ ).

$\therefore$  **S** There will be  $AI^{++}$  (=  $\mathcal{S}$  will occur).

(Good-Chalmers Argument)

(Kurzweil is an “extrapolationist.”)

# Applying this to ... The Singularity Question

So, these super-smart machines that will be built by human-level-smart machines, they can't *possibly* be smart enough to solve the *Entscheidungsproblem*. Hence they'll be just faster at solving problems we can routinely solve? What's so super-smart about *that*?