Rebuilding the Foundations of Math via (the “Theory”) ZFC; ZFC to Axiomatized Arithmetic (the “Theory” PA)

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Intro to Logic
3/21/2019

RAIR
Rensselaer AI and Reasoning Lab

Types of Paradoxes

- Deductive Paradoxes
- Inductive Paradoxes — coming (e.g. The Lottery Paradox & The St Petersburg Paradox)

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https://www.megamillions.com

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1 in 302,575,350

Friday's Hill, Haslemere, 16 June 1902

Dear colleague,

For a year and a half I have been acquainted with your *Grundgesetze der Arithmetik*, but it is only now that I have been able to find the time for the thorough study I intended to make of your work. I find myself in complete agreement with you in all essentials, particularly when you reject any psychological element [[Moment]] in logic and when you place a high value upon an ideography [[Begriffsschrift]] for the foundations of mathematics and of formal logic, which, incidentally, can hardly be distinguished. With regard to many particular questions, I find in your work discussions, distinctions, and definitions that one seeks in vain in the works of other logicians. Especially so far as function is concerned (§ 9 of your *Begriffsschrift*), I have been led on my own to views that are the same even in the details. There is just one point where I have encountered a difficulty. You state (p. 17 [[p. 23 above]]) that a function, too, can act as the indeterminate element. This I formerly believed, but now this view seems doubtful to me because of the following contradiction. Let w be the predicate: to be a predicate that cannot be predicated of itself. Can w be predicated of itself? From each answer its opposite follows. Therefore we must conclude that w is not a predicate. Likewise there is no class (as a totality) of those classes which, each taken as a totality, do not belong to themselves. From this I conclude that under certain circumstances a definable collection [[Menge]] does not form a totality.

I am on the point of finishing a book on the principles of mathematics and in it I should like to discuss your work very thoroughly.[1] I already have your books or shall buy them soon, but I would be very grateful to you if you could send me reprints of your articles in various periodicals. In case this should be impossible, however, I will obtain them from a library.

The exact treatment of logic in fundamental questions, where symbols fail, has remained very much behind; in your works I find the best I know of our time, and therefore I have permitted myself to express my deep respect to you. It is very regrettable that you have not come to publish the second volume of your *Grundgesetze*; I hope that this will still be done.

Very respectfully yours,

BERTRAND RUSSELL

The above contradiction, when expressed in Peano's ideography, reads as follows:

$$w = \mathrm{cls} \cap x \ni (x \sim\varepsilon\, x) . \supset : w \,\varepsilon\, w \,.=.\, w \sim\varepsilon\, w.$$

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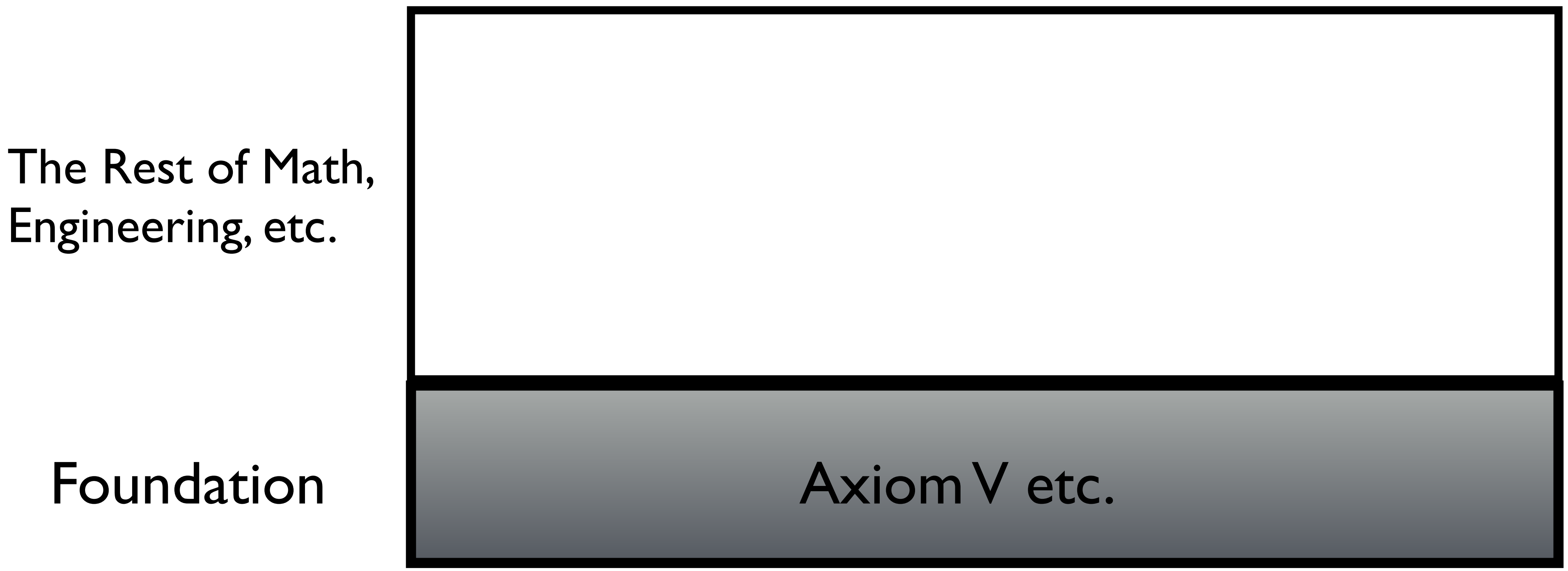
The Foundation Crumbles



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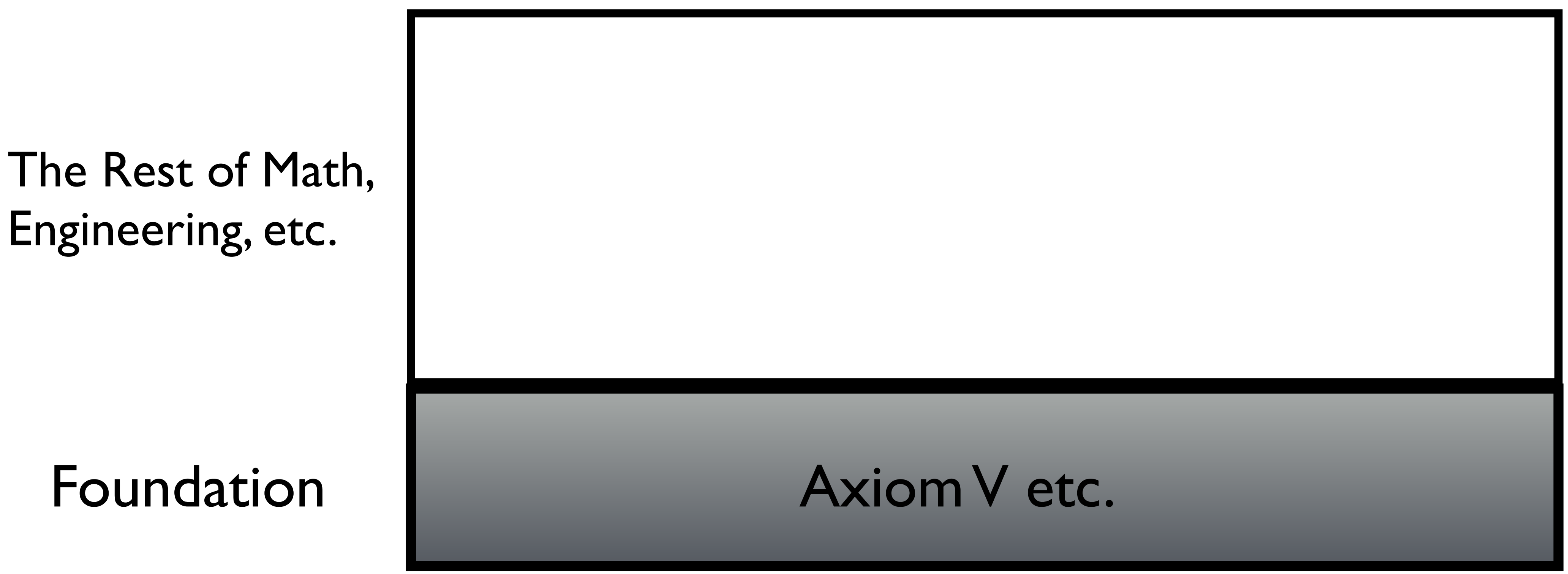

$$\text{Axiom V} \quad \exists x \forall y [y \in x \leftrightarrow \phi(y)]$$

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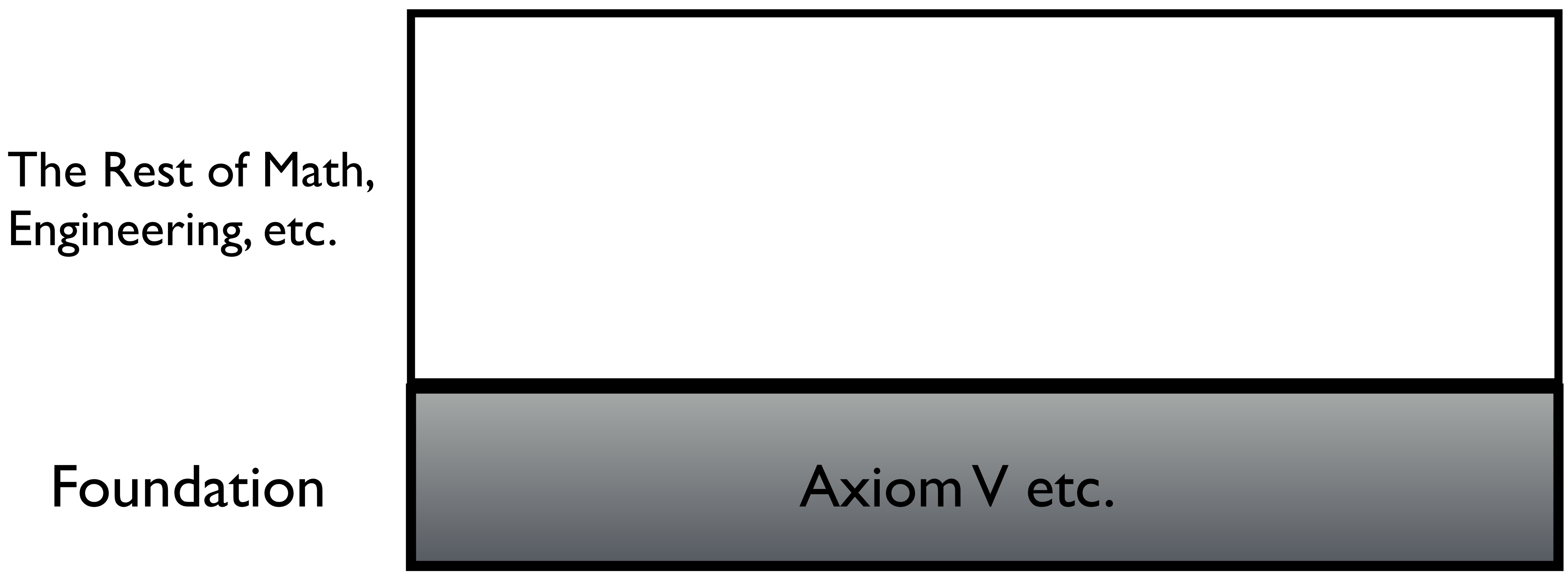

$$\text{Axiom V} \qquad \exists x \forall y [y \in x \leftrightarrow \phi(y)]$$

a formula of arbitrary size in which the variable *y* is free; this formula ascribes a property to *y*

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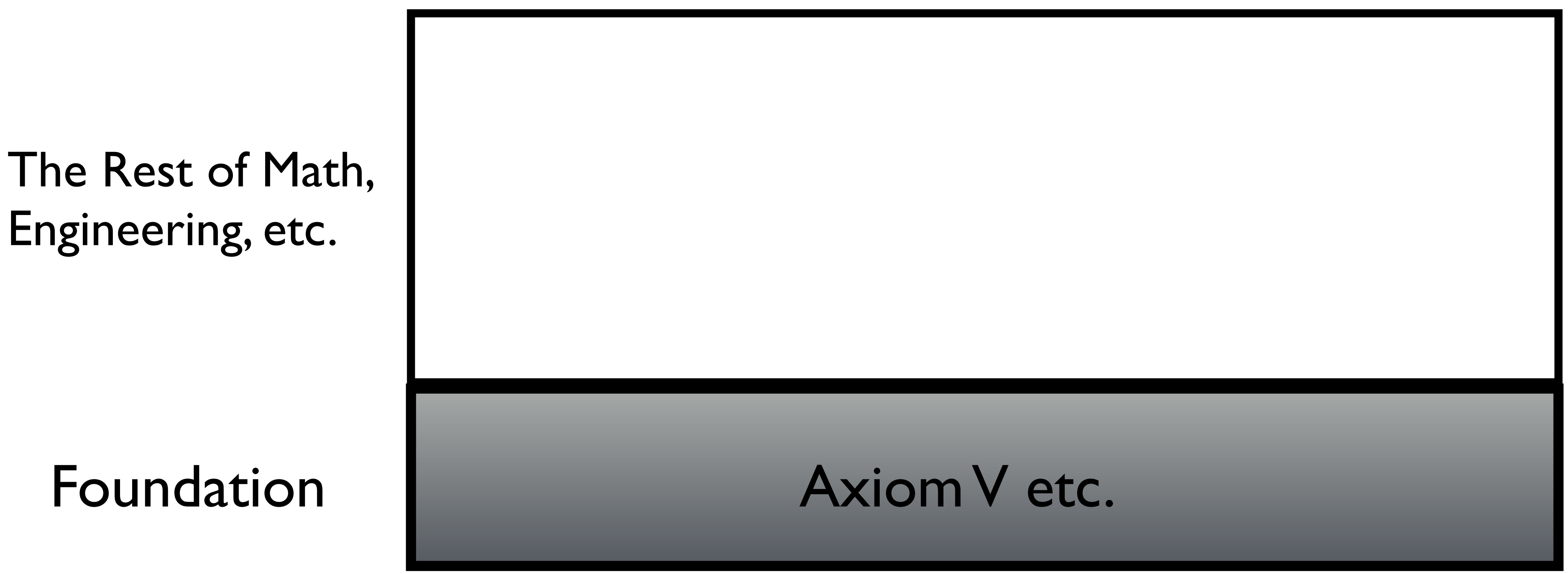

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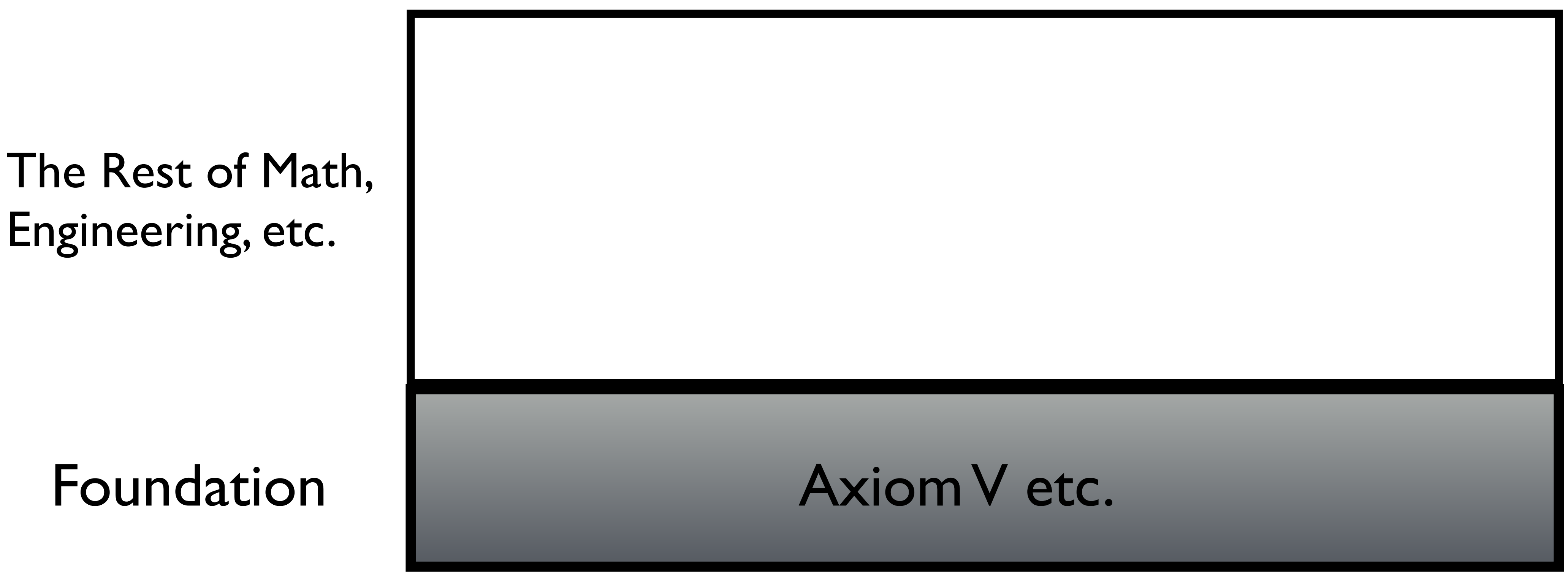

The Foundation Crumbles

The Foundation Crumbles

The Rest of Math,
Engineering, etc.

Foundation

The Foundation Crumbles

The Rest of Math,
Engineering, etc.

Foundation

It’s not just Russell’s Paradox that destroys naïve set theory:

It’s not just Russell’s Paradox that destroys naïve set theory:

Richard’s Paradox …

a

a
b

a
b
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•
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a
b
•
•
•
aa

a

b

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•

aa

ab

a
b
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aa
ab
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a
b
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a
b
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aa
ab
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Doesn't define a real number. ~~aaa~~
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a
b
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aa
ab
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Doesn't define
a real number.
~~aaa~~
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E

a
b
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aa
ab
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Doesn't define a real number. ~~aaa~~
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E

Definition of Richard's *N*:

a
b
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aa
ab
.
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Doesn't define a real number. ~~aaa~~
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.
.
E

Definition of Richard's *N*:

"The real number whose whole part is zero, and whose *n*-th decimal is *p* plus one if the *n*-th decimal of the real number defined by the *n*-th member of *E* is *p* and *p* is neither eight nor nine, and is simply one if this *n*-th decimal is eight or nine."

Definition of Richard's *N*:

a
b
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aa
ab
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Doesn't define a real number. ~~aaa~~
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E

"The real number whose whole part is zero, and whose *n*-th decimal is *p* plus one if the *n*-th decimal of the real number defined by the *n*-th member of *E* is *p* and *p* is neither eight nor nine, and is simply one if this *n*-th decimal is eight or nine."

Proof: *N* is defined by a finite string taken from the English alphabet, so *N* is in the sequence *E*. But on the other hand, by definition of *N*, for every *m*, *N* differs from the *m*-th element of *E* in at least one decimal place; so *N* is not any element of *E*. Contradiction! **QED**

a
b
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.
.
aa
ab
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~~aaa~~
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a
b
⋮
aa
ab
⋮
~~aaa~~
⋮

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a
b
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aa
ab
.
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~~aaa~~
.
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.

E

u_1
u_2
.
.
.
u_k
u_{k+1}
.
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.

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a
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E

u_1
u_2
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u_k
u_{k+1}
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Suppose *N* is u_m.

a
b
•
•
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aa
ab
•
•
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~~aaa~~
•
•
•

E

u_1
u_2
•
•
•
u_k
u_{k+1}
•
•
•

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Suppose *N* is u_m.

Suppose u_m is 0.xxxx…xxx…

a
b
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aa
ab
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~~aaa~~
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E

u_1
u_2
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u_k
u_{k+1}
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Suppose *N* is u_m.

Suppose u_m is 0.xxxx...xxx...

mth

a
b
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.
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aa
ab
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~~aaa~~
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E

u_1
u_2
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u_k
u_{k+1}
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Suppose *N* is u_m.

Suppose u_m is 0.xxxx…xxx…

mth

Suppose u_m is 0.xxxx…8xx…

mth

a
b
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•
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aa
ab
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~~aaa~~
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E

u_1
u_2
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u_k
u_{k+1}
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Suppose u_m is 0.xxxx...xxx...

mth

Suppose u_m is 0.xxxx...8xx...

mth

a
b
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aa
ab
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~~aaa~~
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E

u_1
u_2
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u_k
u_{k+1}
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Suppose *N* is u_m.

Suppose u_m is 0.xxxx...xxx...

mth

Suppose u_m is 0.xxxx...8xx...

mth

Then *N* is 0.xxxx...1xx...

mth

a

b

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aa

ab

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~~aaa~~

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E

u_1

u_2

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.
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u_k

u_{k+1}

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Suppose *N* is u_m.

Suppose u_m is 0.xxxx…xxx…

mth

Suppose u_m is 0.xxxx…8xx…

mth

Then *N* is 0.xxxx…1xx…

mth

Since 8 ≠ 1, *N* can't be u_m!

a
b
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aa
ab
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~~aaa~~
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E

u_1
u_2
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u_k
u_{k+1}
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Suppose *N* is u_m.

Suppose u_m is 0.xxxx…xxx…

mth

Suppose u_m is 0.xxxx…8xx…

mth

Then *N* is 0.xxxx…1xx…

mth

Since 8 $\neq$ 1, *N* can't be u_m!

a
b
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aa
ab
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~~aaa~~
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E

u_1
u_2
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u_k
u_{k+1}
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Suppose *N* is u_m.

Suppose u_m is 0.xxxx…xxx…

mth

Suppose u_m is 0.xxxx…8xx…

mth

Suppose u_m is 0.xxxx…5xx…

mth

Then *N* is 0.xxxx…1xx…

mth

Since 8 ≠ 1, *N* can't be u_m!

a
b
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aa
ab
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E

u_1
u_2
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u_k
u_{k+1}
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Suppose *N* is u_m.

Suppose u_m is 0.xxxx...xxx...

mth

Suppose u_m is 0.xxxx...8xx...

mth

Suppose u_m is 0.xxxx...5xx...

mth

Then *N* is 0.xxxx...1xx...

mth

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a
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E

u_1
u_2
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u_k
u_{k+1}
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Suppose *N* is u_m.

Suppose u_m is 0.xxxx...xxx...

mth

Suppose u_m is 0.xxxx...8xx...

mth

Then *N* is 0.xxxx...1xx...

mth

Suppose u_m is 0.xxxx...5xx...

mth

Then *N* is 0.xxxx...6xx...

mth

Since 8 ≠ 1, *N* can't be u_m!

a
b
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aa
ab
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~~aaa~~
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E

u_1
u_2
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u_k
u_{k+1}
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Suppose *N* is u_m.

Suppose u_m is 0.xxxx...xxx...

$\uparrow$ mth

Suppose u_m is 0.xxxx...8xx...

$\uparrow$ mth

Then *N* is 0.xxxx...1xx...

$\uparrow$ mth

Since 8 ≠ 1, *N* can't be u_m!

Suppose u_m is 0.xxxx...5xx...

$\uparrow$ mth

Then *N* is 0.xxxx...6xx...

$\uparrow$ mth

Since 5 ≠ 6, *N* can't be u_m!

The Foundation Rebuilt

The Rest of Math,
Engineering, etc.

New Foundation

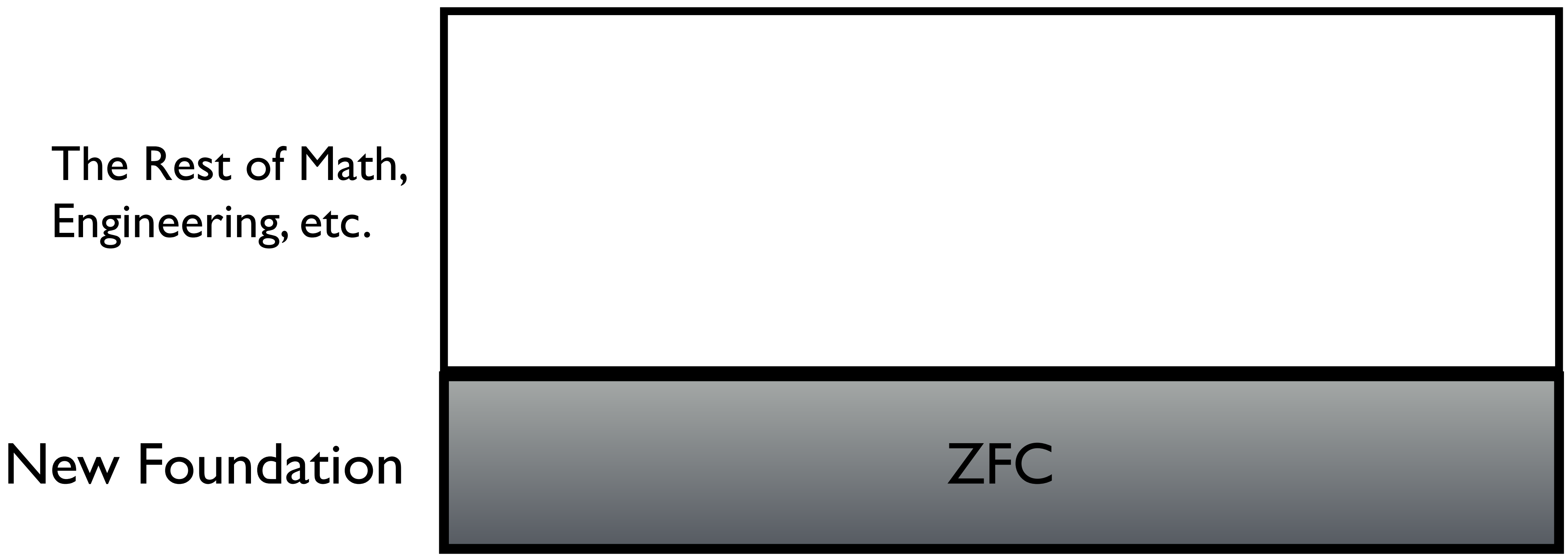
The Foundation Rebuilt

The Rest of Math,
Engineering, etc.

New Foundation

ZFC

The Foundation Rebuilt



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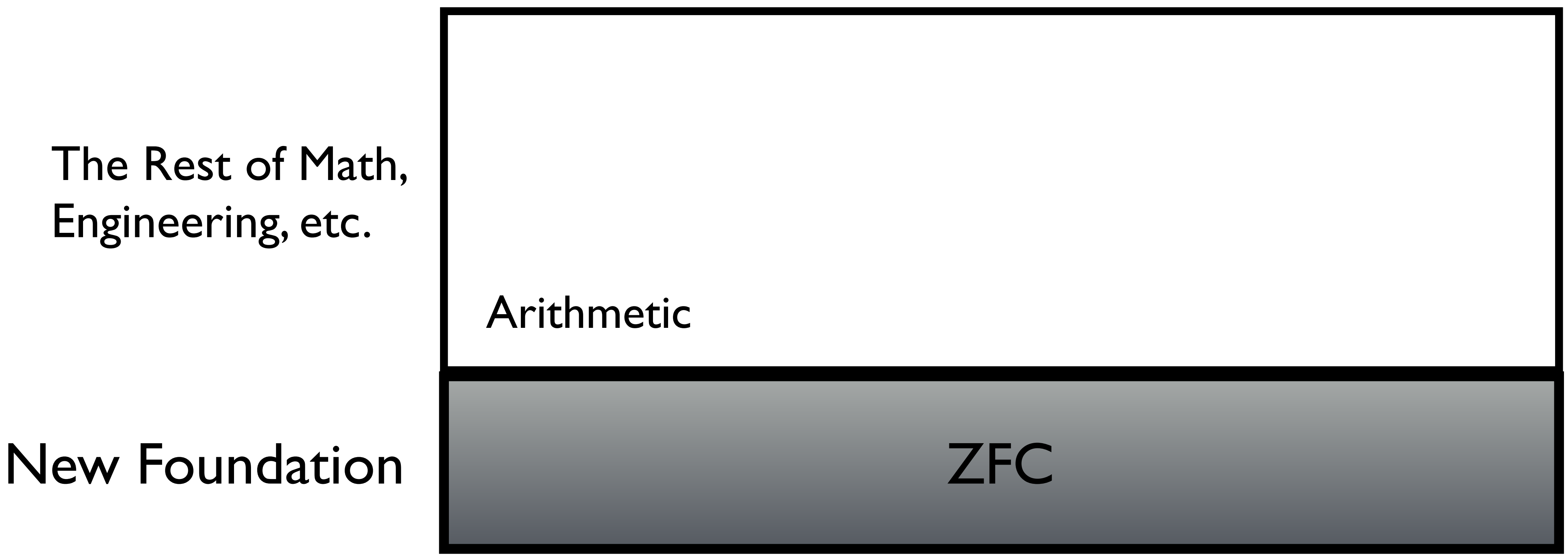

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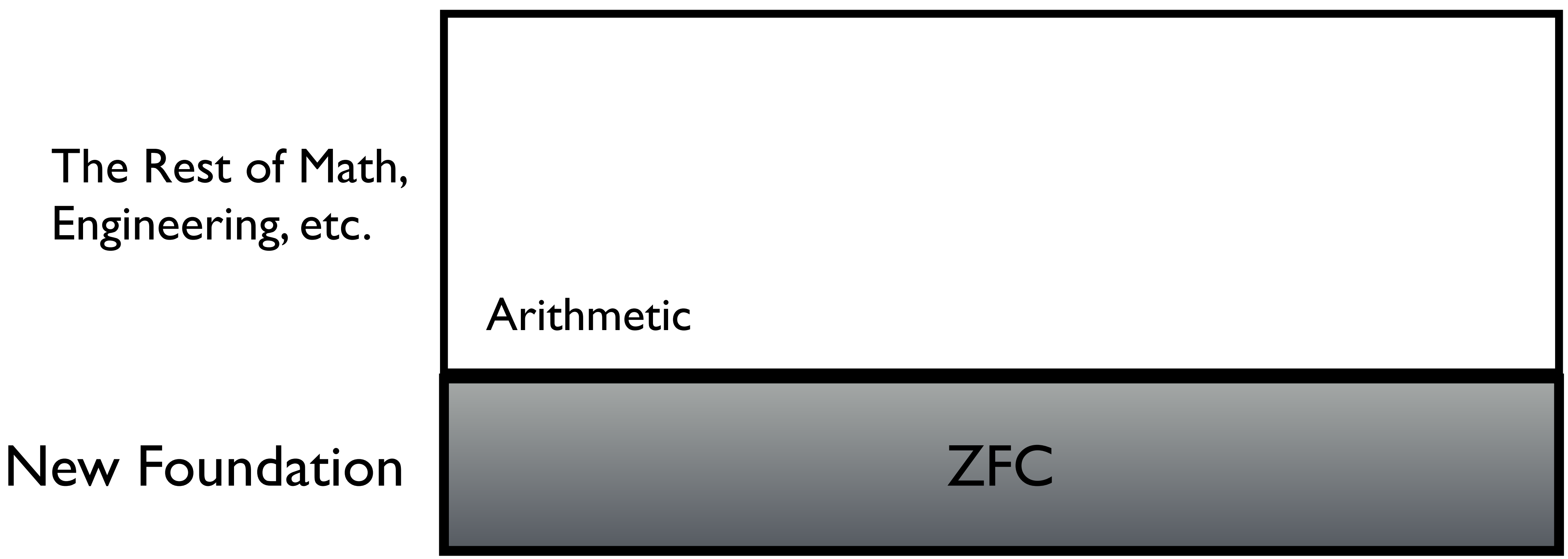
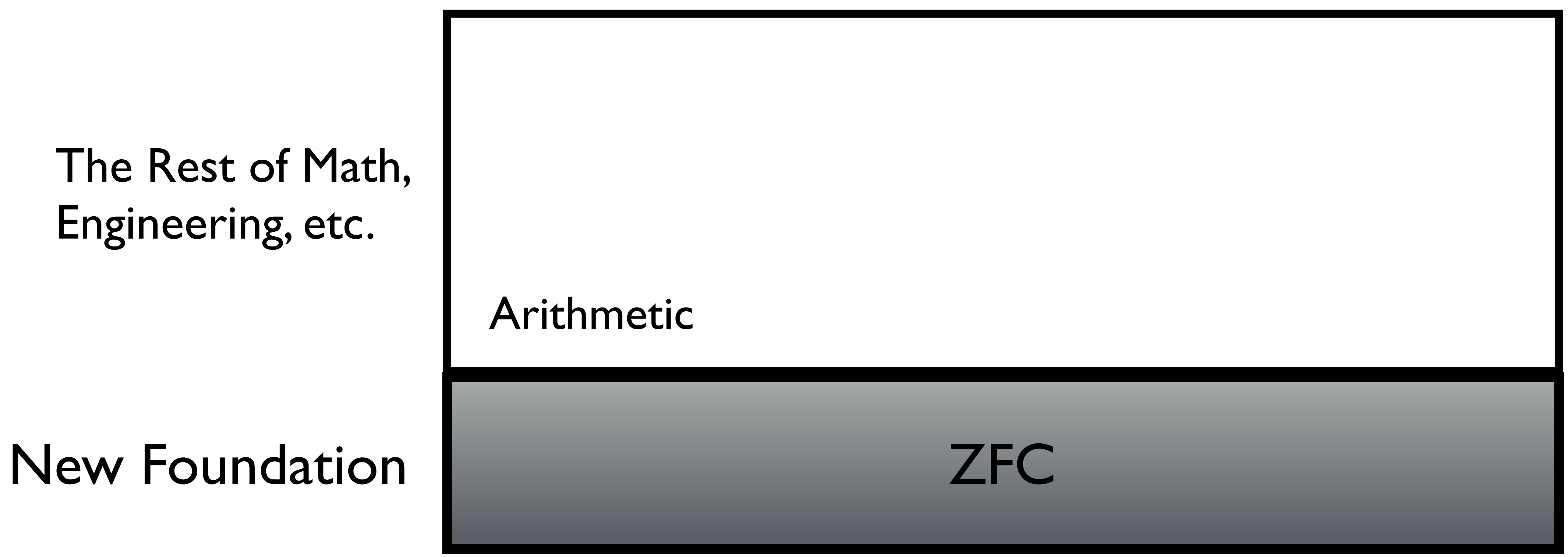


So what are the axioms in ZFC?

The Foundation Rebuilt



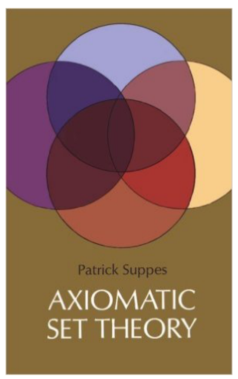
So what are the axioms in ZFC?

Russell's Paradox … to ZFC

$$\vdash \neg\exists x \forall y (y \in x \leftrightarrow y \notin y)$$

(Russell's Theorem; poor Frege!)

http://plato.stanford.edu/entries/russell-paradox/#HOTP



Supplant Cantor's/Frege's Axiom V with the Axiom Schema of Separation (& put on our thinking caps …) and you try to show Theorem 1 from Suppes:

$$\vdash \forall x (x \notin \emptyset)$$

Russell's Paradox … to ZFC

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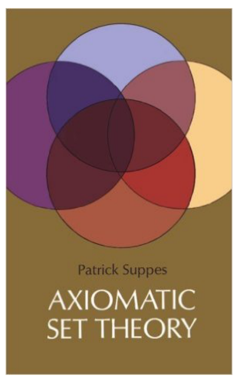

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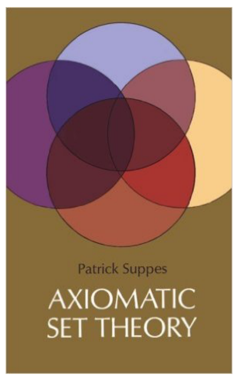

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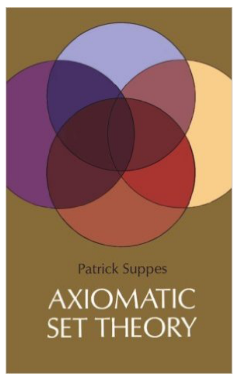

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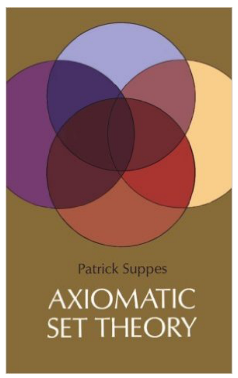

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$$\vdash \neg\exists x \forall y (y \in x \leftrightarrow y \notin y)$$

(Russell's Theorem; poor Frege!) √

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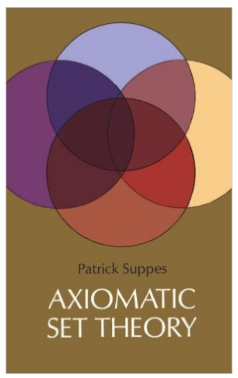

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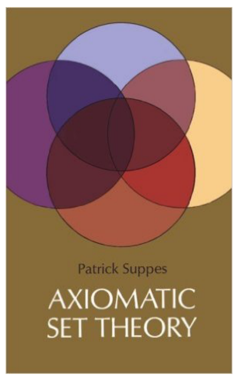

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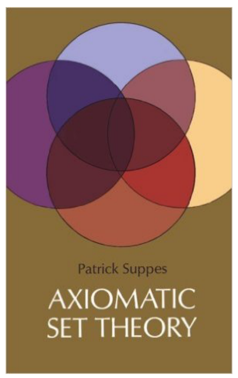

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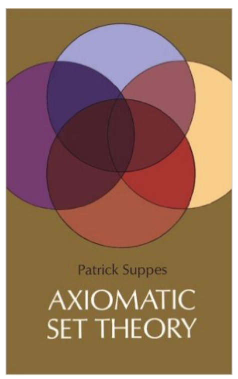

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With this definition, can you prove (Theorem 3) that every set is a subset of itself?

Formal Natural-Number Arithmetic …

Q (= Robinson Arithmetic)

A1	$\forall x(0 \neq s(x))$
A2	$\forall x \forall y(s(x) = s(y) \rightarrow x = y)$
A3	$\forall x(x \neq 0 \rightarrow \exists y(x = s(y))$
A4	$\forall x(x + 0 = x)$
A5	$\forall x \forall y(x + s(y) = s(x + y))$
A6	$\forall x(x \times 0 = 0)$
A7	$\forall x \forall y(x \times s(y) = (x \times y) + x)$

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And, every sentence that is the universal closure of an instance of

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where $\phi(x)$ is open wff with variable x, and perhaps others, free.

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This open wff $\phi(x)$ expresses the arithmetic property 'even.'

Revisiting: Do we just manufacture mathematics?

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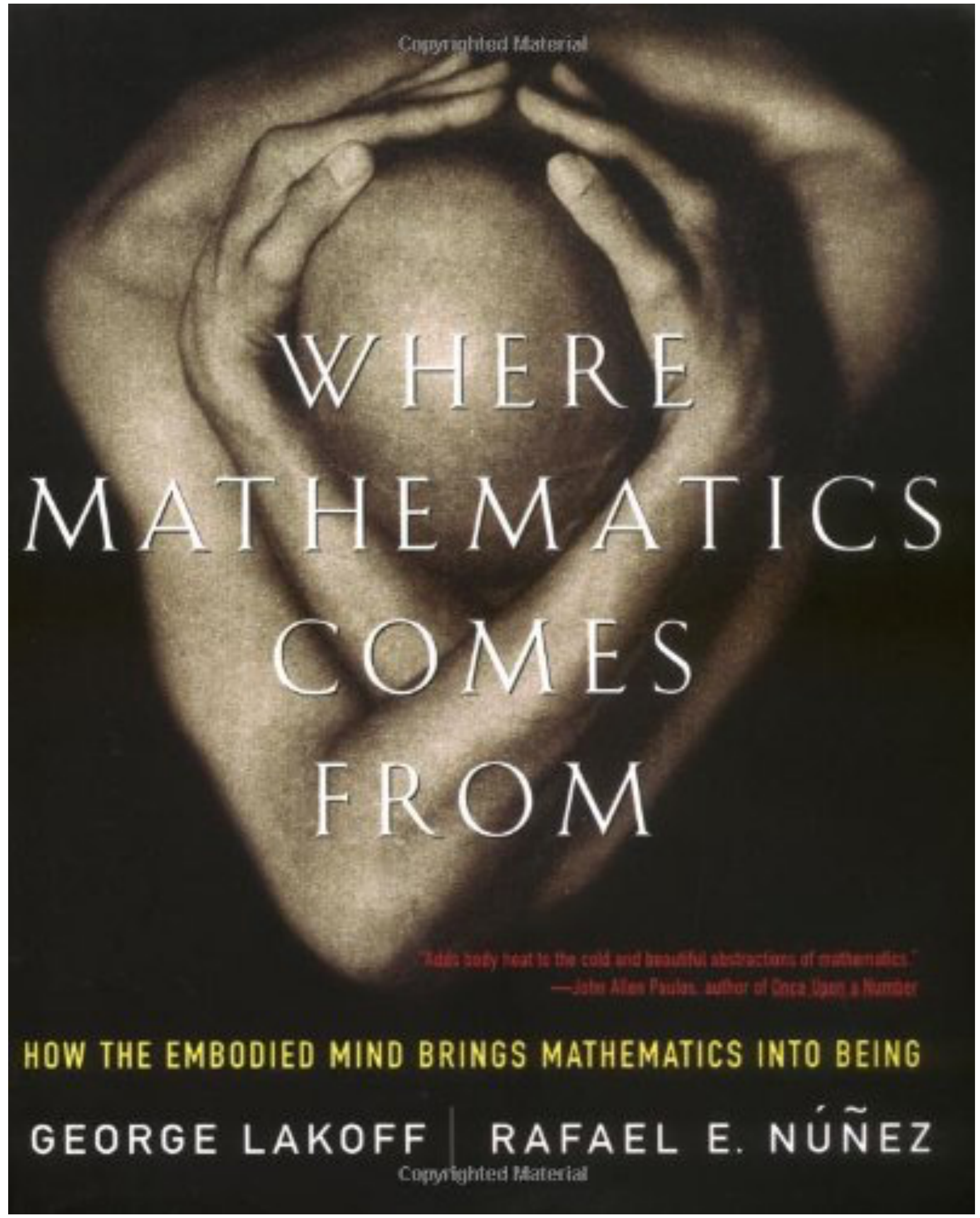

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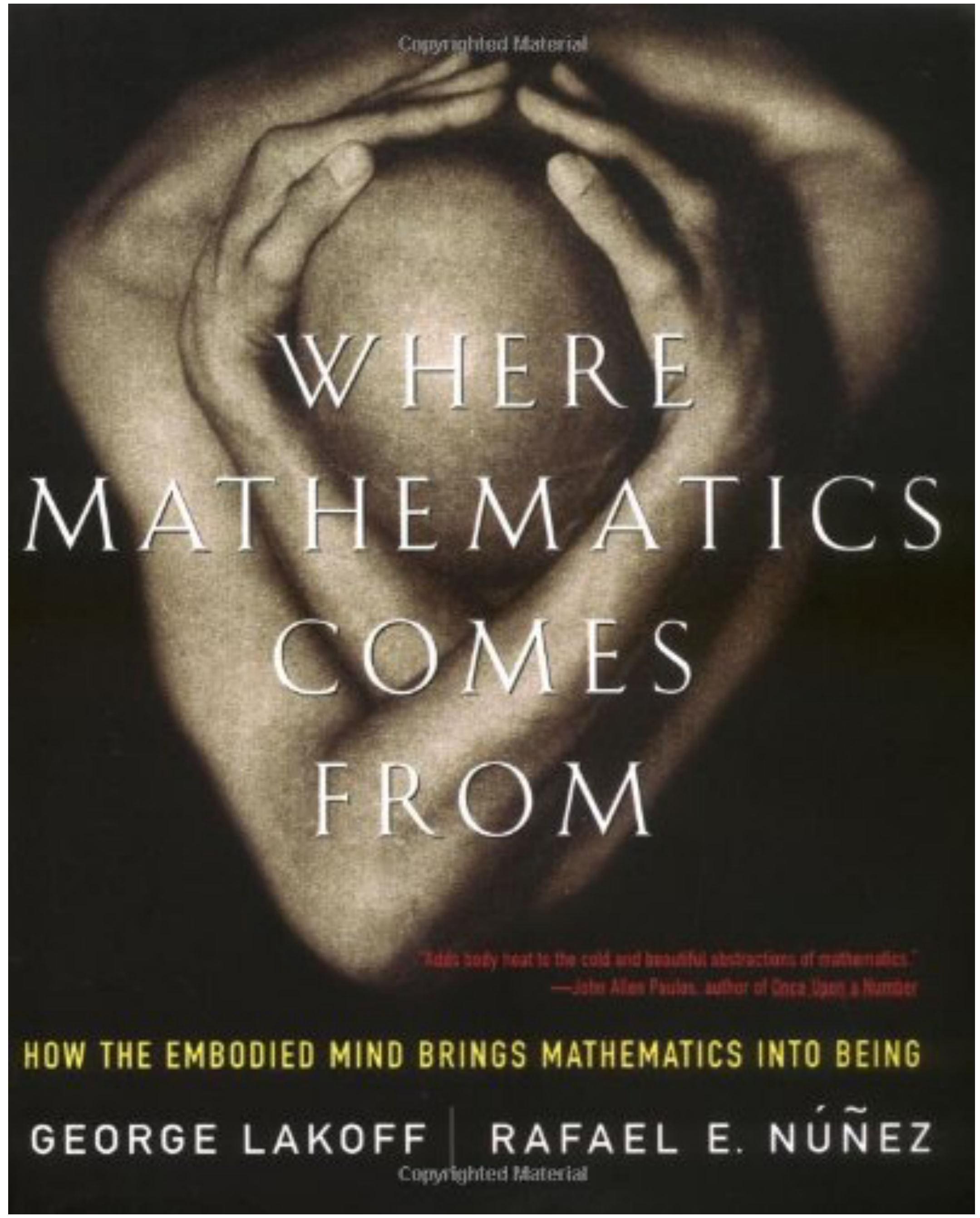

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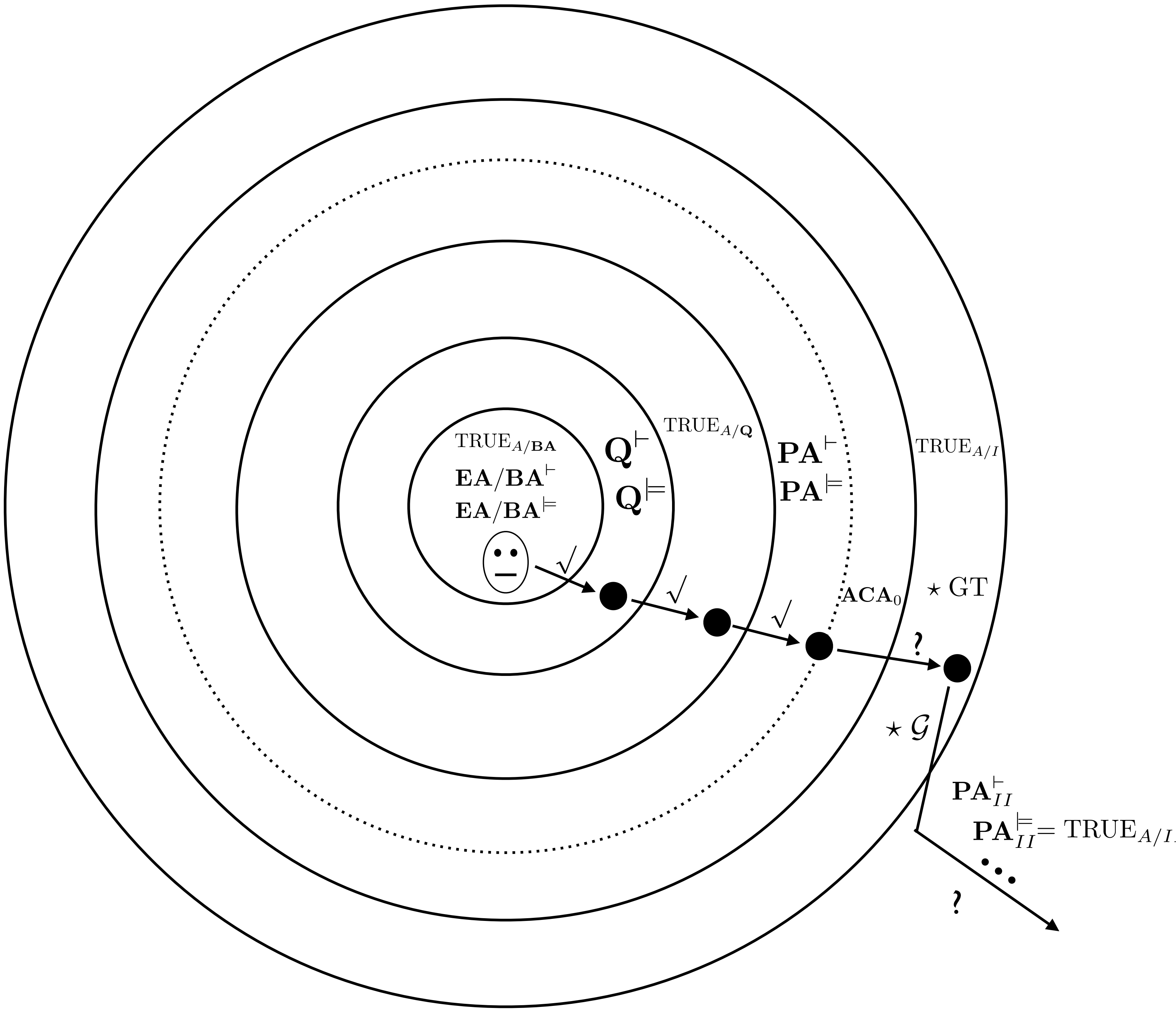
Astrologic

(Aliens & Angels on the Same "Race Track")



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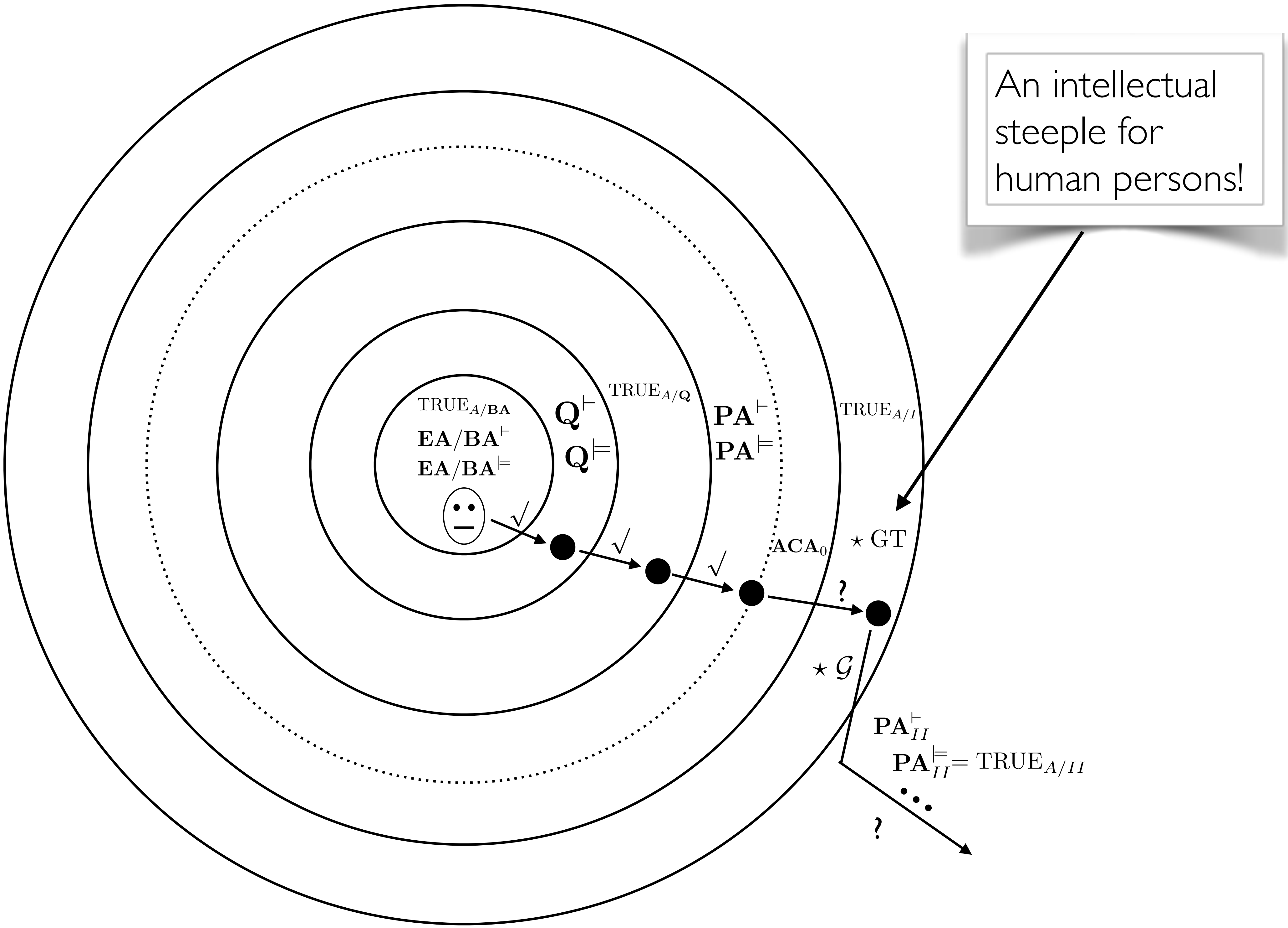

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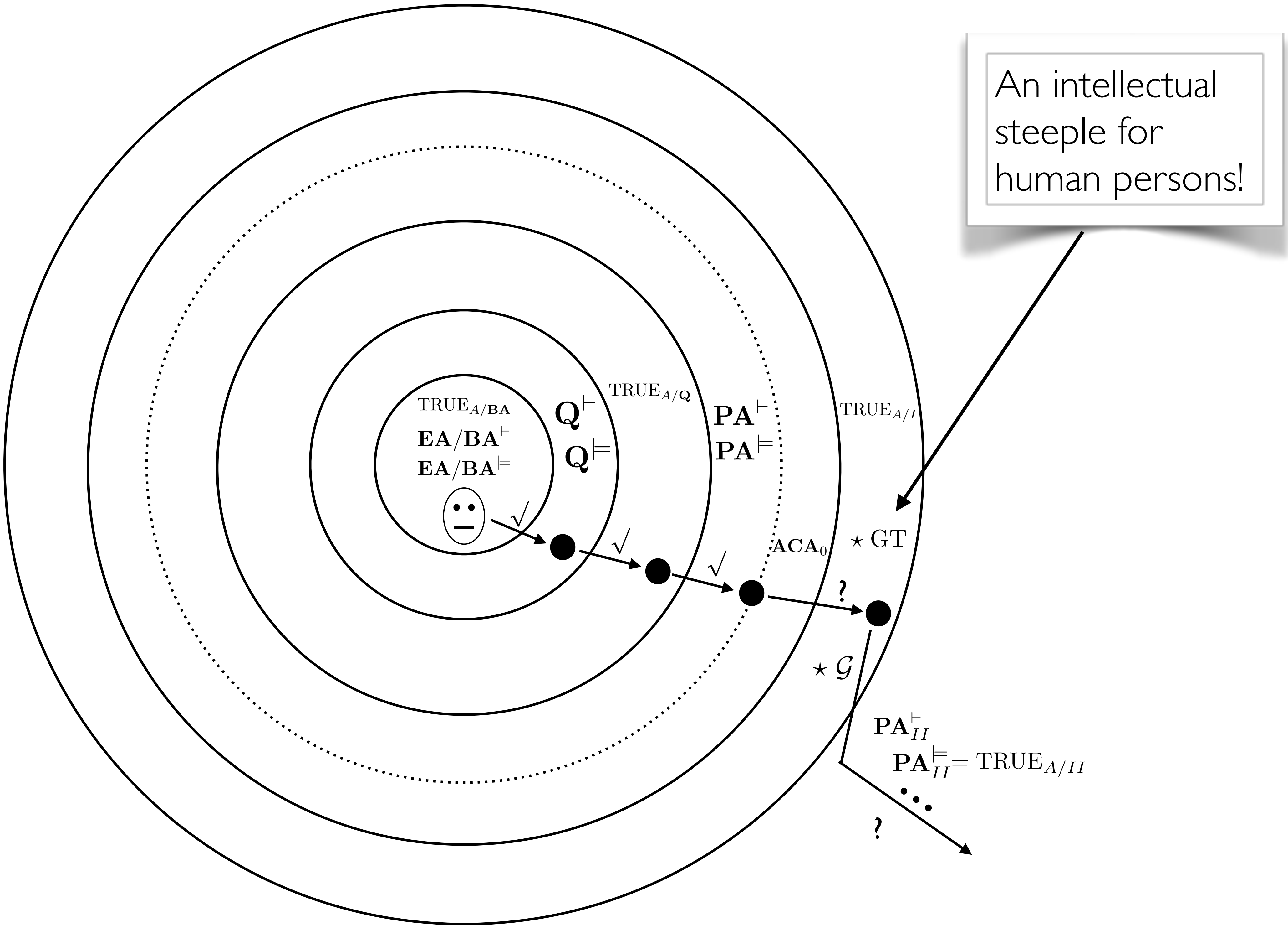

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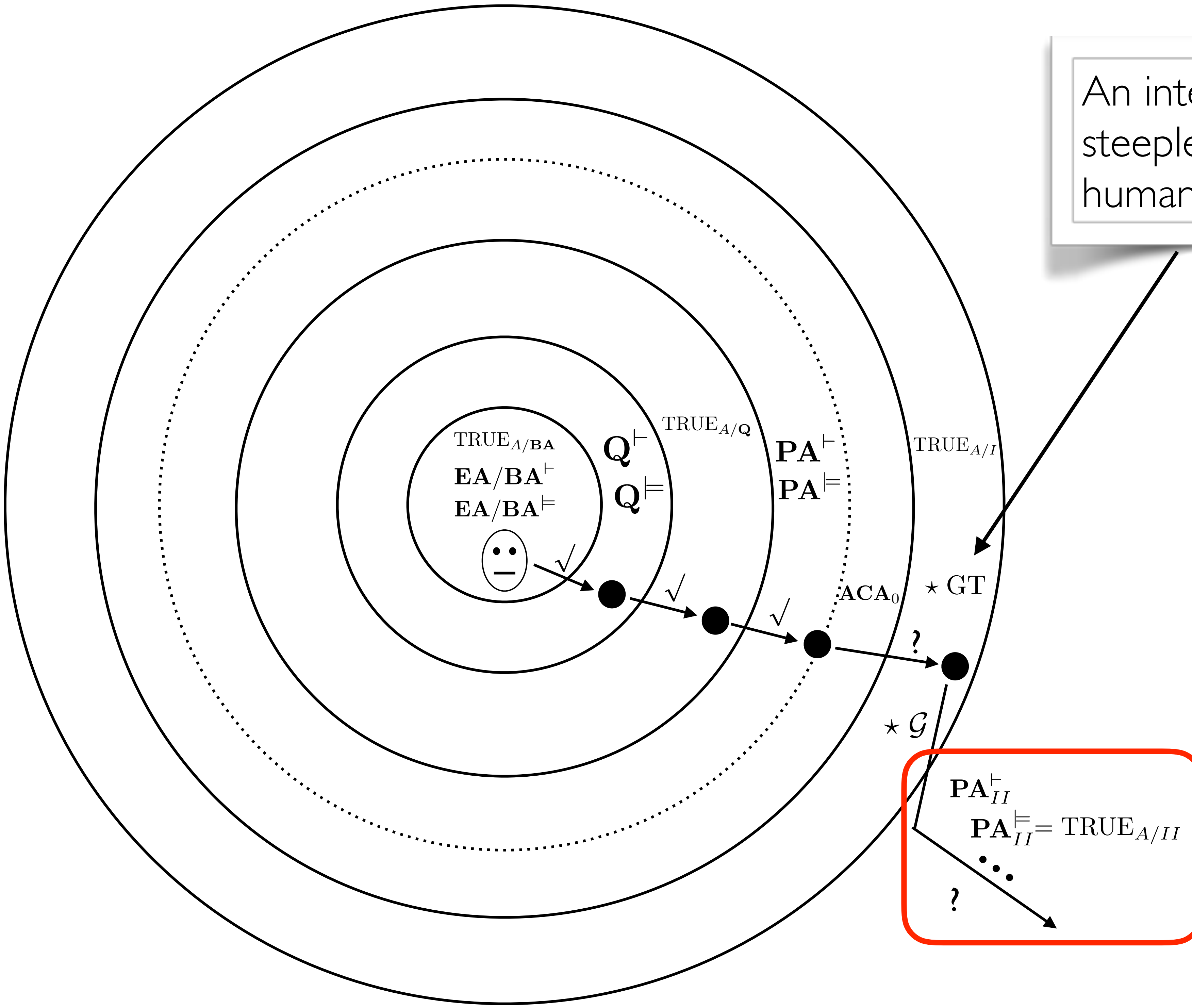

An intellectual steeple for human persons!

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