Modal Logic: First Steps

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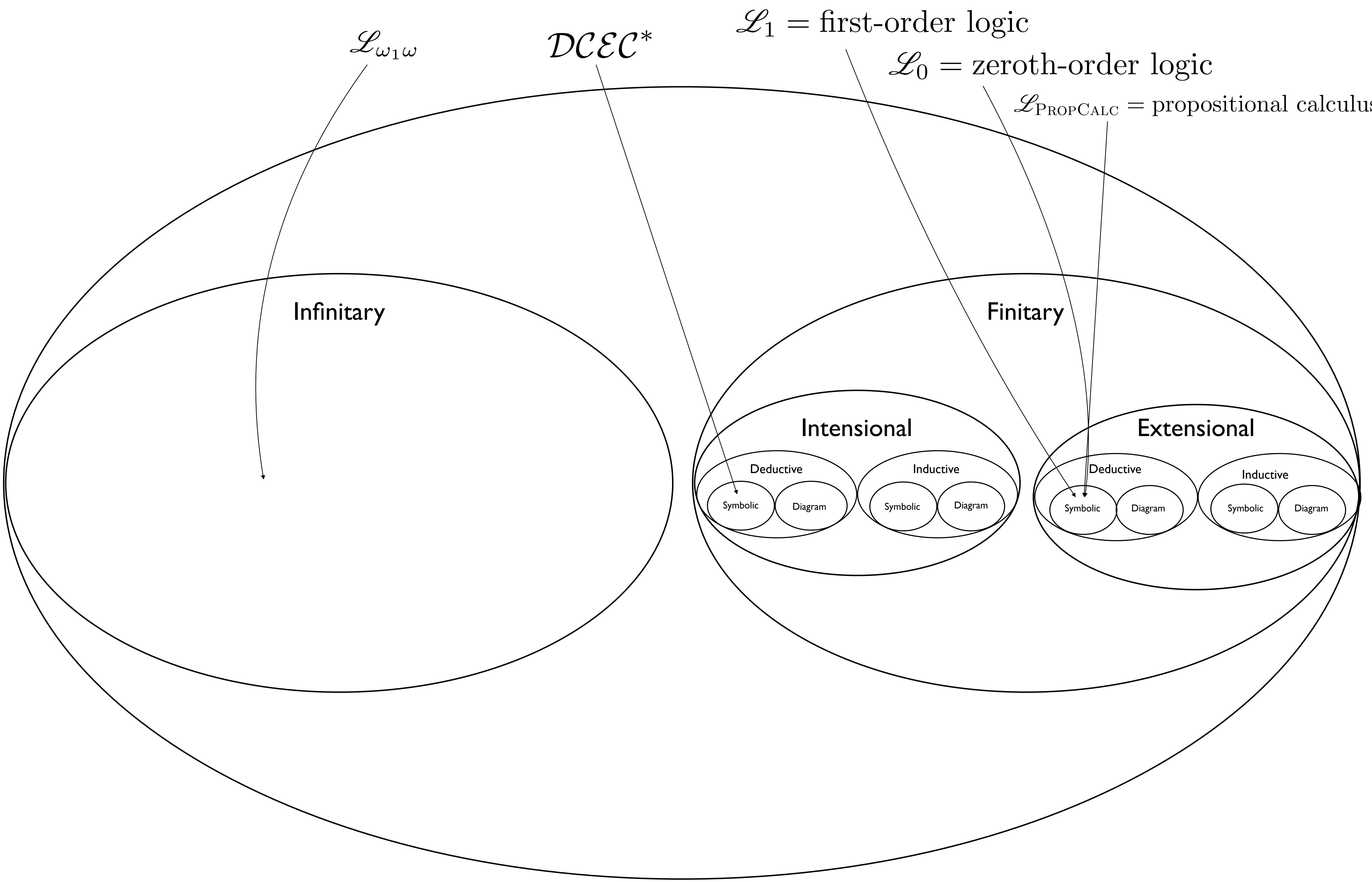
Intro to (Formal) Logic (& AI) — (IFLAI)
4/2/2020

The Universe of Logics



The Smallest Infinitary Logic

$$\mathscr{L}_{\omega_1 \omega}$$

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$$\mathscr{L}_{\omega_1 \omega}$$

$$\bigvee \Phi (\text{where } \Phi \coloneqq \phi_1, \phi_2, \ldots) \text{ is } \phi_1 \vee \phi_2 \ldots \infty$$

$$\bigwedge \Phi (\text{where } \Phi \coloneqq \phi_1, \phi_2, \ldots) \text{ is } \phi_1 \wedge \phi_2 \ldots \infty$$

The Smallest Infinitary Logic

$$\mathscr{L}_{\omega_1 \omega}$$

$$\bigvee \Phi (\text{where } \Phi := \phi_1, \phi_2, \ldots) \text{ is } \phi_1 \vee \phi_2 \ldots \infty$$

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Finitude *can* be captured in *this* logic (but not in $\mathscr{L}_1$; that's why no one won $1000). How?

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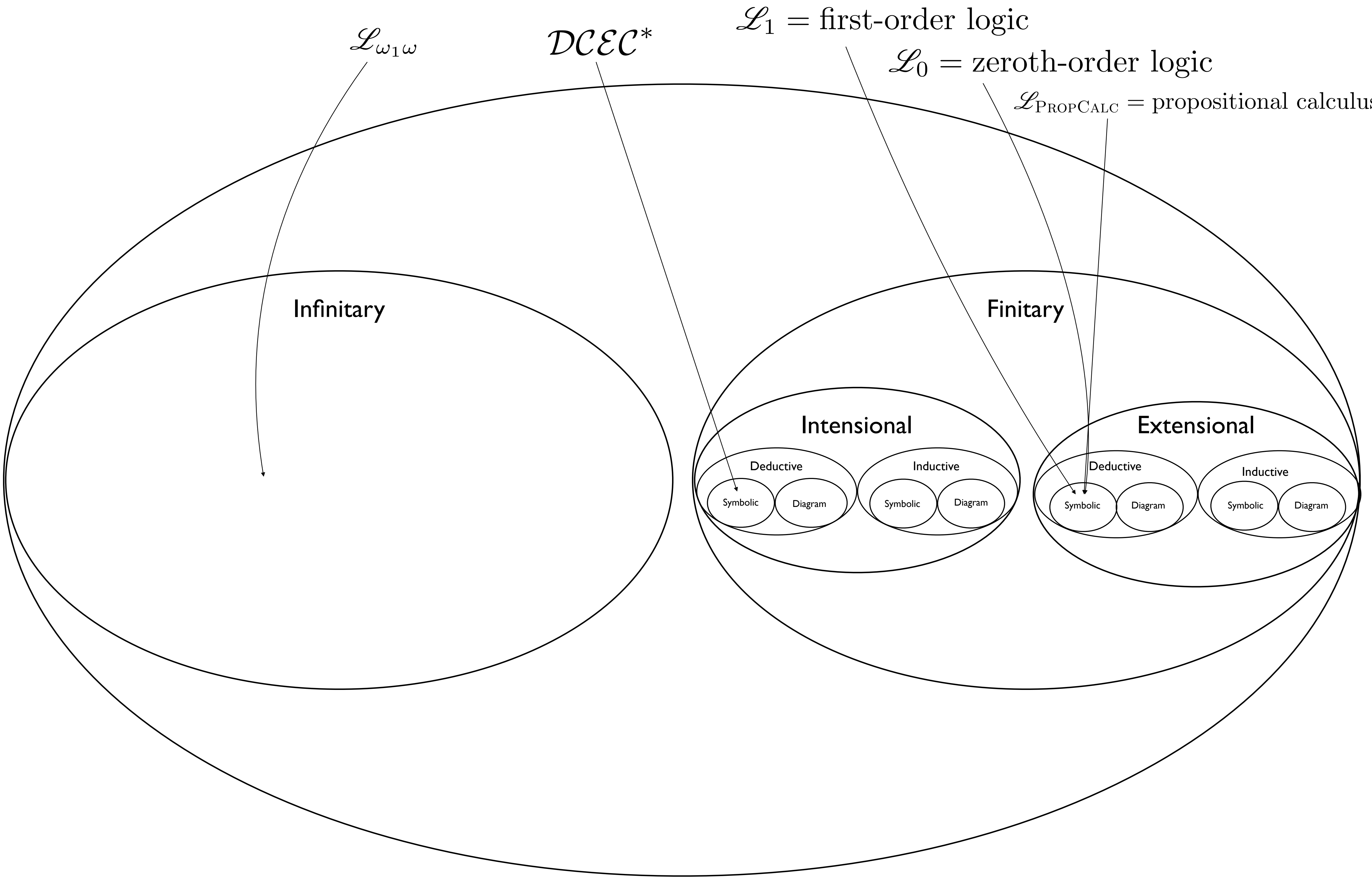

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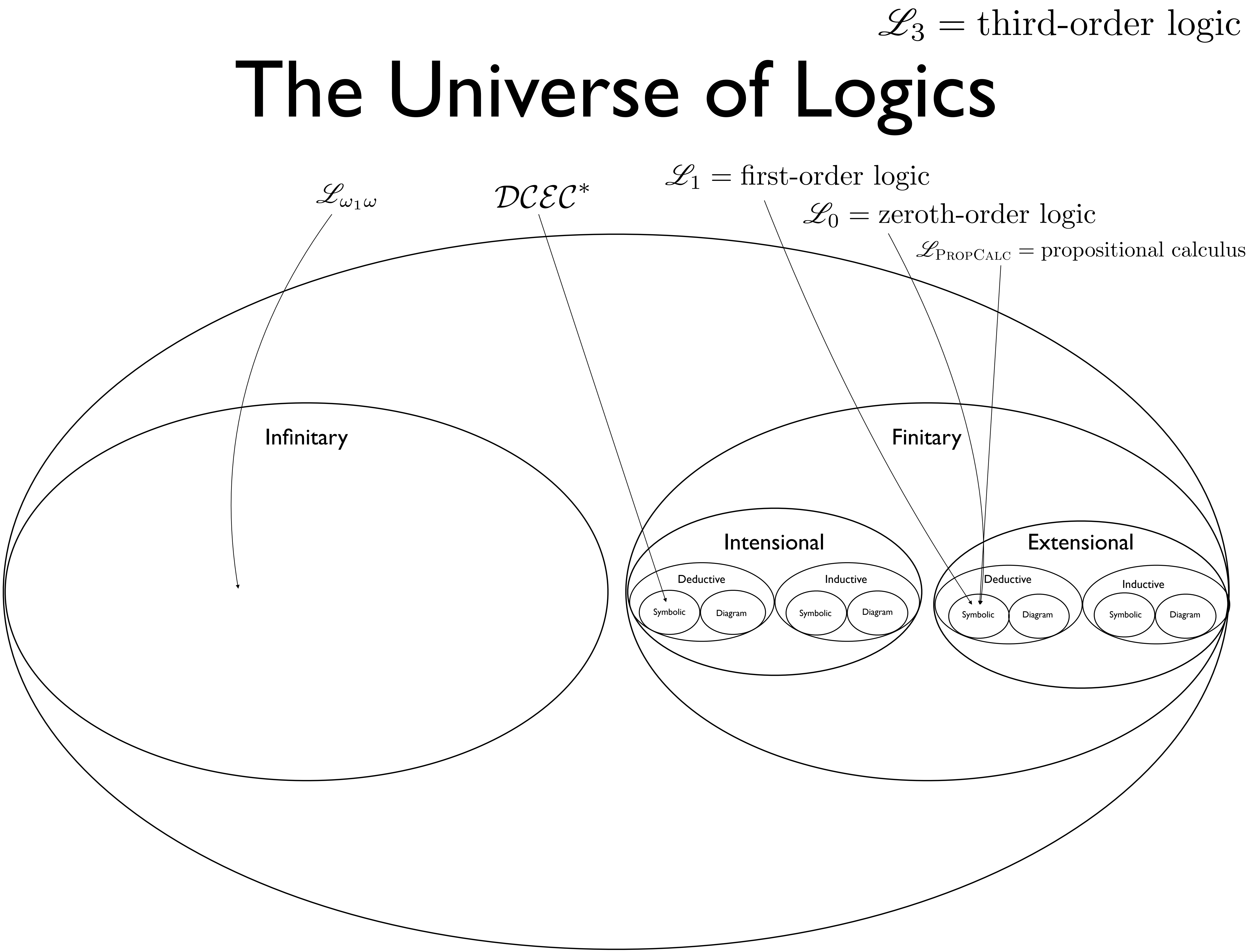

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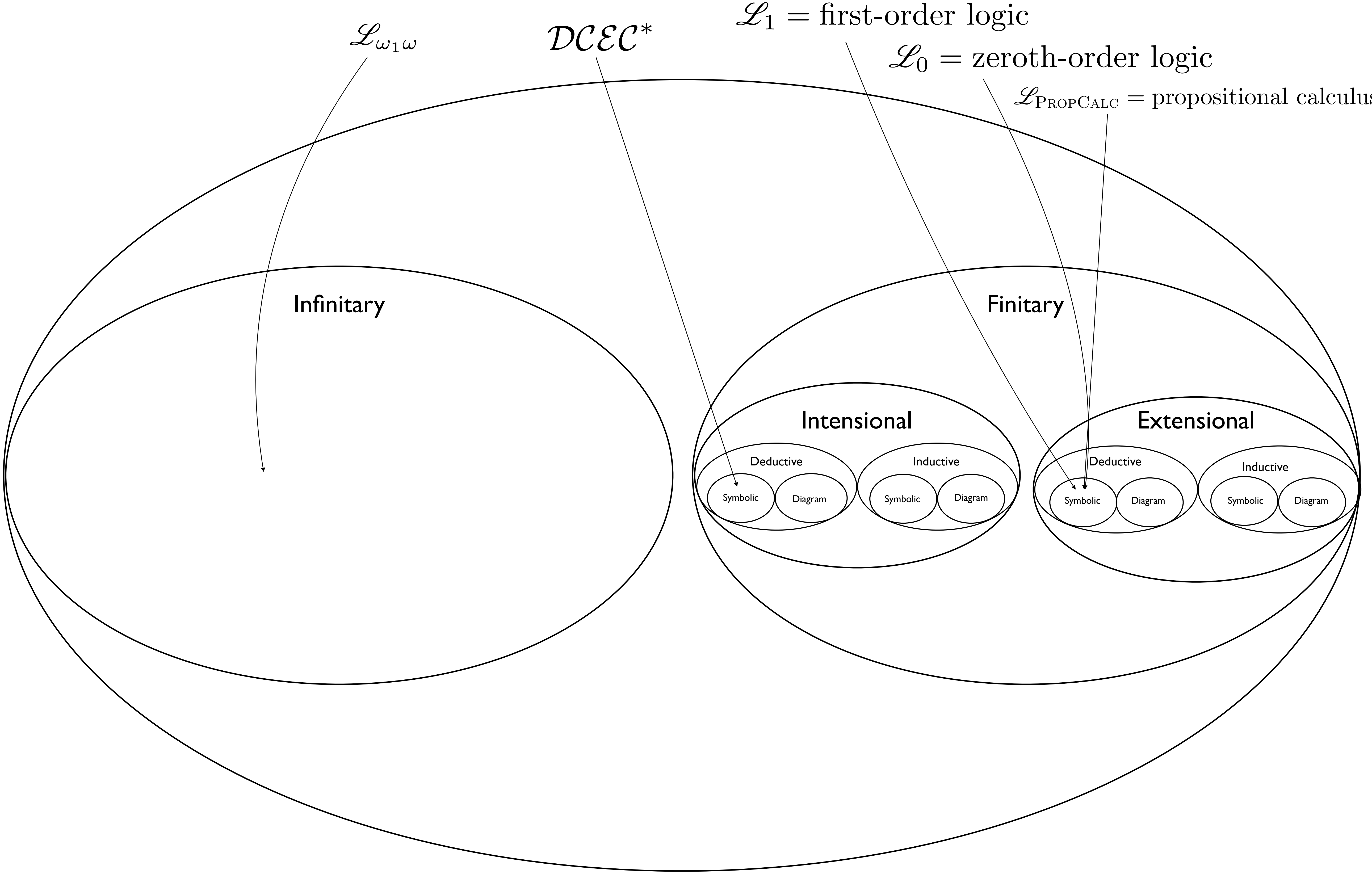
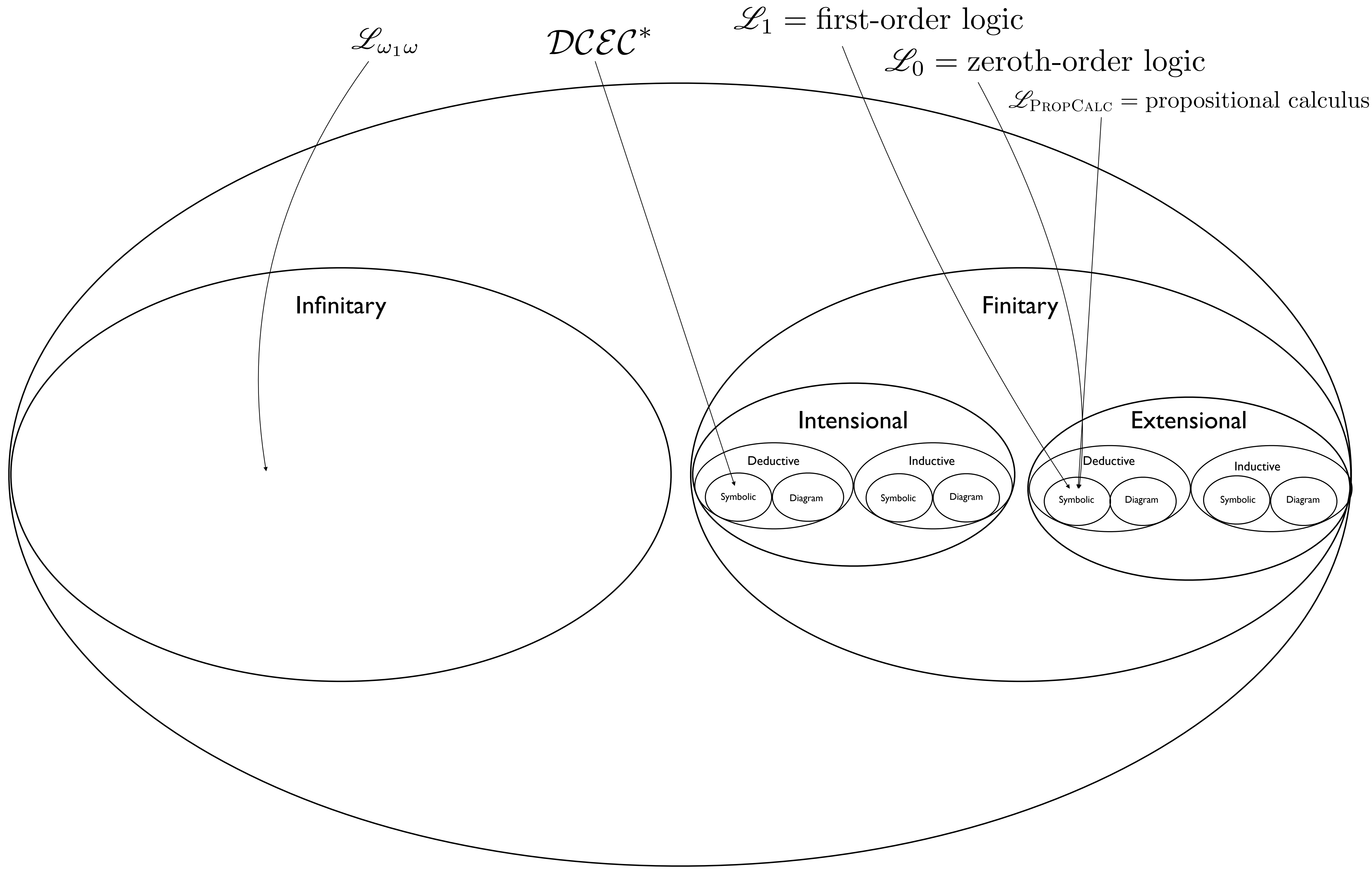


$\{\mathbf{K}, \mathbf{T}, \mathbf{M} = \mathbf{D} \text{ (Deontic)}, \mathbf{S4}, \mathbf{S5}\}$ $\mathscr{L}_2 =$ second-order logic $\mathscr{L}_3 =$ third-order logic

The Universe of Logics



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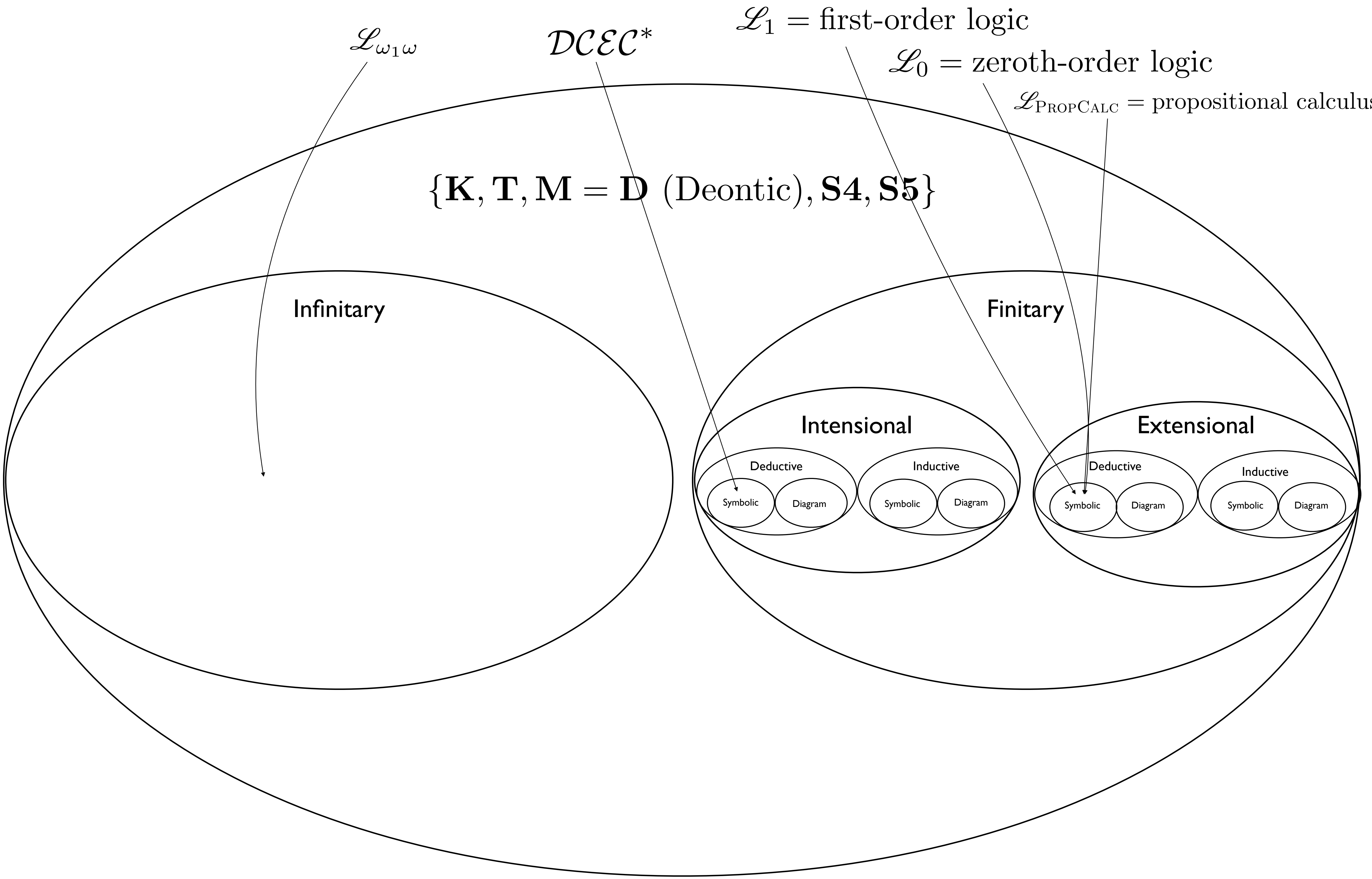

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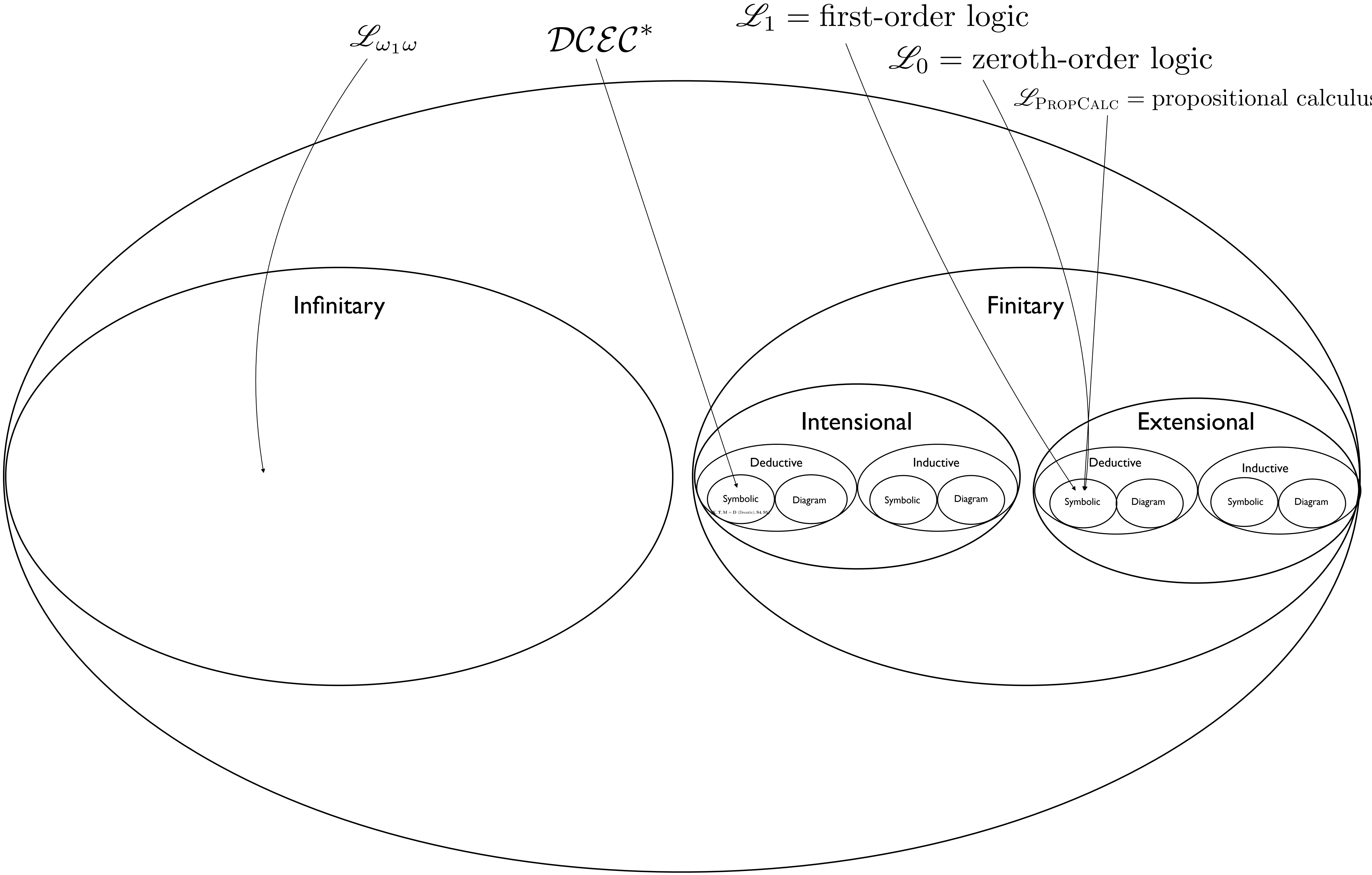
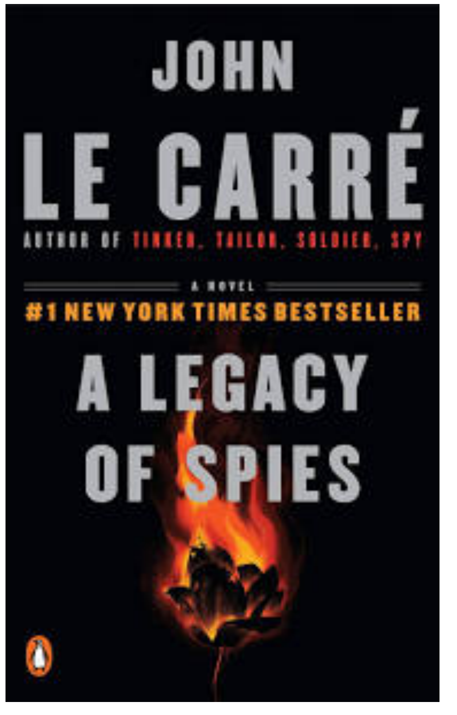
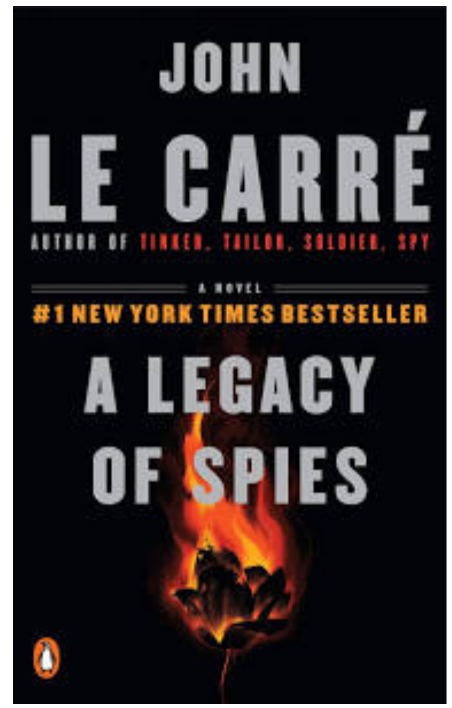


Uh oh

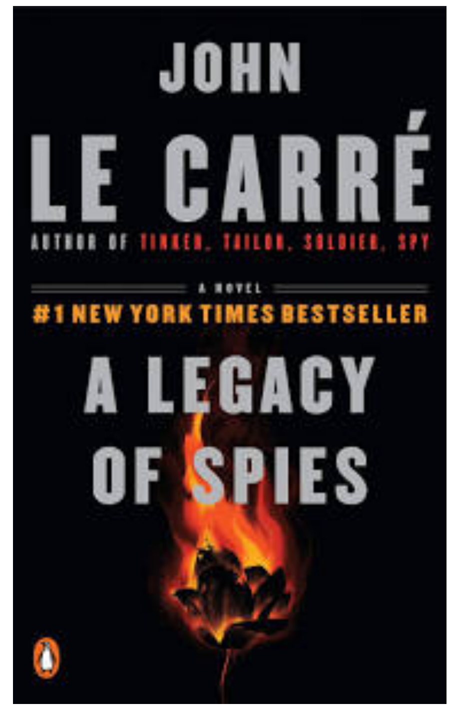

JOHN
LE CARRÉ
AUTHOR OF TINKER, TAILOR, SOLDIER, SPY
A NOVEL
#1 NEW YORK TIMES BESTSELLER
A LEGACY
OF SPIES

Uh oh



	(1)	$lecarre = cornwall$	fact
	(2)	$authOf(legacyofspies) = lecarre$	fact
	(3)	$Knows(alvin, authOf(legacyofspies) = lecarre)$	fact
$\therefore$	(4)	$Knows(alvin, authOf(legacyofspies) = cornwall)$	fact

Uh oh



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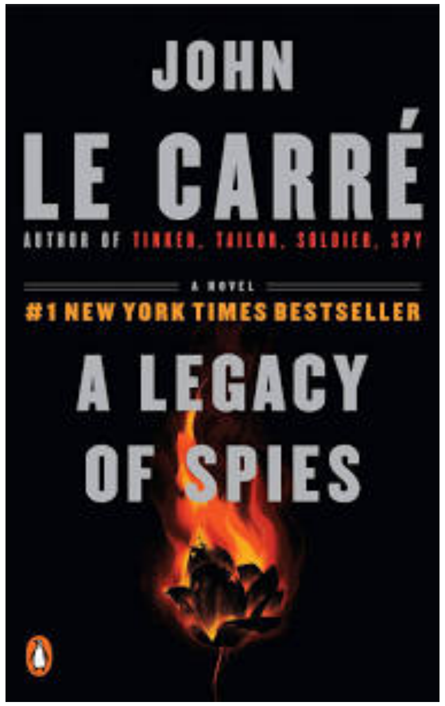

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This is only the *first* wave of some *really* bad news that befalls those in AI (& CogSci) who might think that extensional logic is going to work in the realm of human-level cognition …

Given the Web, What Is Intelligence, Really?

Article (PDF Available) *in* Metaphilosophy 43(4) · July 2012
DOI: 10.1111/j.1467-9973.2012.01760.x

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Naveen Sundar Govindarajulu

Abstract

This article argues that existing systems on the Web cannot approach human-level intelligence, as envisioned by Descartes, without being able to achieve genuine problem solving on unseen problems. The article argues that this entails committing to a strong intensional logic. In addition to revising extant arguments in favor of intensional systems, it presents a novel mathematical argument to show why extensional systems can never hope to capture the inherent complexity of natural language. The argument makes its case by focusing on representing, with increasing degrees of complexity, knowledge in a first-order language. Nevertheless, the attempts at representation fail to achieve consistency, making the case for an intensional representation system for natural language clear.

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Let's investigate the first theorem in this paper, in HyperSlate™ …

Modal Logic to Save Lives

https://en.wikipedia.org/wiki/Air_France_Flight_447

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	(1)	$\mathbf{Knows}(ai, \mathbf{Knows}(pilot2, nosepulledup))$	fact
$\therefore$	(2)	$\mathbf{Knows}(pilot2, nosepulledup)$	from (1)
$\therefore$	(3)	$nosepulledup$	from (2) in $\mathbf{S4}$
	(4)	$nosepulledup \rightarrow aioverride!$	premise
$\therefore$	(5)	$aioverride!$	from (3), (4) $\mathscr{L}_{\mathrm{PC}}$

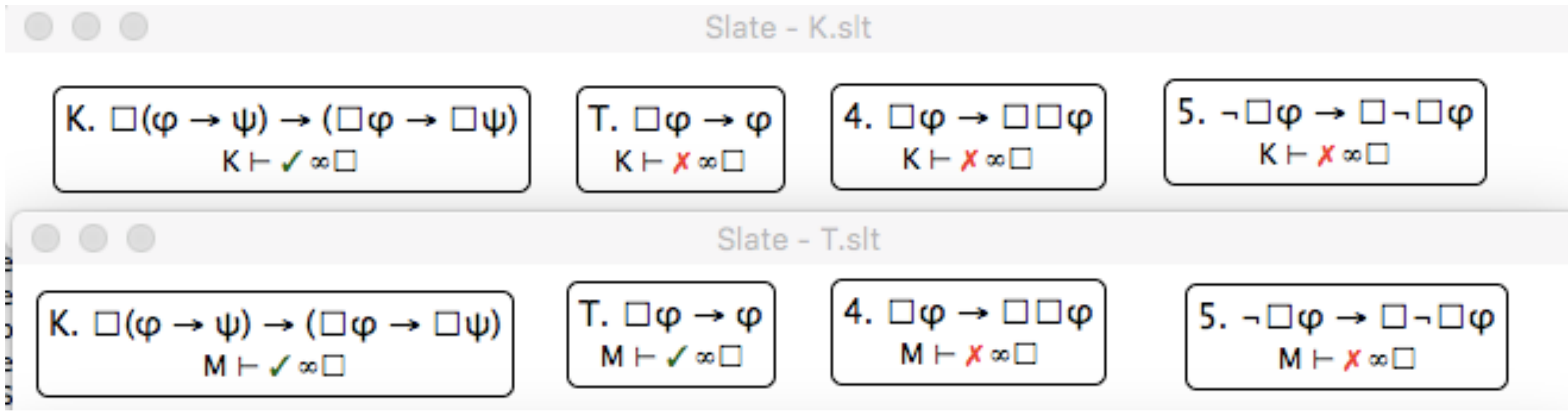
Encapsulation to Memorize

K

Encapsulation to Memorize

K

T

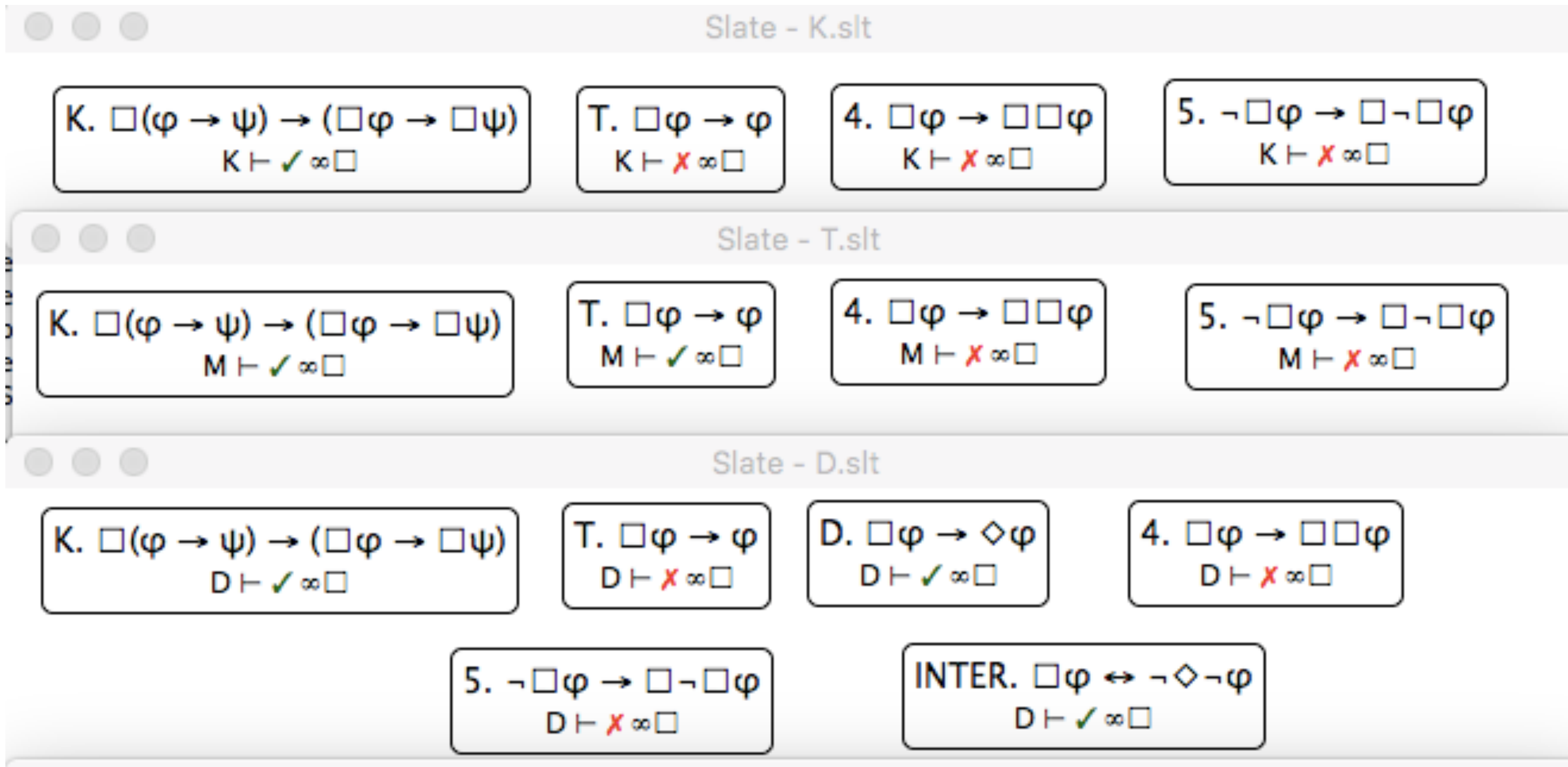


Encapsulation to Memorize

K

T

D



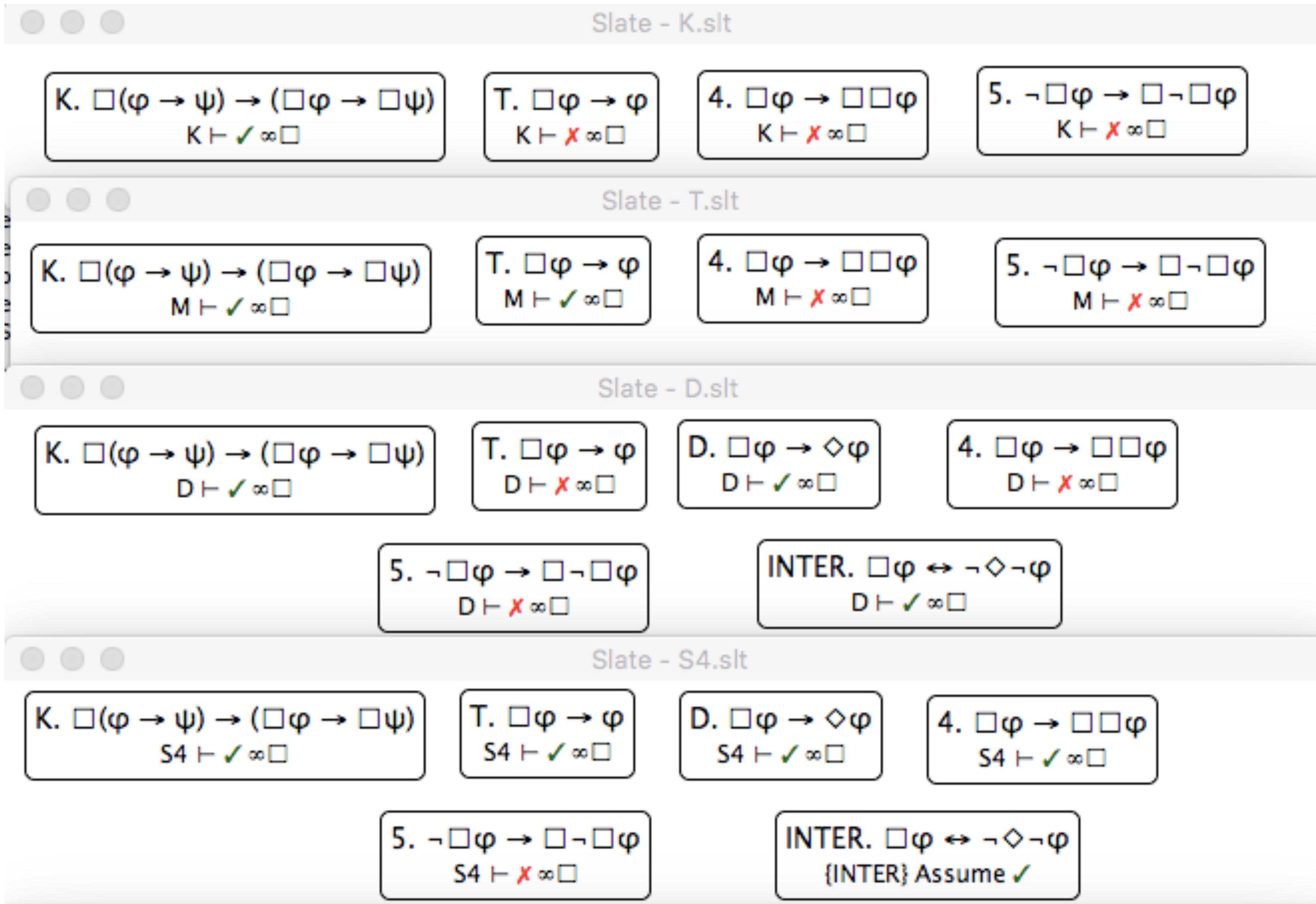
Encapsulation to Memorize

K

T

D

4 = S4



Encapsulation to Memorize

K

T

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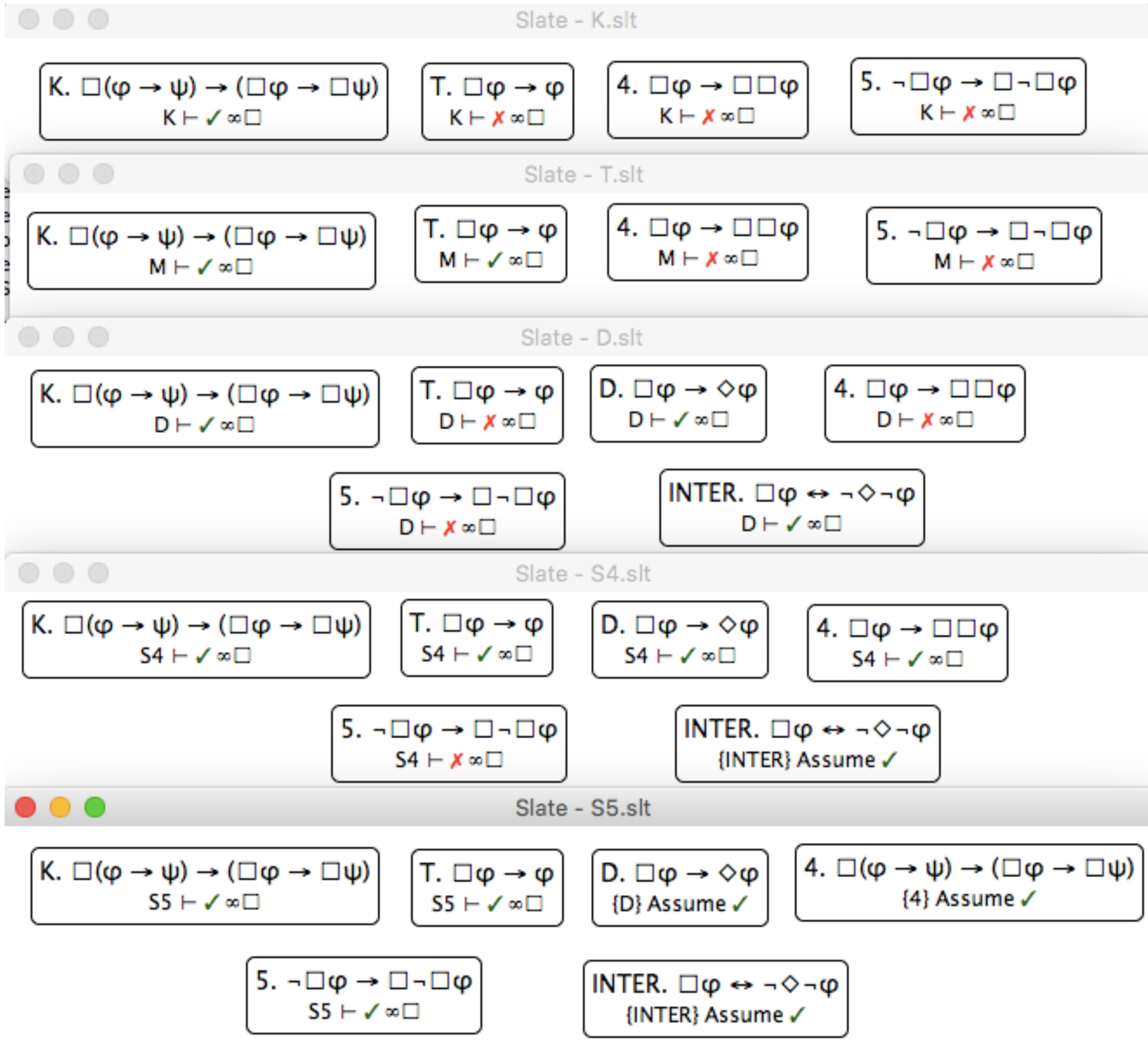
4 = S4

5 = S5



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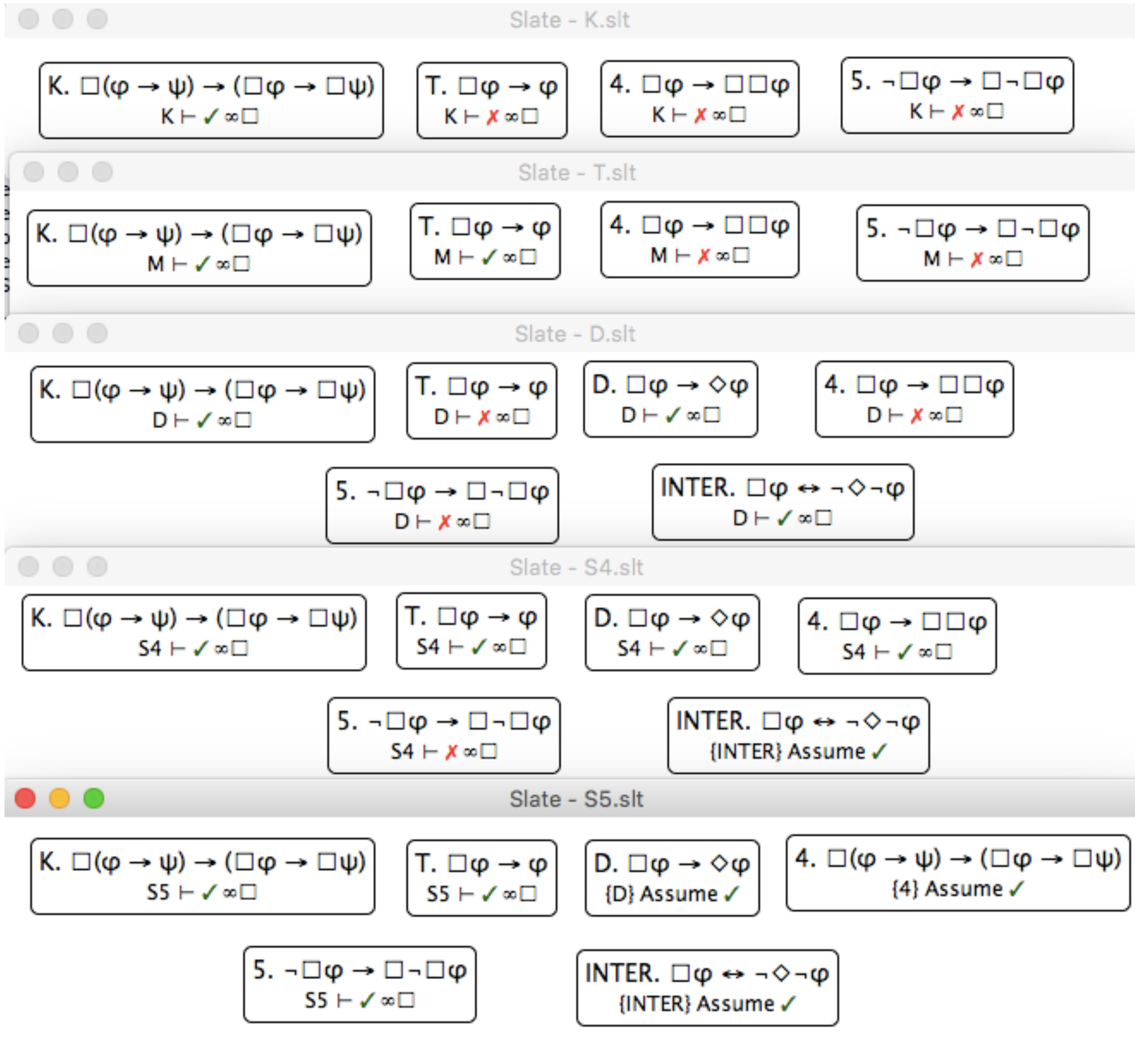

HyperSlate™

Encapsulation to Memorize

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