FOL II: universal intro

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Intro to Logic
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What *is* Logic?

- The key to becoming rational.

- "The science of reasoning." — so the not-unreasonable slogan goes.

- The only invincible subject there is.

- The basis for the formal sciences (from mathematics to game theory to decision theory to probability calculi to axiomatic physics ….) — and hence the basis for disciplines based on the formal sciences (e.g., engineering, computer science).

- The way of escape from shallow content and context to pure, immaterial, and immortal form and structure (which is why the exotic, imaginary, and seemingly non-sensical is so pedagogically useful).

- The most challenging subject there is.

- One of the chief differentiators between dogs and monkeys versus you (let alone bears and you); and mindless machines (like Deep Blue & Watson) versus you.

- A key to riches.

- The key to divining the meaning of life (and other such big questions).

- The better way to program computers; and fundamentally the *only* way to *reliably* program computers.

- One of two fundamental approaches to studying minds, and replicating/simulating minds in machines…

- The thing many creatures of fiction have mastered — have you (as a New Yorker)?…

Re Test 1 Grades

- 2 AI-created problems correct plus 1 human-created problem: A

- more than above: A+

- ????: B

- ????: C

Test 1 Solutions …

Analyze the (forthcoming) solution videos!

HyperGrader
Required Problems:
Self-paced, yes, and deadline
now in countdown — but
interconnected! E.g., again …
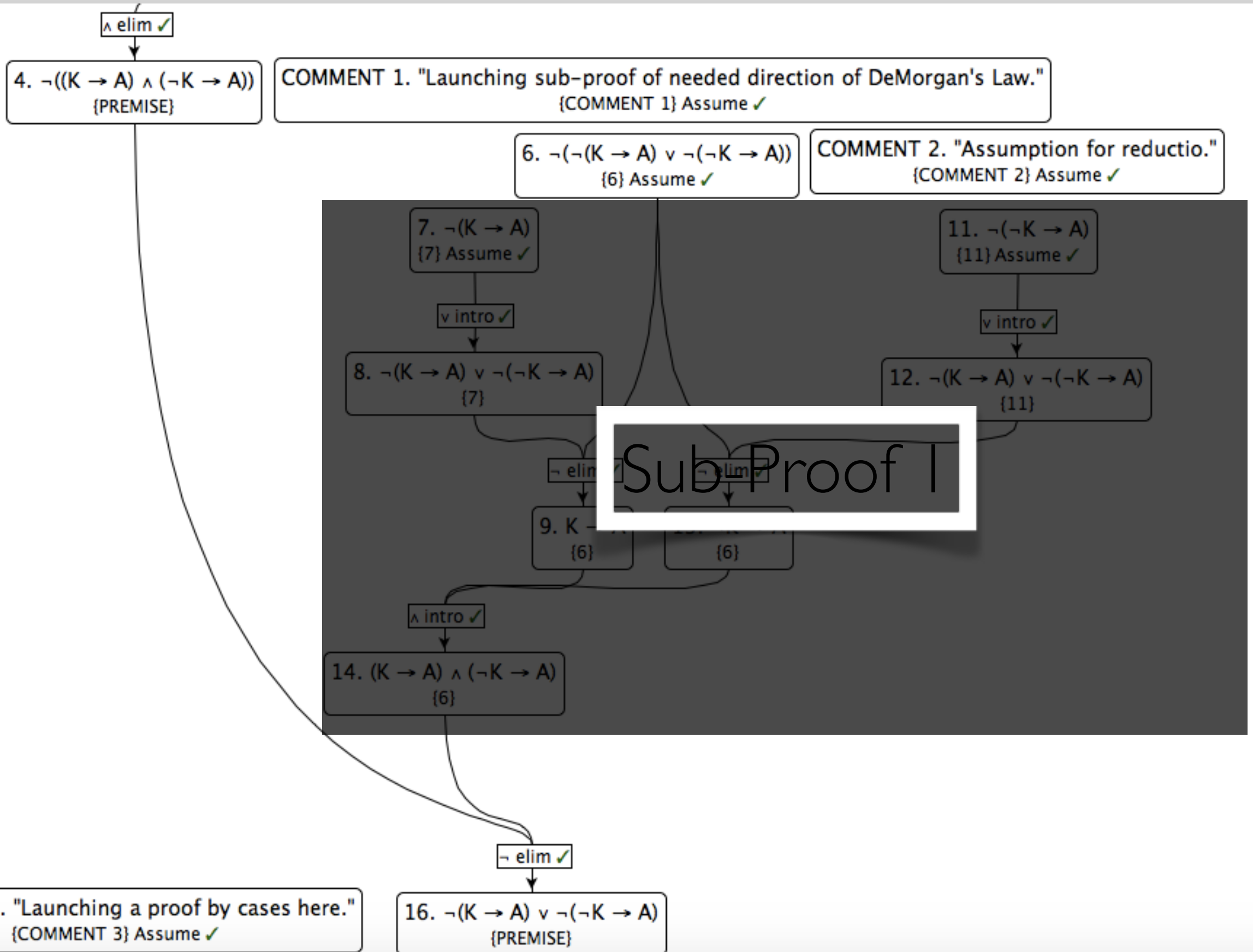
BogusBiconditional

tertium_non_datur

Disj_Elim

BogusBiconditional

BogusBiconditional

tertium_non_datur

Disj_Elim

Further Comments on Proof Plan for KingAce2

∧ elim ✓

4. ¬((K → A) ∧ (¬K → A))
{PREMISE}

COMMENT 1. "Launching sub-proof of needed direction of DeMorgan's Law."
{COMMENT 1} Assume ✓

6. ¬(¬(K → A) ∨ ¬(¬K → A))
{6} Assume ✓

COMMENT 2. "Assumption for reductio."
{COMMENT 2} Assume ✓

7. ¬(K → A)
{7} Assume ✓

11. ¬(¬K → A)
{11} Assume ✓

∨ intro ✓

∨ intro ✓

8. ¬(K → A) ∨ ¬(¬K → A)
{7}

12. ¬(K → A) ∨ ¬(¬K → A)
{11}

¬ elim

¬ elim

Sub-Proof I

9. K →
{6}

{6}

∧ intro ✓

14. (K → A) ∧ (¬K → A)
{6}

¬ elim ✓

COMMENT 3. "Launching a proof by cases here."
{COMMENT 3} Assume ✓

16. ¬(K → A) ∨ ¬(¬K → A)
{PREMISE}

COMMENT 3. "Launching a proof by cases here."
{COMMENT 3} Assume ✓

16. ¬(K → A) ∨ ¬(¬K → A)
{PREMISE}

17. ¬(K → A)
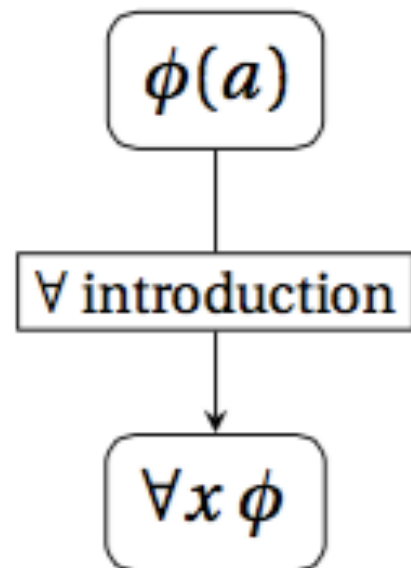{17} Assume ✓

18. ¬(¬K → A)
{18} Assume ✓

Sub-Proof 2

GOAL. ¬A
{PREMISE}

Next New (*Not*-So-Easy!) Inference Rule in FOL

- **universal introduction**

  - If something *a* is an *R*, and the constant/name *a* is *genuinely arbitrary*, then we can deduce that everything is an *R*.
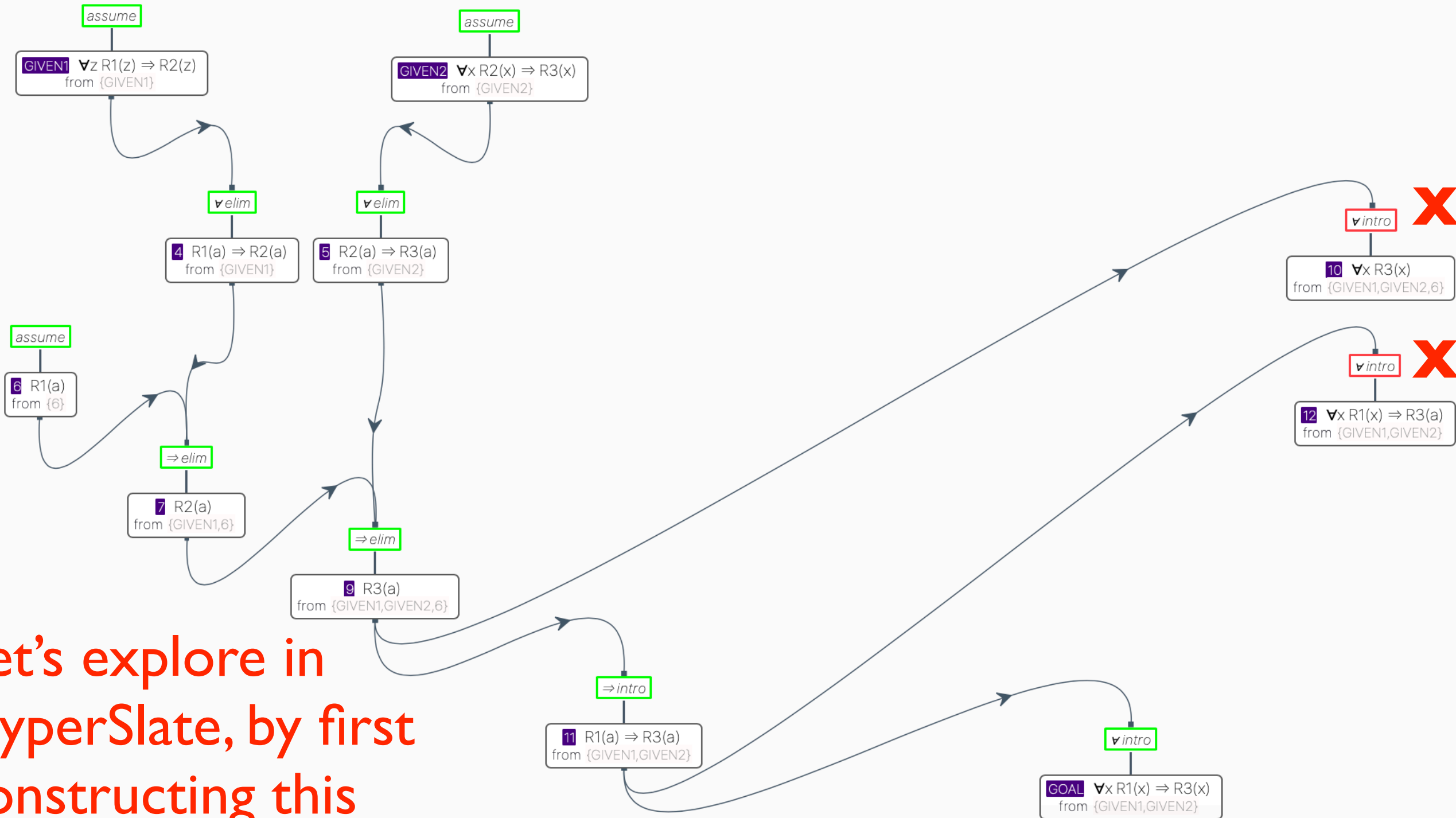
The Inference Schema

$\phi(a)$

∀ introduction

provided that a does not appear free in any in-scope assumption of ϕ, and that no occurrence of a appear in the inferred $\forall x\, \phi$

(3.16)

$\forall x\, \phi$

(Why the provisos?)

universal intro Example/Tutorial

assume

GIVEN1 ∀z R1(z) ⇒ R2(z)
from {GIVEN1}

assume

GIVEN2 ∀x R2(x) ⇒ R3(x)
from {GIVEN2}

∀ elim

4 R1(a) ⇒ R2(a)
from {GIVEN1}

∀ elim

5 R2(a) ⇒ R3(a)
from {GIVEN2}

∀ intro ✗

10 ∀x R3(x)
from {GIVEN1,GIVEN2,6}

assume

6 R1(a)
from {6}

⇒ elim

7 R2(a)
from {GIVEN1,6}

∀ intro ✗

12 ∀x R1(x) ⇒ R3(a)
from {GIVEN1,GIVEN2}

⇒ elim

9 R3(a)
from {GIVEN1,GIVEN2,6}

⇒ intro

11 R1(a) ⇒ R3(a)
from {GIVEN1,GIVEN2}

∀ intro

GOAL ∀x R1(x) ⇒ R3(x)
from {GIVEN1,GIVEN2}

Let's explore in HyperSlate, by first constructing this example from scratch …

Suggested Practice Problems in HyperSlate!

$$\{\forall x(R(x) \leftrightarrow S(x)), \forall x R(x)\} \vdash \forall x S(x)\} \; ?$$

$$\{\forall x[\texttt{Norsk}(x) \rightarrow \forall y(\texttt{Svensk}(y) \rightarrow \texttt{Smarter}(x,y))]\} \vdash \forall x, y[(\texttt{Norsk}(x) \wedge \texttt{Svensk}(y)) \rightarrow \texttt{Smarter}(x,y)] \; ?$$

$$\{\forall x, y[(\texttt{Norsk}(x) \wedge (\texttt{Svensk}(y)) \rightarrow \texttt{Smarter}(x,y)],$$

$$\forall x, y[(\texttt{Svensk}(x) \wedge (\texttt{Dansk}(y)) \rightarrow \texttt{Smarter}(x,y)]\} \vdash$$

$$\forall x, y[(\texttt{Norsk}(x) \wedge (\texttt{Dansk}(y)) \rightarrow \texttt{Smarter}(x,y)] \; ?$$