FOL III

Selmer Bringsjord

Rensselaer AI & Reasoning (RAIR) Lab
Department of Cognitive Science
Department of Computer Science
Lally School of Management & Technology
Rensselaer Polytechnic Institute (RPI)
Troy, New York 12180 USA

Intro to Logic
2/27/2020

RAIR
Rensselaer AI and Reasoning Lab

$\mathscr{L}_1$

FOL III

Selmer Bringsjord

Rensselaer AI & Reasoning (RAIR) Lab
Department of Cognitive Science
Department of Computer Science
Lally School of Management & Technology
Rensselaer Polytechnic Institute (RPI)
Troy, New York 12180 USA

Intro to Logic
2/27/2020

R A I R
Rensselaer AI and Reasoning Lab

Live-action on HyperGrader …

ThxForThePCOracle

ThxForThePCOracle

Please attempt that now-ish; thx.

all

Download: LAMA-BDLA_021720.pdf

Test Problems

- Test_1_G2
- Test_1_G1
- Test_1_G3
- KK1point3Strings
- Special_Problem_3b
- EmmaHelpedToo

Problems

- Pop_2
- Pop_1
- Bonus_1
- KnightKnave_SmullyanKKProblem1.1
- AthenCfromAthenBandBthenC
- BiconditionalIntroByChaining
- BogusBiconditional

☐ ThxForThePCOracle

This straightforward problem is quickly solved with a minimum of tedium, courtesy of the PC (entailment) provability oracle, use of which is allowed to remain in your finished proof (but no use of any other oracle can be in the finished proof). This oracle is for the logic $\mathscr{L}_{PC}$. Any learning of formal logic, at more than a trivial level, without the availability of the kind of AI embodied by this oracle (and more powerful ones farther up the ladder of extensional logic), is not only pedagogically unwise, but also, for the learner, downright painful.

Deadline 22 Apr 2020 23:59:59 EST

Solve

☑ Explosion

Here your task is to prove the explosion inference schema/rule in HyperSlate™; the schema says that from a contradiction anything whatsoever follows. Expressed in meta-logic, you are to prove $\{\phi \land \neg\phi\} \vdash \psi$. As explained in class, for whatever reason, while the general proof technique of *reductio ad absurdum* (= proof by contradiction, indirect proof, etc.) is covered in high-school math (at least it is in the textbooks, e.g. solid ones for Geometry and Algebra 2), explosion for some reason is almost invariably left aside, despite the indisputable dual fact that explosion is valid in classical logic and mathematics, and that it can be extremely useful. (No deadline for now.)

Solve

☑ GreenCheeseMoon1

rpi.logicamodernapproach.com

Plated | News | SUNY System ...ess - Logon | Ultra Hardwar...are Products | Screen Door L...y Von Morris | Screen Door ...d Von Morris | Apple | Amazon | eBay | Yahoo! | .Mac

Slate by Technolutions | xSwarm - Naval Postgr... | https://pdfs.semantics... | DeceptECAI2020 | kryten.mm.rpi.edu/SBr... | Sign in to hypergrader... | ThxForThePCOracle | Check your email

HyperGrader™ | Problems | HyperSlate | My Submissions | Leader Board | Selmer.Bringsjord@gmail.com (longsnowflake876)

Open in HyperSlate

ThxForThePCOracle

This straightforward problem is quickly solved with a minimum of tedium, courtesy of the PC (entailment) provability oracle, use of which is allowed to remain in your finished proof (but no use of any other oracle can be in the finished proof). This oracle is for the logic $\mathscr{L}_{PC}$. Any learning of formal logic, at more than a trivial level, without the availability of the kind of AI embodied by this oracle (and more powerful ones farther up the ladder of extensional logic), is not only pedagogically unwise, but also, for the learner, downright painful.

Deadline 22 Apr 2020 23:59:59 EST

Days Hours Minutes Seconds

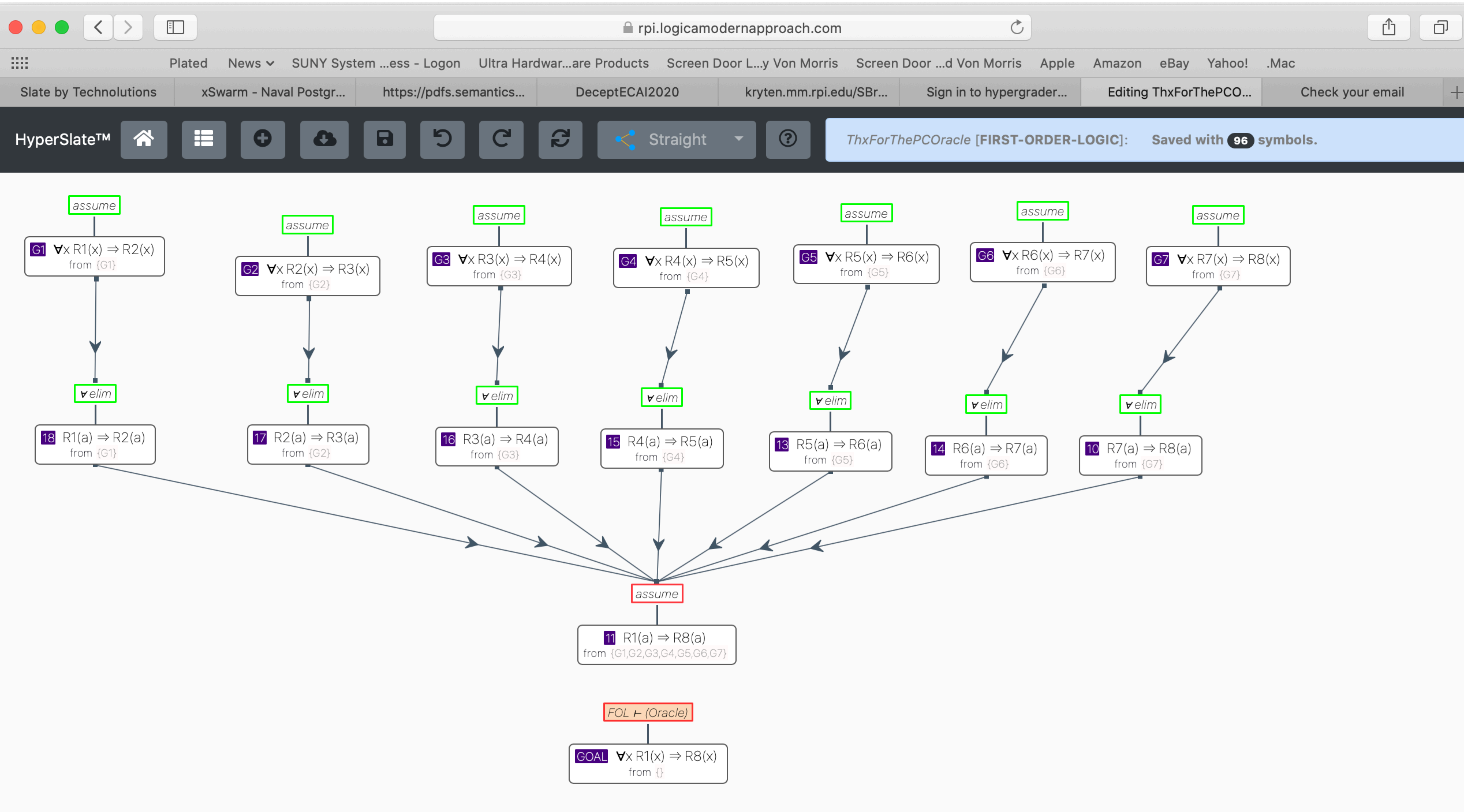
55:15:58:23

Problem Type: SIMPLE

Difficulty: 1

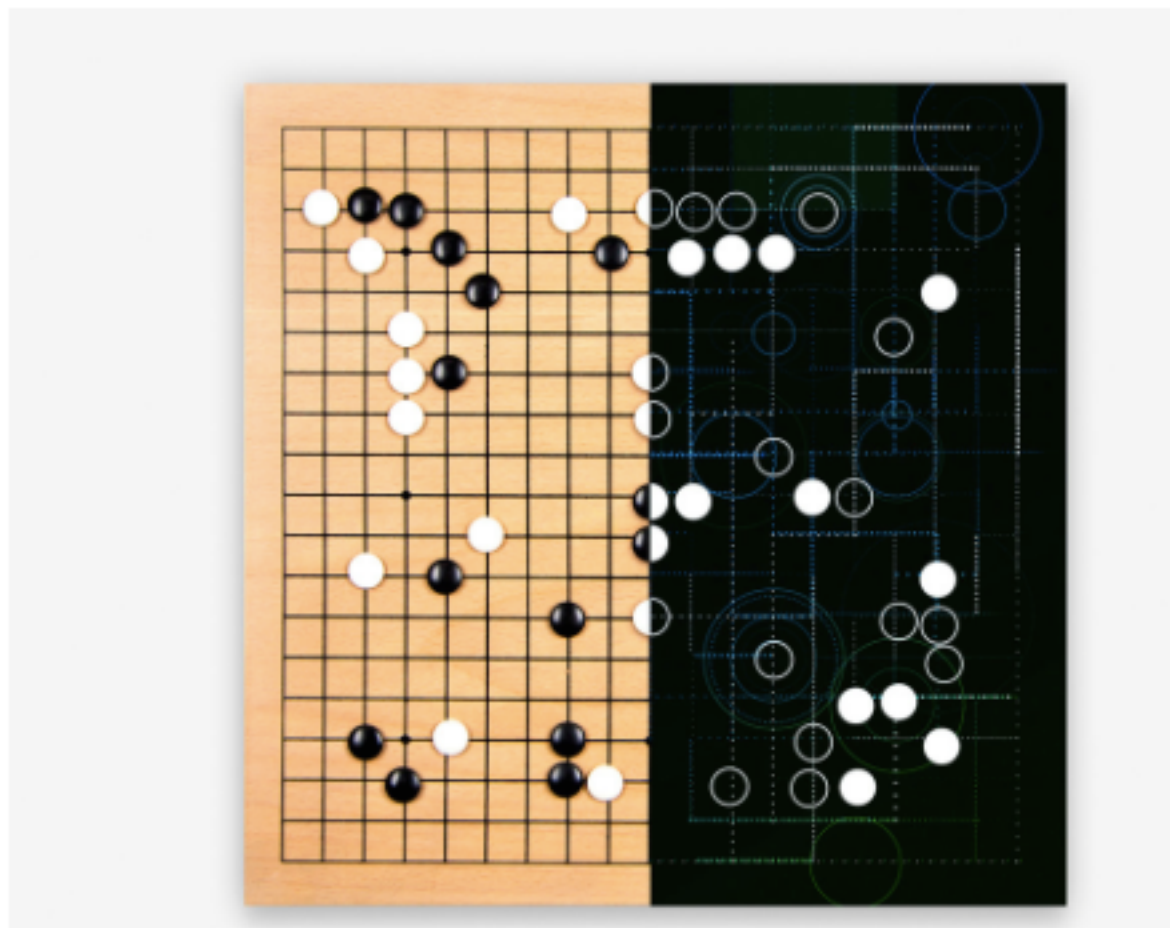
Points: 10

Leaderboard

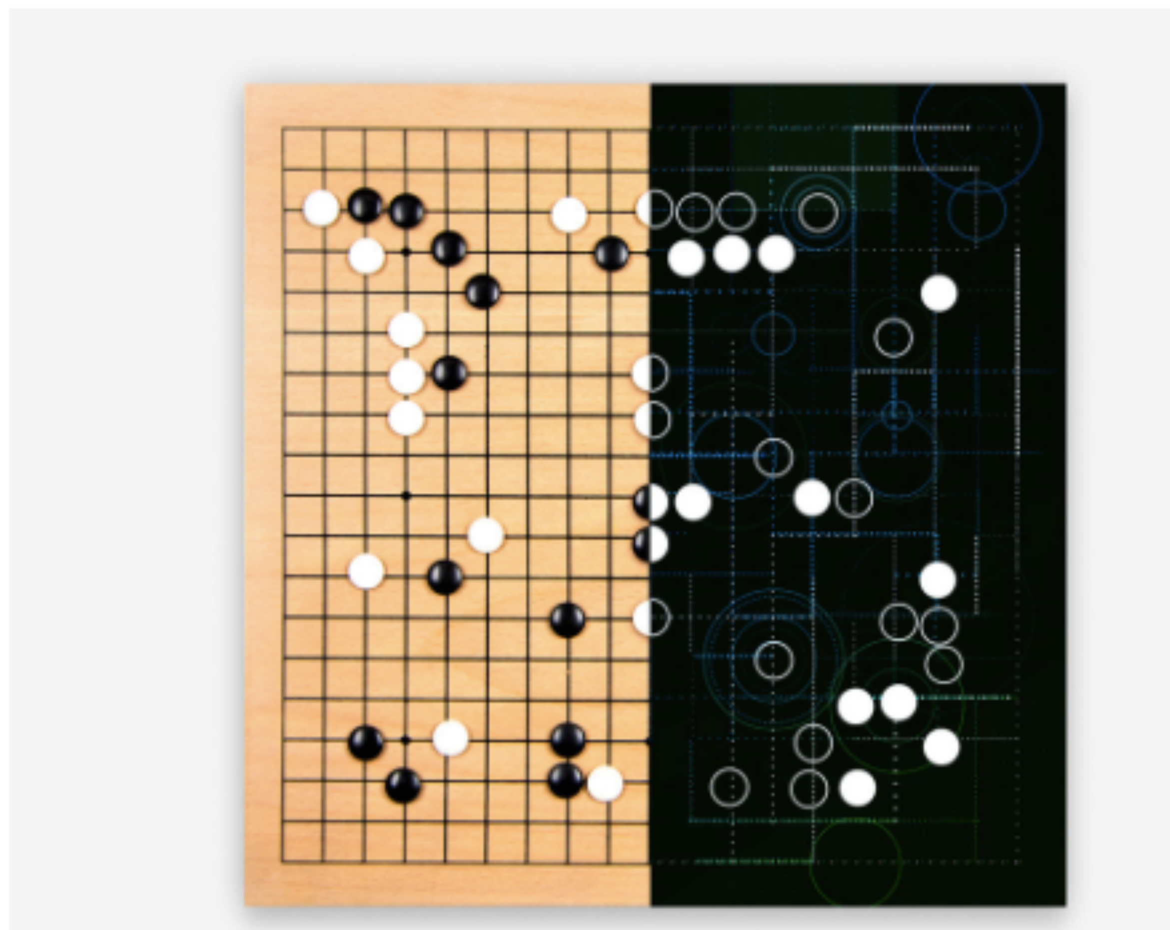

rpi.logicamodernapproach.com
Plated
News
SUNY System ...ess - Logon
Ultra Hardwar...are Products
Screen Door L...y Von Morris
Screen Door ...d Von Morris
Apple
Amazon
eBay
Yahoo!
.Mac
Slate by Technolutions
xSwarm - Naval Postgr...
https://pdfs.semantics...
DeceptECAI2020
kryten.mm.rpi.edu/SBr...
Sign in to hypergrader...
Editing ThxForThePCO...
Check your email
HyperSlate™
Straight
ThxForThePCOracle [FIRST-ORDER-LOGIC]: Saved with 96 symbols.
assume
G1 ∀x R1(x) ⇒ R2(x) from {G1}
assume
G2 ∀x R2(x) ⇒ R3(x) from {G2}
assume
G3 ∀x R3(x) ⇒ R4(x) from {G3}
assume
G4 ∀x R4(x) ⇒ R5(x) from {G4}
assume
G5 ∀x R5(x) ⇒ R6(x) from {G5}
assume
G6 ∀x R6(x) ⇒ R7(x) from {G6}
assume
G7 ∀x R7(x) ⇒ R8(x) from {G7}
∀elim
18 R1(a) ⇒ R2(a) from {G1}
∀elim
17 R2(a) ⇒ R3(a) from {G2}
∀elim
16 R3(a) ⇒ R4(a) from {G3}
∀elim
15 R4(a) ⇒ R5(a) from {G4}
∀elim
13 R5(a) ⇒ R6(a) from {G5}
∀elim
14 R6(a) ⇒ R7(a) from {G6}
∀elim
10 R7(a) ⇒ R8(a) from {G7}
assume
11 R1(a) ⇒ R8(a) from {G1,G2,G3,G4,G5,G6,G7}
FOL ⊢ (Oracle)
GOAL ∀x R1(x) ⇒ R8(x) from {}

Interlude re
Formal Logic & Games …

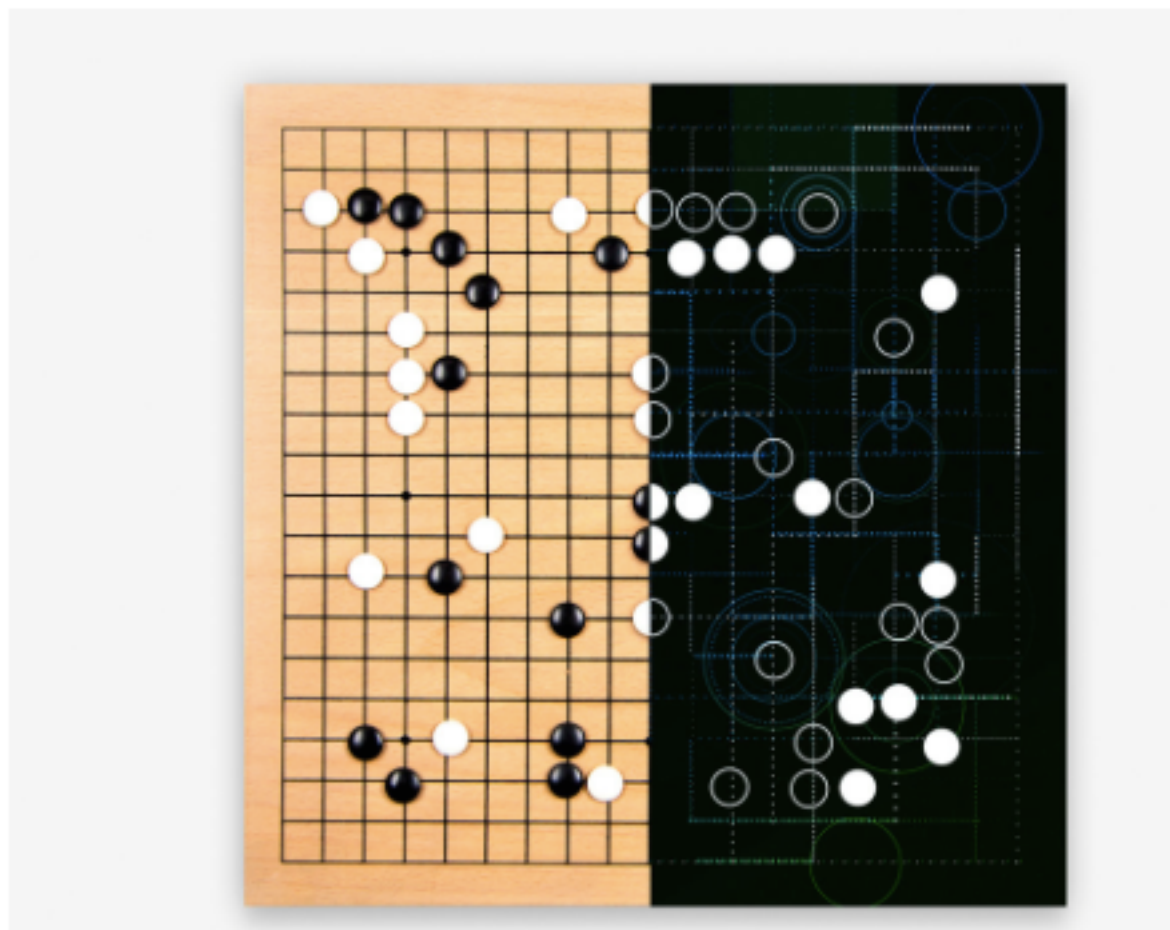
IN A HUGE BREAKTHROUGH, GOOGLE'S AI BEATS A TOP PLAYER AT THE GAME OF GO



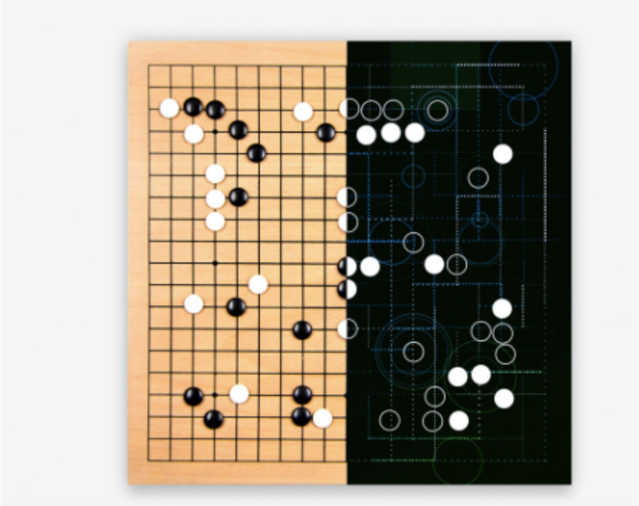
IN A HUGE BREAKTHROUGH, GOOGLE'S AI BEATS A TOP PLAYER AT THE GAME OF GO



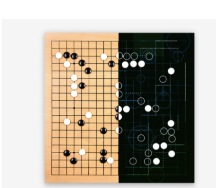
IN A HUGE BREAKTHROUGH, GOOGLE'S AI BEATS A TOP PLAYER AT THE GAME OF GO



IN A HUGE BREAKTHROUGH, GOOGLE'S AI BEATS A TOP PLAYER AT THE GAME OF GO



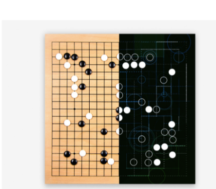
IN A HUGE BREAKTHROUGH, GOOGLE'S AI BEATS A TOP PLAYER AT THE GAME OF GO



The *Entscheidungsproblem*

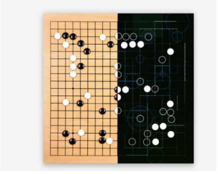
The *Entscheidungsproblem*

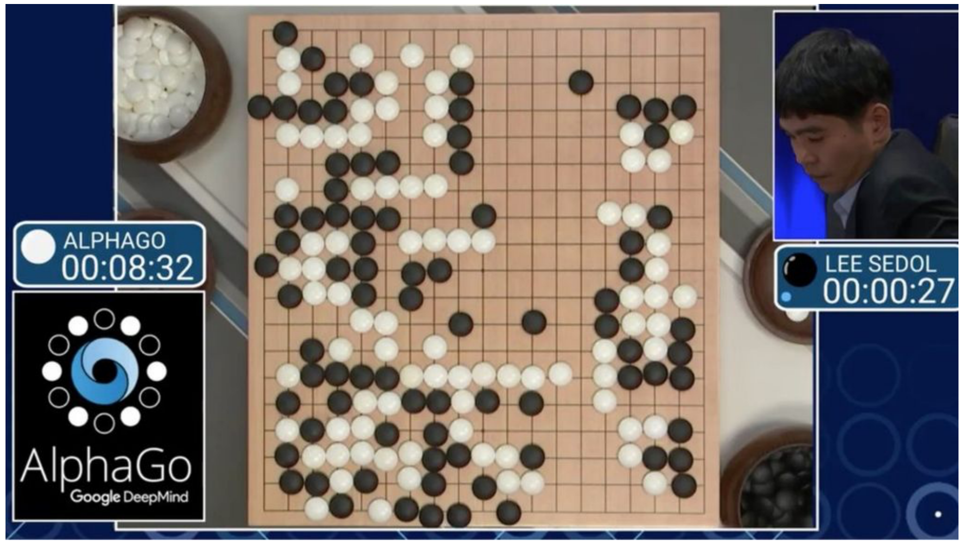
IN A HUGE BREAKTHROUGH, GOOGLE'S AI BEATS A TOP PLAYER AT THE GAME OF GO

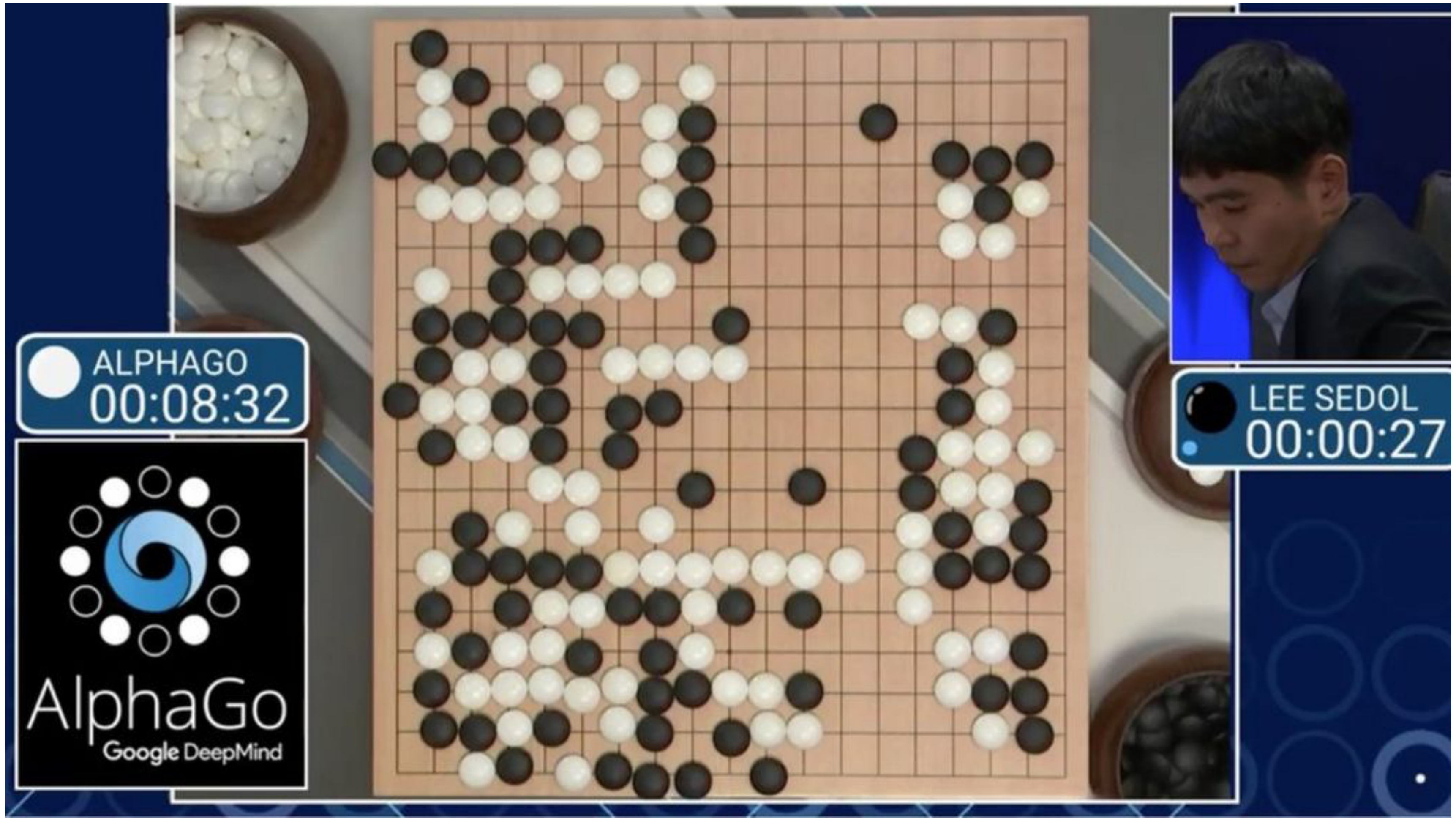


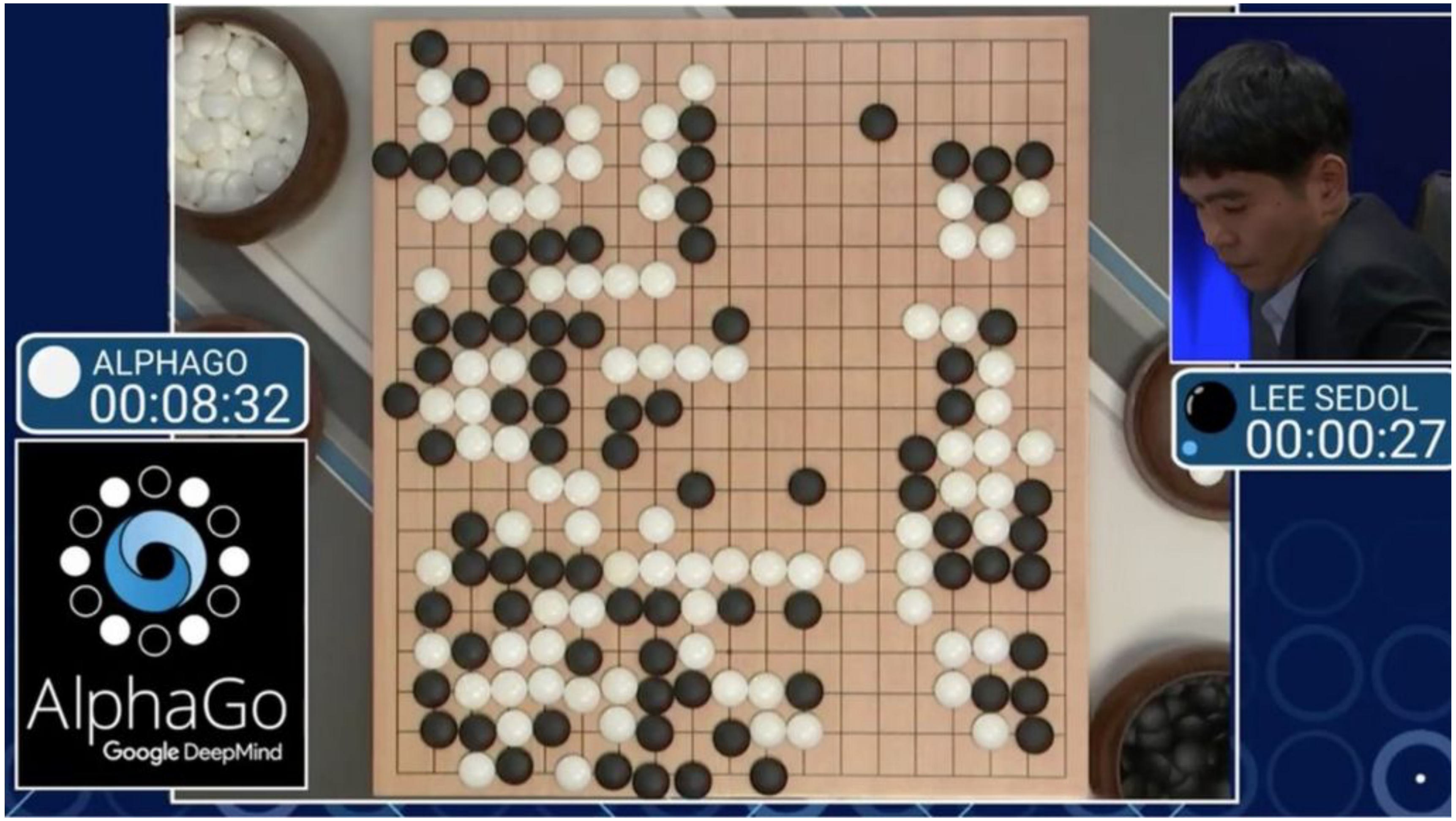
The *Entscheidungsproblem*

IN A HUGE BREAKTHROUGH, GOOGLE'S AI BEATS A TOP PLAYER AT THE GAME OF GO

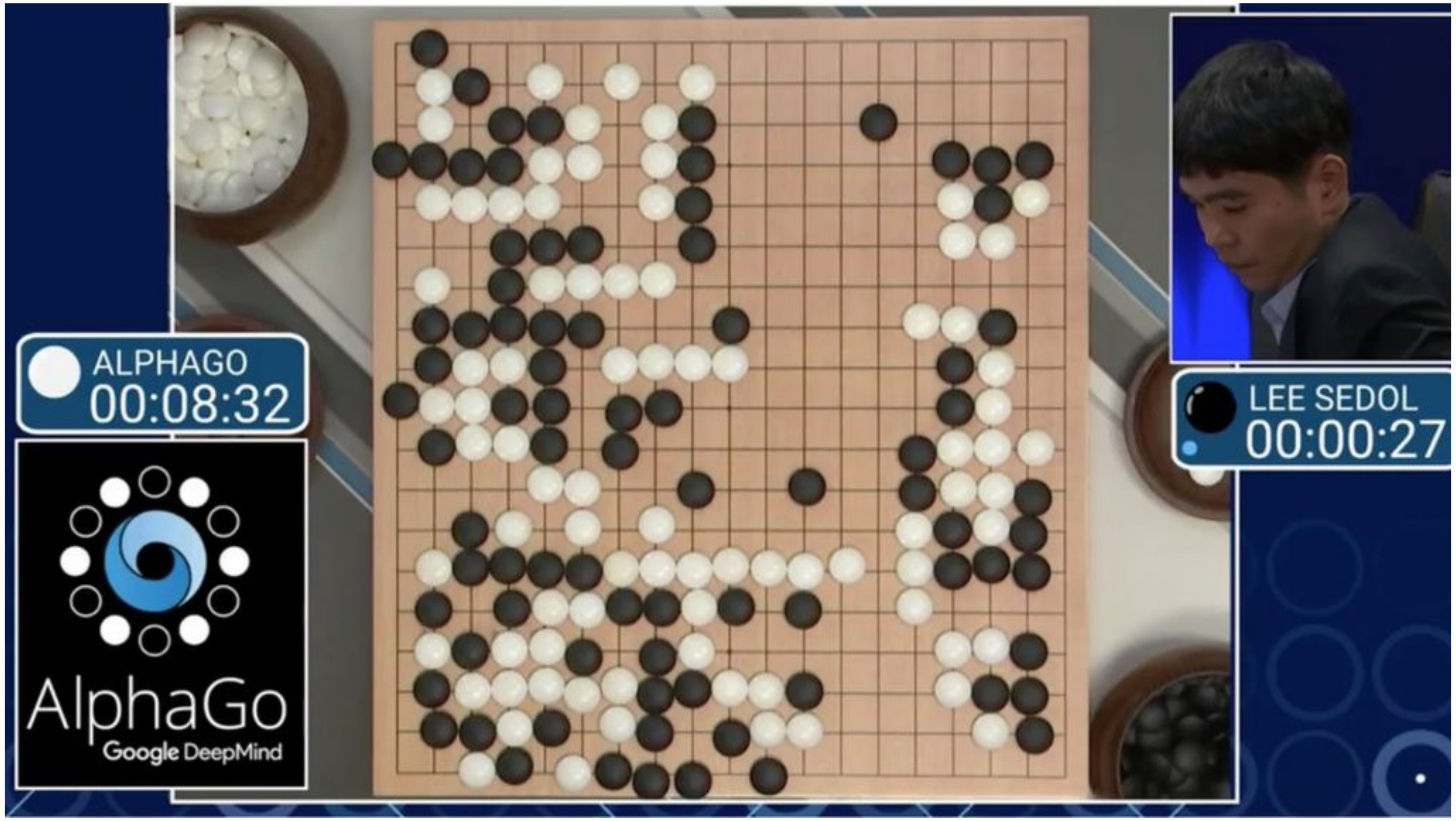



ALPHAGO
00:08:32
AlphaGo
Google DeepMind
LEE SEDOL
00:00:27


ALPHAGO
00:08:32
AlphaGo
Google DeepMind
LEE SEDOL
00:00:27

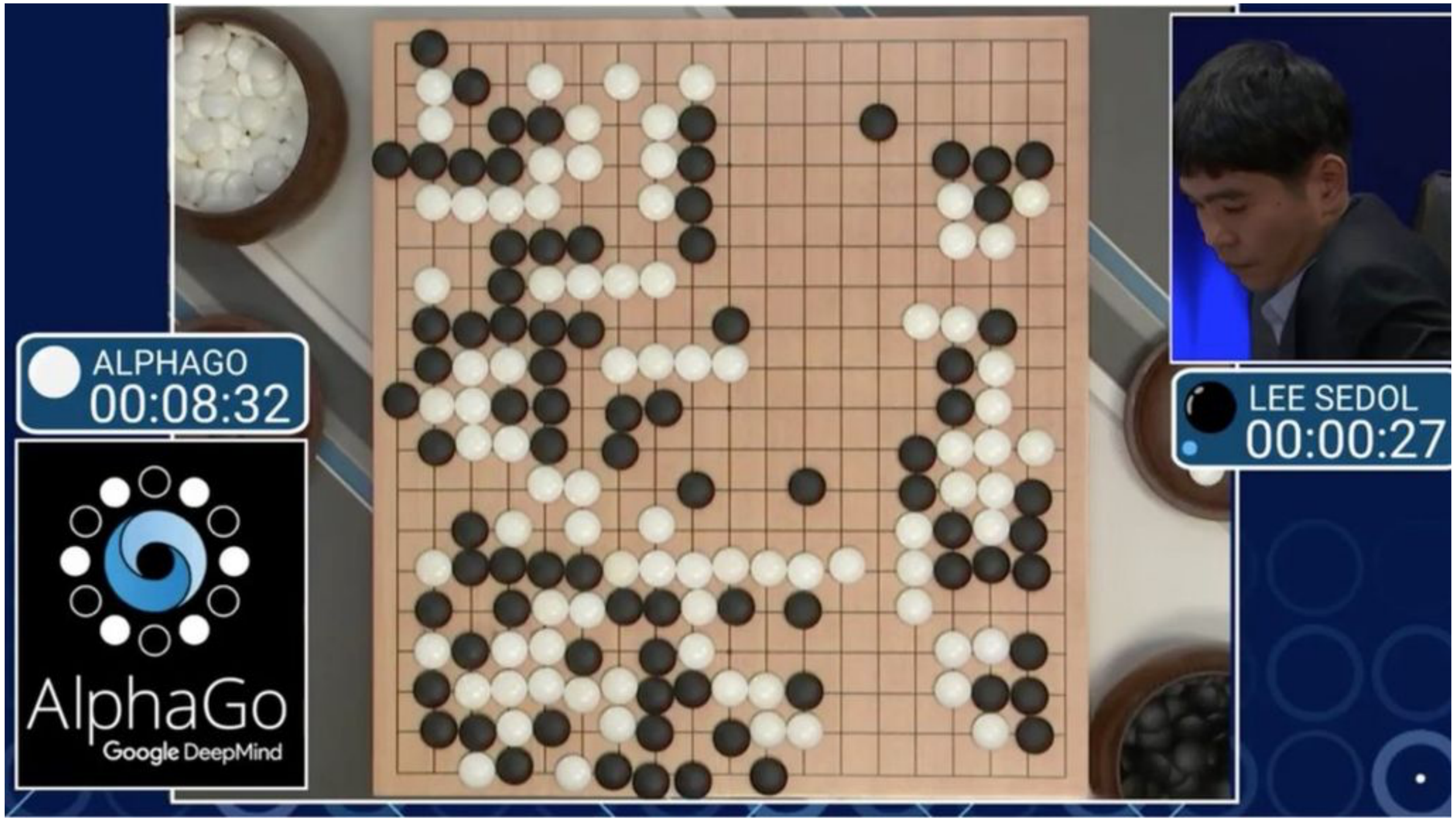


Praiseworthy AI *simplicter*, perhaps.



Praiseworthy AI *simplicter*, perhaps.

But certainly *not* AI = HI!



Praiseworthy AI *simplicter*, perhaps.

But certainly *not* AI = HI!

"AlphaGo, from the perspective of South, how many majuscule Roman letters are in black? Why do you say that?"

Logico-Mathematical Landscape that Has Them Turning in Their Graves

Super-Serious Human Cognitive Power

Serious Human Cognitive Power

Entscheidungsproblem

Mere Calculative Cognitive Power

Logico-Mathematical Landscape that Has Them Turning in Their Graves

Super-Serious Human Cognitive Power

Serious Human Cognitive Power

Descartes

Entscheidungsproblem

Mere Calculative Cognitive Power

Logico-Mathematical Landscape that Has Them Turning in Their Graves

Super-Serious Human Cognitive Power

Serious Human Cognitive Power

Descartes

Leibniz

Entscheidungsproblem

Mere Calculative Cognitive Power

Logico-Mathematical Landscape that Has Them Turning in Their Graves

Super-Serious Human Cognitive Power

Serious Human Cognitive Power

Descartes

Leibniz

Church

Entscheidungsproblem

Mere Calculative Cognitive Power

Logico-Mathematical Landscape that Has Them Turning in Their Graves

Super-Serious Human Cognitive Power

Serious Human Cognitive Power


Descartes


Leibniz


Church


Gödel

Entscheidungsproblem

Mere Calculative Cognitive Power

Logico-Mathematical Landscape that Has Them Turning in Their Graves

Super-Serious Human Cognitive Power

Serious Human Cognitive Power

Descartes

Leibniz

Church

Gödel

Turing

Entscheidungsproblem

Mere Calculative Cognitive Power

Logico-Mathematical Landscape that Has Them Turning in Their Graves

Super-Serious Human Cognitive Power

Serious Human Cognitive Power


Descartes


Leibniz


Church


Gödel


Turing

Entscheidungsproblem

Mere Calculative Cognitive Power

Logico-Mathematical Landscape that Has Them Turning in Their Graves

Super-Serious Human Cognitive Power

Serious Human Cognitive Power

Descartes

Leibniz

Church

Gödel

Turing

Entscheidungsproblem

Mere Calculative Cognitive Power

Logico-Mathematical Landscape that Has Them Turning in Their Graves

Super-Serious Human Cognitive Power

Serious Human Cognitive Power

Descartes

Leibniz

Church

Gödel

Turing

Entscheidungsproblem

Mere Calculative Cognitive Power

Logico-Mathematical Landscape that Has Them Turning in Their Graves

Super-Serious Human Cognitive Power

Serious Human Cognitive Power

Descartes

Leibniz

Church

Gödel

Turing

Entscheidungsproblem

Mere Calculative Cognitive Power

Logico-Mathematical Landscape that Has Them Turning in Their Graves

Super-Serious Human Cognitive Power

Serious Human Cognitive Power

Descartes

Leibniz

Church

Gödel

Entscheidungsproblem

Mere Calculative Cognitive Power

Logico-Mathematical Landscape that Has Them Turning in Their Graves

Analytical Hierarchy

Serious Human Cognitive Power

Descartes

Leibniz

Church

Gödel

Entscheidungsproblem

Mere Calculative Cognitive Power

Logico-Mathematical Landscape that Has Them Turning in Their Graves

Analytical Hierarchy

Arithmetical Hierarchy


Descartes


Leibniz


Church


Gödel

Entscheidungsproblem

Mere Calculative Cognitive Power

Logico-Mathematical Landscape that Has Them Turning in Their Graves

Analytical Hierarchy

Arithmetical Hierarchy


Descartes


Leibniz


Church


Gödel

Entscheidungsproblem

Polynomial Hierarchy

Logico-Mathematical Landscape that Has Them Turning in Their Graves

Analytical Hierarchy

Arithmetical Hierarchy


Descartes


Leibniz


Church


Gödel

Entscheidungsproblem

Polynomial Hierarchy

$$\mathbf{P} \subseteq \mathbf{NP} \subseteq \mathbf{PSPACE} = \mathbf{NPSPACE} \subseteq \mathbf{EXPTIME} \subseteq \mathbf{NEXPTIME} \subseteq \mathbf{EXPSPACE}$$

Logico-Mathematical Landscape that Has Them Turning in Their Graves

Analytical Hierarchy

Arithmetical Hierarchy


Descartes


Leibniz


Church


Gödel

$\vdots$

Π_2

Σ_2

Π_1

Σ_1

Σ_0

Entscheidungsproblem

Polynomial Hierarchy

$$\mathbf{P} \subseteq \mathbf{NP} \subseteq \mathbf{PSPACE} = \mathbf{NPSPACE} \subseteq \mathbf{EXPTIME} \subseteq \mathbf{NEXPTIME} \subseteq \mathbf{EXPSPACE}$$

Logico-Mathematical Landscape that Has Them Turning in Their Graves

Analytical Hierarchy

Arithmetical Hierarchy

Descartes

Leibniz

Church

Gödel

Go:AlphaGo

$\vdots$

Π_2

Σ_2

Π_1

Σ_1

Σ_0

Entscheidungsproblem

Polynomial Hierarchy

$$\mathbf{P} \subseteq \mathbf{NP} \subseteq \mathbf{PSPACE} = \mathbf{NPSPACE} \subseteq \mathbf{EXPTIME} \subseteq \mathbf{NEXPTIME} \subseteq \mathbf{EXPSPACE}$$

Logico-Mathematical Landscape that Has Them Turning in Their Graves

Analytical Hierarchy

Arithmetical Hierarchy

Descartes

Leibniz

Church

Gödel

Jeopardy!:

Go: AlphaGo

$\vdots$

Π_2

Σ_2

Π_1

Σ_1

Σ_0

Entscheidungsproblem

Polynomial Hierarchy

$$\mathbf{P} \subseteq \mathbf{NP} \subseteq \mathbf{PSPACE} = \mathbf{NPSPACE} \subseteq \mathbf{EXPTIME} \subseteq \mathbf{NEXPTIME} \subseteq \mathbf{EXPSPACE}$$

Logico-Mathematical Landscape that Has Them Turning in Their Graves

Analytical Hierarchy

Arithmetical Hierarchy


Descartes


Leibniz


Church


Gödel

Chess: Deep Blue

Jeopardy!:

Go: AlphaGo

$\vdots$

Π_2

Σ_2

Π_1

Σ_1

Σ_0

Entscheidungsproblem

Polynomial Hierarchy

$$\mathbf{P} \subseteq \mathbf{NP} \subseteq \mathbf{PSPACE} = \mathbf{NPSPACE} \subseteq \mathbf{EXPTIME} \subseteq \mathbf{NEXPTIME} \subseteq \mathbf{EXPSPACE}$$

Logico-Mathematical Landscape that Has Them Turning in Their Graves

Analytical Hierarchy

Arithmetical Hierarchy

Descartes

Leibniz

Church

Gödel

Checkers: Chinook

Chess: Deep Blue

Jeopardy!:

Go: AlphaGo

$\vdots$

Π_2

Σ_2

Π_1

Σ_1

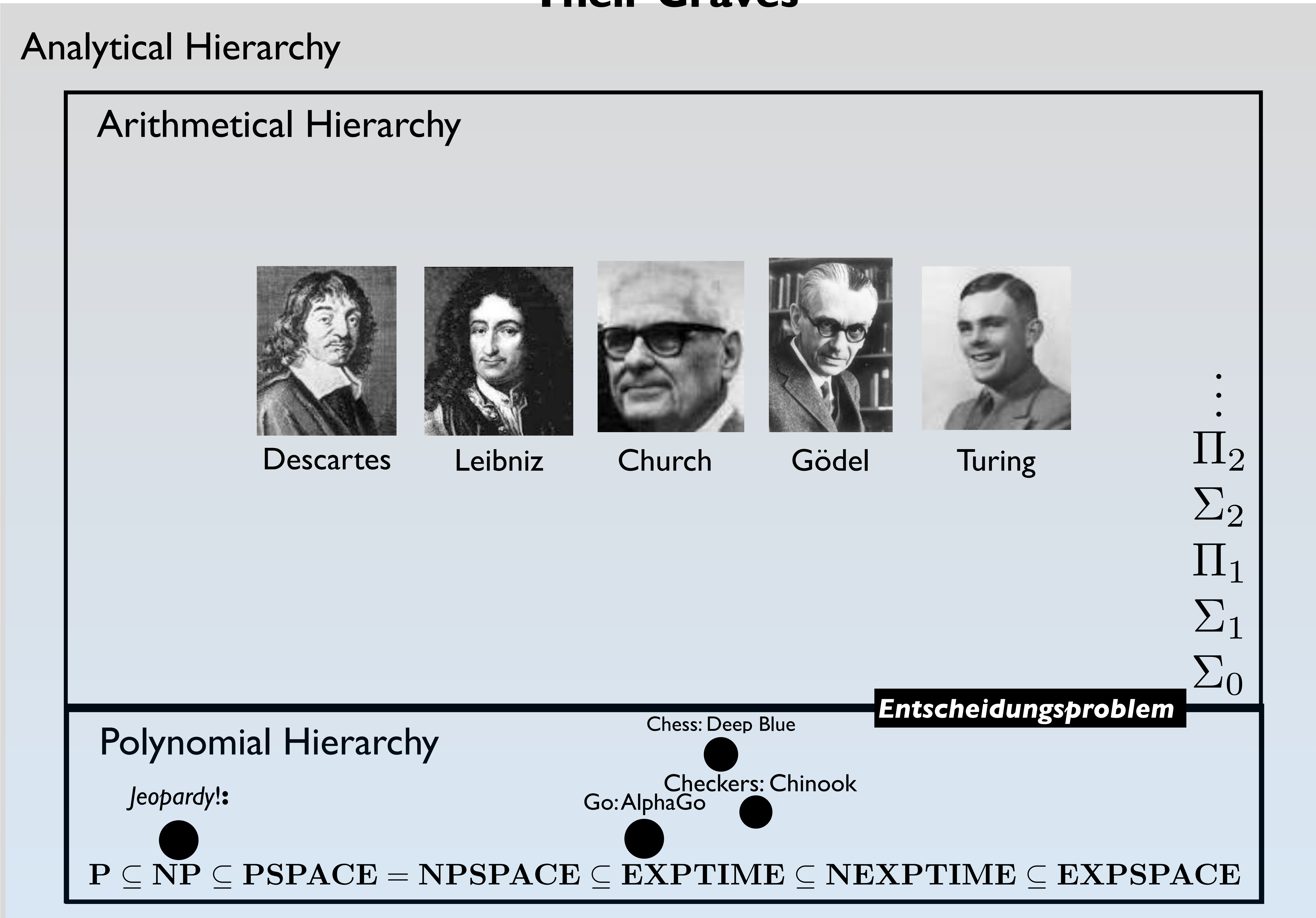
Σ_0

Entscheidungsproblem

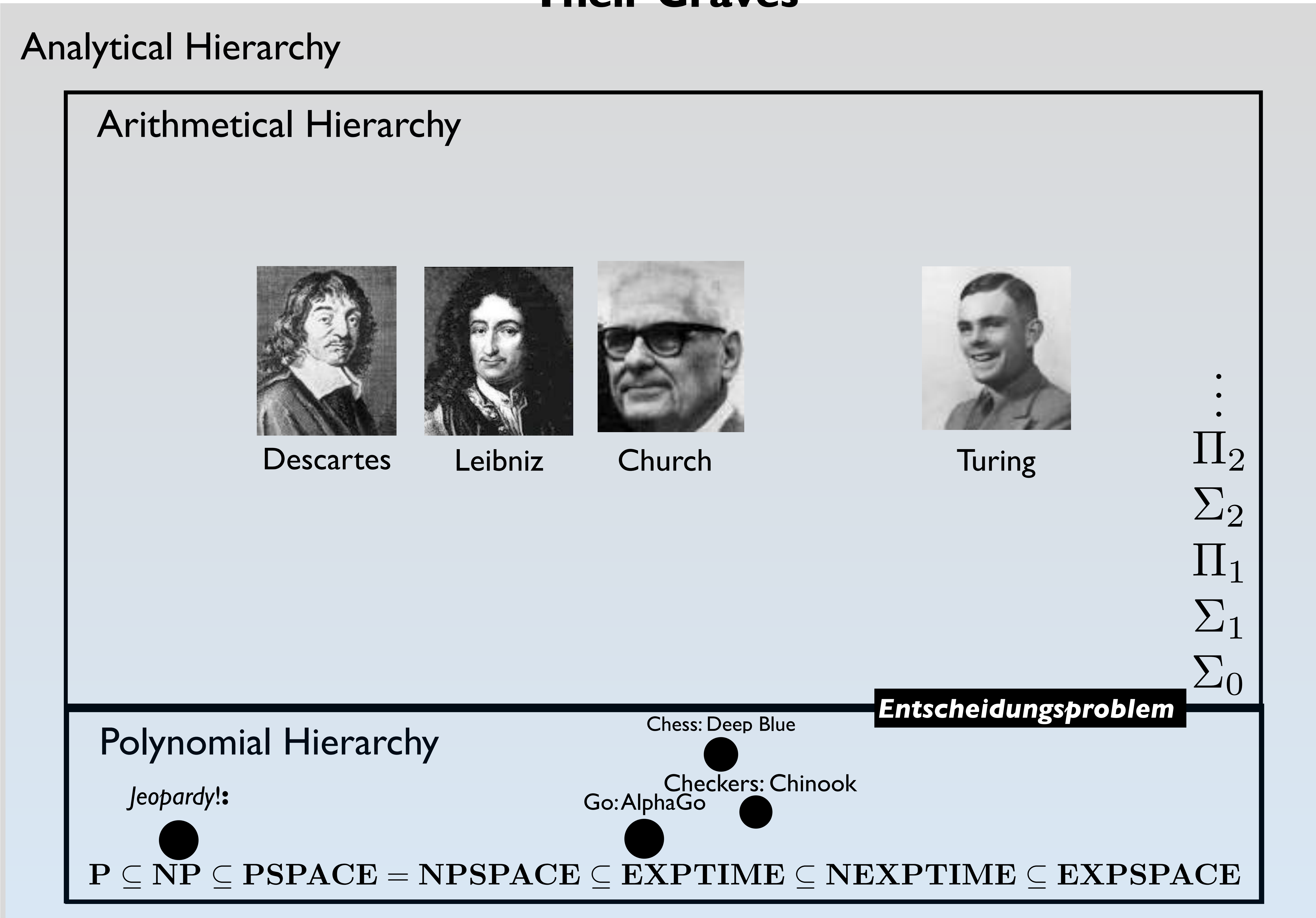
Polynomial Hierarchy

$$\mathbf{P} \subseteq \mathbf{NP} \subseteq \mathbf{PSPACE} = \mathbf{NPSPACE} \subseteq \mathbf{EXPTIME} \subseteq \mathbf{NEXPTIME} \subseteq \mathbf{EXPSPACE}$$

Logico-Mathematical Landscape that Has Them Turning in Their Graves



Logico-Mathematical Landscape that Has Them Turning in Their Graves



Logico-Mathematical Landscape that Has Them Turning in Their Graves

Analytical Hierarchy

Arithmetical Hierarchy

Descartes

Leibniz

Turing

$\vdots$

Π_2

Σ_2

Π_1

Σ_1

Σ_0

Entscheidungsproblem

Polynomial Hierarchy

Chess: Deep Blue

Checkers: Chinook

Jeopardy!:

Go: AlphaGo

$$\mathbf{P} \subseteq \mathbf{NP} \subseteq \mathbf{PSPACE} = \mathbf{NPSPACE} \subseteq \mathbf{EXPTIME} \subseteq \mathbf{NEXPTIME} \subseteq \mathbf{EXPSPACE}$$

Logico-Mathematical Landscape that Has Them Turning in Their Graves

Analytical Hierarchy

Arithmetical Hierarchy

Leibniz

Turing

$\vdots$

Π_2

Σ_2

Π_1

Σ_1

Σ_0

Entscheidungsproblem

Polynomial Hierarchy

Chess: Deep Blue

Checkers: Chinook

Jeopardy!:

Go: AlphaGo

$$\mathbf{P} \subseteq \mathbf{NP} \subseteq \mathbf{PSPACE} = \mathbf{NPSPACE} \subseteq \mathbf{EXPTIME} \subseteq \mathbf{NEXPTIME} \subseteq \mathbf{EXPSPACE}$$

Logico-Mathematical Landscape that Has Them Turning in Their Graves

Analytical Hierarchy

Arithmetical Hierarchy

Leibniz

$\vdots$

Π_2

Σ_2

Π_1

Σ_1

Σ_0

Entscheidungsproblem

Polynomial Hierarchy

Chess: Deep Blue

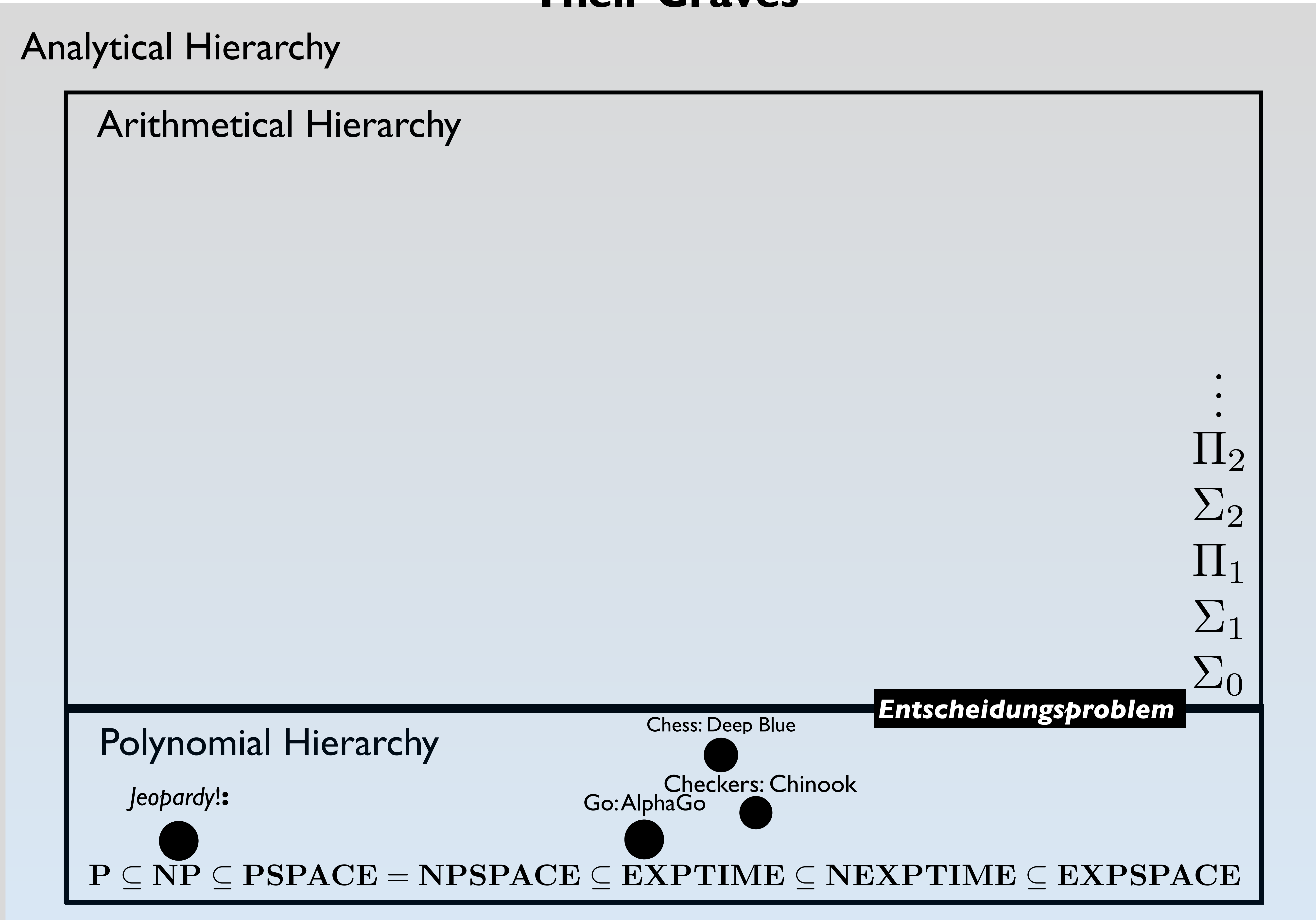
Checkers: Chinook

Jeopardy!:

Go: AlphaGo

$$\mathbf{P} \subseteq \mathbf{NP} \subseteq \mathbf{PSPACE} = \mathbf{NPSPACE} \subseteq \mathbf{EXPTIME} \subseteq \mathbf{NEXPTIME} \subseteq \mathbf{EXPSPACE}$$

Logico-Mathematical Landscape that Has Them Turning in Their Graves



Rengo Kriegspiel

usgo.org

U.S. Go Congress | Crosstabs | Free E-Journal | Upcoming Events | Ratings | Kids & Teens | Latest Go News

AGA Homepage
+What Is Go?
Ratings
+Membership and Chapters
AGA Chapter Email List
Professionals
+Play Go
+Tournaments
+Learn More
+Teach Others
+Outreach
+Kids & Teens
American Go Foundation
Latest Go News
+About the AGA
Donate to the AGA
AGA Go Database
US Go Congress
Go Foundation ... rchive
+Administrators Only

Site Search

American Go E-Journal

US Go Congress Goes a Little Crazy

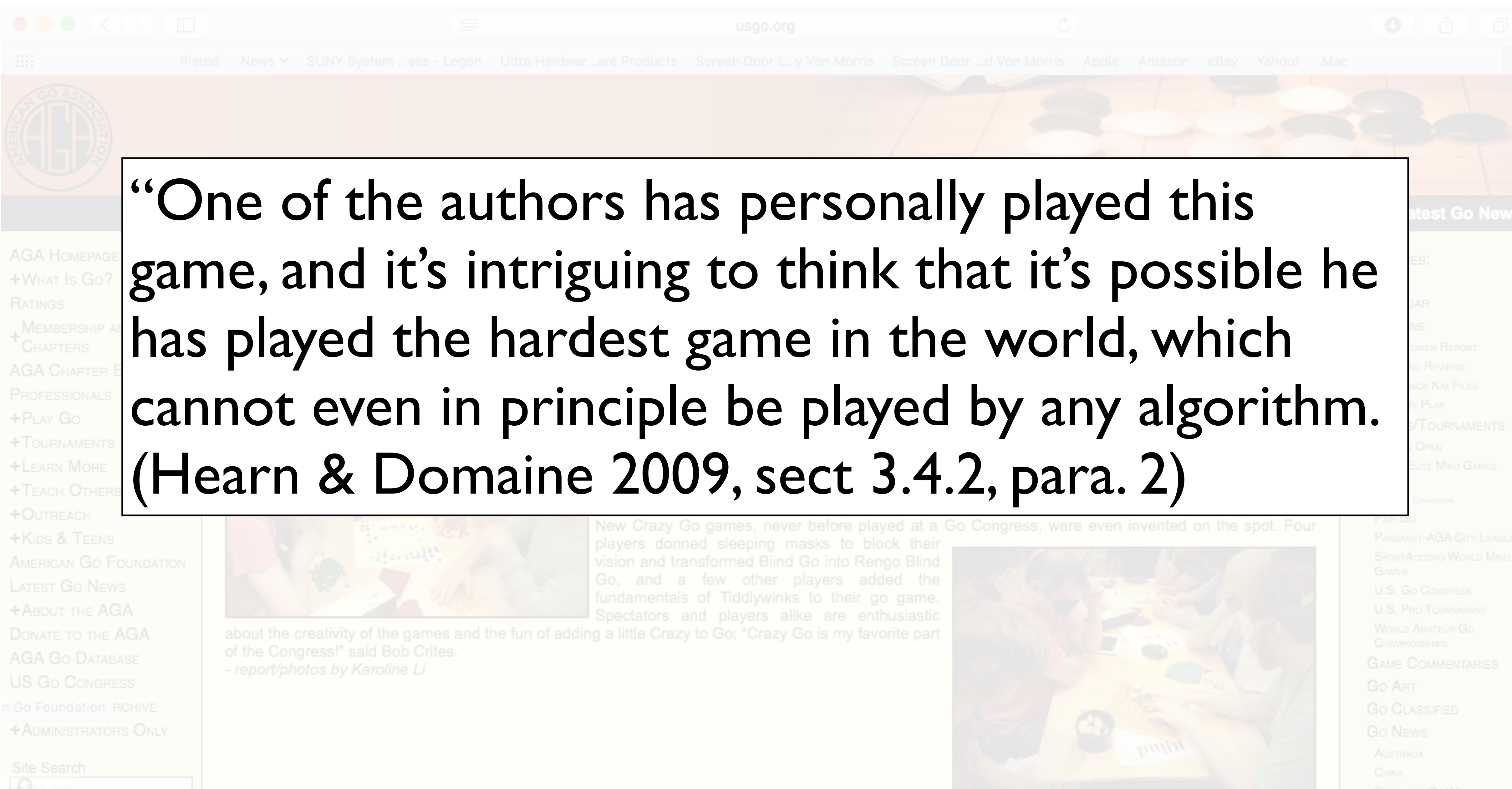
Wednesday August 13, 2014

"White plays capturing black, putting herself and black into atari," calls Crazy Go TD Terry Benson. He officiated several games of Rengo Kriegspiel on Tuesday evening – a pair go game in which all four players face away from the main board and play their stones on their own empty board in front of them; the only clues about where their opponents — and even their partner — have played comes when they make an illegal move, or play where their own team or their opponents already have stones. Rengo Kriegspiel is only one of dozens of variants on the game of go that were played by an enthusiastic crowd of around 100 players. Familiar games include Magnetic Go, 4 Color Go, Tessellation Go, 3D Go, Spiral Go, and Blind Go. "After all these years, it's still crazy," said TD and Crazy Go founder Terry Benson. New Crazy Go games, never before played at a Go Congress, were even invented on the spot. Four players donned sleeping masks to block their vision and transformed Blind Go into Rengo Blind Go, and a few other players added the fundamentals of Tiddlywinks to their go game. Spectators and players alike are enthusiastic about the creativity of the games and the fun of adding a little Crazy to Go; "Crazy Go is my favorite part of the Congress!" said Bob Crites.

- report/photos by Karoline Li

Categories:
All
Calendar
Columns
John Power Report
Redmond Reviews
The Janice Kim Files
Why We Play
Events/Tournaments
Cotsen Open
IMSA Elite Mind Games
Other
N.A. Go Convention
Pair Go
Pandanet-AGA City League
SportAccord World Mind Games
U.S. Go Congress
U.S. Pro Tournament
World Amateur Go Championships
Game Commentaries
Go Art
Go Classified
Go News
Australia
China
Computer Go/AI

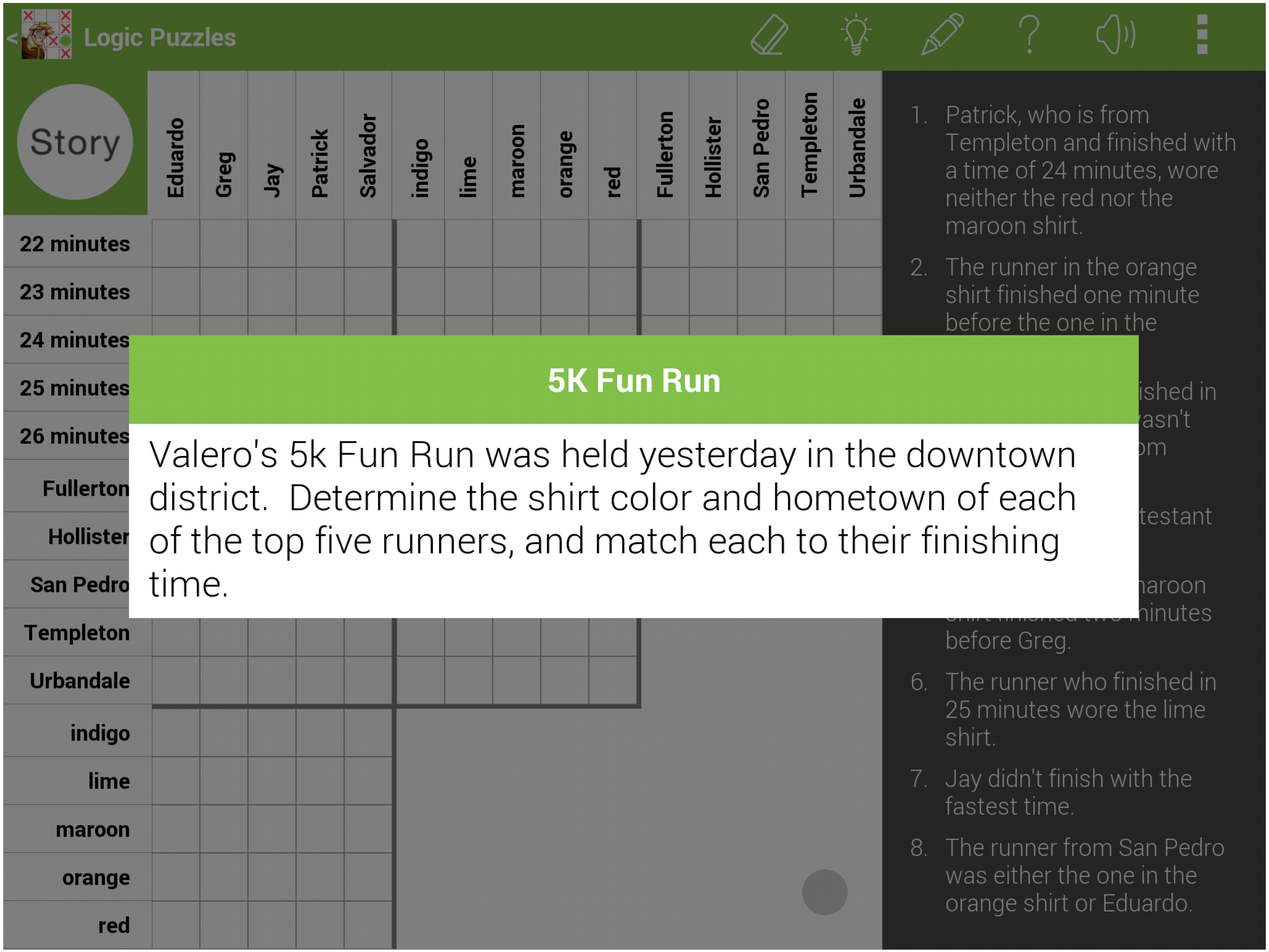
Rengo Kriegspiel



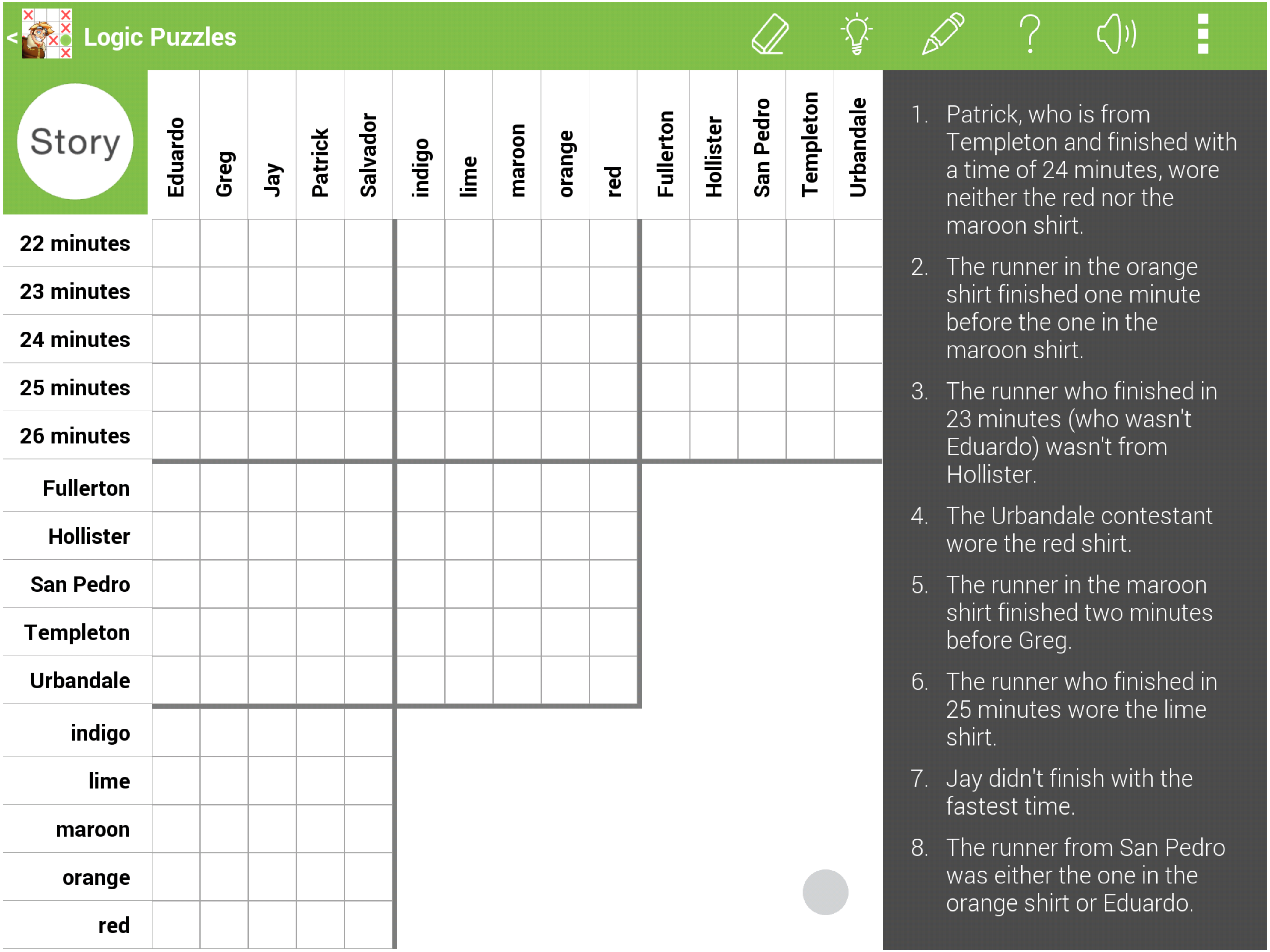
"One of the authors has personally played this game, and it's intriguing to think that it's possible he has played the hardest game in the world, which cannot even in principle be played by any algorithm. (Hearn & Domaine 2009, sect 3.4.2, para. 2)

But starting simpler …

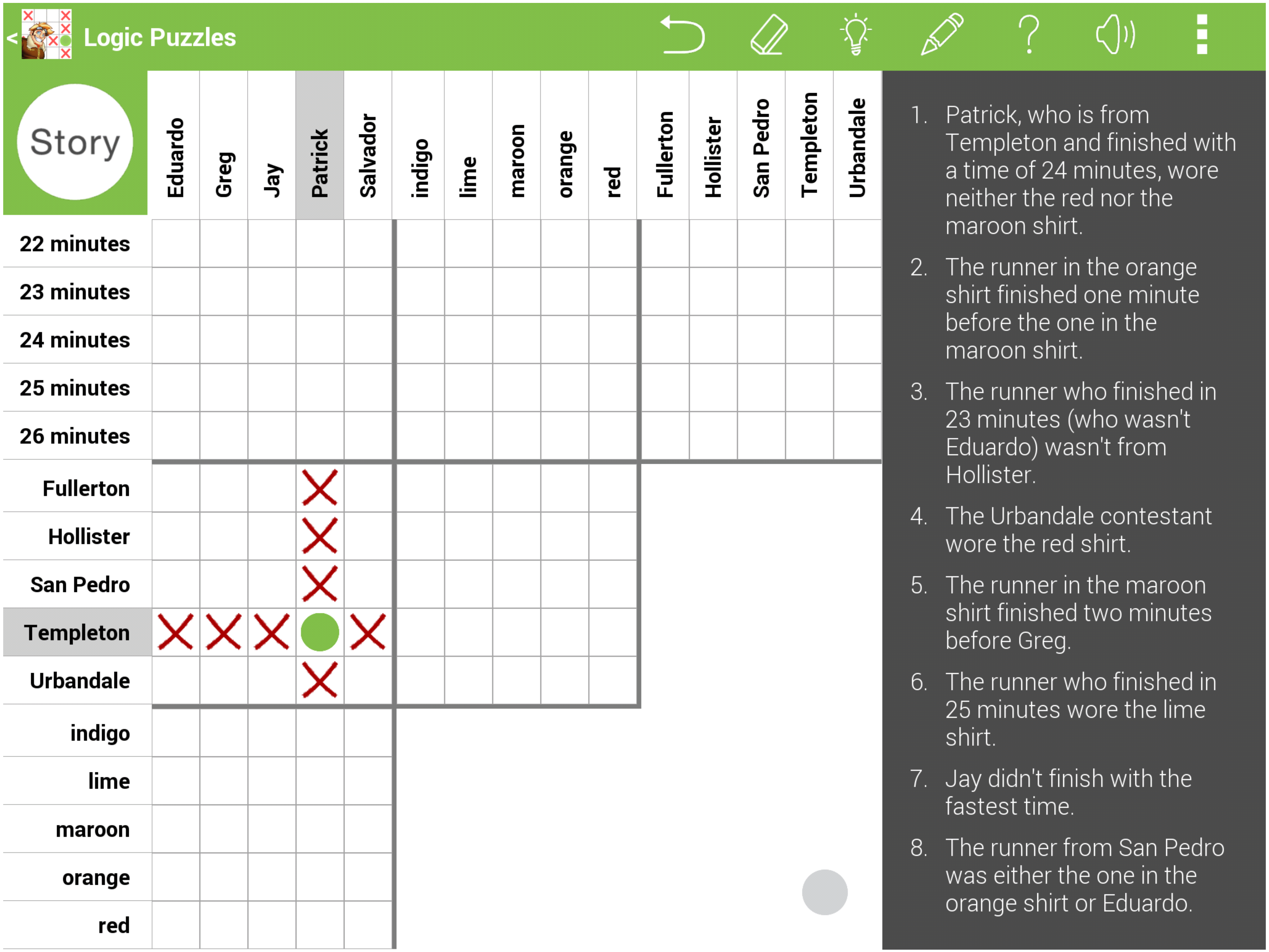
Tabular "Deduction" (Example)



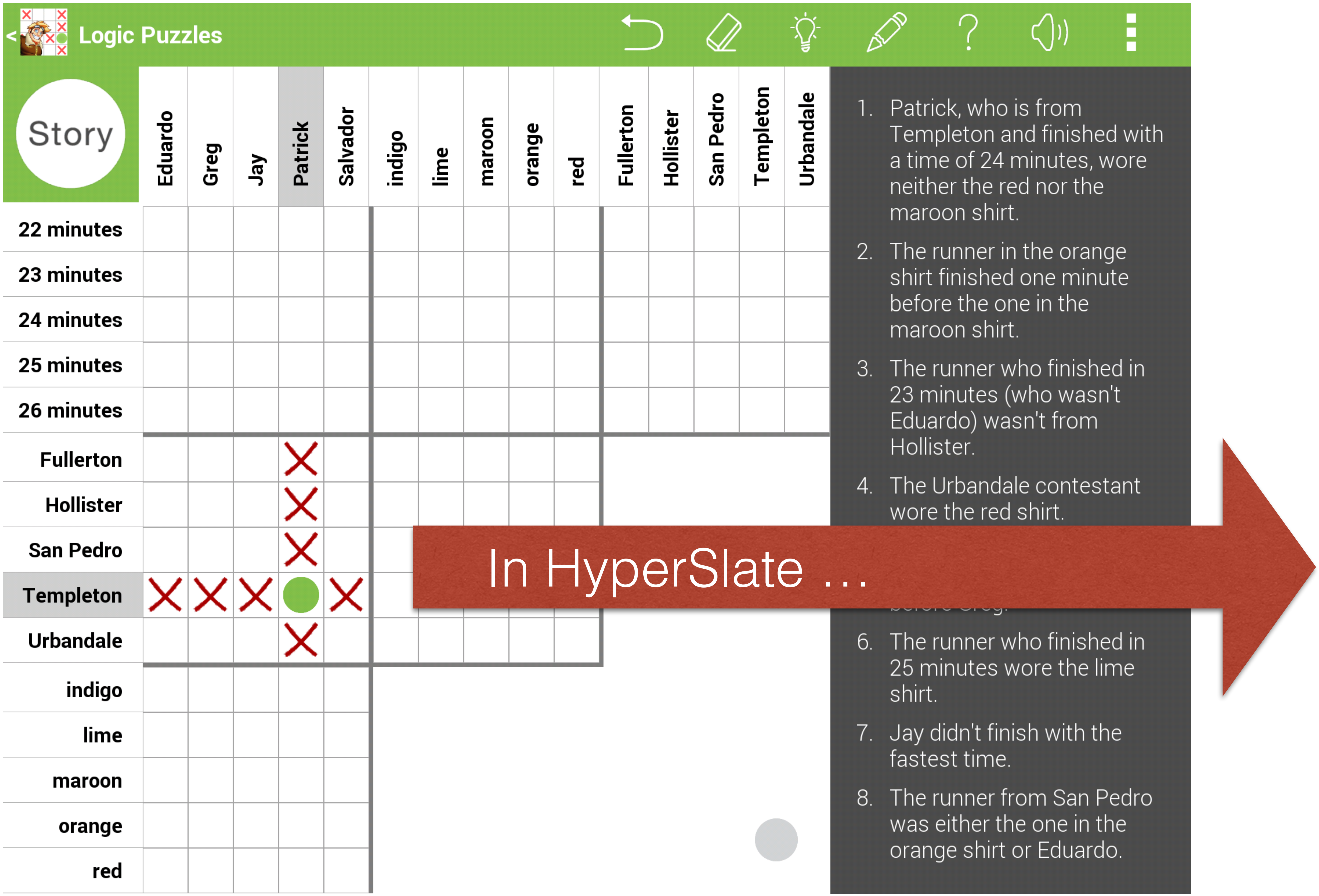
Tabular "Deduction" (Example)



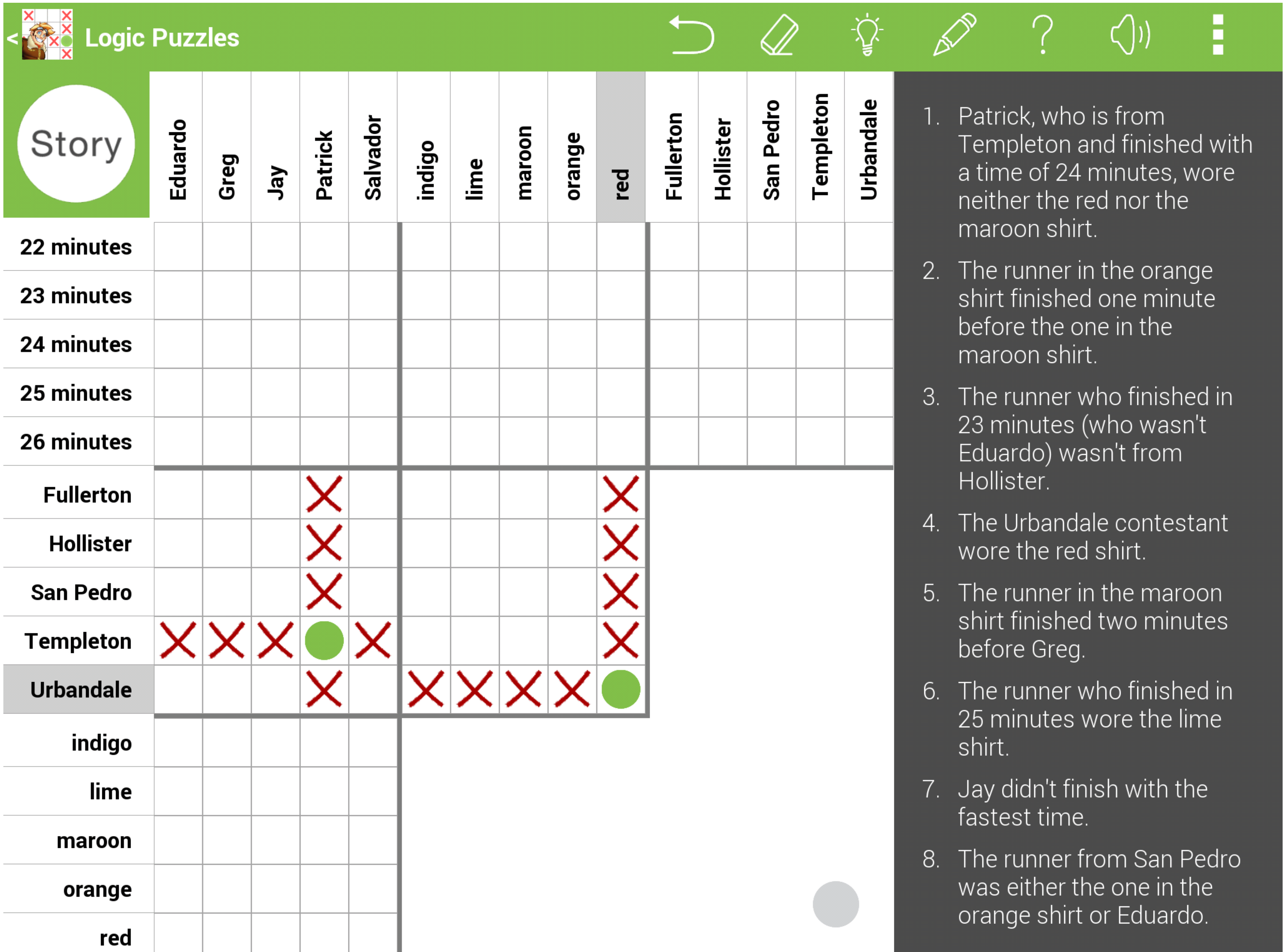
Tabular "Deduction" (Example)



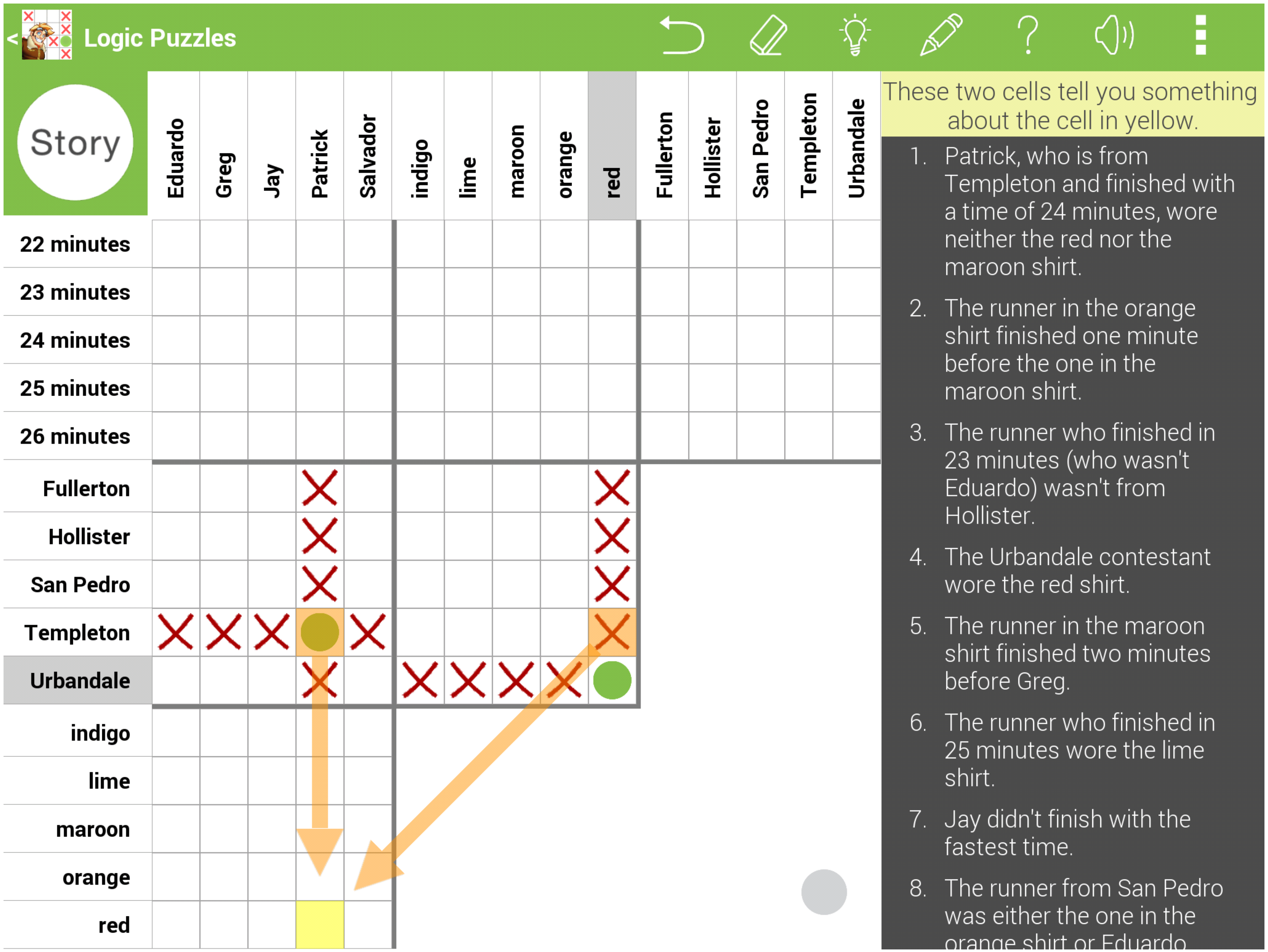
Tabular "Deduction" (Example)



Tabular "Deduction" (Example)



Tabular “Deduction” (Example)



Tabular "Deduction": It's Taught!

Example

Grace, Dylan, Kira, and Diego are each wearing different colored shirts. Grace's shirt is red. Dylan's shirt is not white. Kira's shirt is not green. Diego's shirt is not yellow or white. What color shirt is each person wearing?

First, make a chart to show what you know.

- Each shirt is a different color.
- Grace's shirt is red.
- Dylan's shirt is not white.
- Kira's shirt is not green.
- Diego's shirt is not yellow or white.

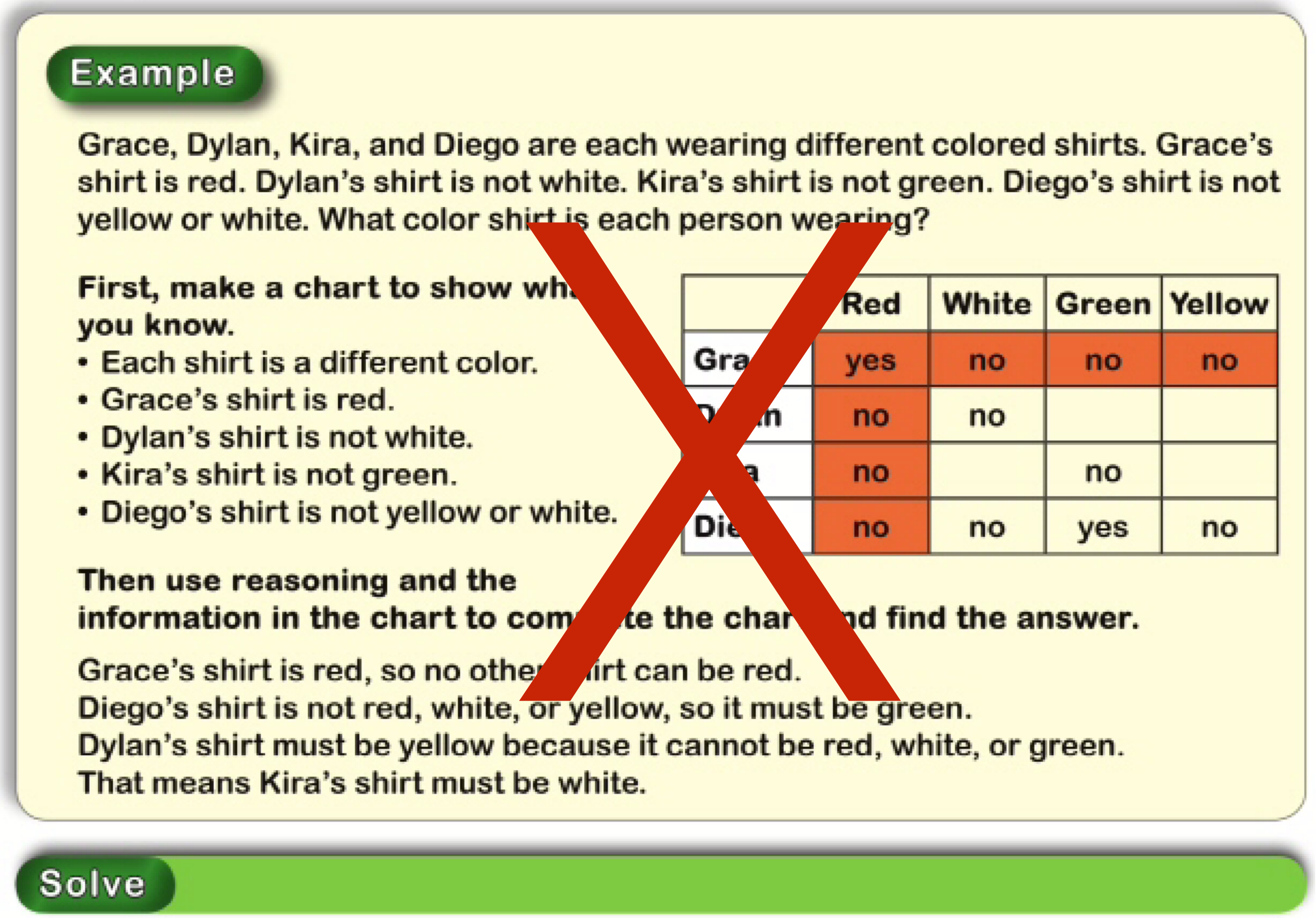
	Red	White	Green	Yellow
Grace	yes	no	no	no
Dylan	no	no		
Kira	no		no	
Diego	no	no	yes	no

Then use reasoning and the information in the chart to complete the chart and find the answer.

Grace's shirt is red, so no other shirt can be red.
Diego's shirt is not red, white, or yellow, so it must be green.
Dylan's shirt must be yellow because it cannot be red, white, or green.
That means Kira's shirt must be white.

Solve

Tabular "Deduction": It's Taught!



Example

Grace, Dylan, Kira, and Diego are each wearing different colored shirts. Grace's shirt is red. Dylan's shirt is not white. Kira's shirt is not green. Diego's shirt is not yellow or white. What color shirt is each person wearing?

First, make a chart to show wh… you know.

- Each shirt is a different color.
- Grace's shirt is red.
- Dylan's shirt is not white.
- Kira's shirt is not green.
- Diego's shirt is not yellow or white.

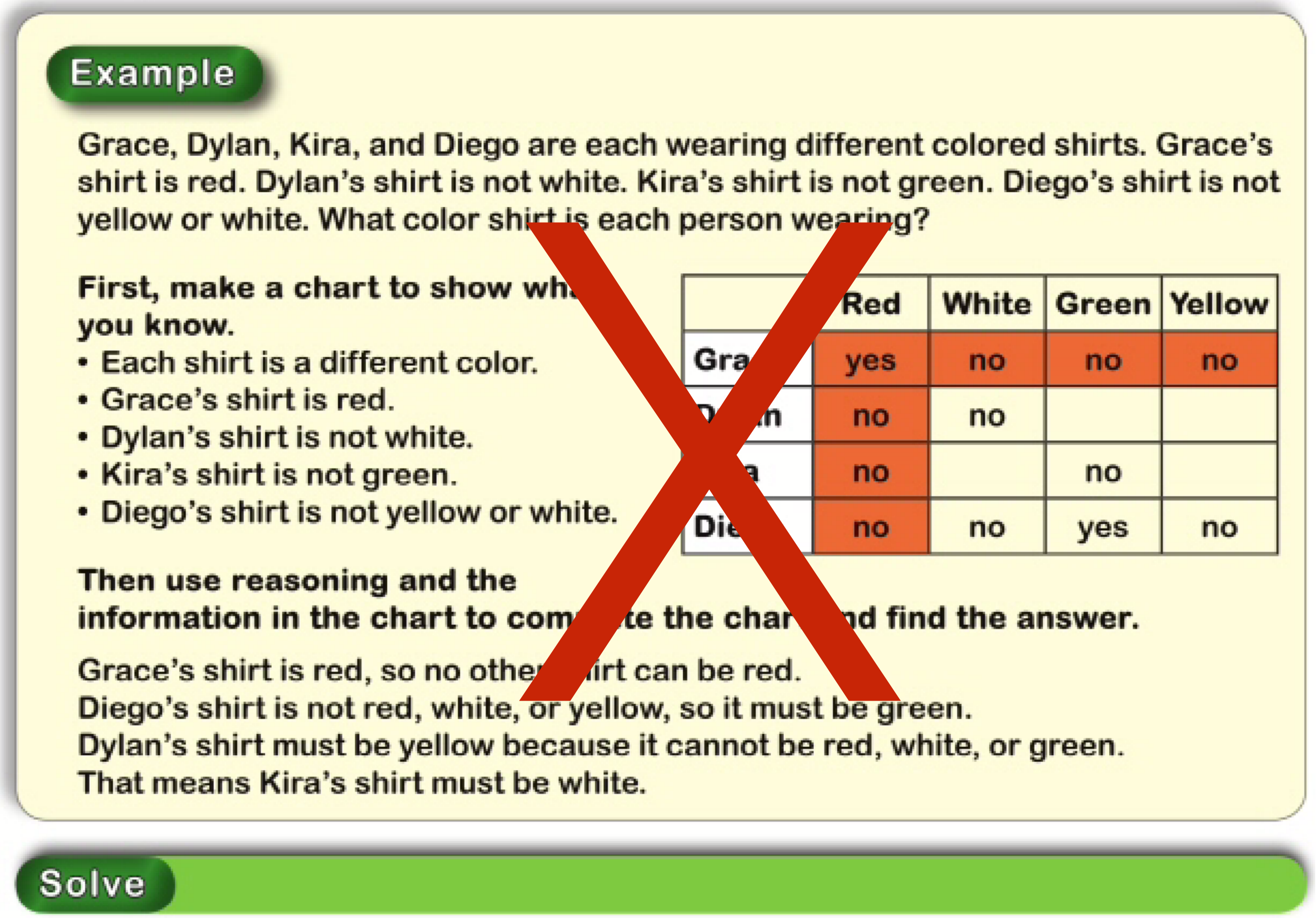
	Red	White	Green	Yellow
Gra…	yes	no	no	no
D…n	no	no		
…a	no		no	
Di…	no	no	yes	no

Then use reasoning and the information in the chart to com…te the char… nd find the answer.

Grace's shirt is red, so no othe… irt can be red.
Diego's shirt is not red, white, or yellow, so it must be green.
Dylan's shirt must be yellow because it cannot be red, white, or green.
That means Kira's shirt must be white.

Solve

Tabular "Deduction": It's Taught!



Example

Grace, Dylan, Kira, and Diego are each wearing different colored shirts. Grace's shirt is red. Dylan's shirt is not white. Kira's shirt is not green. Diego's shirt is not yellow or white. What color shirt is each person wearing?

First, make a chart to show wh[illegible] you know.

- Each shirt is a different color.
- Grace's shirt is red.
- Dylan's shirt is not white.
- Kira's shirt is not green.
- Diego's shirt is not yellow or white.

	Red	White	Green	Yellow
Gra[illegible]	yes	no	no	no
[illegible]	no	no		
[illegible]	no		no	
Di[illegible]	no	no	yes	no

Then use reasoning and the information in the chart to com[illegible]te the char[illegible]nd find the answer.

Grace's shirt is red, so no othe[illegible]irt can be red.
Diego's shirt is not red, white, or yellow, so it must be green.
Dylan's shirt must be yellow because it cannot be red, white, or green.
That means Kira's shirt must be white.

Solve

IMHO very bad idea—if *before* real learning of deduction to answer "Why, exactly? Prove it!"

Tabular "deduction" not the skill that's needed.

8:29 AM iPad 8:29 AM

Recall from Lesson 4-8 that the complex numbers $a + bi$ and $a - bi$ are conjugates. Similarly, the irrational numbers $a + \sqrt{b}$ and $a - \sqrt{b}$ are conjugates. If a complex number or an irrational number is a root of a polynomial equation with rational coefficients, so is its conjugate.

TAKE NOTE Theorem

Conjugate Root Theorem

If $P(x)$ is a polynomial with *rational* coefficients, then irrational roots of $P(x) = 0$ that have the form $a + \sqrt{b}$ occur in conjugate pairs. That is, if $a + \sqrt{b}$ is an irrational root with a and b rational, then $a - \sqrt{b}$ is also a root.

If $P(x)$ is a polynomial with *real* coefficients, then the complex roots of $P(x) = 0$ occur in conjugate pairs. That is, if $a + bi$ is a complex root with a and b real, then $a - bi$ is also a root.

Every quadratic polynomial equation has two roots, every cubic polynomial equation has three roots, and so on.

This result is related to the *Fundamental Theorem of Algebra*. The German mathematician Carl Friedrich Gauss (1777–1855) is credited with proving this theorem.

TAKE NOTE Theorem

The Fundamental Theorem of Algebra

If $P(x)$ is a polynomial of degree $n \geq 1$, then $P(x) = 0$ has exactly n roots, including multiple and complex roots.

iPad

From Algebra 2

Theorems About Roots of Polynomial Equations

Practice and Problem-Solving Exercises - Contir

Determine whether each of the following statements is *always*, *sometimes*, or *never* true.

41. A polynomial function with real coefficients has real zeros.

42. Polynomial functions with complex coefficients have one complex zero.

43. A polynomial function that does not intercept the x-axis has complex roots only.

44. **Reasoning** A 4th-degree polynomial function has zeros at 3 and $5 - i$. Can $4 + i$ also be a zero of the function? Explain your reasoning.

45. **Open-Ended** Write a polynomial function that has four possible rational zeros but no actual rational zeros.

46. **Reasoning** Show that the Fundamental Theorem of Algebra must be true for all quadratic polynomial functions.

C • Challenge

47. Use the Fundamental Theorem of Algebra and the Conjugate Root Theorem to show that any odd degree polynomial equation with real coefficients has at least one real root.

48. **Reasoning** What is the maximum number of points of intersection between the graphs of a quartic and a quintic polynomial function?

49. **Reasoning** What is the least possible degree of a polynomial with rational coefficients, leading coefficient 1, constant term 5, and zeros at $\sqrt{2}$ and $\sqrt{3}$? Show that such a polynomial has a rational zero and indicate this zero.

Tabular "deduction" not the skill that's needed.

8:29 AM iPad 8:29 AM 91

Recall from Lesson 4-8 that the complex numbers $a + bi$ and $a - bi$ are conjugates. Similarly, the irrational numbers $a + \sqrt{b}$ and $a - \sqrt{b}$ are conjugates. If a complex number or an irrational number is a root of a polynomial equation with rational coefficients, so is its conjugate.

TAKE NOTE Theorem

Conjugate Root Theorem

If $P(x)$ is a polynomial with *rational* coefficients, then irrational roots of $P(x) = 0$ that have the form $a + \sqrt{b}$ occur in conjugate pairs. That is, if $a + \sqrt{b}$ is an irrational root with a and b rational, then $a - \sqrt{b}$ is also a root.

If $P(x)$ is a polynomial with *real* coefficients, then the complex roots of $P(x) = 0$ occur in conjugate pairs. That is, if $a + bi$ is a complex root with a and b real, then $a - bi$ is also a root.

Every quadratic polynomial equation has two roots, every cubic polynomial equation has three roots, and so on.

This result is related to the *Fundamental Theorem of Algebra*. The German mathematician Carl Friedrich Gauss (1777–1855) is credited with proving this theorem.

TAKE NOTE Theorem

The Fundamental Theorem of Algebra

If $P(x)$ is a polynomial of degree $n \geq 1$, then $P(x) = 0$ has exactly n roots, including multiple and complex roots.

iPad

From Algebra 2

Practice and Problem-Solving Exercises - Contir

Theorems About Roots of Polynomial Equations

Determine whether each of the following statements is *always*, *sometimes*, or *never* true.

41. A polynomial function with real coefficients has real zeros.

42. Polynomial functions with complex coefficients have one complex zero.

43. A polynomial function that does not intercept the x-axis has complex roots only.

44. **Reasoning** A 4th-degree polynomial function has zeros at 3 and $5 - i$. Can $4 + i$ also be a zero of the function? Explain your reasoning.

45. **Open-Ended** Write a polynomial function that has four possible rational zeros but no actual rational zeros.

46. **Reasoning** Show that the Fundamental Theorem of Algebra must be true for all quadratic polynomial functions.

C • Challenge

47. Use the Fundamental Theorem of Algebra and the Conjugate Root Theorem to show that any odd degree polynomial equation with real coefficients has at least one real root.

48. **Reasoning** What is the maximum number of points of intersection between the graphs of a quartic and a quintic polynomial function?

49. **Reasoning** What is the least possible degree of a polynomial with rational coefficients, leading coefficient 1, constant term 5, and zeros at $\sqrt{2}$ and $\sqrt{3}$? Show that such a polynomial has a rational zero and indicate this zero.

Back to FOL …

Our Final New Inference Rule in FOL

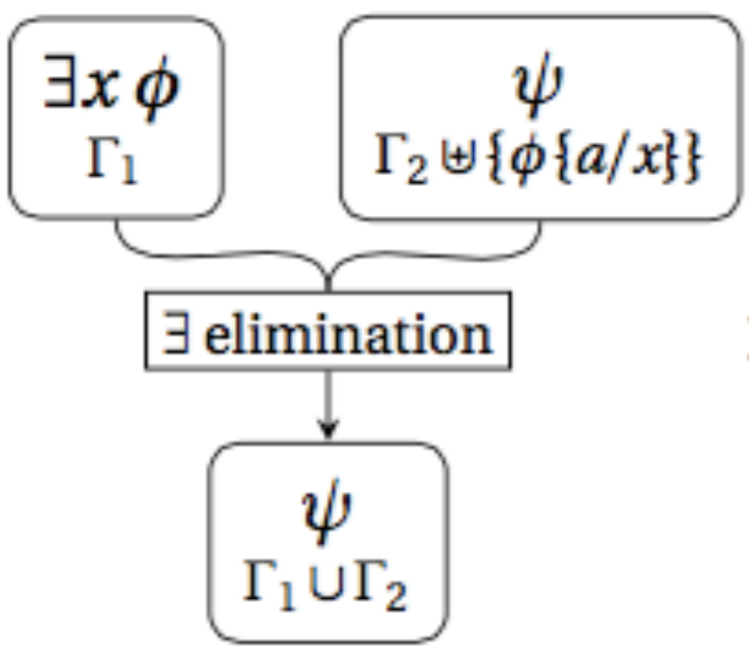
Our Final New Inference Rule in FOL

- `existential elimination` (intuitively put):

Our Final New Inference Rule in FOL

- `existential elimination` (intuitively put):
  - If we know that (i) there's something *x* which is an *R*, and (ii) on the supposition that *a* is an arbitrary representative (a "witness") of such an *x* we can prove *P*, then we are permitted to deduce *P* from (i) alone.

existential elimination, precise version:



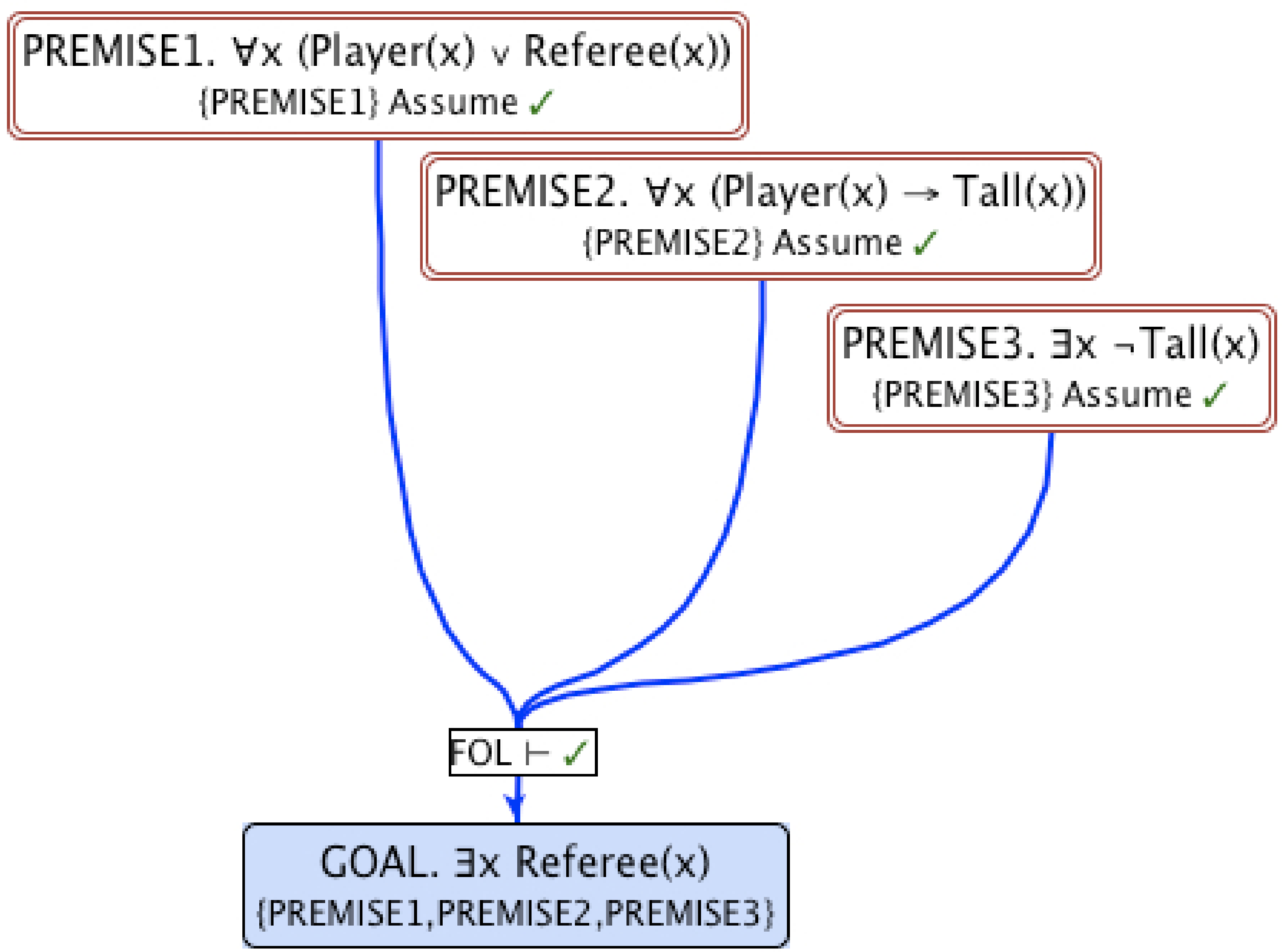
provided that a does not appear free in Γ_1, Γ_2, or ψ

Starting State of Workspace

(Assumes a domain of e.g. players on a March-madness basketball court.)

Starting State of Workspace

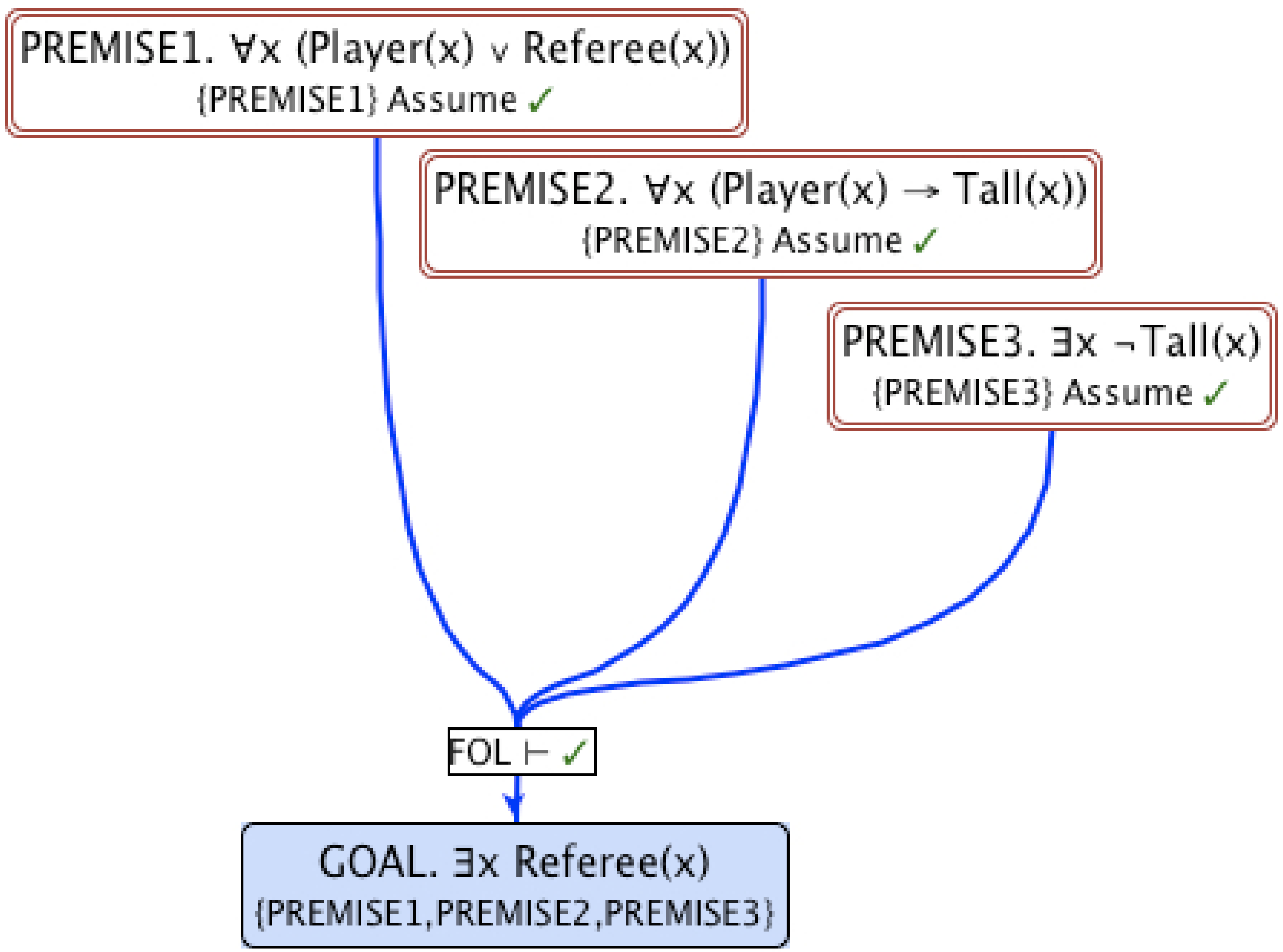
(Assumes a domain of e.g. players on a March-madness basketball court.)



Starting State of Workspace

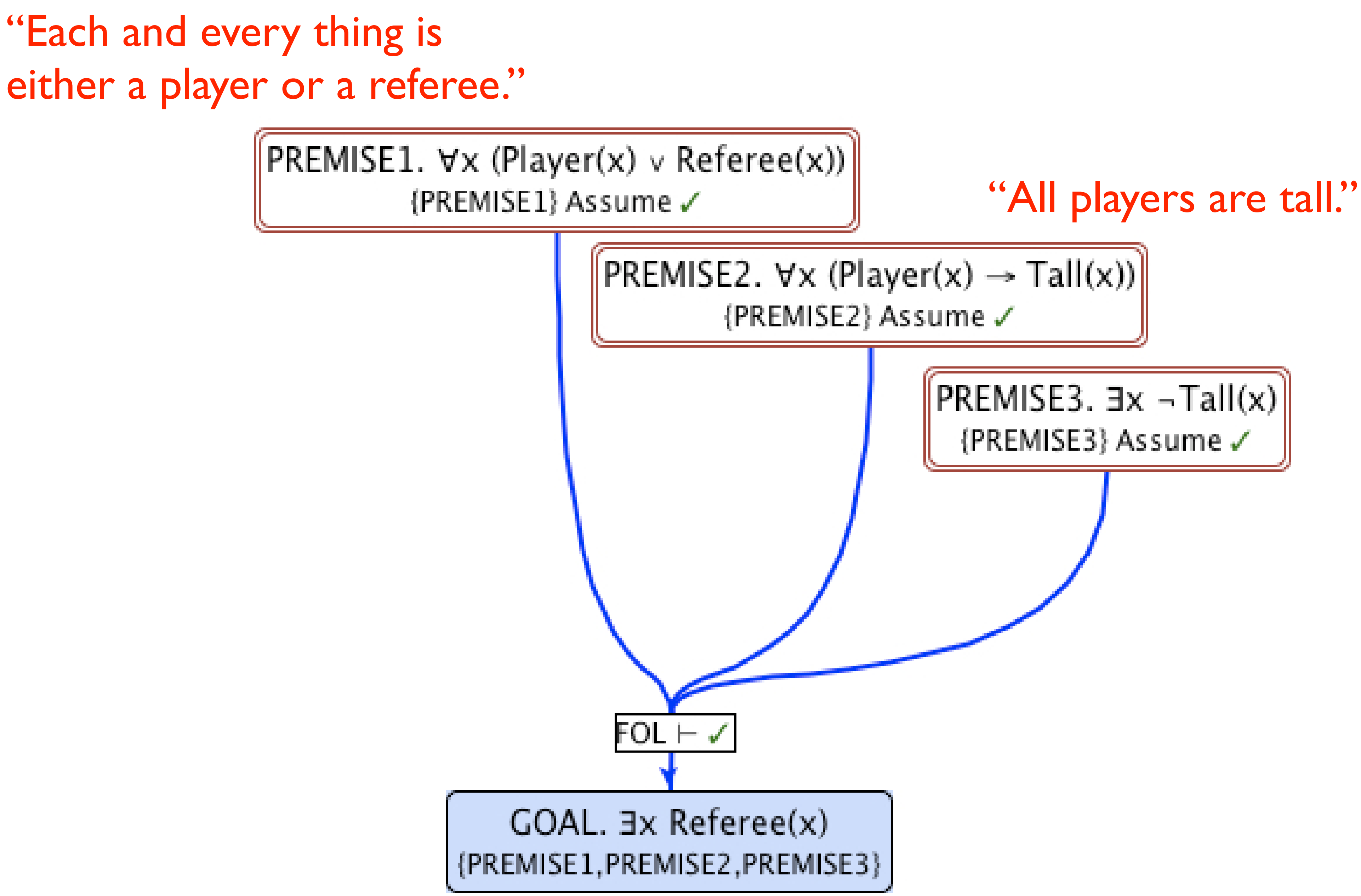
(Assumes a domain of e.g. players on a March-madness basketball court.)

"Each and every thing is either a player or a referee."



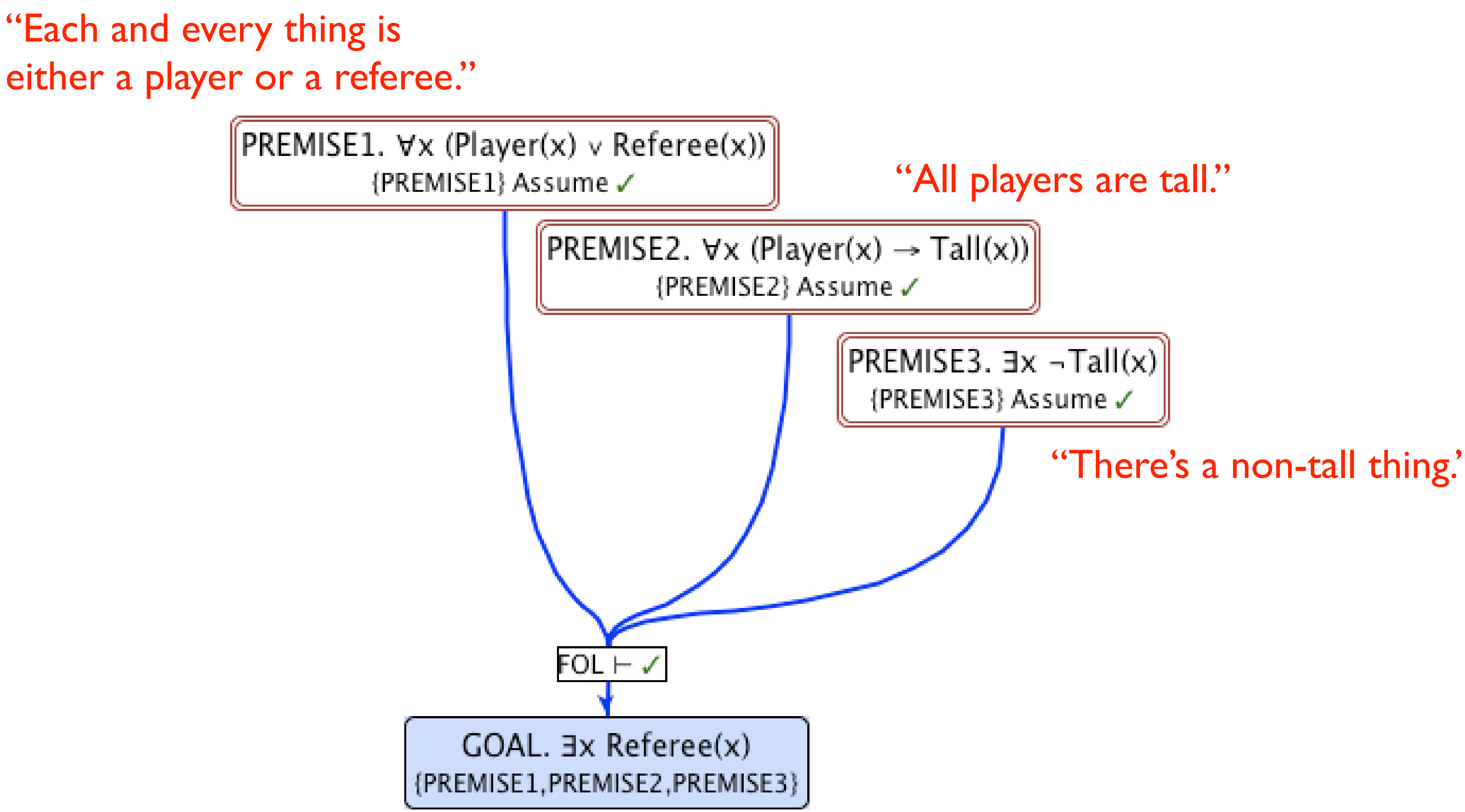
Starting State of Workspace

(Assumes a domain of e.g. players on a March-madness basketball court.)



Starting State of Workspace

(Assumes a domain of e.g. players on a March-madness basketball court.)



Step 1

PREMISE1. $\forall x\ (Player(x) \vee Referee(x))$
{PREMISE1} Assume ✓

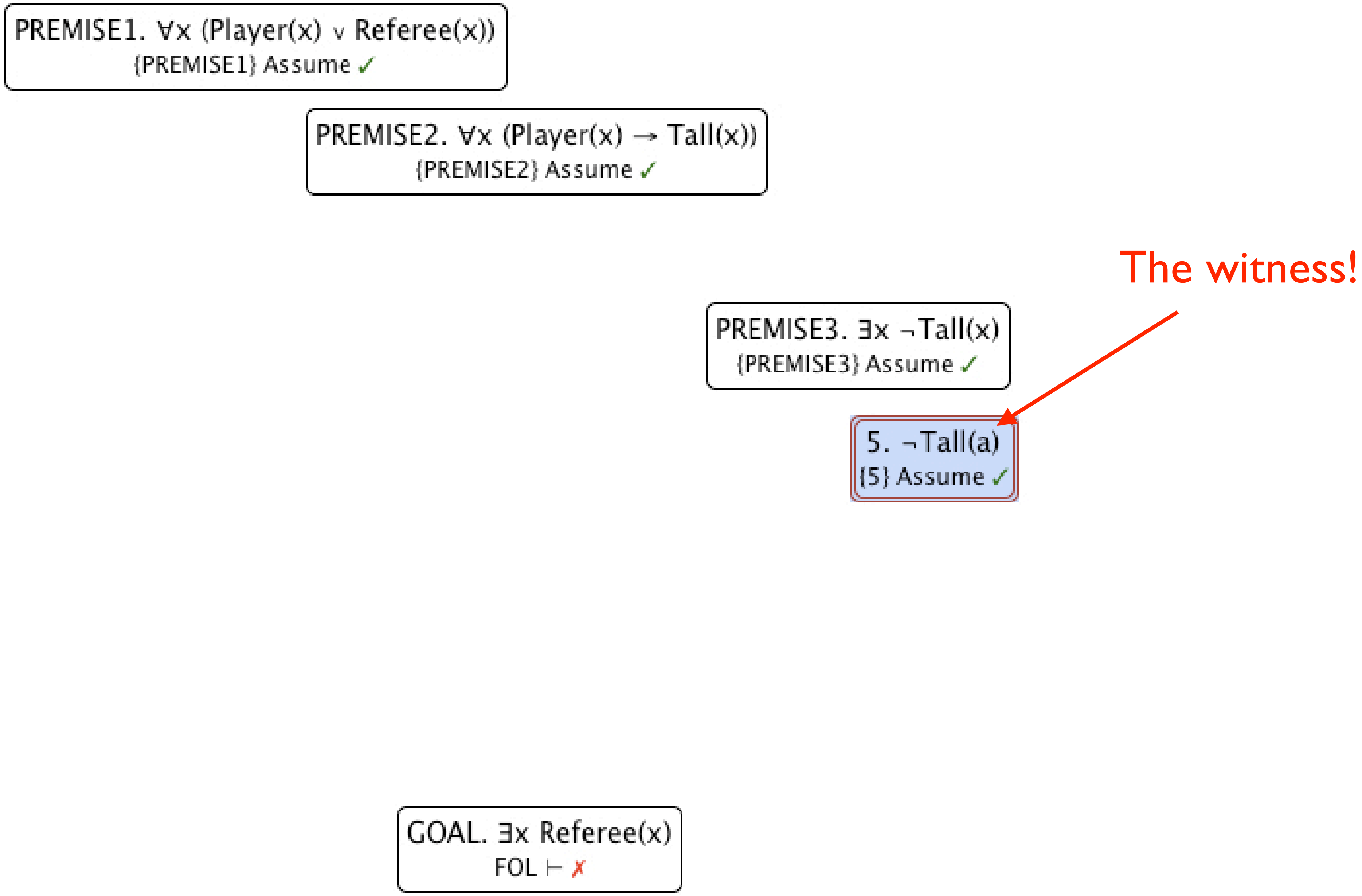
PREMISE2. $\forall x\ (Player(x) \rightarrow Tall(x))$
{PREMISE2} Assume ✓

PREMISE3. $\exists x\ \neg Tall(x)$
{PREMISE3} Assume ✓

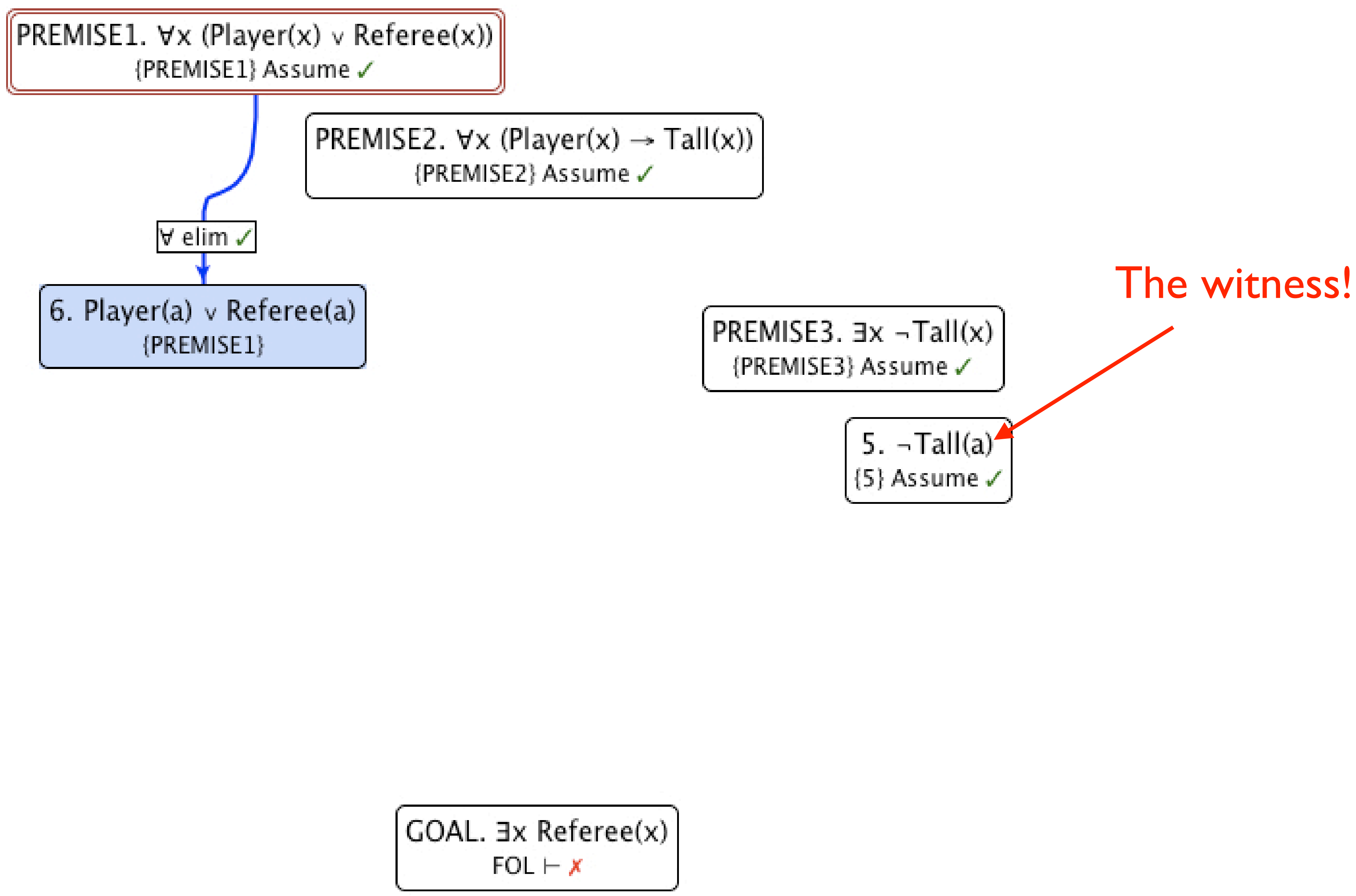
5. $\neg Tall(a)$
{5} Assume ✓

GOAL. $\exists x\ Referee(x)$
FOL $\vdash$ ✗

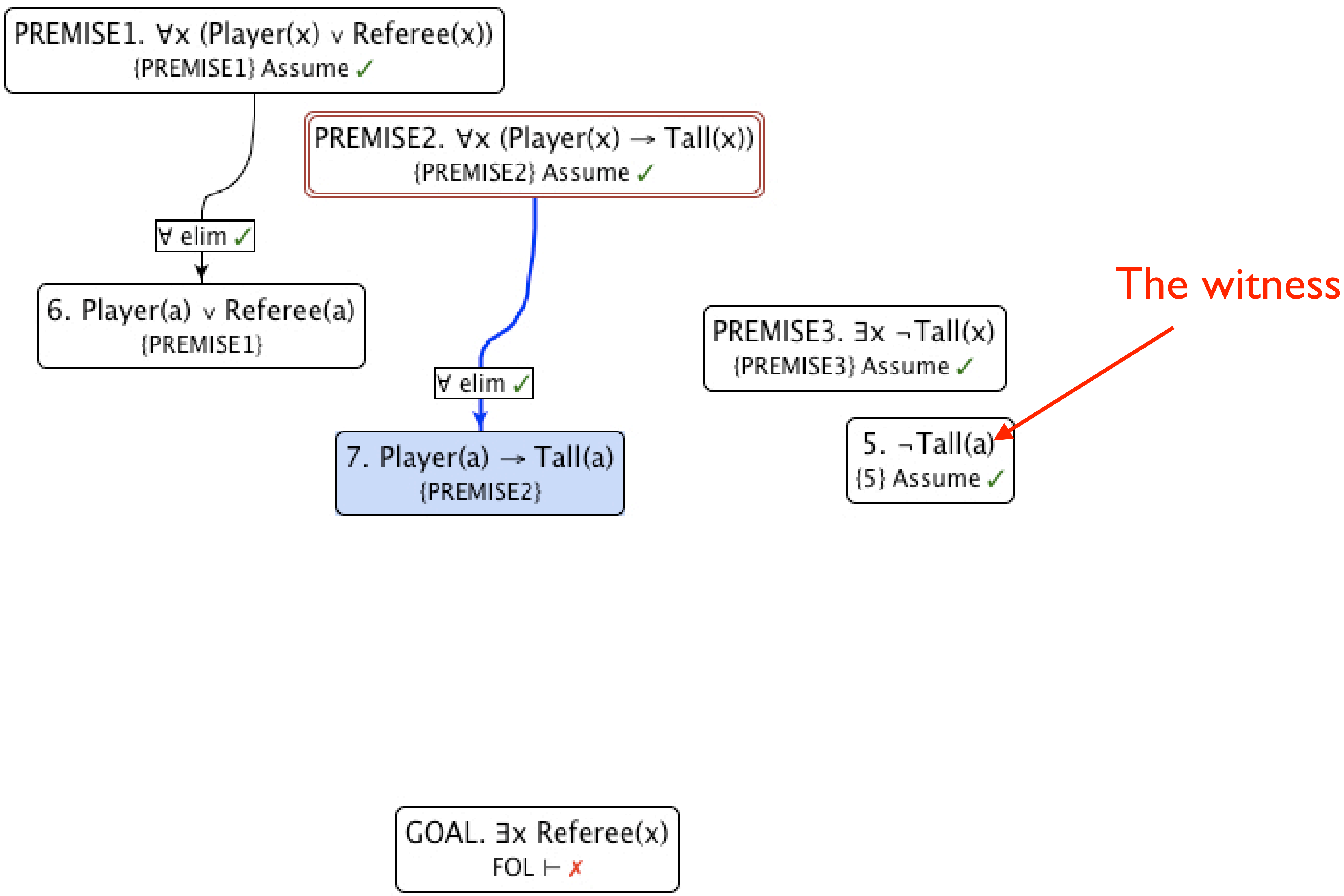
Step 1


PREMISE1. ∀x (Player(x) ∨ Referee(x))
{PREMISE1} Assume ✓
PREMISE2. ∀x (Player(x) → Tall(x))
{PREMISE2} Assume ✓
The witness!
PREMISE3. ∃x ¬Tall(x)
{PREMISE3} Assume ✓
5. ¬Tall(a)
{5} Assume ✓
GOAL. ∃x Referee(x)
FOL ⊢ ✗

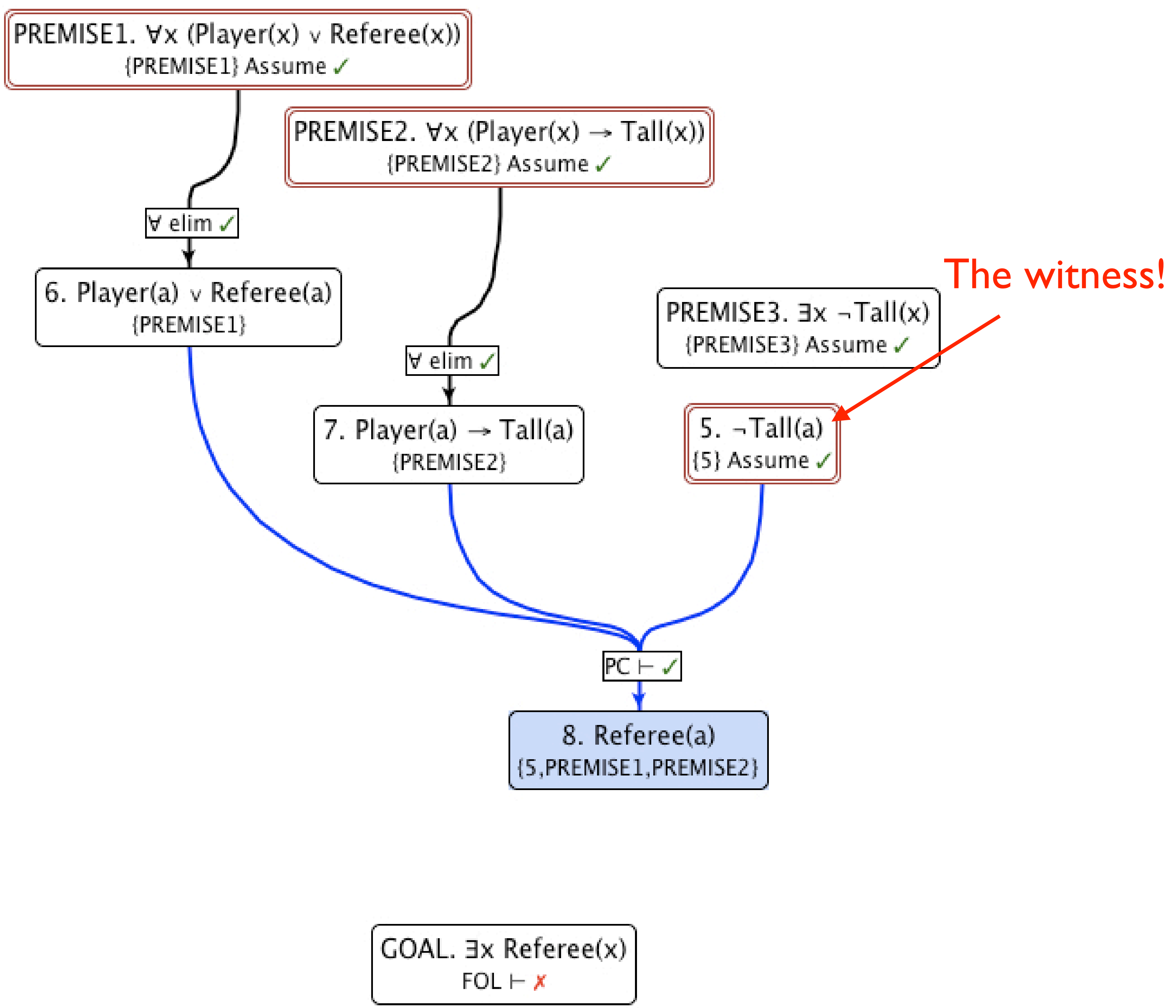
Step 2


PREMISE1. ∀x (Player(x) ∨ Referee(x))
{PREMISE1} Assume ✓
PREMISE2. ∀x (Player(x) → Tall(x))
{PREMISE2} Assume ✓
∀ elim ✓
6. Player(a) ∨ Referee(a)
{PREMISE1}
The witness!
PREMISE3. ∃x ¬Tall(x)
{PREMISE3} Assume ✓
5. ¬Tall(a)
{5} Assume ✓
GOAL. ∃x Referee(x)
FOL ⊢ ✗

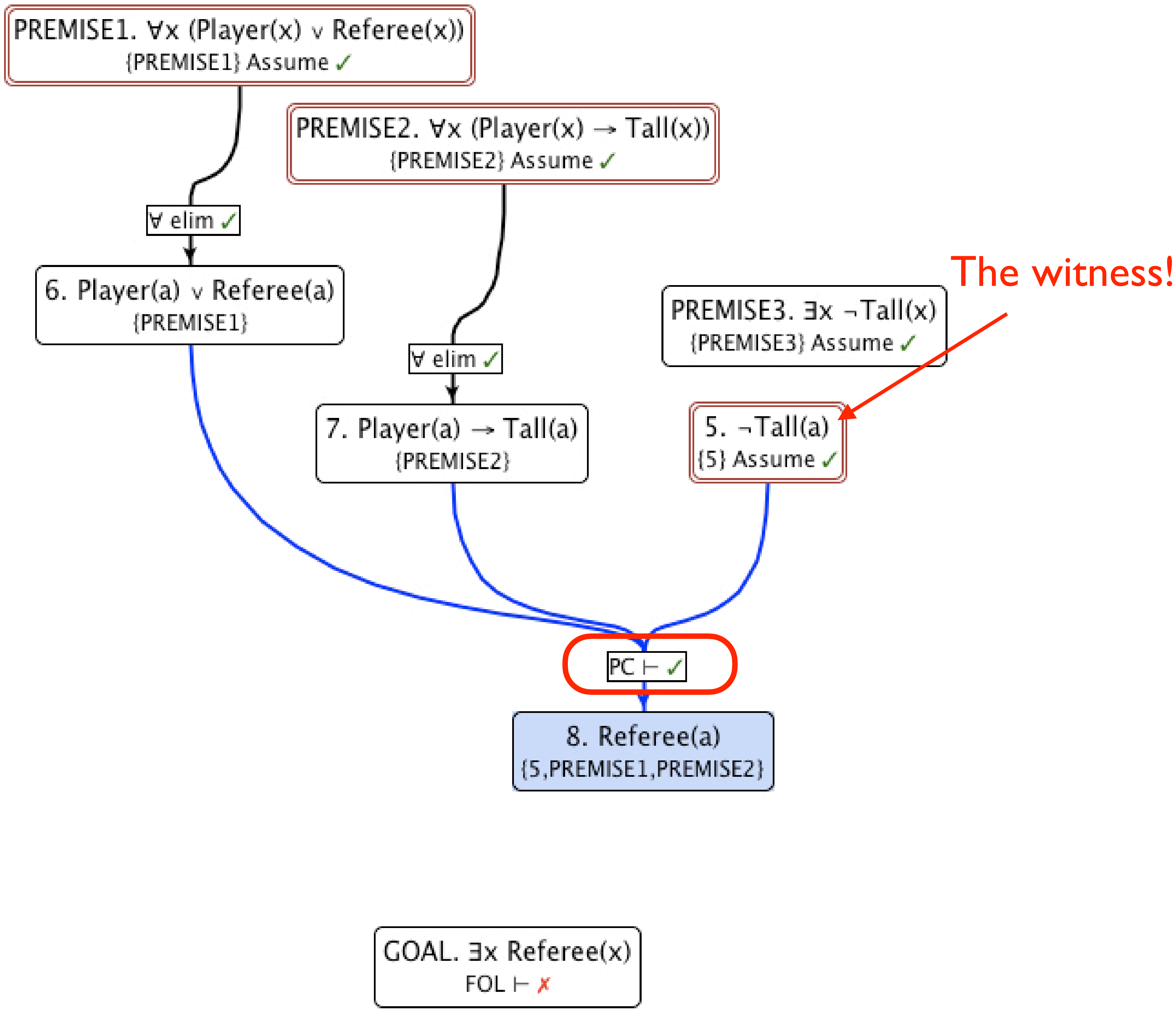
Step 3


PREMISE1. ∀x (Player(x) ∨ Referee(x))
{PREMISE1} Assume ✓
PREMISE2. ∀x (Player(x) → Tall(x))
{PREMISE2} Assume ✓
∀ elim ✓
6. Player(a) ∨ Referee(a)
{PREMISE1}
∀ elim ✓
7. Player(a) → Tall(a)
{PREMISE2}
PREMISE3. ∃x ¬Tall(x)
{PREMISE3} Assume ✓
5. ¬Tall(a)
{5} Assume ✓
The witness!
GOAL. ∃x Referee(x)
FOL ⊢ ✗

Step 4

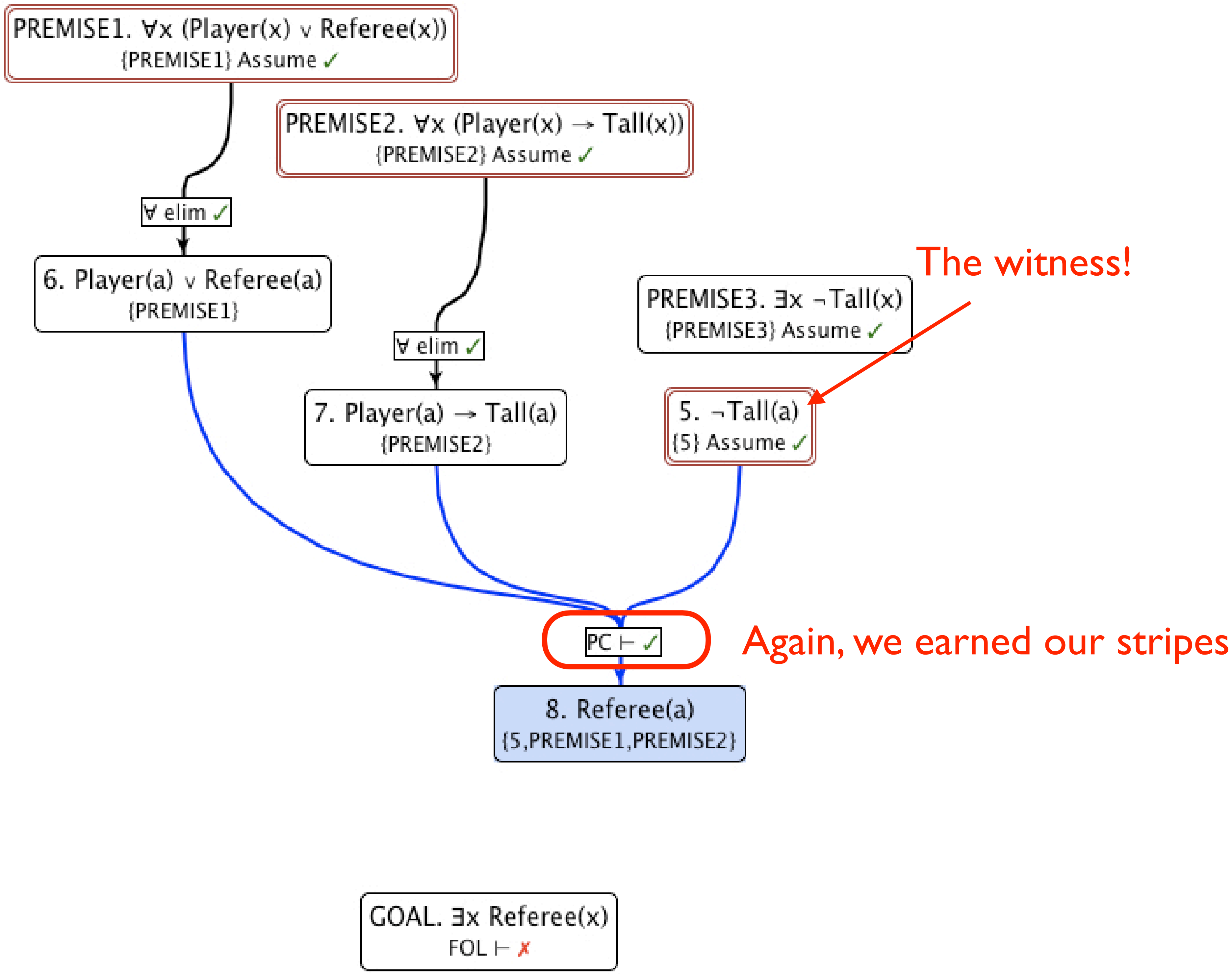

PREMISE1. ∀x (Player(x) ∨ Referee(x))
{PREMISE1} Assume ✓
PREMISE2. ∀x (Player(x) → Tall(x))
{PREMISE2} Assume ✓
∀ elim ✓
6. Player(a) ∨ Referee(a)
{PREMISE1}
∀ elim ✓
PREMISE3. ∃x ¬Tall(x)
{PREMISE3} Assume ✓
The witness!
7. Player(a) → Tall(a)
{PREMISE2}
5. ¬Tall(a)
{5} Assume ✓
PC ⊢ ✓
8. Referee(a)
{5,PREMISE1,PREMISE2}
GOAL. ∃x Referee(x)
FOL ⊢ ✗

Step 4

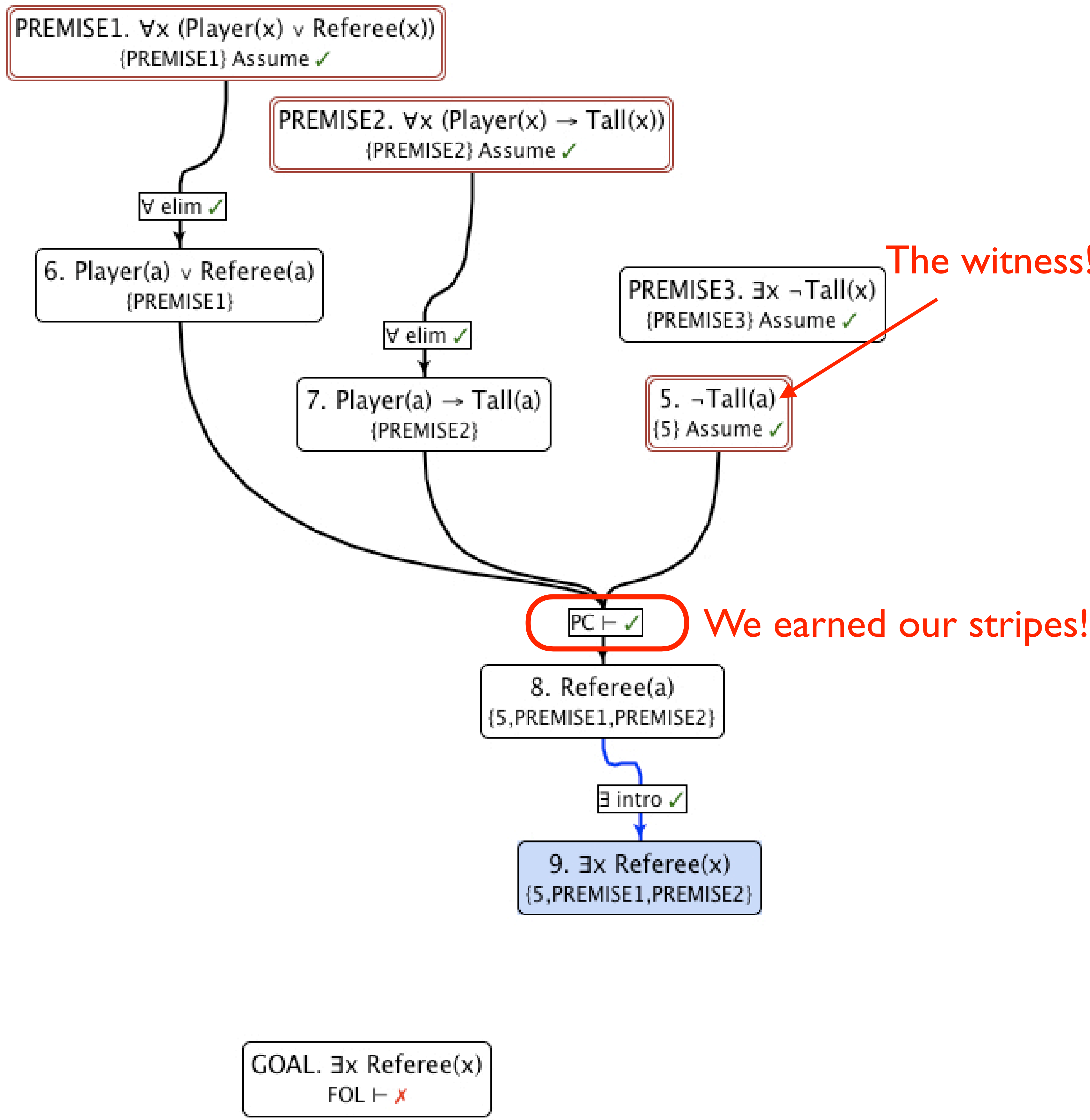


PREMISE1. ∀x (Player(x) ∨ Referee(x))
{PREMISE1} Assume ✓
PREMISE2. ∀x (Player(x) → Tall(x))
{PREMISE2} Assume ✓
∀ elim ✓
6. Player(a) ∨ Referee(a)
{PREMISE1}
The witness!
PREMISE3. ∃x ¬Tall(x)
{PREMISE3} Assume ✓
∀ elim ✓
7. Player(a) → Tall(a)
{PREMISE2}
5. ¬Tall(a)
{5} Assume ✓
PC ⊢ ✓
8. Referee(a)
{5,PREMISE1,PREMISE2}
GOAL. ∃x Referee(x)
FOL ⊢ ✗

Step 4


PREMISE1. ∀x (Player(x) ∨ Referee(x))
{PREMISE1} Assume ✓
PREMISE2. ∀x (Player(x) → Tall(x))
{PREMISE2} Assume ✓
∀ elim ✓
6. Player(a) ∨ Referee(a)
{PREMISE1}
∀ elim ✓
PREMISE3. ∃x ¬Tall(x)
{PREMISE3} Assume ✓
The witness!
7. Player(a) → Tall(a)
{PREMISE2}
5. ¬Tall(a)
{5} Assume ✓
PC ⊢ ✓
Again, we earned our stripes!
8. Referee(a)
{5,PREMISE1,PREMISE2}
GOAL. ∃x Referee(x)
FOL ⊢ ✗

Step 5

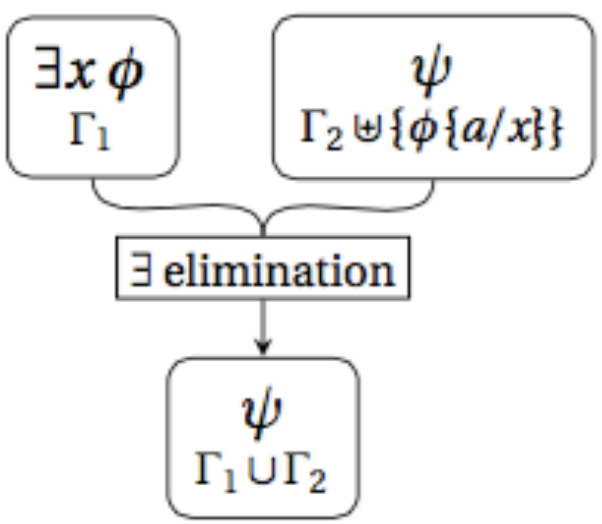


PREMISE1. ∀x (Player(x) ∨ Referee(x))
{PREMISE1} Assume ✓
PREMISE2. ∀x (Player(x) → Tall(x))
{PREMISE2} Assume ✓
∀ elim ✓
6. Player(a) ∨ Referee(a)
{PREMISE1}
∀ elim ✓
PREMISE3. ∃x ¬Tall(x)
{PREMISE3} Assume ✓
The witness!
7. Player(a) → Tall(a)
{PREMISE2}
5. ¬Tall(a)
{5} Assume ✓
PC ⊢ ✓
We earned our stripes!
8. Referee(a)
{5,PREMISE1,PREMISE2}
∃ intro ✓
9. ∃x Referee(x)
{5,PREMISE1,PREMISE2}
GOAL. ∃x Referee(x)
FOL ⊢ ✗

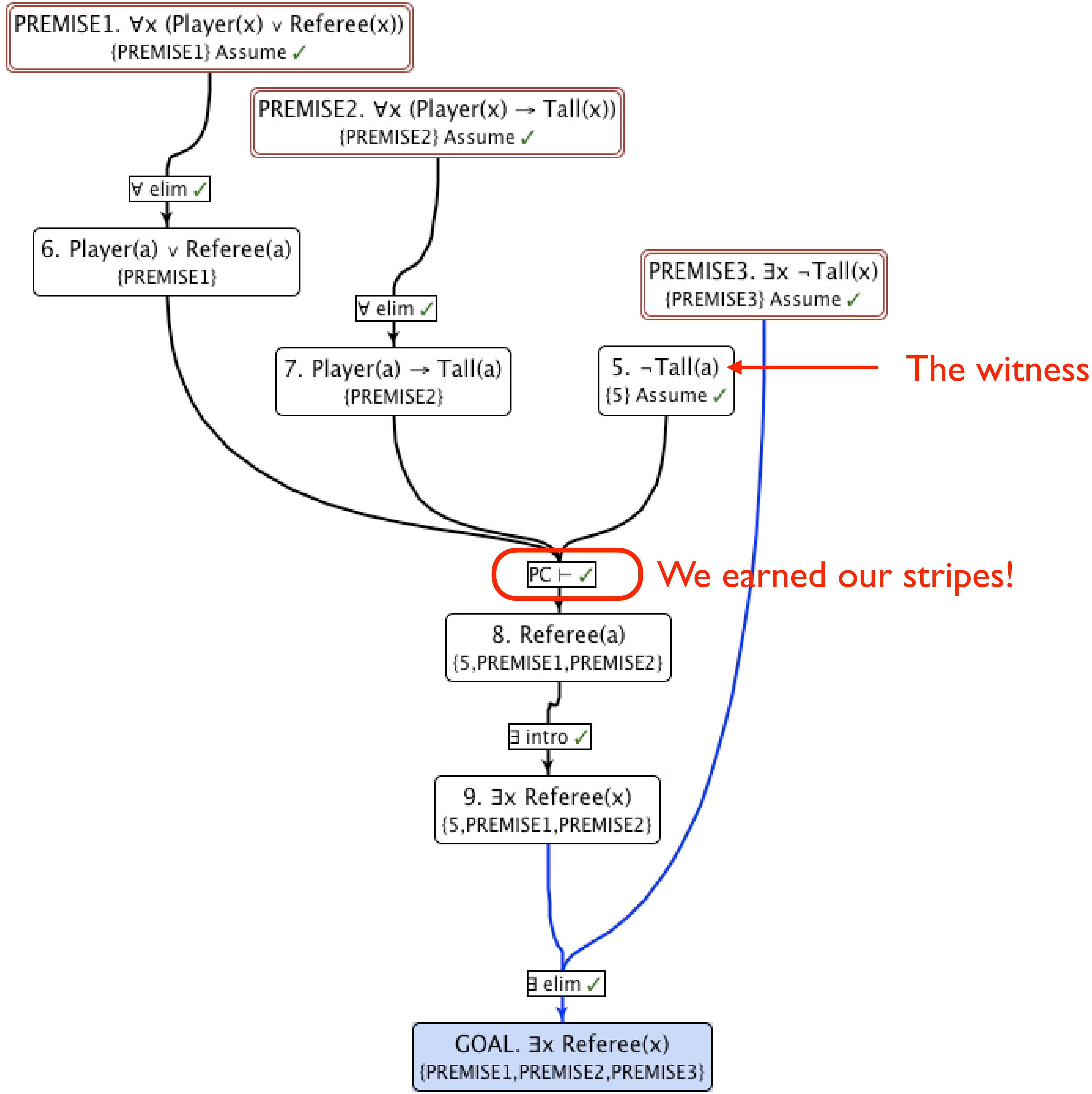
Step 6


PREMISE1. ∀x (Player(x) ∨ Referee(x))
{PREMISE1} Assume ✓
PREMISE2. ∀x (Player(x) → Tall(x))
{PREMISE2} Assume ✓
∀ elim ✓
6. Player(a) ∨ Referee(a)
{PREMISE1}
PREMISE3. ∃x ¬Tall(x)
{PREMISE3} Assume ✓
∀ elim ✓
7. Player(a) → Tall(a)
{PREMISE2}
5. ¬Tall(a)
{5} Assume ✓
The witness!
PC ⊢ ✓
We earned our stripes!
8. Referee(a)
{5,PREMISE1,PREMISE2}
∃ intro ✓
9. ∃x Referee(x)
{5,PREMISE1,PREMISE2}
∃ elim ✓
GOAL. ∃x Referee(x)
{PREMISE1,PREMISE2,PREMISE3}

Step 6

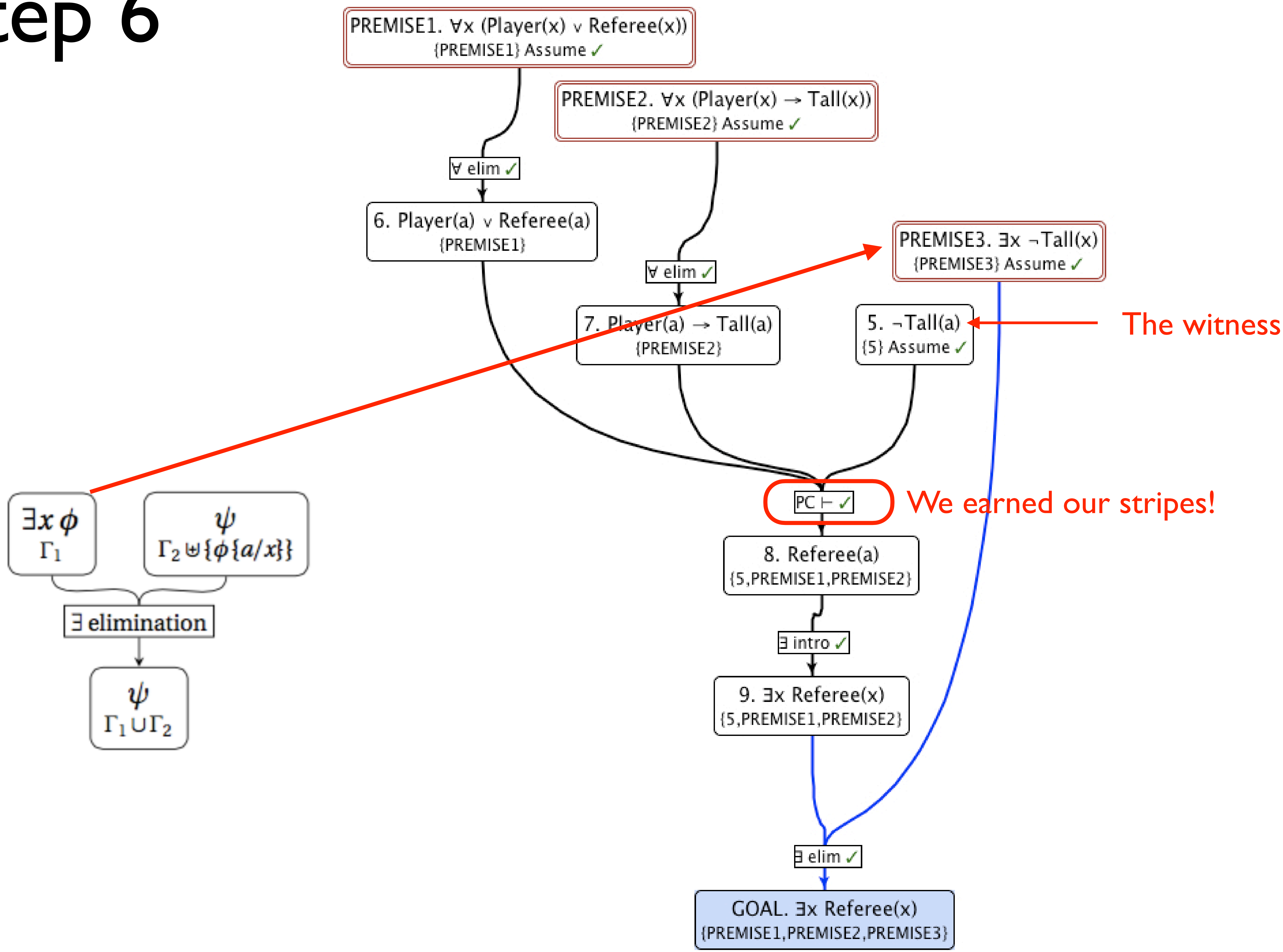


$\exists x\, \phi$
Γ_1
ψ
$\Gamma_2 \uplus \{\phi\{a/x\}\}$
∃ elimination
ψ
$\Gamma_1 \cup \Gamma_2$

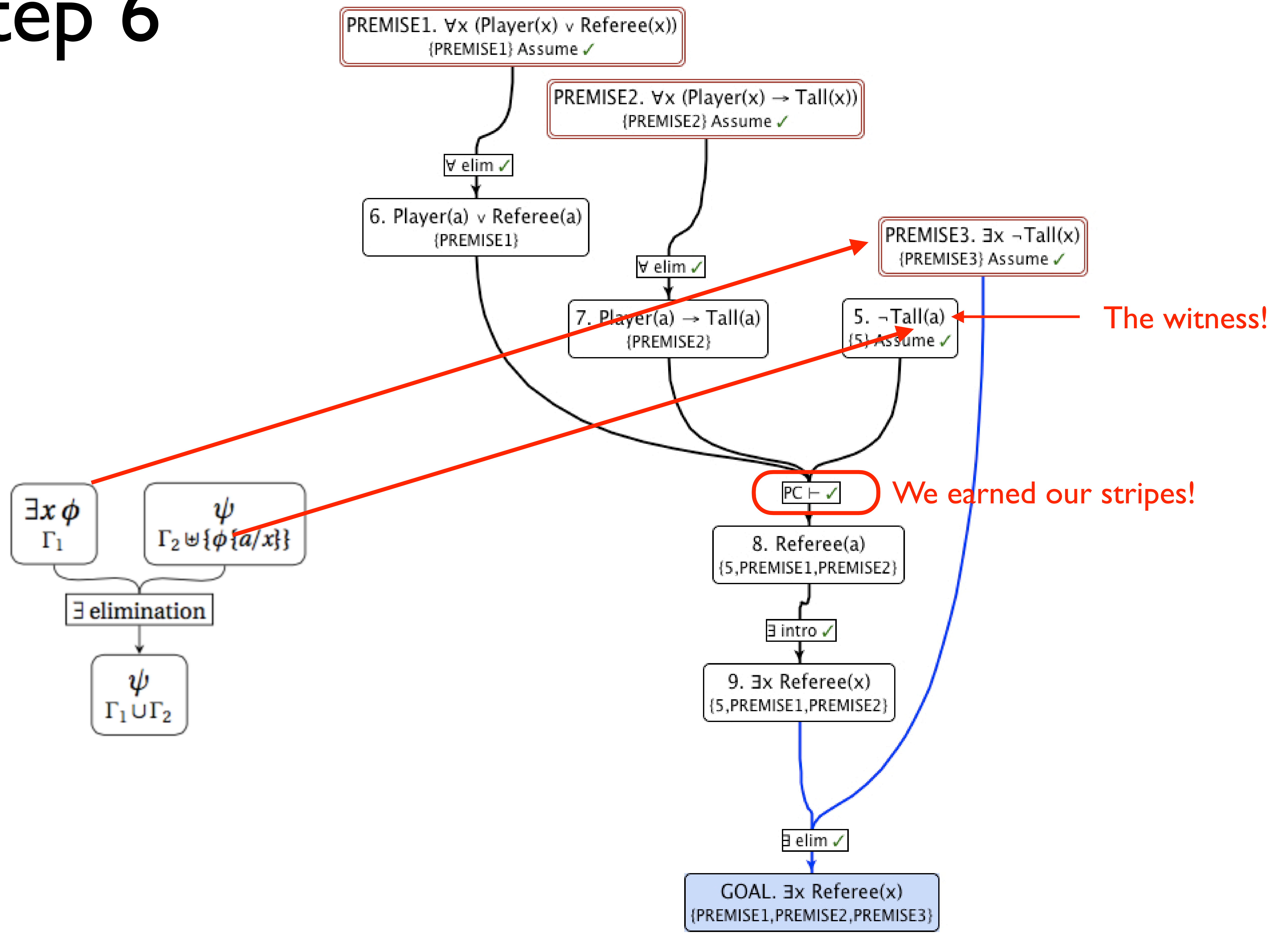


PREMISE1. ∀x (Player(x) ∨ Referee(x))
{PREMISE1} Assume ✓
PREMISE2. ∀x (Player(x) → Tall(x))
{PREMISE2} Assume ✓
∀ elim ✓
6. Player(a) ∨ Referee(a)
{PREMISE1}
PREMISE3. ∃x ¬Tall(x)
{PREMISE3} Assume ✓
∀ elim ✓
7. Player(a) → Tall(a)
{PREMISE2}
5. ¬Tall(a)
{5} Assume ✓
The witness!
PC ⊢ ✓
We earned our stripes!
8. Referee(a)
{5,PREMISE1,PREMISE2}
∃ intro ✓
9. ∃x Referee(x)
{5,PREMISE1,PREMISE2}
∃ elim ✓
GOAL. ∃x Referee(x)
{PREMISE1,PREMISE2,PREMISE3}

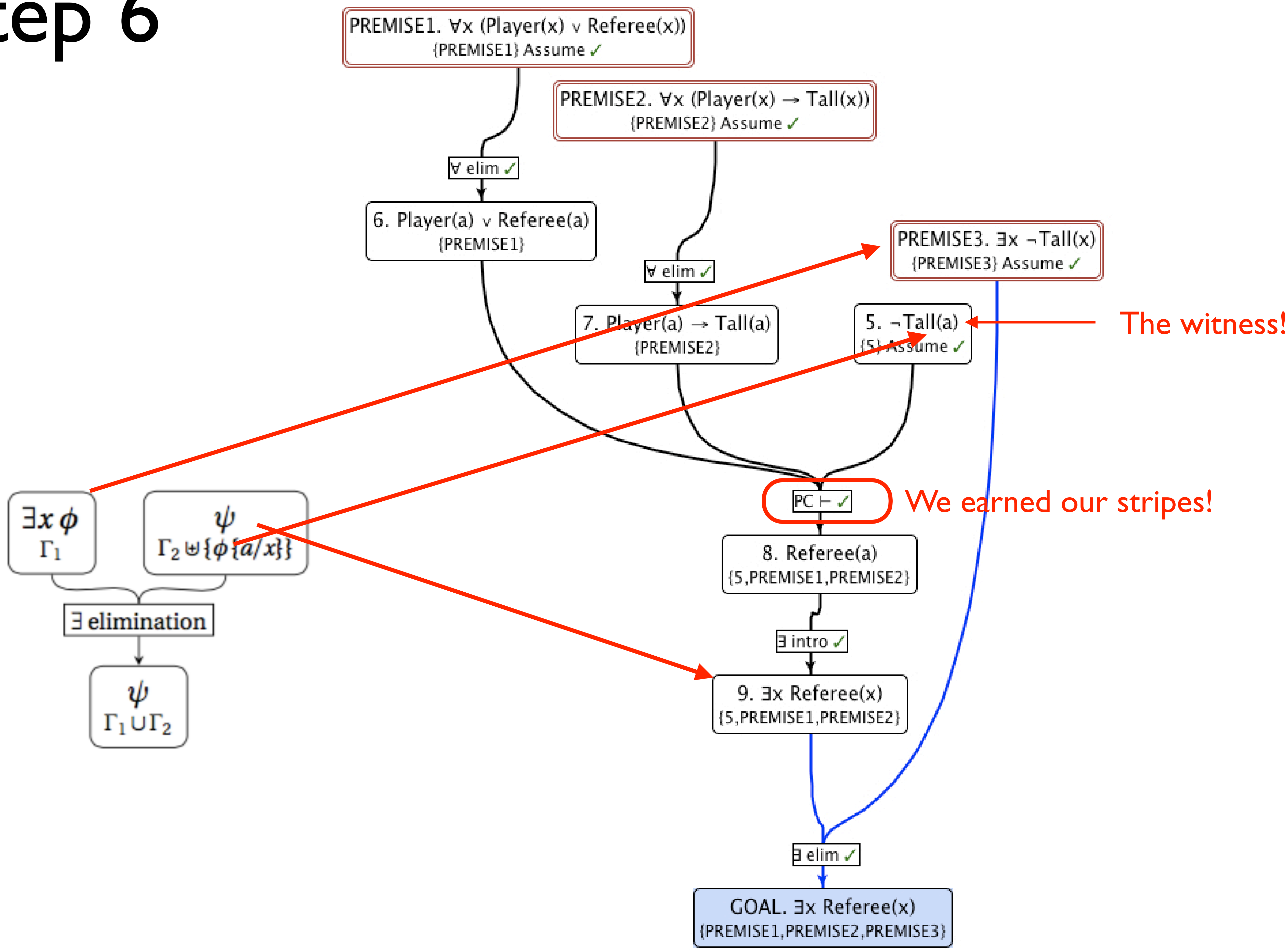
Step 6


PREMISE1. ∀x (Player(x) ∨ Referee(x))
{PREMISE1} Assume ✓
PREMISE2. ∀x (Player(x) → Tall(x))
{PREMISE2} Assume ✓
∀ elim ✓
6. Player(a) ∨ Referee(a)
{PREMISE1}
∀ elim ✓
PREMISE3. ∃x ¬Tall(x)
{PREMISE3} Assume ✓
7. Player(a) → Tall(a)
{PREMISE2}
5. ¬Tall(a)
{5} Assume ✓
The witness!
PC ⊢ ✓
We earned our stripes!
∃x φ
Γ1
ψ
Γ2 ⊎ {φ{a/x}}
∃ elimination
ψ
Γ1 ∪ Γ2
8. Referee(a)
{5,PREMISE1,PREMISE2}
∃ intro ✓
9. ∃x Referee(x)
{5,PREMISE1,PREMISE2}
∃ elim ✓
GOAL. ∃x Referee(x)
{PREMISE1,PREMISE2,PREMISE3}

Step 6

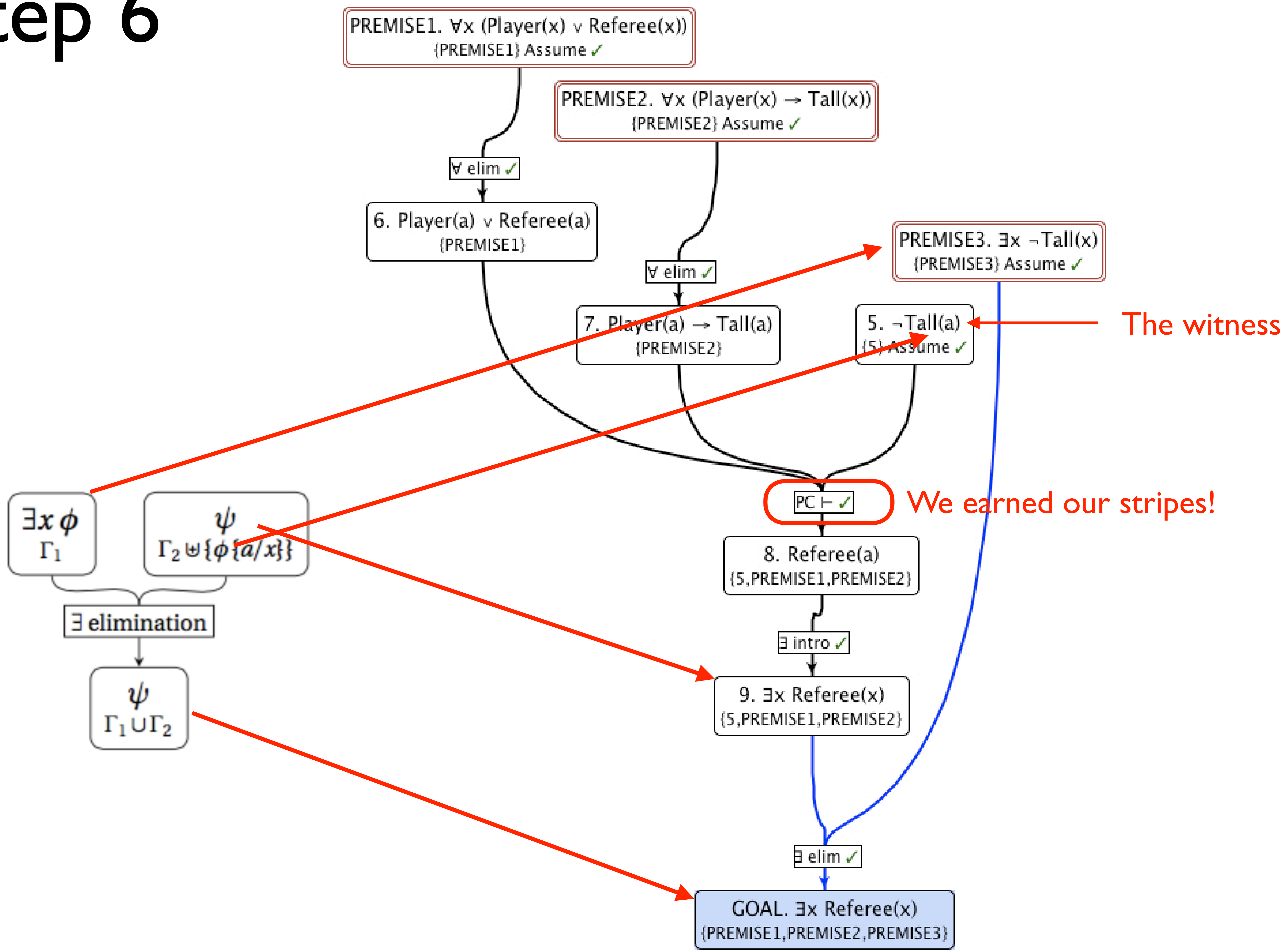

PREMISE1. ∀x (Player(x) ∨ Referee(x))
{PREMISE1} Assume ✓
PREMISE2. ∀x (Player(x) → Tall(x))
{PREMISE2} Assume ✓
∀ elim ✓
6. Player(a) ∨ Referee(a)
{PREMISE1}
PREMISE3. ∃x ¬Tall(x)
{PREMISE3} Assume ✓
∀ elim ✓
7. Player(a) → Tall(a)
{PREMISE2}
5. ¬Tall(a)
{5} Assume ✓
The witness!
∃x φ
Γ1
ψ
Γ2 ⊎ {φ{a/x}}
PC ⊢ ✓
We earned our stripes!
8. Referee(a)
{5,PREMISE1,PREMISE2}
∃ elimination
∃ intro ✓
ψ
Γ1 ∪ Γ2
9. ∃x Referee(x)
{5,PREMISE1,PREMISE2}
∃ elim ✓
GOAL. ∃x Referee(x)
{PREMISE1,PREMISE2,PREMISE3}

Step 6



PREMISE1. ∀x (Player(x) ∨ Referee(x))
{PREMISE1} Assume ✓
PREMISE2. ∀x (Player(x) → Tall(x))
{PREMISE2} Assume ✓
∀ elim ✓
6. Player(a) ∨ Referee(a)
{PREMISE1}
PREMISE3. ∃x ¬Tall(x)
{PREMISE3} Assume ✓
∀ elim ✓
7. Player(a) → Tall(a)
{PREMISE2}
5. ¬Tall(a)
{5} Assume ✓
The witness!
PC ⊢ ✓
We earned our stripes!
8. Referee(a)
{5,PREMISE1,PREMISE2}
∃ intro ✓
9. ∃x Referee(x)
{5,PREMISE1,PREMISE2}
∃ elim ✓
GOAL. ∃x Referee(x)
{PREMISE1,PREMISE2,PREMISE3}
∃x φ
Γ1
ψ
Γ2 ⊎ {φ{a/x}}
∃ elimination
ψ
Γ1 ∪ Γ2

Step 6


PREMISE1. ∀x (Player(x) ∨ Referee(x))
{PREMISE1} Assume ✓
PREMISE2. ∀x (Player(x) → Tall(x))
{PREMISE2} Assume ✓
∀ elim ✓
6. Player(a) ∨ Referee(a)
{PREMISE1}
PREMISE3. ∃x ¬Tall(x)
{PREMISE3} Assume ✓
∀ elim ✓
7. Player(a) → Tall(a)
{PREMISE2}
5. ¬Tall(a)
{5} Assume ✓
The witness!
PC ⊢ ✓
We earned our stripes!
8. Referee(a)
{5,PREMISE1,PREMISE2}
∃ intro ✓
9. ∃x Referee(x)
{5,PREMISE1,PREMISE2}
∃ elim ✓
GOAL. ∃x Referee(x)
{PREMISE1,PREMISE2,PREMISE3}
∃x φ
Γ1
ψ
Γ2 ⊎ {φ{a/x}}
∃ elimination
ψ
Γ1 ∪ Γ2

$$\{\forall \mathtt{x}(\mathtt{Scared}(\mathtt{x}) \leftrightarrow \mathtt{Small}(\mathtt{x})), \exists \mathtt{x} \neg \mathtt{Scared}(x)\} \vdash \exists \mathtt{x} \neg \mathtt{Small}(x)$$

$$\{\exists \mathtt{x}, \mathtt{y} \mathtt{Contiguous}(\mathtt{x}, \mathtt{y}), \forall \mathtt{x}, \mathtt{y}(\mathtt{Contiguous}(\mathtt{x}, \mathtt{y}) \rightarrow \neg \mathtt{SameCountry}(\mathtt{x}, \mathtt{y}))\} \vdash \exists \mathtt{x}, \mathtt{y} \neg \mathtt{SameCountry}(\mathtt{x}, \mathtt{y})$$