Resolution Now In Textbook; Exhortation; Truth Trees;

FOL IV: Layered Quantification and Measuring Intelligence Using This Phenomenon

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Troy, New York 12180 USA

Intro to (Formal) Logic
3/2/2020

Exhortation …

Make sure you're up-to-date today-ish, fully, on HyperGrader's current (Homework) Problems, due April 22. FOL problems are forthcoming.

New Required Problem: `FregTHEN2` ...

Truth Trees vs. Truth Tables

Truth Trees vs. Truth Tables

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Truth Trees vs. Truth Tables

Violent breakage between tabular calculation and proof construction.

Truth Trees vs. ~~Truth Tables~~

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First very simple: truth-tree for *modus ponens* …

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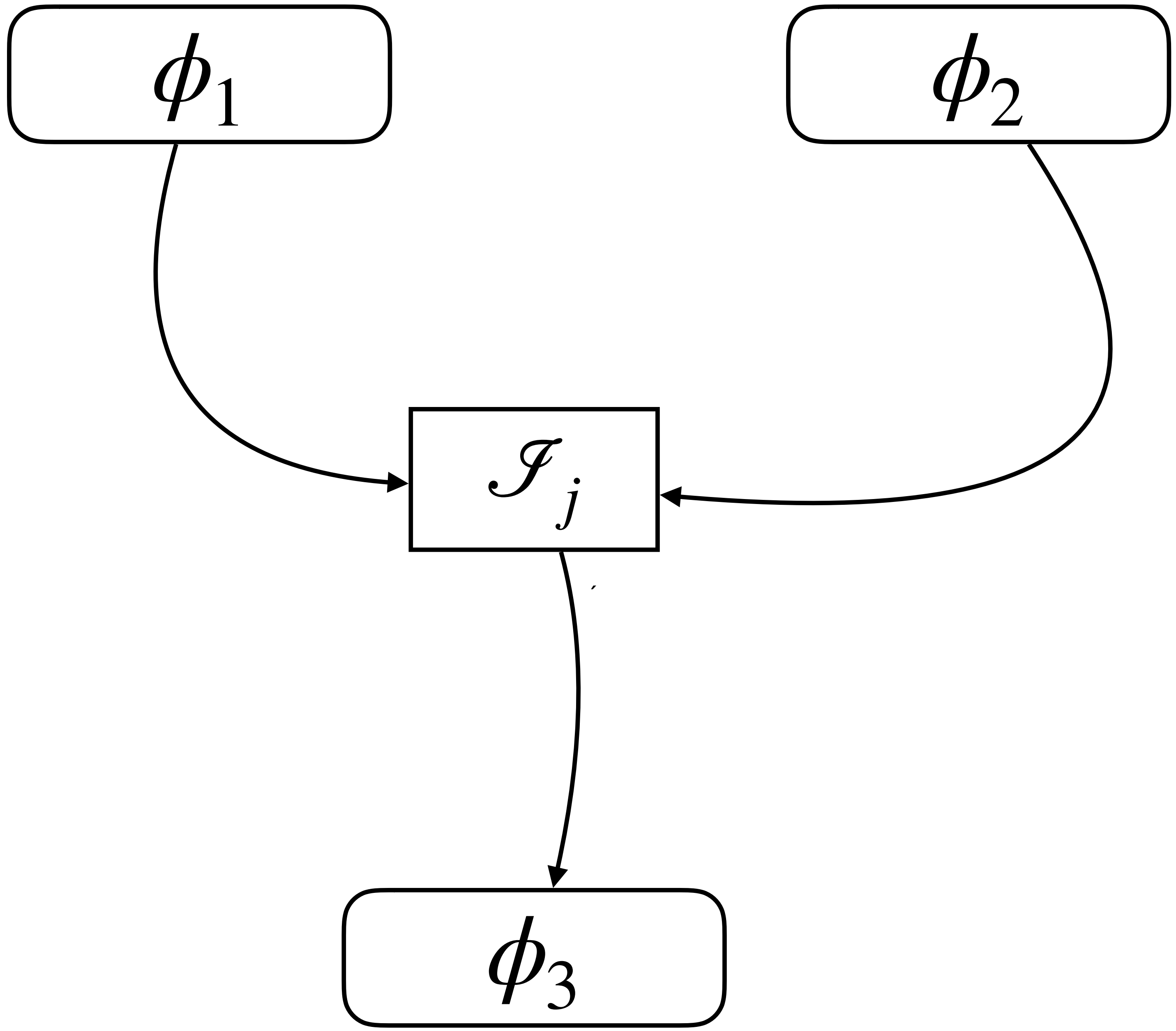
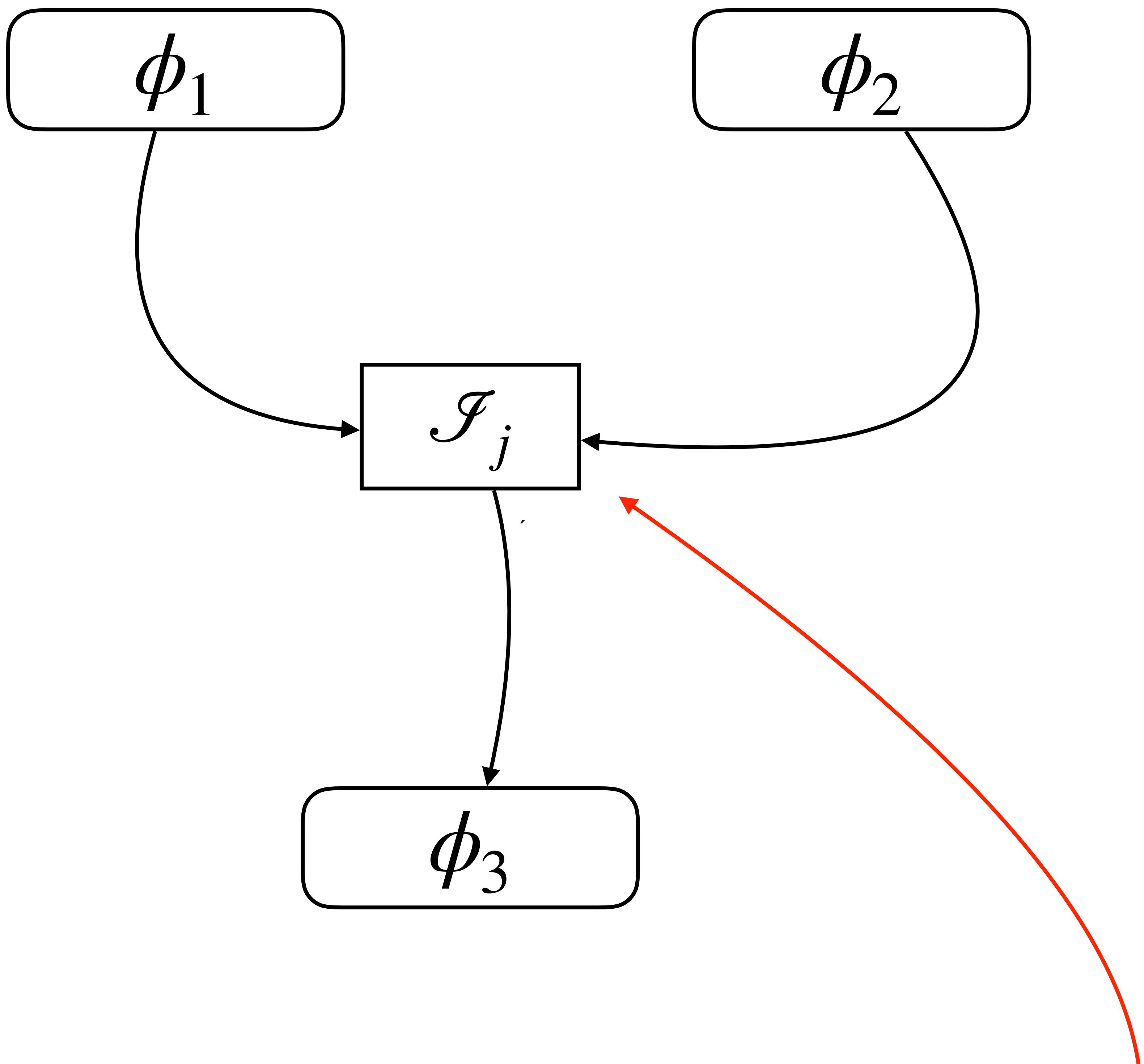
First very simple: truth-tree for *modus ponens* …



ϕ_1
ϕ_2
$\mathscr{I}_j$
ϕ_3



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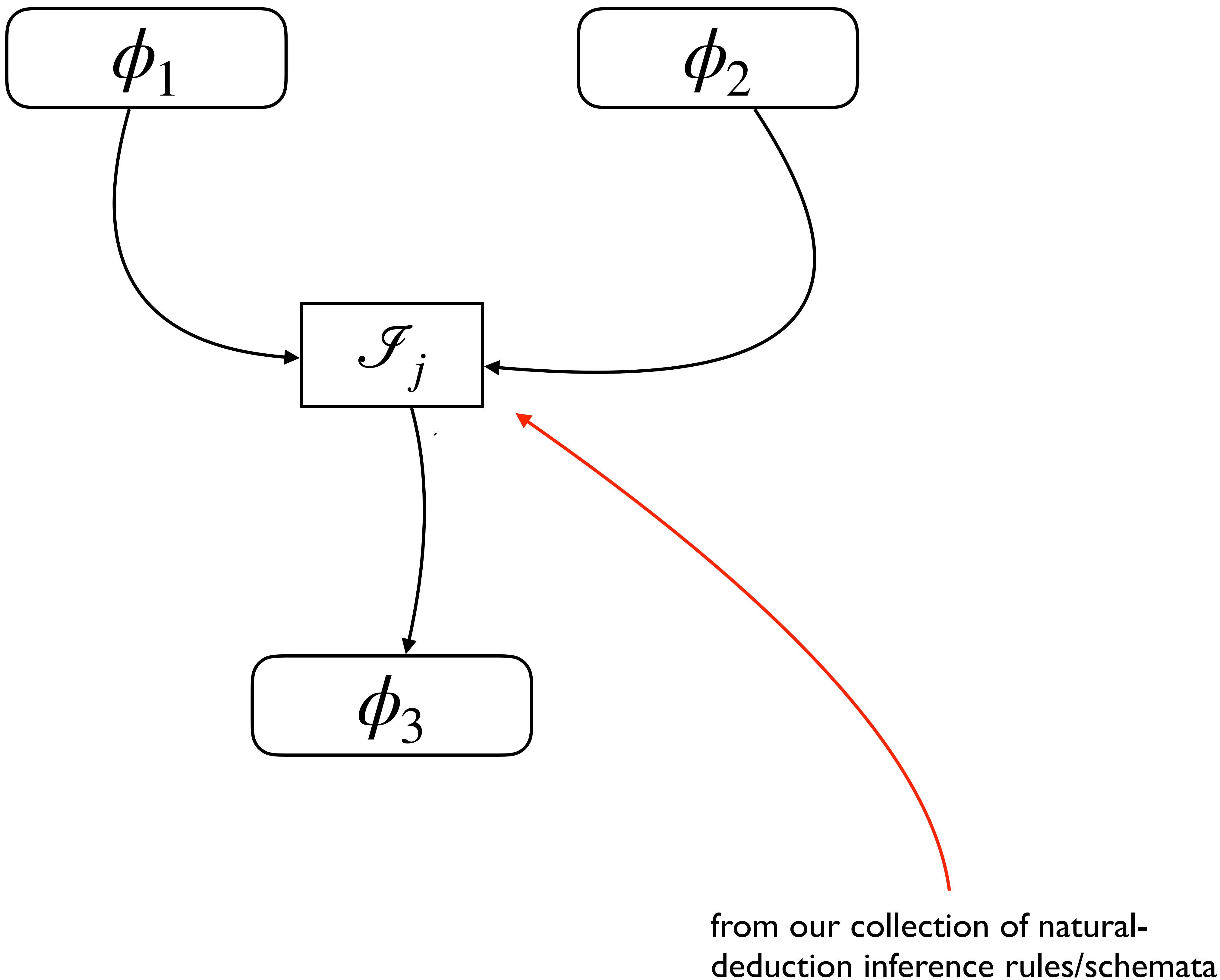
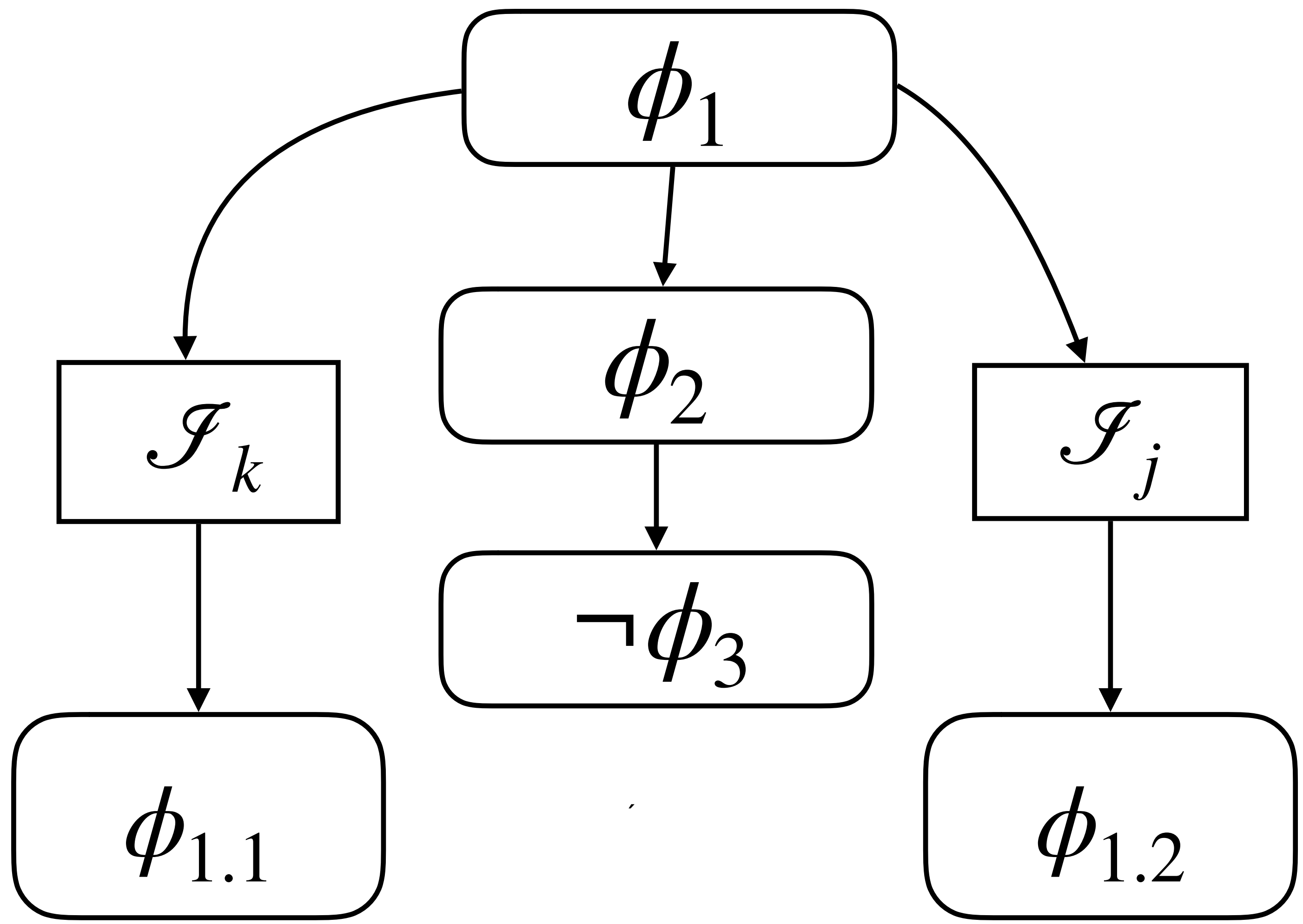
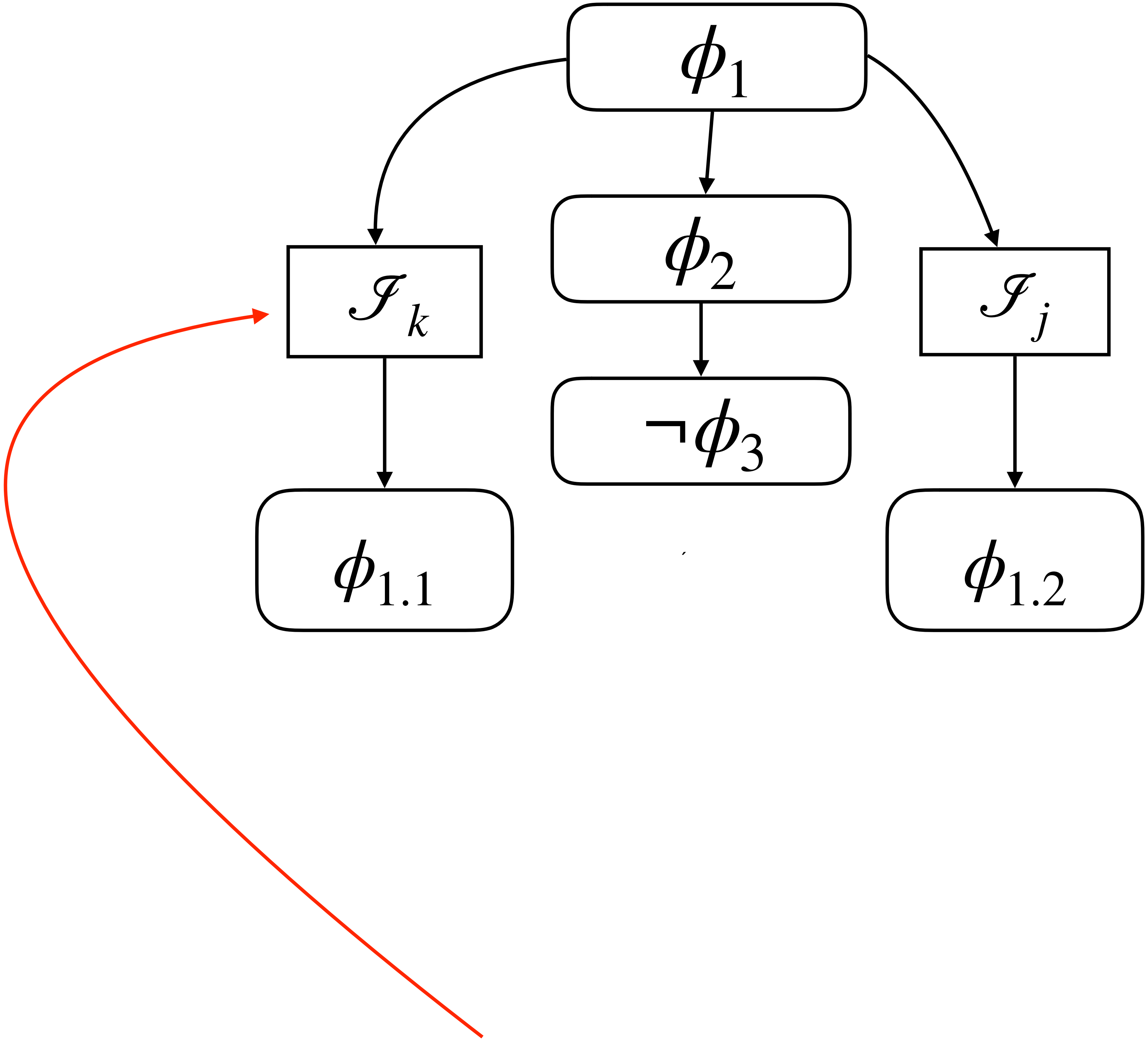

ϕ_1
ϕ_2
$\mathscr{I}_j$
ϕ_3
from our collection of natural-
deduction inference rules/schemata



ϕ_1
$\mathcal{I}_k$
ϕ_2
$\mathcal{I}_j$
$\neg\phi_3$
$\phi_{1.1}$
$\phi_{1.2}$



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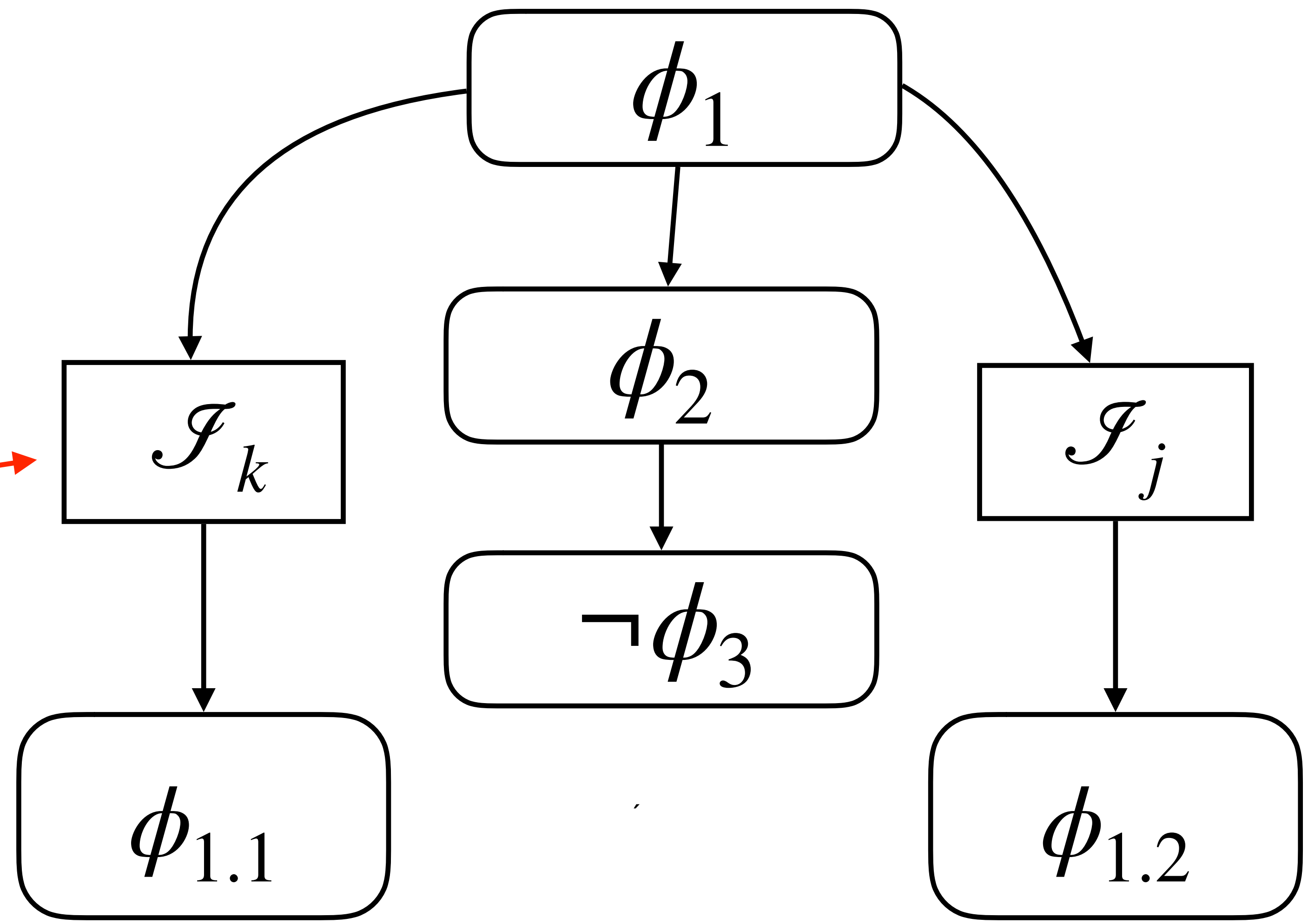
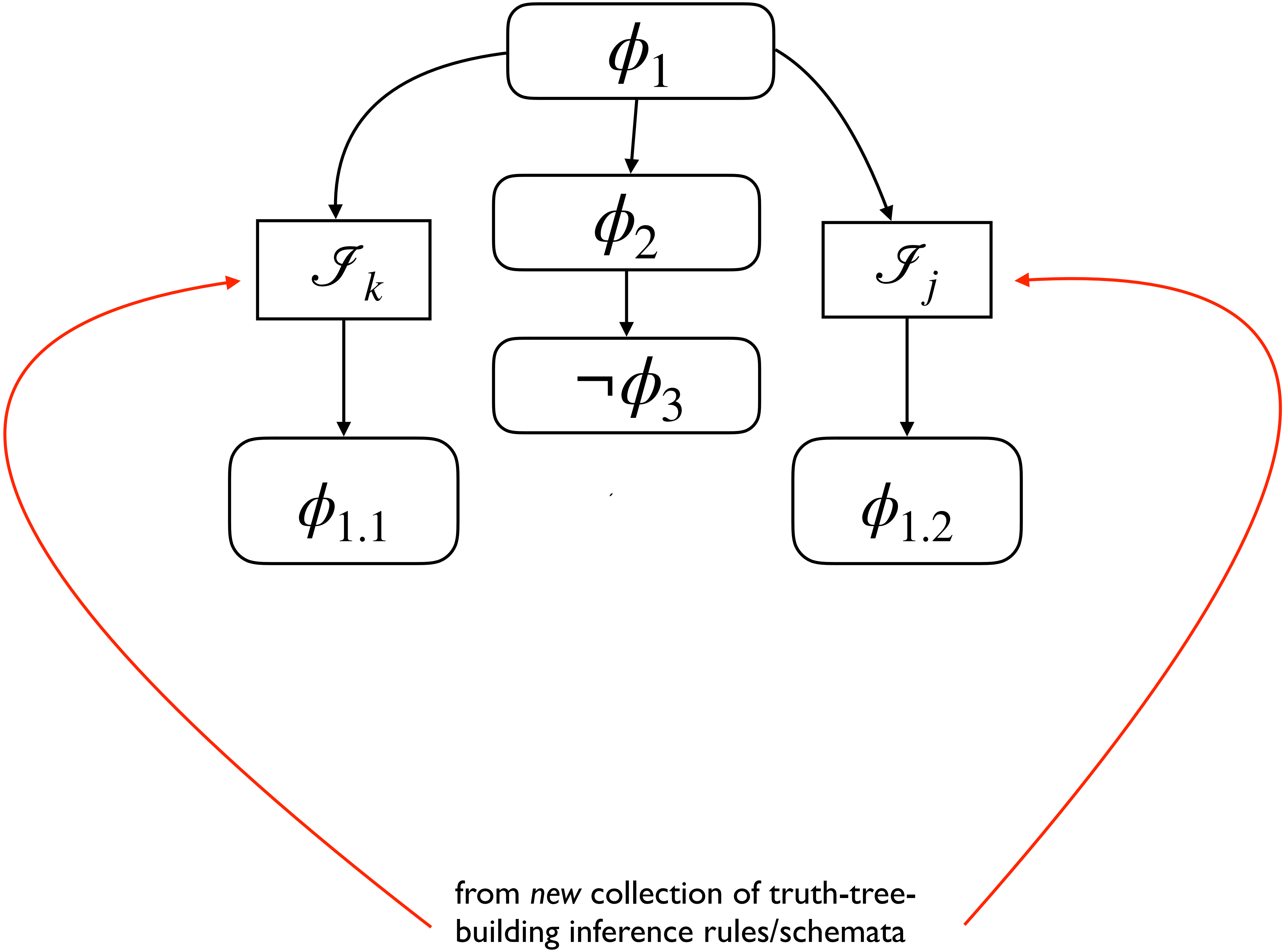


from *new* collection of truth-tree-building inference rules/schemata



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from *new* collection of truth-tree-building inference rules/schemata

$\{P \rightarrow Q, P\} \vdash Q$

GIVEN1. $P \rightarrow Q$

PC $\vdash$ ✗

GIVEN2. P

PC $\vdash$ ✗

NEGATED GOAL. $\neg Q$

PC $\vdash$ ✗

$\{P \to Q, P\} \vdash Q$

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4. $\neg P$

$\{4\}$ Assume ✓

5. Q

$\{5\}$ Assume ✓

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Either way, a contradiction!

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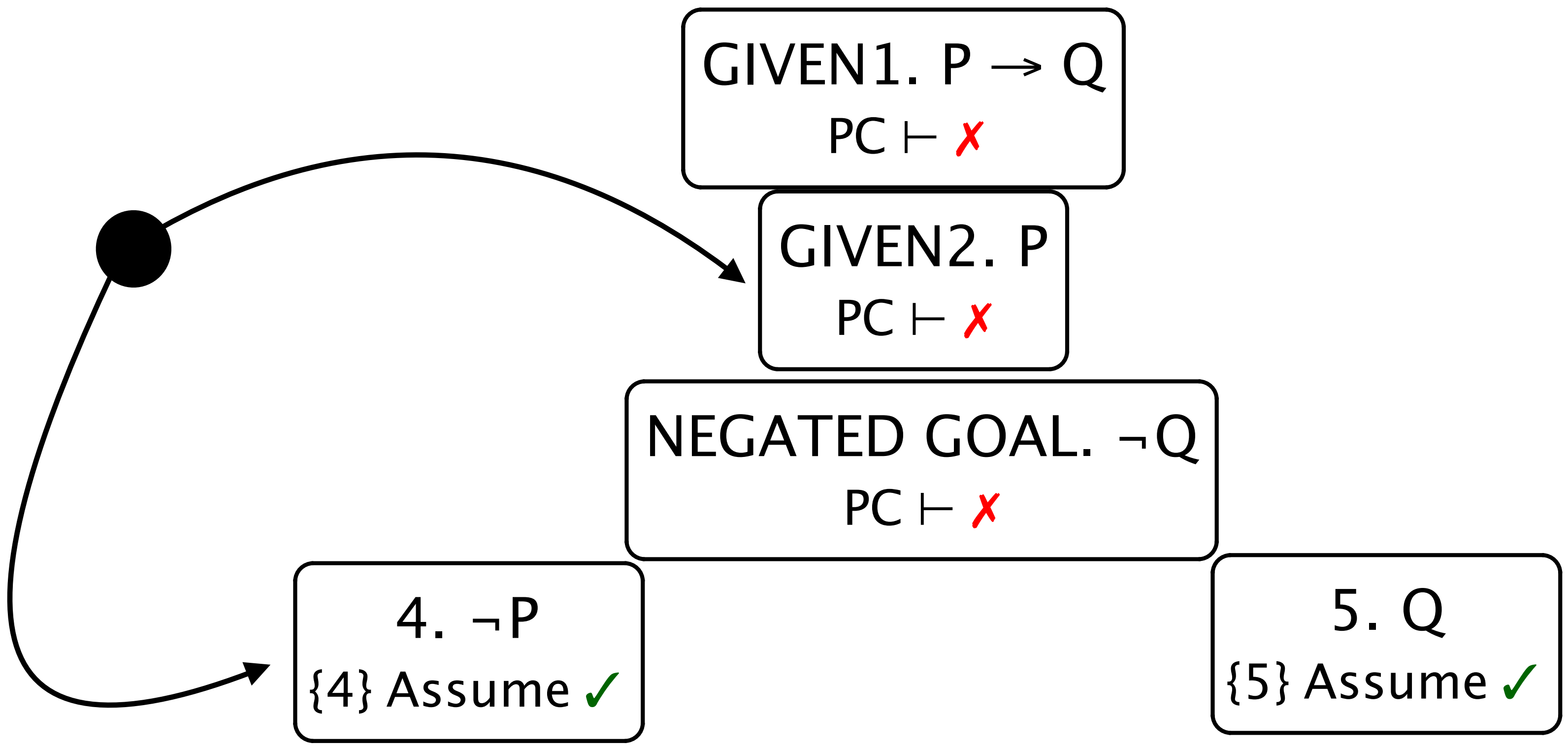

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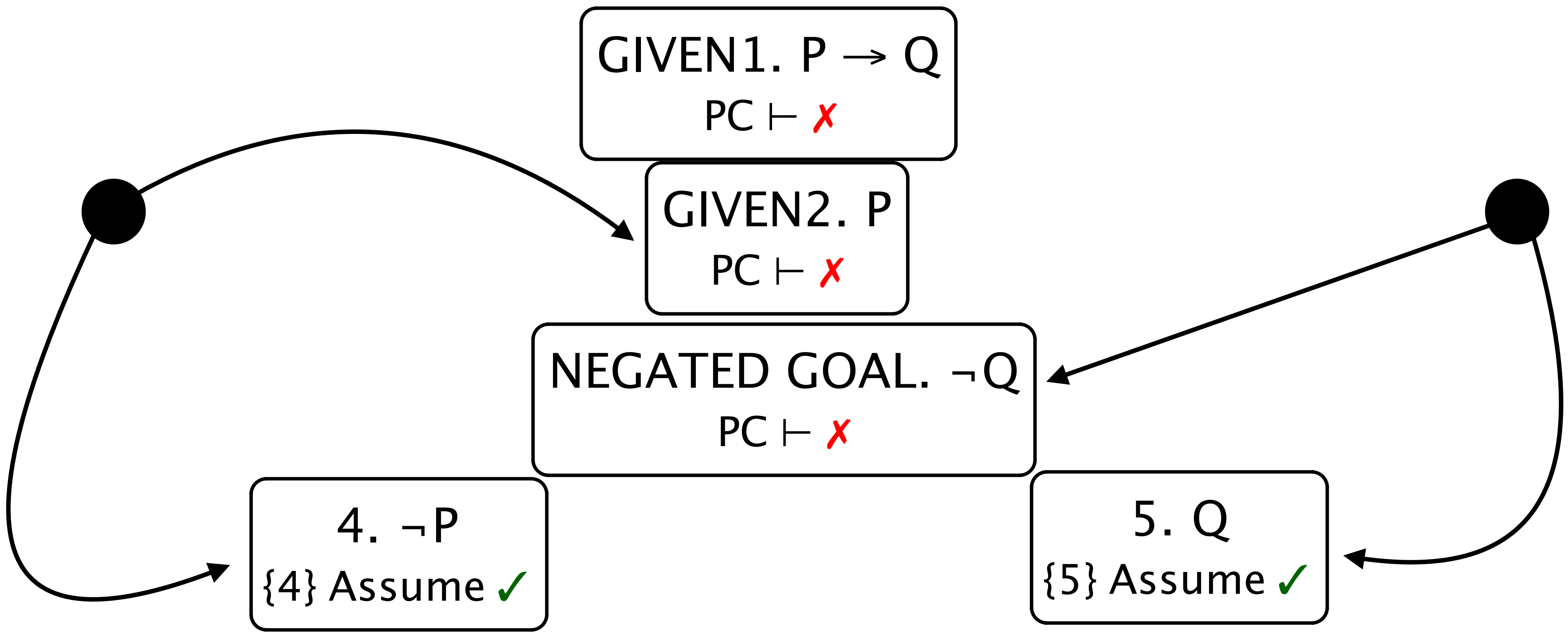

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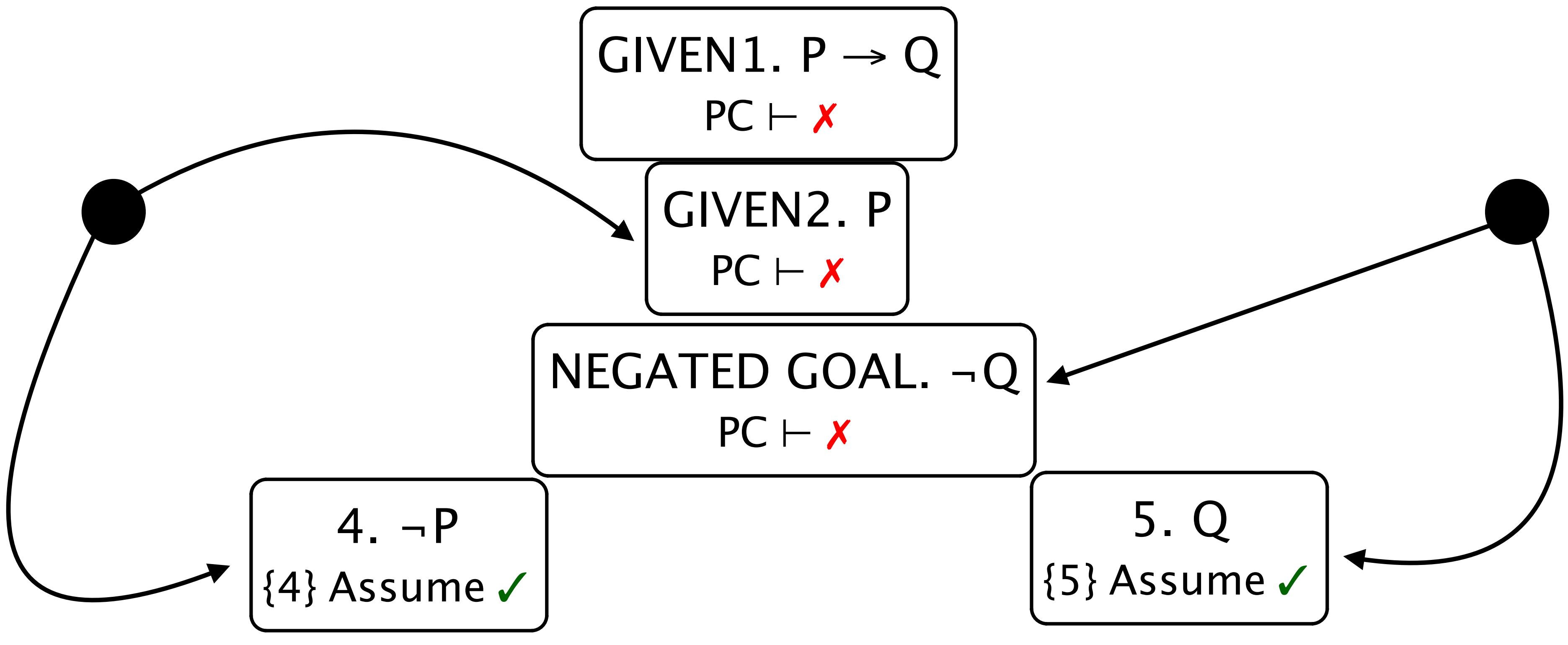
$\{P \to Q, P\} \vdash Q$



Either way, a contradiction!

Therefore the entailment holds!

Slightly Harder Truth Tree

Slightly Harder Truth Tree

$$\vdash (\mathtt{P} \rightarrow (\mathtt{Q} \rightarrow \mathtt{R})) \rightarrow ((\mathtt{P} \rightarrow \mathtt{Q}) \rightarrow (\mathtt{P} \rightarrow \mathtt{R}))$$

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(This is the axiom THEN-2 in Frege's (brutal) axiomatization of the propositional calculus.)

Frege

Slightly Harder Truth Tree

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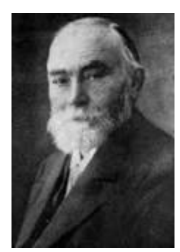

Frege

https://en.wikipedia.org/wiki/Frege%27s_propositional_calculus

Slightly Harder Truth Tree

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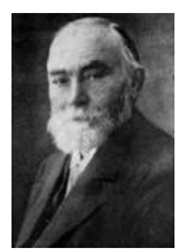

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The Rules of the Game!

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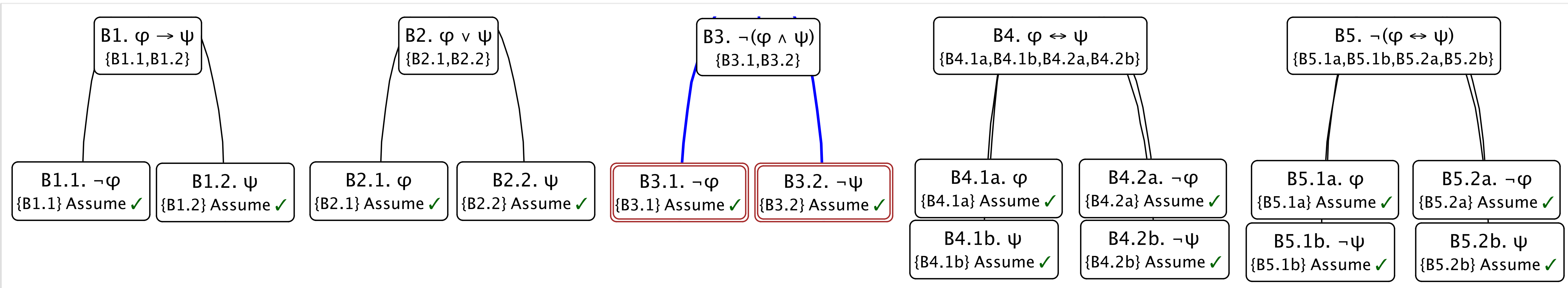
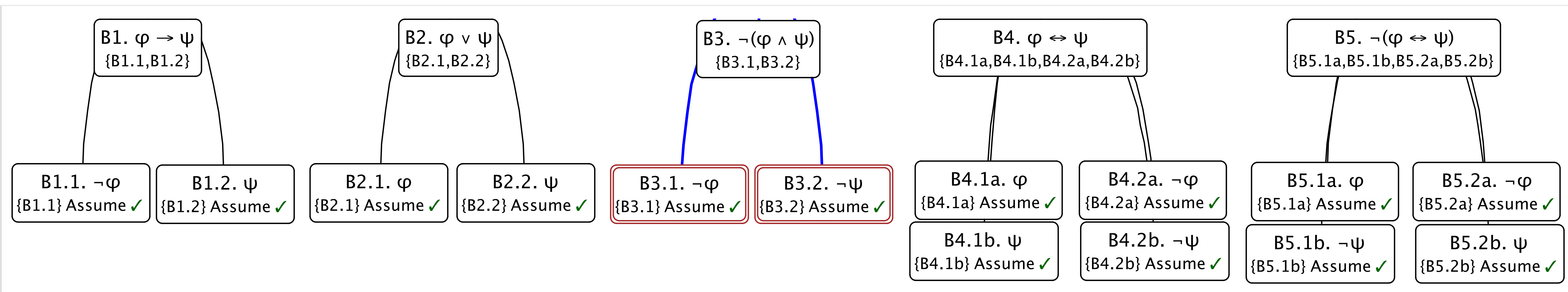
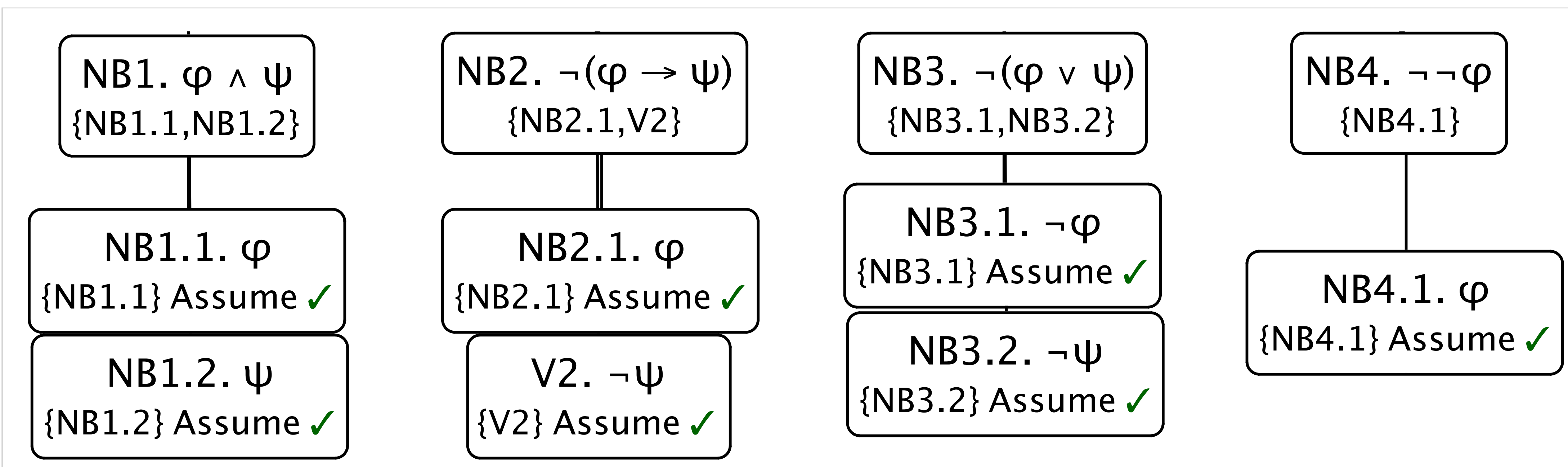


The Rules of the Game!





The Rules of the Game!

The Rules of the Game!

Questions?

Theorem?

Let ϕ be a theorem in the propositional calculus $= \mathcal{L}_{\mathrm{PC}}$.
Then the truth-tree algorithm will lead to no open branches.

Measuring Intelligence & AI/The Singularity

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The Singularity (superhuman machine intelligence) is near!!

Measuring Intelligence & AI/The Singularity

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Is that so? And how are you measuring intelligence, pray tell?

Measuring Intelligence & AI/The Singularity

Is that so? And how are you measuring intelligence, pray tell?

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Polynomial Hierarchy

Measuring Intelligence & AI/The Singularity

Polynomial Hierarchy

$$\mathbf{P} \subseteq \mathbf{NP} \subseteq \mathbf{PSPACE} = \mathbf{NPSPACE} \subseteq \mathbf{EXPTIME} \subseteq \mathbf{NEXPTIME} \subseteq \mathbf{EXPSPACE}$$

Measuring Intelligence & AI/The Singularity

Checkers:Chinook

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Go:AlphaGo

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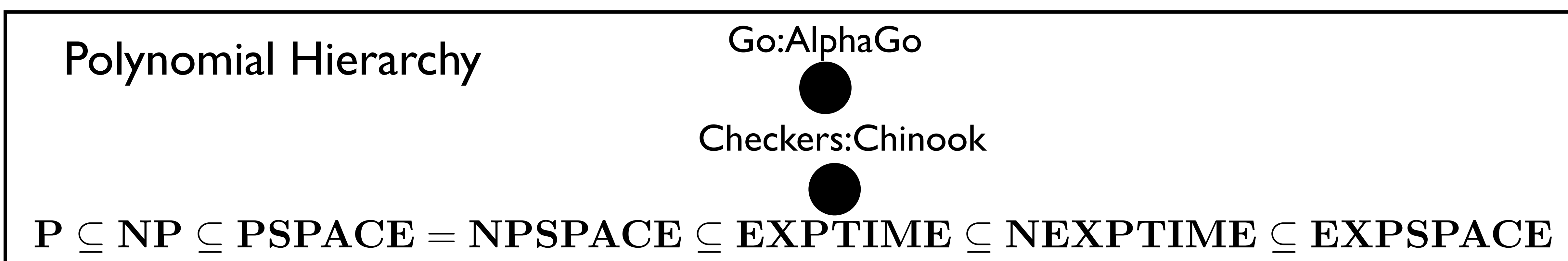
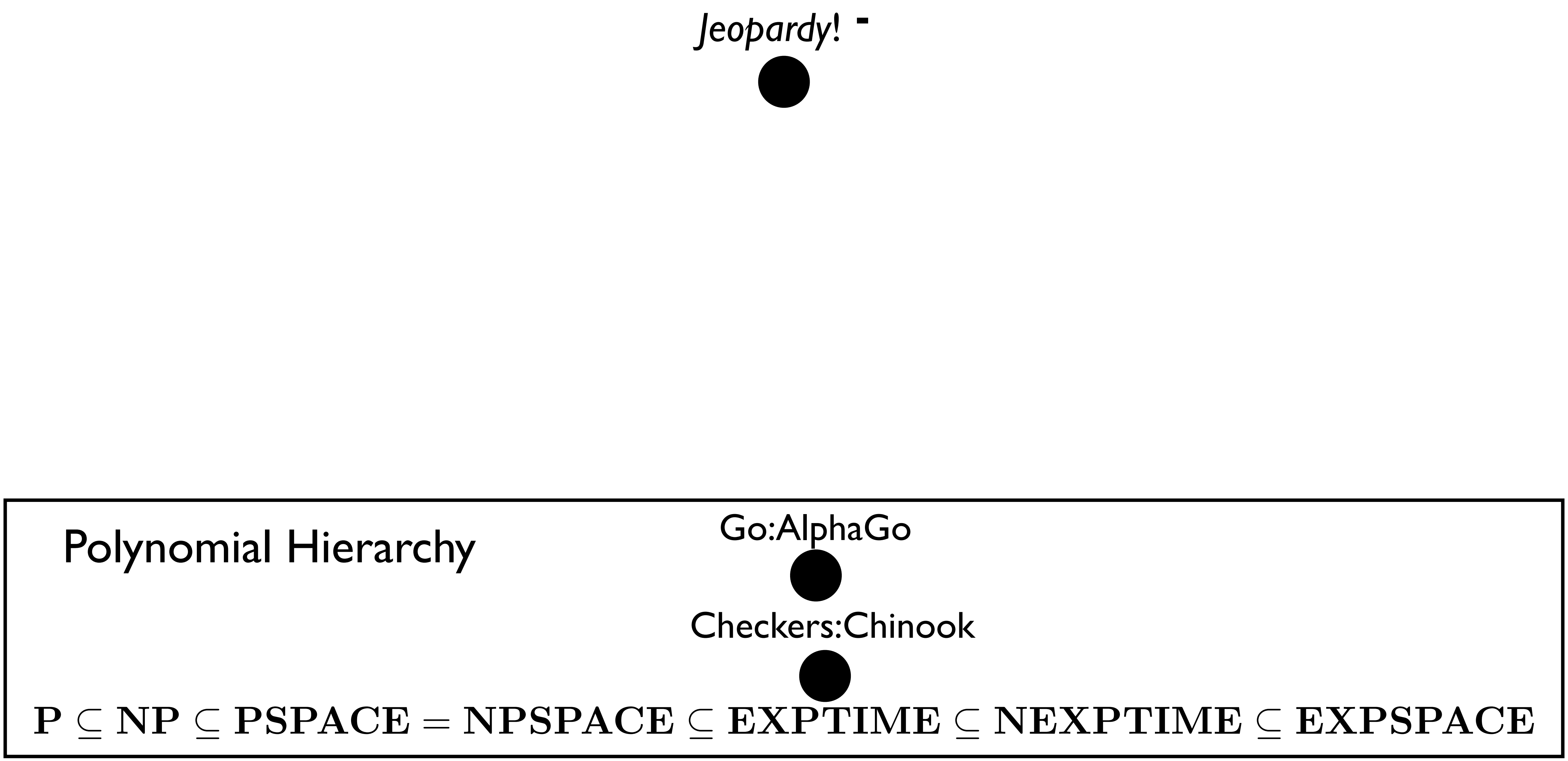


Measuring Intelligence & AI/The Singularity



Measuring Intelligence & AI/The Singularity

Polynomial Hierarchy

Go:AlphaGo

Jeopardy! -

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Measuring Intelligence & AI/The Singularity

Arithmetical Hierarchy

Polynomial Hierarchy

Jeopardy!

Go:AlphaGo

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Measuring Intelligence & AI/The Singularity

Arithmetical Hierarchy

$\vdots$

Π_2

Σ_2

Π_1

Σ_1

Σ_0

Polynomial Hierarchy

Go:AlphaGo

Jeopardy!

Checkers:Chinook

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Measuring Intelligence & AI/The Singularity

Arithmetical Hierarchy

$\vdots$

"Hey, do these two Java programs compute the very same function?" Π_2

Σ_2

Π_1

Σ_1

Σ_0

Polynomial Hierarchy

Go:AlphaGo

Jeopardy! -

Checkers:Chinook

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Measuring Intelligence & AI/The Singularity

Analytical Hierarchy

Arithmetical Hierarchy

$\vdots$

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Σ_2

Π_1

Σ_1

Σ_0

Polynomial Hierarchy

Go:AlphaGo

Jeopardy!

Checkers:Chinook

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Measuring Intelligence & AI/The Singularity

Analytical Hierarchy

Arithmetical Hierarchy

This, all of this, is derived from consideration of first-order logic and second-order logic, with an emphasis on *quantification* and proof.

“Hey, do these two Java programs compute the very same function?”

$$\begin{array}{c} \vdots \\ \Pi_2 \\ \Sigma_2 \\ \Pi_1 \\ \Sigma_1 \\ \Sigma_0 \end{array}$$

 

Polynomial Hierarchy

Go:AlphaGo

Jeopardy! -

Checkers:Chinook

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An “Advanced” Topic for Measuring Intelligence …

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$$\exists x \exists y (x \neq y)$$

An "Advanced" Topic for Measuring Intelligence …

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$\exists x \exists y (x \neq y)$ at least two things

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$\exists x \exists y \exists z (x \neq y \land y \neq z \land x \neq z)$

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$$\vdots$$

$$\underline{\phi_n}$$

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$\underline{\phi_n}$ domain of at least *n* things

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$$\exists x \forall y (y = x)$$

An "Advanced" Topic for Measuring Intelligence …

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$\vdots$

$\underline{\phi_n}$ domain of at least *n* things

$\exists x \forall y (y = x)$ at most one thing

$\exists x \exists y \forall z (z = x \lor z = y)$

An "Advanced" Topic for Measuring Intelligence …

- FOL formulae that (only) enforce domain size:

$$\exists x \exists y (x \neq y) \quad \text{at least two things}$$

$$\exists x \exists y \exists z (x \neq y \land y \neq z \land x \neq z) \quad \text{at least three things}$$

$$\vdots$$

$$\underline{\phi_n} \qquad \text{domain of at least } n \text{ things}$$

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$\vdots$

$\underline{\phi_n}$ domain of at least *n* things

$\exists x \forall y (y = x)$ at most one thing

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$$\vdots$$

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$\vdots$

$\overline{\phi_n}$

CLEVR

https://cs.stanford.edu/people/jcjohns/clevr/

Addition to RAIR-Lab Interoperability for AI …

RAIR
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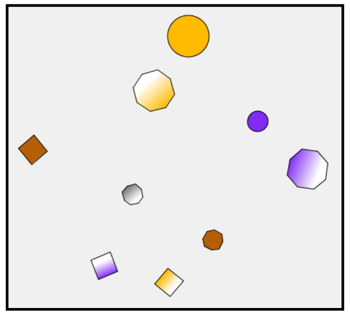
$\mathcal{CC}$

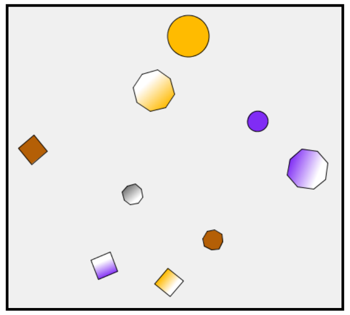
$\cdots$



"AI, are there more than two spheres? Answer & justify."

$\mathscr{C}\mathscr{C}$

...



“AI, are there more than two spheres? Answer & justify.”

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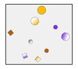

• • •

"AI, are there more than two spheres? Answer & justify."

$\mathcal{CC}$

Φ

$\ldots$

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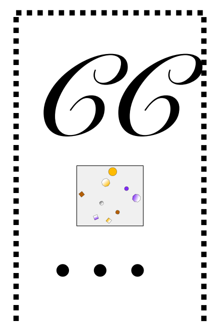

“AI, are there more than two spheres? Answer & justify.”

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U.S. NAVAL RESEARCH LABORATORY

$\mathcal{CC}$

Φ

• • •

"AI, are there more than two spheres? Answer & justify."

$\mathcal{C}\mathcal{C}$

Φ'

$\ldots$

"AI, are there more than two spheres? Answer & justify."

Φ'

$\mathcal{C}\mathcal{C}$

Φ

$\cdots$

"AI, are there more than two spheres? Answer & justify."

AI: "Yes! And here's the proof."

Φ'

$\mathcal{CC}$

Φ

$\cdots$

"AI, are there more than two spheres? Answer & justify."

AI: "Yes! And here's the proof."

So we go from VQA to VQAJV!

Φ'

$\mathcal{CC}$

Φ

$\cdots$

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Measuring AI Intelligence via (in part) Logic:Quantification

Toby Walsh: “The Singularity May Never Be Near”
(http://arxiv.org/pdf/1602.06462v1.pdf)

Measuring AI Intelligence via (in part) Logic:Quantification

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(http://arxiv.org/pdf/1602.06462v1.pdf)

"I will not tackle here head on what we mean by measuring the intelligence of machines (or of humans). I will simply suppose there is such a property as intelligence, that it can be measured and compared, and that the technological singularity is when this measure increases exponentially fast in an appropriate and reasonable scale." (p. 1)

But logico-mathematical definitions of intelligence for animals, humans, machines, aliens, gods … *are* possible; recall our consideration of the *Entscheidungsproblem*. We can specifically challenge today's AI on the basis of simple quantification and simple deduction …

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Given this, is it true that there are two red blookers? Why, exactly?

$$
\begin{aligned}
&\forall x \forall y \forall z\ \{[x \neq y \wedge y \neq z \wedge x \neq z \wedge Cx \wedge Cy \wedge Cz\ \wedge \\
&Tz' \wedge \\
&\exists w_1 \exists w_2\ (w_1 \neq w_2 \wedge Aw_1 \wedge Aw_2 \wedge Gz'w_1 \wedge Gz'w_2)\ \wedge \\
&\forall u_1 \forall u_2 \forall u_3\ ((Gz'u_1 \wedge Gz'u_2 \wedge Gz'u_3 \wedge C^b u_1 \wedge C^b u_2 \wedge C^b u_3) \rightarrow \\
&\forall v((Gz'v \wedge C^b v) \rightarrow (v = u_1 \vee v = u_2 \vee v = u_3)))] \\
&\rightarrow \\
&(Gxz' \wedge Gyz' \wedge Gzz')\}
\end{aligned}
$$

Every three cylinders glower at any triangular prism that glowers at at least two arches and at at most three cubes.

$$
\forall x \forall y \forall z \forall z' \left\{
\begin{array}{c}
\left[
\begin{array}{c}
x \neq y \wedge y \neq z \wedge x \neq z \\
\wedge \\
Cx \wedge Cy \wedge Cz \\
\wedge \\
Tz' \\
\wedge \\
\exists w_1 \exists w_2 \ (w_1 \neq w_2 \wedge Aw_1 \wedge Aw_2 \wedge Gz'w_1 \wedge Gz'w_2) \\
\wedge \\
\forall u_1 \forall u_2 \forall u_3 \ \left(
\begin{array}{c}
\left[Gz'u_1 \wedge Gz'u_2 \wedge Gz'u_3 \wedge C^b u_1 \wedge C^b u_2 \wedge C^b u_3 \right] \\
\rightarrow \\
\forall v \left[(Gz'v \wedge C^b v) \rightarrow (v = u_1 \vee v = u_2 \vee v = u_3) \right]
\end{array}
\right)
\end{array}
\right] \\
\rightarrow \\
(Gxz' \wedge Gyz' \wedge Gzz')
\end{array}
\right\}
$$