Propositional Calculus I:

The Formal Language,
The Prop. Calc. Oracle (= AI),
Application to Some Motivating Problems

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Intro to (Formal) Logic
1/30/2020

How'd We Arrive Here?

(Selmer's Leibnizian Whirlwind History of Logic)

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Intro to Logic
1/27/2020

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Questions …?

Intro to Logic
1/27/2020

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Logistics …

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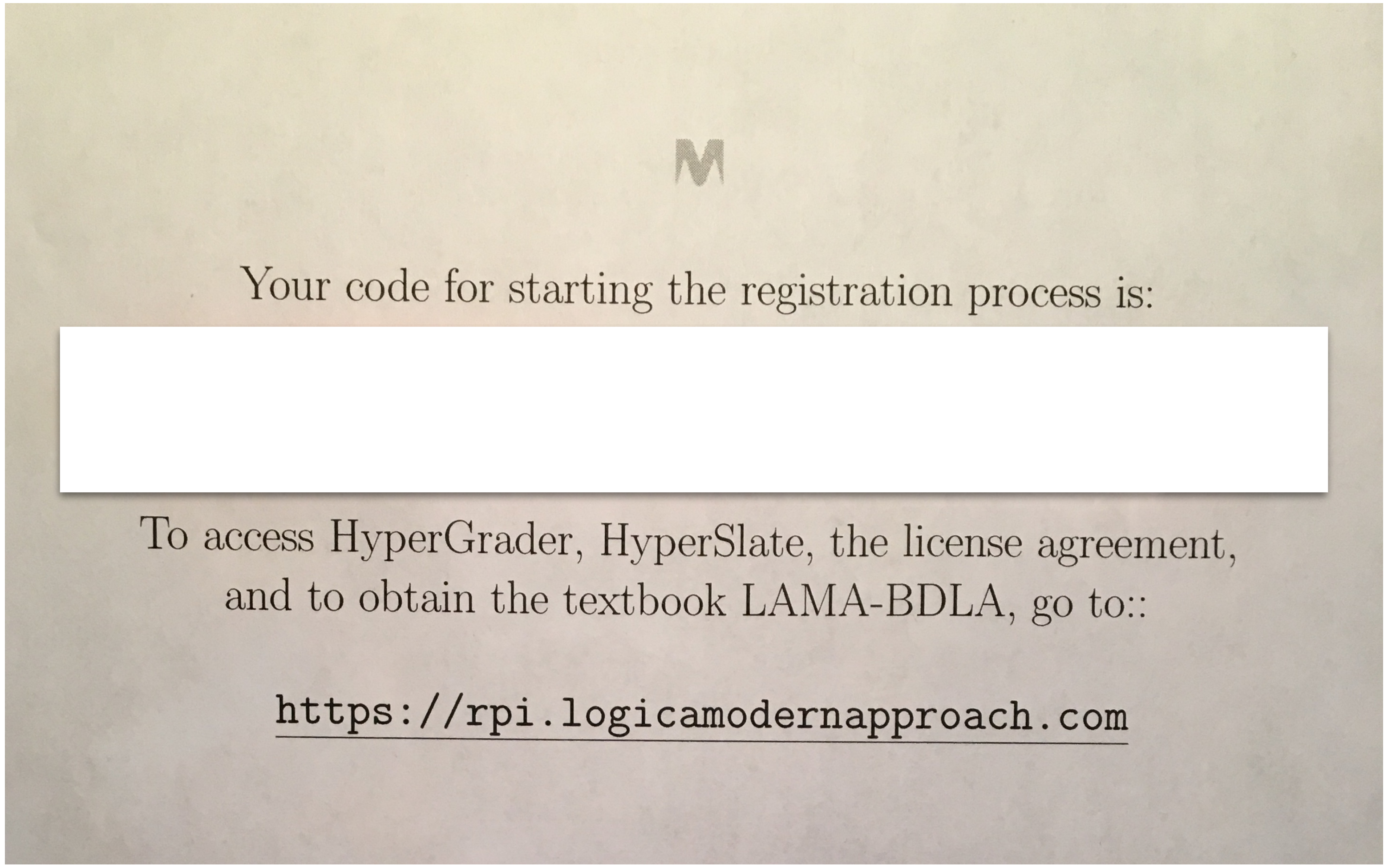
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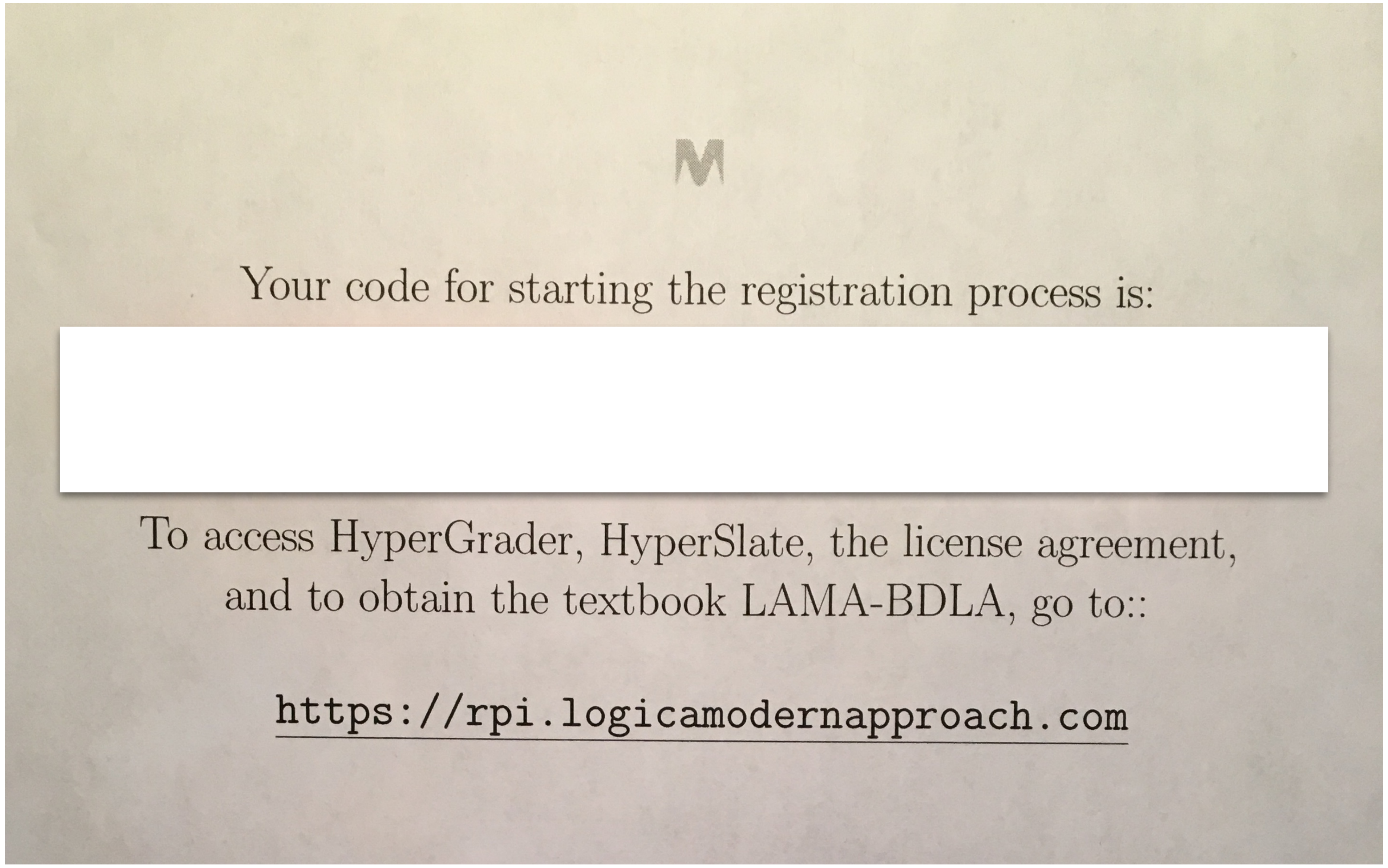
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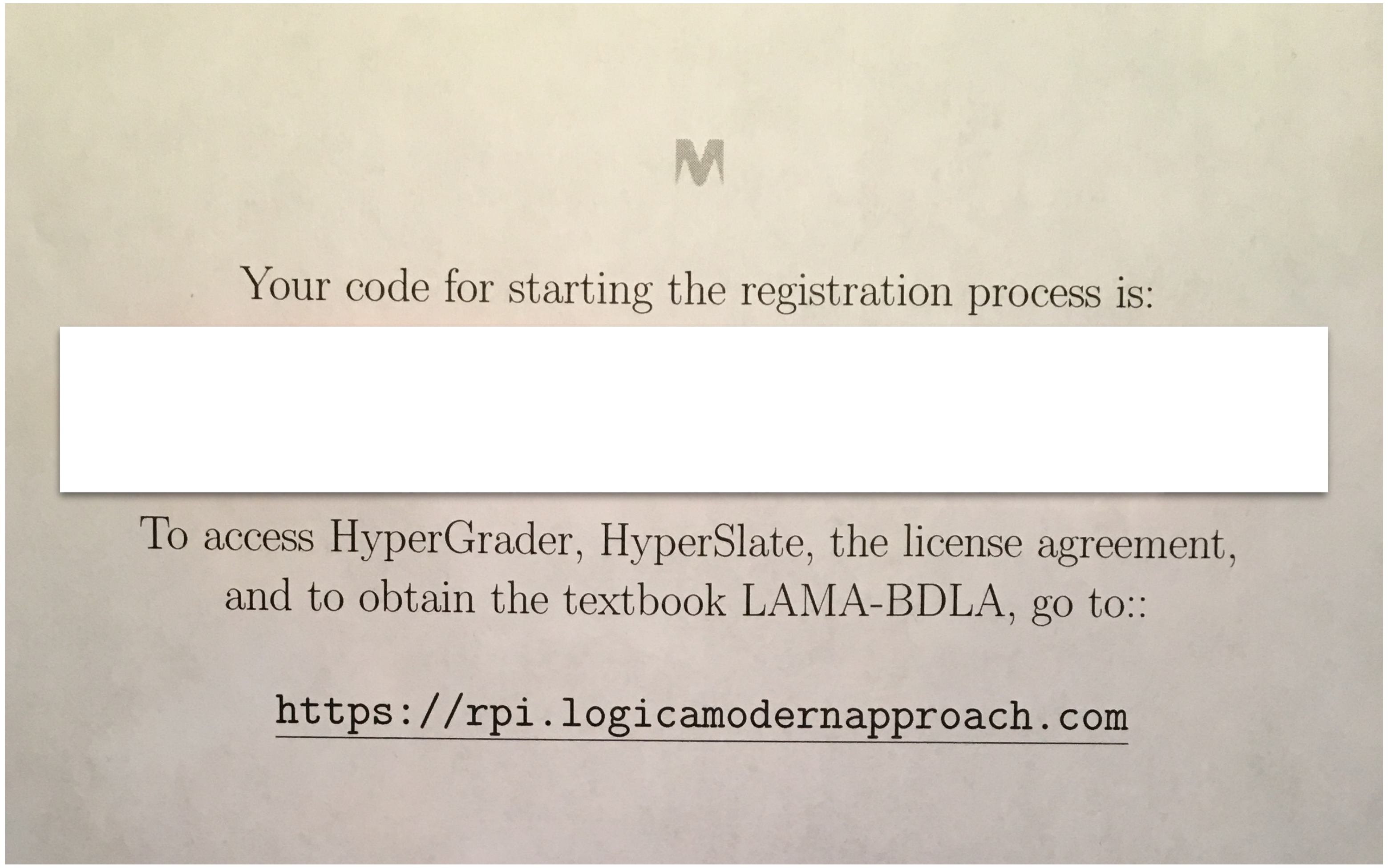
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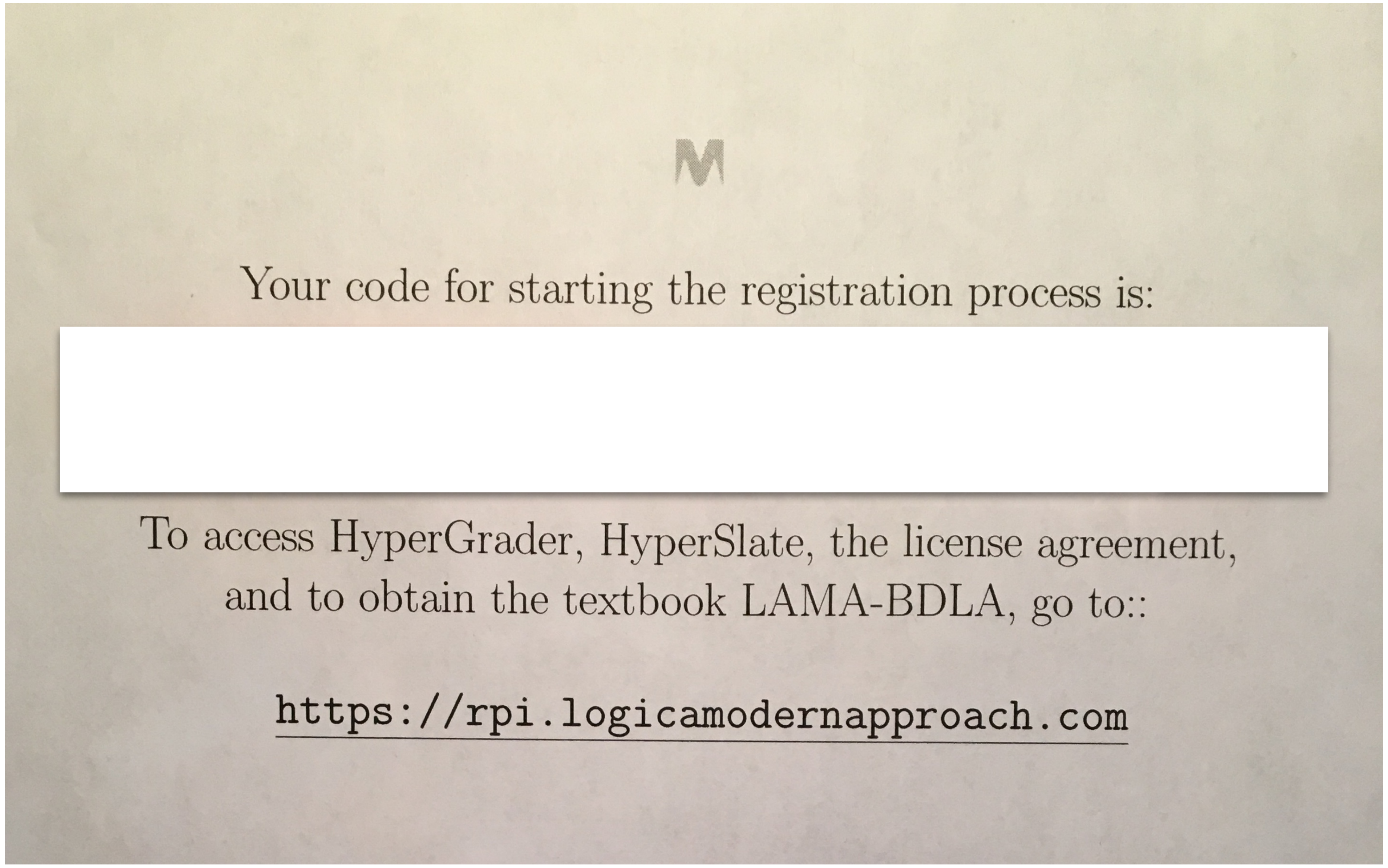
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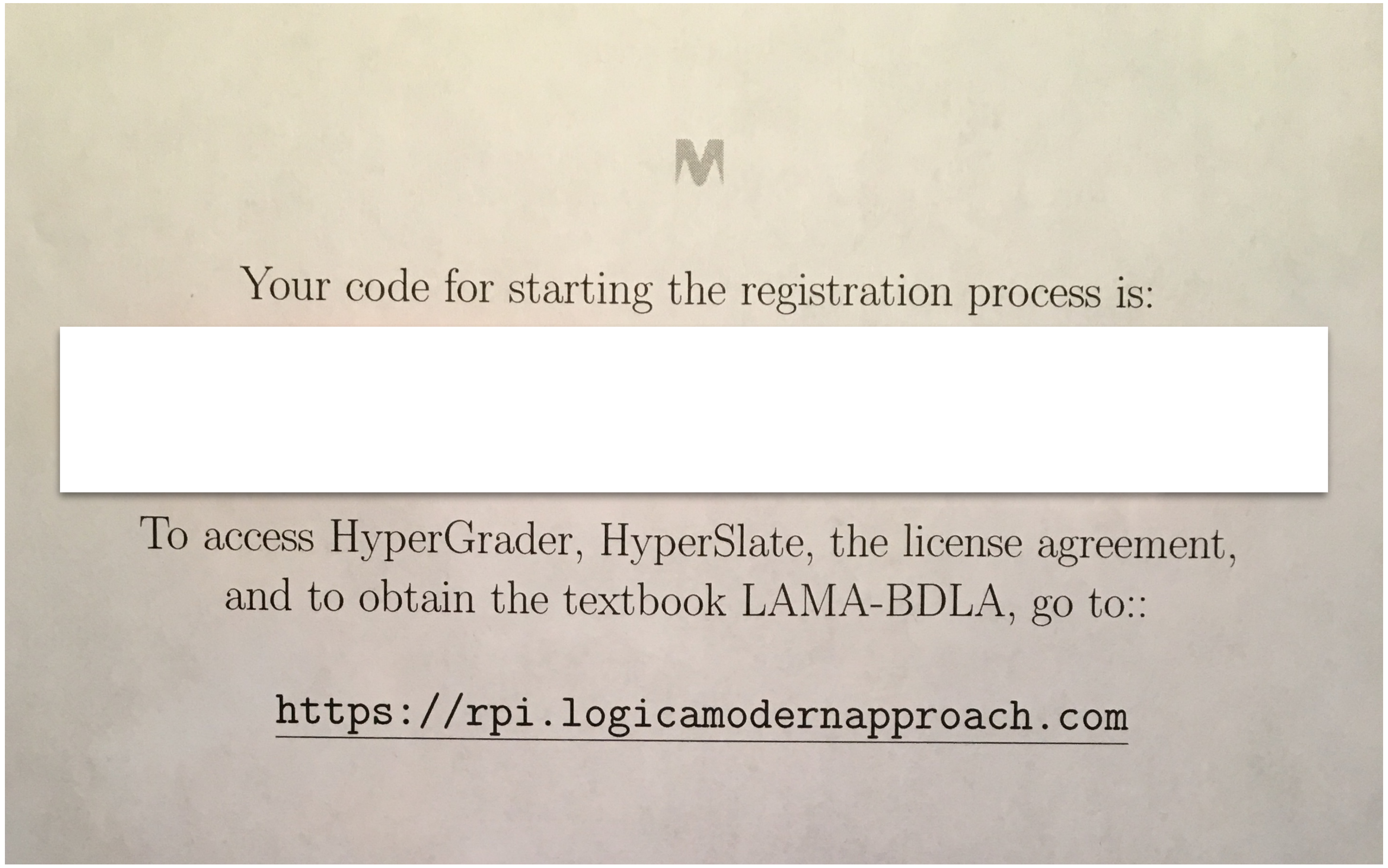
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Watch that the link doesn't end up being classified as spam.

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Intro to Logic
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Micro-homily:

Micro-homily:

skipping to ~ p. 34!

Micro-homily:

skipping to ~ p. 34!

Micro-homily:

skipping to ~ p. 34!

M. Chi: Self-testers end up being self-made.

Micro-homily:

skipping to ~ p. 34!

M. Chi: Self-testers end up being self-made.

Micro-homily:

skipping to ~ p. 34!

M. Chi: Self-testers end up being self-made.

"What category of English sentences does logic focus on?"

The Formal Language

CHAPTER 2. PROPOSITIONAL CALCULUS

Syntax	**Formula Type**	**Sample Representation**
$\mathsf{P}, \mathsf{P}_1, \mathsf{P}_2, \mathsf{Q}, \mathsf{Q}_1, \ldots$	Atomic Formulas	"Larry is lucky." as L_l
$\neg\phi$	Negation	"Gary isn't lucky." as $\neg\mathsf{L}_g$
$\phi_1 \wedge \ldots \wedge \phi_n$	Conjunction	"Both Larry and Carl are lucky." as $\mathsf{L}_l \wedge \mathsf{L}_c$
$\phi_1 \vee \ldots \vee \phi_n$	Disjunction	"Either Billy is lucky or Alvin is." as $\mathsf{L}_b \vee \mathsf{L}_a$
$\phi \rightarrow \psi$	Conditional (Implication)	"If Ron is lucky, so is Frank." as $\mathsf{L}_r \rightarrow L_f$
$\phi \leftrightarrow \psi$	Biconditional (Coimplication)	"Tim is lucky if and only if Kim is." as $\mathsf{L}_t \longleftrightarrow L_k$

Table 2.1: Syntax of the Propositional Calculus. Note that ϕ, ψ, and ϕ_i stand for arbitrary formulas.

The Formal Language

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Table 2.1: Syntax of the Propositional Calculus. Note that ϕ, ψ, and ϕ_i stand for arbitrary formulas.

Exercise: Is this language Roger-decidable? Prove it!

The Formal Language
(presented as formal grammar)

$$Formula \Rightarrow AtomicFormula$$

$$\mid (Formula\ Connective\ Formula)$$

$$\mid \neg\ Formula$$

$$AtomicFormula \Rightarrow \mathtt{P}_1 \mid \mathtt{P}_2 \mid \mathtt{P}_3 \mid \ldots$$

$$Connective \Rightarrow \wedge \mid \vee \mid \rightarrow \mid \leftrightarrow$$

The Formal Language

(presented as formal grammar)

$$
\begin{array}{lcl}
\mathit{Formula} & \Rightarrow & \mathit{AtomicFormula} \\
 & \mid & (\mathit{Formula}\ \ \mathit{Connective}\ \ \mathit{Formula}) \\
 & \mid & \neg\ \mathit{Formula}
\end{array}
$$

$$
\mathit{AtomicFormula} \Rightarrow \mathrm{P}_1 \mid \mathrm{P}_2 \mid \mathrm{P}_3 \mid \ldots
$$

$$
\mathit{Connective} \Rightarrow \wedge \mid \vee \mid \rightarrow \mid \leftrightarrow
$$

Exercise: Is this language Roger-decidable? Prove it!

As S-expressions

Formula	$\Rightarrow$	*AtomicFormula*
	$\mid$	(*Formula Connective Formula*)
	$\mid$	$\neg$ *Formula*
AtomicFormula	$\Rightarrow$	$\mathtt{P}_1 \mid \mathtt{P}_2 \mid \mathtt{P}_3 \mid \ldots$
Connective	$\Rightarrow$	$\wedge \mid \vee \mid \rightarrow \mid \leftrightarrow$

As S-expressions

$Formula$	$\Rightarrow$	$AtomicFormula$
	$\mid$	$(Formula\ Connective\ Formula)$
	$\mid$	$\neg\ Formula$
$AtomicFormula$	$\Rightarrow$	$\texttt{P}_1 \mid \texttt{P}_2 \mid \texttt{P}_3 \mid \ldots$
$Connective$	$\Rightarrow$	$\land \mid \lor \mid \rightarrow \mid \leftrightarrow$

As S-expressions

Formula	$\Rightarrow$	*AtomicFormula*
	$\mid$	$($*Formula Connective Formula*$)$
	$\mid$	$\neg$ *Formula*
AtomicFormula	$\Rightarrow$	$\mathtt{P}_1 \mid \mathtt{P}_2 \mid \mathtt{P}_3 \mid \ldots$
Connective	$\Rightarrow$	$\wedge \mid \vee \mid \rightarrow \mid \leftrightarrow$

p

As S-expressions

$$
\begin{array}{lcl}
\textit{Formula} & \Rightarrow & \textit{AtomicFormula} \\
 & \mid & (\textit{Formula}\ \textit{Connective}\ \textit{Formula}) \\
 & \mid & \neg\ \textit{Formula}
\end{array}
$$

$$
\textit{AtomicFormula} \Rightarrow \texttt{P}_1 \mid \texttt{P}_2 \mid \texttt{P}_3 \mid \ldots
$$

$$
\textit{Connective} \Rightarrow \wedge \mid \vee \mid \rightarrow \mid \leftrightarrow
$$

p P

As S-expressions

$\mathit{Formula}$	$\Rightarrow$	$\mathit{AtomicFormula}$
	$\mid$	$(\mathit{Formula}\ \ \mathit{Connective}\ \ \mathit{Formula})$
	$\mid$	$\neg\ \mathit{Formula}$
$\mathit{AtomicFormula}$	$\Rightarrow$	$\mathtt{P}_1 \mid \mathtt{P}_2 \mid \mathtt{P}_3 \mid \ldots$
$\mathit{Connective}$	$\Rightarrow$	$\wedge \mid \vee \mid \rightarrow \mid \leftrightarrow$

`p`   `P`   `bradyisleaving`

As S-expressions

$$
\begin{array}{lll}
\textit{Formula} & \Rightarrow & \textit{AtomicFormula} \\
 & | & (\textit{Formula Connective Formula}) \\
 & | & \neg\ \textit{Formula}
\end{array}
$$

$$\textit{AtomicFormula} \Rightarrow \mathtt{P}_1 \mid \mathtt{P}_2 \mid \mathtt{P}_3 \mid \ldots$$

$$\textit{Connective} \Rightarrow \wedge \mid \vee \mid \rightarrow \mid \leftrightarrow$$

`p   P   bradyisleaving   P26`

As S-expressions

$$
\begin{array}{lcl}
\mathit{Formula} & \Rightarrow & \mathit{AtomicFormula} \\
 & | & (\mathit{Formula}\ \ \mathit{Connective}\ \ \mathit{Formula}) \\
 & | & \neg\ \mathit{Formula} \\
\hline
\mathit{AtomicFormula} & \Rightarrow & \mathtt{P}_1 \mid \mathtt{P}_2 \mid \mathtt{P}_3 \mid \ldots \\
\hline
\mathit{Connective} & \Rightarrow & \wedge \mid \vee \mid \rightarrow \mid \leftrightarrow
\end{array}
$$

`p   P   bradyisleaving   P26   ...`

As S-expressions

Formula	$\Rightarrow$	*AtomicFormula*
	$\mid$	(*Formula* *Connective* *Formula*)
	$\mid$	$\neg$ *Formula*
AtomicFormula	$\Rightarrow$	$\mathtt{P}_1 \mid \mathtt{P}_2 \mid \mathtt{P}_3 \mid \ldots$
Connective	$\Rightarrow$	$\wedge \mid \vee \mid \rightarrow \mid \leftrightarrow$

`p   P   bradyisleaving   P26   ...`

As S-expressions

Formula	$\Rightarrow$	*AtomicFormula*
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AtomicFormula	$\Rightarrow$	$\texttt{P}_1 \mid \texttt{P}_2 \mid \texttt{P}_3 \mid \ldots$
Connective	$\Rightarrow$	$\wedge \mid \vee \mid \rightarrow \mid \leftrightarrow$

`p   P   bradyisleaving   P26   ...`

`(not p)`

As S-expressions

Formula	$\Rightarrow$	*AtomicFormula*
	$\mid$	(*Formula Connective Formula*)
	$\mid$	$\neg$ *Formula*
AtomicFormula	$\Rightarrow$	$\texttt{P}_1 \mid \texttt{P}_2 \mid \texttt{P}_3 \mid \ldots$
Connective	$\Rightarrow$	$\wedge \mid \vee \mid \rightarrow \mid \leftrightarrow$

```
p   P   bradyisleaving   P26   ...

(not p)  (not P)
```

As S-expressions

$$
\begin{array}{lcl}
\mathit{Formula} & \Rightarrow & \mathit{AtomicFormula} \\
 & | & (\mathit{Formula}\ \mathit{Connective}\ \mathit{Formula}) \\
 & | & \neg\ \mathit{Formula} \\
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\mathit{AtomicFormula} & \Rightarrow & \mathtt{P}_1 \mid \mathtt{P}_2 \mid \mathtt{P}_3 \mid \ldots \\
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\mathit{Connective} & \Rightarrow & \wedge \mid \vee \mid \rightarrow \mid \leftrightarrow
\end{array}
$$

```
p   P   bradyisleaving   P26   ...

(not p)  (not P)  (not P26)
```

As S-expressions

Formula	$\Rightarrow$	*AtomicFormula*
	$\mid$	$(Formula\ Connective\ Formula)$
	$\mid$	$\neg\ Formula$
AtomicFormula	$\Rightarrow$	$\mathtt{P}_1 \mid \mathtt{P}_2 \mid \mathtt{P}_3 \mid \ldots$
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```
p   P   bradyisleaving   P26   ...

(not p)  (not P)  (not P26)    ...
```

As S-expressions

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\hline
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\end{array}
$$

`p   P   bradyisleaving   P26   ...`

`(not p)  (not P)  (not P26)   ...`

As S-expressions

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\textit{Formula} & \Rightarrow & \textit{AtomicFormula} \\
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\end{array}
$$

```
p   P   bradyisleaving   P26   ...

(not p)  (not P)  (not P26)    ...

(and P Q)
```

As S-expressions

$$
\begin{array}{lcl}
\textit{Formula} & \Rightarrow & \textit{AtomicFormula} \\
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\end{array}
$$

```
p   P   bradyisleaving   P26   ...

(not p)  (not P)  (not P26)    ...

(and P Q)  (or P Q)
```

As S-expressions

$$
\begin{array}{lcl}
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\mathit{Connective} & \Rightarrow & \wedge \mid \vee \mid \rightarrow \mid \leftrightarrow
\end{array}
$$

```
p   P   bradyisleaving   P26   ...

(not p)  (not P)  (not P26)    ...

(and P Q)  (or P Q) (if P Q)
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As S-expressions

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\textit{Formula} & \Rightarrow & \textit{AtomicFormula} \\
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\end{array}
$$

```
p   P   bradyisleaving   P26   ...

(not p)  (not P)  (not P26)   ...

(and P Q)  (or P Q)  (if P Q)  (iff P Q)
```

Better Formal Language: Pure Predicate Calculus (presented via formal grammar)

$$
\begin{array}{lcl}
\textit{Formula} & \Rightarrow & \textit{AtomicFormula} \\
 & \mid & (\textit{Formula}\ \textit{Connective}\ \textit{Formula}) \\
 & \mid & \neg\ \textit{Formula}
\end{array}
$$

$$
\textit{AtomicFormula} \Rightarrow (\textit{Predicate}\ \textit{Term}_1 \ldots \textit{Term}_k)
$$

$$
\begin{array}{lcl}
\textit{Term} & \Rightarrow & (\textit{Function}\ \textit{Term}_1\ \ldots\ \textit{Term}_k) \\
 & \mid & \textit{Constant} \\
 & \mid & \textit{Variable}
\end{array}
$$

$$
\textit{Connective} \Rightarrow \wedge \mid \vee \mid \rightarrow \mid \leftrightarrow
$$

$$
\begin{array}{lcl}
\textit{Predicate} & \Rightarrow & P_1 \mid P_2 \mid P_3\ \ldots \\
\textit{Constant} & \Rightarrow & c_1 \mid c_2 \mid c_3\ \ldots \\
\textit{Variable} & \Rightarrow & v_1 \mid v_2 \mid v_3\ \ldots \\
\textit{Function} & \Rightarrow & f_1 \mid f_2 \mid f_3\ \ldots
\end{array}
$$

Better Formal Language: Pure Predicate Calculus (presented via formal grammar)

Formula	$\Rightarrow$	*AtomicFormula*
	$\mid$	($Formula\ Connective\ Formula$)
	$\mid$	$\neg$ *Formula*
AtomicFormula	$\Rightarrow$	$(Predicate\ Term_1 \ldots Term_k)$
Term	$\Rightarrow$	$(Function\ Term_1\ \ldots\ Term_k)$
	$\mid$	*Constant*
	$\mid$	*Variable*
Connective	$\Rightarrow$	$\wedge \mid \vee \mid \rightarrow \mid \leftrightarrow$
Predicate	$\Rightarrow$	$P_1 \mid P_2 \mid P_3\ \ldots$
Constant	$\Rightarrow$	$c_1 \mid c_2 \mid c_3\ \ldots$
Variable	$\Rightarrow$	$v_1 \mid v_2 \mid v_3\ \ldots$
Function	$\Rightarrow$	$f_1 \mid f_2 \mid f_3\ \ldots$

Exercise: Is this language also Roger-decidable? Prove it!

“NYS 1” Revisited

Given the statements

$\neg a \vee \neg b$

b

$c \rightarrow a$

which one of the following statements must also be true?

c

$\neg b$

$\neg c$

h

a

none of the above

"NYS 1" Revisited

Given the statements

$\neg a \vee \neg b$

b

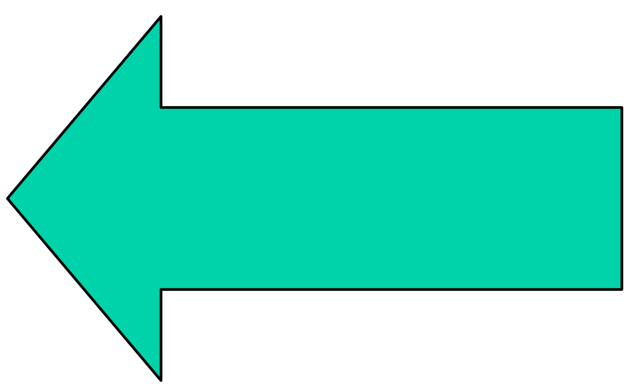
$c \rightarrow a$

which one of the following statements must also be true?

c

$\neg b$

$\neg c$

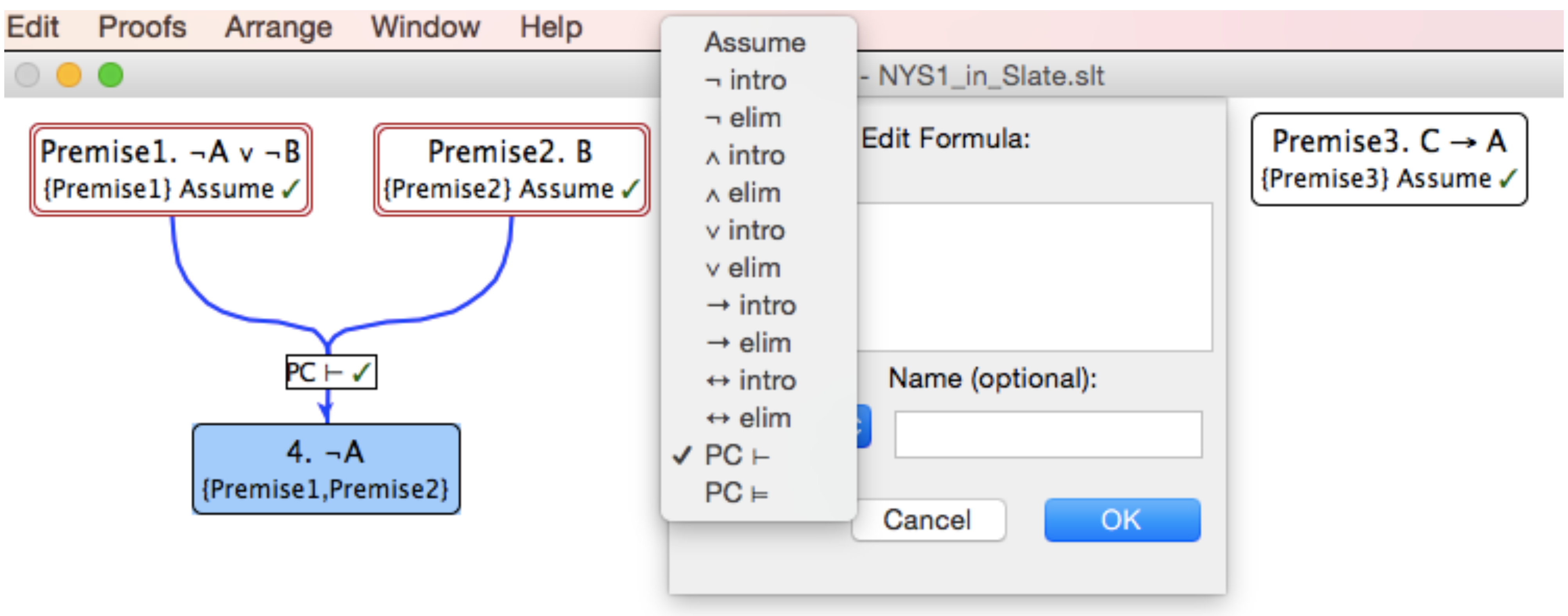


h

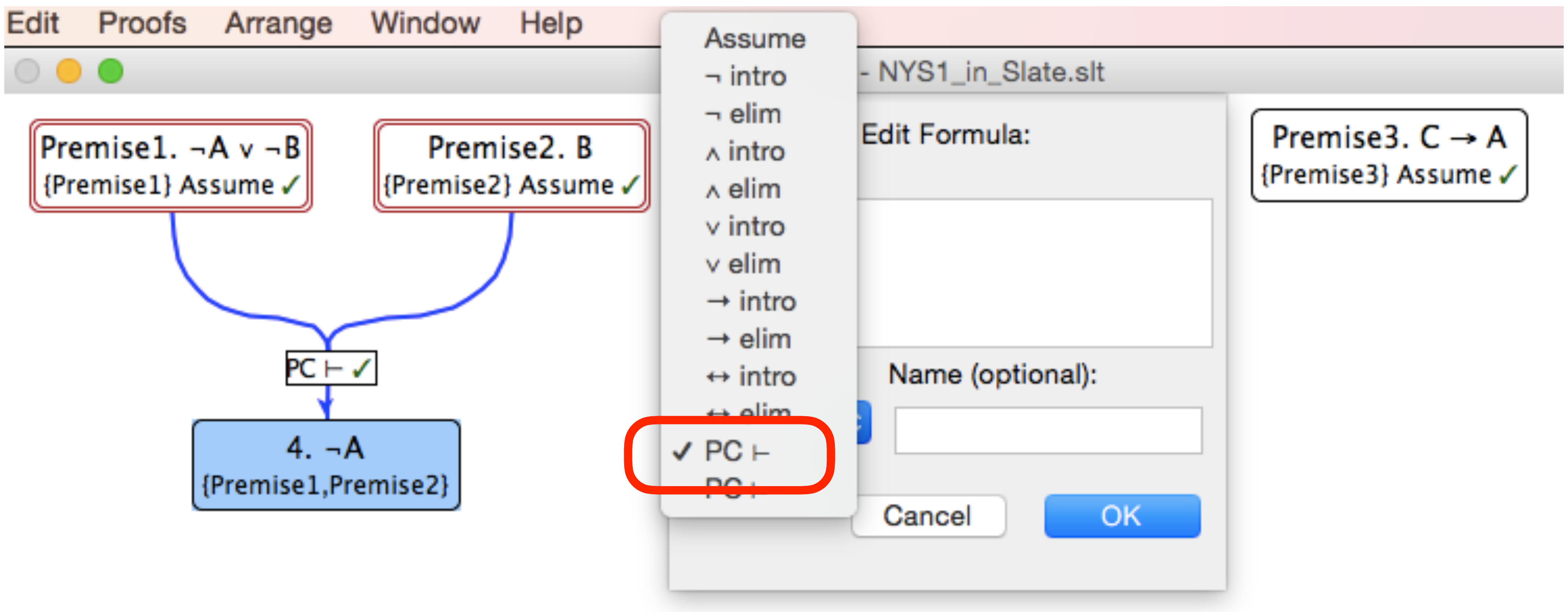
a

none of the above

Our First Rule of Inference: PC (Entailment) Oracle



Our First Rule of Inference: PC (Entailment) Oracle



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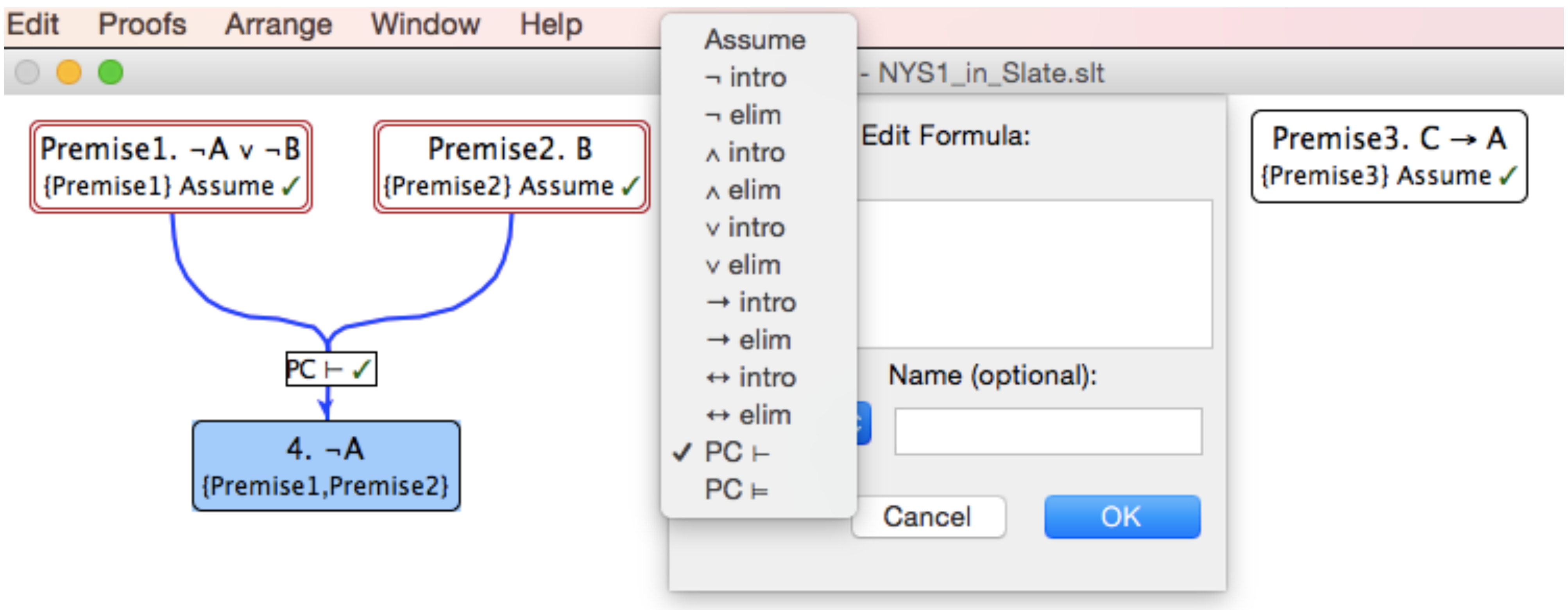

Our First Rule of Inference: PC (Entailment) Oracle

Our First Rule of Inference: PC (Entailment) Oracle

“NYS 3” Revisited

Given the statements

$\neg\neg c$

$c \rightarrow a$

$\neg a \vee b$

$b \rightarrow d$

$\neg(d \vee e)$

which one of the following statements must also be true?

$\neg c$

e

h

$\neg a$

all of the above

"NYS 3" Revisited

Given the statements

$\neg\neg c$

$c \rightarrow a$

$\neg a \vee b$

$b \rightarrow d$

$\neg(d \vee e)$

which one of the following statements must also be true?

$\neg c$

e

h

$\neg a$

all of the above

“NYS 3” Revisited

Given the statements

$\neg\neg c$

$c \rightarrow a$

$\neg a \vee b$

$b \rightarrow d$

$\neg(d \vee e)$

Show in <u>HyperSlate</u> that each of the first four options can be proved using the PC entailment oracle.

which one of the following statements must also be true?

$\neg c$

e

h

$\neg a$

all of the above