Propositional Calculus II:

Two more Rules of Inference/Inference Schemata (`conditional elim` = *modus ponens*, *proof by cases*), Application to Additional Motivating Problems

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Troy, New York 12180 USA

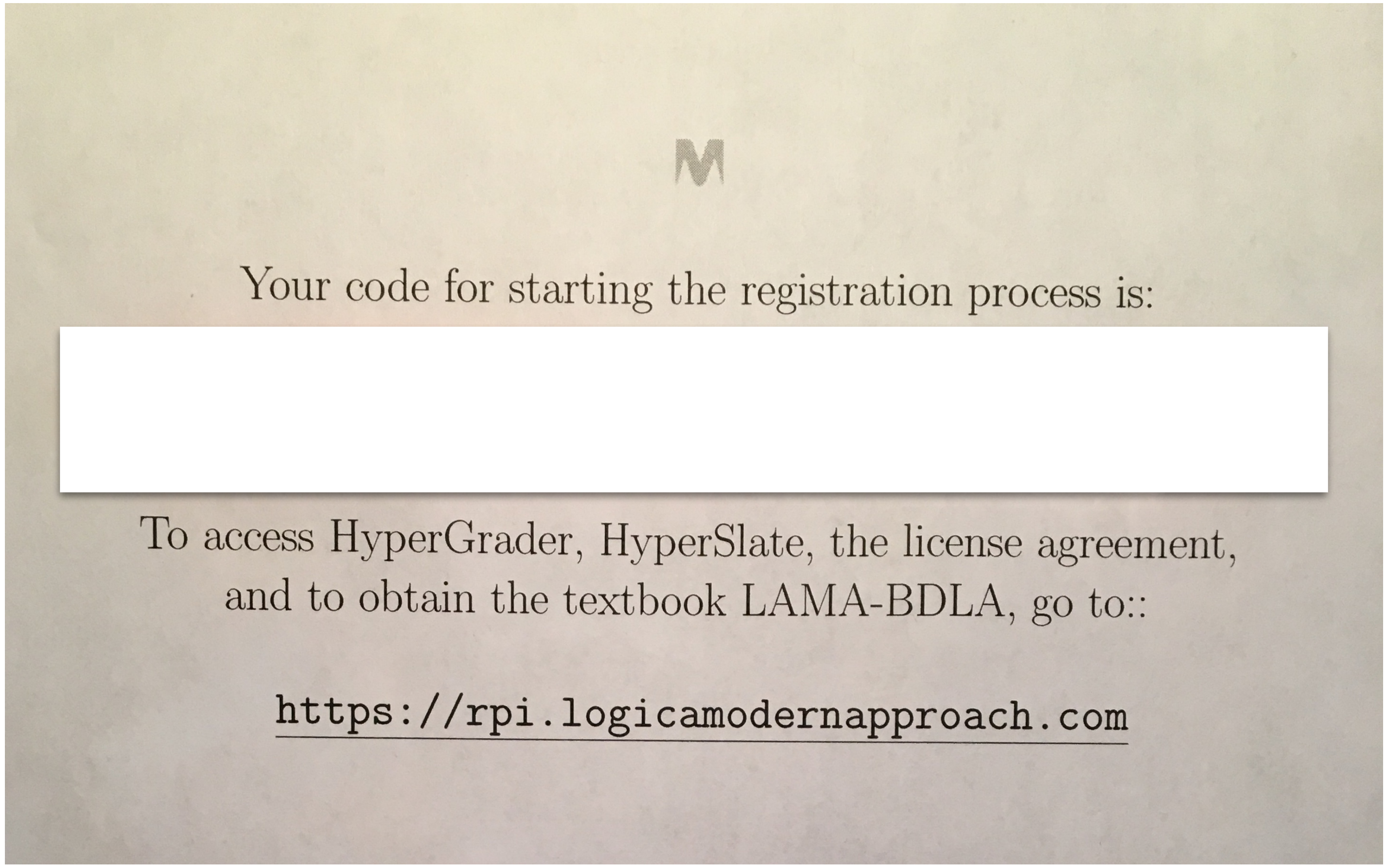
Intro to Logic
2/3/2020

R A I R
Rensselaer AI and Reasoning Lab

No, it's *not* logical to throw a bomb from the Kansas City 49-yard line @ 3rd and 10, with 1:49 to go.

Re-re-re…orientation w.r.t. web pages …

The Starting Code to Purchase in Bookstore


Your code for starting the registration process is:

To access HyperGrader, HyperSlate, the license agreement, and to obtain the textbook LAMA-BDLA, go to::

https://rpi.logicamodernapproach.com

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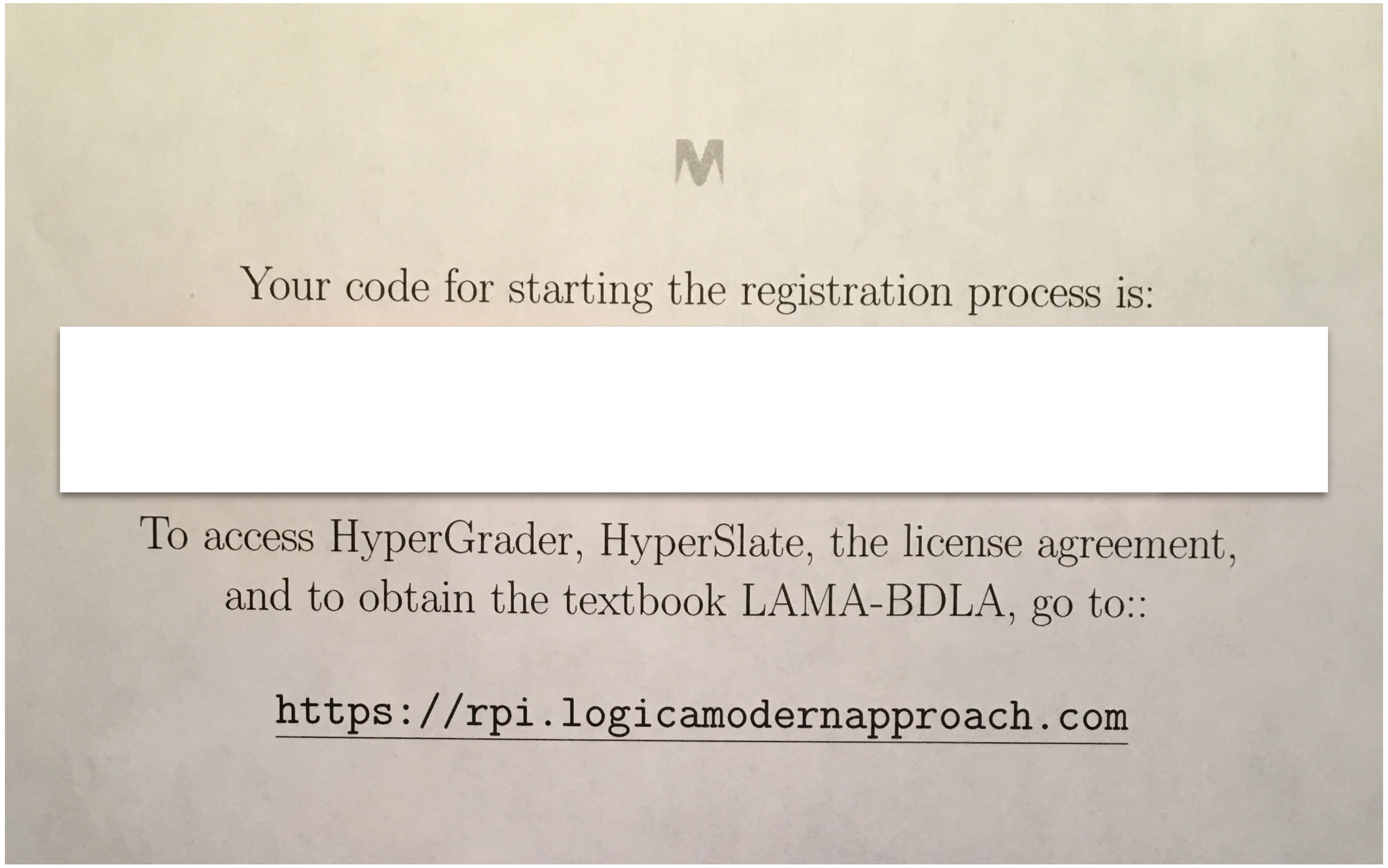
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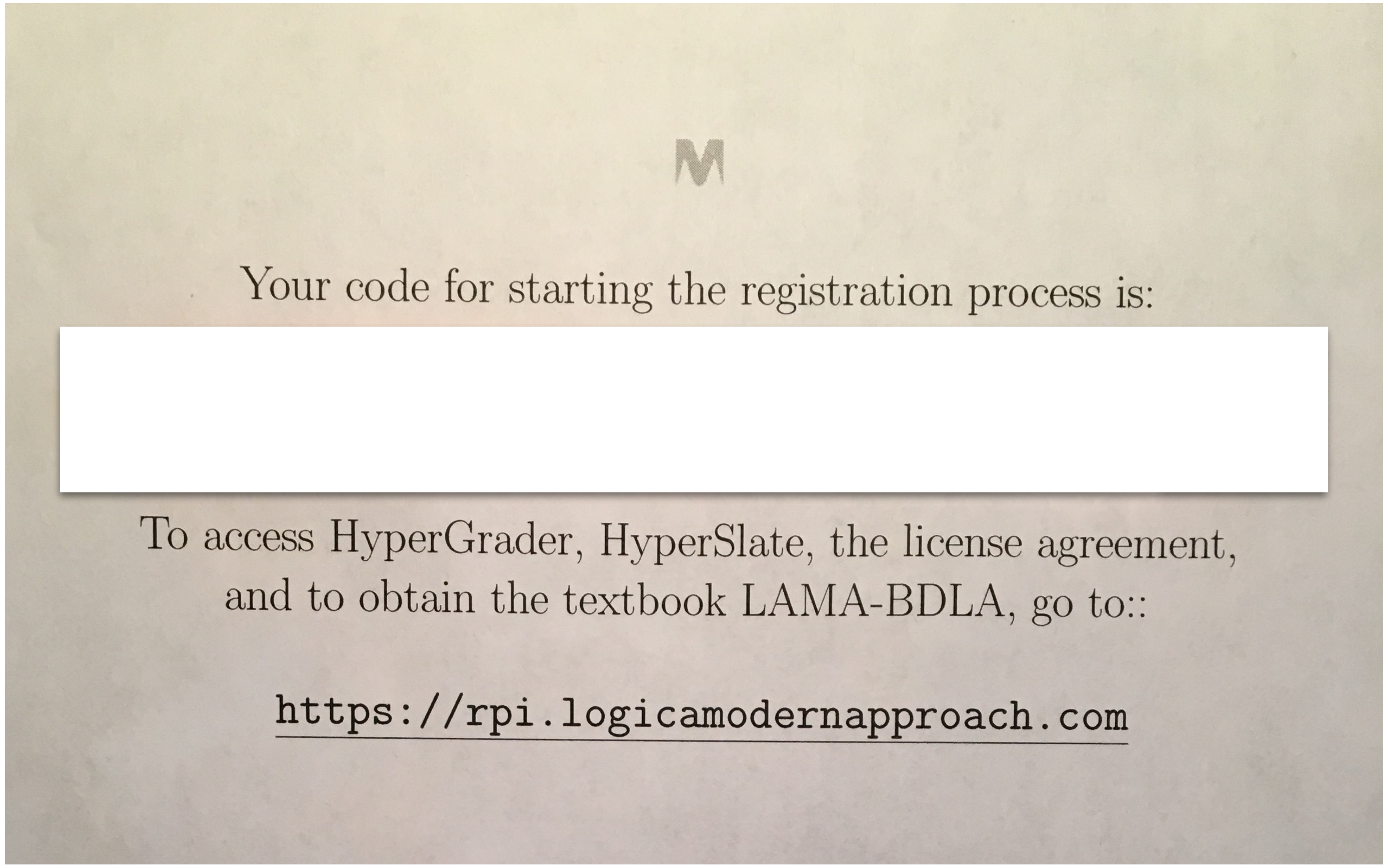

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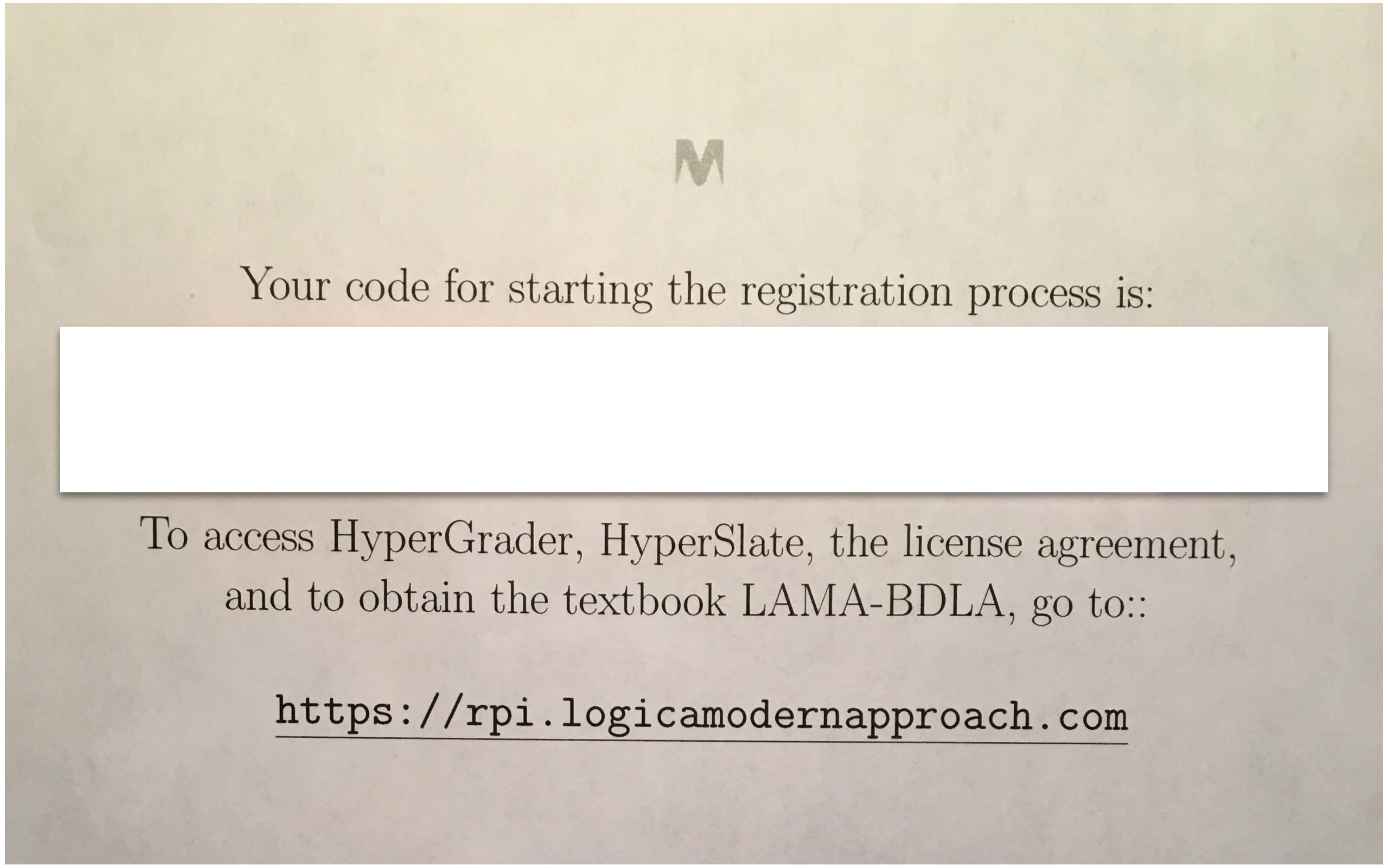

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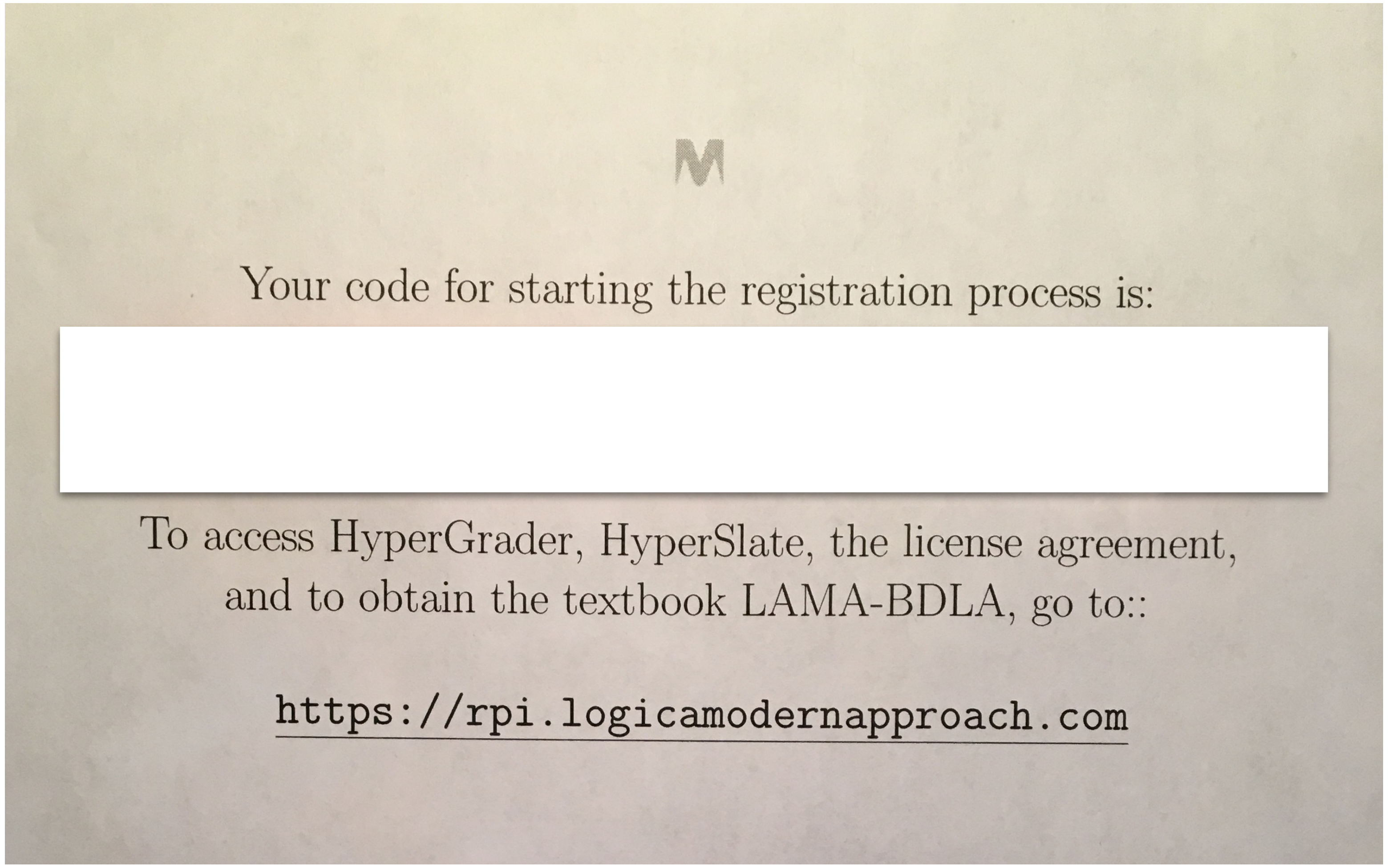
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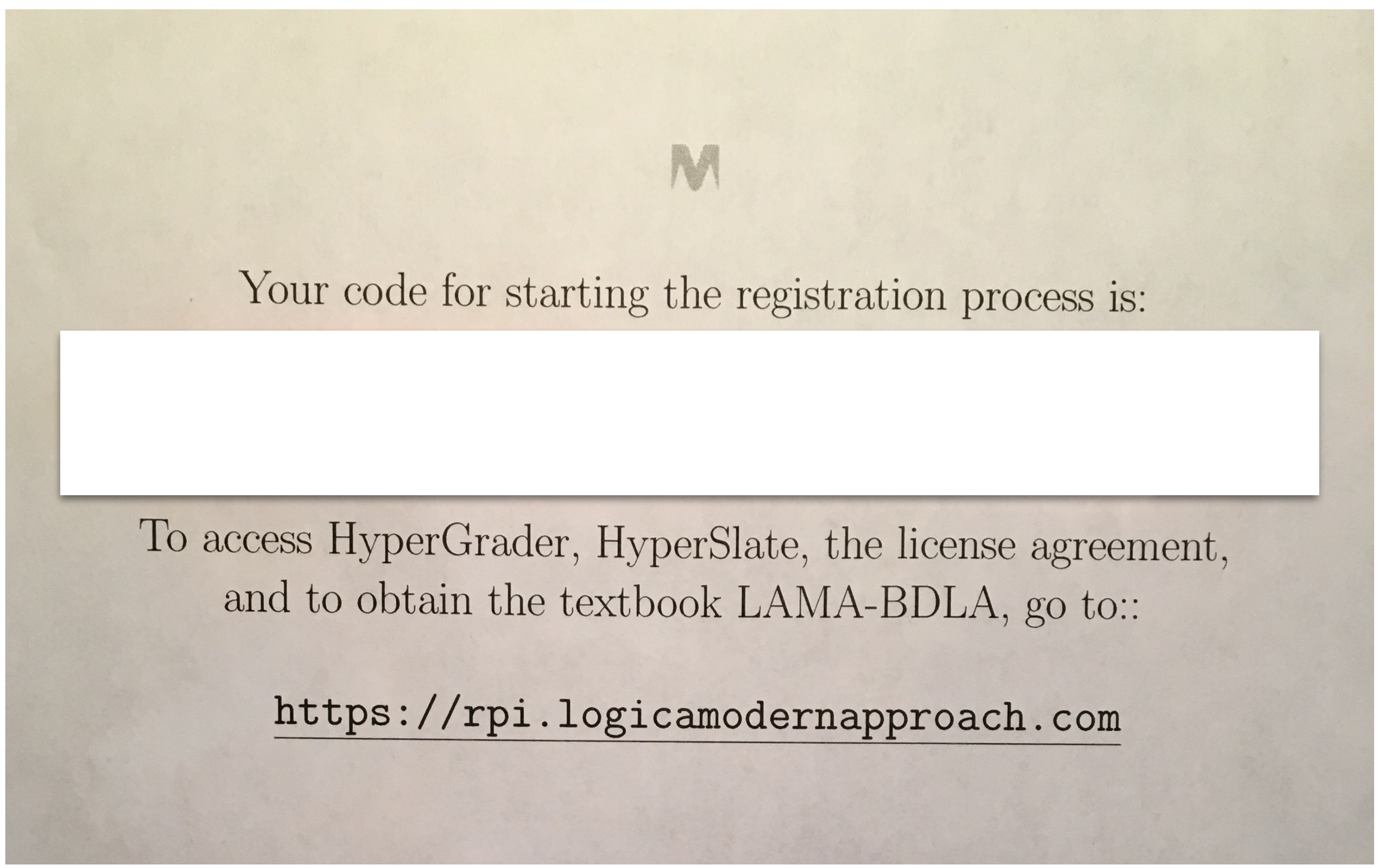
Watch that the link doesn't end up being classified as spam.

The Starting Code Purchased in Bookstore Should By Now've Been Used to Register & Subsequently Sign In

First batch of prop. calc. (Homework) Problems in; easiest starting place: `switching_conjuncts_fine`.


M

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E-Housekeeping Pts, Redux

E-Housekeeping Pts, Redux

- Must input your RIN.

E-Housekeeping Pts, Redux

- Must input your RIN.
- Make sure OS fully up-to-date.

E-Housekeeping Pts, Redux

- Must input your RIN.
- Make sure OS fully up-to-date.
- Make sure browser fully up-to-date.

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- Must input your RIN.
- Make sure OS fully up-to-date.
- Make sure browser fully up-to-date.
- Chrome best (but I use Safari).

E-Housekeeping Pts, Redux

- Must input your RIN.
- Make sure OS fully up-to-date.
- Make sure browser fully up-to-date.
- Chrome best (but I use Safari).
- Always work in the same browser window with multiple tabs; must do this with email and HyperGrader™ & HyperSlate™.

Propositional Calculus II:

More Rules of Inference

(`conditional elim` = *modus ponens*, *proof by cases*),

Application to Additional Motivating Problems

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Last time we introduced and and lauded the power of **oracles**, and questions … and now … picking up where we left off …

"NYS 3" Revisited

Given the statements

$\neg\neg c$

$c \rightarrow a$

$\neg a \vee b$

$b \rightarrow d$

$\neg(d \vee e)$

which one of the following statements must also be true?

$\neg c$

e

h

$\neg a$

all of the above

"NYS 3" Revisited

Given the statements

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$\neg(d \vee e)$

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e

h

$\neg a$

all of the above

“NYS 3” Revisited

Given the statements

$\neg\neg c$

$c \rightarrow a$

$\neg a \vee b$

$b \rightarrow d$

$\neg(d \vee e)$

After last class, should have done … Exercise: Show in HyperSlate™ that each of the first four options can be proved using the PC entailment oracle.

which one of the following statements must also be true?

$\neg c$

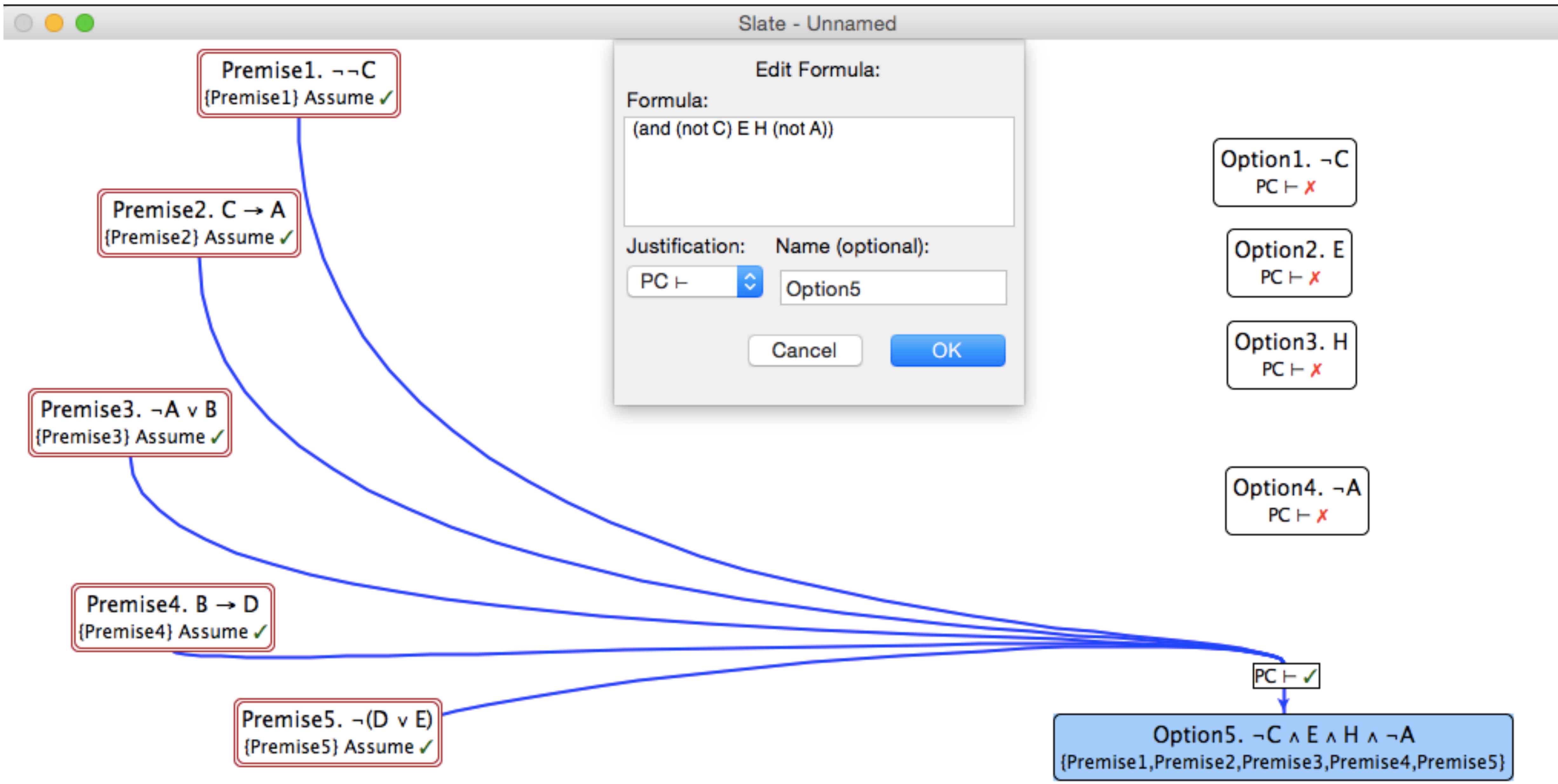
e

h

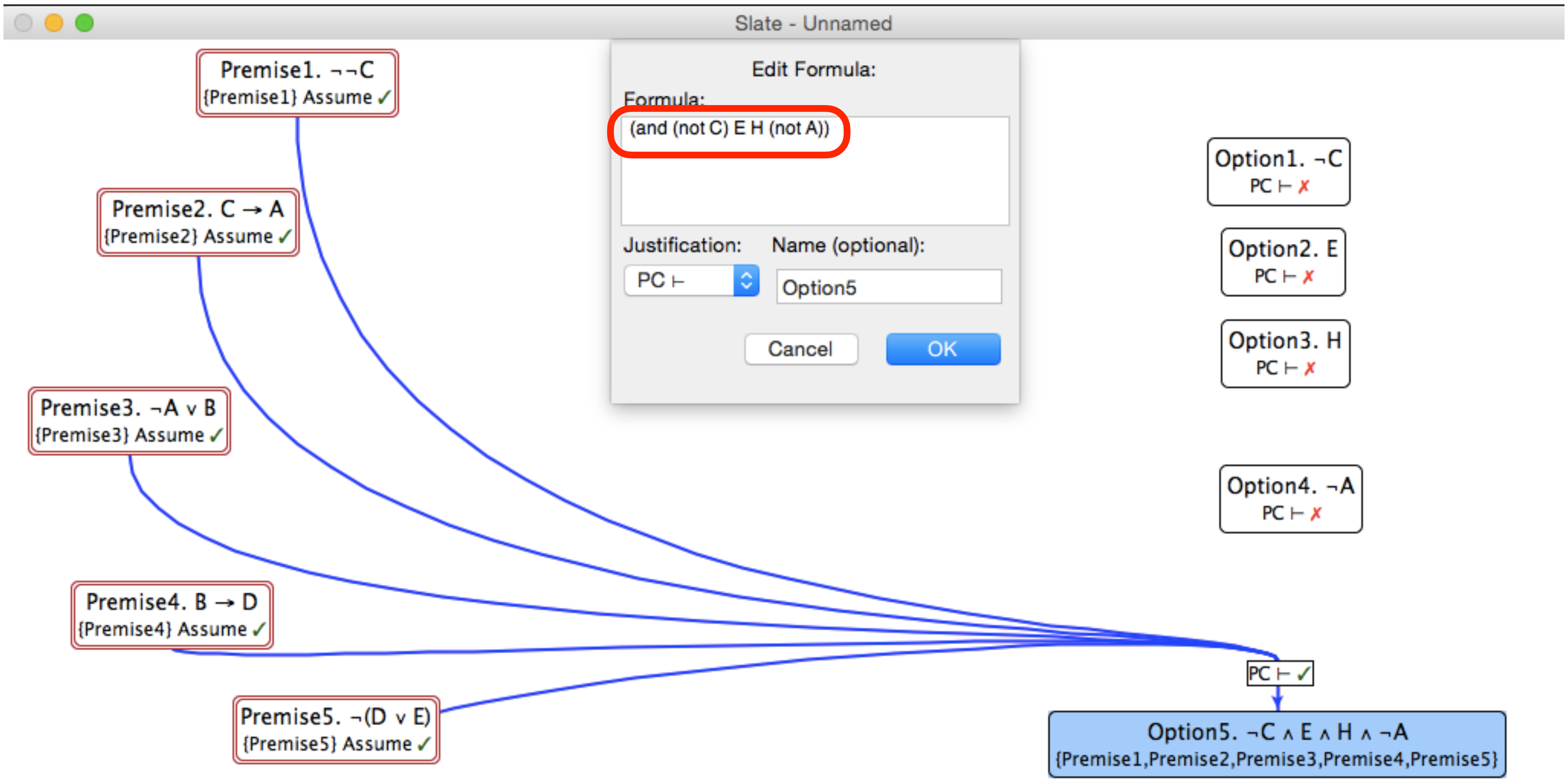
$\neg a$

all of the above

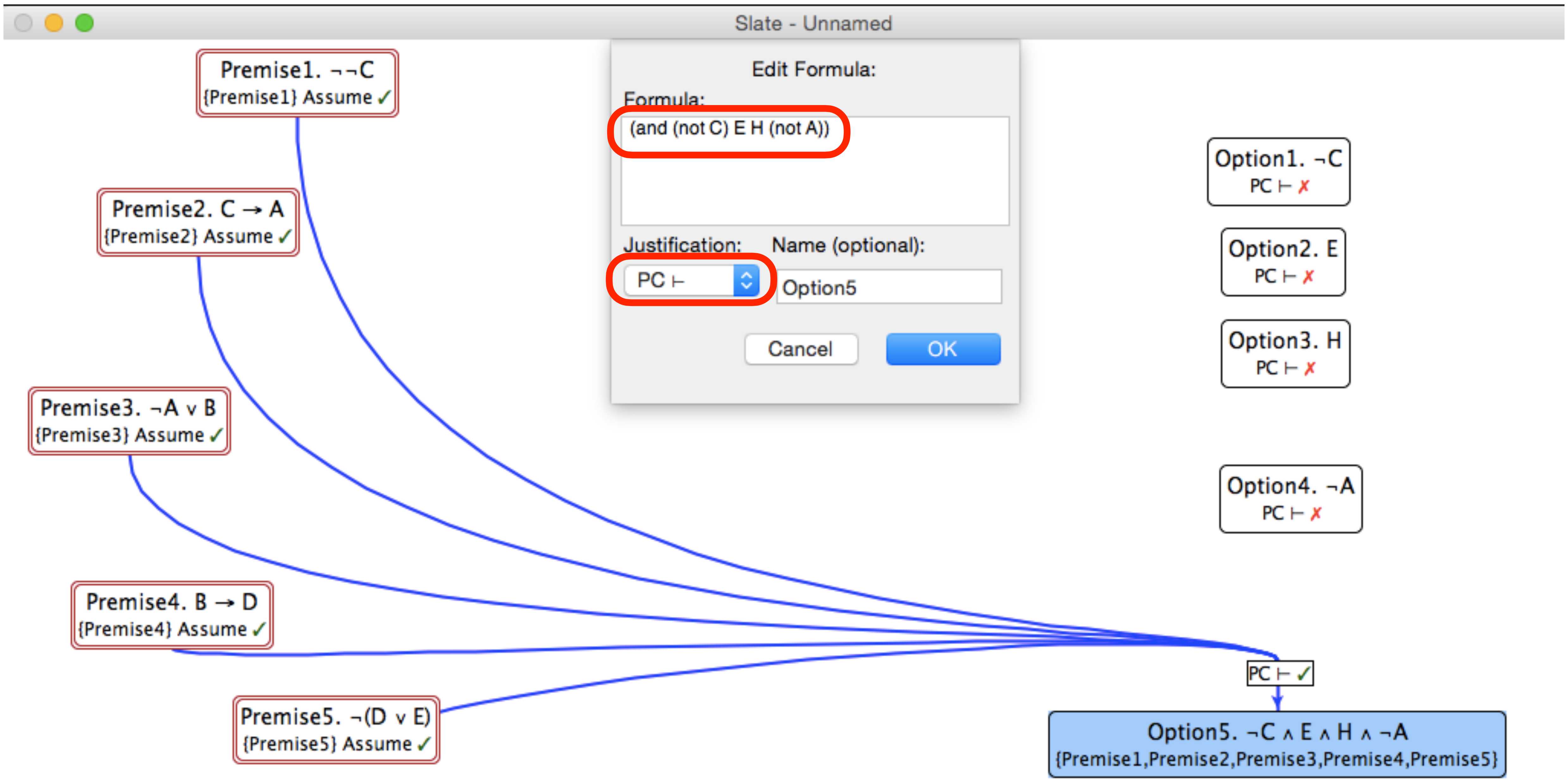
(and (not C) E H (not A))



(and (not C) E H (not A))



(and (not C) E H (not A))



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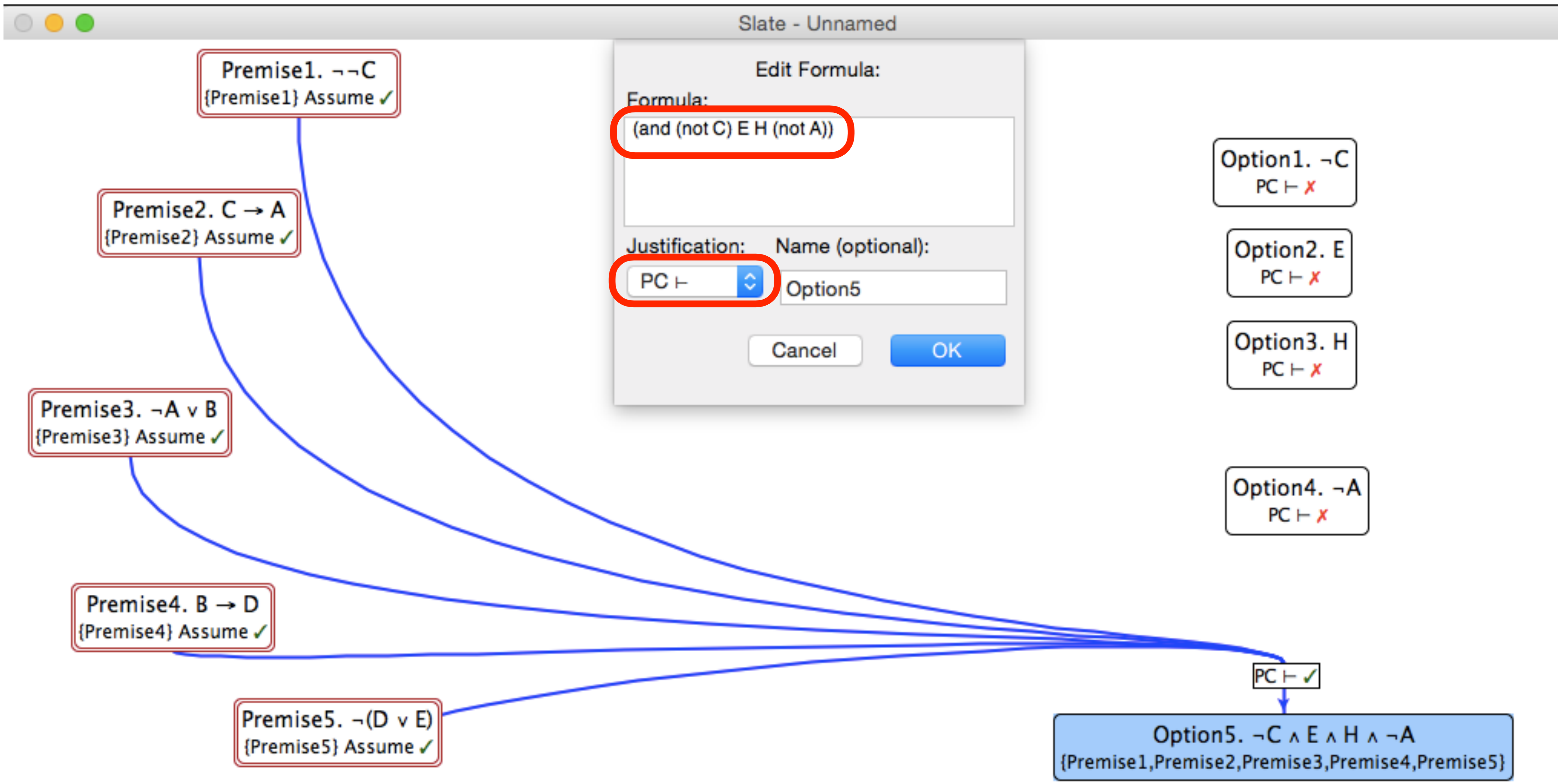

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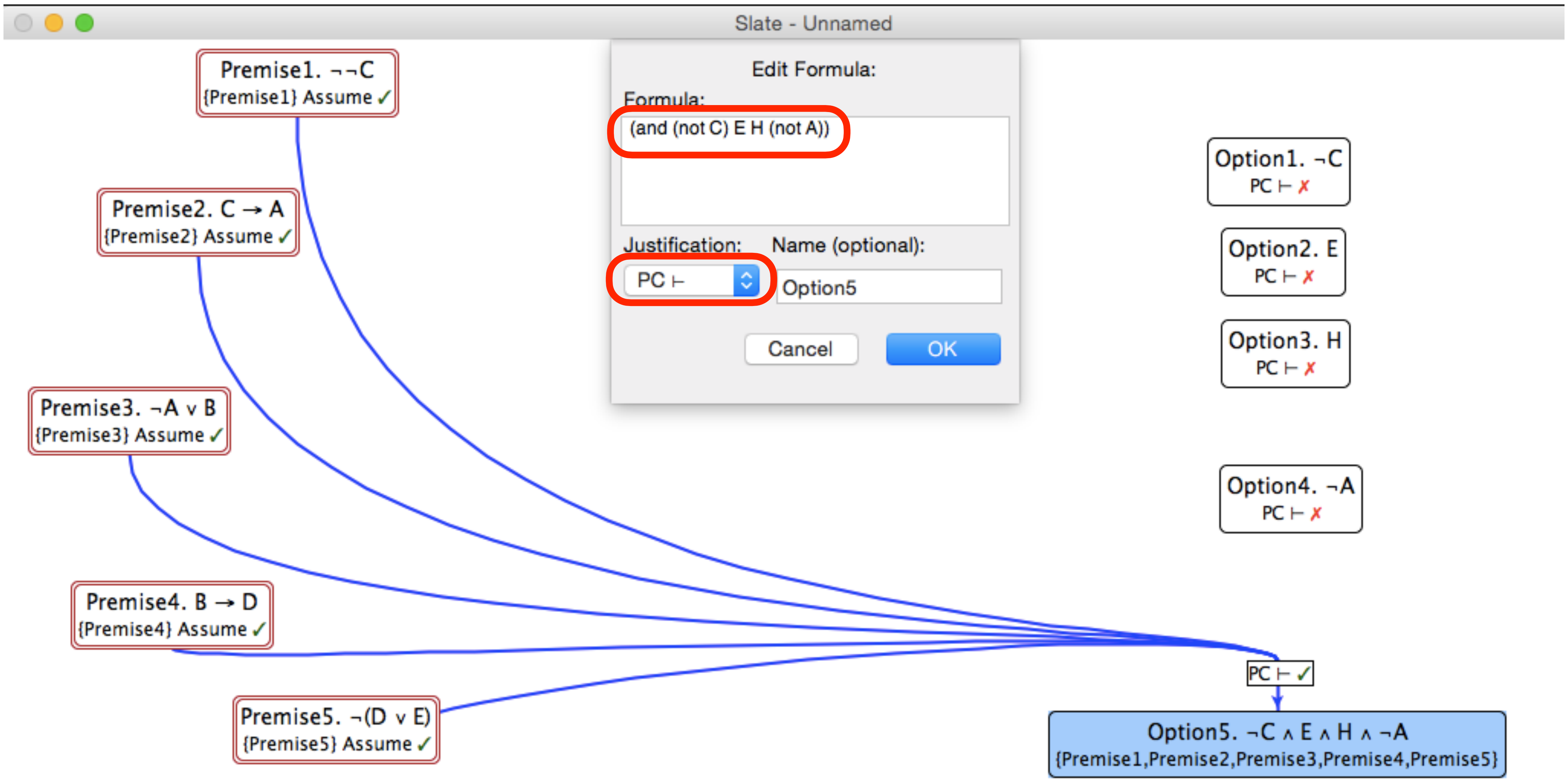

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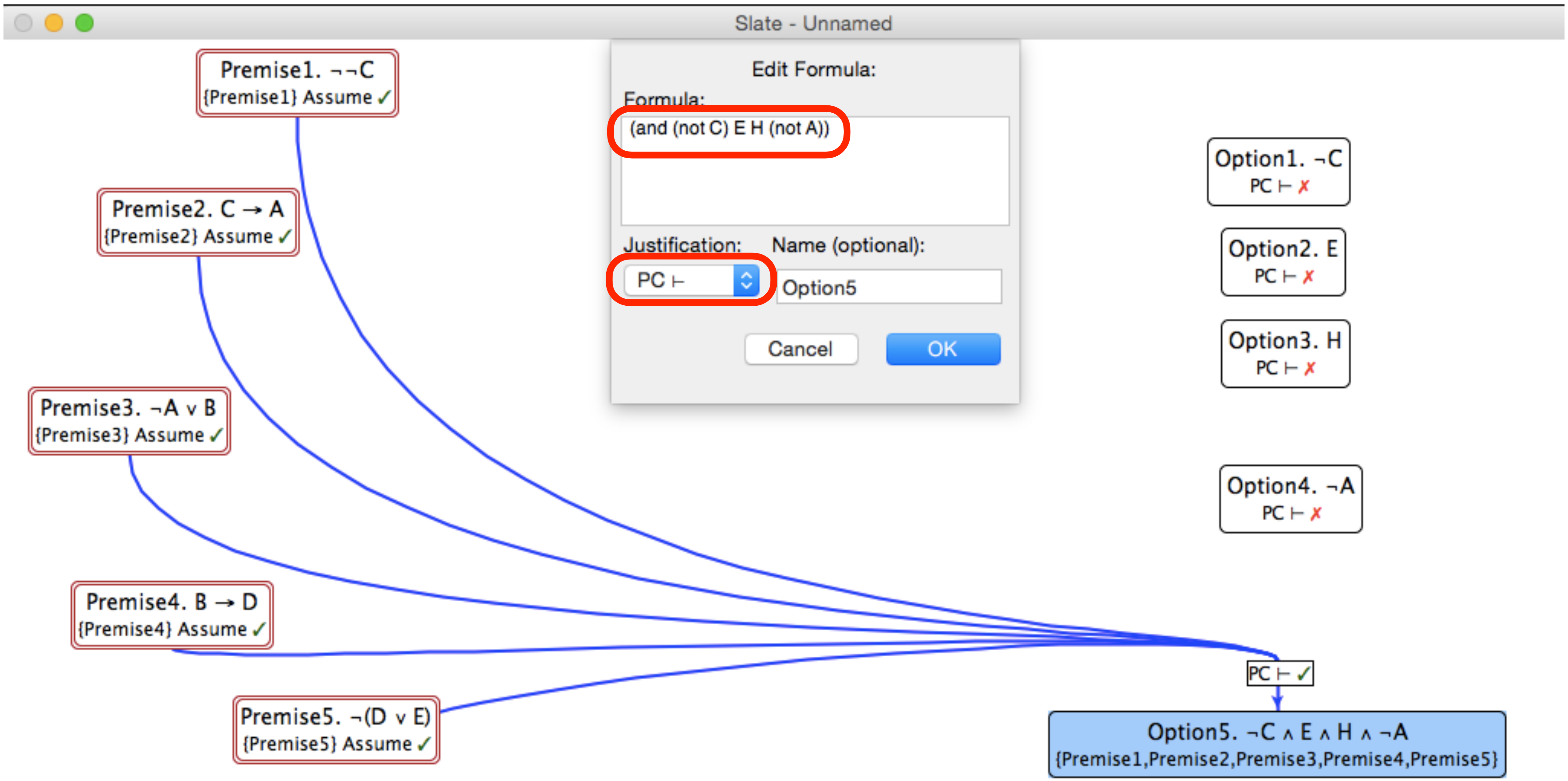

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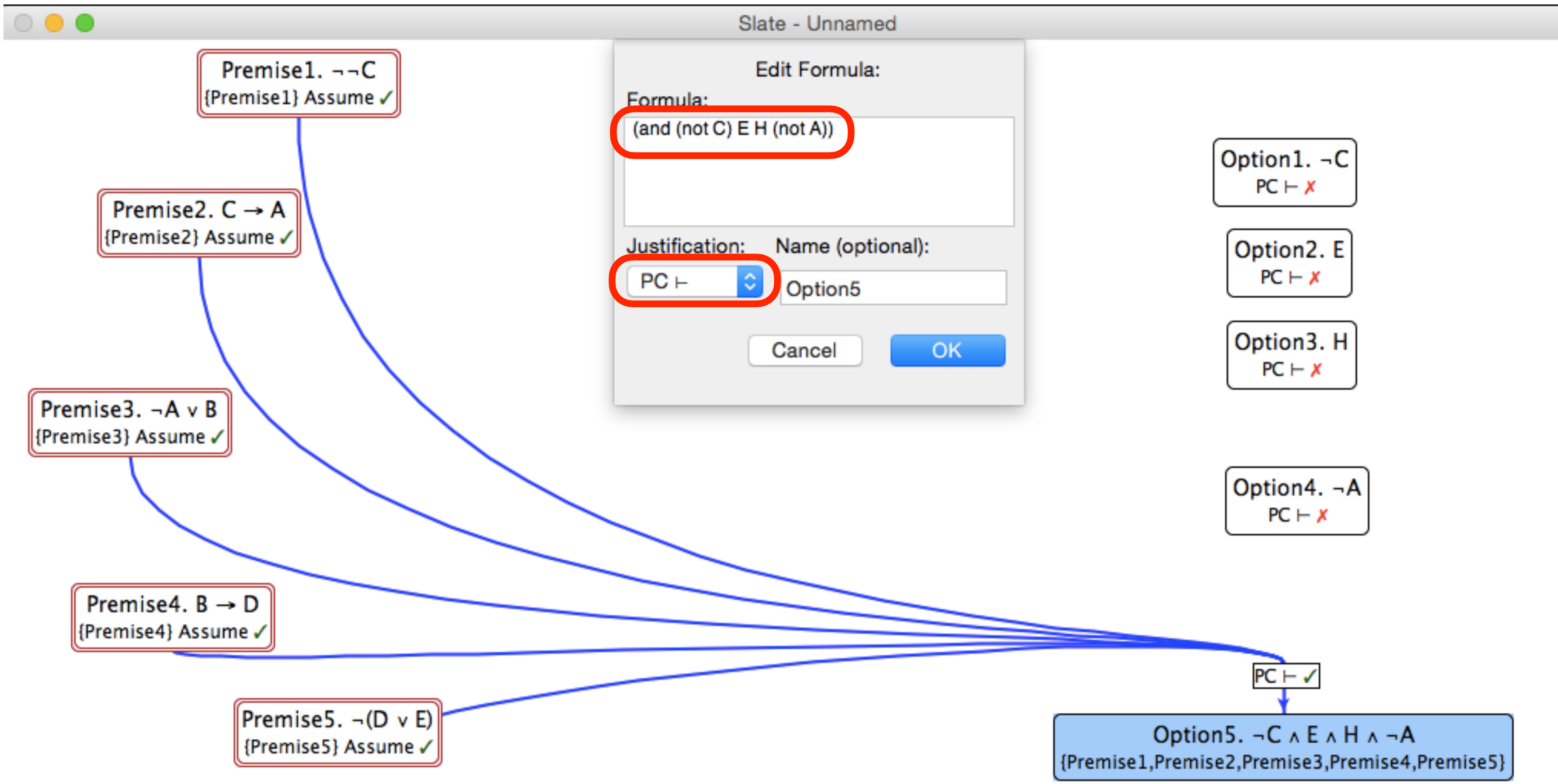
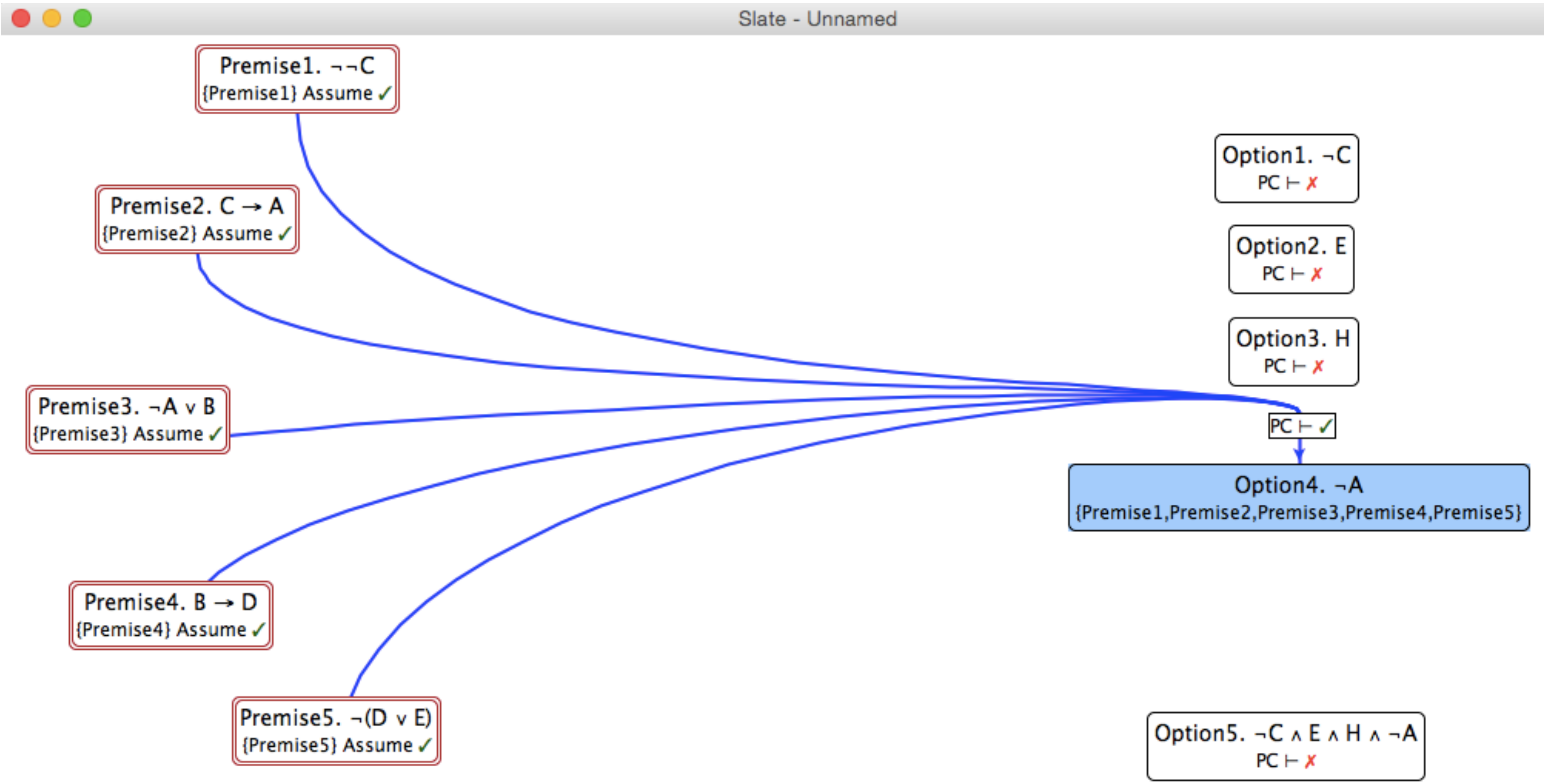


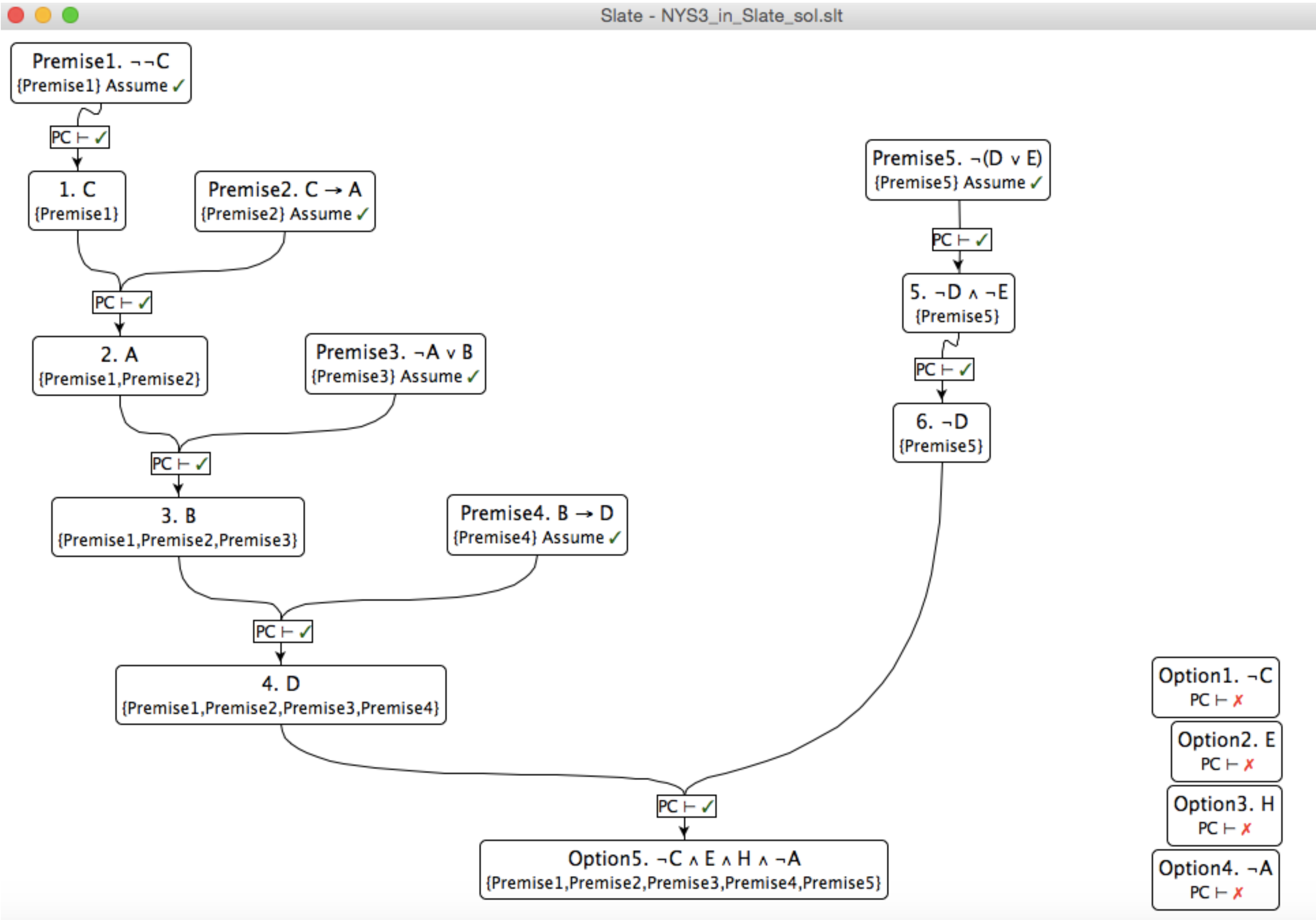

Slate - Unnamed
Premise1. ¬¬C
{Premise1} Assume ✓
Premise2. C → A
{Premise2} Assume ✓
Premise3. ¬A ∨ B
{Premise3} Assume ✓
Premise4. B → D
{Premise4} Assume ✓
Premise5. ¬(D ∨ E)
{Premise5} Assume ✓
Option1. ¬C
PC ⊢ ✗
Option2. E
PC ⊢ ✗
Option3. H
PC ⊢ ✗
PC ⊢ ✓
Option4. ¬A
{Premise1,Premise2,Premise3,Premise4,Premise5}
Option5. ¬C ∧ E ∧ H ∧ ¬A
PC ⊢ ✗

Proof Plan …

Proof Plan …

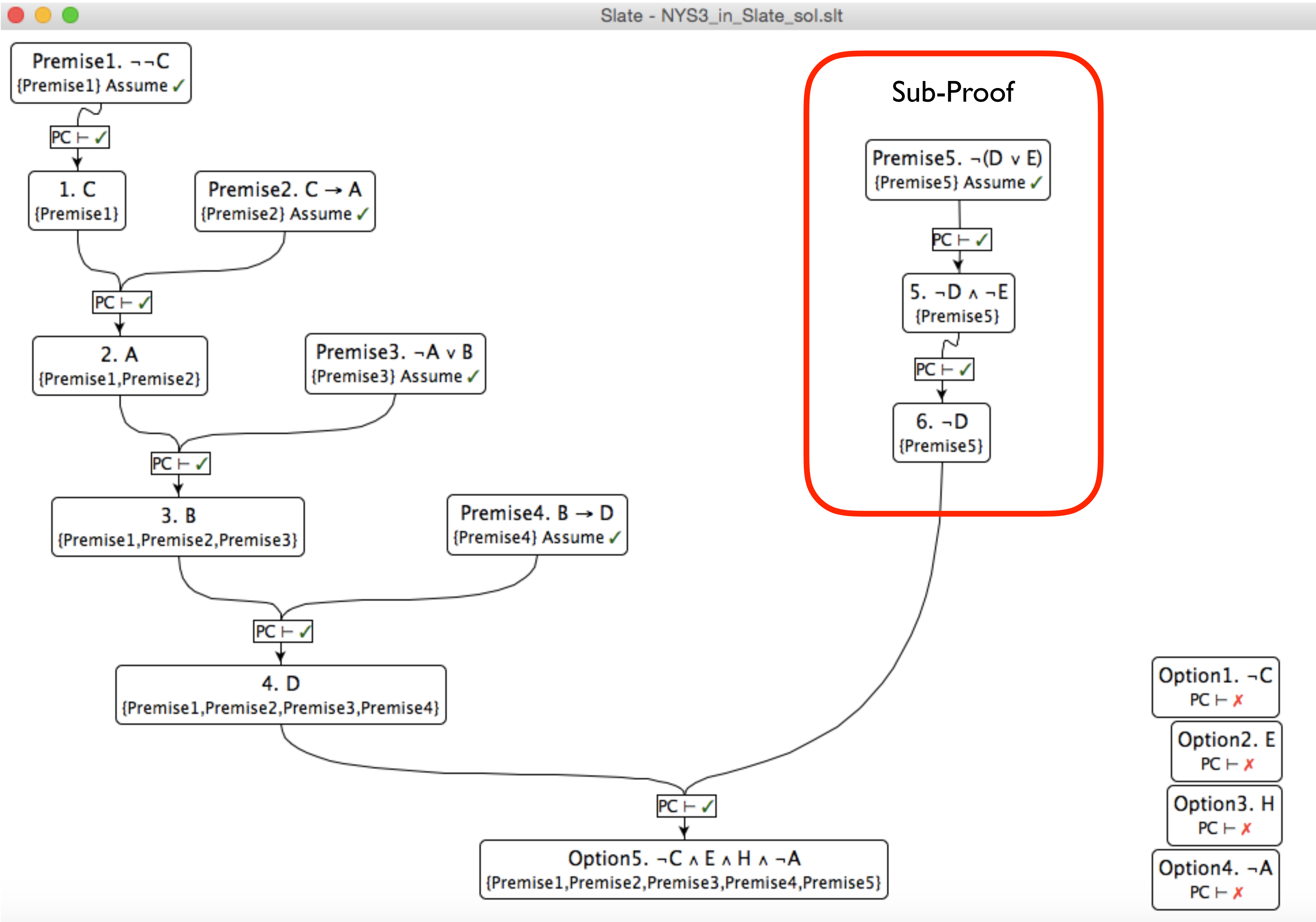
Proof Plan …

Proof Plan …



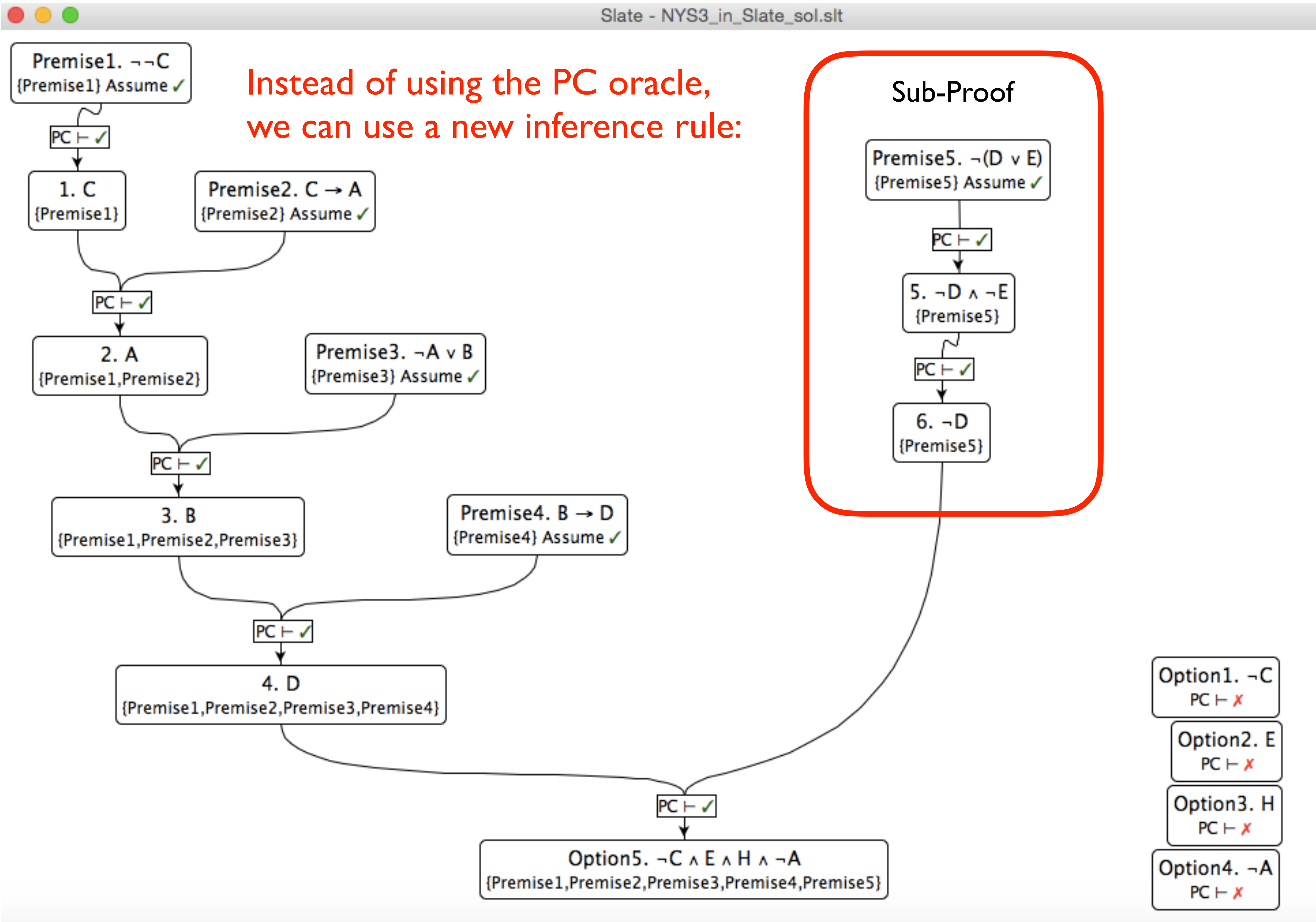
Slate - NYS3_in_Slate_sol.slt
Premise1. ¬¬C
{Premise1} Assume ✓
PC ⊢ ✓
1. C
{Premise1}
Premise2. C → A
{Premise2} Assume ✓
PC ⊢ ✓
2. A
{Premise1,Premise2}
Premise3. ¬A ∨ B
{Premise3} Assume ✓
PC ⊢ ✓
3. B
{Premise1,Premise2,Premise3}
Premise4. B → D
{Premise4} Assume ✓
PC ⊢ ✓
4. D
{Premise1,Premise2,Premise3,Premise4}
Premise5. ¬(D ∨ E)
{Premise5} Assume ✓
PC ⊢ ✓
5. ¬D ∧ ¬E
{Premise5}
PC ⊢ ✓
6. ¬D
{Premise5}
PC ⊢ ✓
Option5. ¬C ∧ E ∧ H ∧ ¬A
{Premise1,Premise2,Premise3,Premise4,Premise5}
Option1. ¬C
PC ⊢ ✗
Option2. E
PC ⊢ ✗
Option3. H
PC ⊢ ✗
Option4. ¬A
PC ⊢ ✗

Proof Plan …

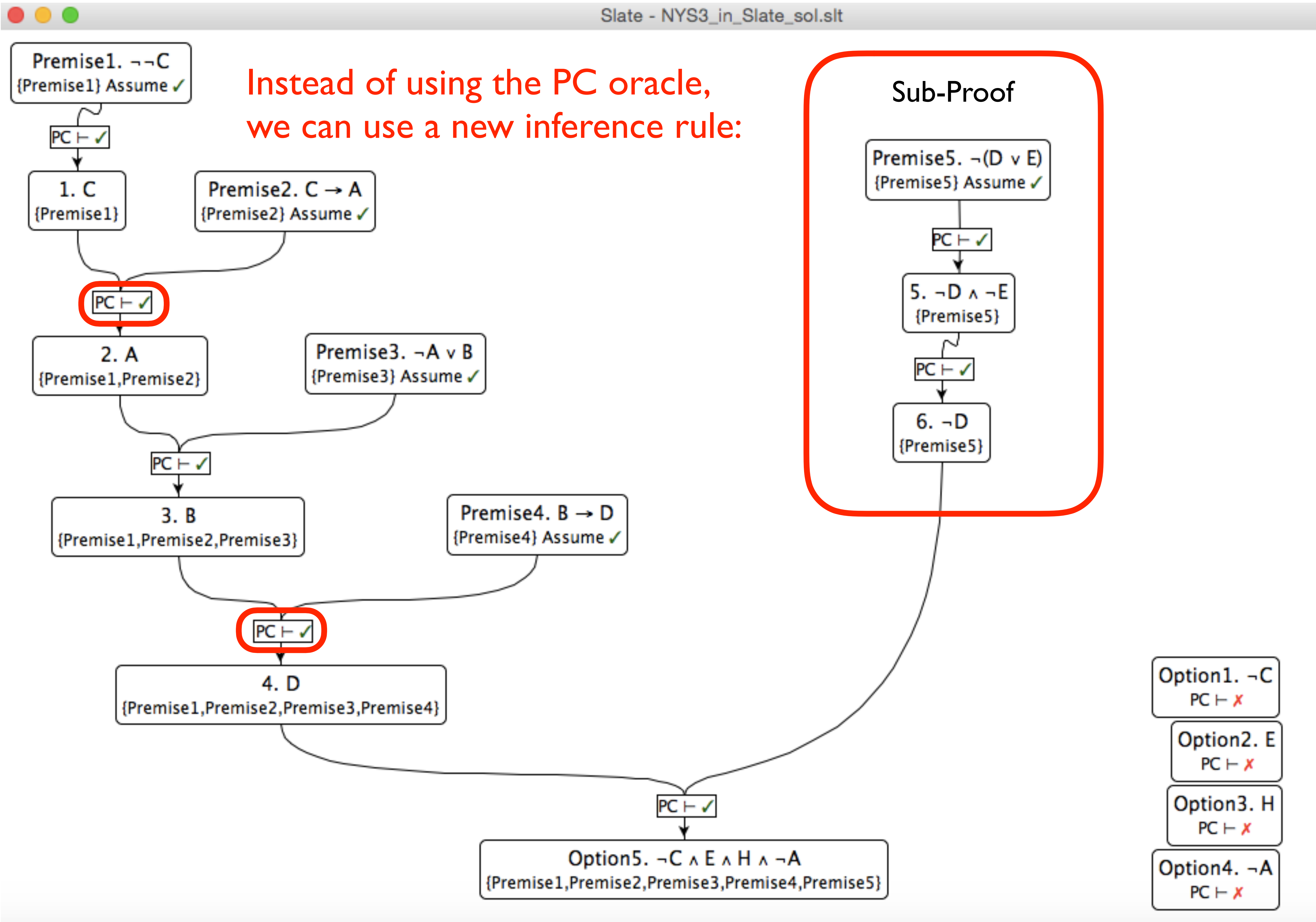


Slate - NYS3_in_Slate_sol.slt
Premise1. ¬¬C
{Premise1} Assume ✓
PC ⊢ ✓
1. C
{Premise1}
Premise2. C → A
{Premise2} Assume ✓
PC ⊢ ✓
2. A
{Premise1,Premise2}
Premise3. ¬A ∨ B
{Premise3} Assume ✓
PC ⊢ ✓
3. B
{Premise1,Premise2,Premise3}
Premise4. B → D
{Premise4} Assume ✓
PC ⊢ ✓
4. D
{Premise1,Premise2,Premise3,Premise4}
Sub-Proof
Premise5. ¬(D ∨ E)
{Premise5} Assume ✓
PC ⊢ ✓
5. ¬D ∧ ¬E
{Premise5}
PC ⊢ ✓
6. ¬D
{Premise5}
PC ⊢ ✓
Option5. ¬C ∧ E ∧ H ∧ ¬A
{Premise1,Premise2,Premise3,Premise4,Premise5}
Option1. ¬C
PC ⊢ ✗
Option2. E
PC ⊢ ✗
Option3. H
PC ⊢ ✗
Option4. ¬A
PC ⊢ ✗

Proof Plan …

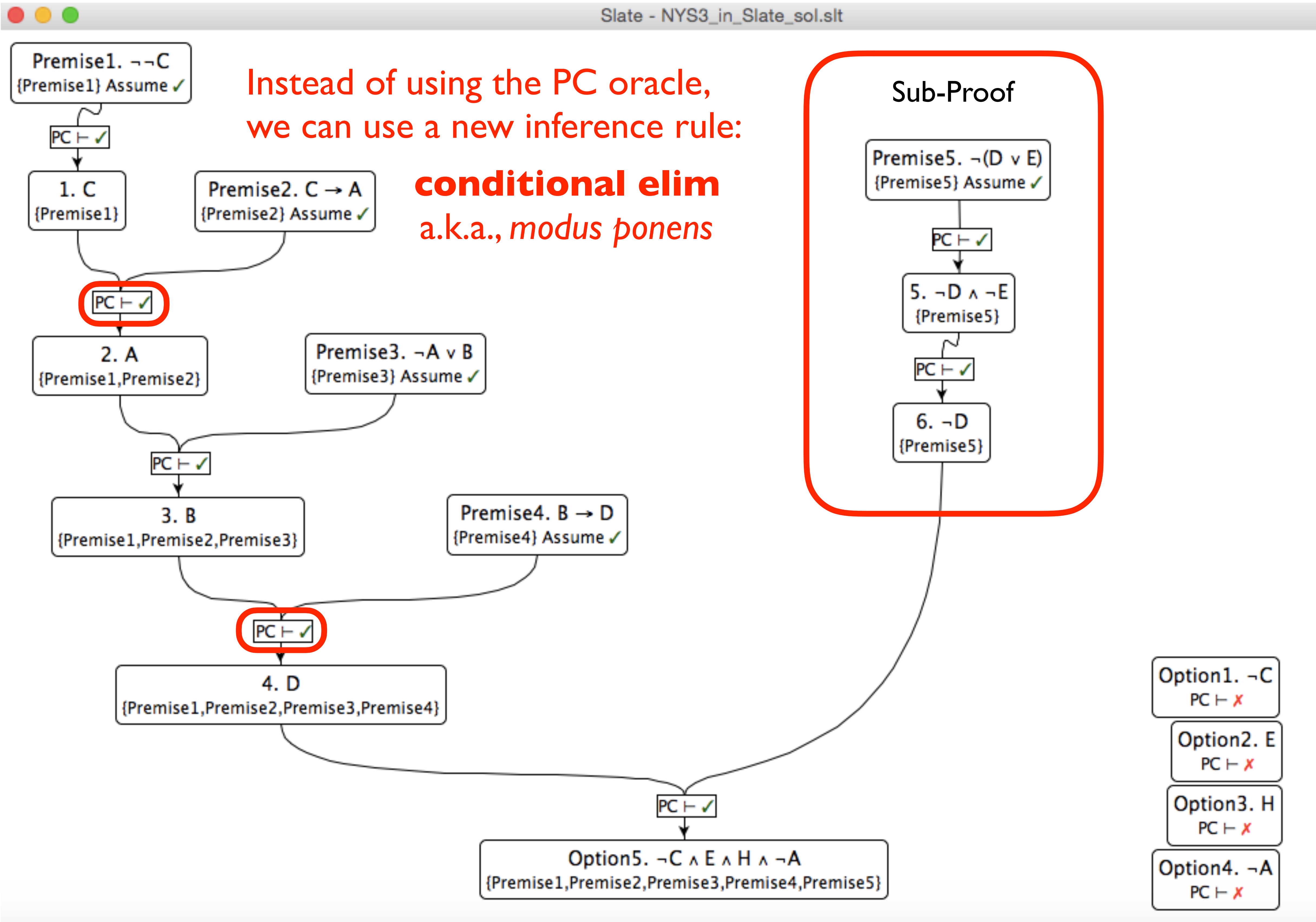

Slate - NYS3_in_Slate_sol.slt
Instead of using the PC oracle,
we can use a new inference rule:
Sub-Proof
Premise1. ¬¬C
{Premise1} Assume ✓
PC ⊢ ✓
1. C
{Premise1}
Premise2. C → A
{Premise2} Assume ✓
2. A
{Premise1,Premise2}
Premise3. ¬A ∨ B
{Premise3} Assume ✓
3. B
{Premise1,Premise2,Premise3}
Premise4. B → D
{Premise4} Assume ✓
4. D
{Premise1,Premise2,Premise3,Premise4}
Premise5. ¬(D ∨ E)
{Premise5} Assume ✓
5. ¬D ∧ ¬E
{Premise5}
6. ¬D
{Premise5}
Option5. ¬C ∧ E ∧ H ∧ ¬A
{Premise1,Premise2,Premise3,Premise4,Premise5}
Option1. ¬C
PC ⊢ ✗
Option2. E
PC ⊢ ✗
Option3. H
PC ⊢ ✗
Option4. ¬A
PC ⊢ ✗

Proof Plan …



Slate - NYS3_in_Slate_sol.slt
Premise1. ¬¬C
{Premise1} Assume ✓
PC ⊢ ✓
1. C
{Premise1}
Premise2. C → A
{Premise2} Assume ✓
PC ⊢ ✓
2. A
{Premise1,Premise2}
Premise3. ¬A ∨ B
{Premise3} Assume ✓
PC ⊢ ✓
3. B
{Premise1,Premise2,Premise3}
Premise4. B → D
{Premise4} Assume ✓
PC ⊢ ✓
4. D
{Premise1,Premise2,Premise3,Premise4}
Instead of using the PC oracle,
we can use a new inference rule:
Sub-Proof
Premise5. ¬(D ∨ E)
{Premise5} Assume ✓
PC ⊢ ✓
5. ¬D ∧ ¬E
{Premise5}
PC ⊢ ✓
6. ¬D
{Premise5}
PC ⊢ ✓
Option5. ¬C ∧ E ∧ H ∧ ¬A
{Premise1,Premise2,Premise3,Premise4,Premise5}
Option1. ¬C
PC ⊢ ✗
Option2. E
PC ⊢ ✗
Option3. H
PC ⊢ ✗
Option4. ¬A
PC ⊢ ✗

Proof Plan …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PC ⊢ ✓
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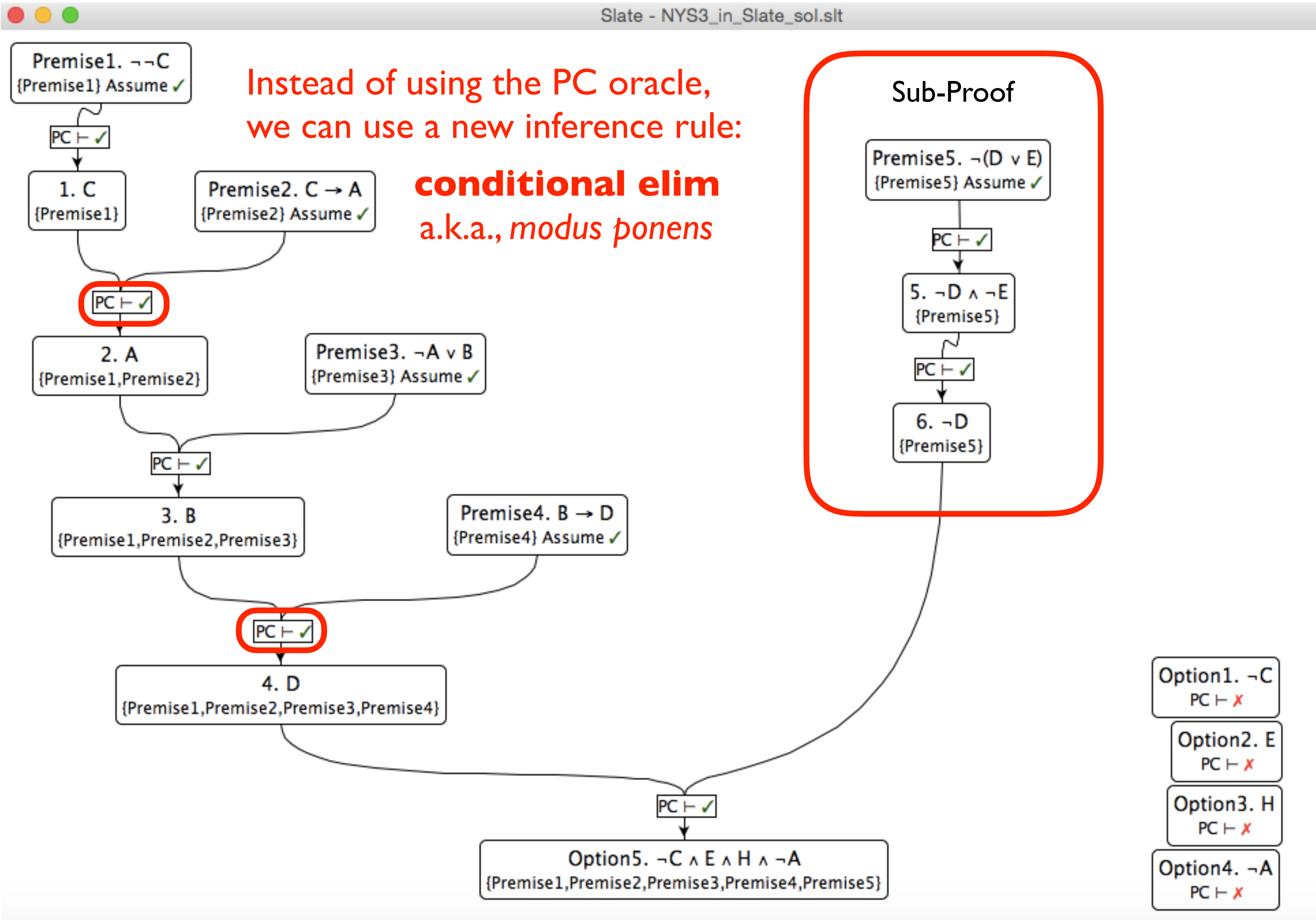
{Premise1}
Premise2. C → A
{Premise2} Assume ✓
Instead of using the PC oracle,
we can use a new inference rule:
conditional elim
a.k.a., modus ponens
PC ⊢ ✓
2. A
{Premise1,Premise2}
Premise3. ¬A ∨ B
{Premise3} Assume ✓
PC ⊢ ✓
3. B
{Premise1,Premise2,Premise3}
Premise4. B → D
{Premise4} Assume ✓
PC ⊢ ✓
4. D
{Premise1,Premise2,Premise3,Premise4}
Sub-Proof
Premise5. ¬(D ∨ E)
{Premise5} Assume ✓
PC ⊢ ✓
5. ¬D ∧ ¬E
{Premise5}
PC ⊢ ✓
6. ¬D
{Premise5}
PC ⊢ ✓
Option5. ¬C ∧ E ∧ H ∧ ¬A
{Premise1,Premise2,Premise3,Premise4,Premise5}
Option1. ¬C
PC ⊢ ✗
Option2. E
PC ⊢ ✗
Option3. H
PC ⊢ ✗
Option4. ¬A
PC ⊢ ✗

Proof Plan …



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PC ⊢ ✓
1. C
{Premise1}
Premise2. C → A
{Premise2} Assume ✓
PC ⊢ ✓
2. A
{Premise1,Premise2}
Premise3. ¬A ∨ B
{Premise3} Assume ✓
PC ⊢ ✓
3. B
{Premise1,Premise2,Premise3}
Premise4. B → D
{Premise4} Assume ✓
PC ⊢ ✓
4. D
{Premise1,Premise2,Premise3,Premise4}
Instead of using the PC oracle,
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Sub-Proof
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{Premise5} Assume ✓
PC ⊢ ✓
5. ¬D ∧ ¬E
{Premise5}
PC ⊢ ✓
6. ¬D
{Premise5}
PC ⊢ ✓
Option5. ¬C ∧ E ∧ H ∧ ¬A
{Premise1,Premise2,Premise3,Premise4,Premise5}
Option1. ¬C
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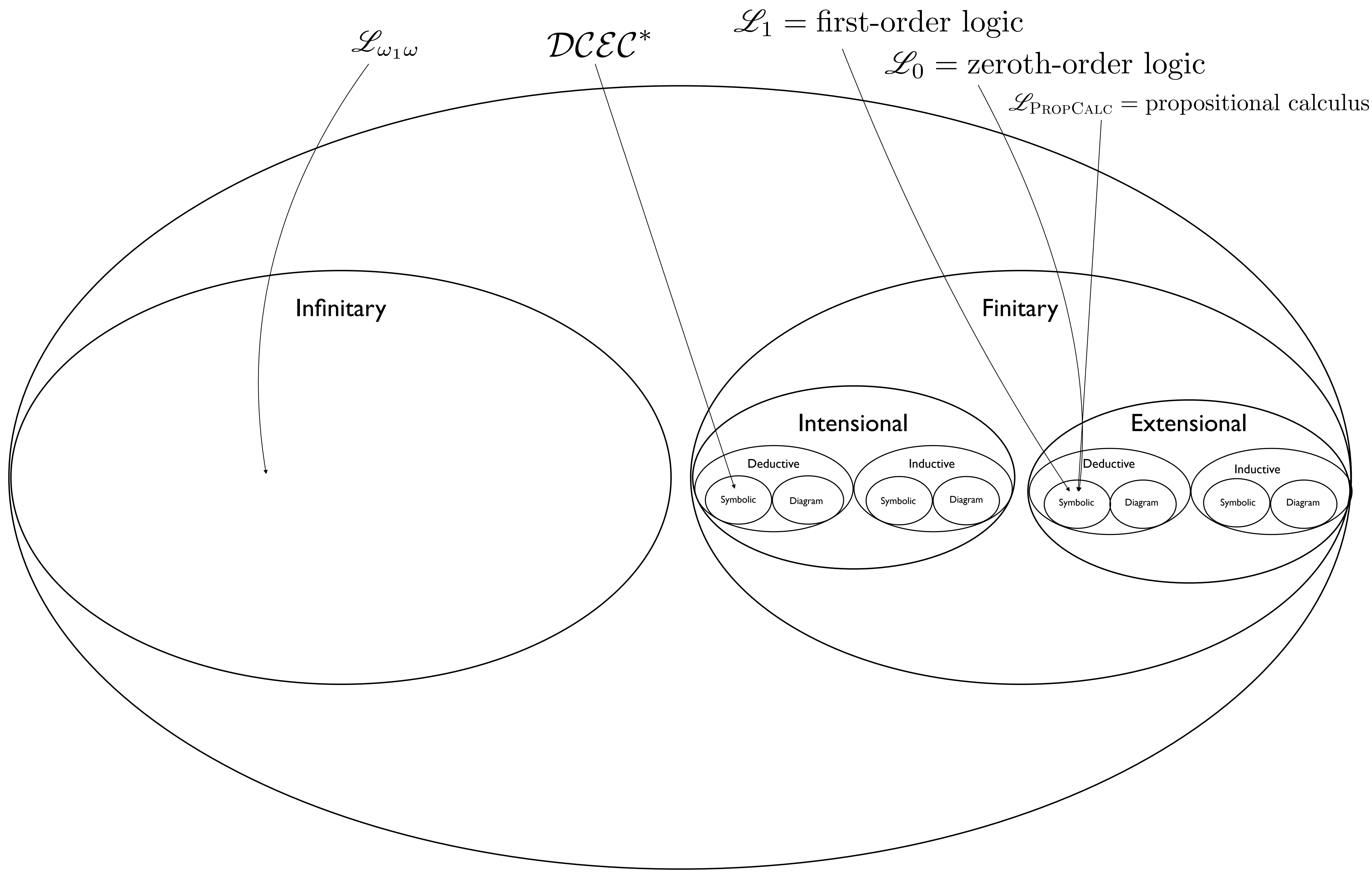
Option2. E
PC ⊢ ✗
Option3. H
PC ⊢ ✗
Option4. ¬A
PC ⊢ ✗

The Universe of Logics



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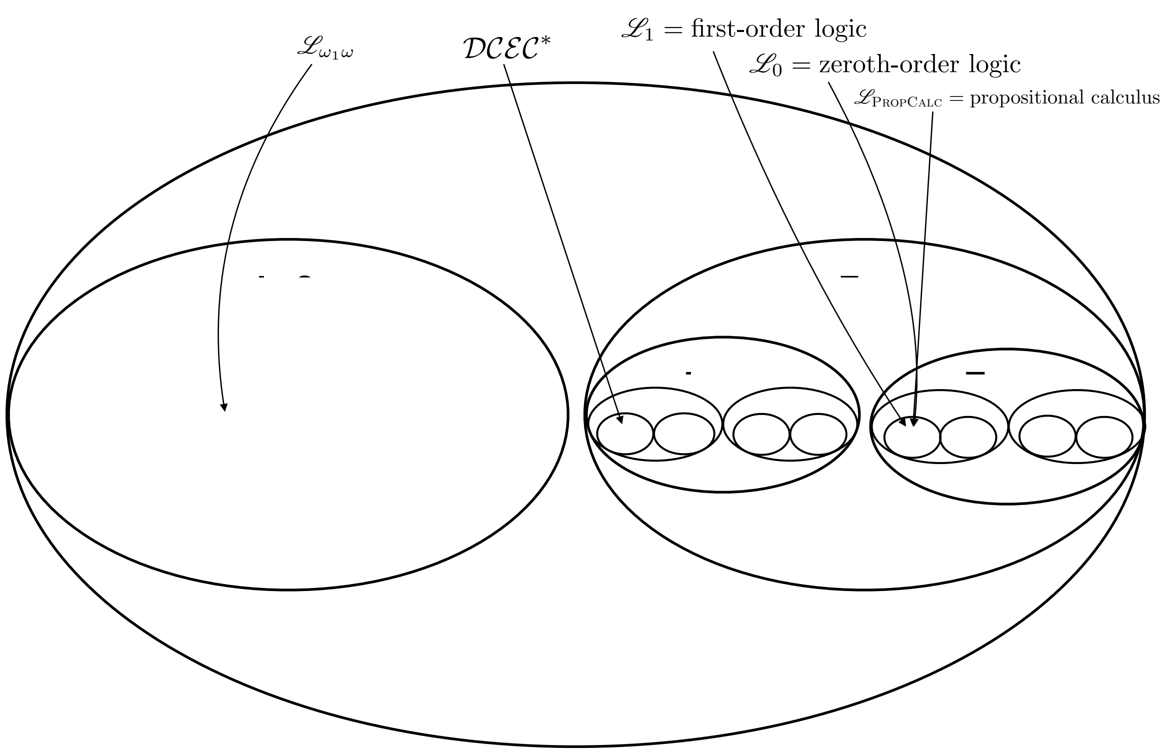

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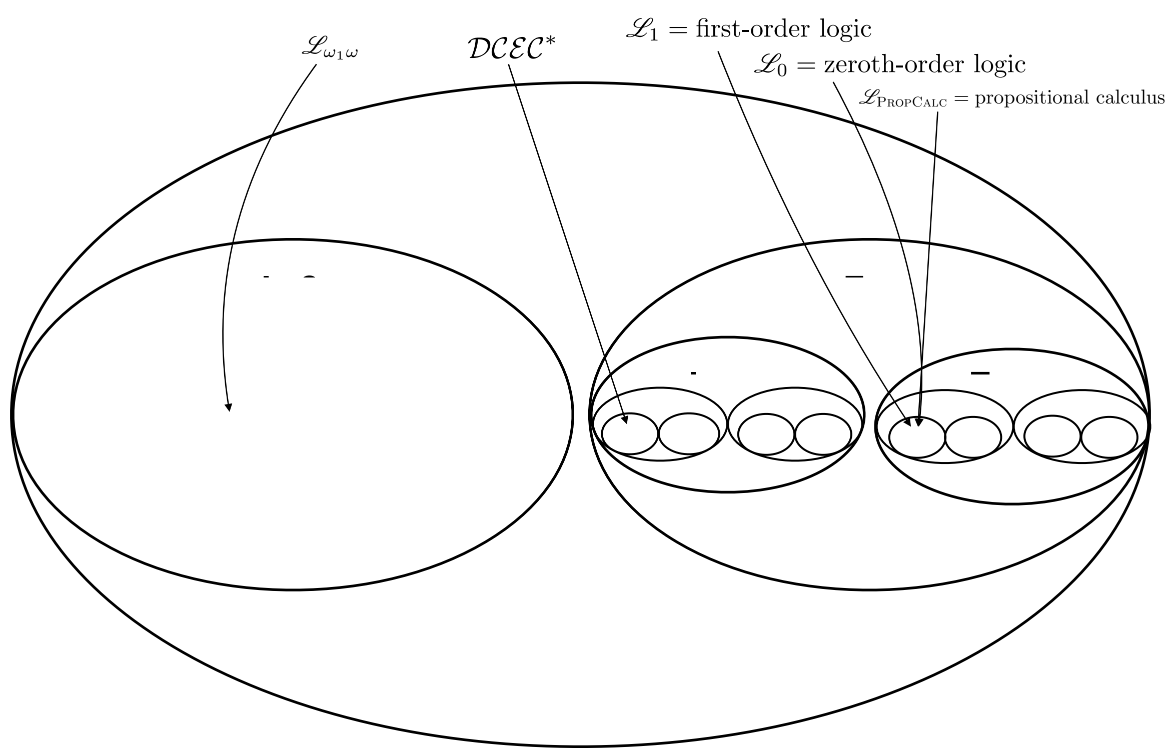
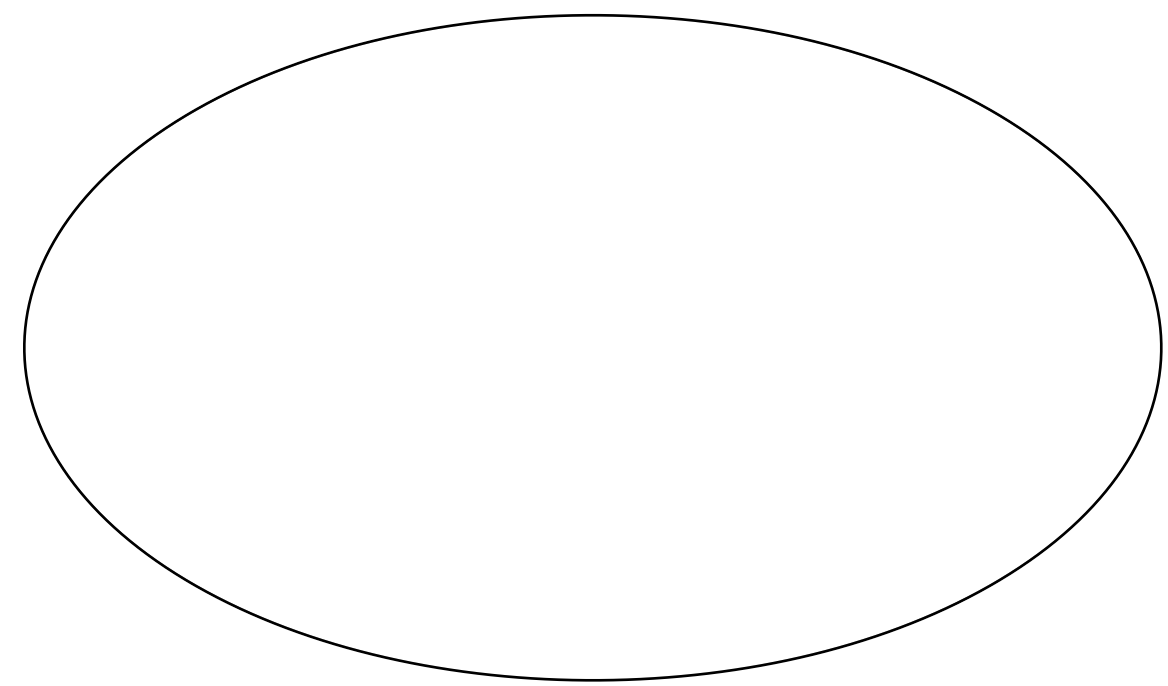


The Physical Universe



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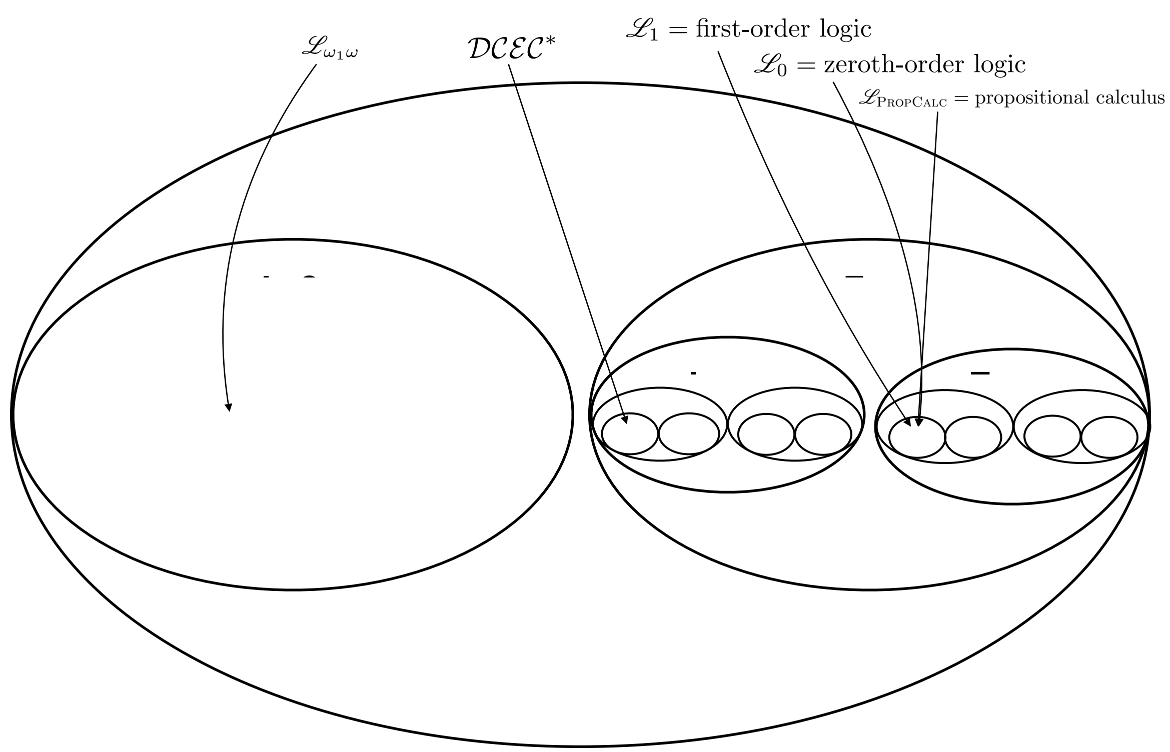

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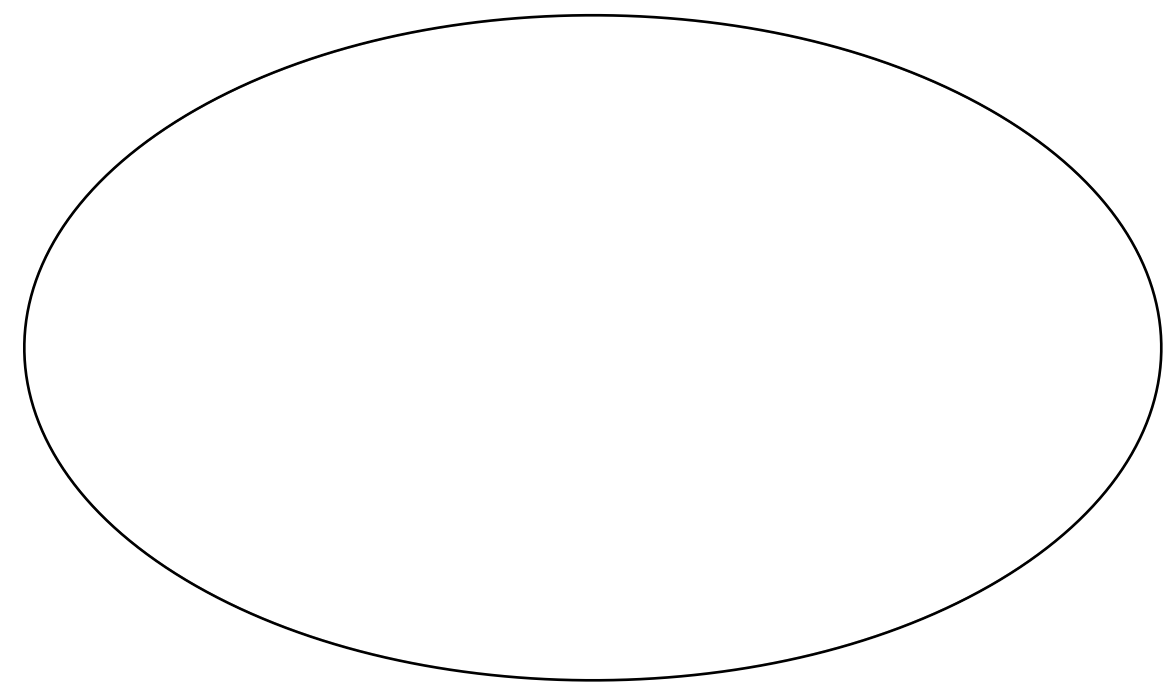
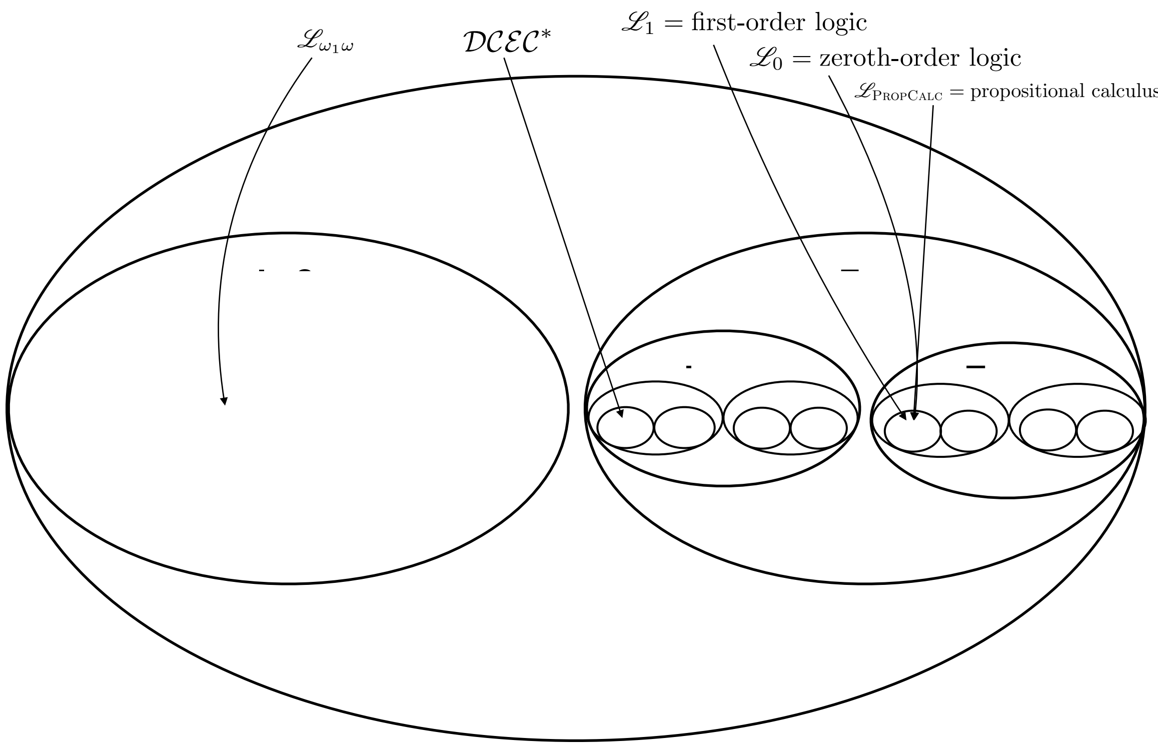
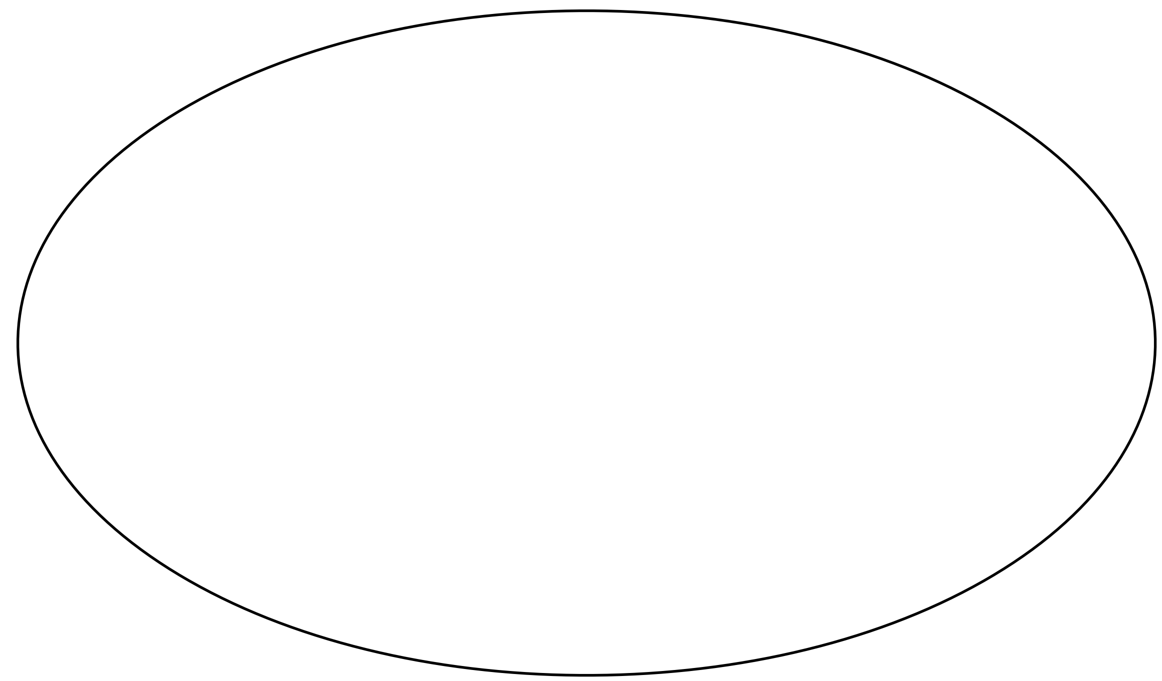


Non-Physical

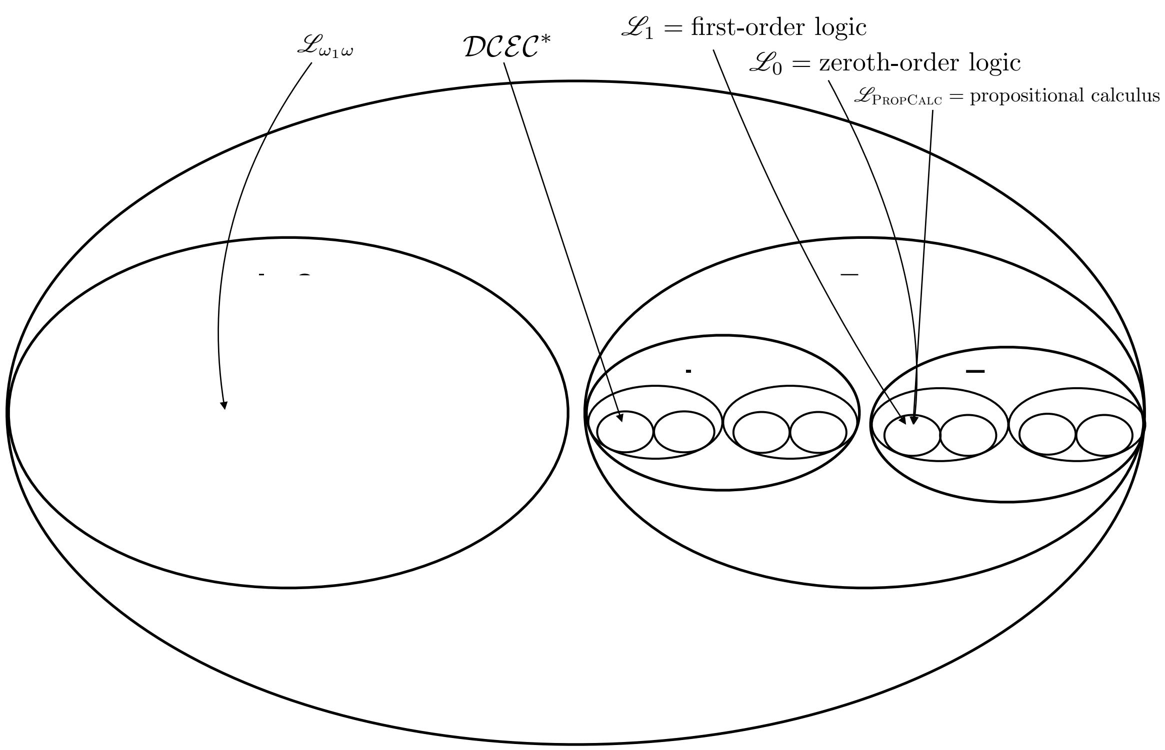
The Universe of Logics



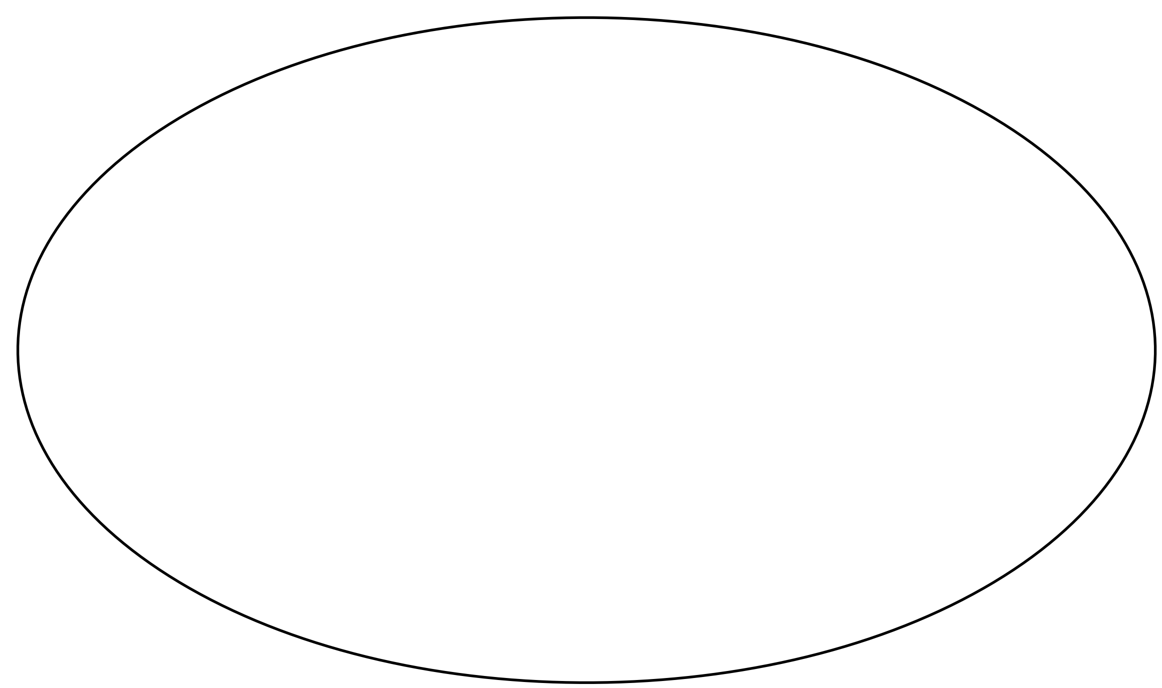
The Physical Universe



The Universe of Logics

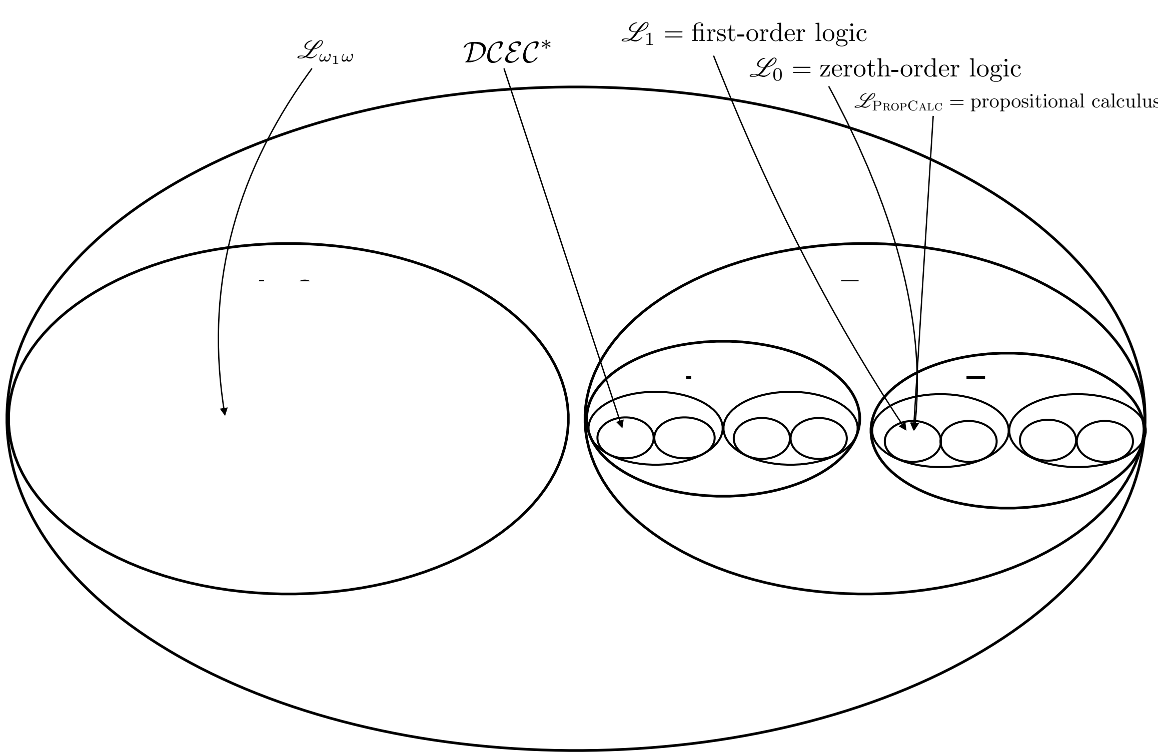


The Physical Universe

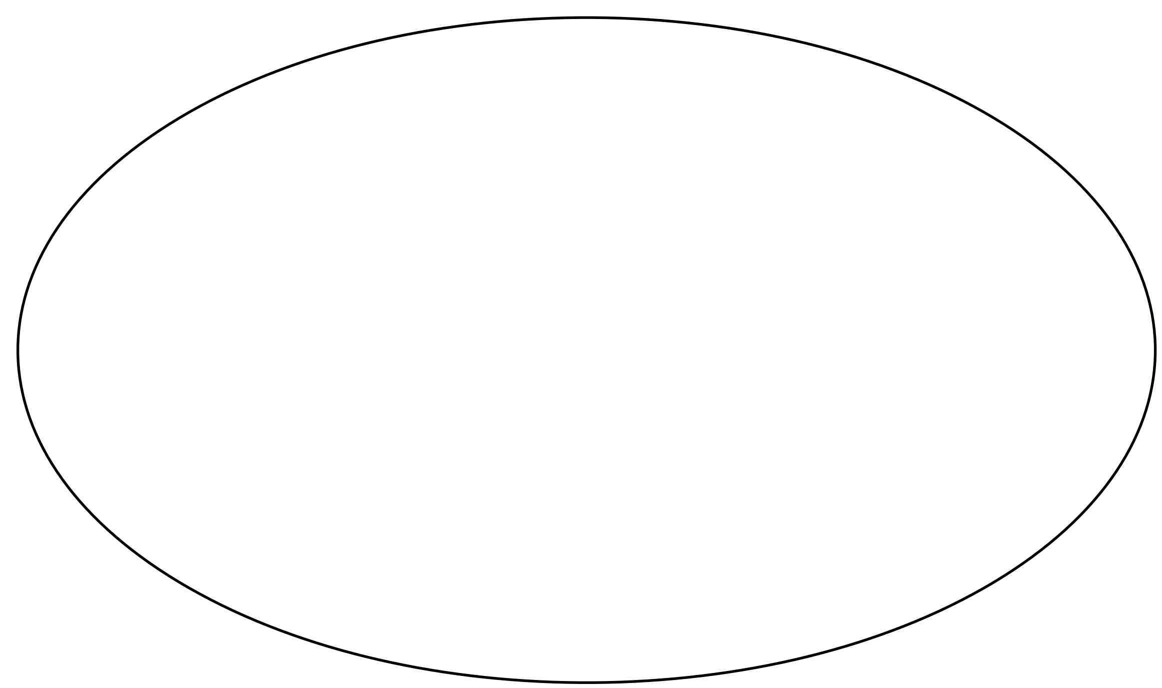


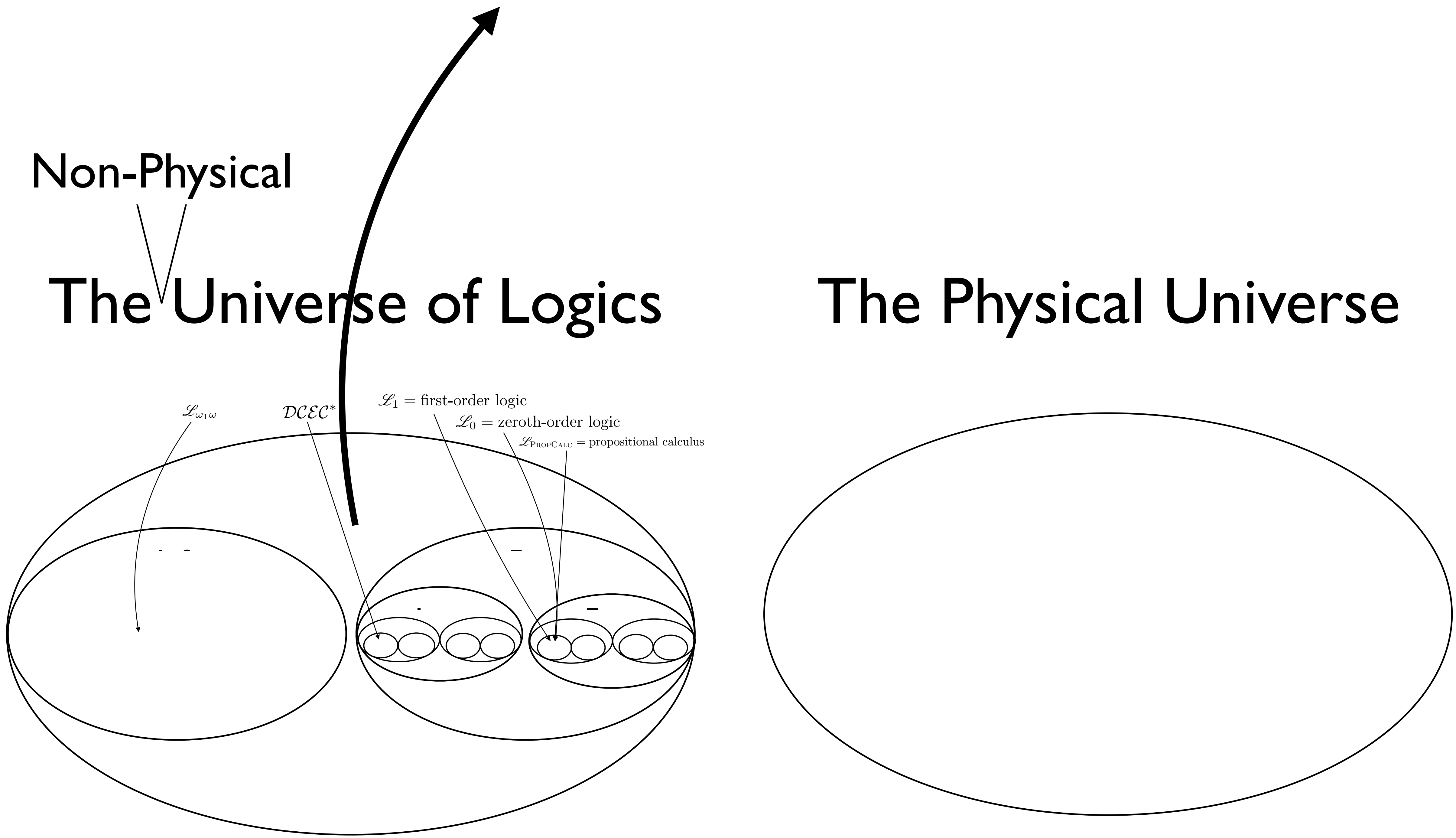
Non-Physical

The Universe of Logics

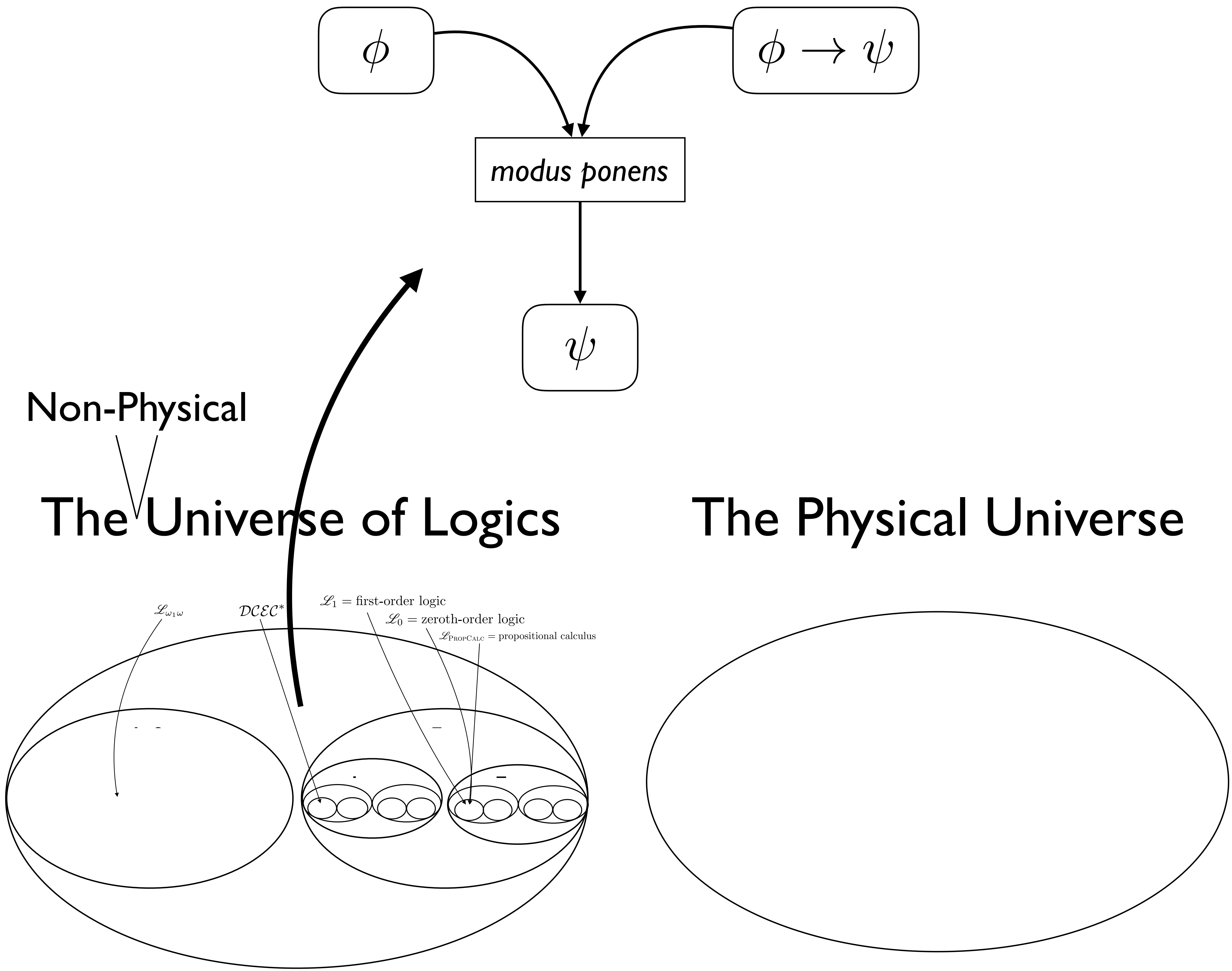


The Physical Universe





Non-Physical
The Universe of Logics
The Physical Universe
$\mathscr{L}_{\omega_1\omega}$
$\mathcal{DCEC}^*$
$\mathscr{L}_1$ = first-order logic
$\mathscr{L}_0$ = zeroth-order logic
$\mathscr{L}_{\text{PropCalc}}$ = propositional calculus



ϕ
$\phi \to \psi$
modus ponens
ψ
Non-Physical
The Universe of Logics
The Physical Universe
$\mathscr{L}_{\omega_1\omega}$
$\mathcal{DCEC}^*$
$\mathscr{L}_1$ = first-order logic
$\mathscr{L}_0$ = zeroth-order logic
$\mathscr{L}_{\text{PropCalc}}$ = propositional calculus

Next problem (King-Ace) …

The Original King-Ace

Suppose that the following premise is true:

> If there is a king in the hand, then there is an ace in the hand, or else if there isn't a king in the hand, then there is an ace.

What can you infer from this premise?

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King-Ace Solved

Proposition: There is *not* an ace in the hand.

Proof: We know that at least one of the if-thens (i.e., at least one of the **conditionals**) is false. So we have two cases to consider, viz., that *K* => *A* is false, and that ¬*K* => *A* is false. Take first the first case; accordingly, suppose that *K* => *A* is false. Then it follows that *K* is true (since when a conditional is false, its antecedent holds but its consequent doesn't), and *A* is false. Now consider the second case, which consists in ¬*K* => *A* being false. Here, in a direct parallel, we know ¬*K* and, once again, ¬*A*. In both of our two cases, which are exhaustive, there is no ace in the hand. The proposition is established. **QED**

King-Ace 2

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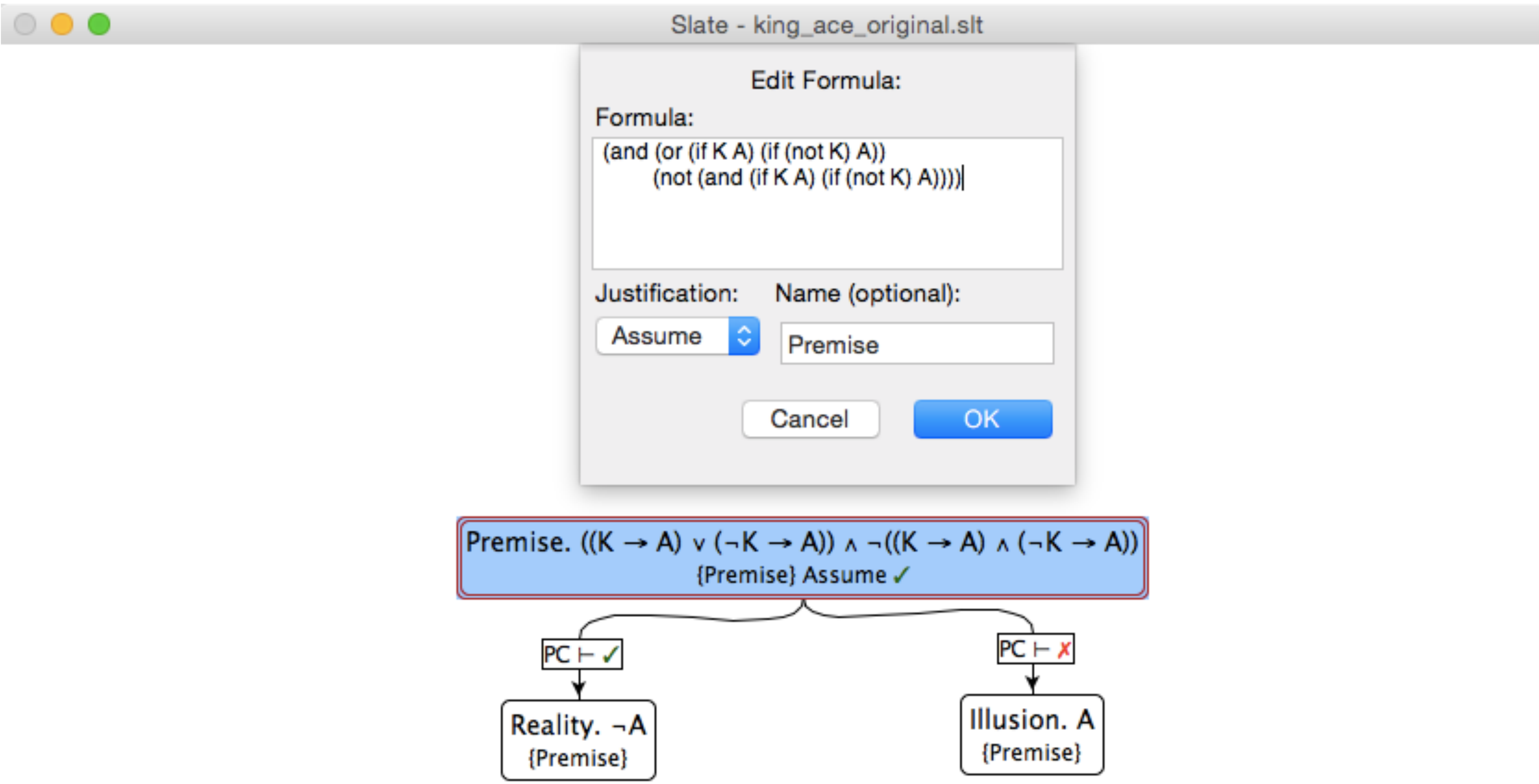
Study the S-expression



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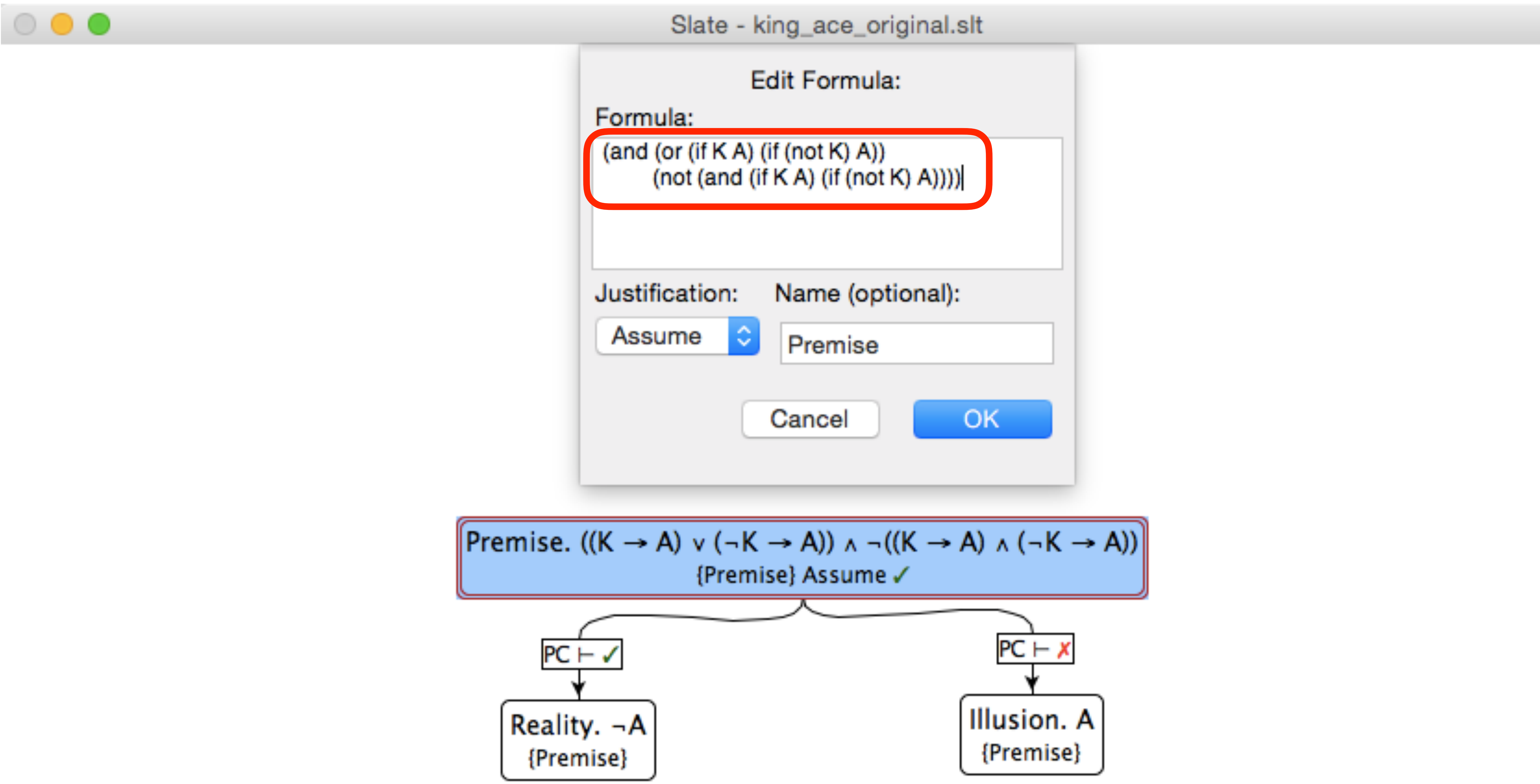

We need another rule of inference to crack this problem … …

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`disjunction elimination`

From ~ p. 54 in LAMA-BDLA

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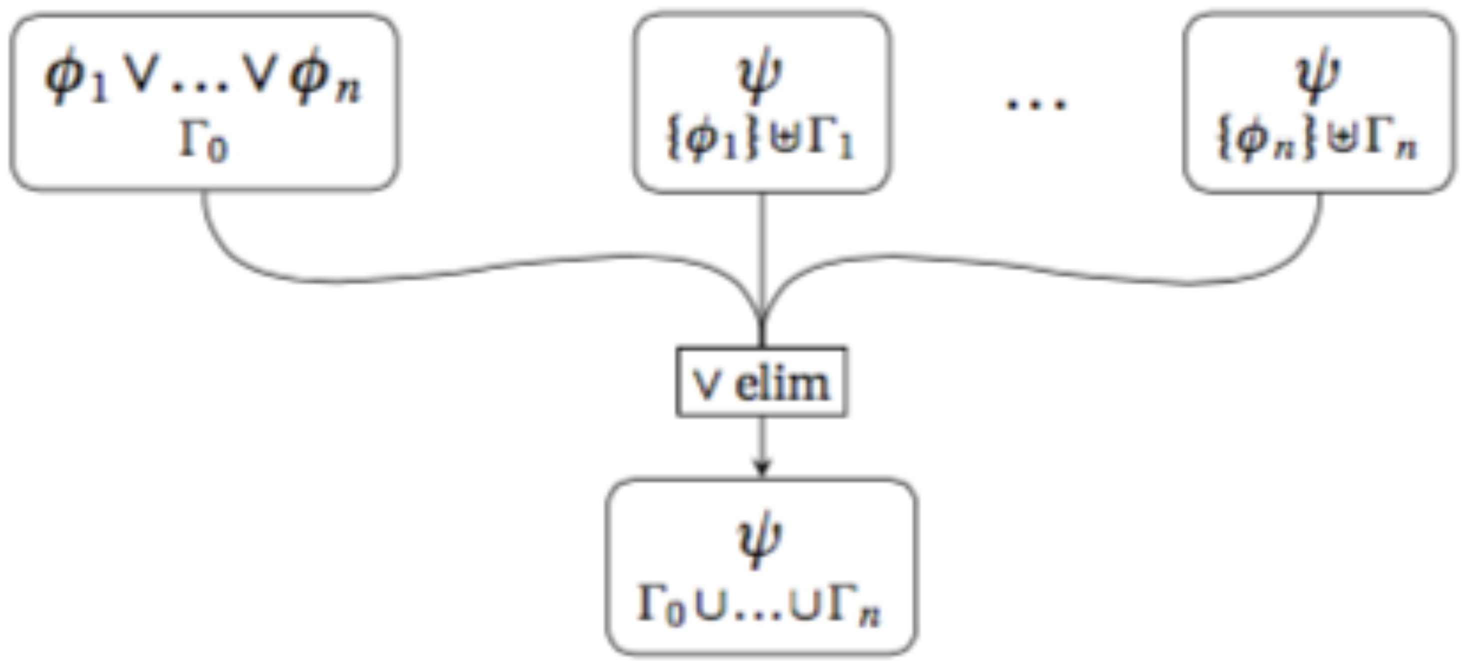

(2.25)

The various Γ_i on the premises of disjunction elimination might make this rule seem more complicated than it really is. Their presence makes it clear that the only assumptions discharged from each line of reasoning is the assumption corresponding to that particular case.

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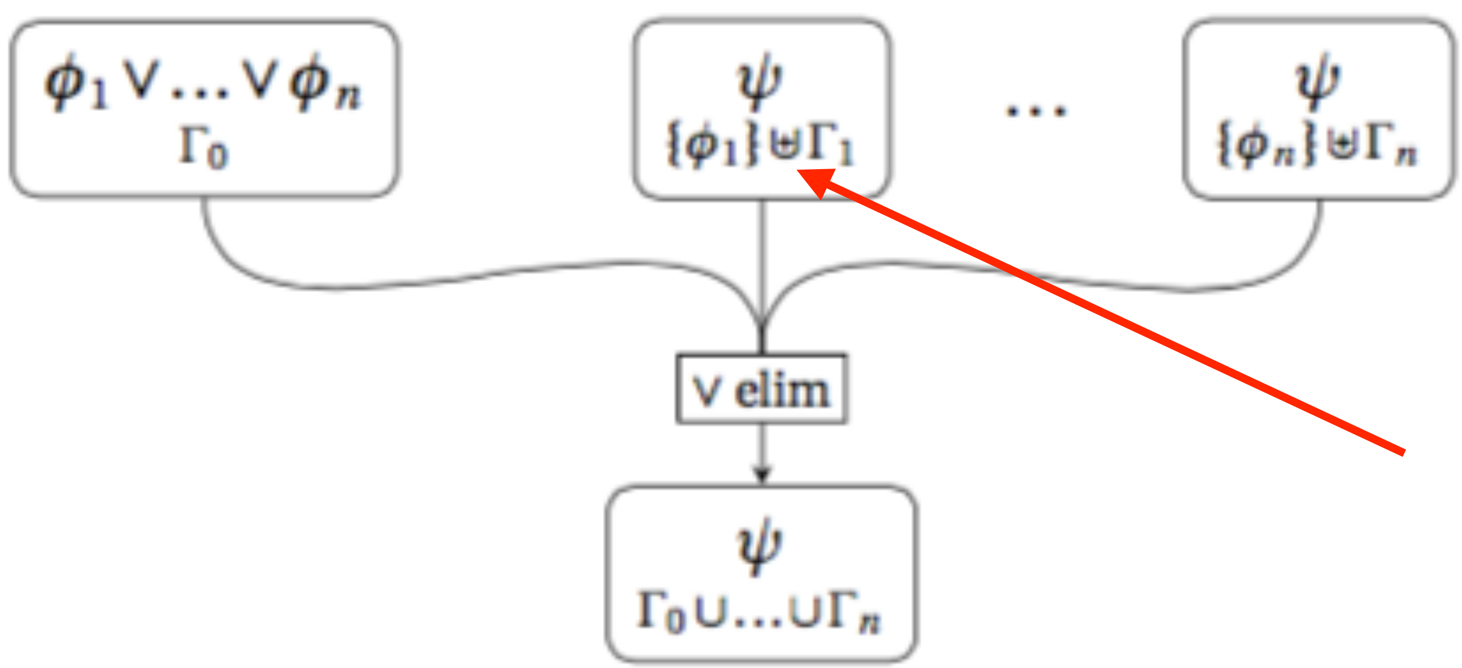

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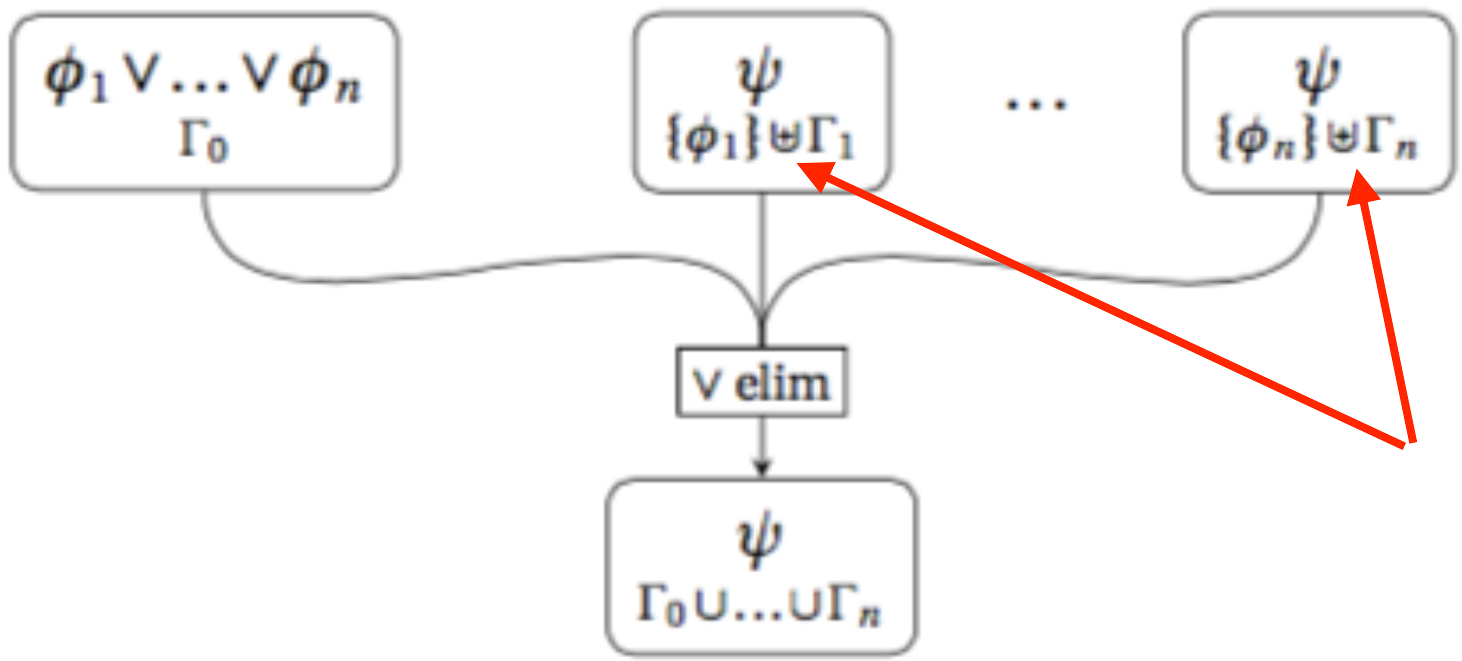

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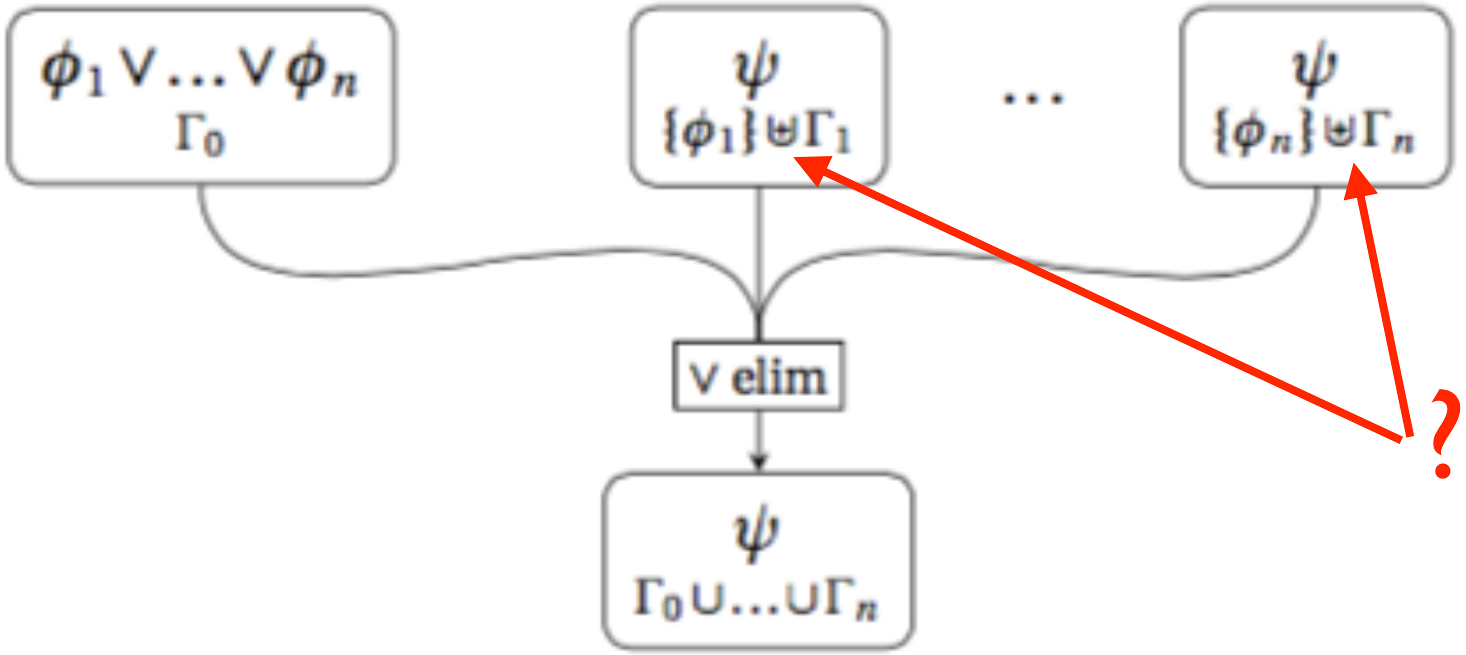

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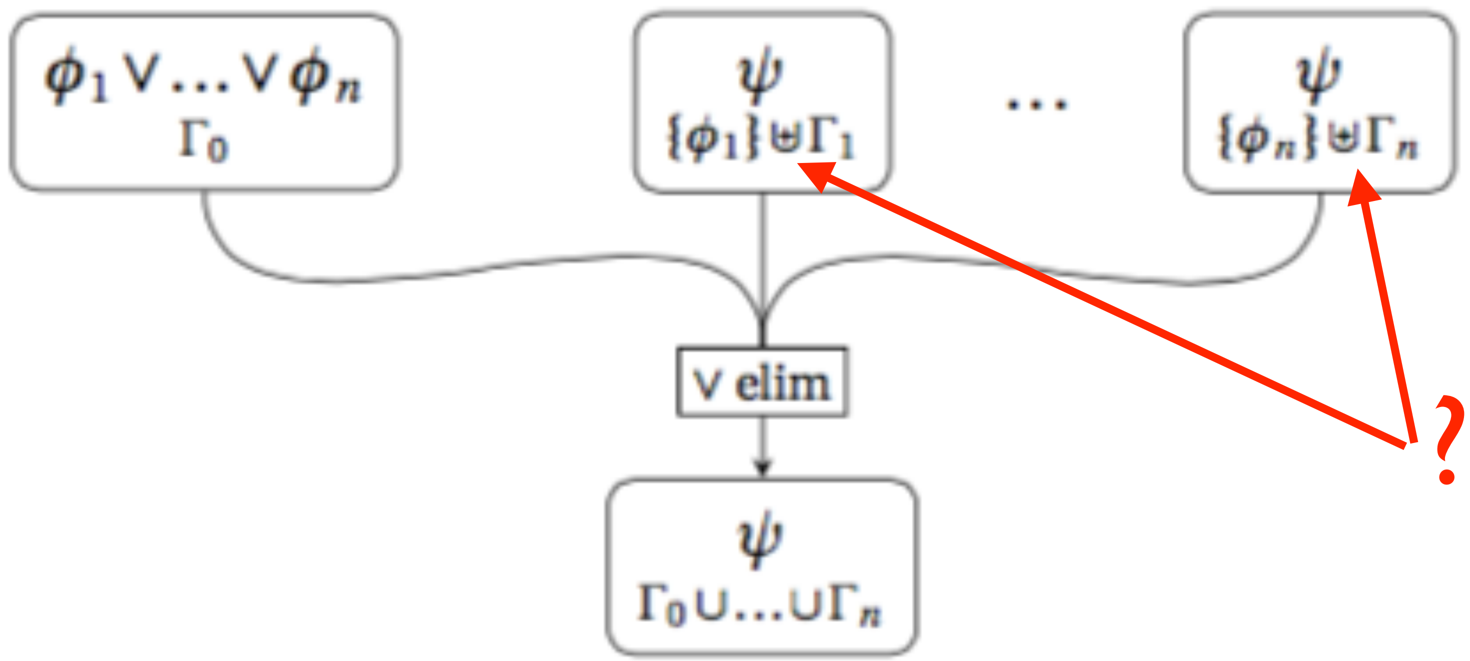

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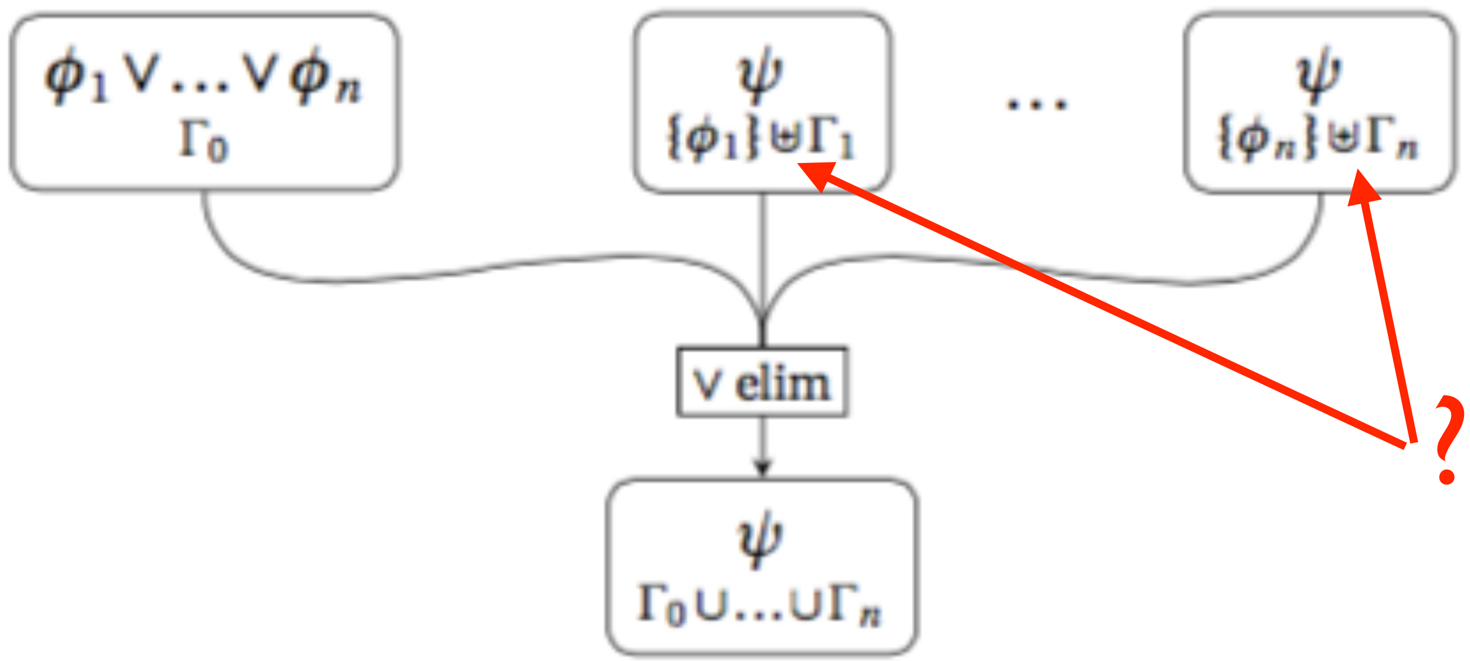

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What can you infer from this premise?

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Exercise (on HyperGrader™): Finish the proof in HyperSlate™ — with no remaining use of an oracle.

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Pop Problem