

Rebuilding the Foundations of Math via (the “Theory”) ZFC; ZFC to Axiomatized Arithmetic (the “Theory” PA)

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Intro to Logic
3/19/2020



Reviewing the situation

...

Types of Paradoxes

- Deductive Paradoxes
- Inductive Paradoxes — coming (e.g. The Lottery Paradox & The St Petersburg Paradox)

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<https://www.megamillions.com>

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1 in 302,575,350

Friday's Hill, Haslemere, 16 June 1902

Dear colleague,

For a year and a half I have been acquainted with your *Grundgesetze der Arithmetik*, but it is only now that I have been able to find the time for the thorough study I intended to make of your work. I find myself in complete agreement with you in all essentials, particularly when you reject any psychological element [Moment] in logic and when you place a high value upon an ideography [Begriffsschrift] for the foundations of mathematics and of formal logic, which, incidentally, can hardly be distinguished. With regard to many particular questions, I find in your work discussions, distinctions, and definitions that one seeks in vain in the works of other logicians. Especially so far as function is concerned (§ 9 of your *Begriffsschrift*), I have been led on my own to views that are the same even in the details. There is just one point where I have encountered a difficulty. You state (p. 17 [p. 23 above]) that a function, too, can act as the indeterminate element. This I formerly believed, but now this view seems doubtful to me because of the following contradiction. Let w be the predicate: to be a predicate that cannot be predicated of itself. Can w be predicated of itself? From each answer its opposite follows. Therefore we must conclude that w is not a predicate. Likewise there is no class (as a totality) of those classes which, each taken as a totality, do not belong to themselves. From this I conclude that under certain circumstances a definable collection [Menge] does not form a totality.

I am on the point of finishing a book on the principles of mathematics and in it I should like to discuss your work very thoroughly.¹ I already have your books or shall buy them soon, but I would be very grateful to you if you could send me reprints of your articles in various periodicals. In case this should be impossible, however, I will obtain them from a library.

The exact treatment of logic in fundamental questions, where symbols fail, has remained very much behind; in your works I find the best I know of our time, and therefore I have permitted myself to express my deep respect to you. It is very regrettable that you have not come to publish the second volume of your *Grundgesetze*; I hope that this will still be done.

Very respectfully yours,

BERTRAND RUSSELL

The above contradiction, when expressed in Peano's ideography, reads as follows:

$$w = \text{cls} \cap x \ni (x \sim_\varepsilon x) . \supset : w \varepsilon w . = . w \sim_\varepsilon w .$$

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Russell's Theorem

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$$\vdash \neg \exists x \forall y (y \in x \leftrightarrow y \notin y)$$

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<http://plato.stanford.edu/entries/russell-paradox/#HOTP>

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FregTHEN2

KnightKnave_SmullyanKKPro
blem1.1AthenCfromAthenBandBthen
C

BiconditionalIntroByChaining

BogusBiconditional

CheatersNeverPropser

Contrapositive_NYS_2

Disj_Syll

GreenCheeseMoon2

HypSyll

LarryIsSomehowSmart

Modus_Tollens

RussellsLetter2Frege

ThxForThePCOracle

Explosion

OnlyMediumOrLargeLlamas

GreenCheeseMoon1

Disj_Elim

kok13_28

KingAce2

kok_13_31

☒ RussellsLetter2Frege

The challenge here is to prove that from Russell's instantiation of Frege's doomed Axiom V a contradiction can be promptly derived. The letter has of course been examined in some detail by S Bringsjord (in the Mar 16 2020 lecture in [the 2020 lecture lineup](#)); it, along with an astoundingly soft-spoken reply from Frege, can be found [here](#). Put meta-logically, your task in the present problem is to build a proof that confirms this:

$$\{\exists x \forall y ((y \in x) \rightarrow (y \notin y))\} \vdash \zeta \wedge \neg \zeta.$$

Make sure you understand that the given here is an instantiation of Frege's Axiom V; i.e. it's an instantiation of

$$\exists x \forall y ((y \in x) \rightarrow \phi(y)).$$

(The notation $\phi(y)$, recall, is the standard way in mathematical logic to say that y is free in ϕ .) **Note:** Your finished proof is allowed to make use the PC-provability oracle (but *only* that oracle).

(Now a brief remark on matters covered by in class by Bringsjord when second-order logic = \mathcal{L}_2 arrives on the scene: Longer term, and certainly constituting evidence of Frege's capacity for ingenious, intricate deduction, it has recently been realized that while Frege himself relied on Axiom V to obtain what is known as **Hume's Principle** (= HP), this reliance is avoidable. That from just HP we can deduce all of Peano Arithmetic (**PA**) (!) is a result Frege can be credited with showing; the result is known today as [Frege's Theorem](#) (= FT). Following the link just given will reward the reader with an understanding of HP, and how how to obtain **PA** from it.)

Deadline 22 Apr 2020 23:59:00 EST

[Solve](#)

The Foundation Crumbles

The Rest of Math,
Engineering, etc.

Foundation



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Axiom V etc.

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Axiom V etc.

Axiom V $\exists x \forall y [y \in x \leftrightarrow \phi(y)]$

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Axiom V $\exists x \forall y [y \in x \leftrightarrow \phi(y)]$

a formula of arbitrary size in which the variable y is free; this formula ascribes a property to y

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Axiom V $\exists x \forall y [y \in x \leftrightarrow \phi(y)]$

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It's not just Russell's Paradox that
destroys naïve set theory:

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destroys naïve set theory:

Richard's Paradox ...

a

a

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Doesn't define
a real number.

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E

Doesn't define
a real number.

Definition of Richard's N :

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E

Doesn't define
a real number.

Doesn't define
a real number.

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aa
ab
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~~aaa~~
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E

Definition of Richard's N :

“The real number whose whole part is zero, and whose n -th decimal is p plus one if the n -th decimal of the real number defined by the n -th member of E is p and p is neither eight nor nine, and is simply one if this n -th decimal is eight or nine.”

Doesn't define
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a
b
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aa
ab
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~~aaa~~
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E

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Proof: N is defined by a finite string taken from the English alphabet, so N is in the sequence E . But on the other hand, by definition of N , for every m , N differs from the m -th element of E in at least one decimal place; so N is not any element of E . Contradiction! **QED**

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 b
 •
 •
 •
 aa
 ab
 •
 •
 •

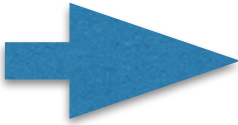
aaa
 •
 •
 •

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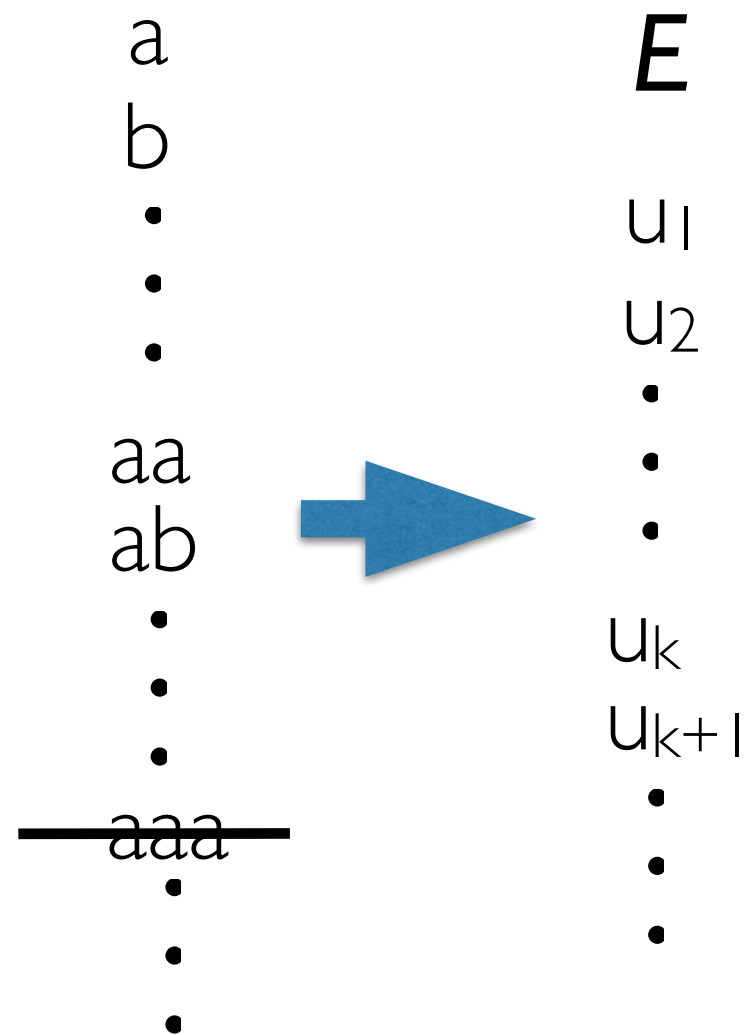
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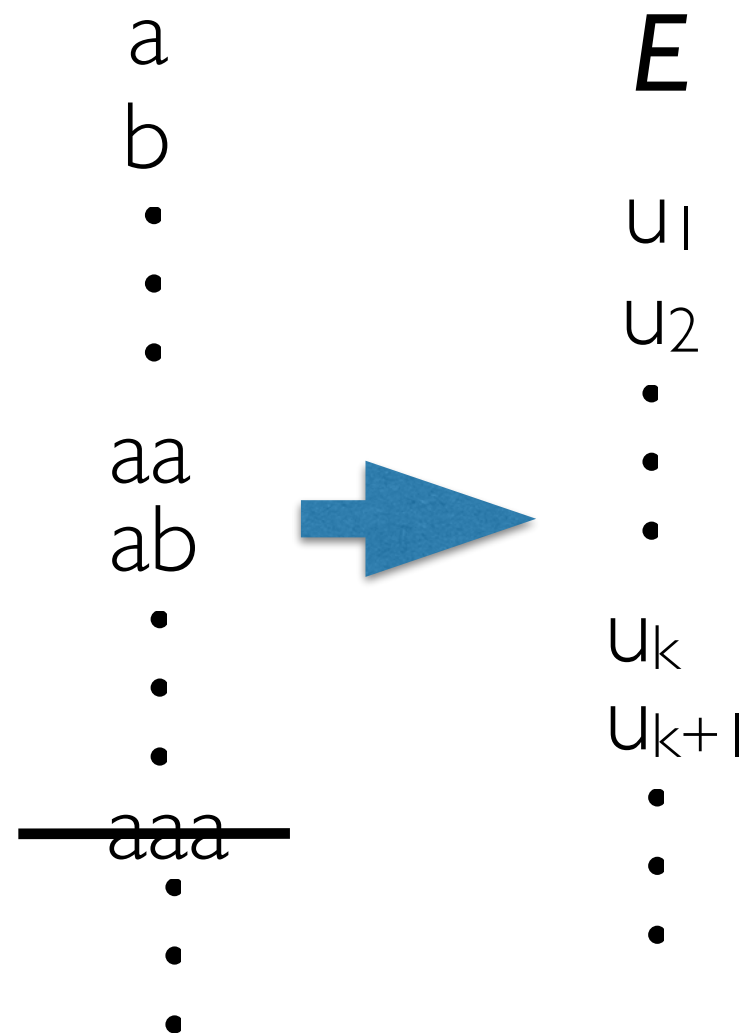
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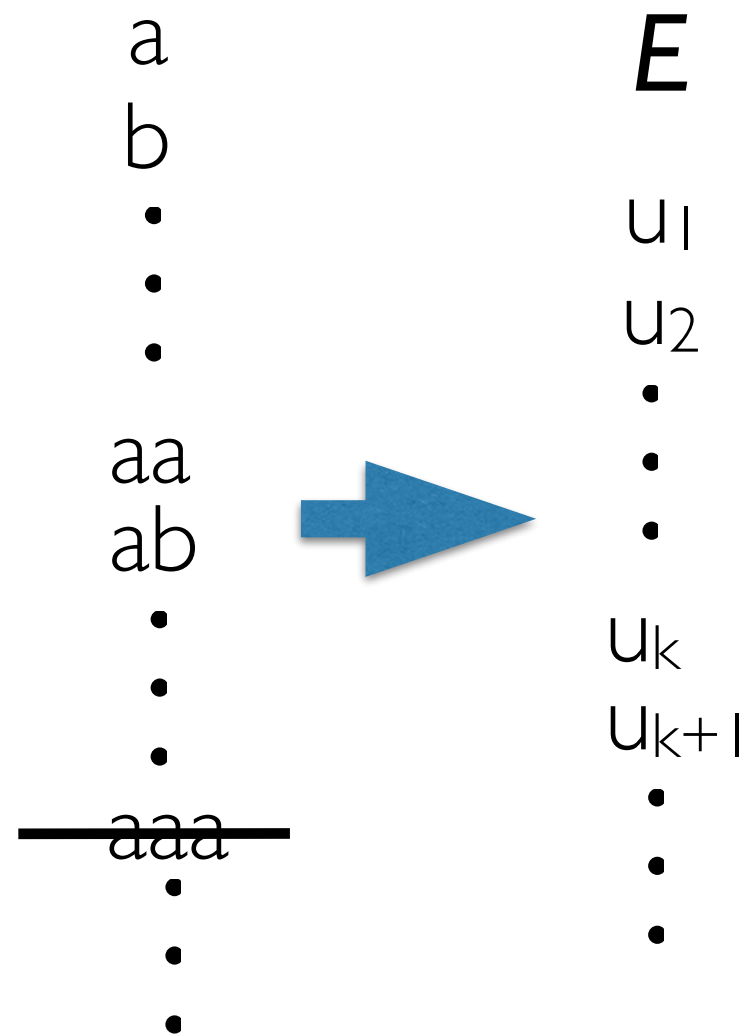


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Suppose N is

u_m .



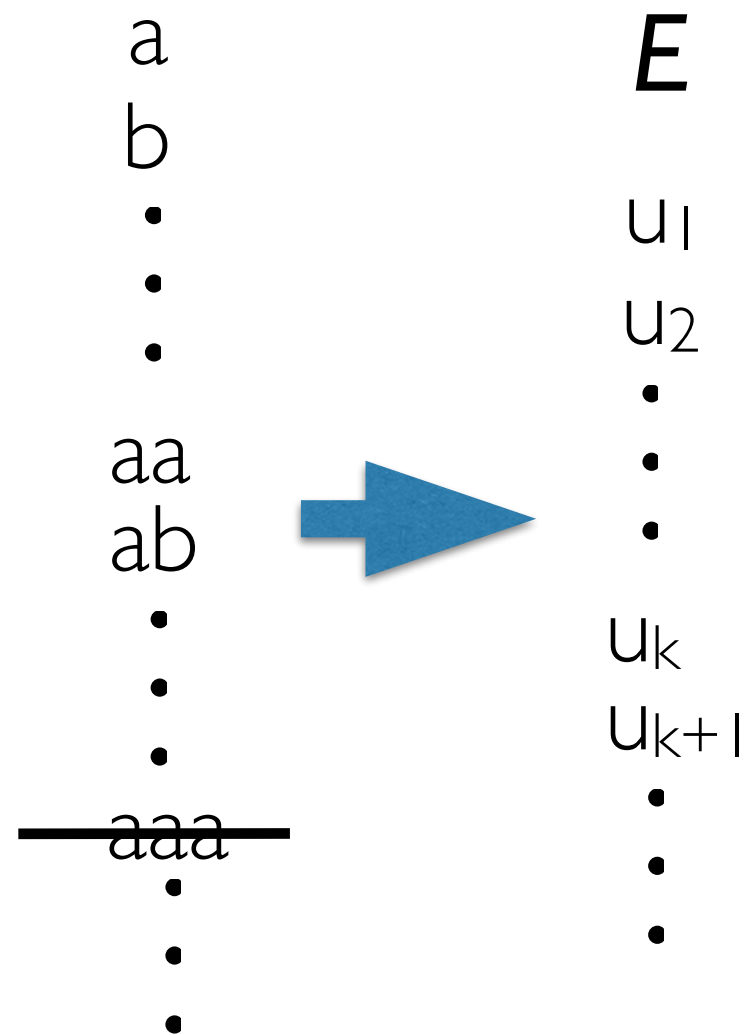
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Suppose N is

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Suppose u_m is 0.xxxx...xxx...



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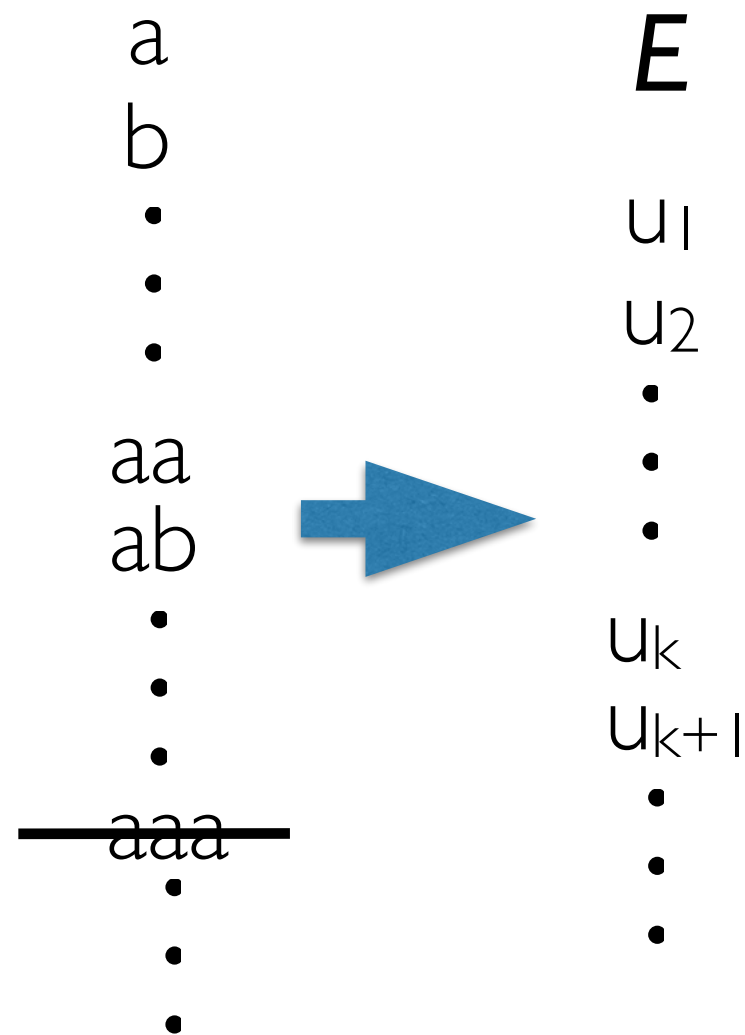
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Suppose N is

u_m .

Suppose u_m is $0.xxxx...xxx...$

↑
mth



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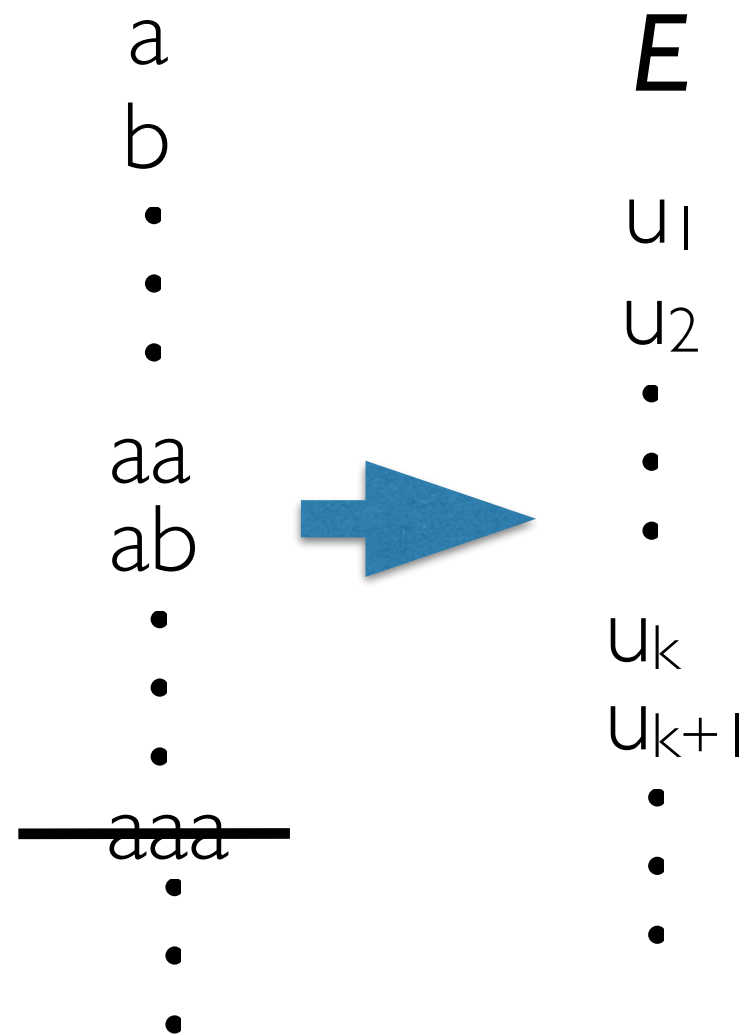
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↑
mth

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↑
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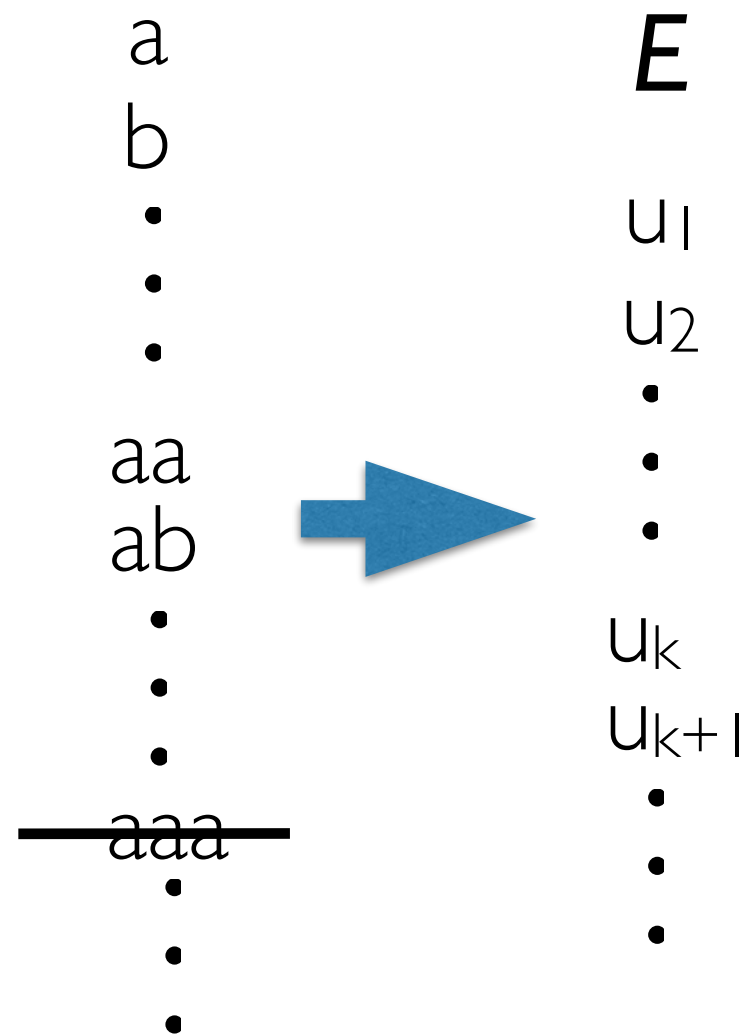
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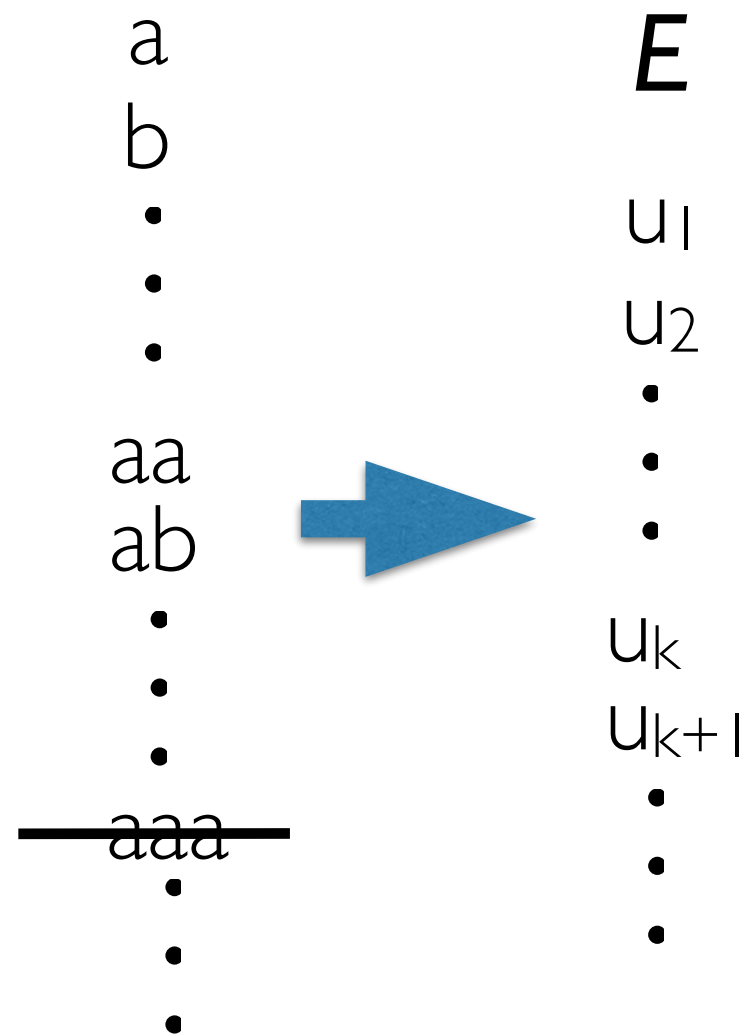
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Suppose u_m is $0.xxxx...8xx...$

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mth

Then N is $0.xxxx...1xx...$

↑
mth



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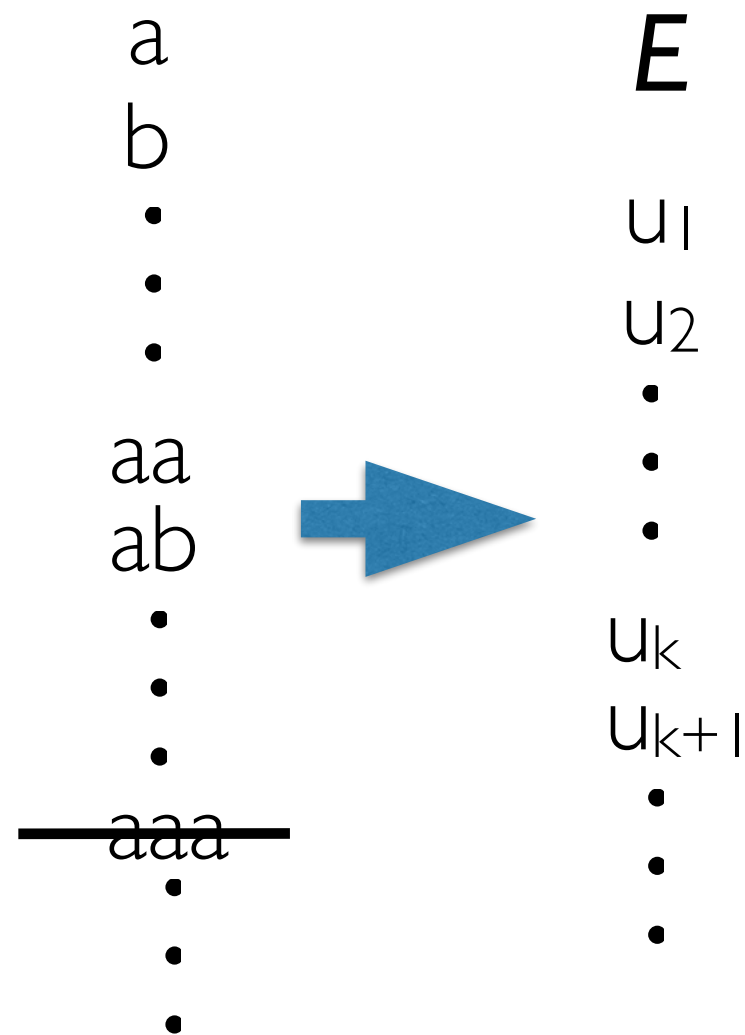
Suppose u_m is $0.xxxx...8xx...$

↑
mth

Then N is $0.xxxx...1xx...$

↑
mth

Since $8 \neq 1$, N can't be u_m !



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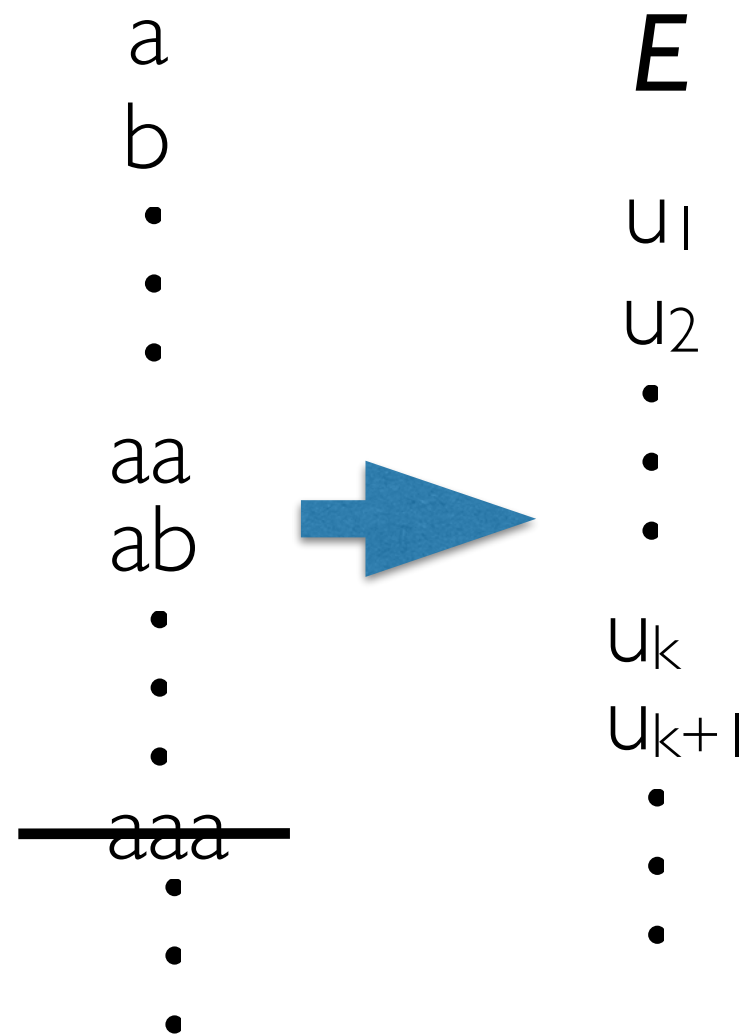
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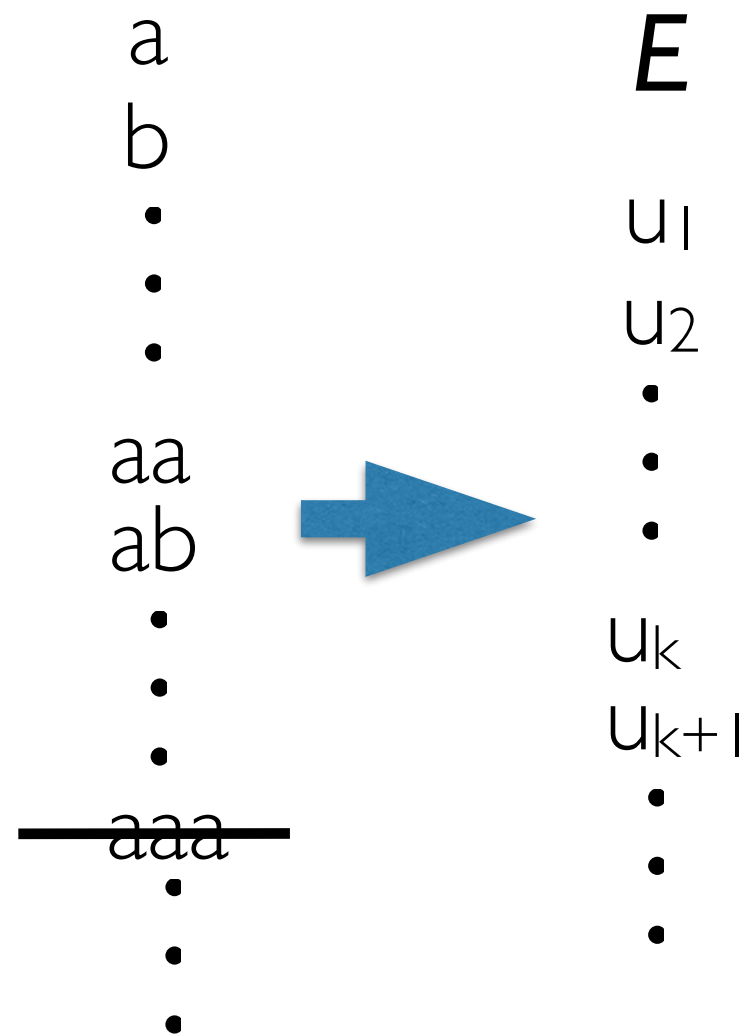
Suppose u_m is $0.xxxx...5xx...$

↑
 m th

Then N is $0.xxxx...1xx...$

↑
 m th

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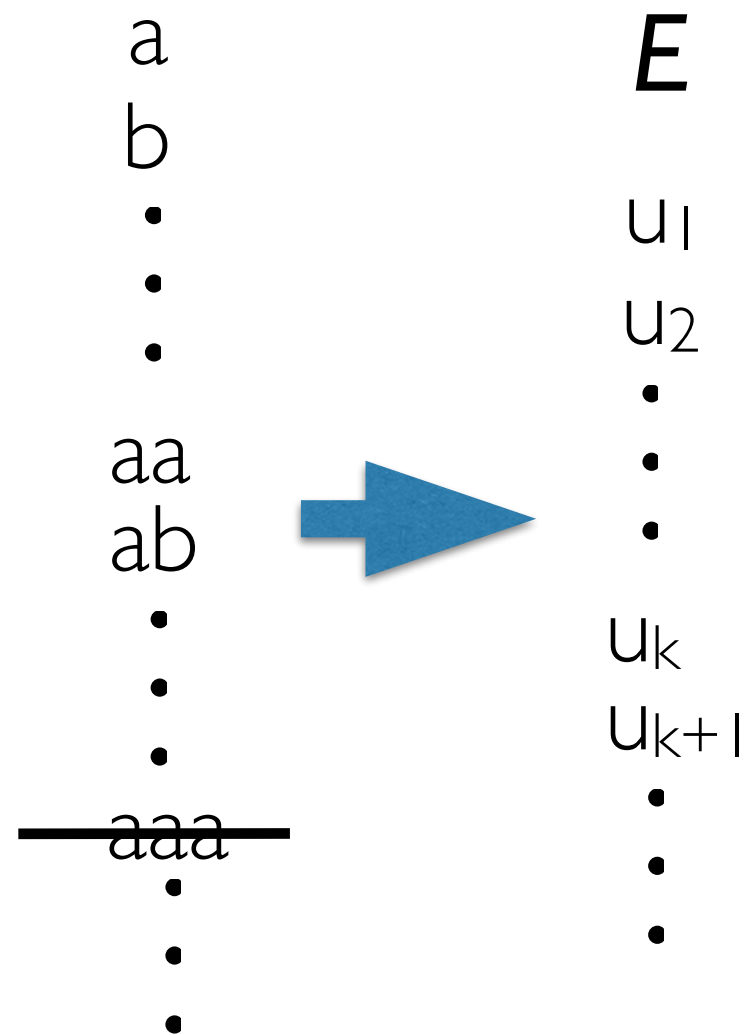
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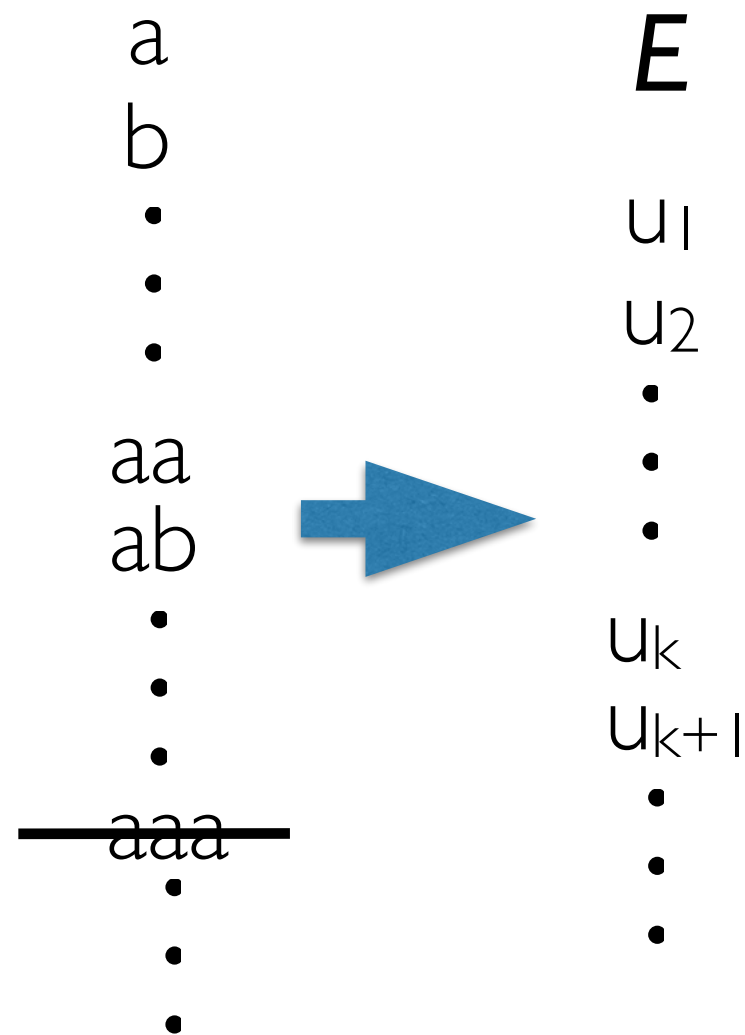
Then N is $0.xxxx...1xx...$

↑
mth

Then N is $0.xxxx...6xx...$

↑
mth

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Suppose u_m is $0.xxxx...xxx...$

↑
mth

Suppose u_m is $0.xxxx...8xx...$

↑
mth

Then N is $0.xxxx...1xx...$

↑
mth

Since $8 \neq 1$, N can't be u_m !

Suppose u_m is $0.xxxx...5xx...$

↑
mth

Then N is $0.xxxx...6xx...$

↑
mth

Since $5 \neq 6$, N can't be u_m !

The Foundation Rebuilt

The Rest of Math,
Engineering, etc.

New Foundation

The Foundation Rebuilt

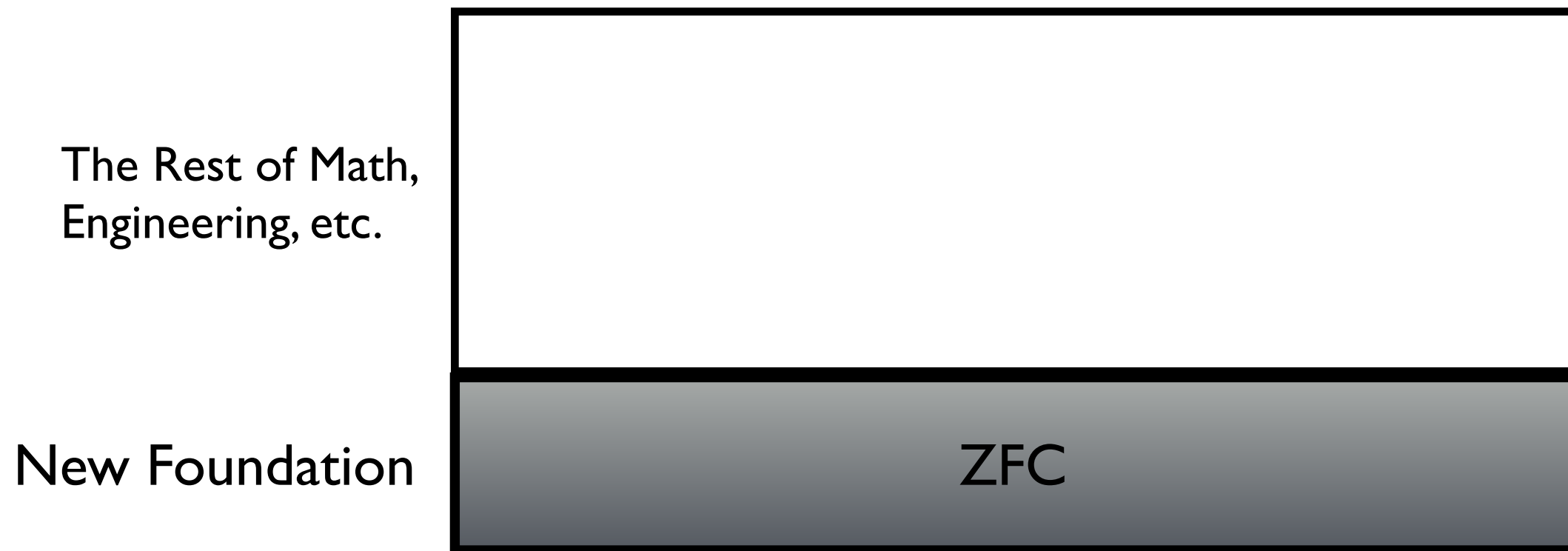
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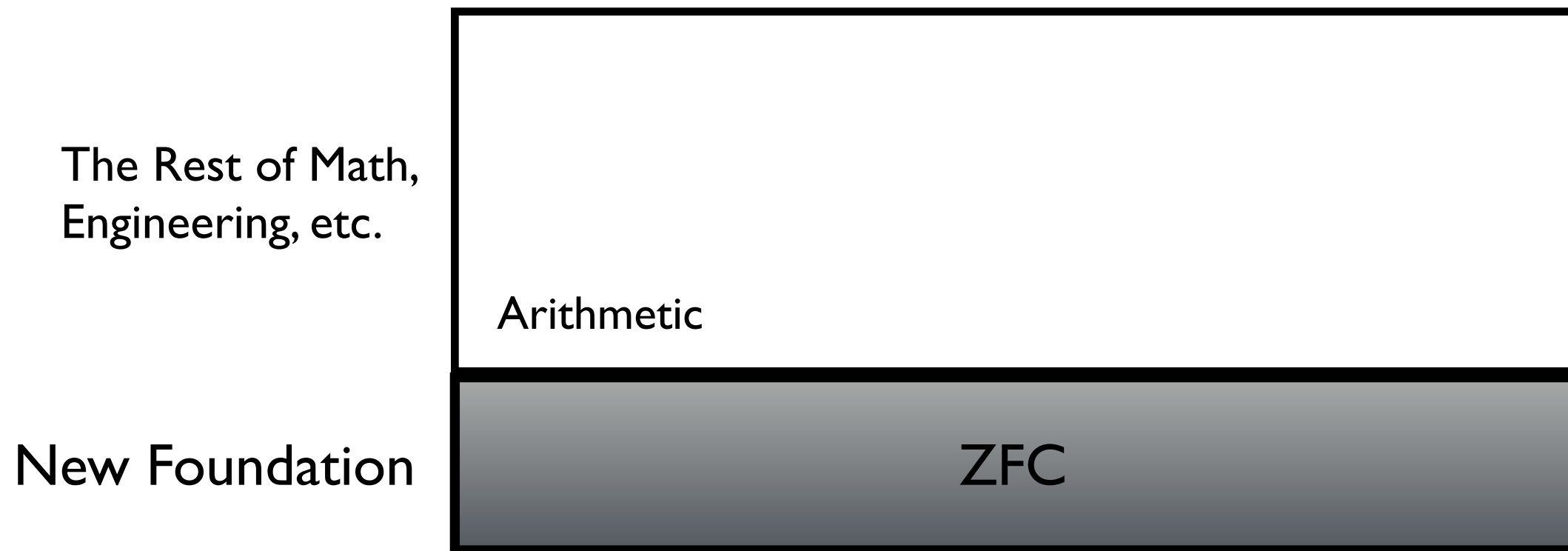


ZFC

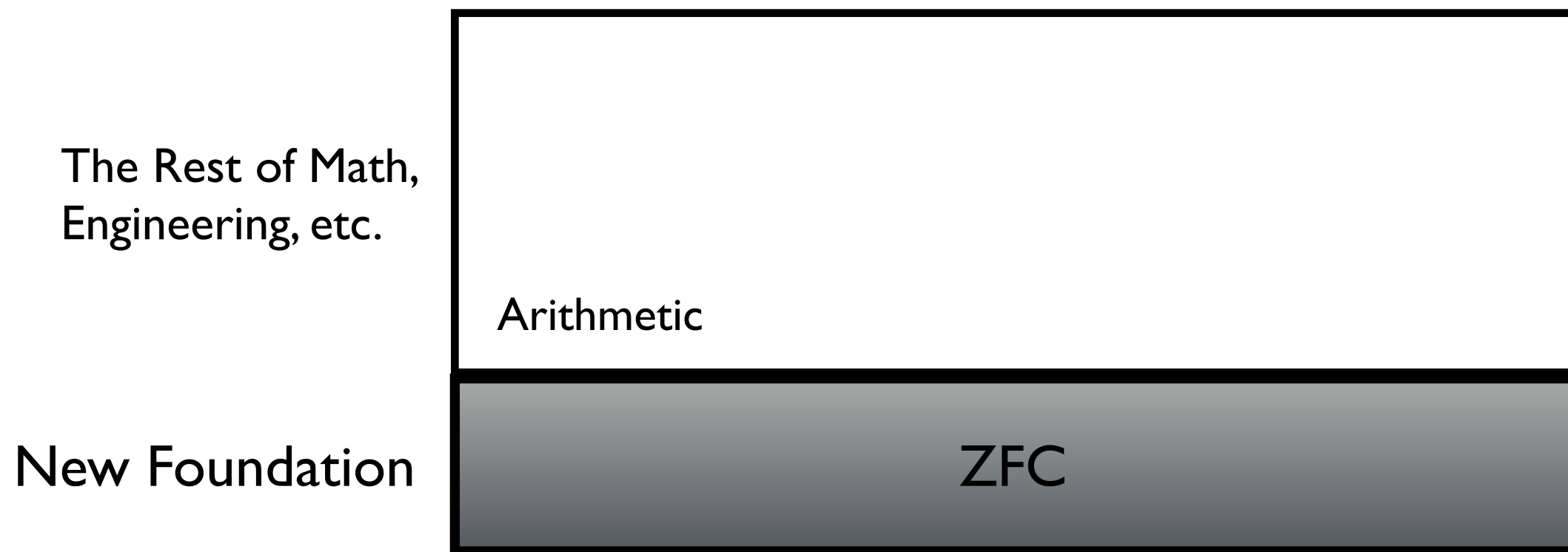
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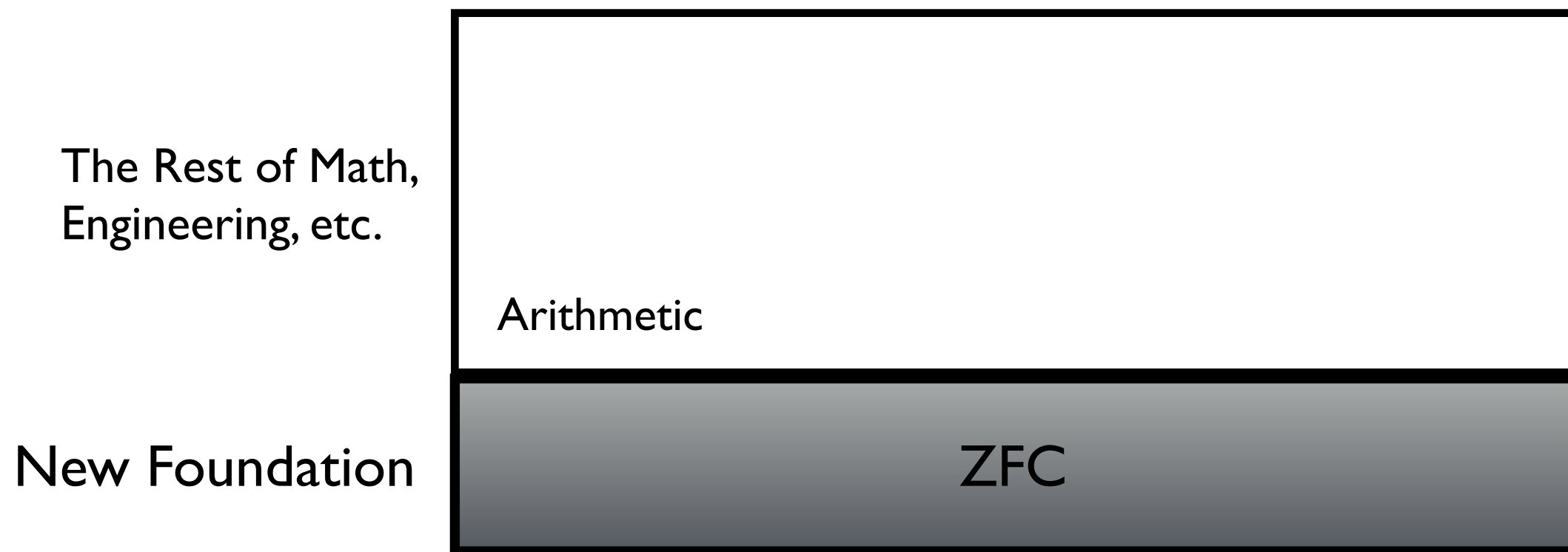


The Foundation Rebuilt



So what are the axioms in ZFC?

The Foundation Rebuilt



So what are the axioms in ZFC?

Axiom *Schema* of Separation (SEP)

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—
SEP

$$\forall x_1 \dots \forall x_k \forall x \exists y \forall z [z \in y \leftrightarrow (z \in x \wedge \phi(z, x_1, \dots, x_k))]$$

where x and y are distinct, and are both distinct from z and the x_i ;
and, as usual for us now, ϕ expresses a property using \in .

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where x and y are distinct, and are both distinct from z and the x_i ;
and, as usual for us now, ϕ expresses a property using \in .

—
“Given *beforehand* some set x and property \mathcal{P}
captured by a formula ϕ that uses \in for its relation,
the set y composed of $\{z \in x : \mathcal{P}(z)\}$ exists.”

Axiom *Schema* of Separation (SEP)

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SEP

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How does this neutralize
Russell's letter to Frege?

Formal Natural- Number Arithmetic ...

Q (= Robinson Arithmetic)

A1 $\forall x(0 \neq s(x))$

A2 $\forall x \forall y (s(x) = s(y) \rightarrow x = y)$

A3 $\forall x (x \neq 0 \rightarrow \exists y (x = s(y)))$

A4 $\forall x (x + 0 = x)$

A5 $\forall x \forall y (x + s(y) = s(x + y))$

A6 $\forall x (x \times 0 = 0)$

A7 $\forall x \forall y (x \times s(y) = (x \times y) + x)$

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A2 $\forall x \forall y (s(x) = s(y) \rightarrow x = y)$

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A6 $\forall x (x \times 0 = 0)$

A7 $\forall x \forall y (x \times s(y) = (x \times y) + x)$

PA (Peano Arithmetic)

$$\text{A1} \quad \forall x(0 \neq s(x))$$

$$\text{A2} \quad \forall x \forall y (s(x) = s(y) \rightarrow x = y)$$

$$\text{A3} \quad \forall x (x \neq 0 \rightarrow \exists y (x = s(y)))$$

$$\text{A4} \quad \forall x (x + 0 = x)$$

$$\text{A5} \quad \forall x \forall y (x + s(y) = s(x + y))$$

$$\text{A6} \quad \forall x (x \times 0 = 0)$$

$$\text{A7} \quad \forall x \forall y (x \times s(y) = (x \times y) + x)$$

And, every sentence that is the universal closure of an instance of

$$([\phi(0) \wedge \forall x(\phi(x) \rightarrow \phi(s(x)))] \rightarrow \forall x \phi(x))$$

where $\phi(x)$ is open wff with variable x , and perhaps others, free.

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This open wff $\phi(x)$ expresses the arithmetic property ‘even.’

Slutten