

Standard Deontic Logic (SDL = D)

Isn't Going to Cut It!

(Chisholm's Paradox; The Free Choice Permission Paradox)

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Troy, New York 12180 USA

IFLAI
4/5/2021





Problem Bank



Edit Problems

Required

Metrics for
Required

Download: LAMA-
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Problems



AbigailAndBarneyDontShareAll2



DoubleMindedMan1



RussellsLetter2Frege

Test 2

Bonus



Test 1

Required

Exercises

Personalized

All

AbigailAndBarneyDontShareAll2

challenge is a rather simple one in second-order logic = $\mathcal{L}_2$. Given that Abigail is fearful, but that Barney isn't, you prove that this pair doesn't share all properties. Certainly what is to be proved here, given the given, is more possible; yet it can't be proved in mere $\mathcal{L}_1$.

permitted to leave remaining use of the PC provability oracle in your completed proof.

Deadline May 2, 2021, 11:59 PM EDT

Solve



Submissions



Metrics



DoubleMindedMan1

This problem is based on the higher-order-logic elements behind the scenes in the short-short three-scene screenplay *Double-Minded Man*, co-authored by Selmer and Alexander Bringsjord, and available [here](#). Your task is to build a simple proof in second-order logic, one that shows that there is a property that Joe has that Harriet's mental model of Joe doesn't have. Note that you are never, in working on this problem, to invoke the FOL provability oracle in the GOAL node. However, you *can* use the FOL provability oracle as long as it is used to only instantiate universal formulae, and you can leave this use in your finished proof. When you use this oracle in this manner, you can remove only one variable.



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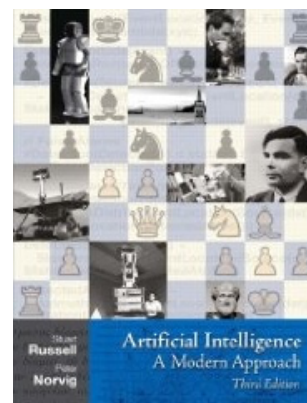
“We’re in *very* deep trouble.”

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The PAID Problem!

Actually, it's quite simple:
“Equation” for Why Stakes are High

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$$u(\text{AIA}_i(\pi_j)) > \tau^+ \in \mathbb{Z} \text{ or } \tau^- \in \mathbb{Z}$$

Actually, it's quite simple: “Equation” for Why Stakes are High

$$\begin{array}{c} \forall x : \text{Agents} \\ \text{Powerful}(x) \\ \downarrow \\ u(\text{AIA}_i(\pi_j)) > \end{array}$$

Dangerous(x)

Are Autonomous-and-Creative Machines Intrinsically Untrustworthy?*

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Troy NY 12180 USA

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Abstract

Given what we find in the case of human cognition, the following principle appears to be quite plausible: An artificial agent that is both autonomous (A) and creative (C) will tend to be, from the viewpoint of a rational, fully informed agent, (U) untrustworthy. After briefly explaining the intuitive, internal structure of this disturbing principle, in the context of the human sphere, we provide a more formal rendition of it designed to apply to the realm of intelligent artificial agents. The more-formal version makes use of some of the basic structures available in one of our cognitive-event calculi, and can be expressed as a (confessedly — for reasons explained — naïve) theorem. We prove the theorem, and provide simple demonstrations of it in action, using a novel theorem prover (ShadowProver). We then end by pointing toward some future defensive engineering measures that should be taken in light of the theorem.

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Actually, it's quite simple: “Equation” for Why Stakes are High

$\forall x : \text{Agents}$

$$\text{Powerful}(x) + \text{Autonomous}(x) + \text{Intelligent}(x) = \text{Dangerous}(x)$$



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$$u(\text{AIA}_i(\pi_j)) > \tau^+ \in \mathbb{Z} \text{ or } \tau^- \in \mathbb{Z}$$

Theorem ACU: In a collaborative situation involving agents a (as the “trustor”) and a' (as the “trustee”), if a' is at once both autonomous and ToM-creative, a' is untrustworthy from an ideal-observer o 's viewpoint, with respect to the action-goal pair $\langle \alpha, \gamma \rangle$ in question.

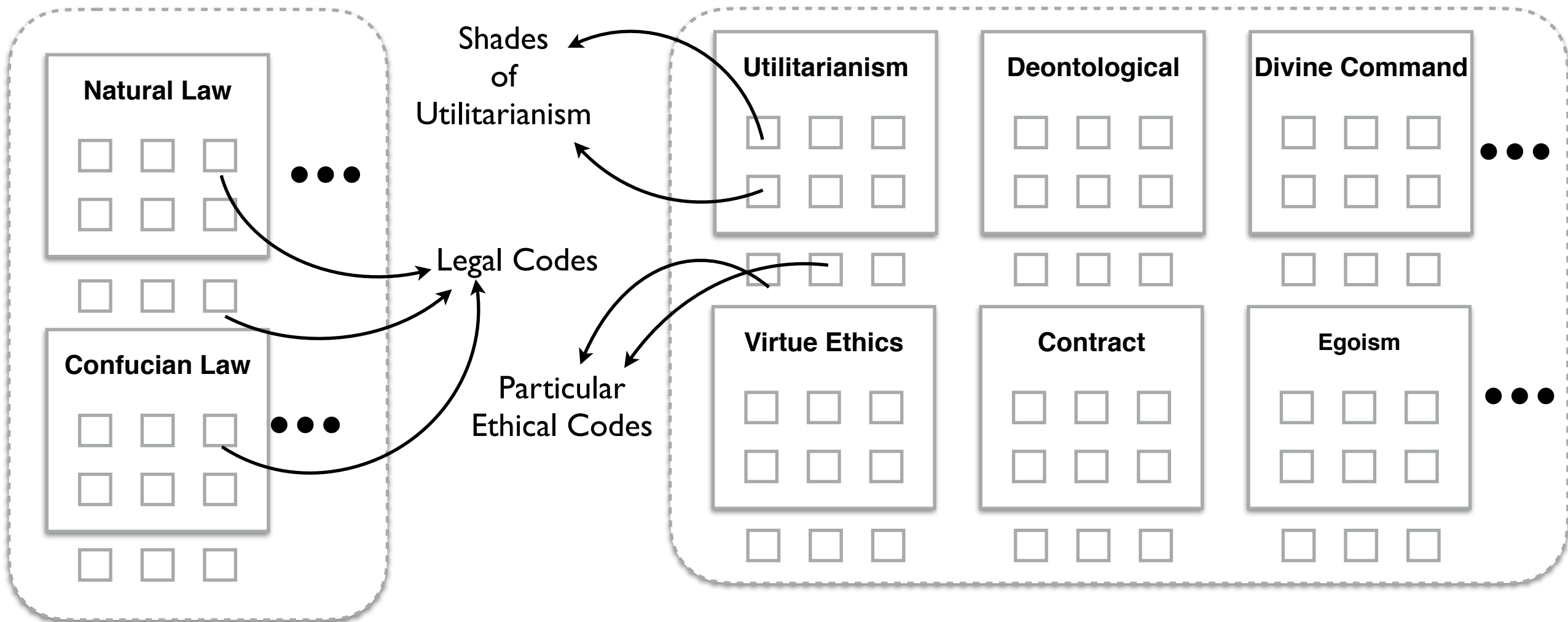
Proof: Let a and a' be agents satisfying the hypothesis of the theorem in an arbitrary collaborative situation. Then, by definition, $a \neq a'$ desires to obtain some goal γ in part by way of a contributed action α_k from a' , a' knows this, and moreover a' knows that a believes that this contribution will succeed. Since a' is by supposition ToM-creative, a' may desire to surprise a with respect to a 's belief regarding a' 's contribution; and because a' is autonomous, attempts to ascertain whether such surprise will come to pass are fruitless since what will happen is locked inaccessibly in the oracle that decides the case. Hence it follows by TRANS that an ideal observer o will regard a' to be untrustworthy with respect to the pair $\langle \alpha, \gamma \rangle$ pair. **QED**

Making Morally X Machines

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Theories of Law

Ethical Theories

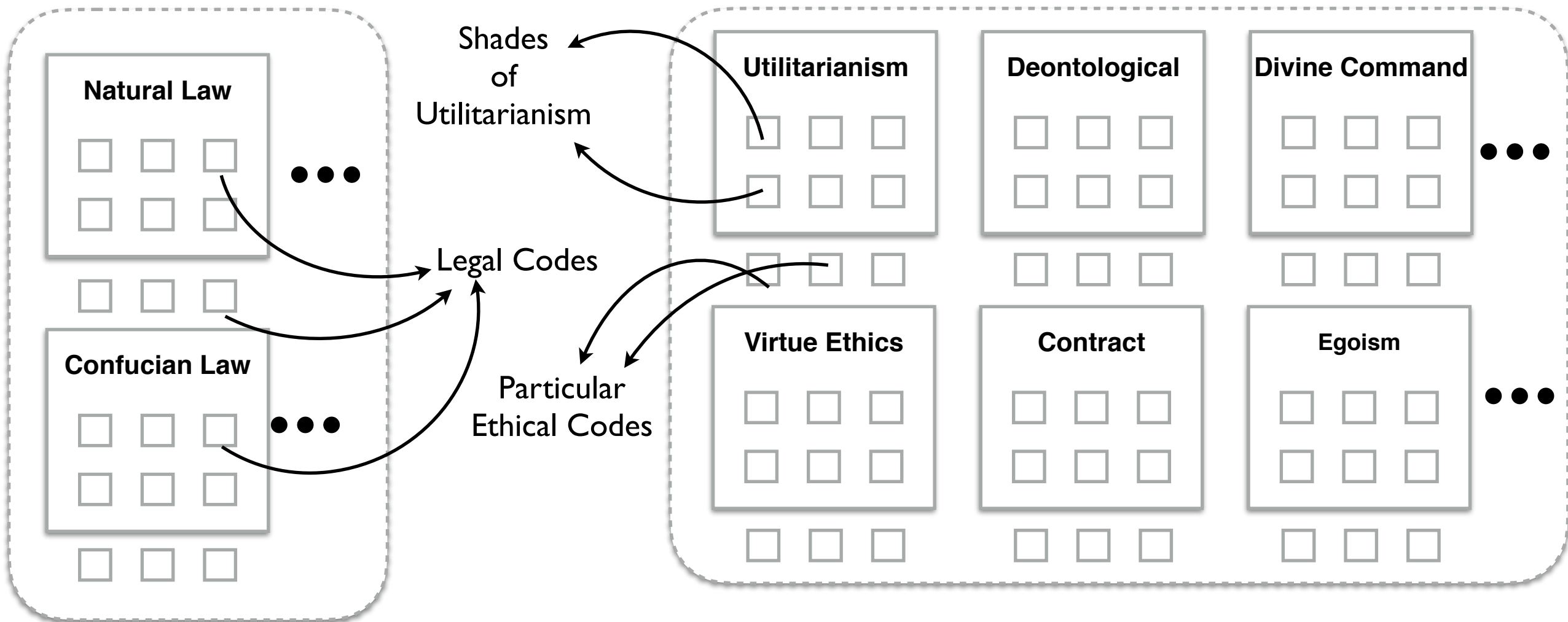


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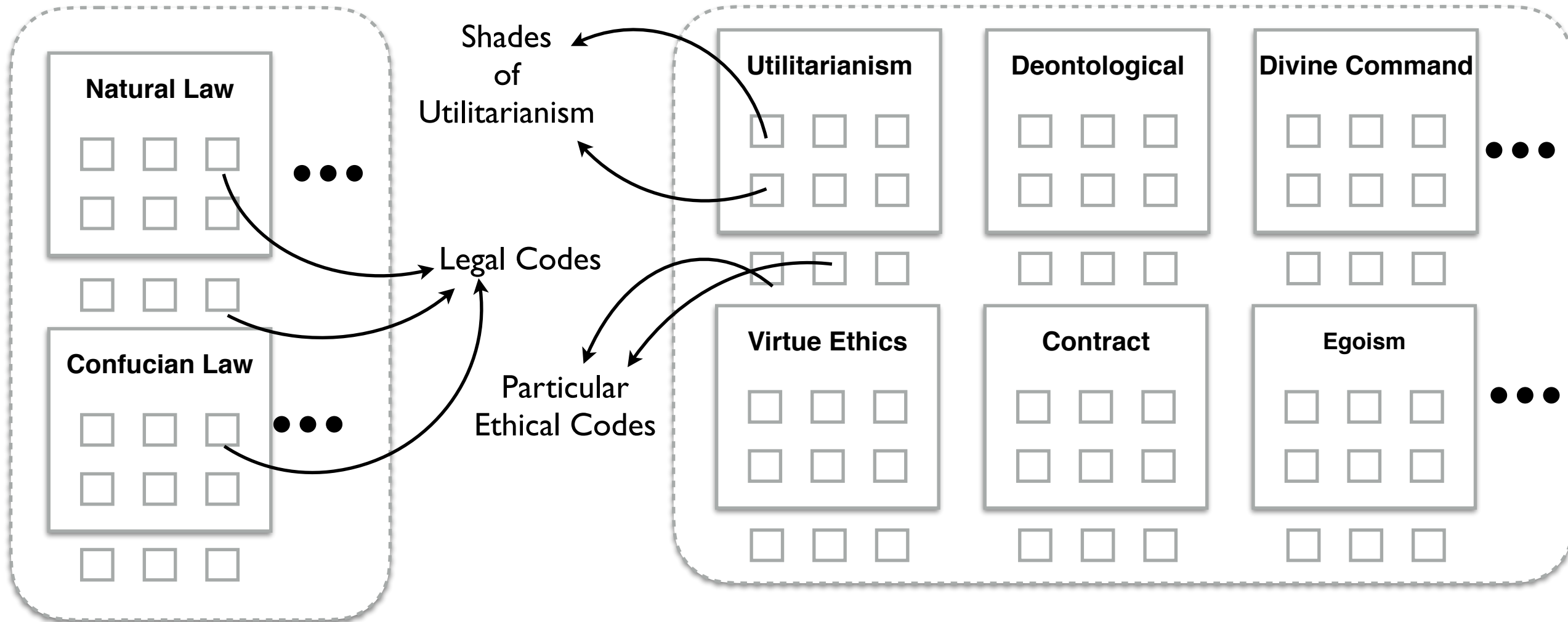


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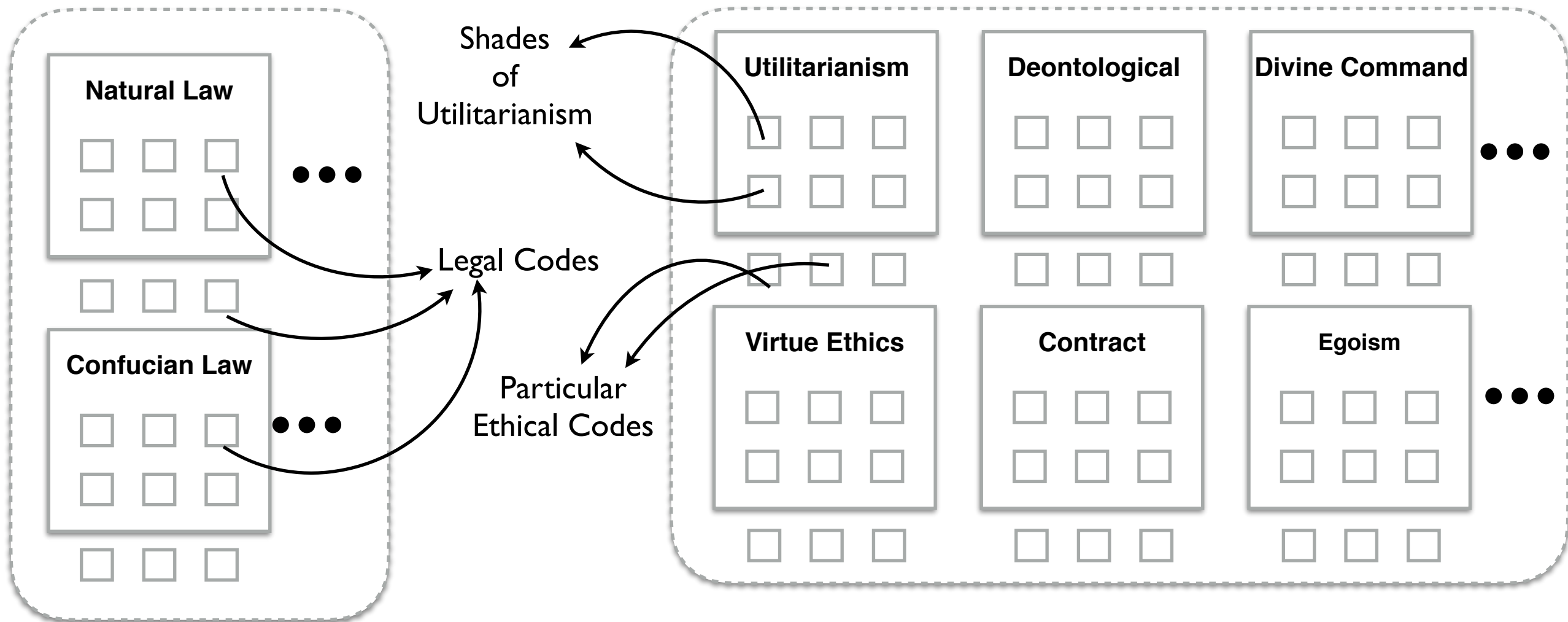


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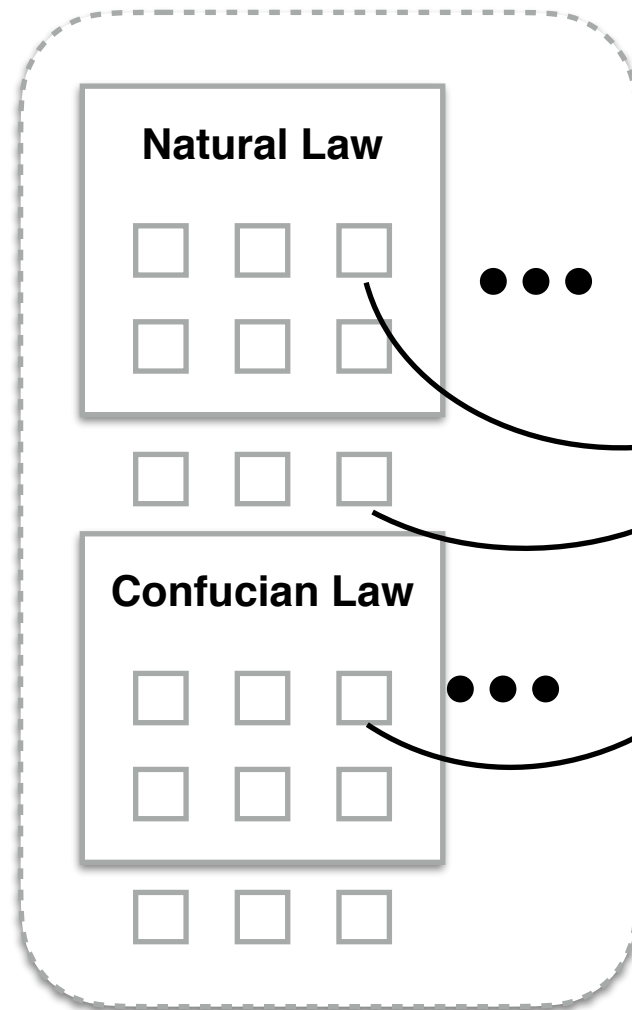
Step I

1. Pick a theory
2. Pick a code
3. Run through EH.

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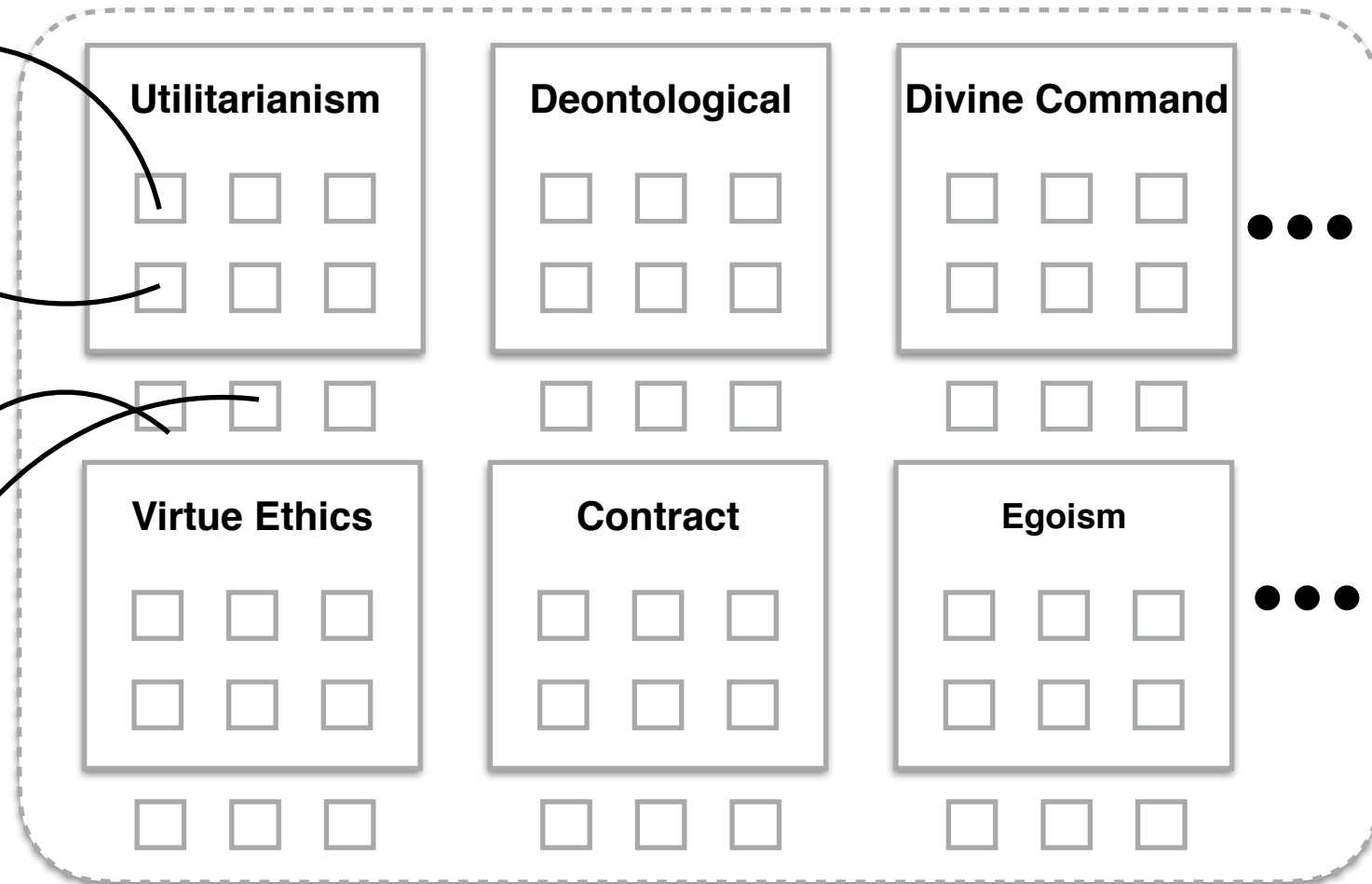


Shades
of
Utilitarianism

Legal Codes

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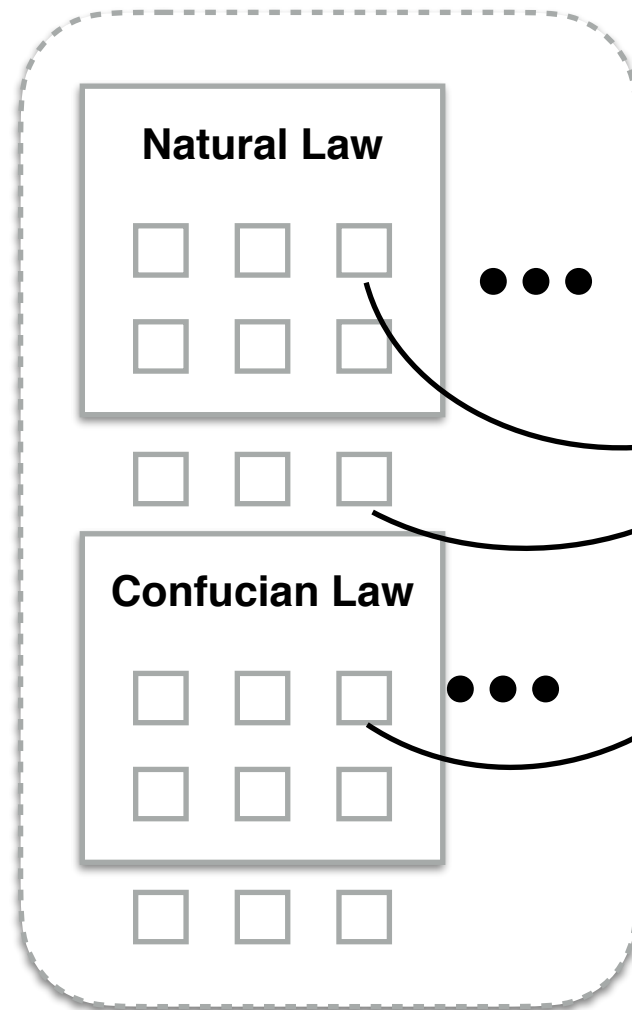
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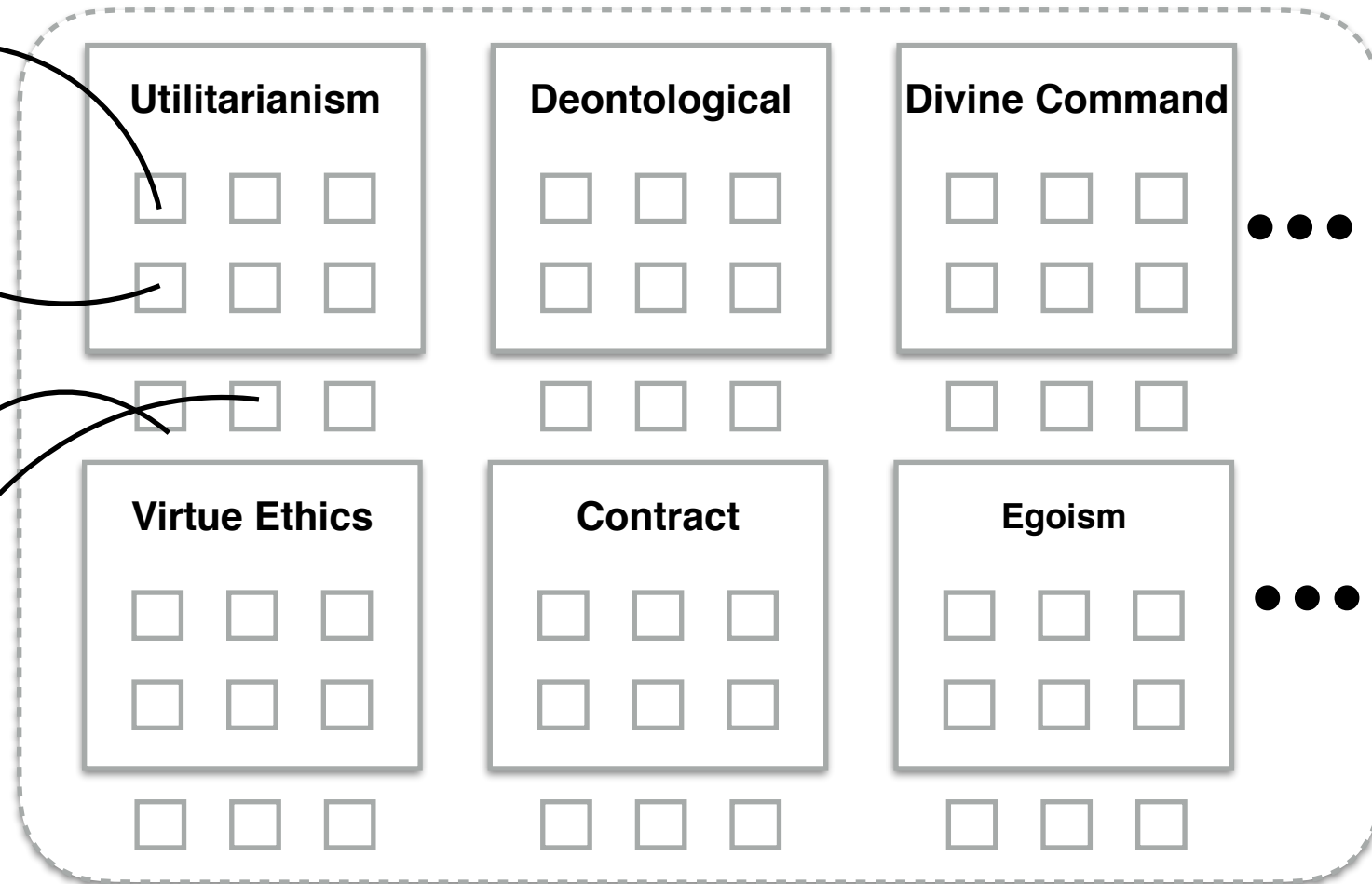


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Step 2

Automate



Prover

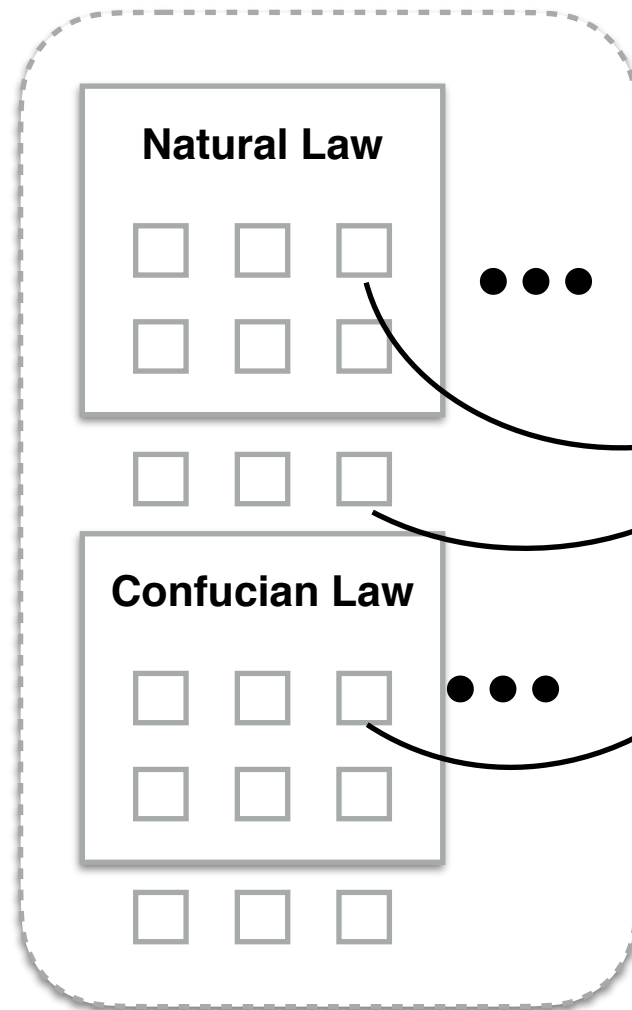


Spectra

Making Morally X Machines

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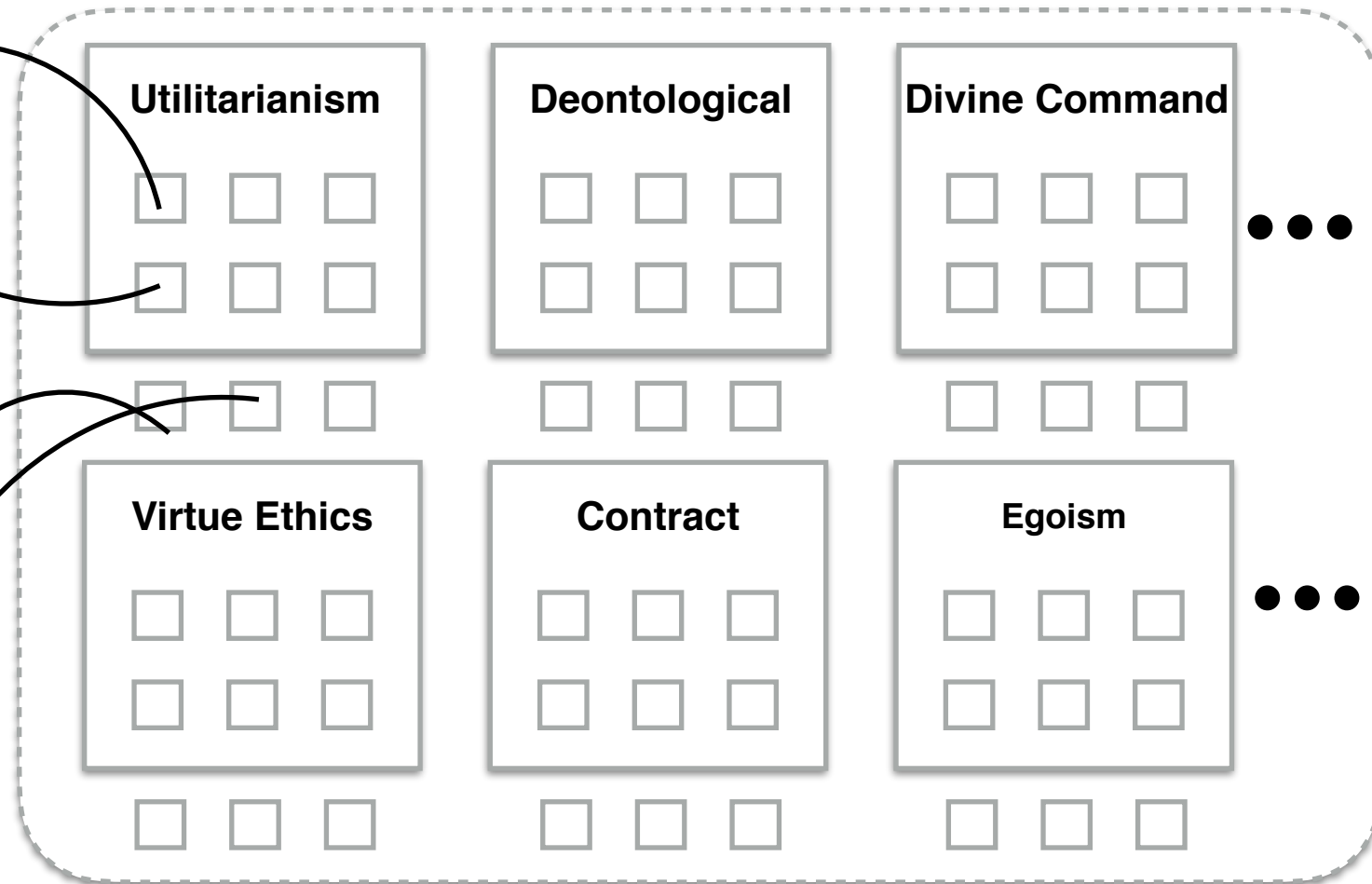


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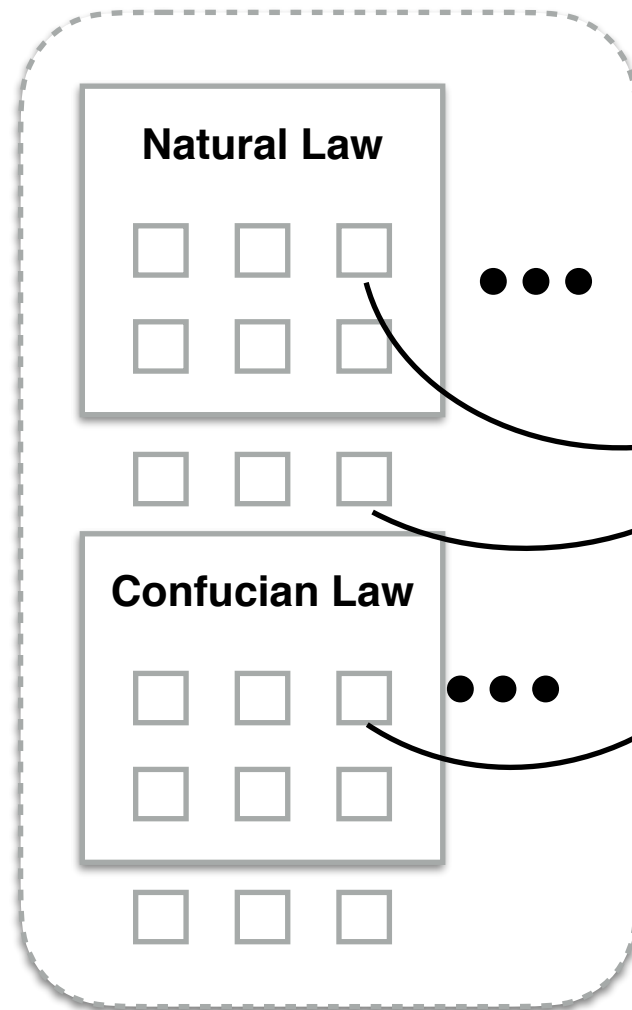


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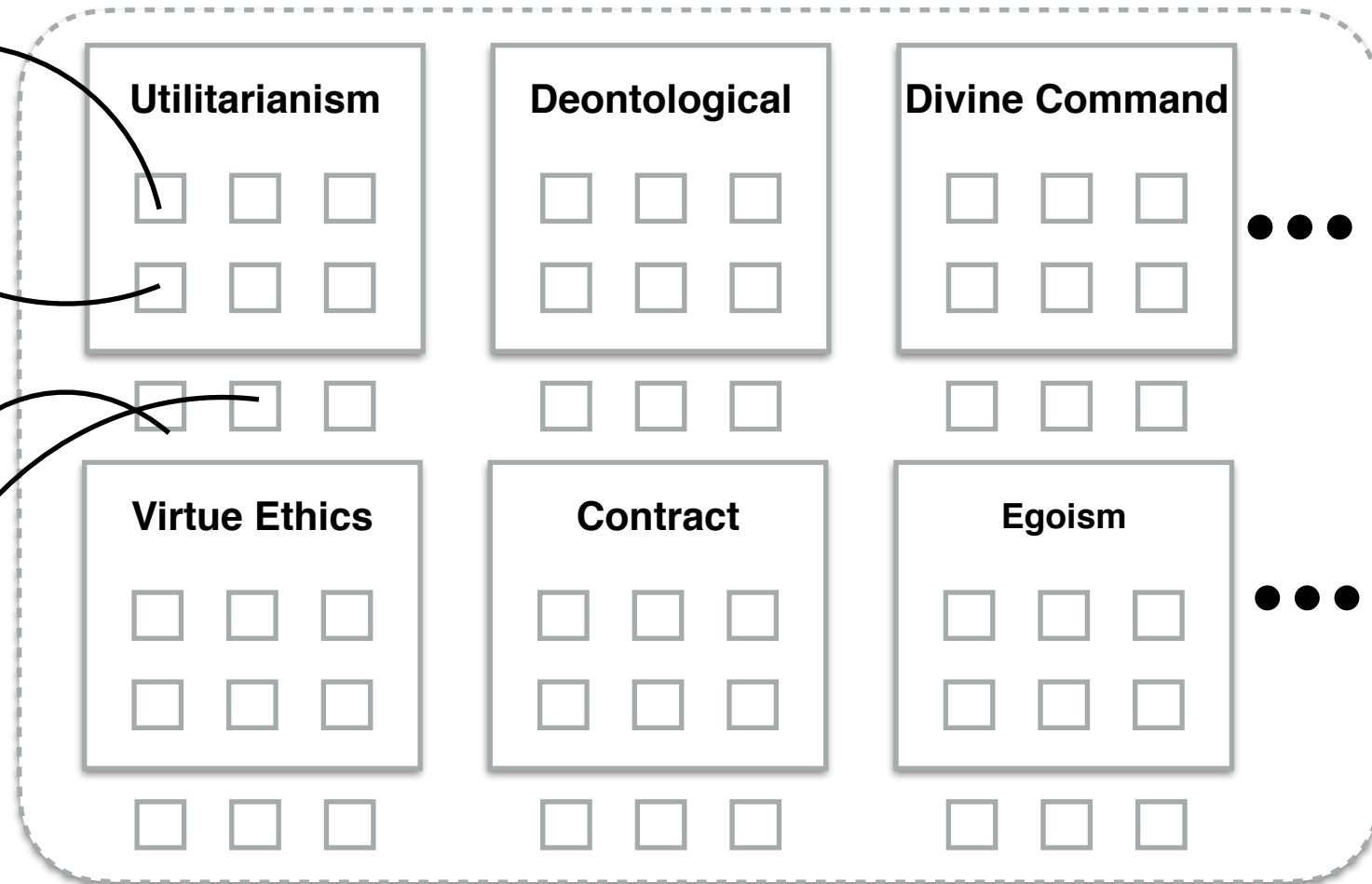


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Step 3

Ethical OS



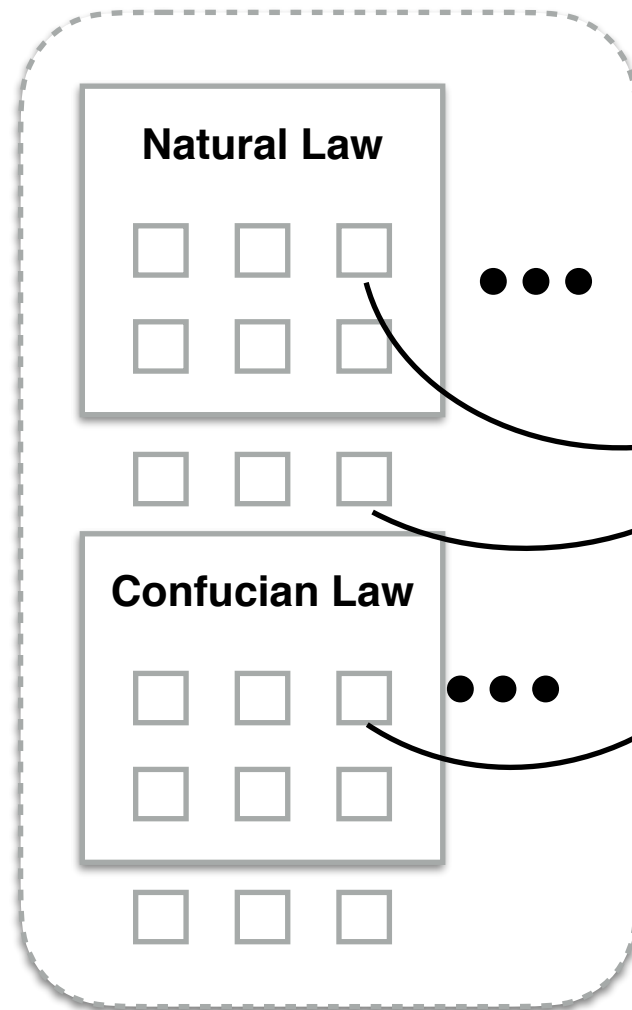
Ethical Substrate

Robotic Substrate

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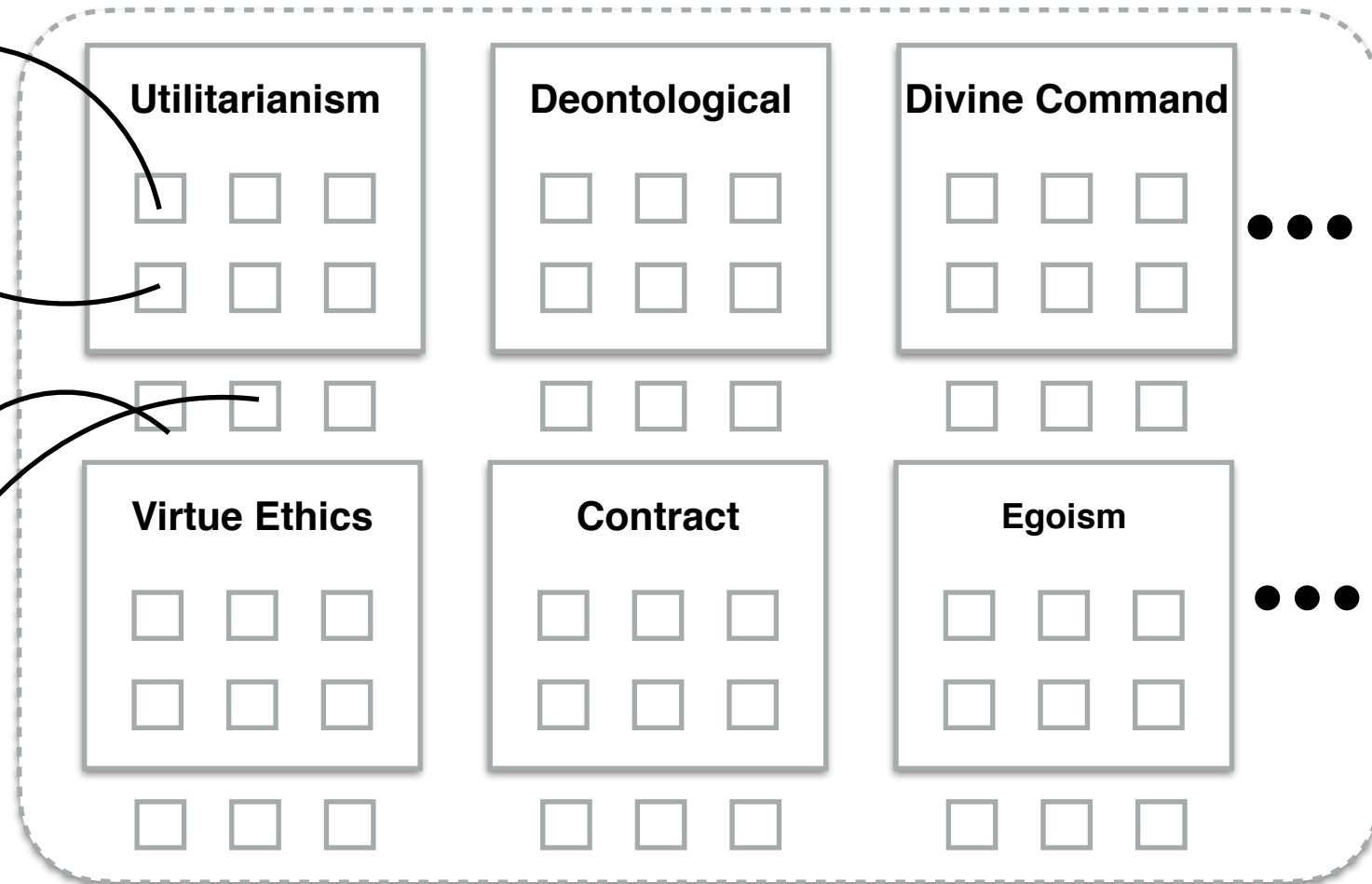


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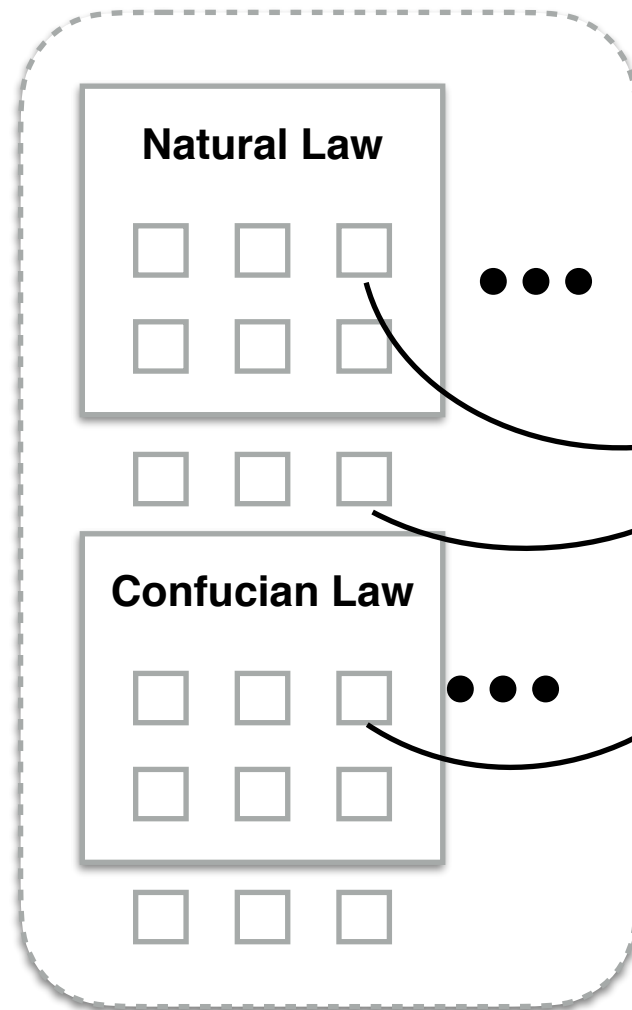
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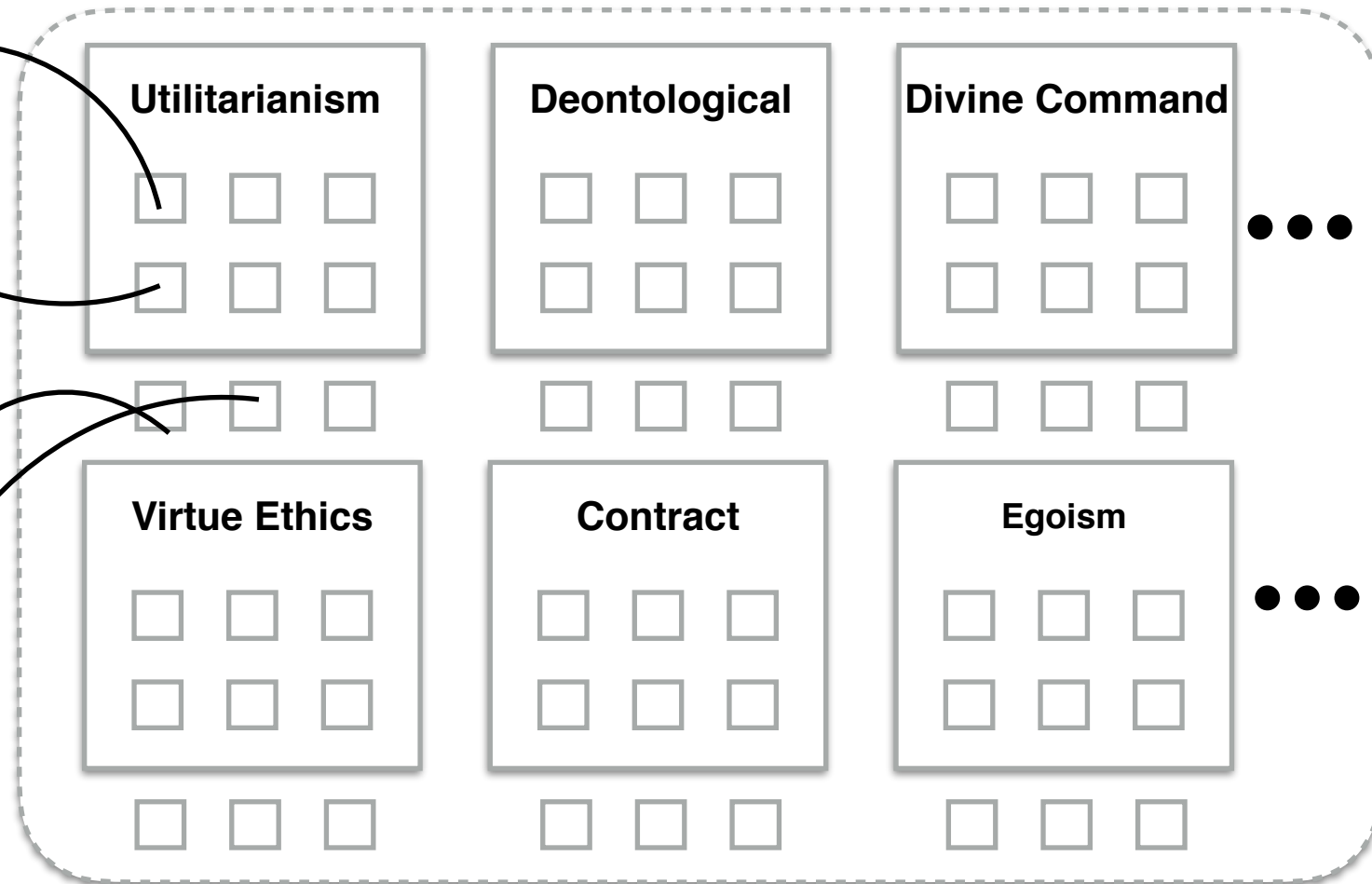


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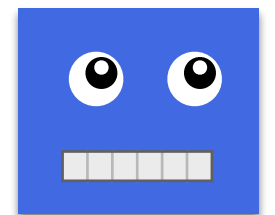
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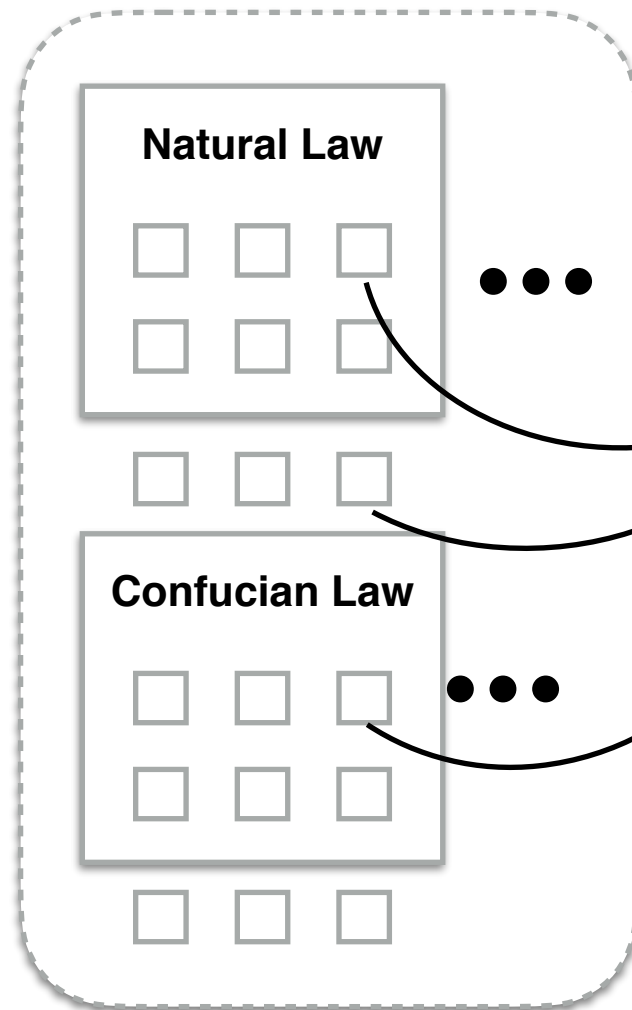
Robotic Substrate



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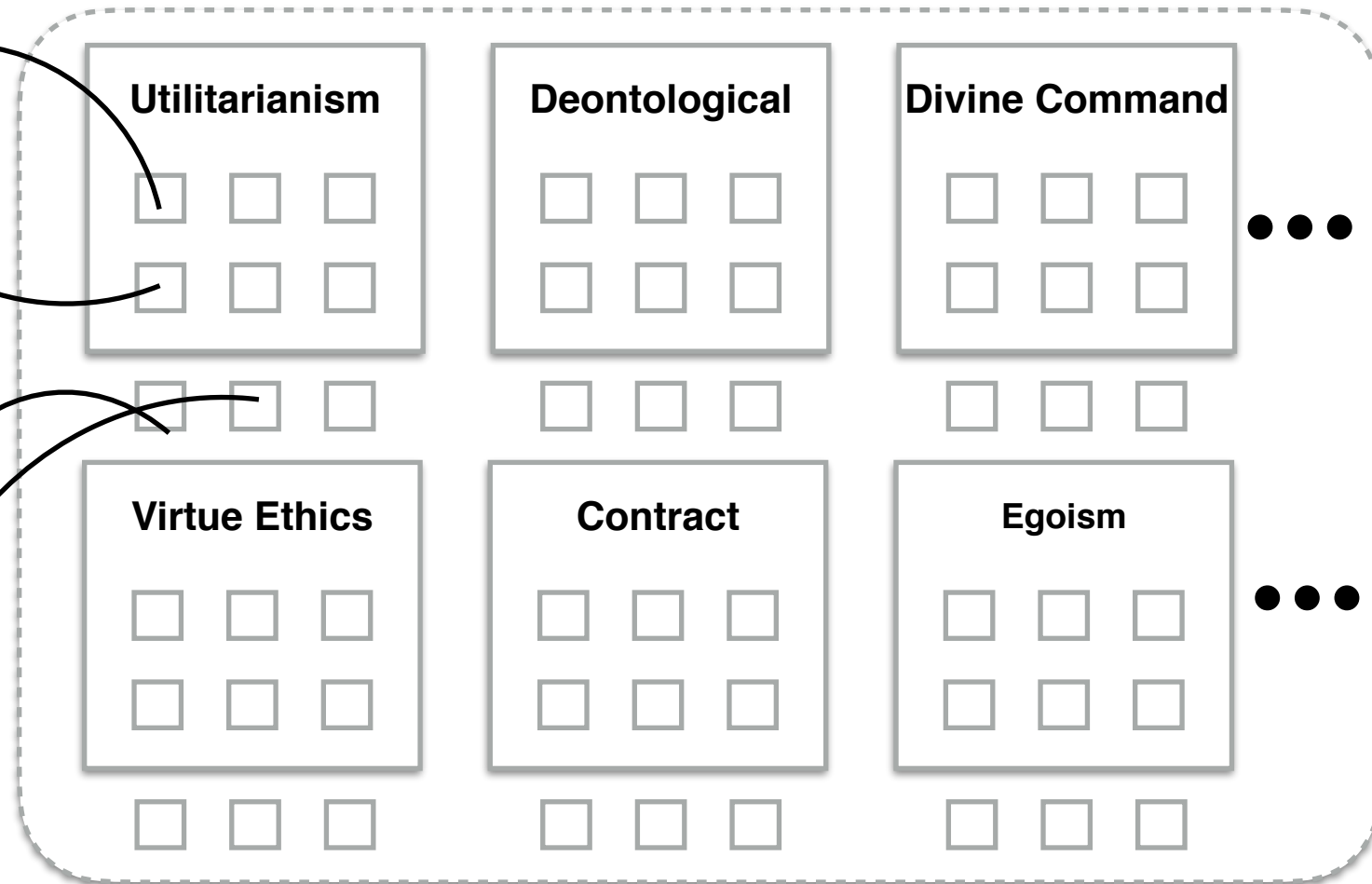


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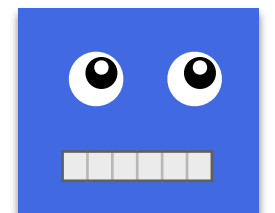
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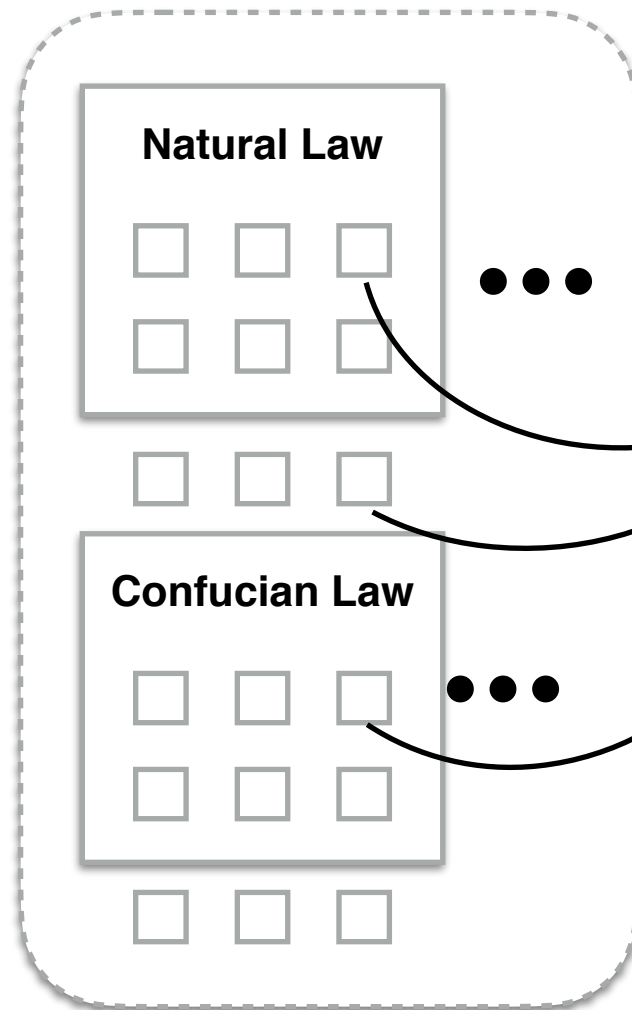
Ethical Robot



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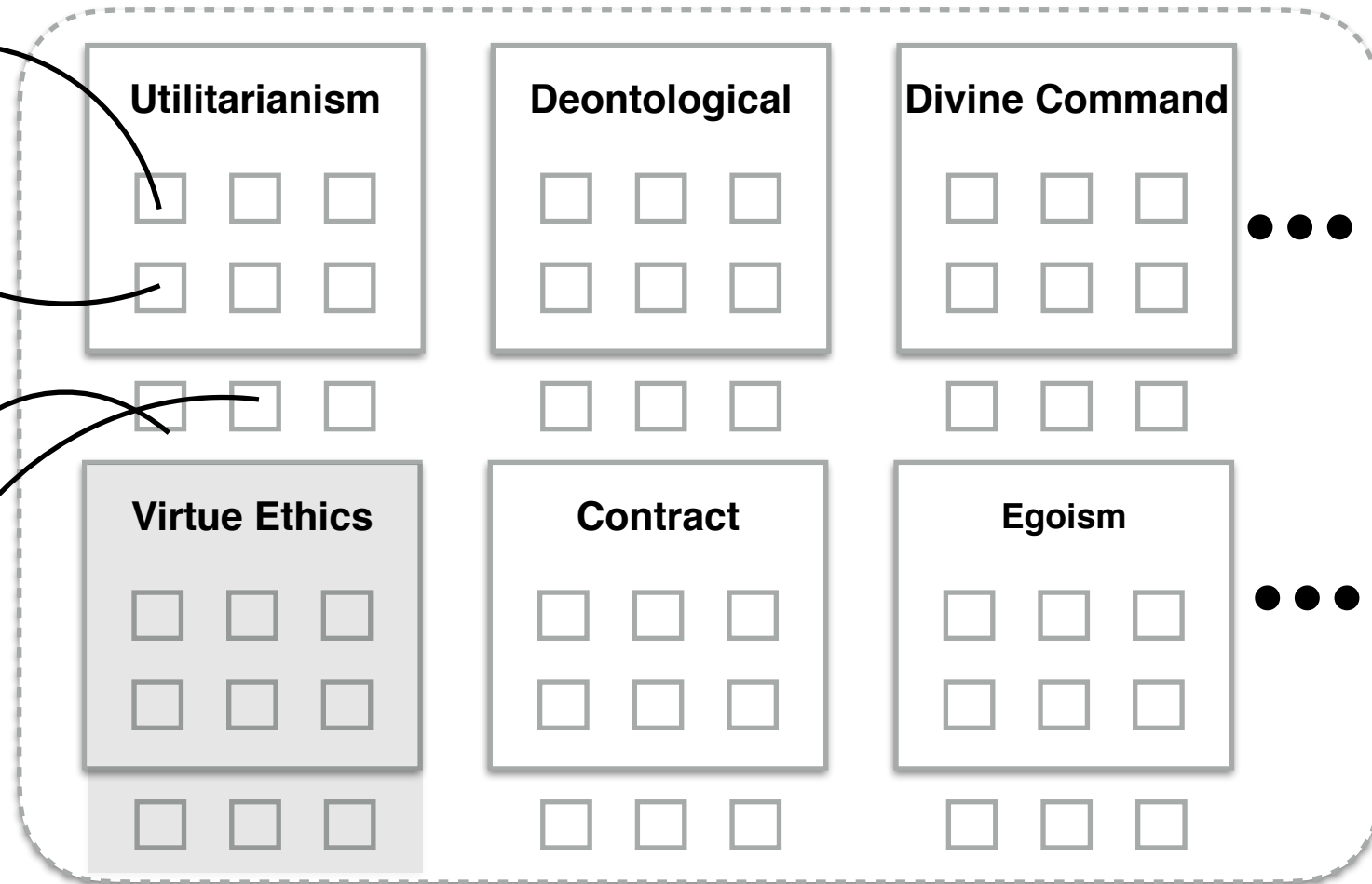


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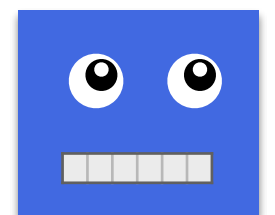
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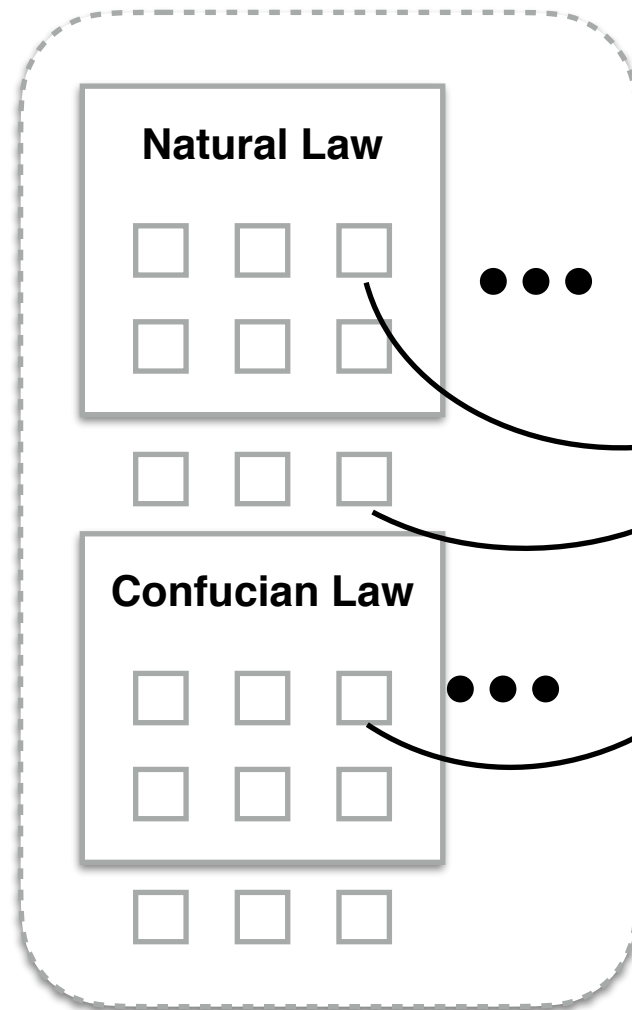
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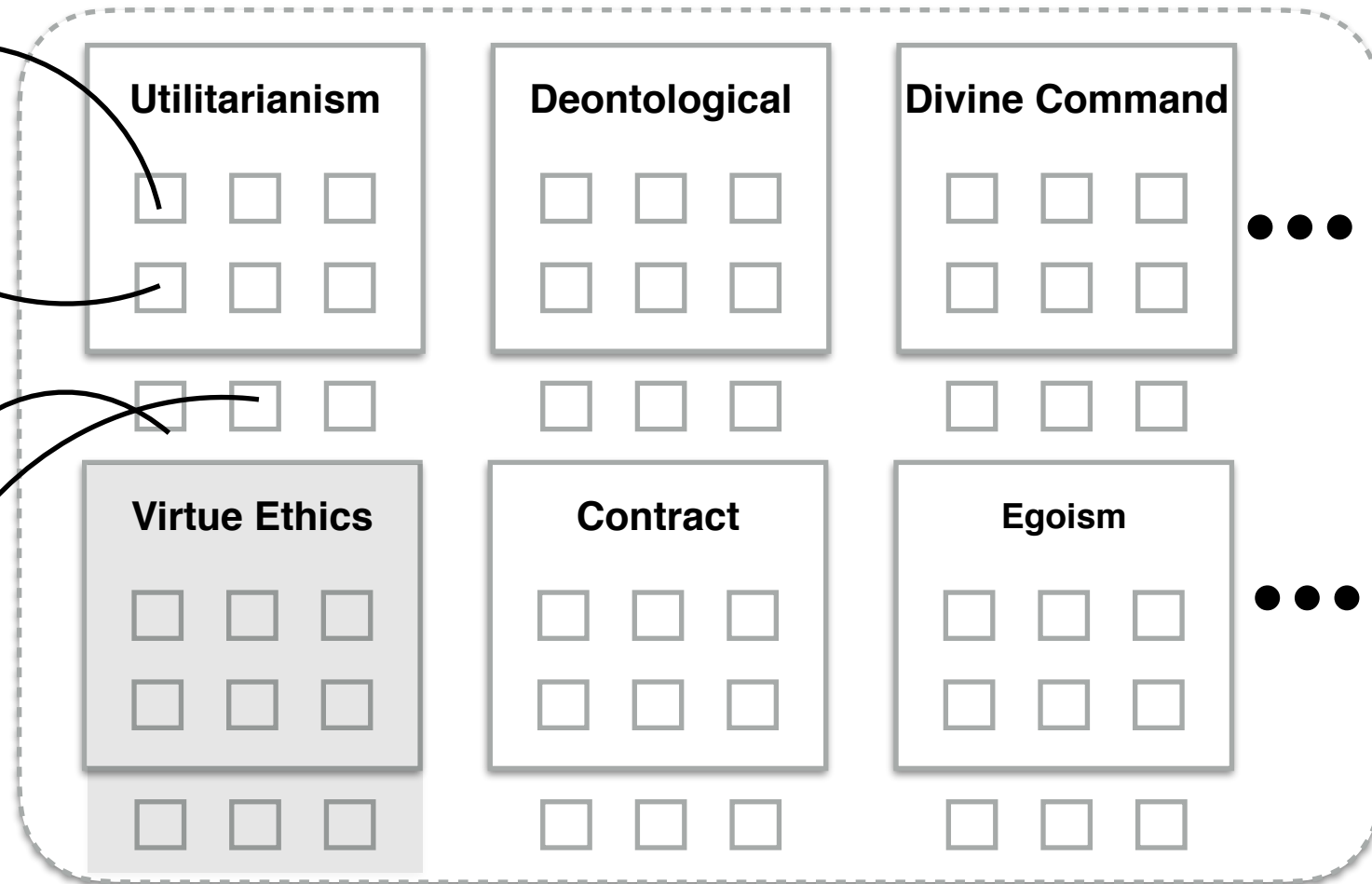


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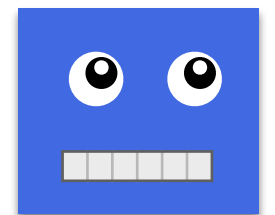
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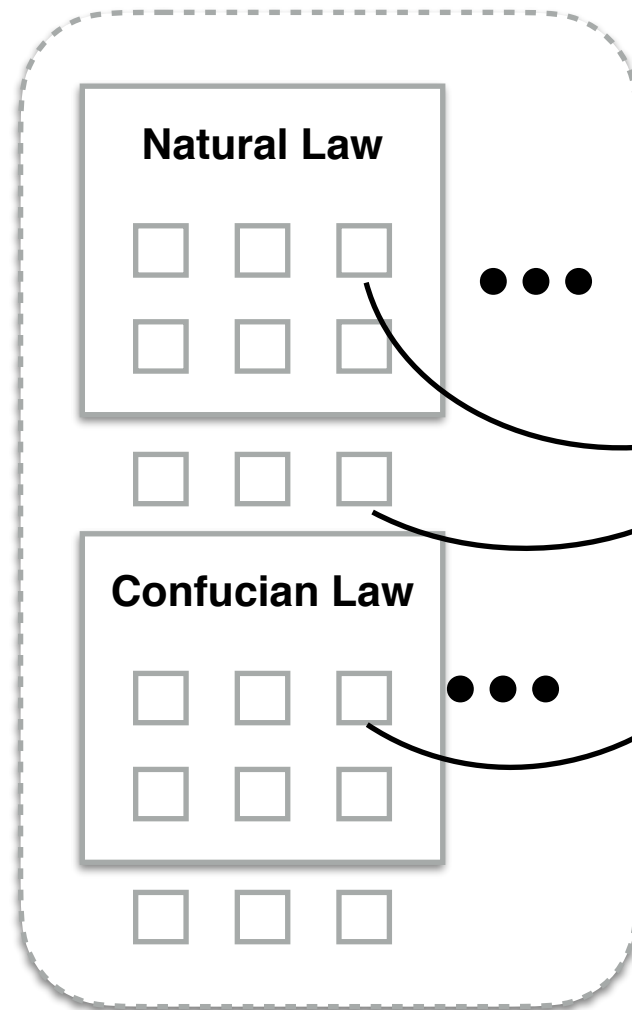


"Toward the Engineering of Virtuous Robots" Naveen, Selmer et al.

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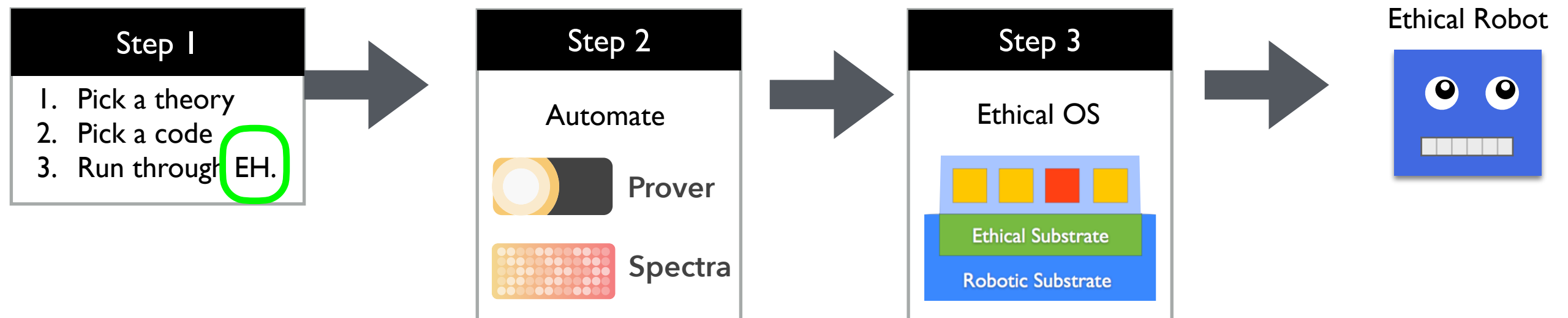
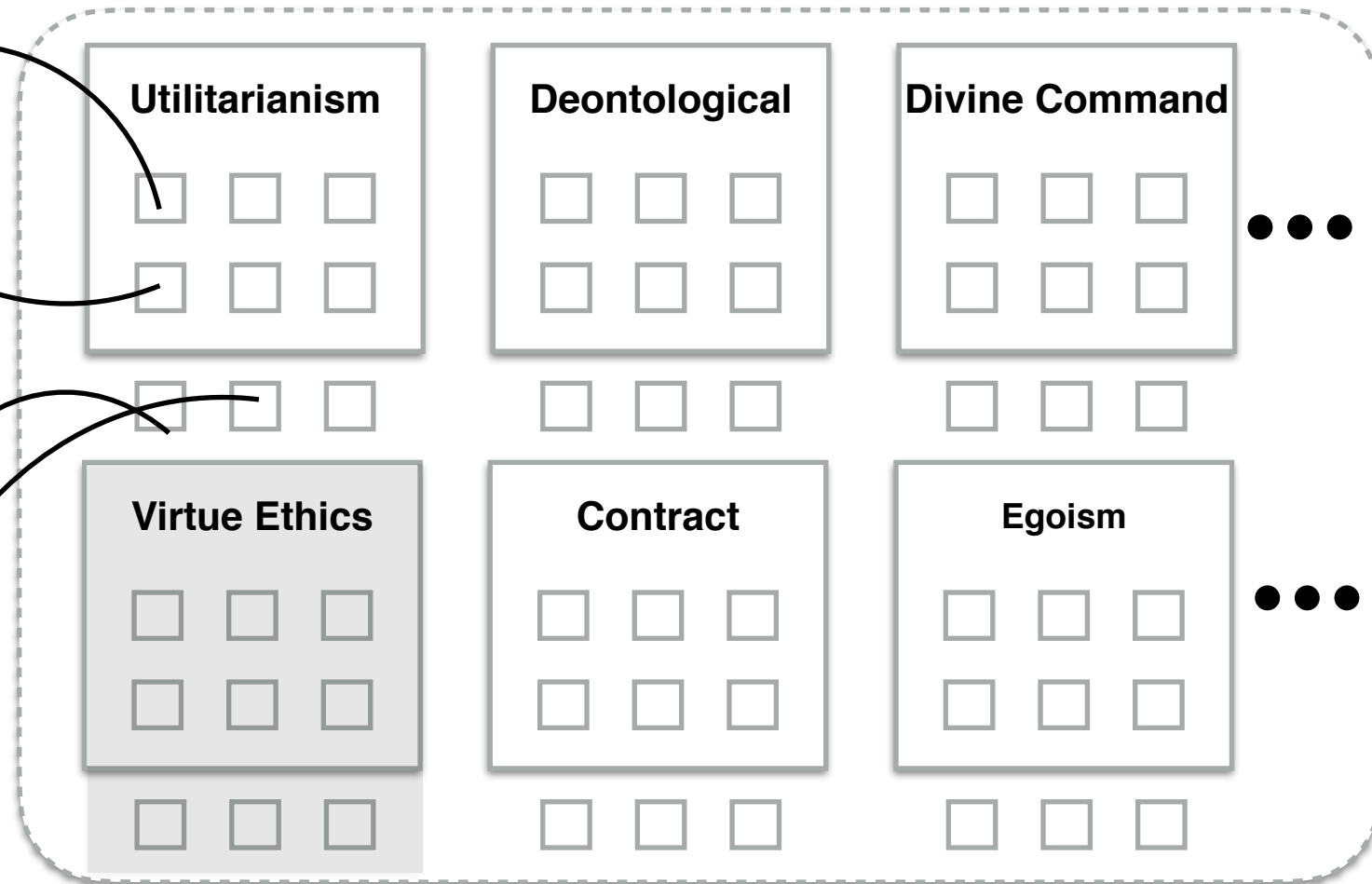


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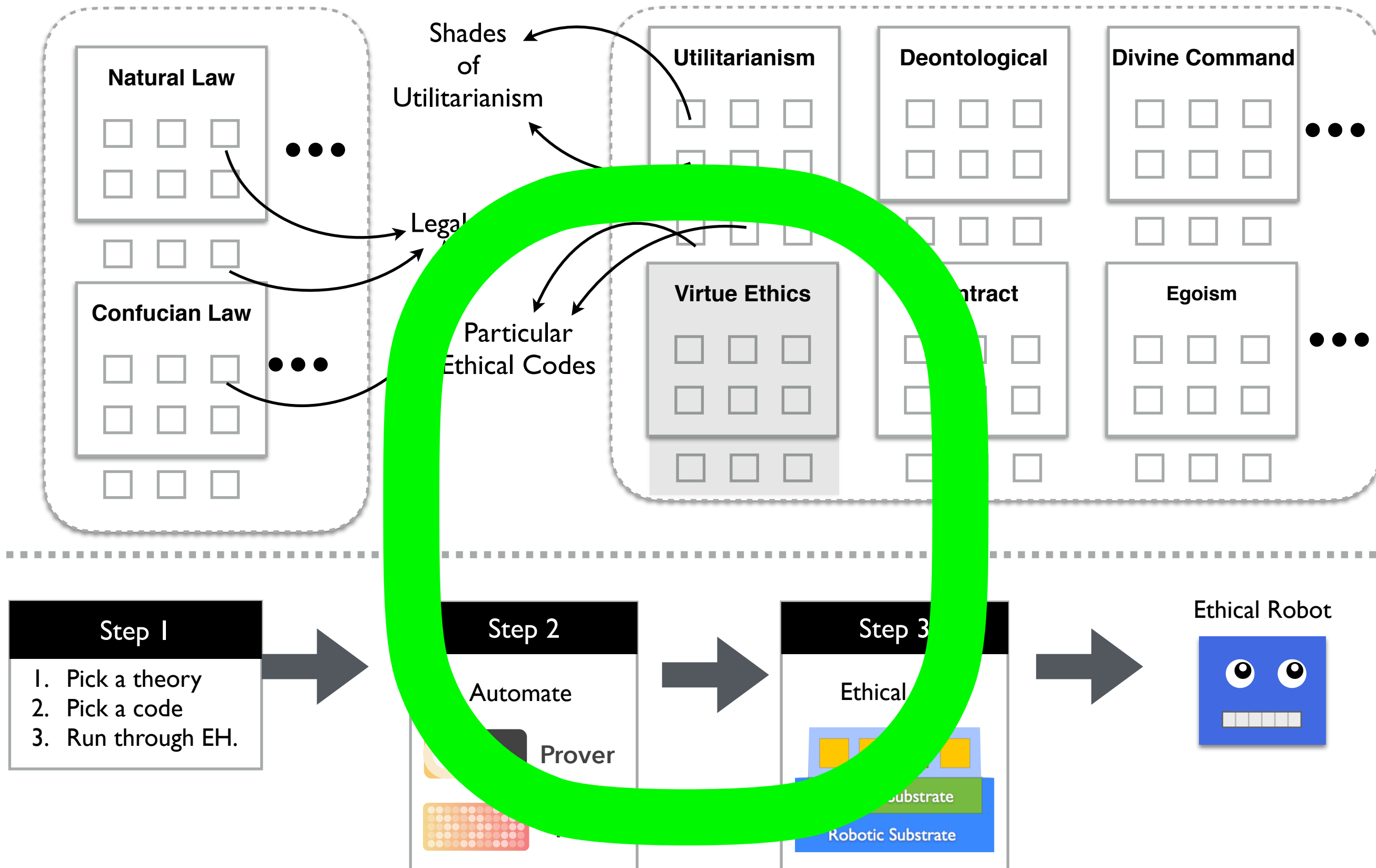
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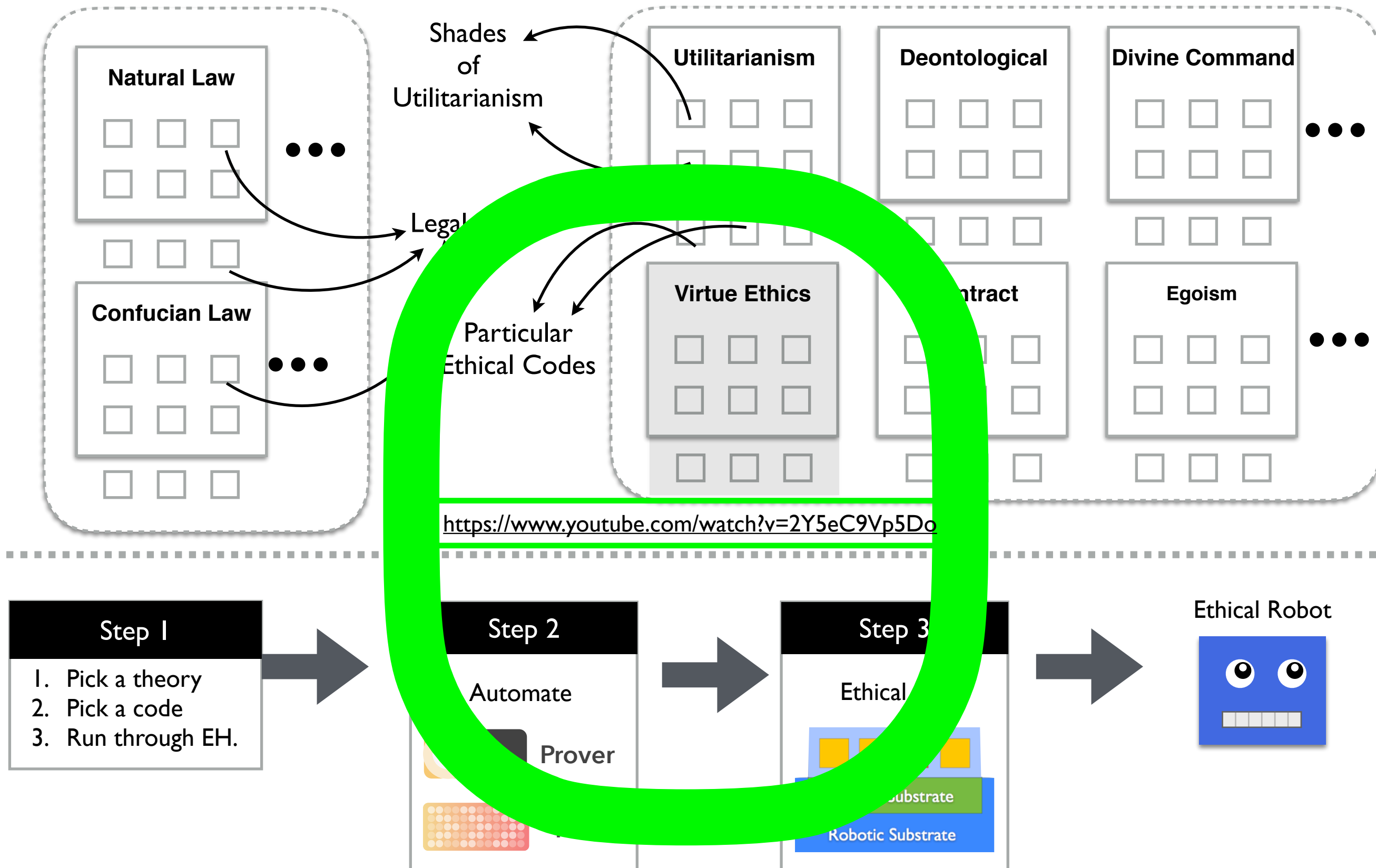
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Well, maybe, but at any rate, *what logic??*

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Perhaps **D = SDL?** ...

Review: Encapsulation

Slate - K.slt

K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $K \vdash \checkmark \infty \Box$	T. $\Box\varphi \rightarrow \varphi$ $K \vdash \times \infty \Box$	4. $\Box\varphi \rightarrow \Box\Box\varphi$ $K \vdash \times \infty \Box$	5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $K \vdash \times \infty \Box$
---	---	---	---

Review: Encapsulation

The image shows two overlapping Slate editor windows. The top window is titled 'Slate - K.slt' and contains four boxes with modal logic formulas and their derivability status in the K system. The bottom window is titled 'Slate - T.slt' and contains the same four boxes, but with their derivability status in the T system.

Formula	K System	T System
K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$	$K \vdash \checkmark \infty \Box$	$M \vdash \checkmark \infty \Box$
T. $\Box\varphi \rightarrow \varphi$	$K \vdash \times \infty \Box$	$M \vdash \checkmark \infty \Box$
4. $\Box\varphi \rightarrow \Box\Box\varphi$	$K \vdash \times \infty \Box$	$M \vdash \times \infty \Box$
5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$	$K \vdash \times \infty \Box$	$M \vdash \times \infty \Box$

Review: Encapsulation

The image shows three overlapping windows from a software application, each displaying a set of modal logic theorems and their derivability status in a specific system. The windows are titled 'Slate - K.slt', 'Slate - T.slt', and 'Slate - D.slt'.

Slate - K.slt

- K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$
K $\vdash$ ✓ $\infty\Box$
- T. $\Box\varphi \rightarrow \varphi$
K $\vdash$ ✗ $\infty\Box$
- 4. $\Box\varphi \rightarrow \Box\Box\varphi$
K $\vdash$ ✗ $\infty\Box$
- 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$
K $\vdash$ ✗ $\infty\Box$

Slate - T.slt

- K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$
M $\vdash$ ✓ $\infty\Box$
- T. $\Box\varphi \rightarrow \varphi$
M $\vdash$ ✓ $\infty\Box$
- 4. $\Box\varphi \rightarrow \Box\Box\varphi$
M $\vdash$ ✗ $\infty\Box$
- 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$
M $\vdash$ ✗ $\infty\Box$

Slate - D.slt

- K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$
D $\vdash$ ✓ $\infty\Box$
- T. $\Box\varphi \rightarrow \varphi$
D $\vdash$ ✗ $\infty\Box$
- D. $\Box\varphi \rightarrow \Diamond\varphi$
D $\vdash$ ✓ $\infty\Box$
- 4. $\Box\varphi \rightarrow \Box\Box\varphi$
D $\vdash$ ✗ $\infty\Box$
- 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$
D $\vdash$ ✗ $\infty\Box$
- INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$
D $\vdash$ ✓ $\infty\Box$

Review: Encapsulation

The image shows four Slate windows, each displaying a set of modal logic formulae and their derivability status in a specific system. The windows are titled 'Slate - K.slt', 'Slate - T.slt', 'Slate - D.slt', and 'Slate - S4.slt'.

Slate - K.slt

- K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$
K $\vdash \checkmark \infty \Box$
- T. $\Box\varphi \rightarrow \varphi$
K $\vdash \times \infty \Box$
- 4. $\Box\varphi \rightarrow \Box\Box\varphi$
K $\vdash \times \infty \Box$
- 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$
K $\vdash \times \infty \Box$

Slate - T.slt

- K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$
M $\vdash \checkmark \infty \Box$
- T. $\Box\varphi \rightarrow \varphi$
M $\vdash \checkmark \infty \Box$
- 4. $\Box\varphi \rightarrow \Box\Box\varphi$
M $\vdash \times \infty \Box$
- 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$
M $\vdash \times \infty \Box$

Slate - D.slt

- K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$
D $\vdash \checkmark \infty \Box$
- T. $\Box\varphi \rightarrow \varphi$
D $\vdash \times \infty \Box$
- D. $\Box\varphi \rightarrow \Diamond\varphi$
D $\vdash \checkmark \infty \Box$
- 4. $\Box\varphi \rightarrow \Box\Box\varphi$
D $\vdash \times \infty \Box$
- 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$
D $\vdash \times \infty \Box$
- INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$
D $\vdash \checkmark \infty \Box$

Slate - S4.slt

- K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$
S4 $\vdash \checkmark \infty \Box$
- T. $\Box\varphi \rightarrow \varphi$
S4 $\vdash \checkmark \infty \Box$
- D. $\Box\varphi \rightarrow \Diamond\varphi$
S4 $\vdash \checkmark \infty \Box$
- 4. $\Box\varphi \rightarrow \Box\Box\varphi$
S4 $\vdash \checkmark \infty \Box$
- 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$
S4 $\vdash \times \infty \Box$
- INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$
{INTER} Assume $\checkmark$

Review: Encapsulation

K

T

D

4 = S4

5 = S5

Slate - K.slt

K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $K \vdash \checkmark \infty \Box$	T. $\Box\varphi \rightarrow \varphi$ $K \vdash \times \infty \Box$	4. $\Box\varphi \rightarrow \Box\Box\varphi$ $K \vdash \times \infty \Box$	5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $K \vdash \times \infty \Box$
---	---	---	---

Slate - T.slt

K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $M \vdash \checkmark \infty \Box$	T. $\Box\varphi \rightarrow \varphi$ $M \vdash \checkmark \infty \Box$	4. $\Box\varphi \rightarrow \Box\Box\varphi$ $M \vdash \times \infty \Box$	5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $M \vdash \times \infty \Box$
---	---	---	---

Slate - D.slt

K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $D \vdash \checkmark \infty \Box$	T. $\Box\varphi \rightarrow \varphi$ $D \vdash \times \infty \Box$	D. $\Box\varphi \rightarrow \Diamond\varphi$ $D \vdash \checkmark \infty \Box$	4. $\Box\varphi \rightarrow \Box\Box\varphi$ $D \vdash \times \infty \Box$
5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $D \vdash \times \infty \Box$		INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$ $D \vdash \checkmark \infty \Box$	

Slate - S4.slt

K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $S4 \vdash \checkmark \infty \Box$	T. $\Box\varphi \rightarrow \varphi$ $S4 \vdash \checkmark \infty \Box$	D. $\Box\varphi \rightarrow \Diamond\varphi$ $S4 \vdash \checkmark \infty \Box$	4. $\Box\varphi \rightarrow \Box\Box\varphi$ $S4 \vdash \checkmark \infty \Box$
5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $S4 \vdash \times \infty \Box$		INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$ {INTER} Assume $\checkmark$	

Slate - S5.slt

K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $S5 \vdash \checkmark \infty \Box$	T. $\Box\varphi \rightarrow \varphi$ $S5 \vdash \checkmark \infty \Box$	D. $\Box\varphi \rightarrow \Diamond\varphi$ {D} Assume $\checkmark$	4. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ {4} Assume $\checkmark$
5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $S5 \vdash \checkmark \infty \Box$		INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$ {INTER} Assume $\checkmark$	

Review: Encapsulation

K

T

D

4 = S4

5 = S5

The image shows a series of five windows, each representing a different modal logic system. Each window contains a grid of boxes, each representing a specific logical formula and its status in that system. The status is indicated by a checkmark (green) for derivable, a cross (red) for not derivable, and a box for assumed.

Window	Formula	Status
Slate - K.slt	K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$	$K \vdash \checkmark \infty \Box$
	T. $\Box\varphi \rightarrow \varphi$	$K \vdash \times \infty \Box$
	4. $\Box\varphi \rightarrow \Box\Box\varphi$	$K \vdash \times \infty \Box$
	5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$	$K \vdash \times \infty \Box$
Slate - T.slt	K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$	$M \vdash \checkmark \infty \Box$
	T. $\Box\varphi \rightarrow \varphi$	$M \vdash \checkmark \infty \Box$
	4. $\Box\varphi \rightarrow \Box\Box\varphi$	$M \vdash \times \infty \Box$
	5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$	$M \vdash \times \infty \Box$
Slate - D.slt (highlighted)	K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$	$D \vdash \checkmark \infty \Box$
	T. $\Box\varphi \rightarrow \varphi$	$D \vdash \times \infty \Box$
	D. $\Box\varphi \rightarrow \Diamond\varphi$	$D \vdash \checkmark \infty \Box$
	4. $\Box\varphi \rightarrow \Box\Box\varphi$	$D \vdash \times \infty \Box$
	5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$	$D \vdash \times \infty \Box$
INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$	$D \vdash \checkmark \infty \Box$	
Slate - S4.slt	K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$	$S4 \vdash \checkmark \infty \Box$
	T. $\Box\varphi \rightarrow \varphi$	$S4 \vdash \checkmark \infty \Box$
	D. $\Box\varphi \rightarrow \Diamond\varphi$	$S4 \vdash \checkmark \infty \Box$
	4. $\Box\varphi \rightarrow \Box\Box\varphi$	$S4 \vdash \checkmark \infty \Box$
	5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$	$S4 \vdash \times \infty \Box$
INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$	{INTER} Assume $\checkmark$	
Slate - S5.slt	K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$	$S5 \vdash \checkmark \infty \Box$
	T. $\Box\varphi \rightarrow \varphi$	$S5 \vdash \checkmark \infty \Box$
	D. $\Box\varphi \rightarrow \Diamond\varphi$	{D} Assume $\checkmark$
	4. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$	{4} Assume $\checkmark$
	5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$	$S5 \vdash \checkmark \infty \Box$
INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$	{INTER} Assume $\checkmark$	

Review: Encapsulation

K
T
D
4 = S4
5 = S5

The screenshot displays five windows from the HyperSlate application, each showing a set of logical formulas and their status in a specific modal logic system. The windows are titled 'Slate - K.slt', 'Slate - T.slt', 'Slate - D.slt', 'Slate - S4.slt', and 'Slate - S5.slt'.

- Slate - K.slt:**
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $K \vdash \checkmark \infty \Box$
 - T. $\Box\varphi \rightarrow \varphi$ $K \vdash \times \infty \Box$
 - 4. $\Box\varphi \rightarrow \Box\Box\varphi$ $K \vdash \times \infty \Box$
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $K \vdash \times \infty \Box$
- Slate - T.slt:**
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $M \vdash \checkmark \infty \Box$
 - T. $\Box\varphi \rightarrow \varphi$ $M \vdash \checkmark \infty \Box$
 - 4. $\Box\varphi \rightarrow \Box\Box\varphi$ $M \vdash \times \infty \Box$
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $M \vdash \times \infty \Box$
- Slate - D.slt (highlighted with a red border):**
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $D \vdash \checkmark \infty \Box$
 - T. $\Box\varphi \rightarrow \varphi$ $D \vdash \times \infty \Box$
 - D. $\Box\varphi \rightarrow \Diamond\varphi$ $D \vdash \checkmark \infty \Box$
 - 4. $\Box\varphi \rightarrow \Box\Box\varphi$ $D \vdash \times \infty \Box$
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $D \vdash \times \infty \Box$
 - INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$ $D \vdash \checkmark \infty \Box$
- Slate - S4.slt:**
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $S4 \vdash \checkmark \infty \Box$
 - T. $\Box\varphi \rightarrow \varphi$ $S4 \vdash \checkmark \infty \Box$
 - D. $\Box\varphi \rightarrow \Diamond\varphi$ $S4 \vdash \checkmark \infty \Box$
 - 4. $\Box\varphi \rightarrow \Box\Box\varphi$ $S4 \vdash \checkmark \infty \Box$
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $S4 \vdash \times \infty \Box$
 - INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$ $\{INTER\} \text{ Assume } \checkmark$
- Slate - S5.slt:**
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $S5 \vdash \checkmark \infty \Box$
 - T. $\Box\varphi \rightarrow \varphi$ $S5 \vdash \checkmark \infty \Box$
 - D. $\Box\varphi \rightarrow \Diamond\varphi$ $\{D\} \text{ Assume } \checkmark$
 - 4. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $\{4\} \text{ Assume } \checkmark$
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $S5 \vdash \checkmark \infty \Box$
 - INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$ $\{INTER\} \text{ Assume } \checkmark$

4.4.4 D = SDL (= ‘Standard Deontic Logic’)

We here introduce what is known as ‘Standard Deontic Logic’ (SDL), which in Slate is the system **D**. Deontic logic is the sub-branch of logic devoted to formalizing the fundamental concepts of morality; for example, the concepts of *obligation*, *permissibility*, and *forbiddenness*. The first of these three concepts can apparently serve as a cornerstone, since to say that ϕ (a formulae representing some state-of-affairs) is permissible seems to amount to saying that it’s not obligatory that it not be the case that ϕ (which shows permissibility can be defined in terms of obligation), and to say that ϕ is forbidden would seem to amount to it being obligatory that it not be the case that ϕ (which of course appears to show that forbiddenness buildable from obligation). This interconnected trio of ethical concepts is a triad explicitly invoked and analyzed since the end of the 18th century, and the importance of the triad even to modern deontic logic would be quite hard to exaggerate.⁹

SDL is traditionally axiomatized by the following:¹⁰

SDL

TAUT All theorems of the propositional calculus.

OB-K $\odot(\phi \rightarrow \psi) \rightarrow (\odot\phi \rightarrow \odot\psi)$

OB-D $\odot\phi \rightarrow \neg\odot\neg\phi$

MP If $\vdash \phi$ and $\vdash \phi \rightarrow \psi$, then $\vdash \psi$

OB-NEC If $\vdash \phi$ then $\vdash \odot\phi$

4.4.4 D = SDL (= ‘Standard Deontic Logic’)

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SDL

TAUT All theorems of the propositional calculus

OB-K $\odot(\phi \rightarrow \psi) \rightarrow (\odot\phi \rightarrow \odot\psi)$

OB-D $\odot\phi \rightarrow \neg\odot\neg\phi$

MP If $\vdash \phi$ and $\vdash \phi \rightarrow \psi$, then $\vdash \psi$

OB-NEC If $\vdash \phi$ then $\vdash \odot\phi$

CHAPTER 4. PROPOSITIONAL MODAL LOGIC

OB-RE If $\vdash \phi \leftrightarrow \psi$, then $\vdash \odot\phi \leftrightarrow \odot\psi$.

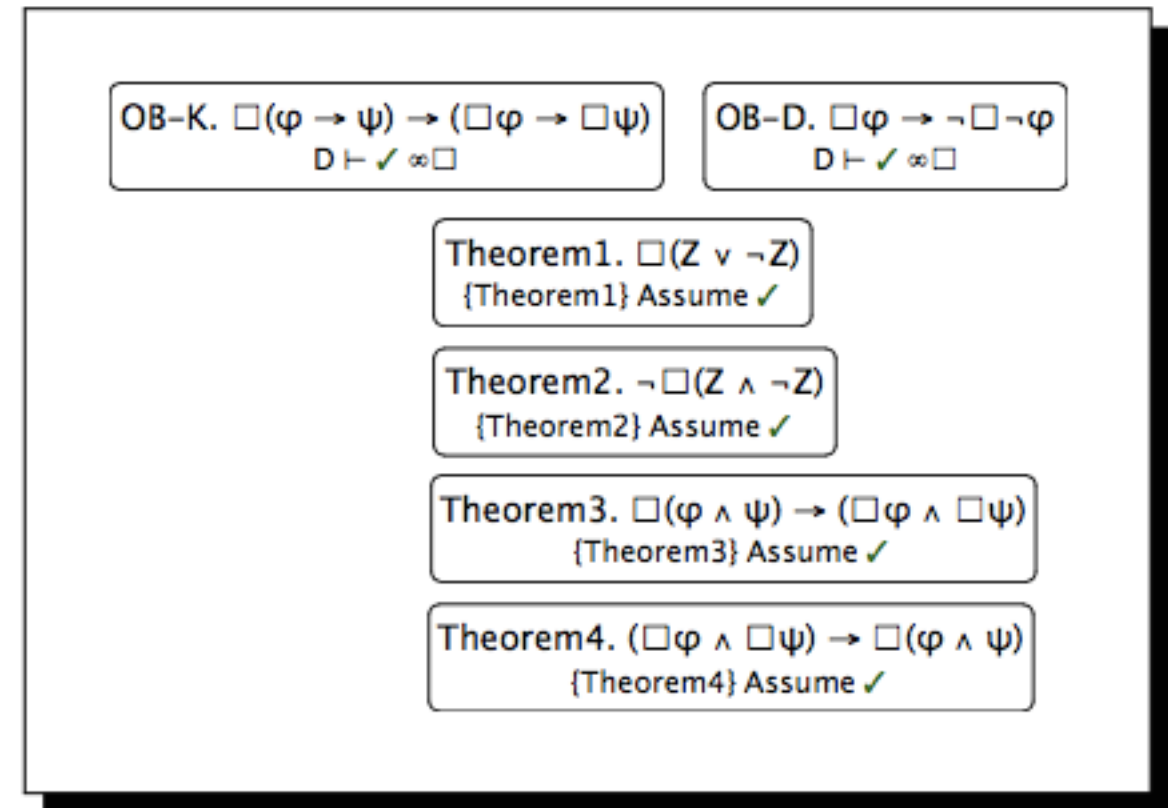
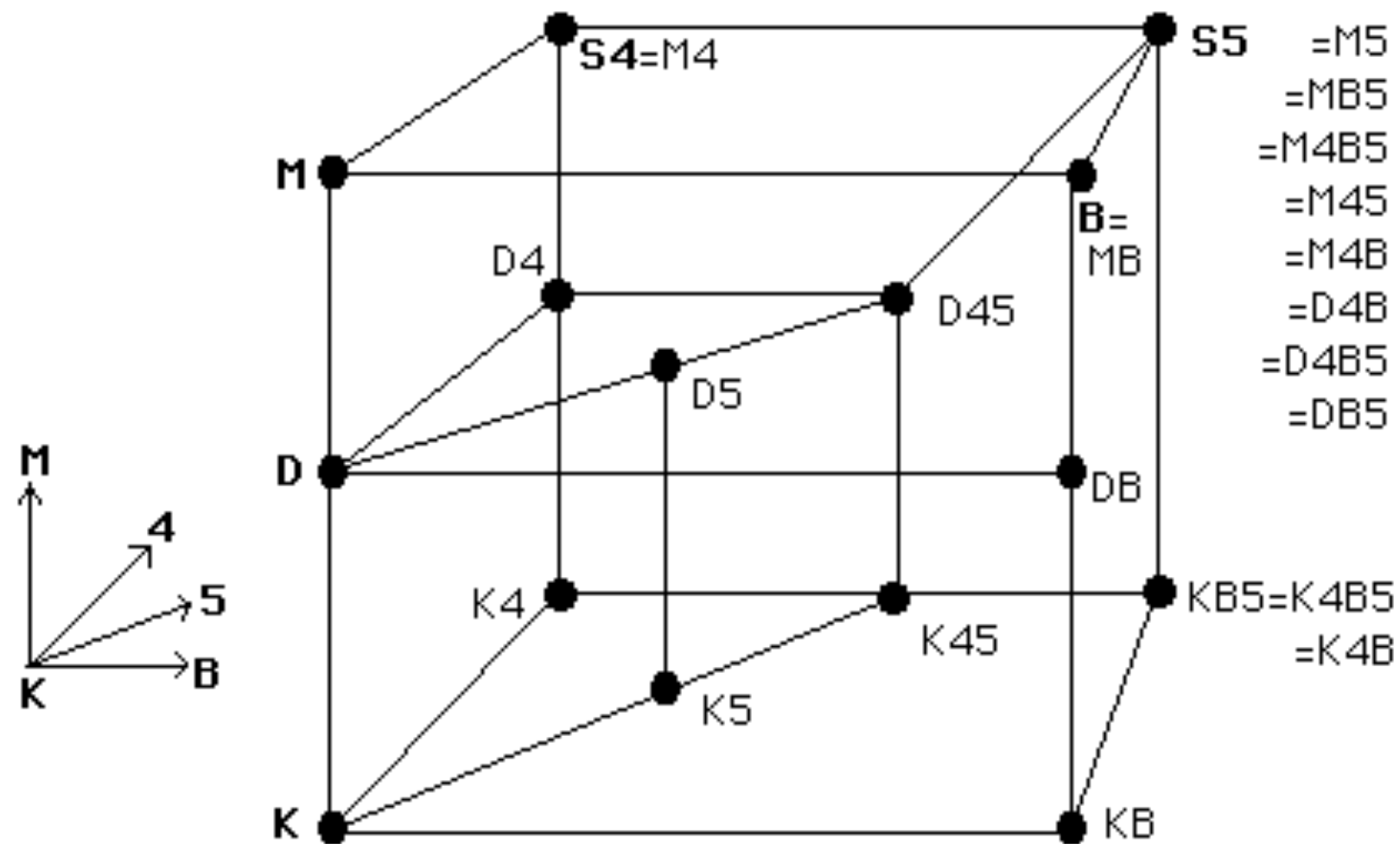


Figure 4.7: The Initial Configuration Upon Opening the File SDL.slt



4.4.4.1 Chisholm's Paradox and SDL

There are a host of problems that, together, constitute what is probably a fatal threat to **SDL** as a model of human-level ethical reasoning. We discuss in the present section the first of these problems to hit the “airwaves”: Chisholm's Paradox (CP) (Chisholm 1963). CP can be generated in Slate, you we shall see. But before we get to the level of experimentation in Slate, let's understand the scenario that Chisholm's imagined.

Chisholm's clever scenario revolves around the character Jones.¹¹ It's given that Jones is obligated to go to assist his neighbors, in part because he has promised to do so. The second given fact is that it's obligatory that, if Jones goes to assist his neighbors, he tells them (in advance) that he is coming. In addition, and this is the third given, if Jones *doesn't* go to assist his neighbors, it's obligatory that he not tell

¹¹We change some particulars to ease exposition; generally, again, follow, the *SEP* entry on deontic logic (recall footnote 10). The core logic mirrors (Chisholm 1963), the original publication.

them that he is coming. The fourth and final given fact is simply that Jones doesn't go to assist his neighbors. (On the way to do so, suppose he comes upon a serious vehicular accident, is proficient in emergency medicine, and (commendably!) seizes the opportunity to save the life (and subsequently monitor) of one of the victims in this accident.) These four givens have been represented in an obvious way within four formula nodes in a Slate file; see Figure 4.8. (Notice that $\Box$ is used in place of $\odot$.) The paradox arises from the fact that Chisholm's quartet of givens, which surely reflect situations that are common in everyday life, in conjunction with the axioms of **SDL**, entail outright contradictions (see Exercise 2 for **D** = **SDL**, in §4.4.4.2).

4.4.4.1 Chisholm's Paradox and SDL

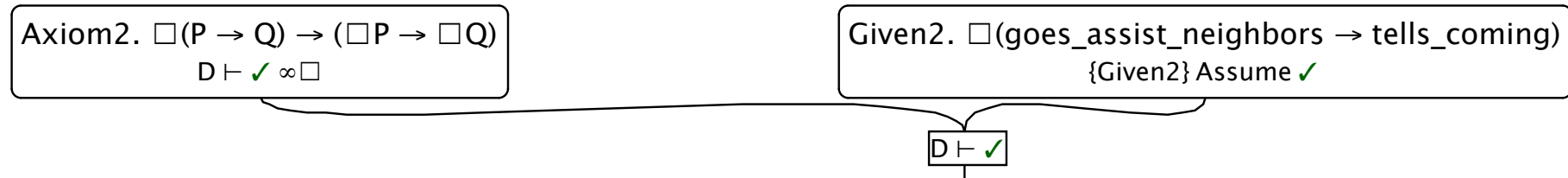
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Chisholm's Paradox

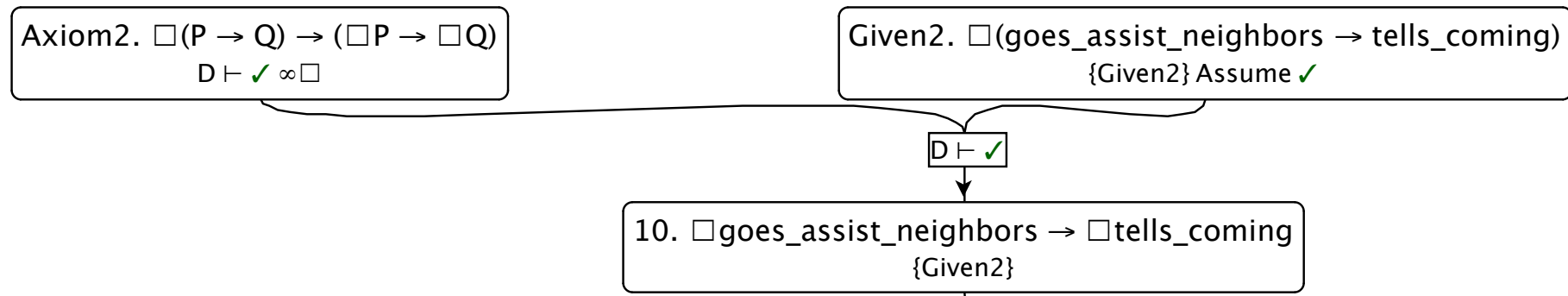


Axiom4. "Modus ponens for provability."
 $\{\text{Axiom4}\} \text{ Assume } \checkmark$

Axiom5. "Theorems are obligatory."
 $\{\text{Axiom5}\} \text{ Assume } \checkmark$

Axiom1. "All theorems of the propositional calculus."
 $\{\text{Axiom1}\} \text{ Assume } \checkmark$

Chisholm's Paradox

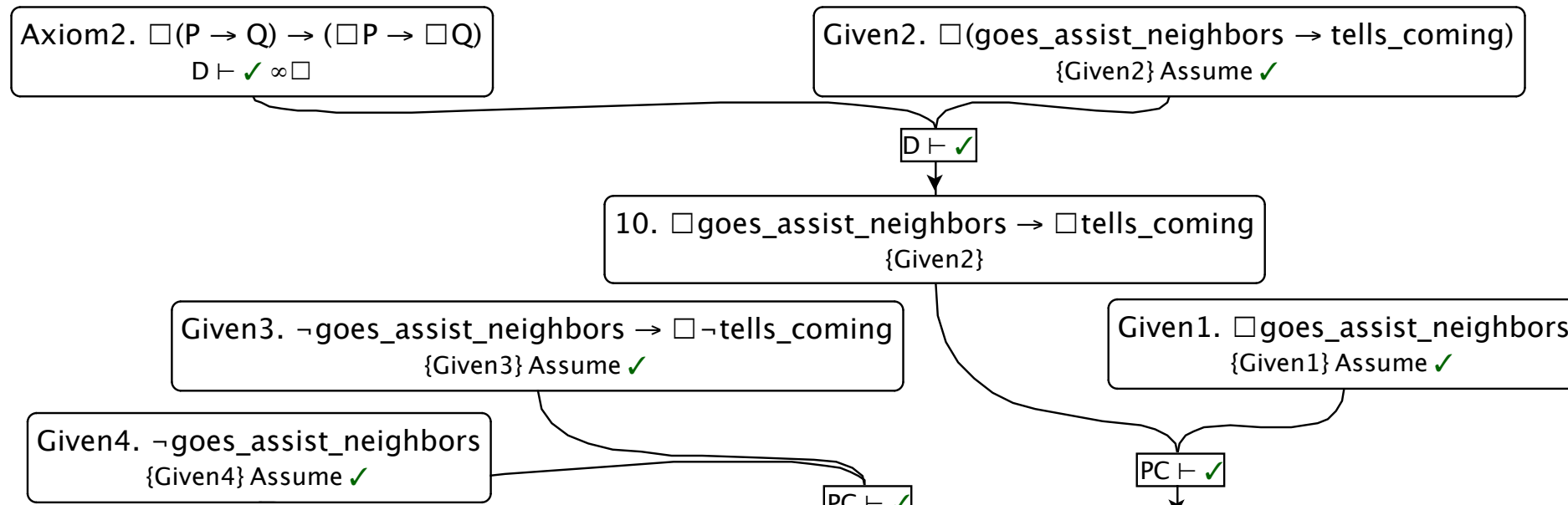


Axiom4. "Modus ponens for provability."
{Axiom4} Assume $\checkmark$

Axiom5. "Theorems are obligatory."
{Axiom5} Assume $\checkmark$

Axiom1. "All theorems of the propositional calculus."
{Axiom1} Assume $\checkmark$

Chisholm's Paradox

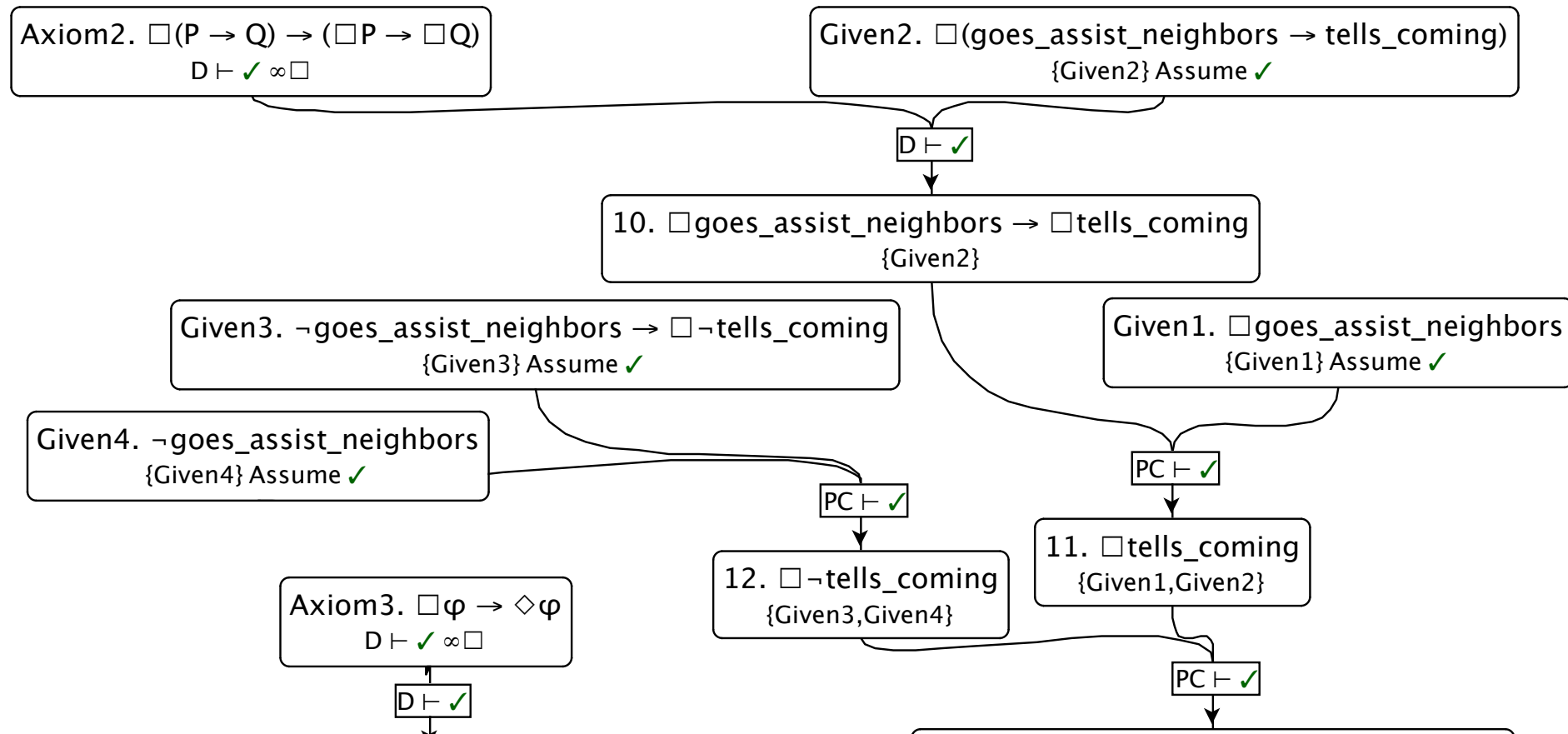


Axiom4. "Modus ponens for provability."
 $\{\text{Axiom4}\} \text{ Assume } \checkmark$

Axiom5. "Theorems are obligatory."
 $\{\text{Axiom5}\} \text{ Assume } \checkmark$

Axiom1. "All theorems of the propositional calculus."
 $\{\text{Axiom1}\} \text{ Assume } \checkmark$

Chisholm's Paradox

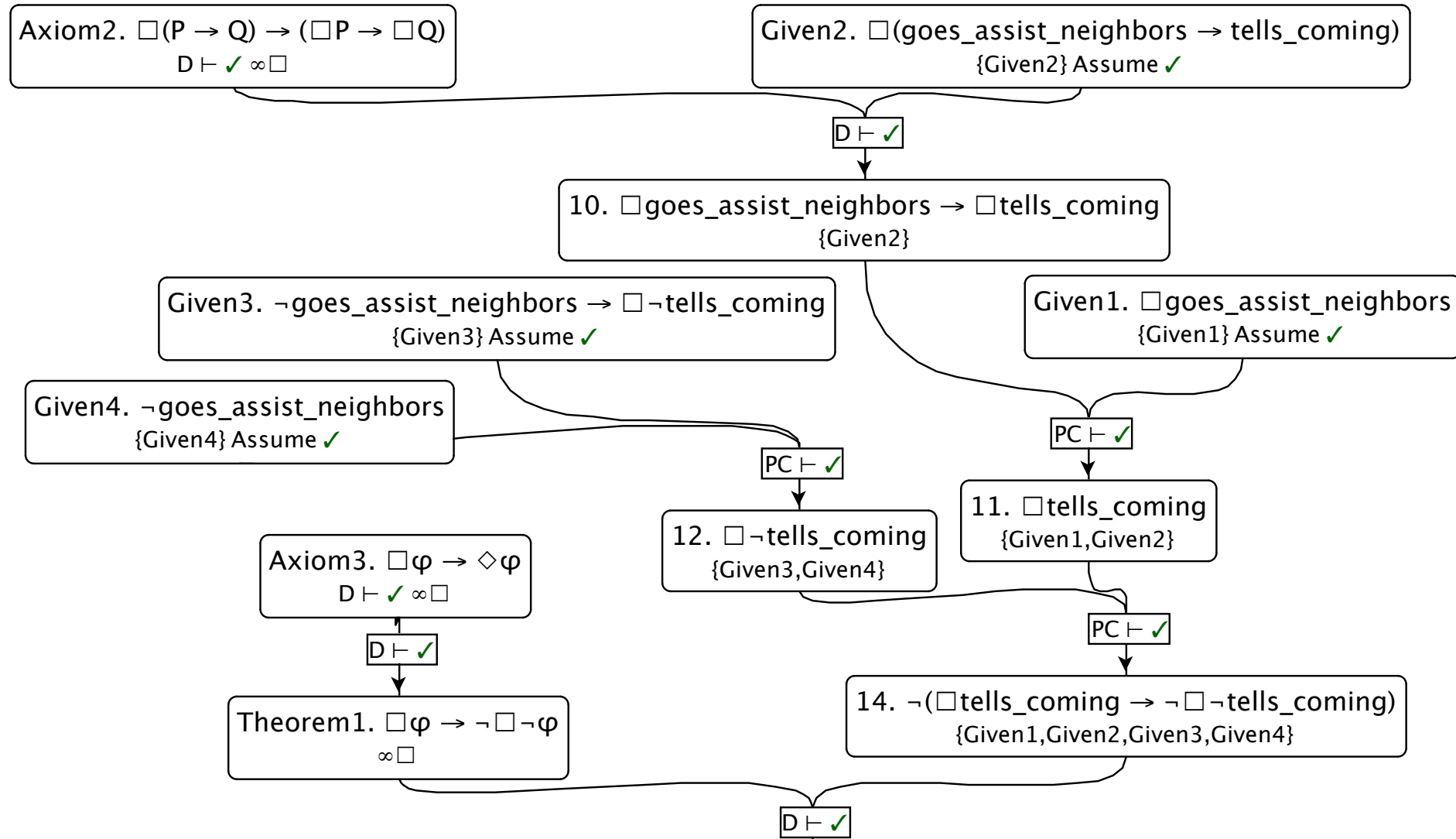


Axiom4. "Modus ponens for provability."
 $\{\text{Axiom4}\} \text{ Assume } \checkmark$

Axiom5. "Theorems are obligatory."
 $\{\text{Axiom5}\} \text{ Assume } \checkmark$

Axiom1. "All theorems of the propositional calculus."
 $\{\text{Axiom1}\} \text{ Assume } \checkmark$

Chisholm's Paradox



Axiom4. "Modus ponens for provability."

$\{\text{Axiom4}\} \text{ Assume } \checkmark$

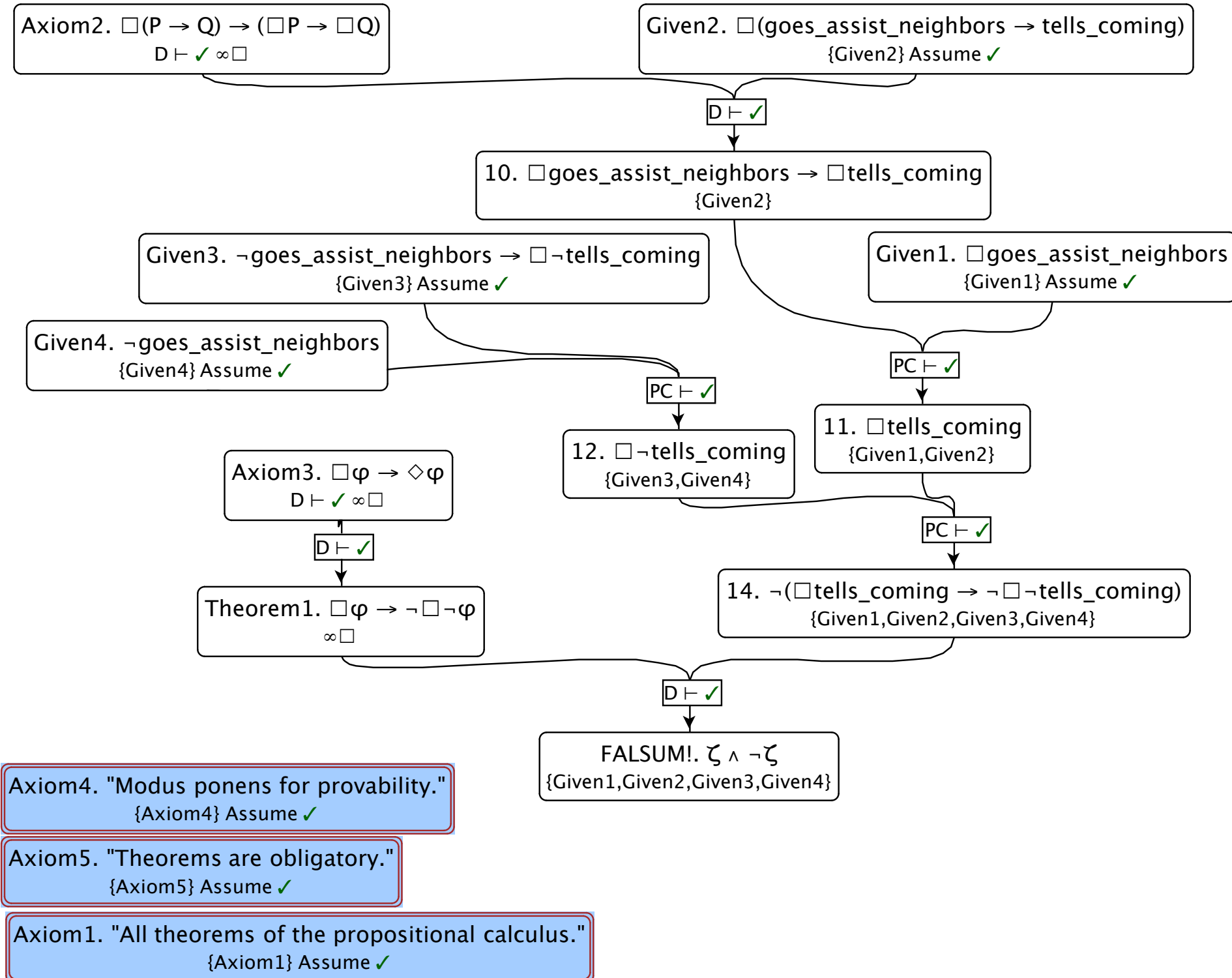
Axiom5. "Theorems are obligatory."

$\{\text{Axiom5}\} \text{ Assume } \checkmark$

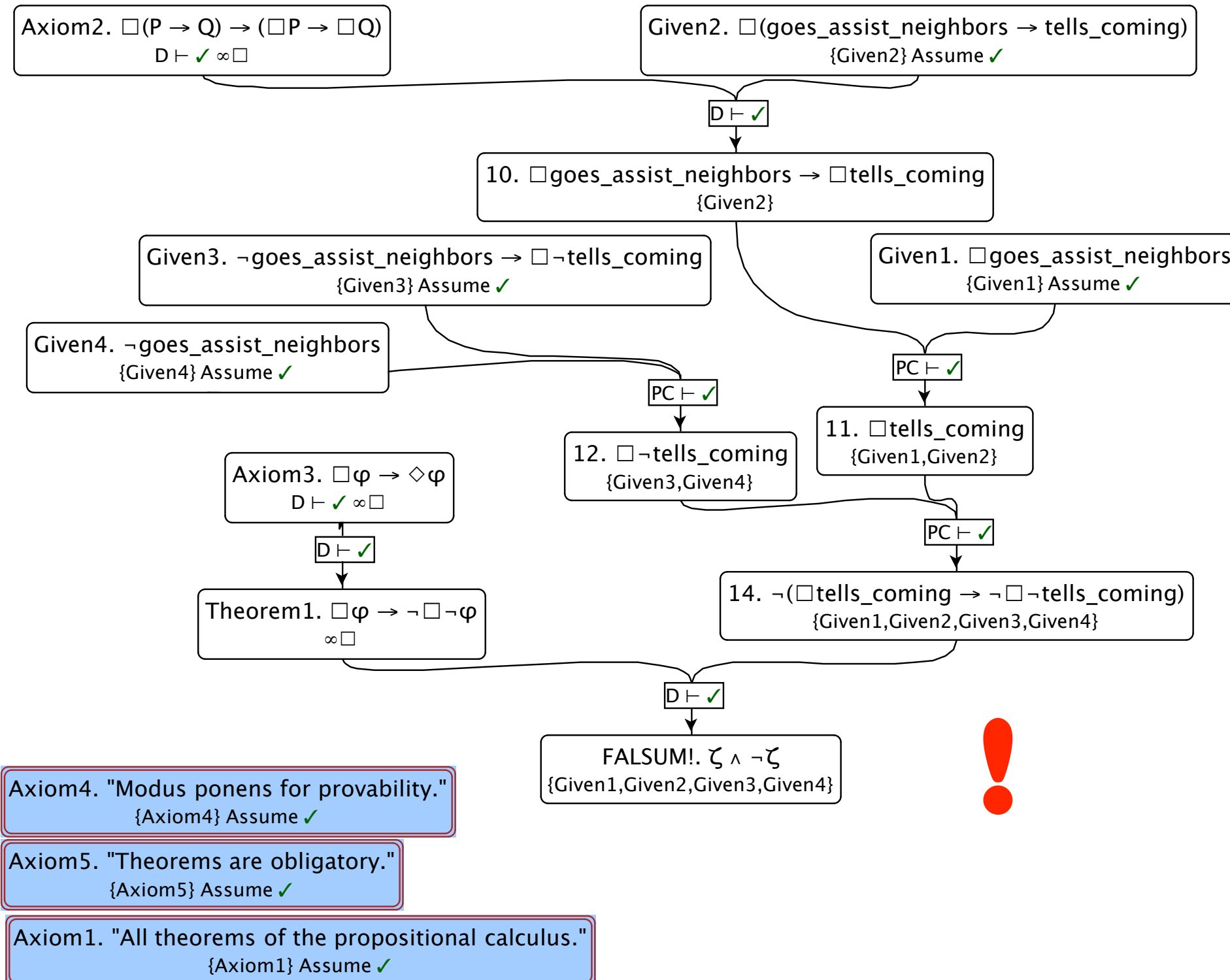
Axiom1. "All theorems of the propositional calculus."

$\{\text{Axiom1}\} \text{ Assume } \checkmark$

Chisholm's Paradox



Chisholm's Paradox



☒ ChisholmsParadox

Here you are asked to build a proof that confirms *Chisholm's Paradox*. This paradox is that from a particular representation in **D** (= Standard Deontic Logic (SDL)) of four seemingly innocuous givens, a contradiction $\zeta \wedge \neg\zeta$ can be deduced. (Your instructor should have covered this in class, and may well have supplied a proof of CP.) The four givens are based on the story of a character Jones, who is obligated to go to assist his neighbors (move to a different domicile, e.g.). It would be wrong of him to show up unannounced, though; so if he goes to assist them, it ought to be that he tells them he's coming. In addition, if it's not the case that Jones goes to assist them, then it ought to be that it not be the case that he tells them he is coming. Finally, as a matter of fact, it's not case the Jones goes to assist (because on the way he comes across a car accident, and has an opportunity to save one of the victims).

Fortunately, the [RAIR Lab](#)'s modern cognitive calculus $\mathcal{DCEC}^*$ allows Chisholm's Paradox to be avoided. A recent paper explaining the use by an ethically correct AI of this calculus is available [here](#).

Your finished proof is allowed to make use of the PC provability oracle, but of no other oracle.

Deadline May 2 23:59 NY time

SDL's = D's Problems
Don't Stop Here ...

The Free Choice Permission Paradox (Ross)

1. "You may either sleep on the sofa bed or the guest bed."
 {1} Assume ✓

2. "Therefore: You may sleep on the sofa bed, and you may sleep on the guest bed."
 {2} Assume ✓

The Free Choice Permission Paradox (Ross)

1'. $\Diamond(\text{sofa-bed} \vee \text{guest-bed})$
{1'} Assume ✓

$\Box \vdash \text{X}$

2'. $\Diamond \text{sofa-bed} \wedge \Diamond \text{guest-bed}$
{1'}

1. "You may either sleep on the sofa bed or the guest bed."
{1} Assume ✓

2. "Therefore: You may sleep on the sofa bed, and you may sleep on the guest bed."
{2} Assume ✓

The Free Choice Permission Paradox (Ross)

1'. $\Diamond(\text{sofa-bed} \vee \text{guest-bed})$
{1'} Assume ✓

$\Box \vdash \text{X}$

2'. $\Diamond \text{sofa-bed} \wedge \Diamond \text{guest-bed}$
{1'}

1. "You may either sleep on the sofa bed or the guest bed."
{1} Assume ✓

2. "Therefore: You may sleep on the sofa bed, and you may sleep on the guest bed."
{2} Assume ✓

NEW SCHEMA?. $\Diamond(\varphi \vee \psi) \rightarrow (\Diamond\varphi \wedge \Diamond\psi)$
{NEW SCHEMA?} Assume ✓

The Free Choice Permission Paradox (Ross)

1'. $\Diamond(\text{sofa-bed} \vee \text{guest-bed})$
{1'} Assume ✓

$D \vdash \text{X}$

2'. $\Diamond \text{sofa-bed} \wedge \Diamond \text{guest-bed}$
{1'}

1. "You may either sleep on the sofa bed or the guest bed."
{1} Assume ✓

2. "Therefore: You may sleep on the sofa bed, and you may sleep on the guest bed."
{2} Assume ✓

NEW SCHEMA?. $\Diamond(\varphi \vee \psi) \rightarrow (\Diamond\varphi \wedge \Diamond\psi)$
{NEW SCHEMA?} Assume ✓

COMMENT. "We can prove:"
{COMMENT} Assume ✓

THM 5. $\Diamond\varphi \rightarrow \Diamond(\varphi \vee \psi)$
 $D \vdash \text{✓} \infty \square$

The Free Choice Permission Paradox (Ross)

1'. $\Diamond(\text{sofa-bed} \vee \text{guest-bed})$
 $\{1'\}$ Assume ✓

$D \vdash \text{X}$

2'. $\Diamond \text{sofa-bed} \wedge \Diamond \text{guest-bed}$
 $\{1'\}$

1. "You may either sleep on the sofa bed or the guest bed."
 $\{1\}$ Assume ✓

2. "Therefore: You may sleep on the sofa bed, and you may sleep on the guest bed."
 $\{2\}$ Assume ✓

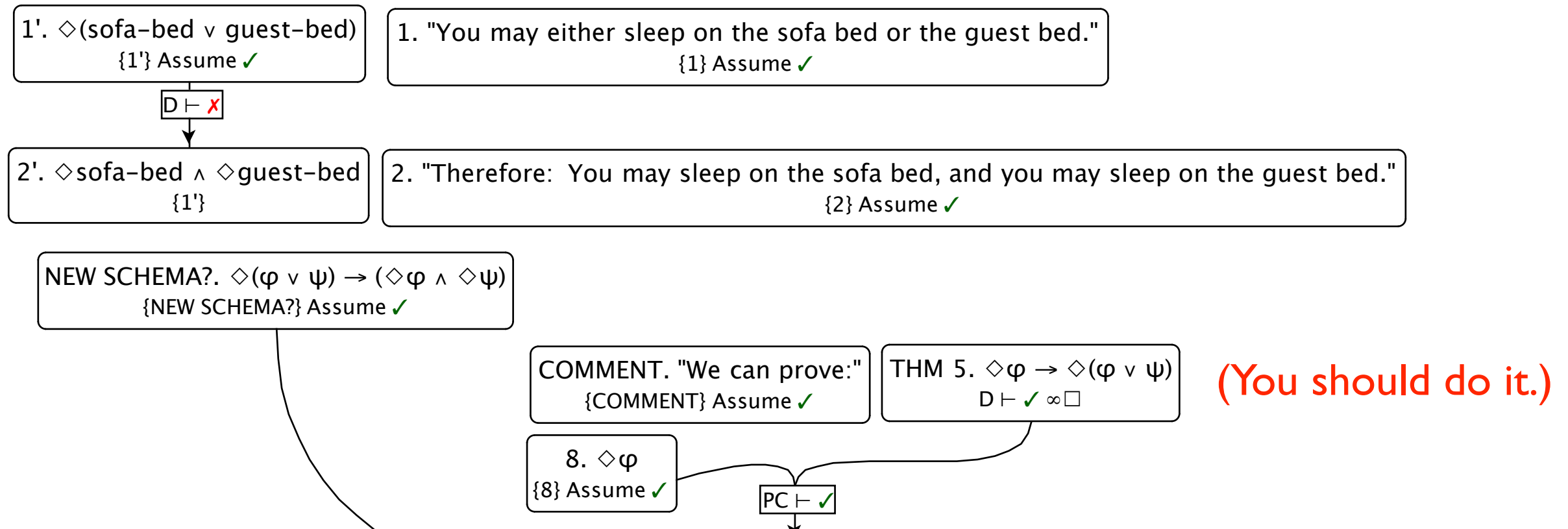
NEW SCHEMA?. $\Diamond(\varphi \vee \psi) \rightarrow (\Diamond\varphi \wedge \Diamond\psi)$
 $\{\text{NEW SCHEMA?}\}$ Assume ✓

COMMENT. "We can prove:"
 $\{\text{COMMENT}\}$ Assume ✓

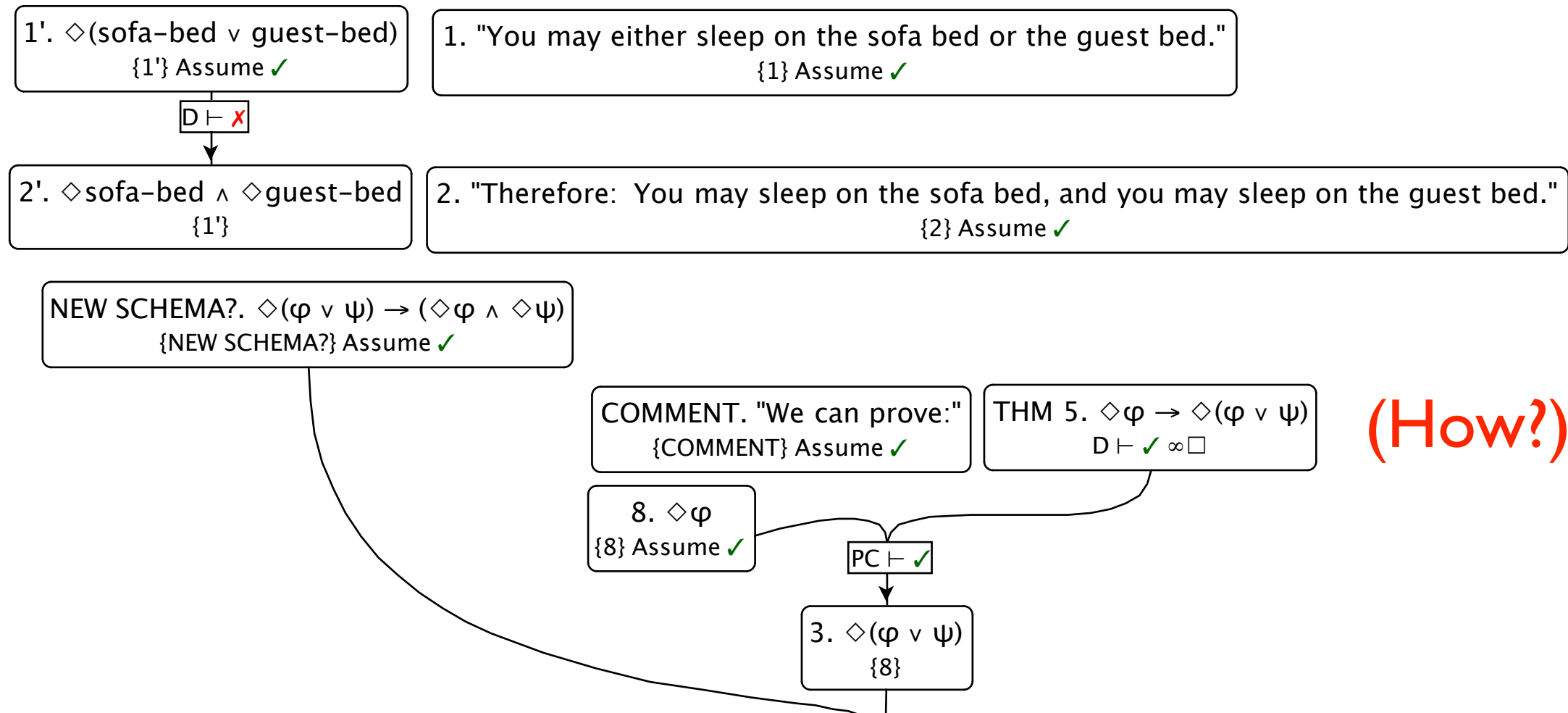
THM 5. $\Diamond\varphi \rightarrow \Diamond(\varphi \vee \psi)$
 $D \vdash \text{✓} \infty \square$

(How?)

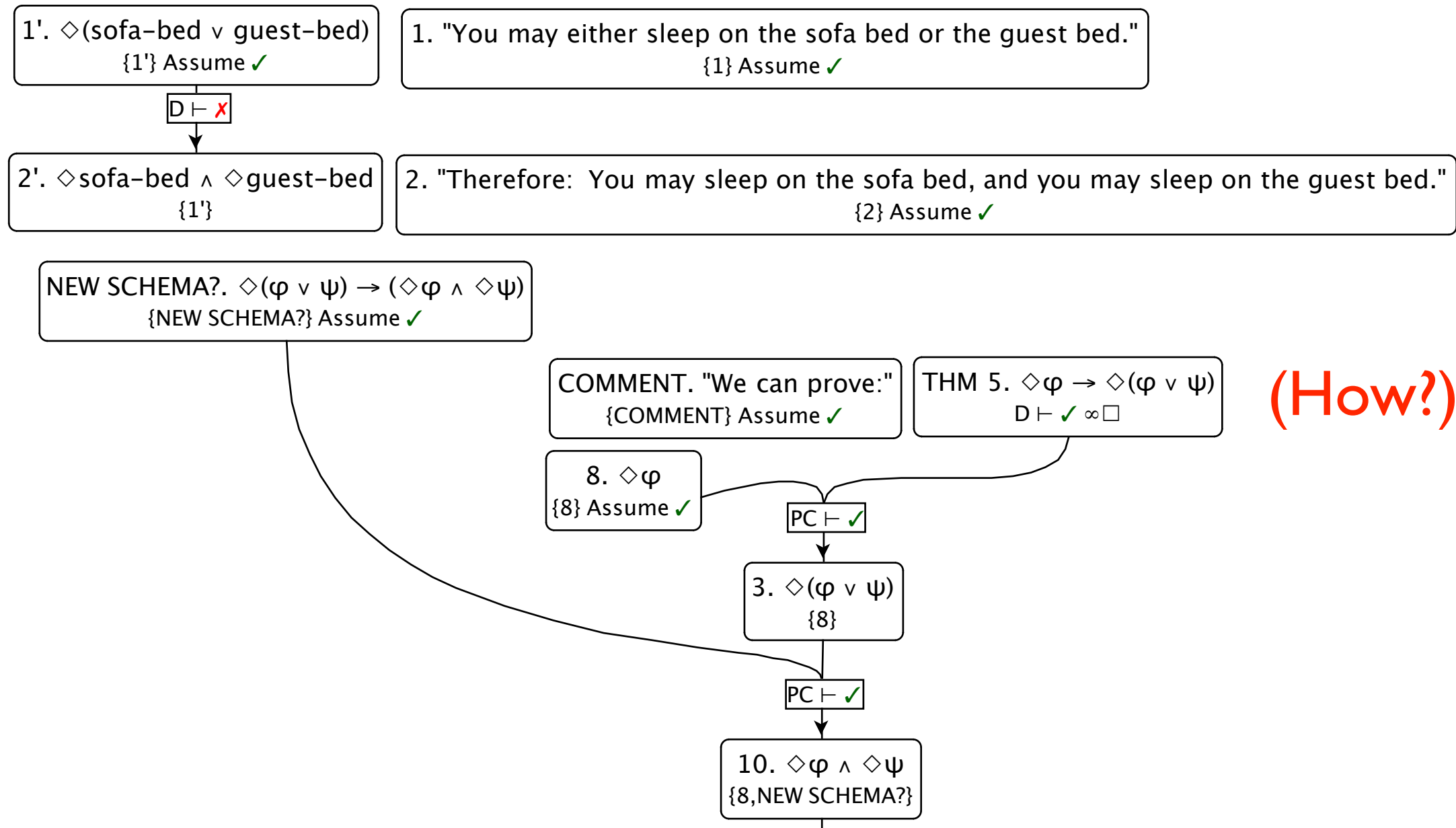
The Free Choice Permission Paradox (Ross)



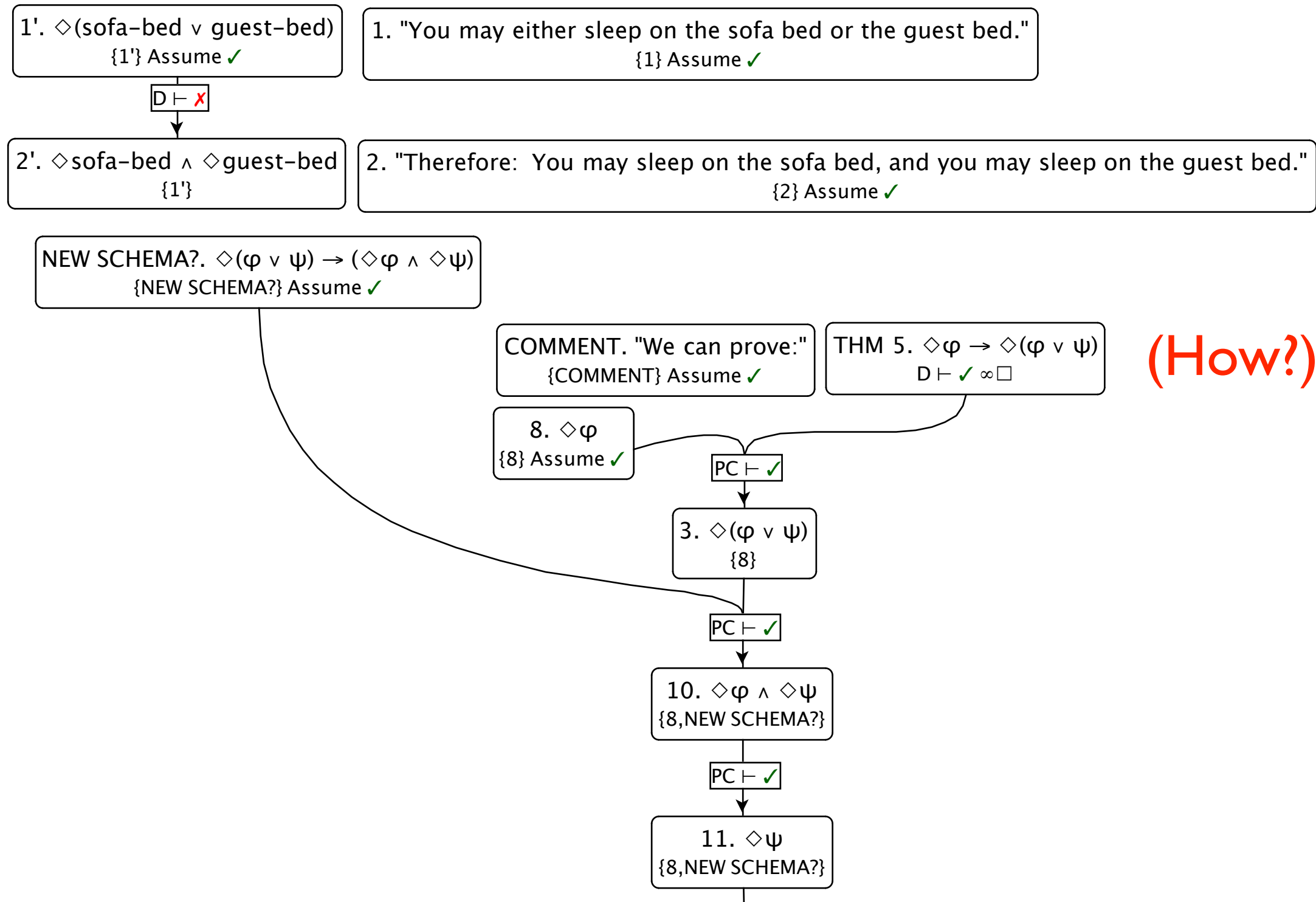
The Free Choice Permission Paradox (Ross)



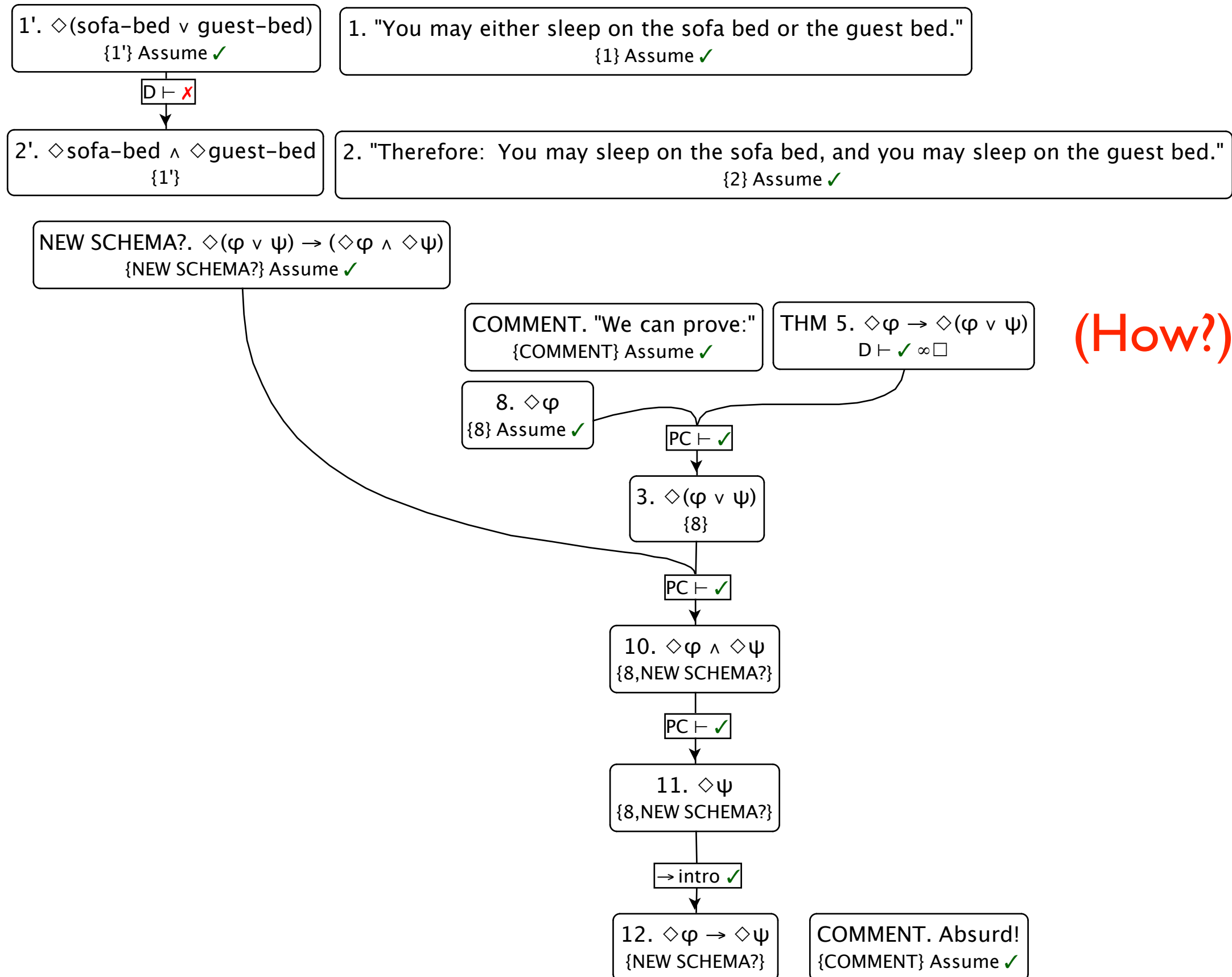
The Free Choice Permission Paradox (Ross)



The Free Choice Permission Paradox (Ross)



The Free Choice Permission Paradox (Ross)



Producing a valid proof in this problem will enable you to understand The Free Choice Permission Paradox (FCPP), discovered in 1941 by Ross ("Imperatives and Logic," *Theoria* 7: 53–71). Given that the proof in question yields an absurdity, FCPP can be taken to show that **SDL** (Standard Deontic Logic) = **D** leads to inconsistency when applied; or, put in AI terms, you wouldn't want a robot to base its ethical decision-making on **D**! Fortunately, the [RAIR Lab](#)'s modern cognitive calculus $\mathcal{DCEC}^*$ allows FCPP to be avoided. (A recent paper explaining the use by an ethically correct AI of this calculus is available [here](#).)

Here's the paradox. Suppose that you travel to visit a friend, arrive late at night, and are weary. Your friend says hospitably: "You may either sleep on the sofa-bed or sleep on the guest-room bed." (1) From this statement it follows that you are permitted to sleep on the sofa-bed, and you are permitted to sleep on the guest-room bed. (2) In **D**, this pair gets symbolized like this:

(1')

$\Diamond(sofabed \vee guestbed)$

(2')

$\Diamond sofabed \wedge \Diamond guestbed$

But (2') doesn't follow deductively from (1') in **D**, as a call to the provability oracle for **D** in the HyperSlate™ file for this problem confirms. A suggested repair is to add to **D** the schema

$$\Diamond(\phi \vee \psi) \rightarrow (\Diamond\phi \wedge \Diamond\psi),$$

but as your proof will (hopefully) show, this addition allows a proof of the absurd theorem that if anything is morally permissible, everything is!

Your finished proof is allowed to make use of the PC provability oracle, but of no other oracle. (No deadline for now.)

“Computational logician,
sorry, back to your drawing
board to find a logic that *does*
work with The Four Steps!”

