

# The Lottery Paradox (and inductive logic)

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Paradoxes are engines of progress in formal logic.

E.g., Russell's Paradox — as we've seen.

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- The Lottery Paradox (apparently) shows, courtesy of its two Sequences (of Reasoning), that a perfectly rational person can indeed have such a belief (upon considering a fair, large lottery).
- Contradiction! — and hence a paradox!

# Types of Paradoxes

- Deductive Paradoxes. The reasoning in question is exclusively deductive.
  - Russell's Paradox
  - The Liar Paradox
  - Richard's Paradox
- Inductive Paradoxes Some of the reasoning in question uses non-deductive reasoning (e.g., probabilistic reasoning, abductive reasoning, analogical reasoning, etc.).

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# Inductive Logic

the hallmark of deductive logic is *proof*, the hallmark of inductive logic is the concept of an *argument*. An exceptionally strong kind of argument is a proof, but plenty of arguments fall short of being proofs — and yet still have considerable force. For instance, consider the following argument  $\alpha_1$ :

- |                  |                     |
|------------------|---------------------|
| (1)              | Tweety is bird.     |
| (2)              | Most birds can fly. |
| <hr/>            |                     |
| $\therefore$ (3) | Tweety can fly.     |

For stark contrast, consider as well this argument ( $\alpha_2$ ):

- |                   |   |
|-------------------|---|
| (1')              | 3 is a positive integer.                  |
| (2')              | All positive integers are greater than 0. |
| <hr/>             |   |
| $\therefore$ (3') | 3 is greater than 0.                      |

The second of these arguments qualifies as an outright proof. That is, using the notation much employed before the present chapter:

$$\{(1'), (2')\} \vdash (3')$$

But in stark contrast, argument  $\alpha_1$  is not a proof that Tweety can fly. The reason is obvious: (3) isn't deduced from the combination of (1) and (2); that is,

$$\{(1), (2)\} \not\vdash (3)$$

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Inductive (new)

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# Inductive-Reasoning Example from Pollock — for Peek Ahead



Imagine the following:

Keith tells you that the morning news predicts rain in Troy today. However, Alvin tells you that the same news report predicted sunshine.

Imagine the following: Keith tells you that the morning news predicts rain in Tucson today. However, Alvin tells you that the same news report predicted sunshine.

Without any other source of information, it would be irrational to place belief in either Keith's or Alvin's statements.

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Further, suppose you happened to watch the noon news report, and that report predicted rain. Then you should believe that it will rain despite your knowledge of Alvin's argument.

# Defeasible Reasoning in OSCAR

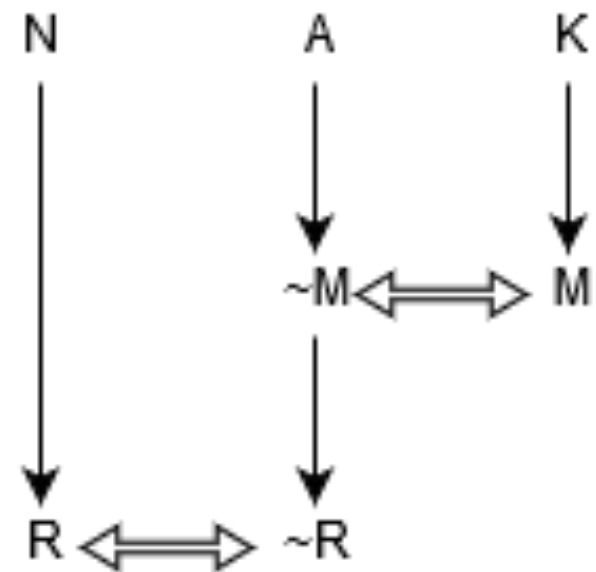
K- Keith says that M

A- Alvin says that  $\sim M$

M- The morning news said that R

R- It is going to rain this afternoon

N- The noon news says that R



All such can be absorbed into our inductive logics  
and our automated inductive reasoners (= our AI).

# In Our Inductive Modal Logic

(1)	$\mathbf{K}(you, \mathbf{S}(keith, \mathbf{S}(m, rain)))$	fact
(2)	$\mathbf{K}(you, \mathbf{S}(alvin, \mathbf{S}(m, \neg rain)))$	fact

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$\therefore$	(3)	$\mathbf{S}(keith, \mathbf{S}(m, rain))$	?
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$\therefore$	(6)	$\mathbf{B}^2(you, \mathbf{S}(m, rain)) \wedge \mathbf{B}^2(you, \mathbf{S}(m, \neg rain))$	



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$\therefore$	(6)	$\mathbf{B}^2(you, \mathbf{S}(m, rain)) \wedge \mathbf{B}^2(you, \mathbf{S}(m, \neg rain))$	
$\therefore$	(7)	$\neg \mathbf{B}^2(you, \mathbf{S}(m, rain)) \wedge \neg \mathbf{B}^2(you, \mathbf{S}(m, \neg rain))$	“Clash” Principle

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$\therefore$	(9)	$\mathbf{S}(noonnews, rain)$	?

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	(8)	$\mathbf{K}(you, \mathbf{S}(noonnews, rain))$	
$\therefore$	(9)	$\mathbf{S}(noonnews, rain)$	?
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$\therefore$	(3)	$\mathbf{S}(keith, \mathbf{S}(m, rain))$	?
$\therefore$	(4)	$\mathbf{S}(alvin, \mathbf{S}(m, \neg rain))$	?
	(5)	$\mathbf{S}(keith, \phi) \rightarrow \mathbf{B}^2(you, \phi)$	Testimonial P1
$\therefore$	(6)	$\mathbf{B}^2(you, \mathbf{S}(m, rain)) \wedge \mathbf{B}^2(you, \mathbf{S}(m, \neg rain))$	
$\therefore$	(7)	$\neg \mathbf{B}^2(you, \mathbf{S}(m, rain)) \wedge \neg \mathbf{B}^2(you, \mathbf{S}(m, \neg rain))$	“Clash” Principle
	(8)	$\mathbf{K}(you, \mathbf{S}(noonnews, rain))$	
$\therefore$	(9)	$\mathbf{S}(noonnews, rain)$	?
	(10)	$\mathbf{S}(noonnews, \phi) \rightarrow \mathbf{B}^3(you, \phi)$	Testimonial P2
$\therefore$	(11)	$\mathbf{B}^3(you, rain)$	

# The Lottery Paradox ...









E: “Please go down to Stewart’s & get the T U.”



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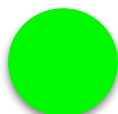
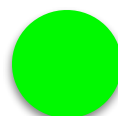
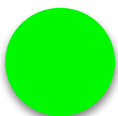
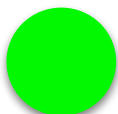
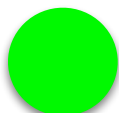
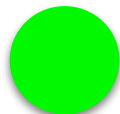
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It would be irrational.”



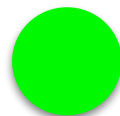
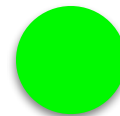
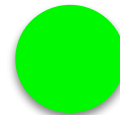
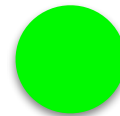
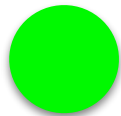
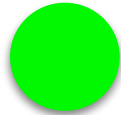
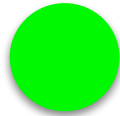
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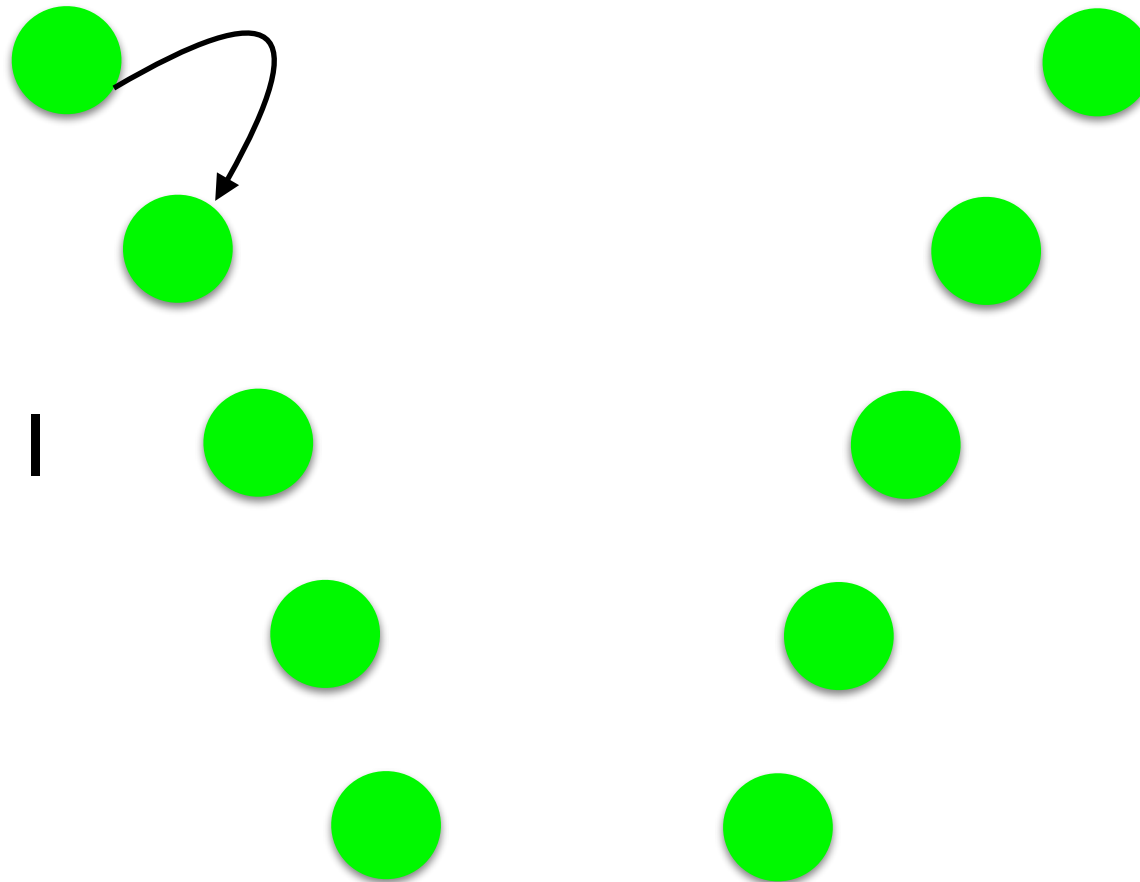
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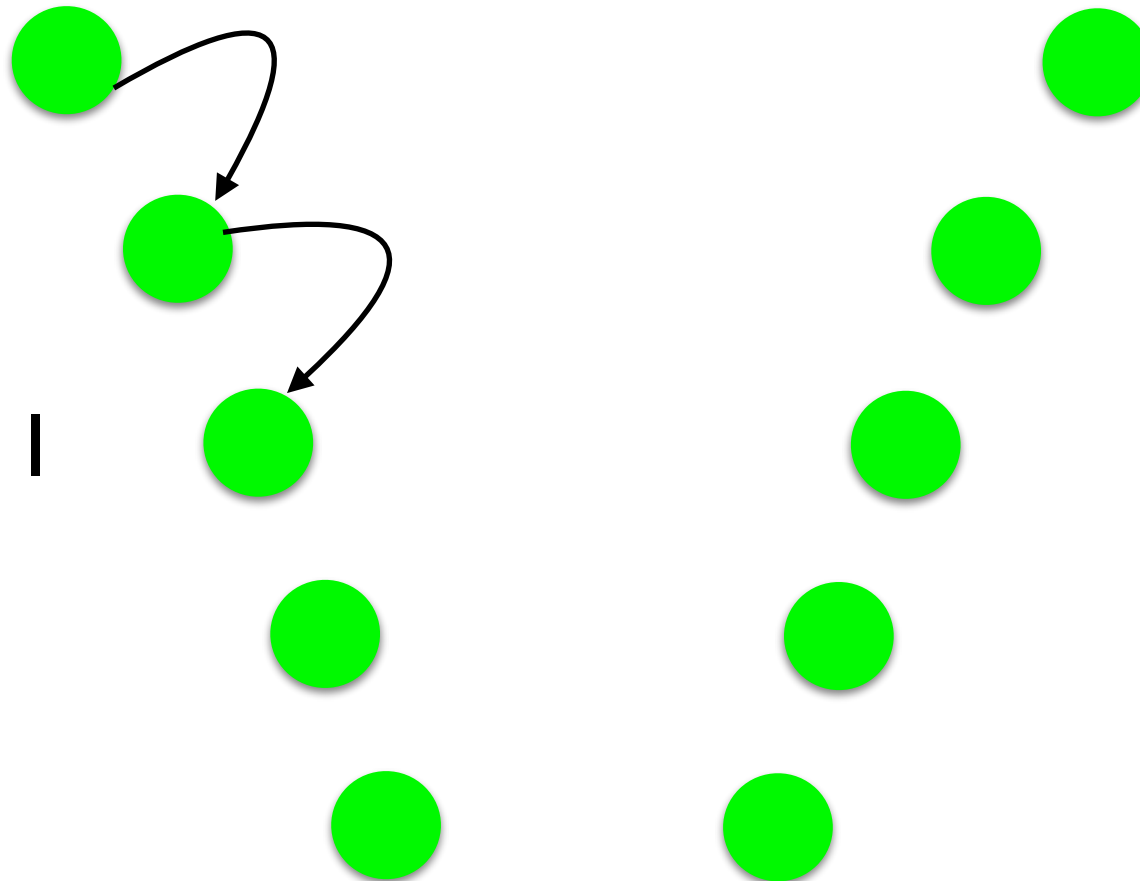
Sequence I



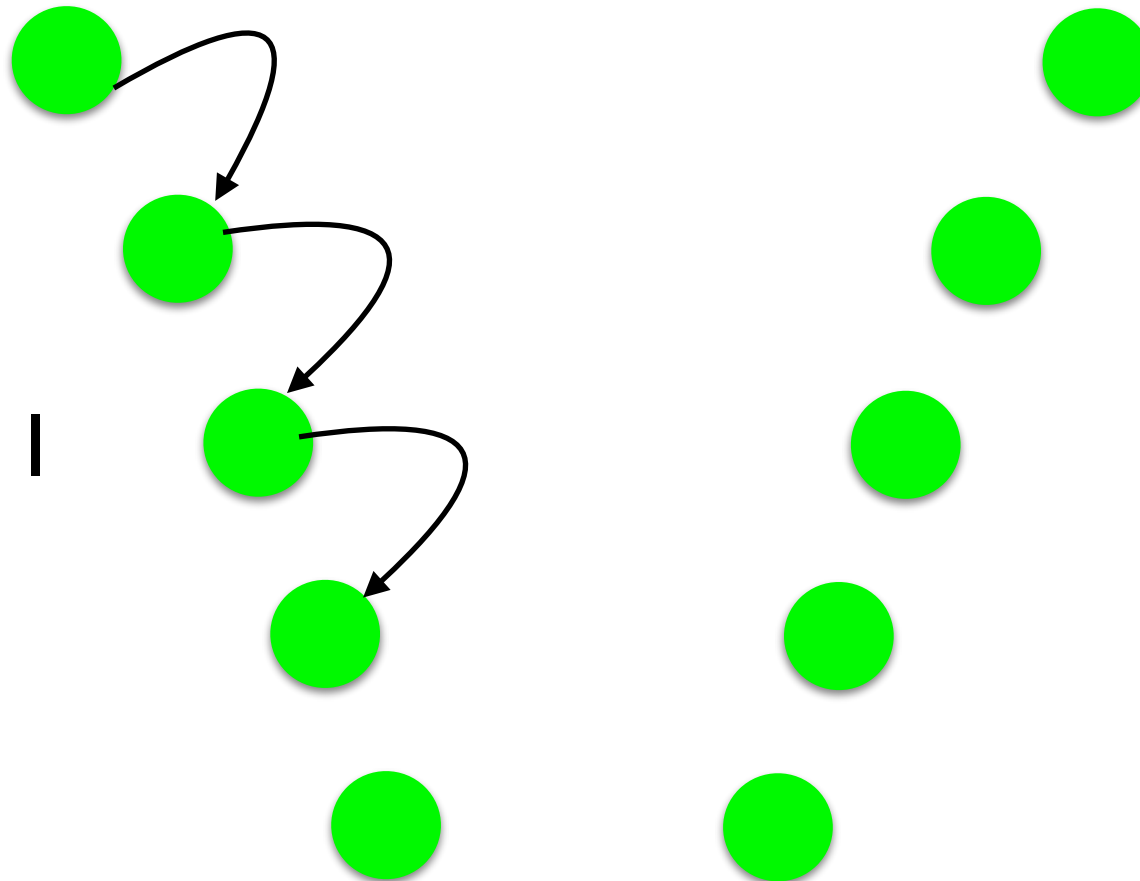
Sequence I



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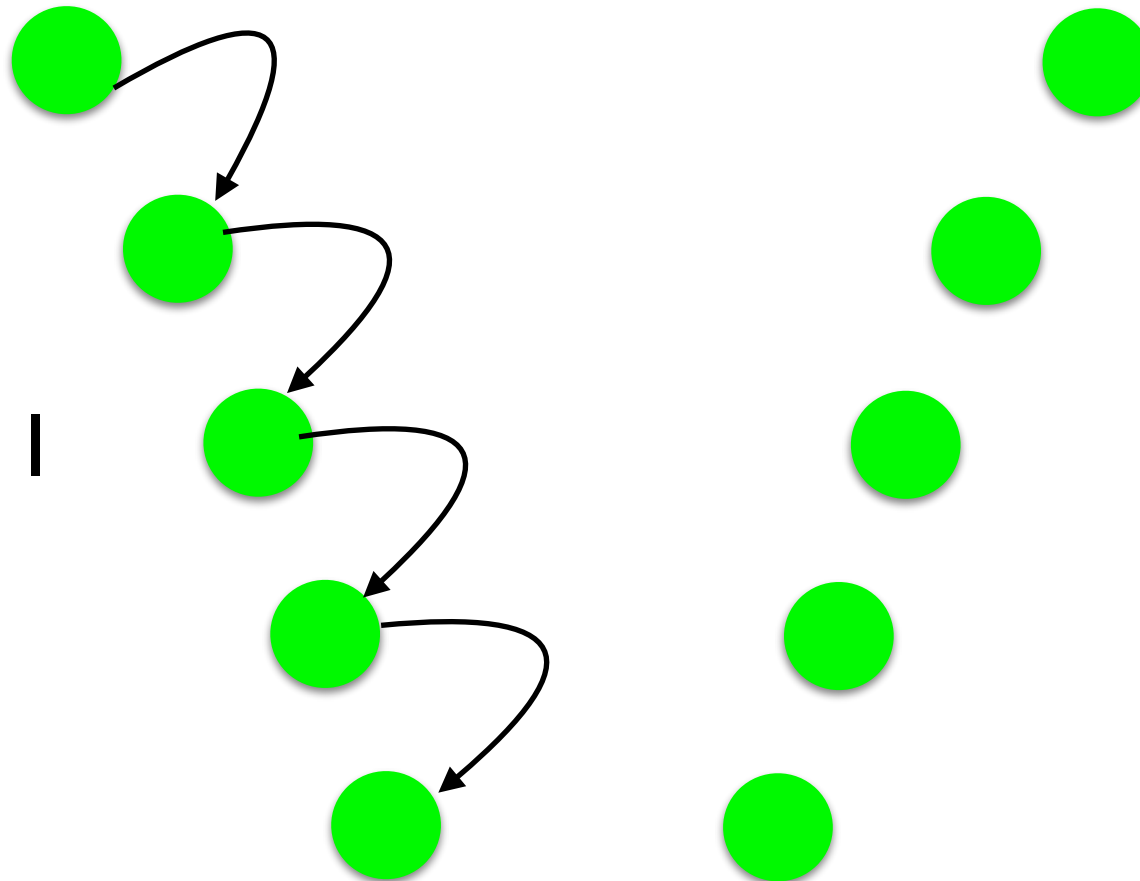


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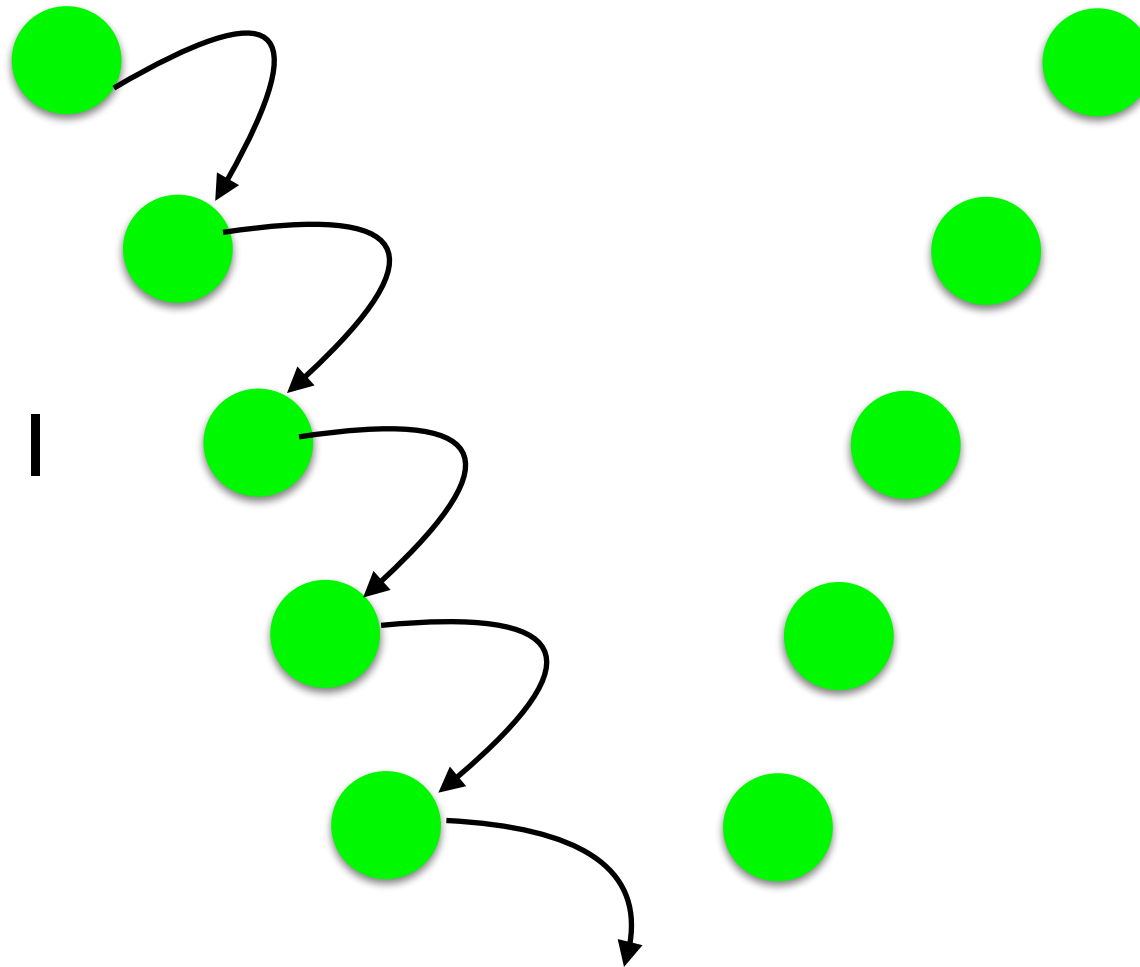




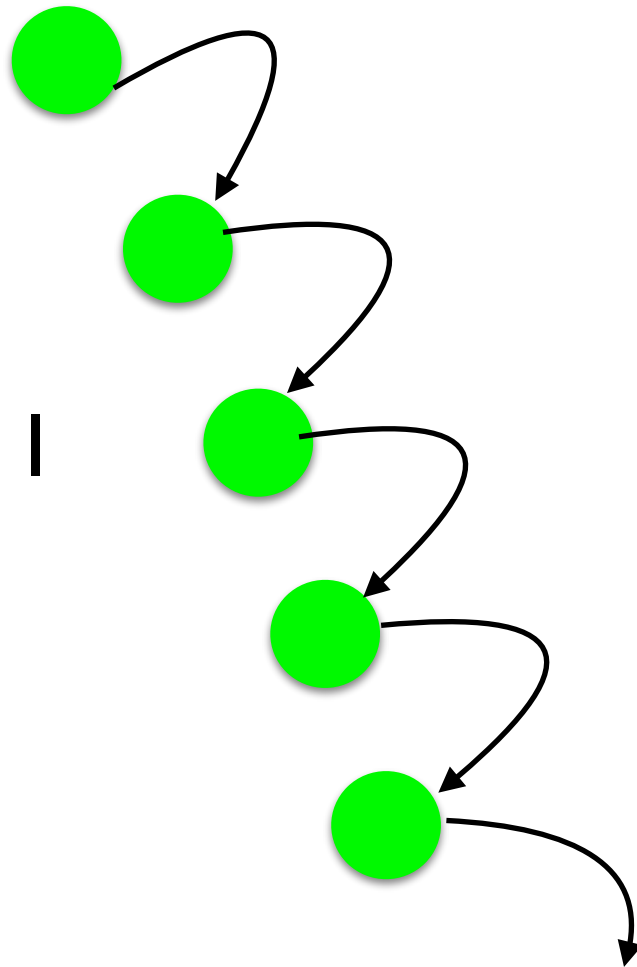
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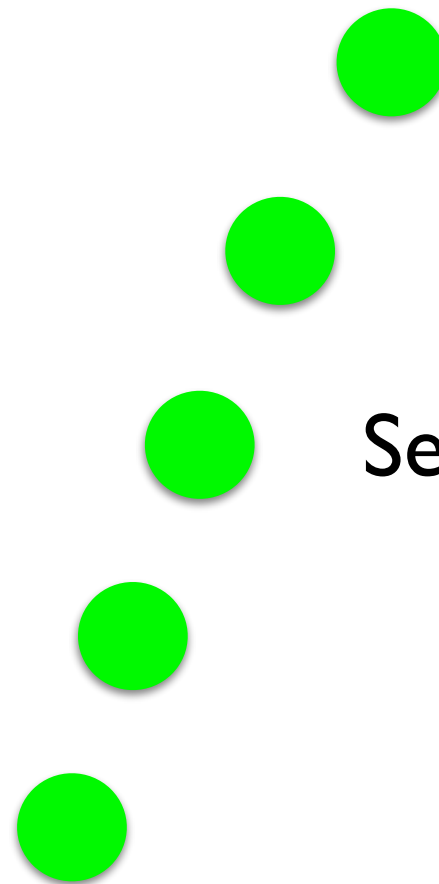
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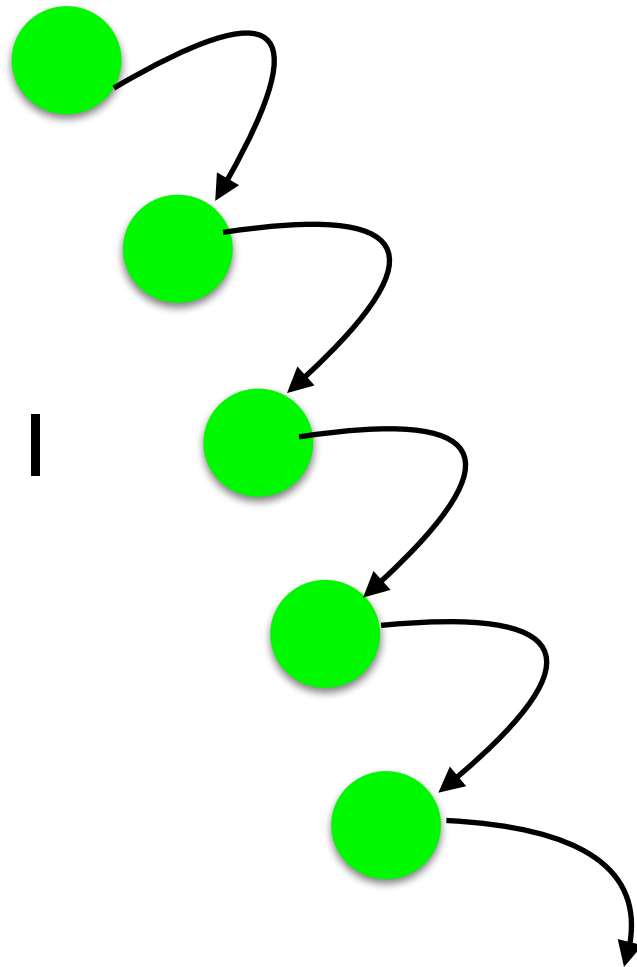
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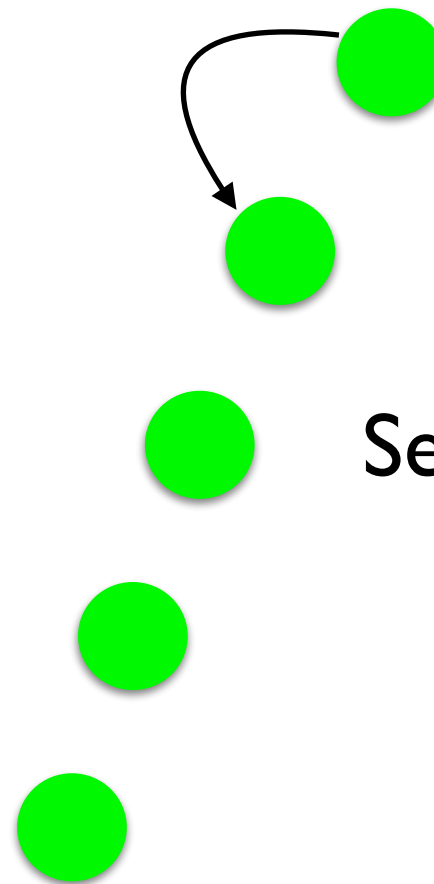
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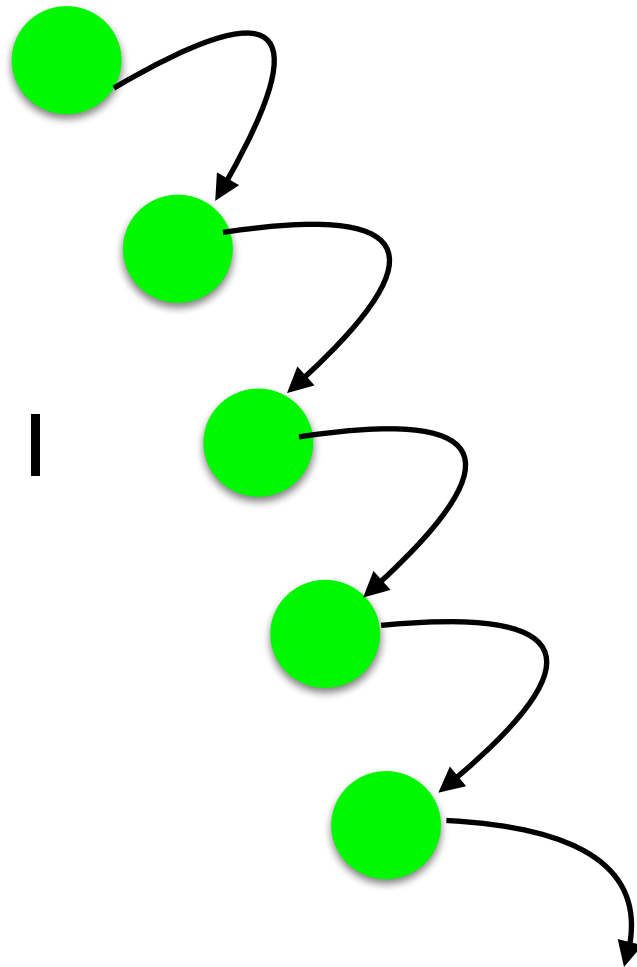
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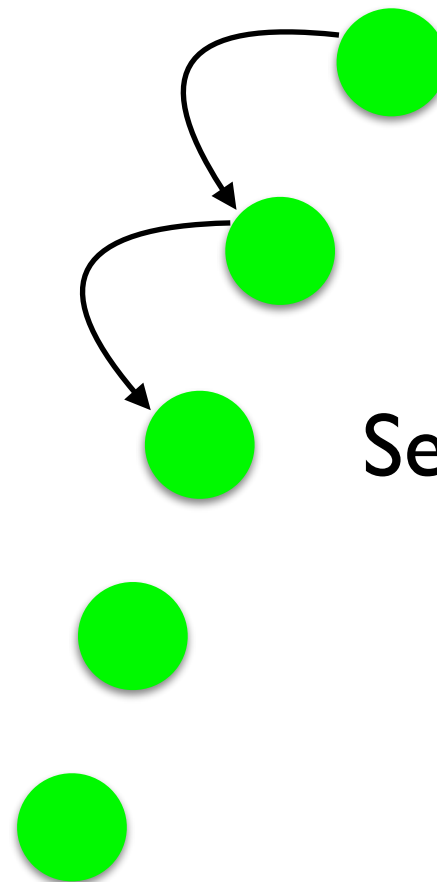
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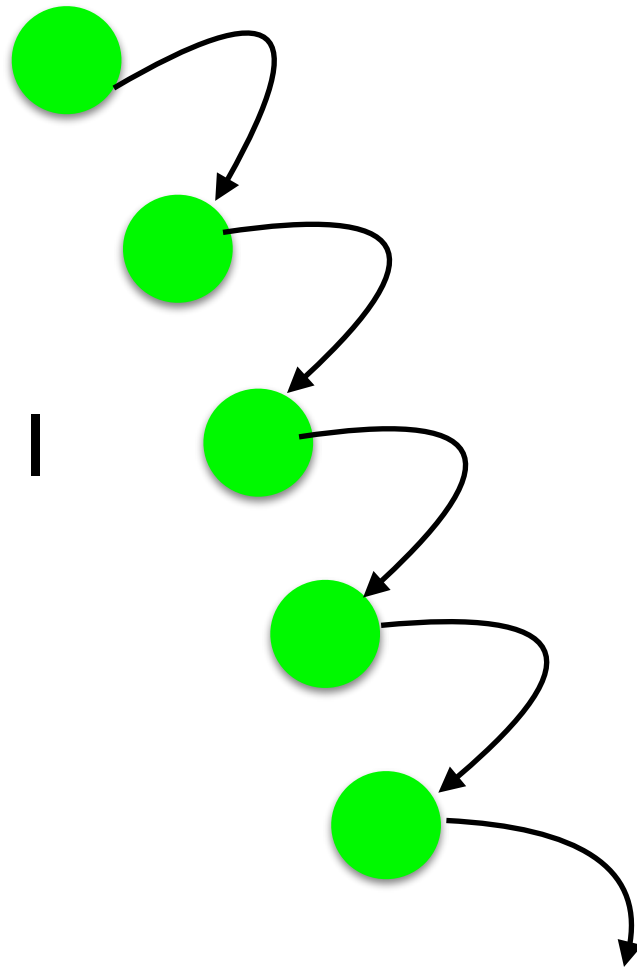
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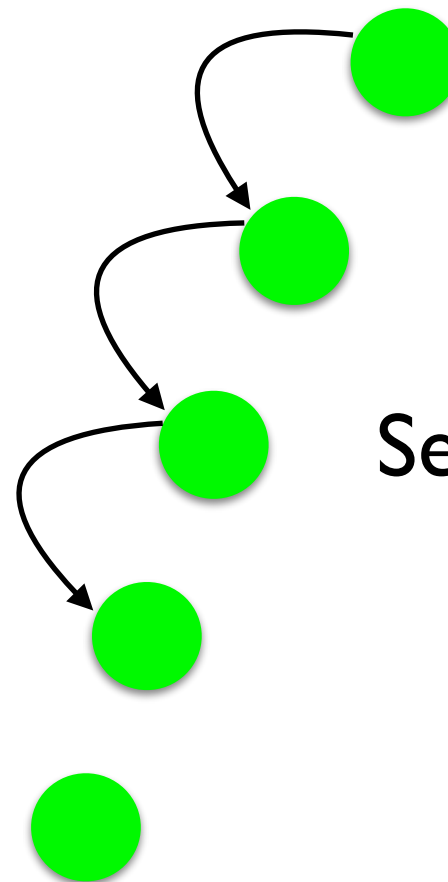
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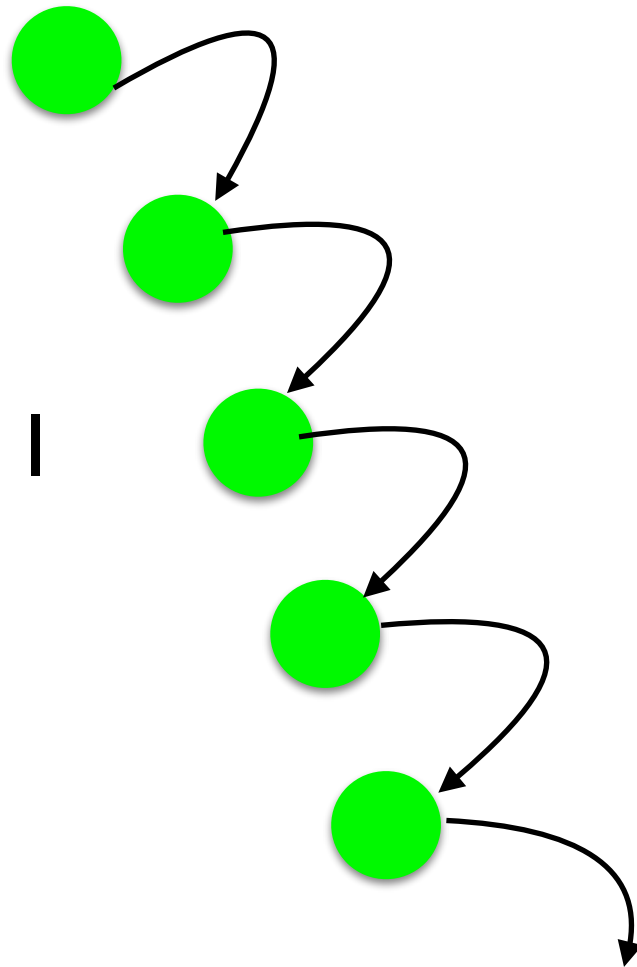
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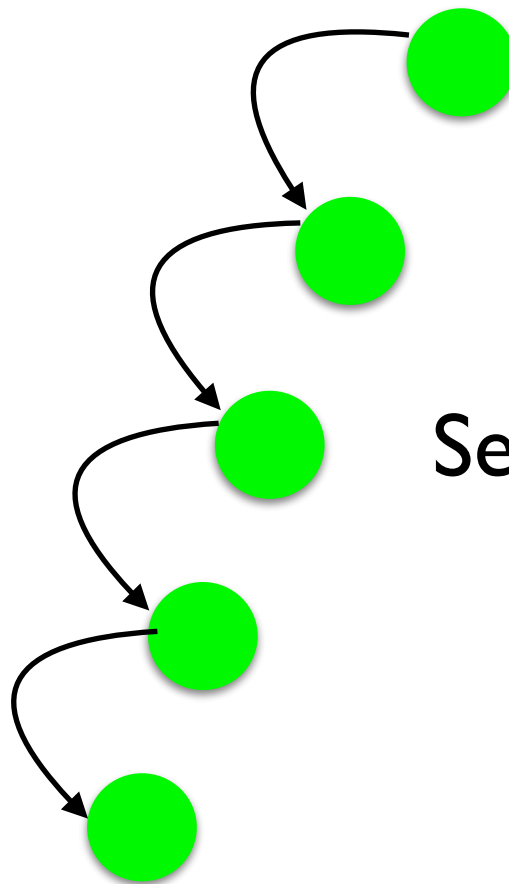
Sequence 2



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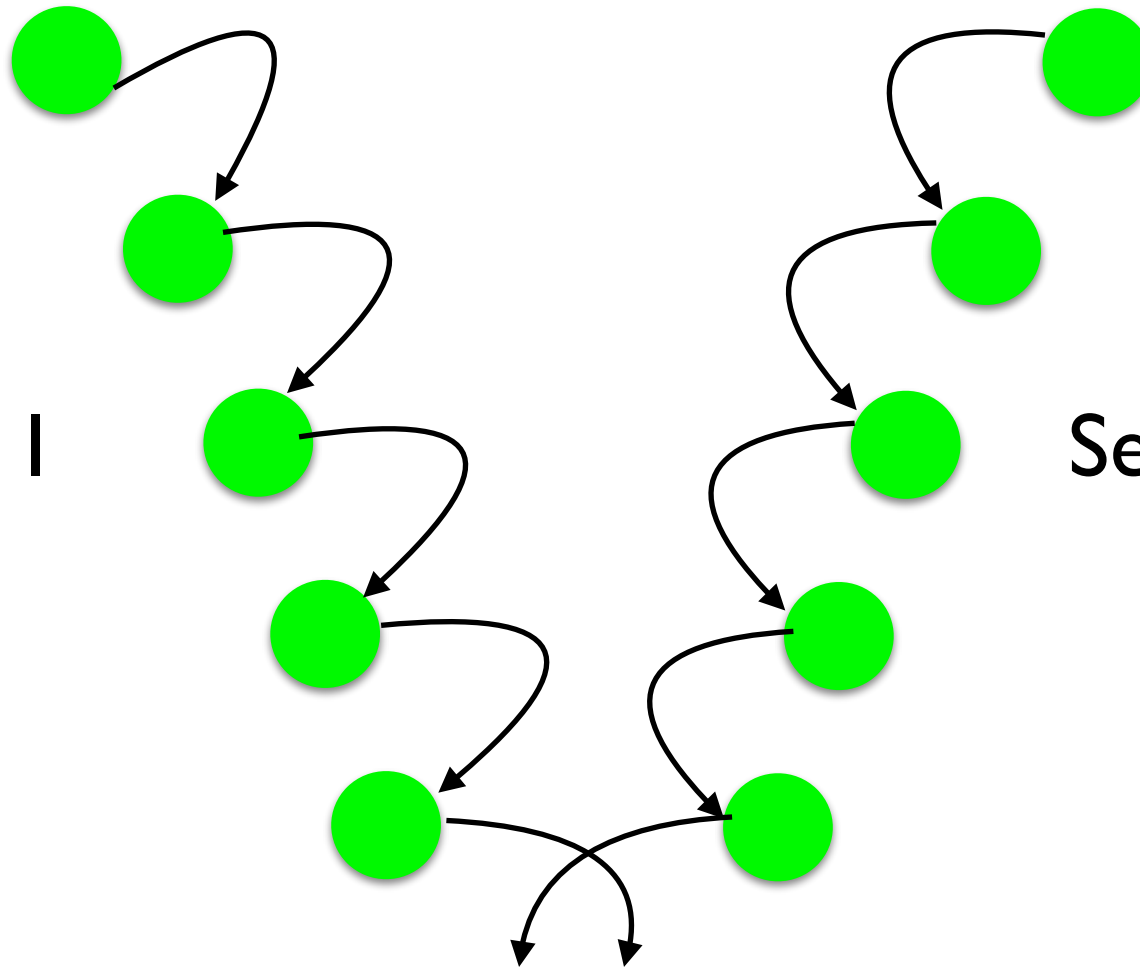


Sequence 2

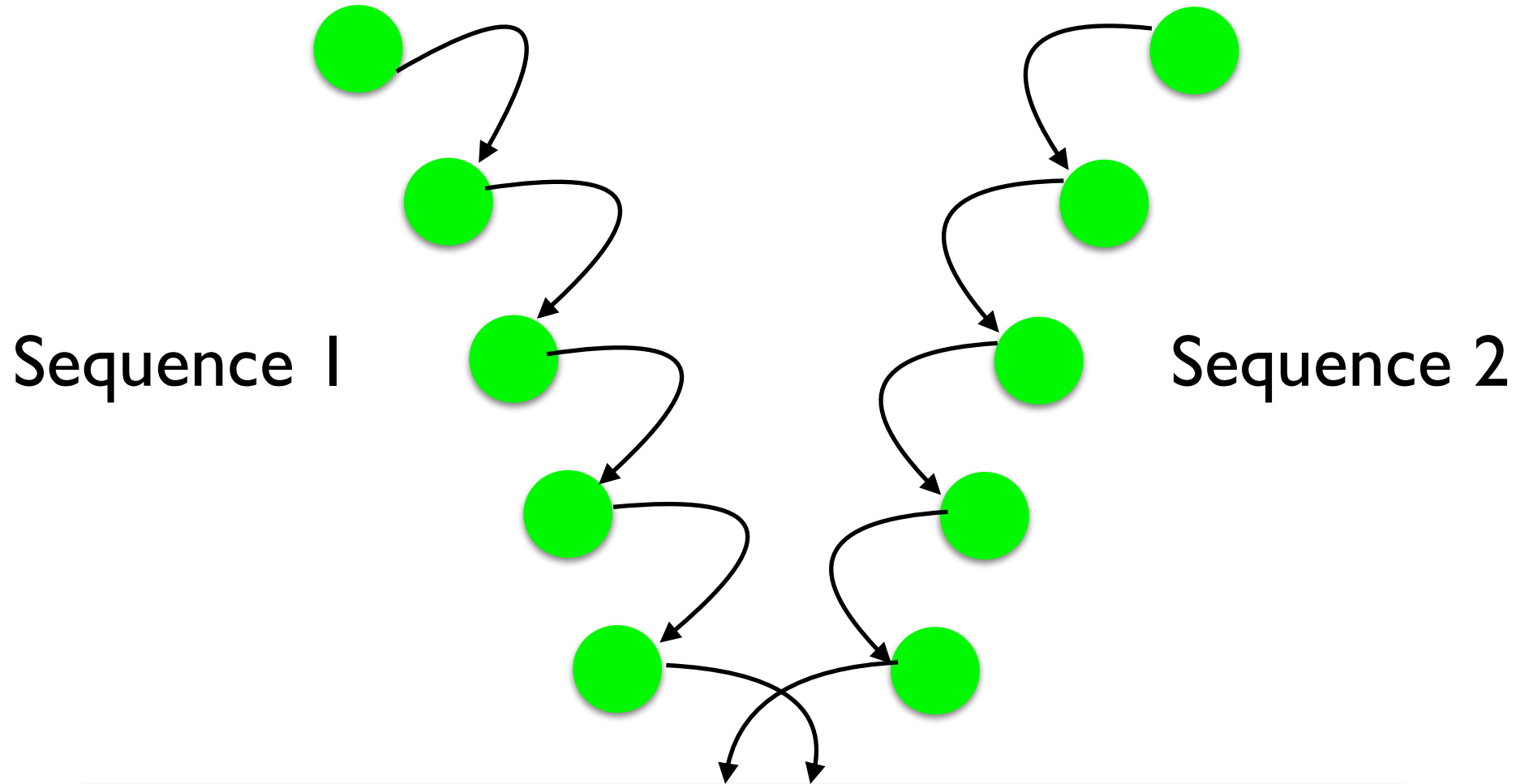


Sequence 1

Sequence 2







Contradiction!

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$$Wt_1 \oplus Wt_2 \oplus \dots \oplus Wt_{1T} \quad (1)$$

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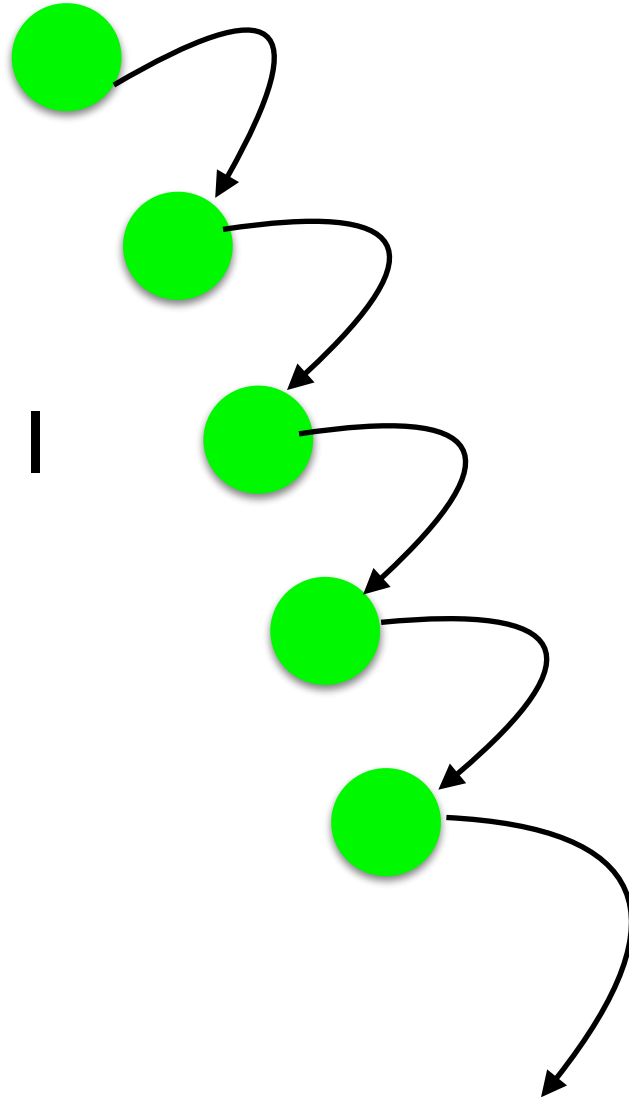
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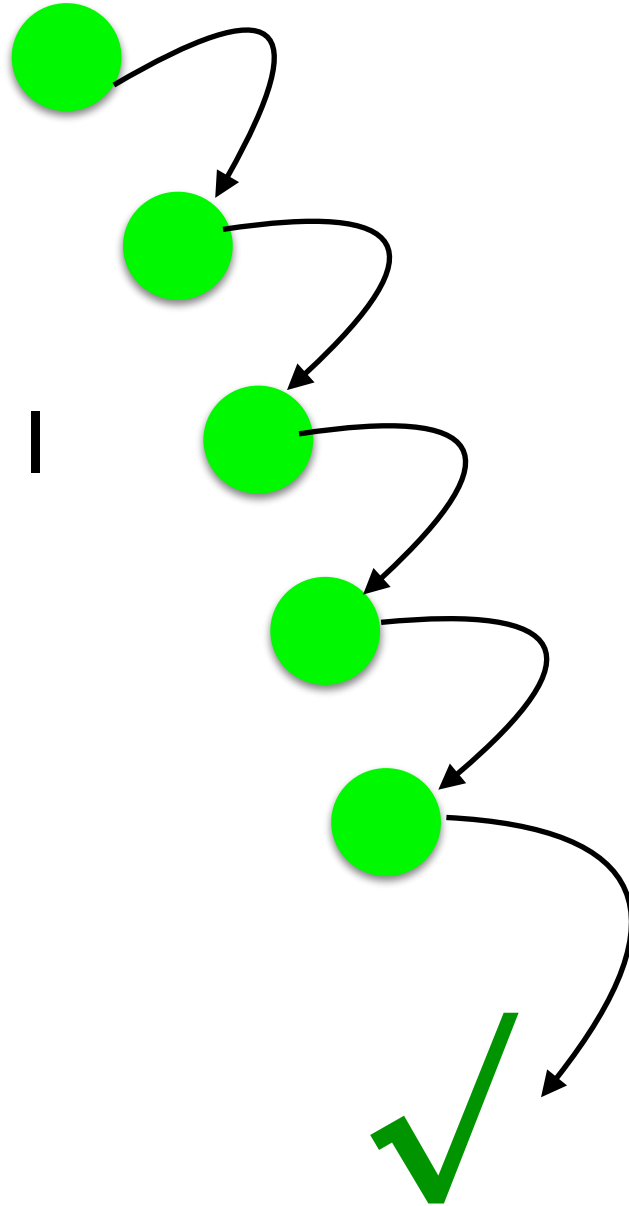
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Sequence I



Sequence I



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For the next step, note that the probability of ticket  $t_i$  winning is lower than, say, the probability that if you walk outside a minute from now you will be flattened on the spot by a door from a 747 that falls from a jet of that type cruising at 35,000 feet. Since you have the rational belief that death won't ensue if you go outside (and have this belief precisely because you believe that the odds of your sudden demise in this manner are vanishingly small), the inference to the rational belief on the part of *a* that  $t_i$  won't win sails through—and this of course works for each ticket. Hence we have:



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Of course, if a rational agent believes *P*, and believes *Q* as well, it follows that that agent will believe the conjunction *P* & *Q*. Applying this principle to (2) yields:

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But (3) is logically equivalent to the statement that there doesn't exist a winning ticket; i.e., (3) is logically equivalent to this result from Sequence 2:

# Sequence 2

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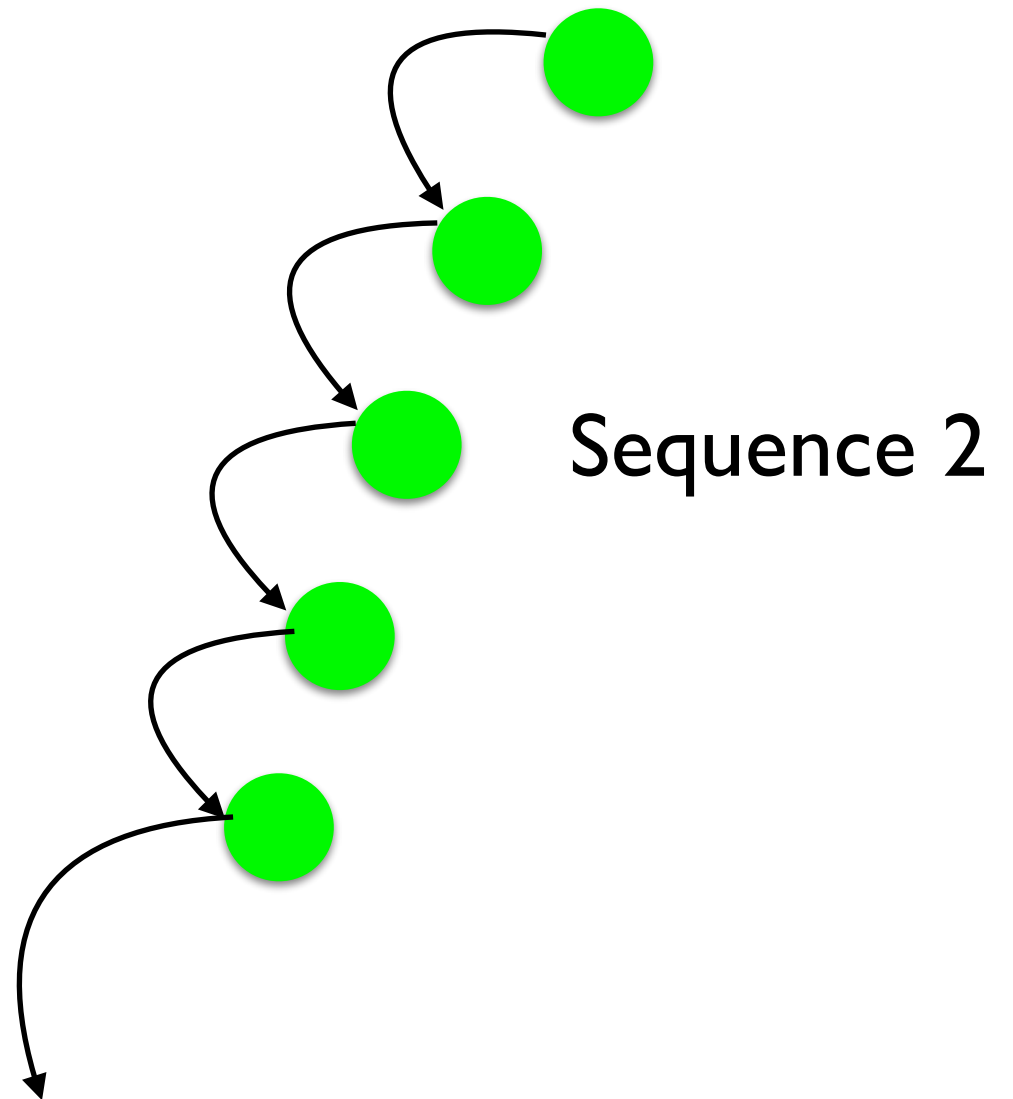
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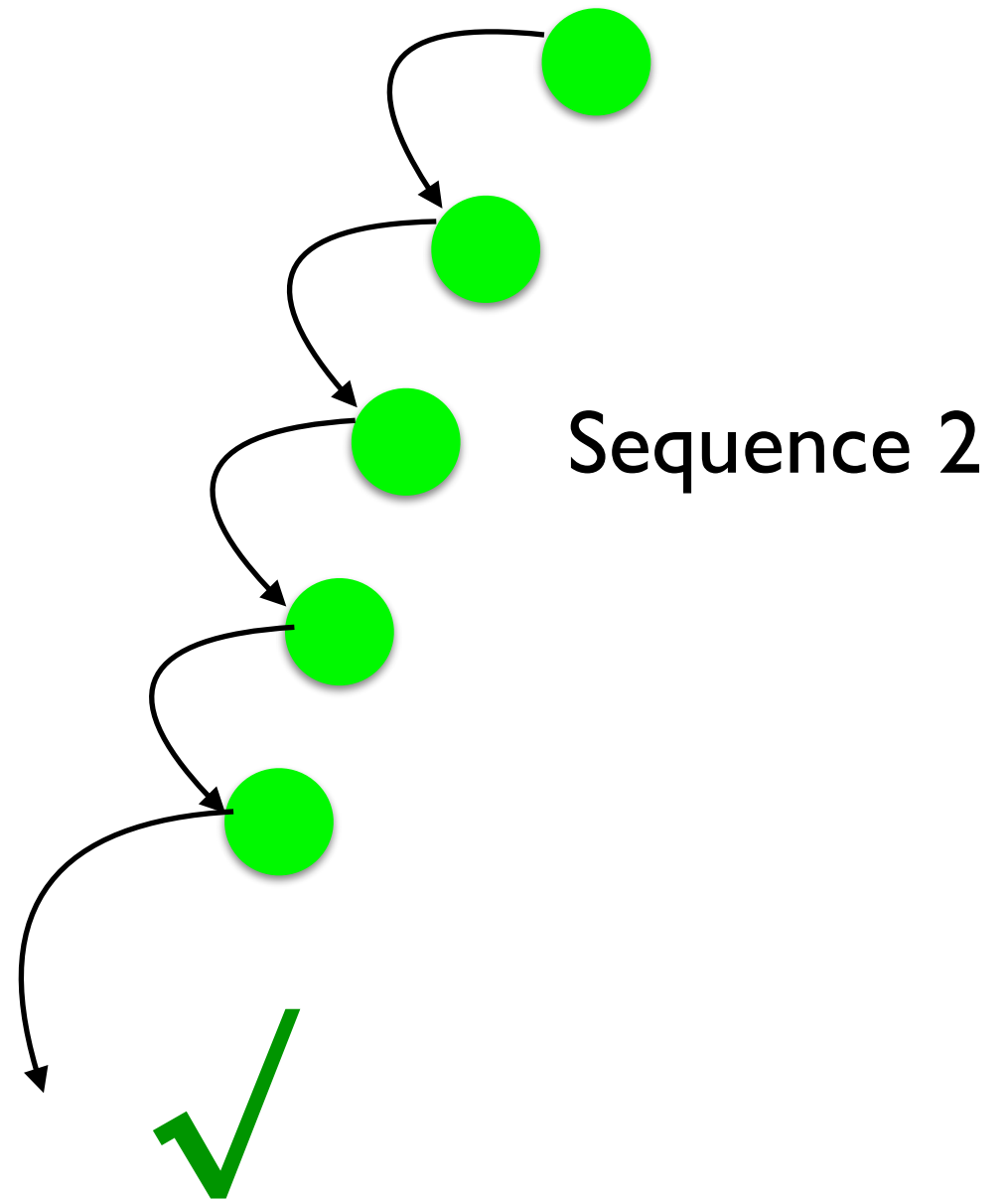
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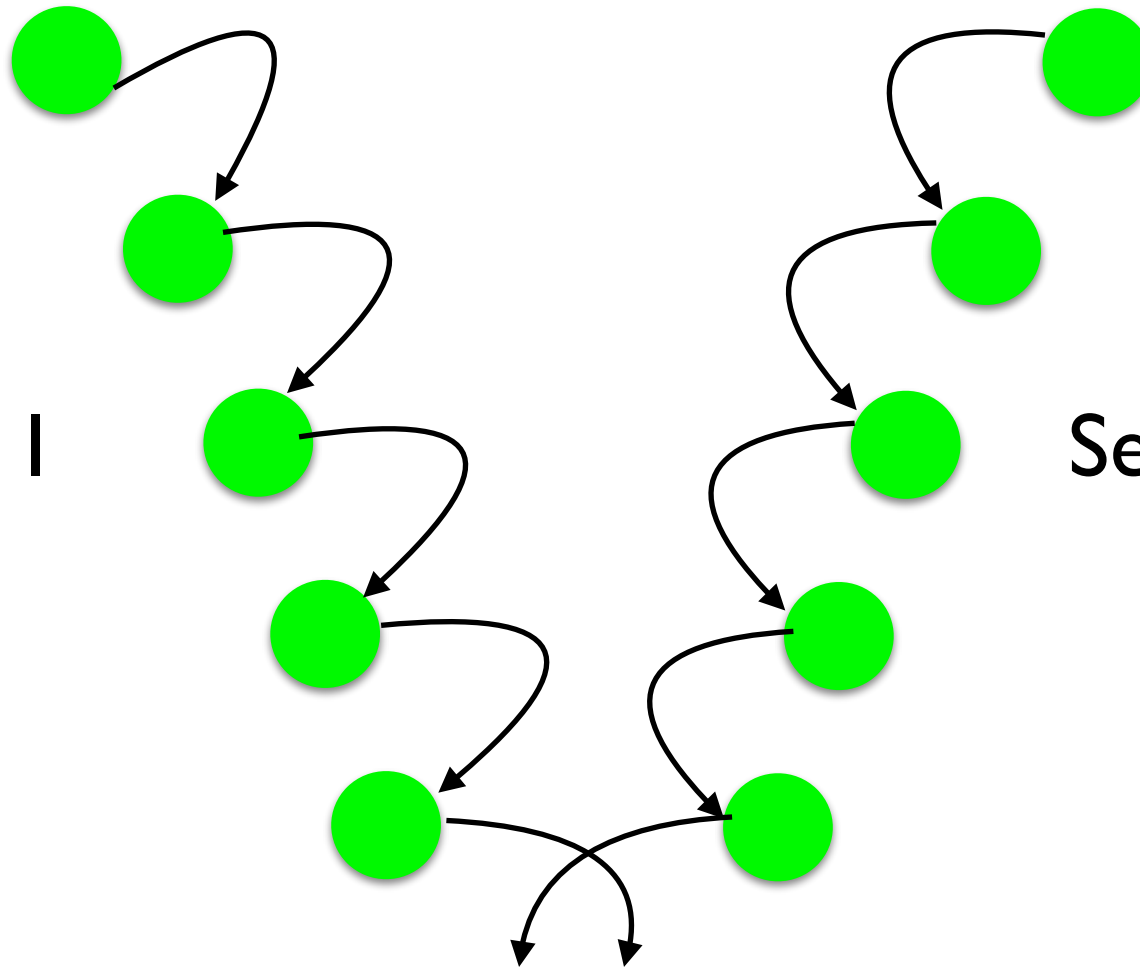
$$\mathbf{B}_a \neg \exists t_i Wt_i \quad (4)$$





Sequence 1

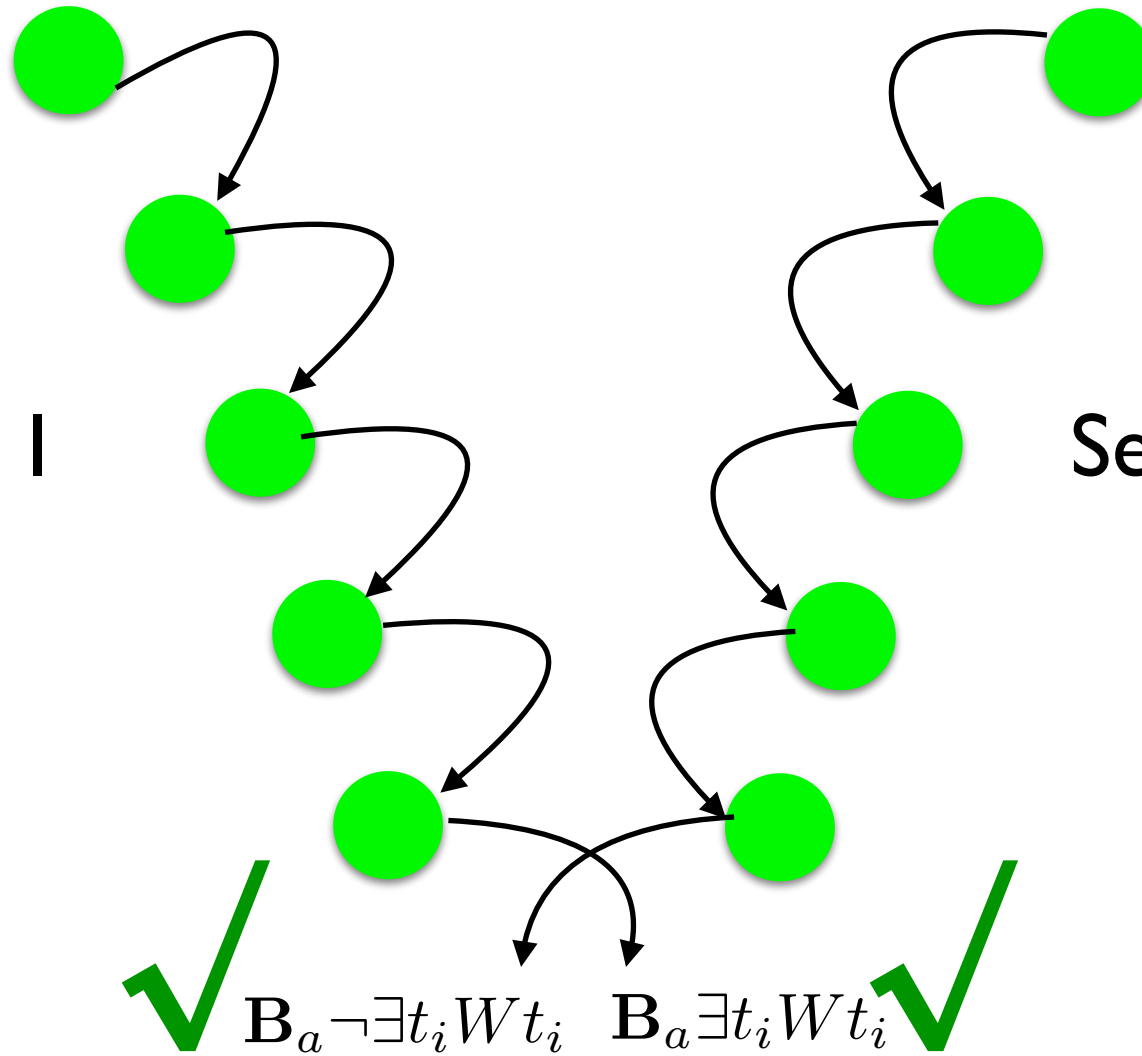
Sequence 2

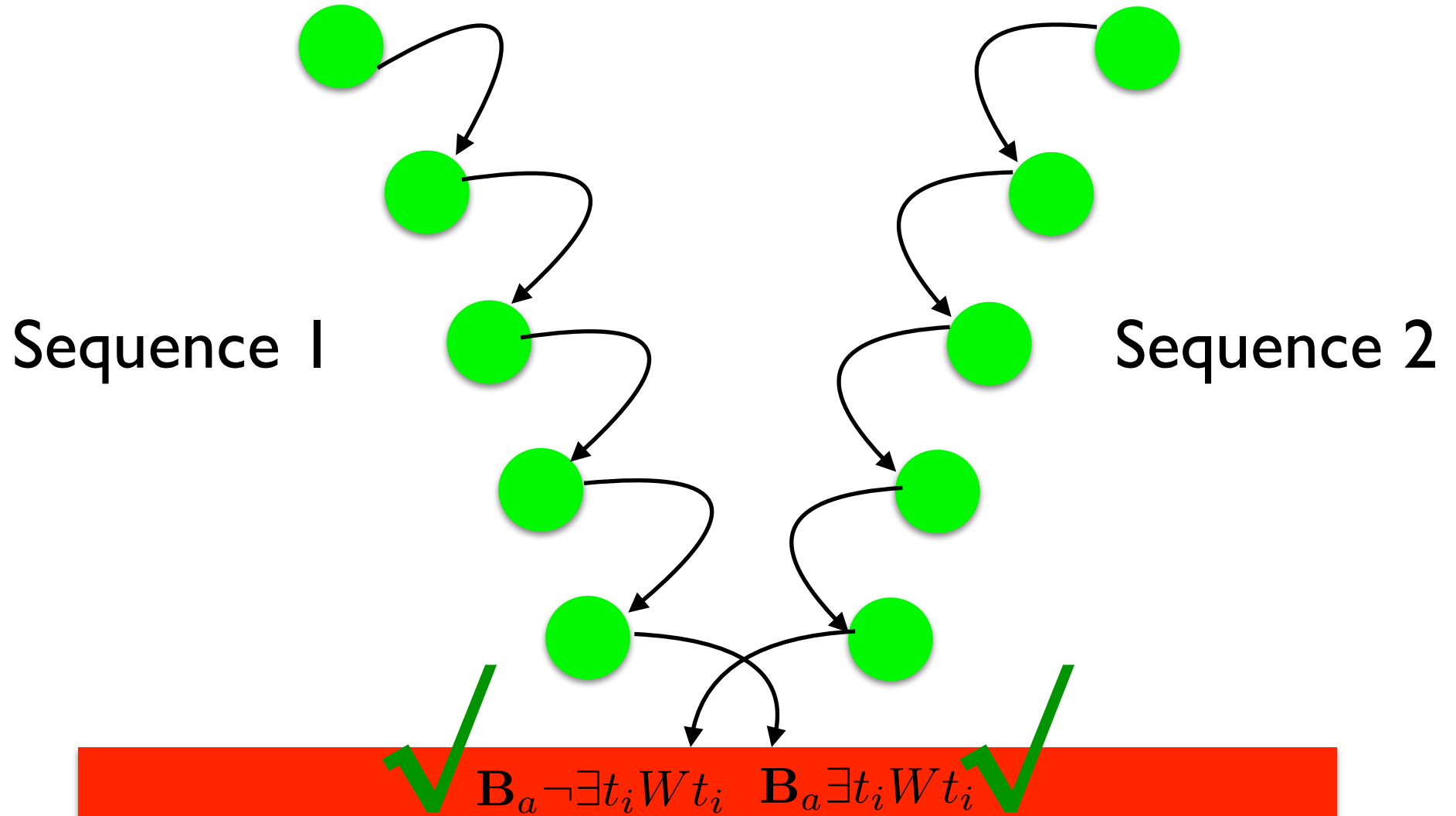




Sequence 1

Sequence 2





(The contradiction we sketched earlier has arrived.)

# A Solution to The Lottery Paradox ...

# Strength-Factor Continuum

Certain

Improbable

Evidently False

Probable

Beyond Reasonable Belief

Certainly False

Counterbalanced

Evident

Beyond Reasonable Doubt

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**Actually, now ...**

# Actually, now ...

English	Value
certain	6
evident	5
overwhelmingly likely = “beyond reasonable doubt” = “one in a million”	4
very likely	3
likely	2
more likely than not	1
counterbalanced	0

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... but let's use the simpler scheme.



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Epistemically Positive

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# Strength-Factor Continuum

Epistemically Positive

(4) Certain

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# Key Principles

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Epistemically Positive

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**Any rational belief that  $p$ , where the basis for  $p$  is at most evident, is at most an evident (= level 3) belief.**

(-1) Improbable

(-2) Beyond Reasonable Doubt Belief

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# Sequence I, “Rigorized”

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Very well; perfectly clear so far. And now we can add another deductive step: Since our rational agent *a* can follow this deduction sequence to this point, and since **D** is assumed to be indubitable, it follows that our rational agent in turn believes (2); i.e., we conclude Sequence I by obtaining the following:

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# Sequence 2, “Rigorized”

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**Deduction preserves strength.**

**Clashes are resolved in favor of higher strength; clashes propagate backwards through inverse deduction; if no higher-strength factors, suspend belief.**

**Any proposition  $p$  such that  $prob(p) < 1$  is at most evident.**

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This is why, to Mega Millions ticket holder:  
“Sorry. I’m rational, and I believe you won’t win.”

To be clear about the effects of the first principle:

$$\vdash \mathbf{B}_a^3 \neg \text{~~!x~~} Wx \wedge \mathbf{B}_a^3 \exists x Wx!$$

$$\vdash \mathbf{B}_a^2 \neg \text{~~!x~~} Wx \wedge \mathbf{B}_a^2 \exists x Wx!$$

$$\vdash \mathbf{B}_a^1 \neg \text{~~!x~~} Wx \wedge \mathbf{B}_a^1 \exists x Wx!$$

**Clashes are resolved in favor of higher strength; clashes propagate backwards through inverse deduction, preserving affirmation/belief of premises as far as is possible; if no higher-strength factors, suspend belief.** (This means that in this case belief at level 4 also shoots down belief at level 2, and level 1. This is sort of bizarre, because to retain the belief (at levels 3, 2, 1) that every particular ticket won't win, the step that gets to believing the existential formula is blocked. Pollock doesn't have steps in his "arguments." Our agents thus ends up believing at all levels that some ticket will win, and believing at all levels 3 and down, of each particular ticket, that it won't win.)





*slutten*