# The Immaterial Paradise, Motivating Paradoxes, Puzzles, and $\mathbb{R}$ ,

Part I

Selmer Bringsjord

Intro to ( $\underline{Formal}$ ) Logic (and  $\underline{AI}$ ) = IFLAII



### Logistics ...

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See Sergei's FAQ.

(Reading the syllabus is helpful.)

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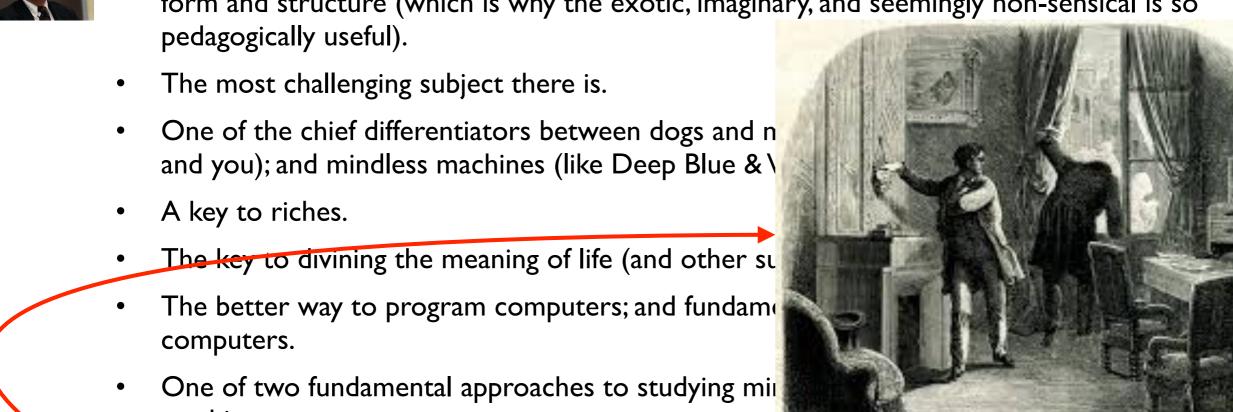
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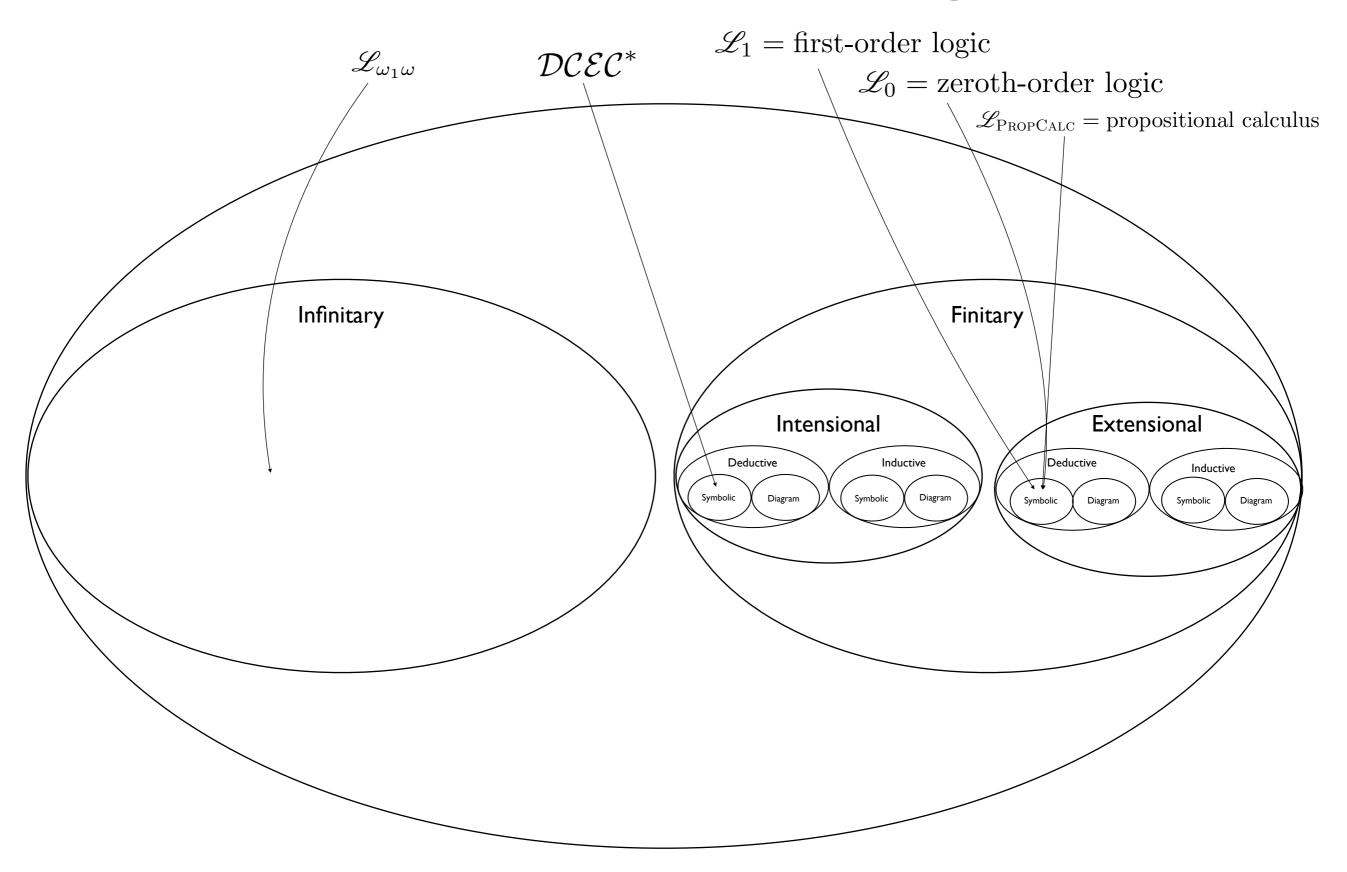
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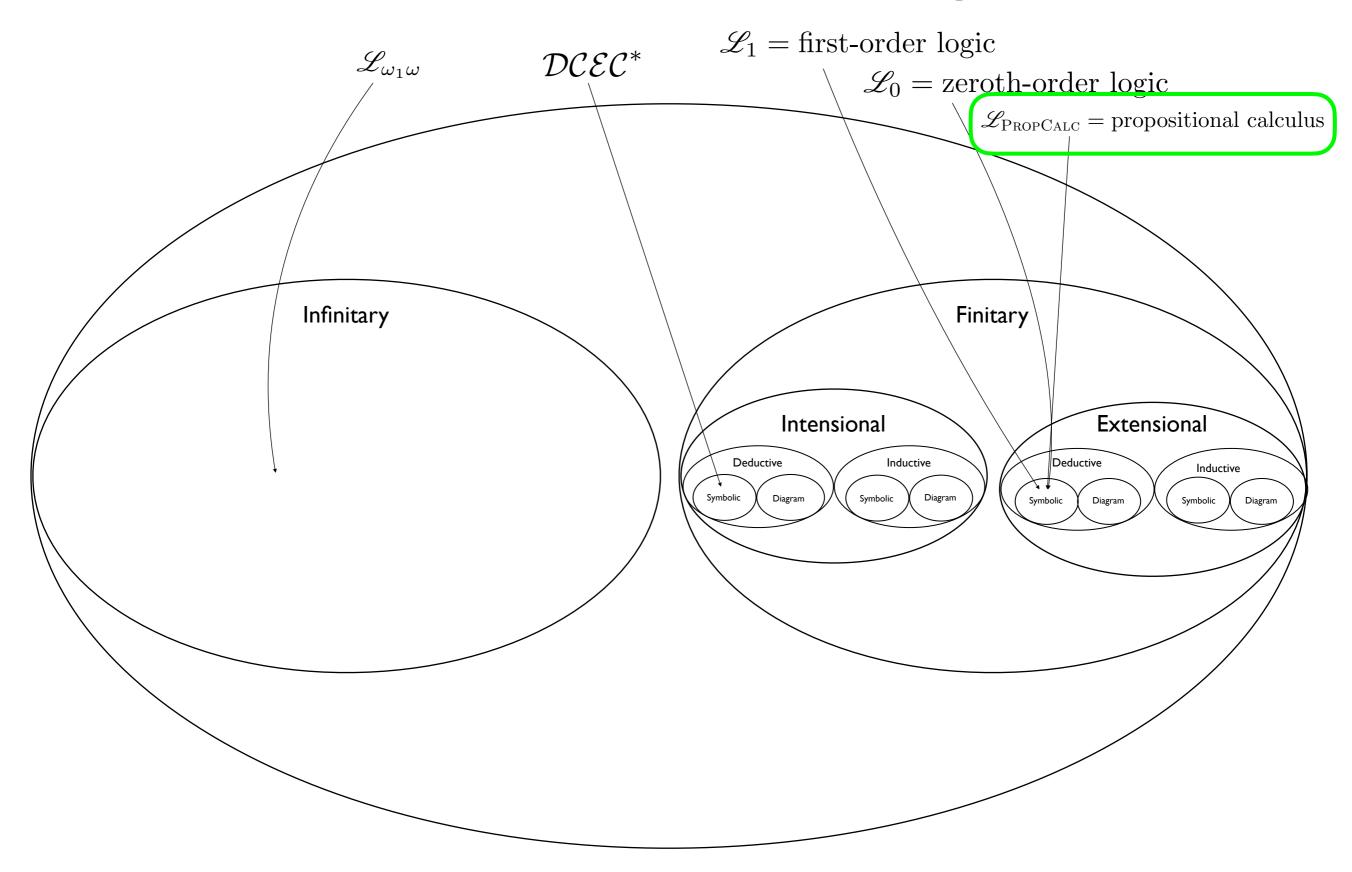




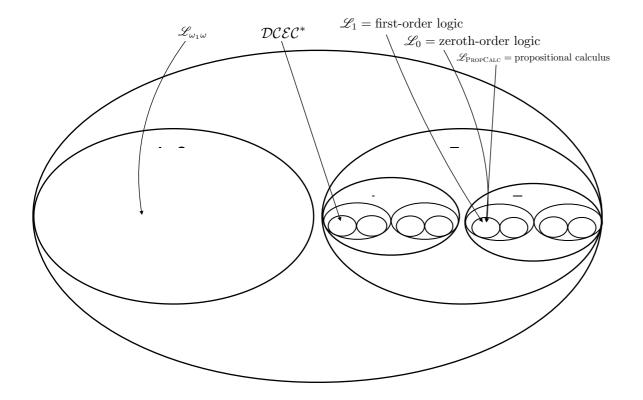
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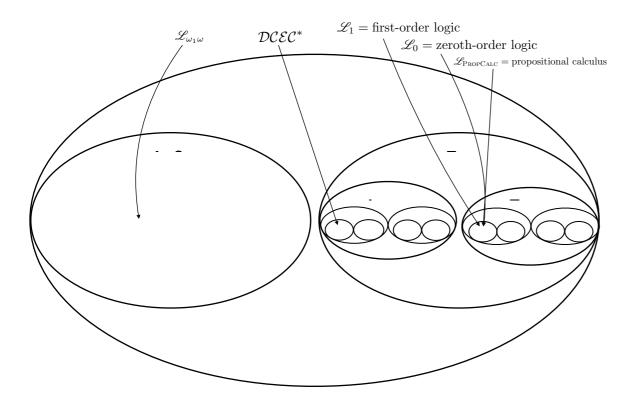


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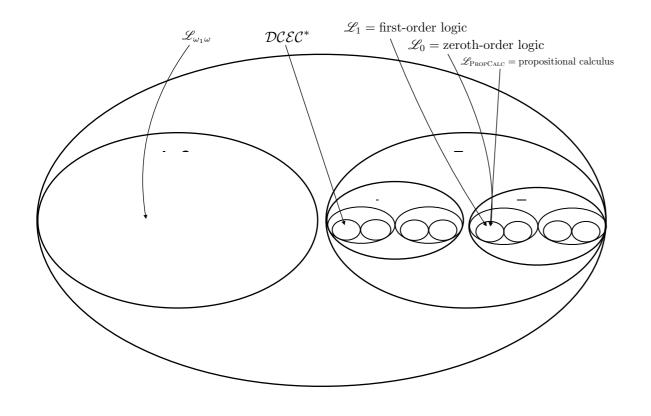


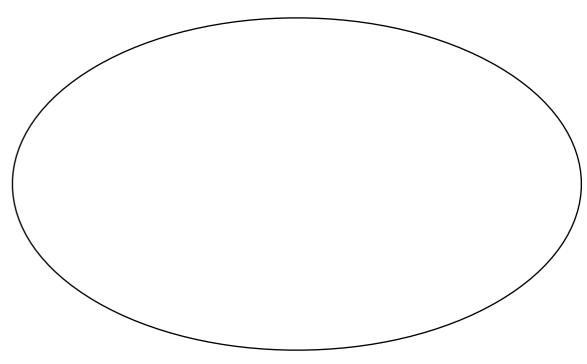
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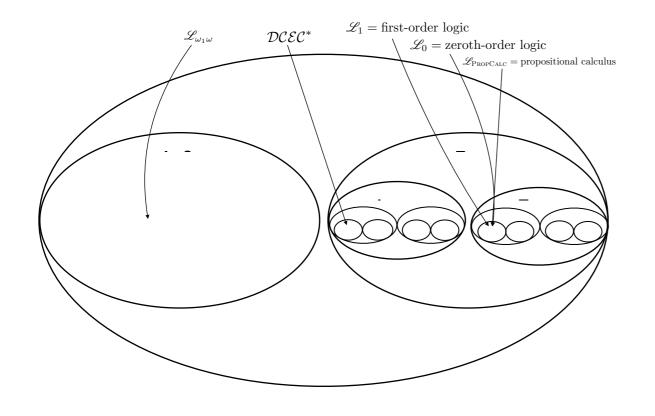


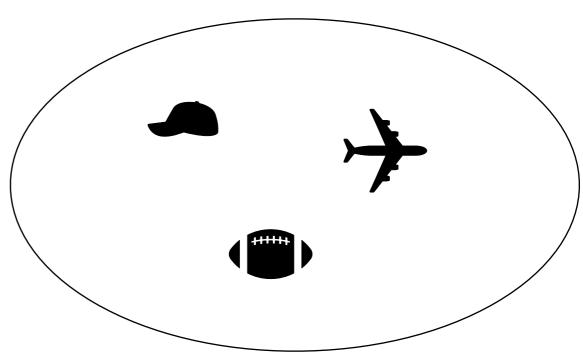
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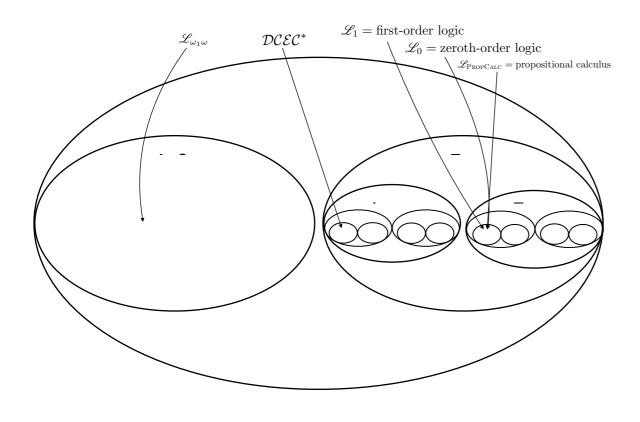


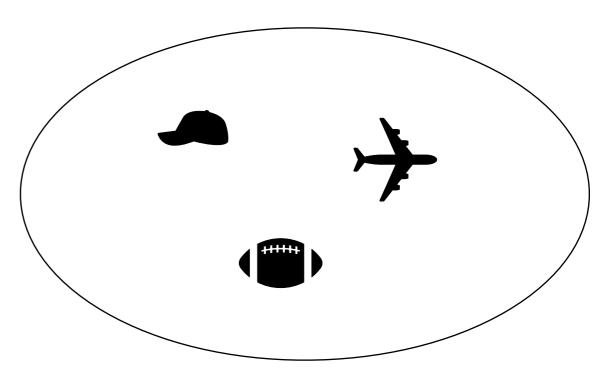
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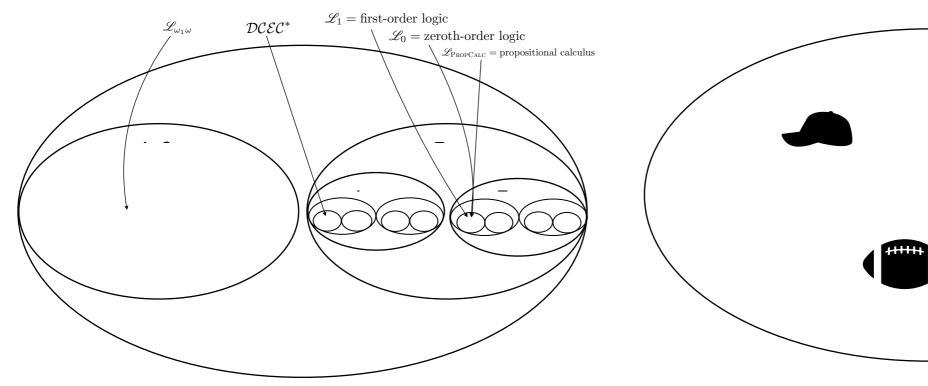
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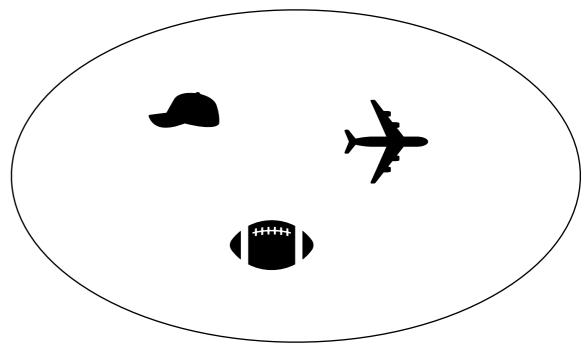






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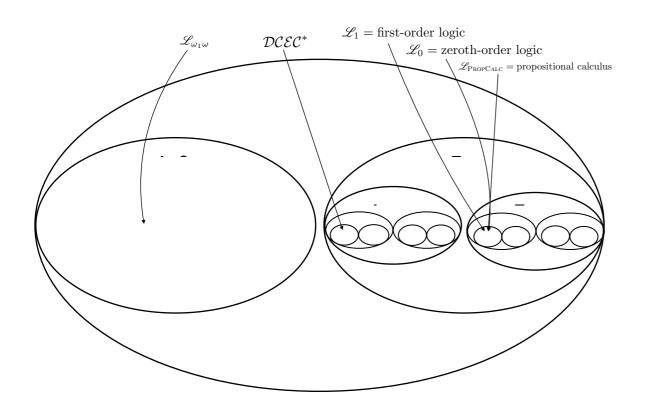


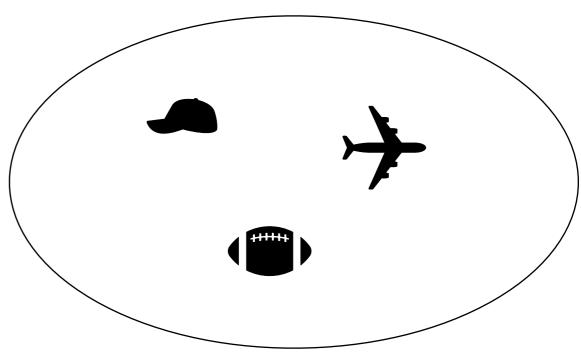


$$\frac{\phi, \phi \to \psi}{\psi}$$

modus ponens

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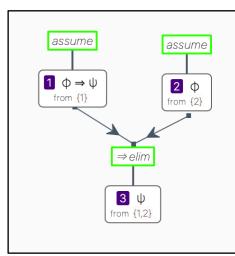




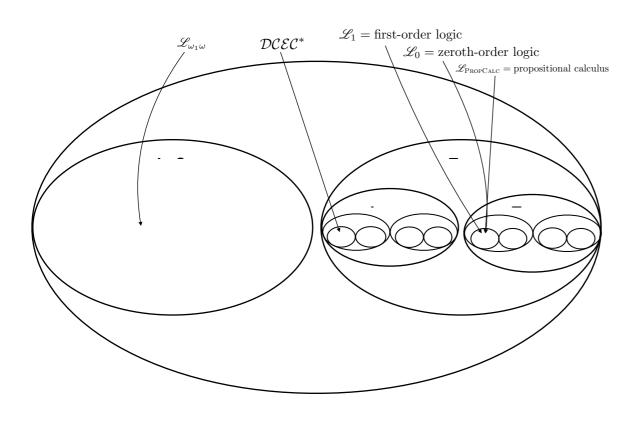


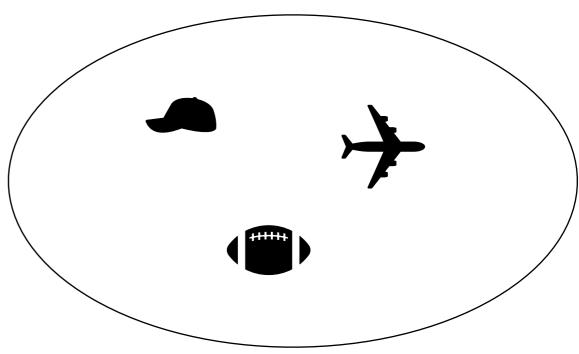
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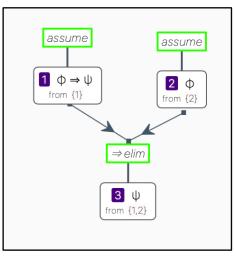






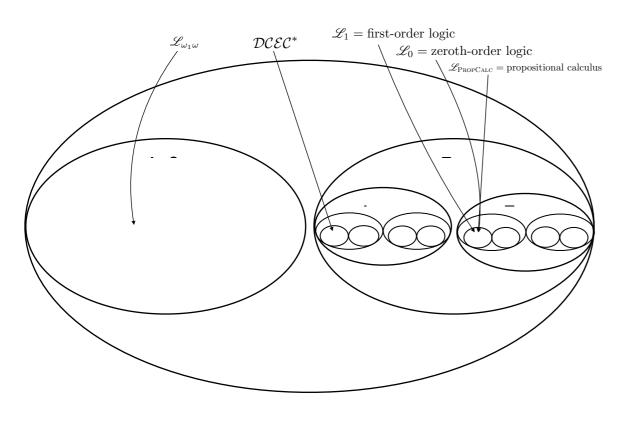
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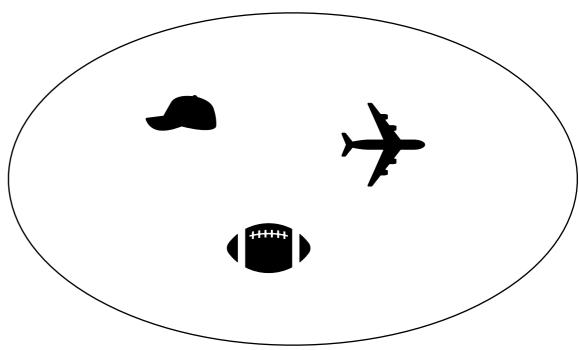
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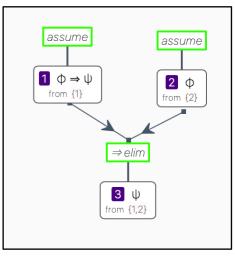


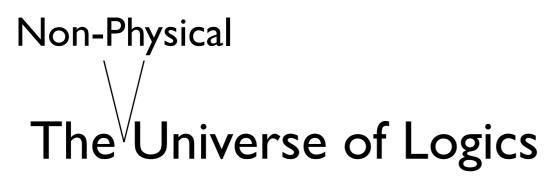




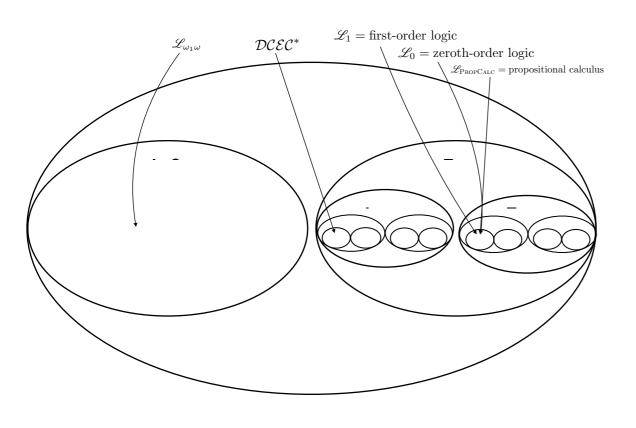
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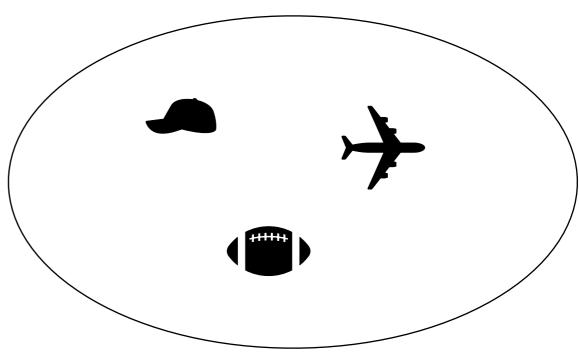
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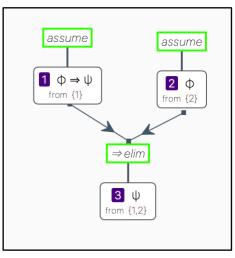


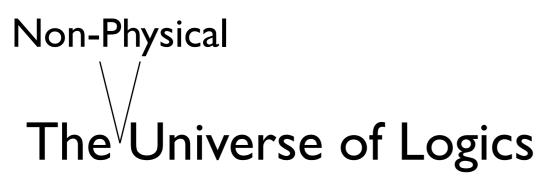


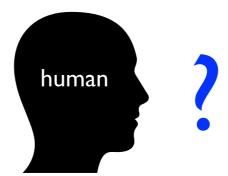


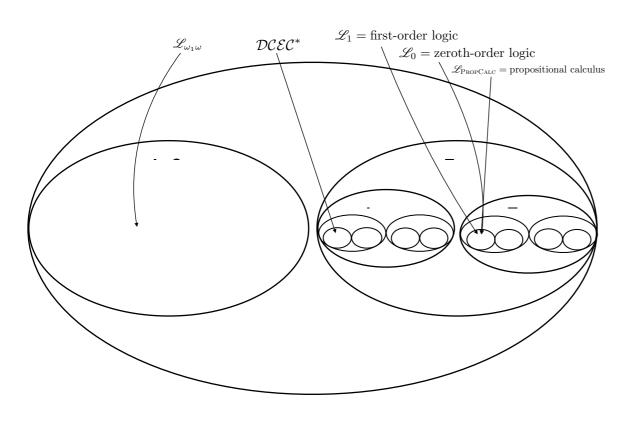
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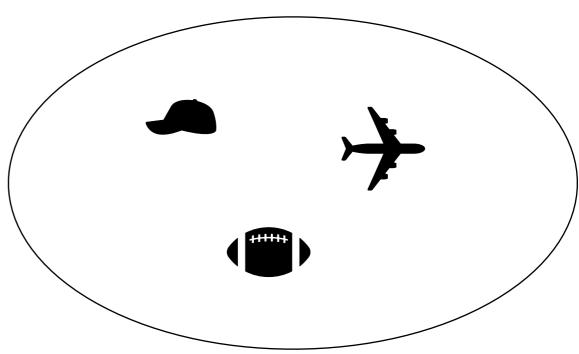
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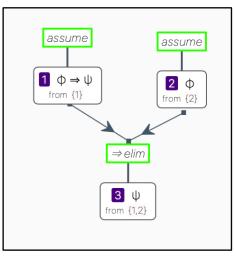


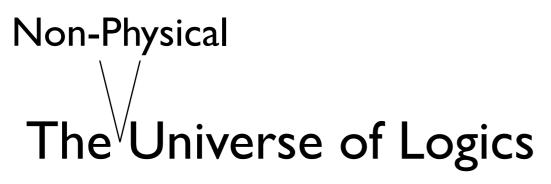


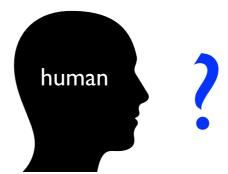


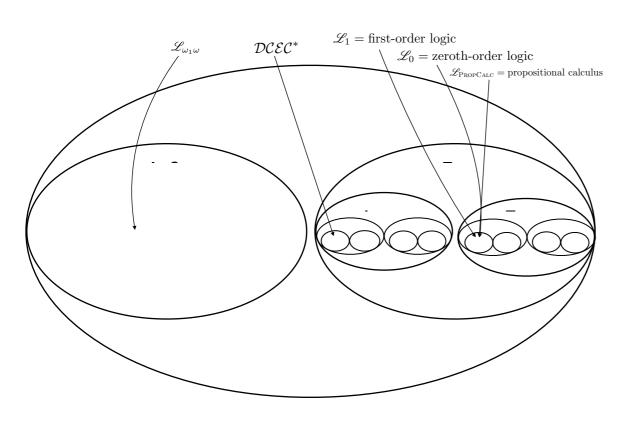
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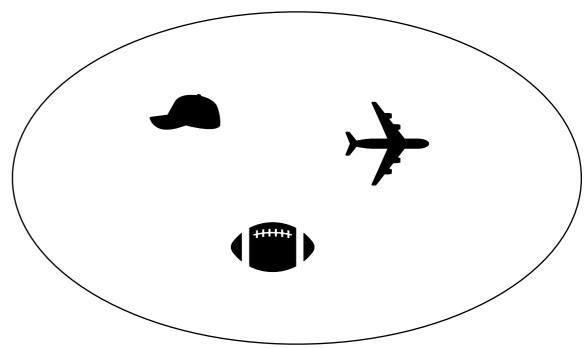
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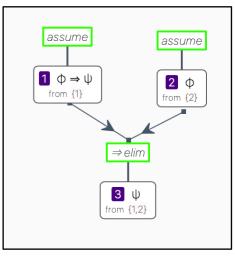






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Really? I'm skeptical. What's so special about us?

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intensional reasoning

recursion

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self-reference

quantification Background Claim

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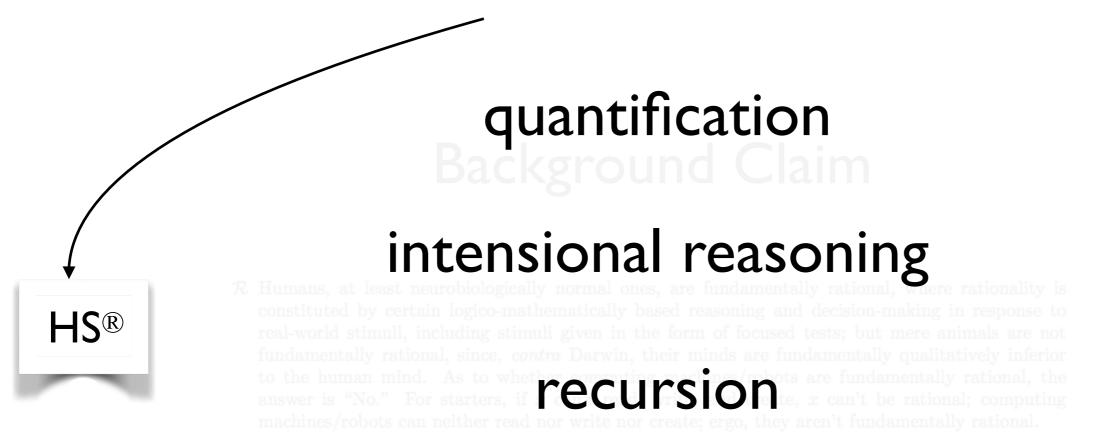
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## intensional reasoning

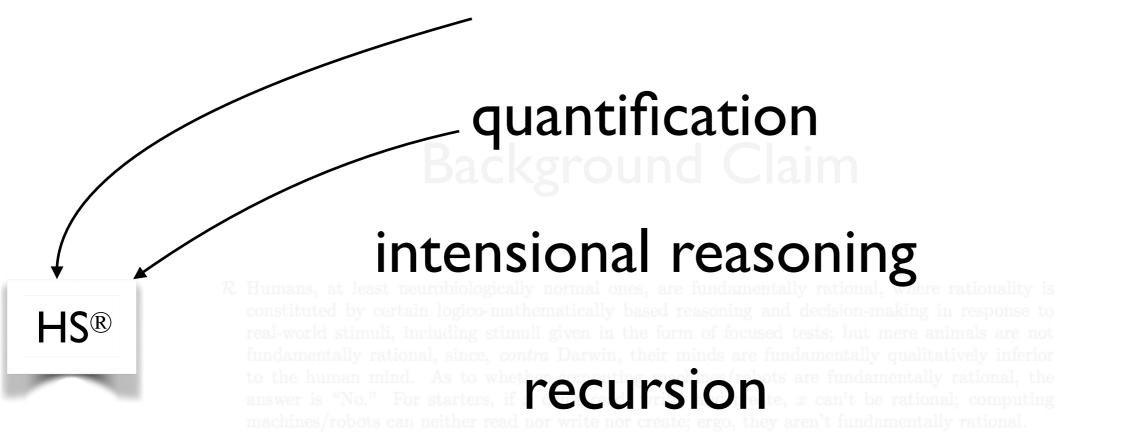
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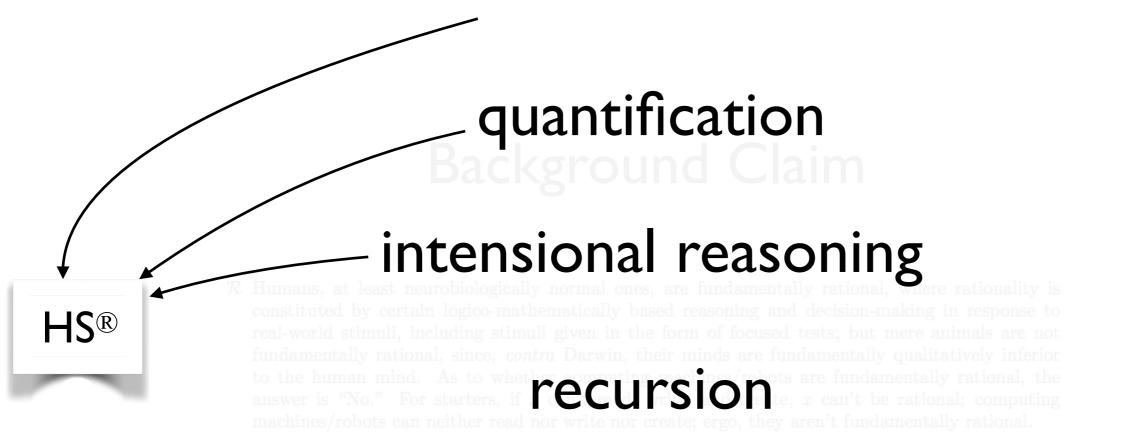
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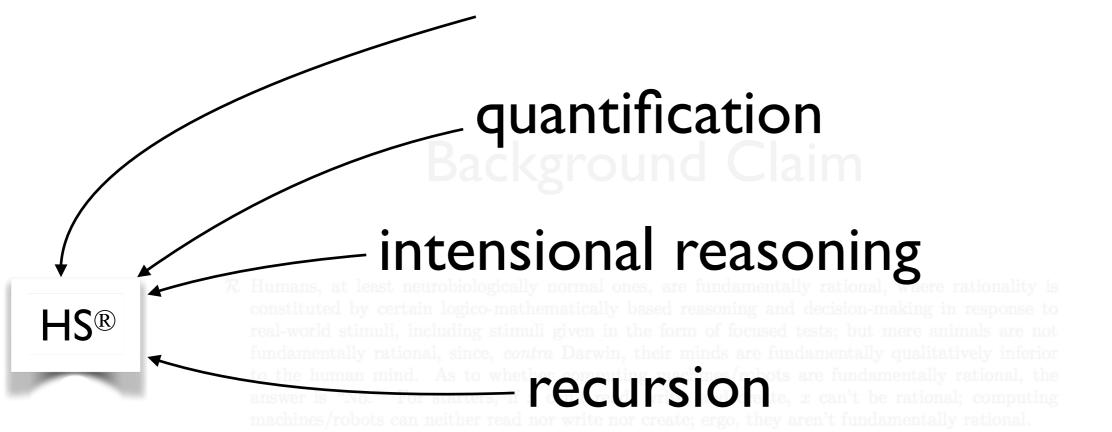
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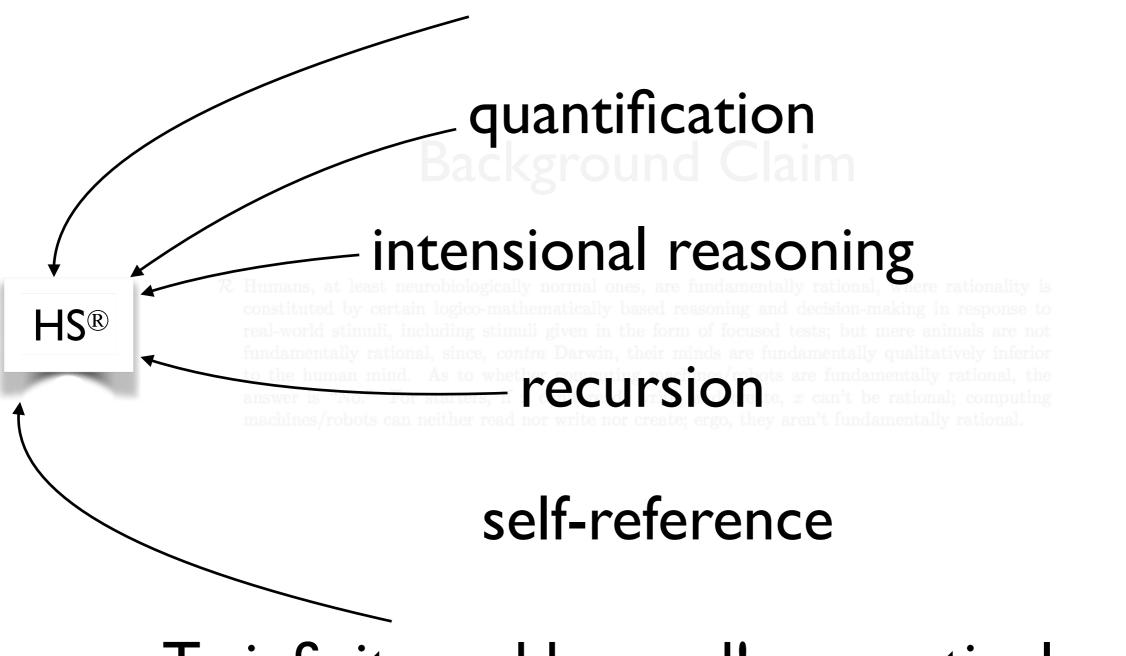
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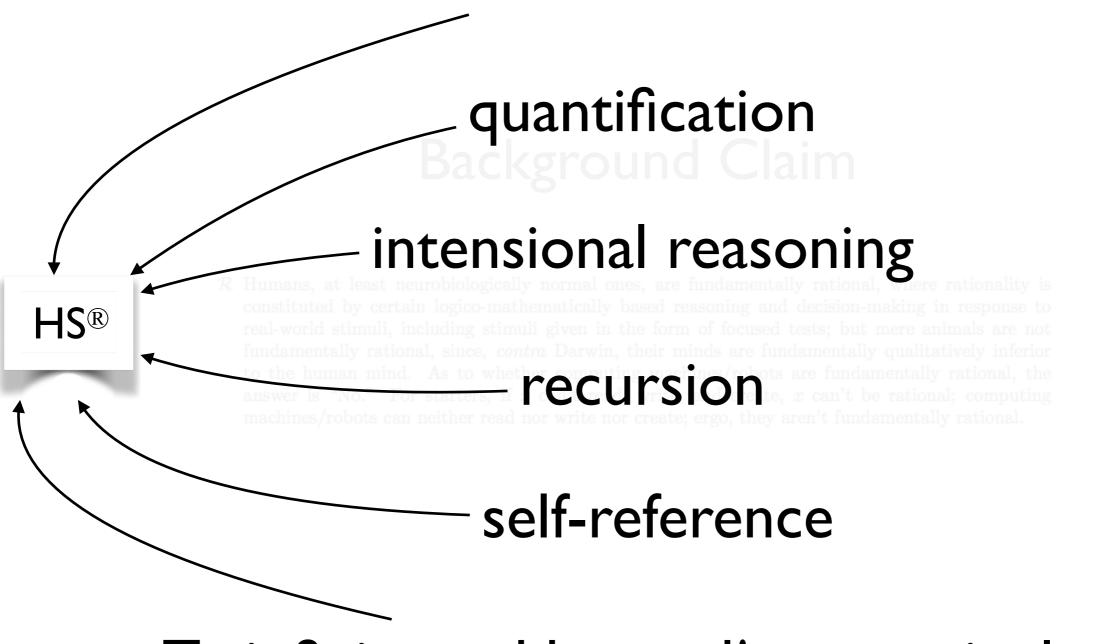


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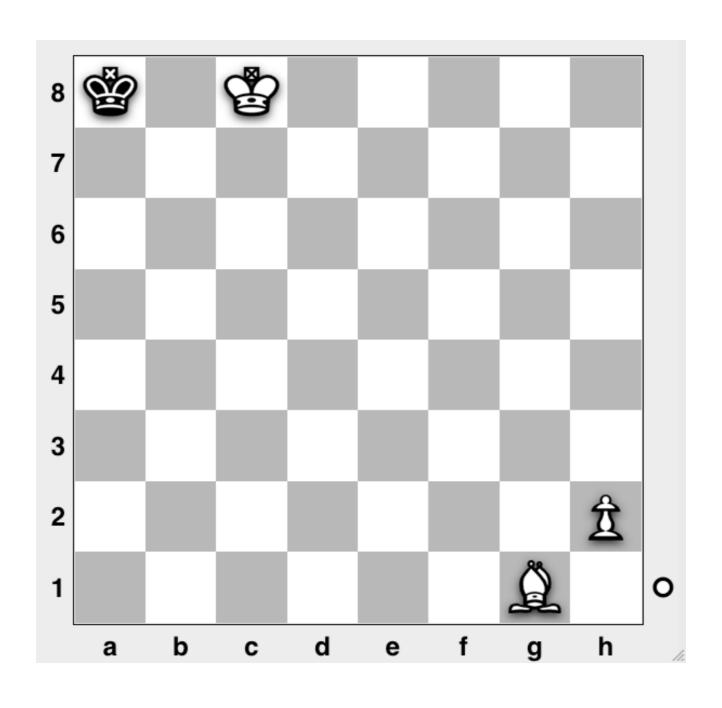


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- One of the chief differentiators between dogs and monkeys versus you (let alone bears and you); and mindless machines (like Deep Blue & Watson) versus you.
- A key to riches.
- The key to divining the meaning of life (and other such big questions).
- The better way to program computers; and fundamentally the *only* way to *reliably* program computers.
- One of two fundamental approaches to studying minds, and replicating/simulating minds in machines...
- The thing many creatures of fiction have mastered have you (as a New Yorker)?...
- •

- The key to becoming rational.
- "The science and engineering of reasoning." so the not-unreasonable slogan goes.
- The only invincible subject there is.
- The basis for the formal sciences (from mathematics to game theory to decision theory to probability calculi to axiomatic physics ....) and hence the basis for disciplines based on the formal sciences (e.g., engineering, computer science).
  - Computer Science!
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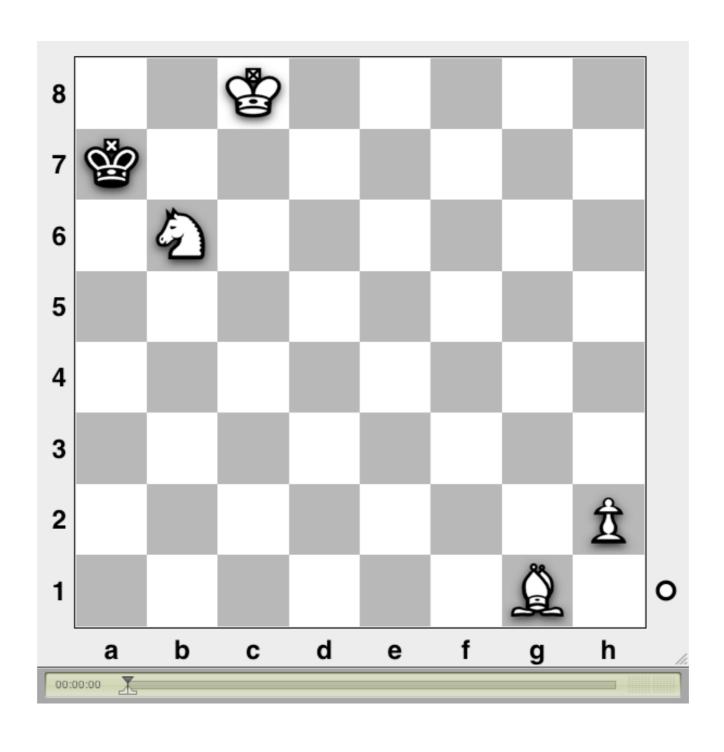
- The key to becoming rational. Or are you already rational? ...
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### It's White's turn. What move did Black just make?



## Aha! (Beyond Deep Blue?)

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**NOTE**: Every card in this game has a capital Roman letter on one side, and a number from 1 to 9, inclusive.

# Simple Selection Task

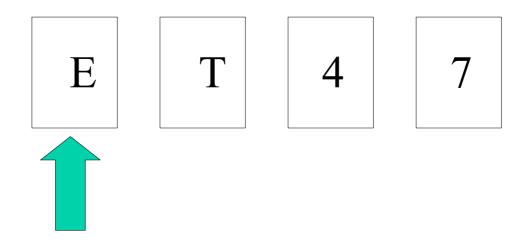
E T 4 7

Suppose I claim that the following rule is true.

If a card has a vowel on one side, it has an even number on the other side.

**NOTE**: Every card in this game has a capital Roman letter on one side, and a number from 1 to 9, inclusive.

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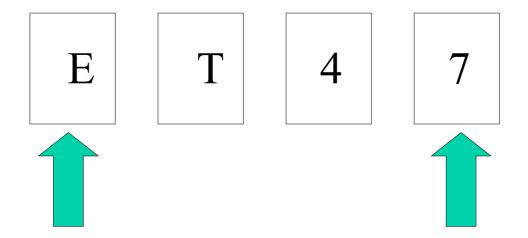


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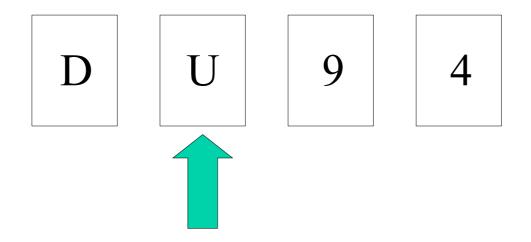
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D U 9 4

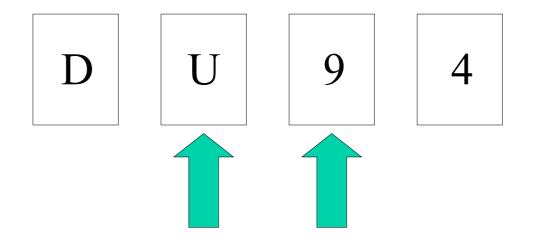
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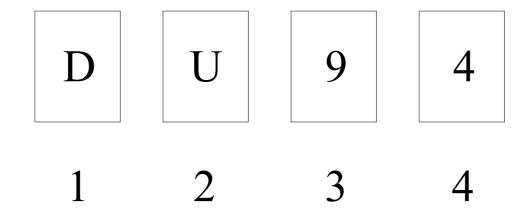
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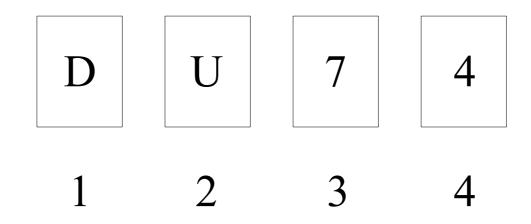


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POLL: Chat your answer now. To suggest flipping a card or cards, just type in the corresponding #/#s, e.g. 'I' would be a suggestion to flip card #I.

# Another Simple Selection Task

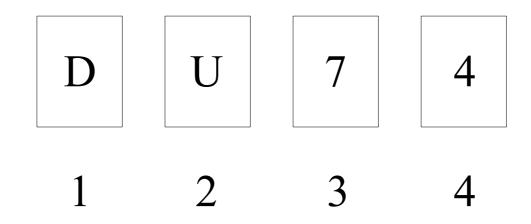


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If a card has a letter on one side, it has a prime number on the other side.

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# Another Simple Selection Task



Suppose I claim that the following rule is true.

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Suppose that the following premise is true:

If there is a king in the hand, then there is an ace in the hand, or else if there isn't a king in the hand, then there is an ace.

What can you infer from this premise?

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# The Original King-Ace

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In fact, what you can infer is that there isn't an ace in the hand!

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#### **TODAY**

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#### STARTING $\geq 2/8/21$

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## King-Ace Solved

(informal proof)

**Proposition**: There is *not* an ace in the hand.

**Proof**: We know that at least one of the if-thens (i.e., at least one of the **conditionals**) is false. So we have two cases to consider, viz., that K => A is false, and that  $\neg K => A$  is false. Take first the first case; accordingly, suppose that K => A is false. Then it follows that K is true (since when a conditional is false, its antecedent holds but its consequent doesn't), and A is false. Now consider the second case, which consists in  $\neg K => A$  being false. Here, in a direct parallel, we know  $\neg K$  and, once again,  $\neg A$ . In both of our two cases, which are exhaustive, there is no ace in the hand. The proposition is established. **QED** 

# Bringsjord I

(I) The following three assertions are either all true or all false:

If Billy helped, Doreen helped.

If Doreen helped, Frank did as well.

If Frank helped, so did Emma.

(2) The following assertion is definitely true: Billy helped.

Can it be inferred from (I) and (2) that Emma helped?

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Can it be inferred from (1) and (2) that Emma helped?

YUP! — & now prove it!

## Bringsjord I: Proof

**Proof**: We have two cases to work from: when the conditionals in (I) are all true, and when they are all false. (In both cases, (2) remains true, and available.) So assume Case I first. In this case, we can simply chain through the conditionals by repeated application of modus ponens to arrive at the conclusion that Emma helped. Now assume Case 2 holds. This immediately implies that the first two conditionals are false; i.e., we have  $\sim$  (B =>D) and  $\sim$  (D => F). Recalling that a conditional fails to hold exactly when its antecedent is true while its consequent is false, we have, in turn: B & ~D, and D & ~F. But then we have a contradiction, viz. ~D & D. Since everything follows ("explosively"!) from a contradiction, we are done. **QED** 

# Logic kan redde menneskehten!