Quantifiers; FOL I; "Proving" God's Existence

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Intro to Logic
3/8/2021

RAIR
Rensselaer AI and Reasoning Lab

Re Test 1…

We will have an *optional* help session on Test 1 etc in the 30 min add-on part of today's class!

HyperGrader® Required Homework Problems:

Self-paced, yes! — but interconnected!

$\boxed{\text{BogusBiconditional}}$

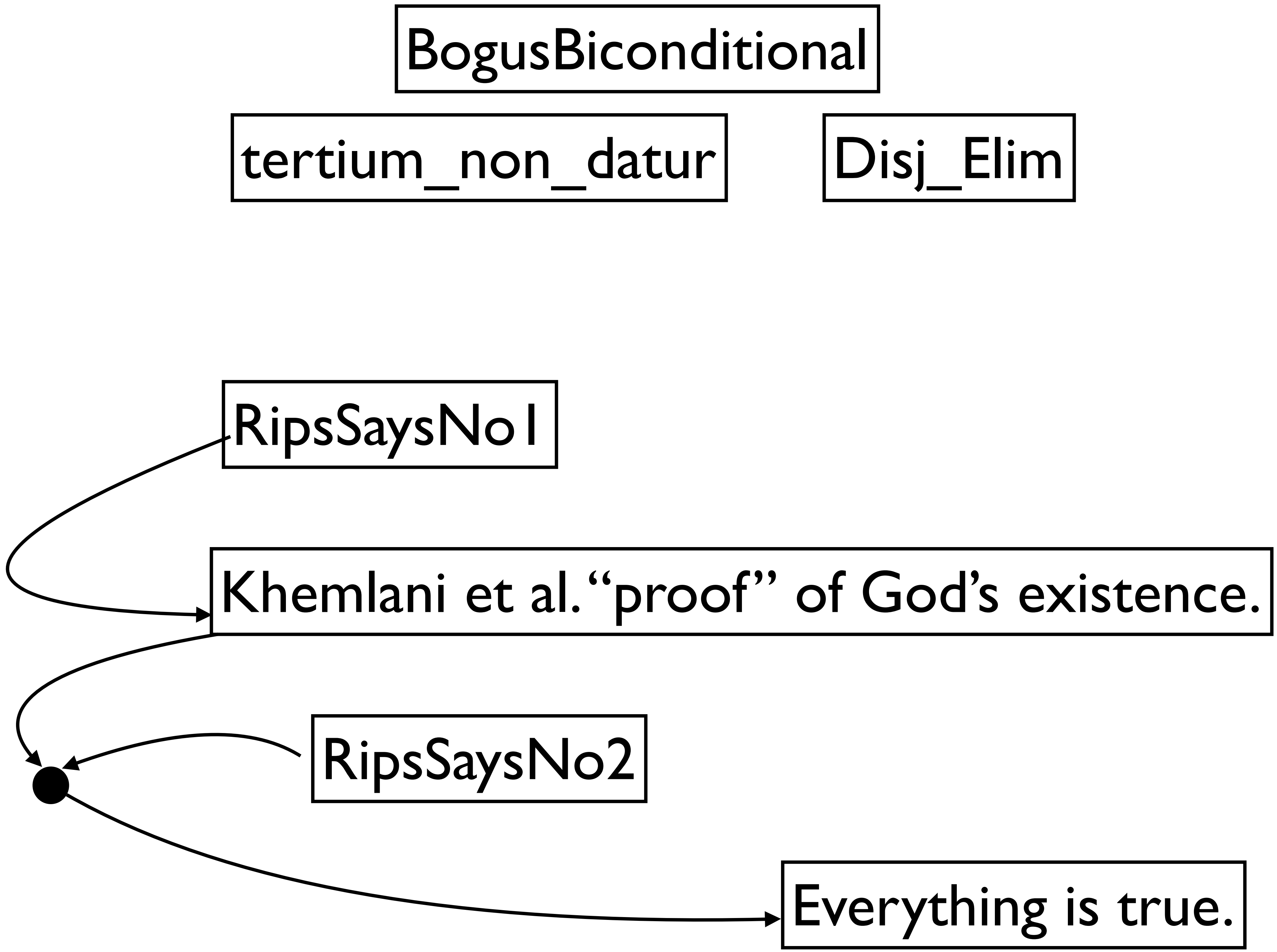
tertium_non_datur

Disj_Elim

BogusBiconditional

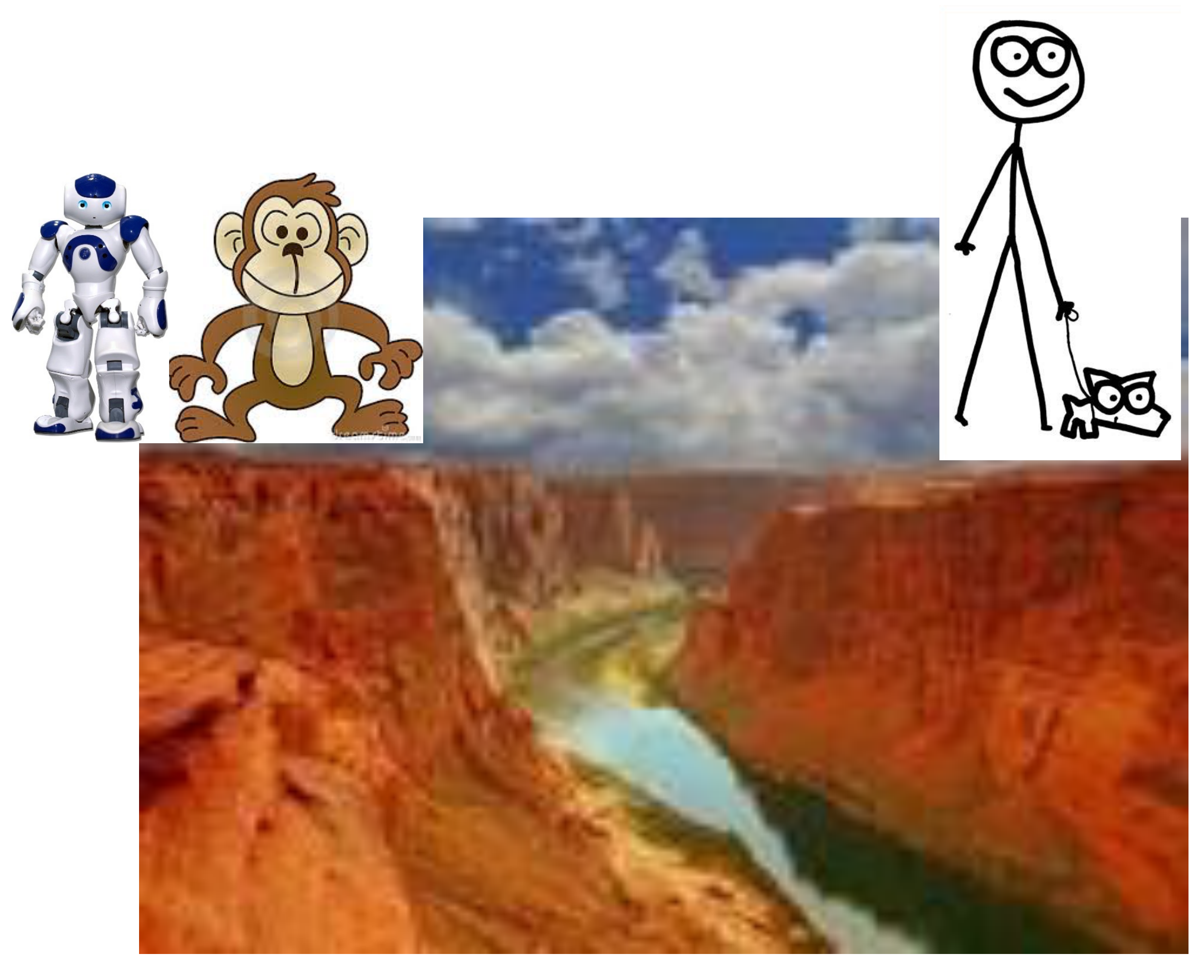
RipsSaysNo1

RipsSaysNo2


BogusBiconditional
tertium_non_datur
Disj_Elim
RipsSaysNo1
Khemlani et al. "proof" of God's existence.
RipsSaysNo2
Everything is true.

Quantifiers (etc) …

The Canyon of Discontinuity (or Darwin's Dread)



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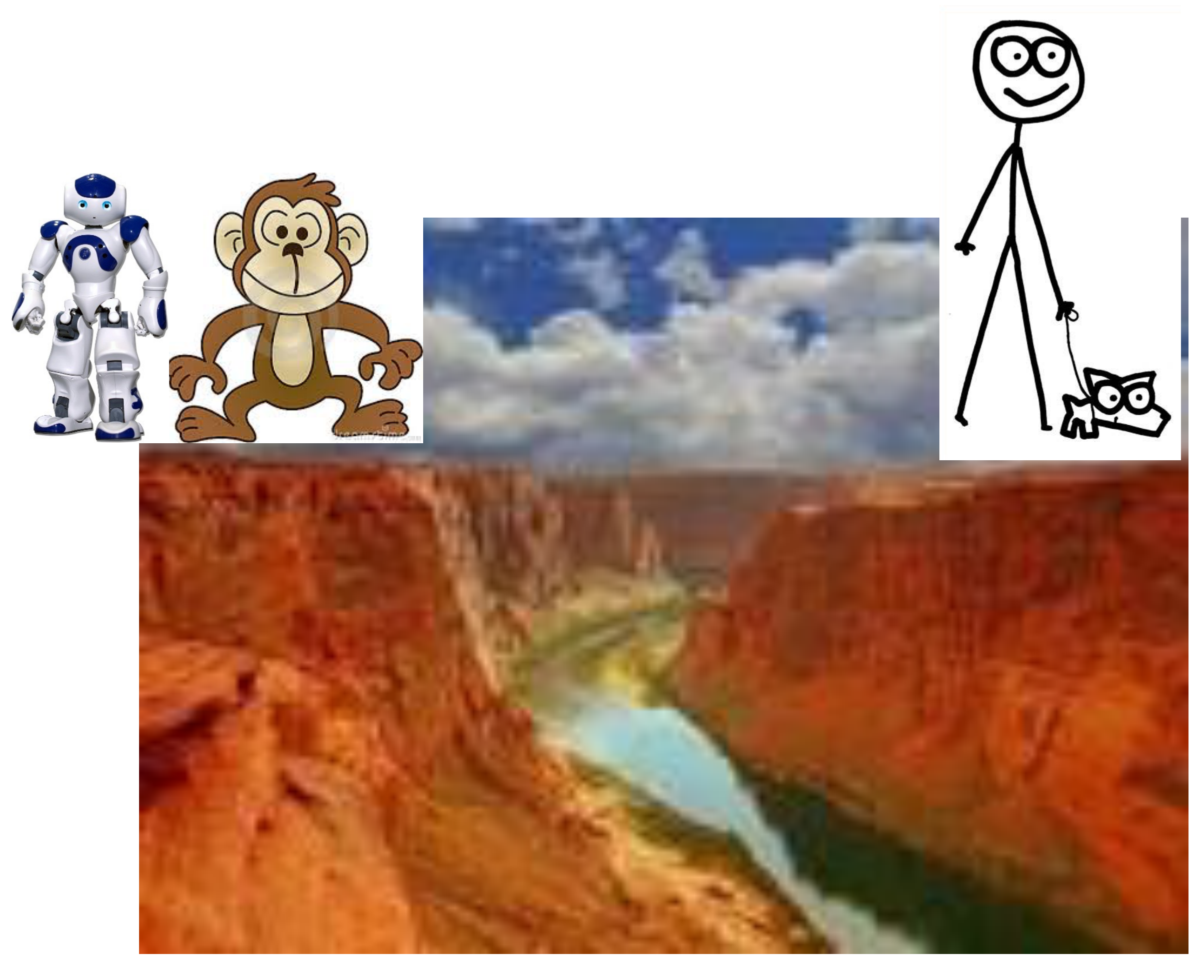

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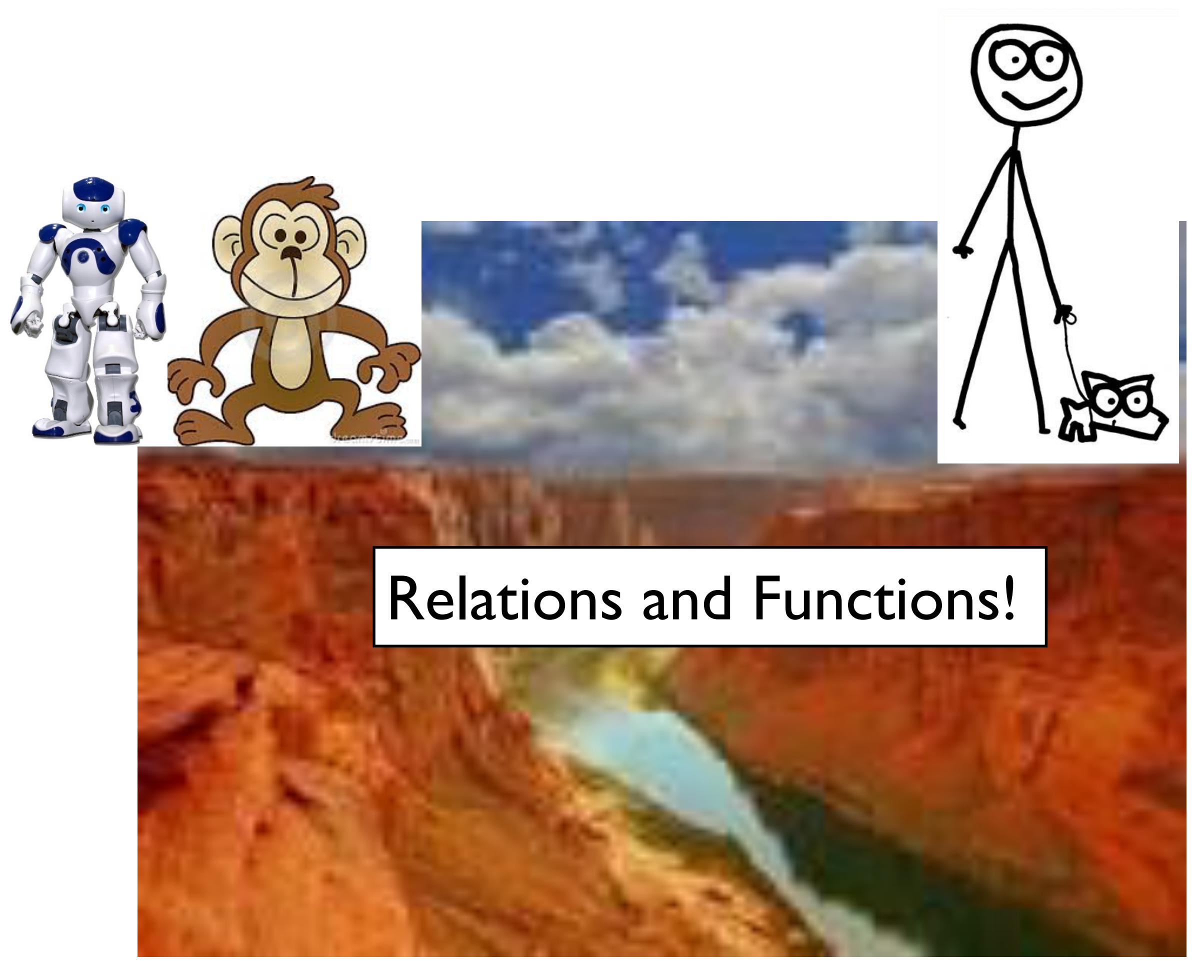

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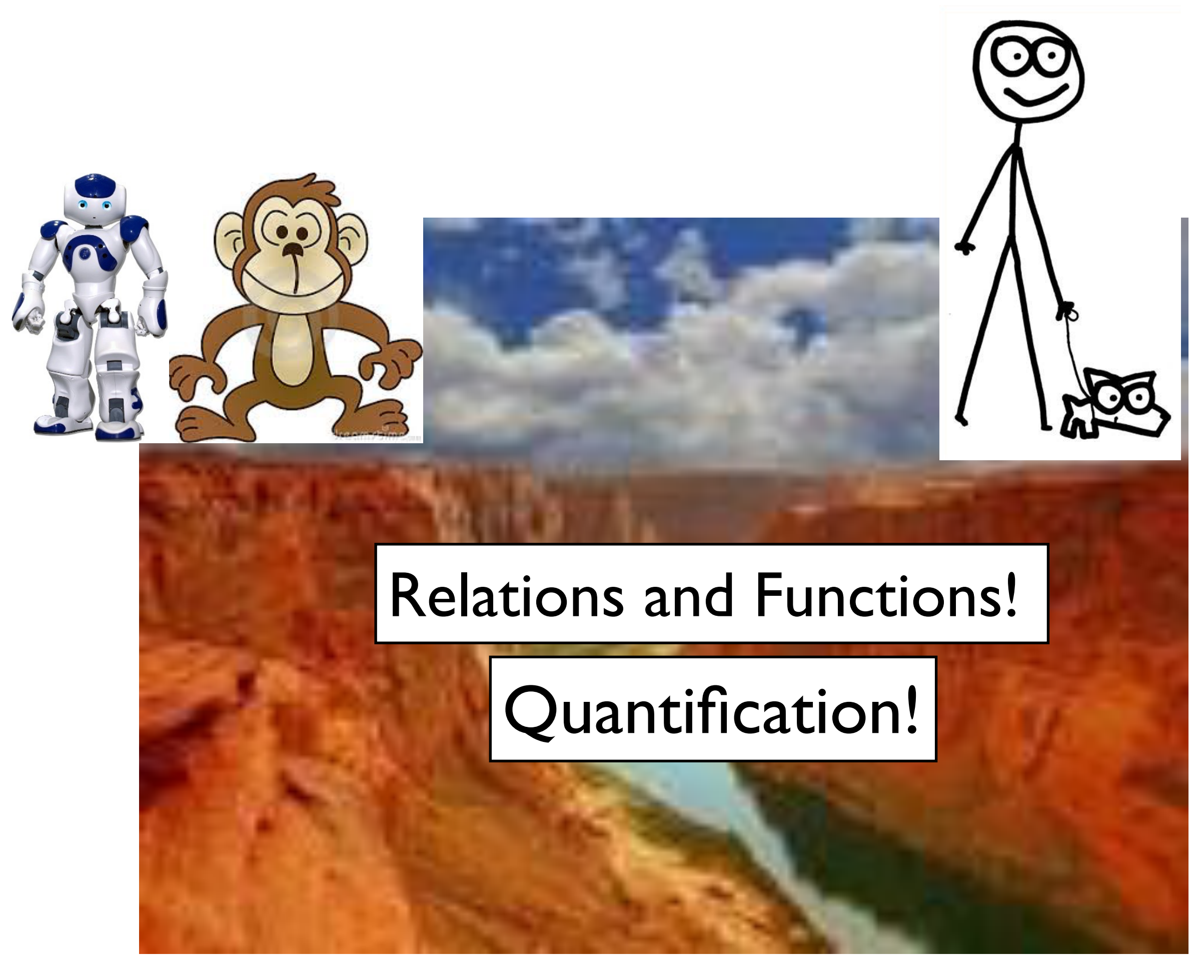

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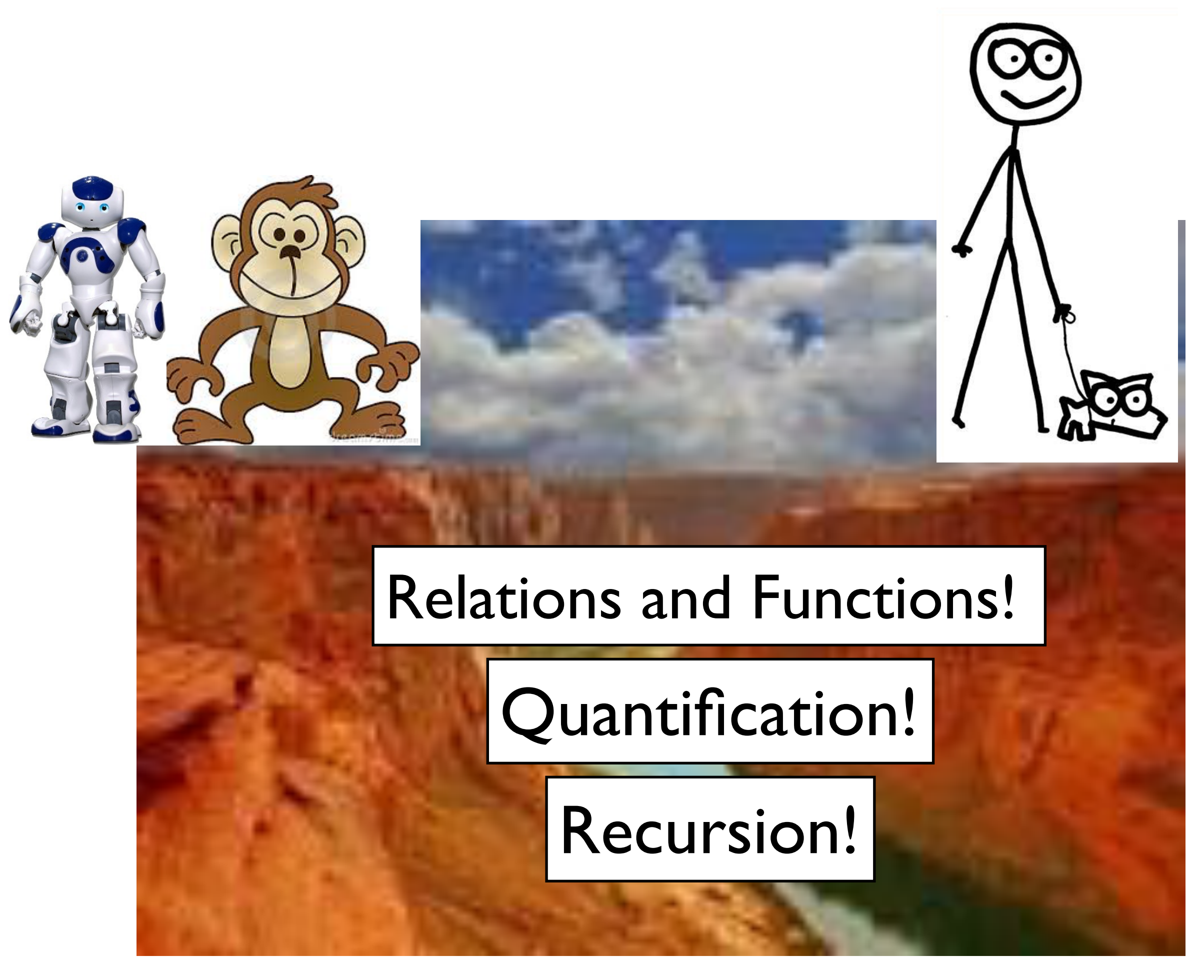

The Canyon of Discontinuity (or Darwin's Dread)

Quantification!

Karkooking Problem …

Everyone karkooks anyone who karkooks someone.

Alvin karkooks Bill.

Can you infer that everyone karkooks Bill?

ANSWER:

JUSTIFICATION:

Karkooking Problem …

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ANSWER:

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Relations and Functions!

Alvin karkooks Bill.

Quantification!

Can you infer that everyone karkooks Bill?

Recursion!

ANSWER:

JUSTIFICATION:

Two Proposed Arguments; Valid?

- All mammals walk.
- Whales are mammals.
- Therefore:
- Whales walk.

- All of the Frenchmen in the room are wine-drinkers.
- Some of the wine-drinkers in the room are gourmets.
- Therefore:
- Some of the Frenchmen in the room are gourmets.

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We can of course easily symbolize and settle the matter in HyperSlate® (PC oracle permitted now)! (Show this.). Doing so is *impossible* in the prop calc, and likewise impossible in zeroth-order logic!

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- All mammals walk. $\forall x(M(x) \rightarrow W(x))$
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- Some of the wine-drinkers in the room are gourmets. $\exists x(W(x) \wedge G(x))$
- Therefore:
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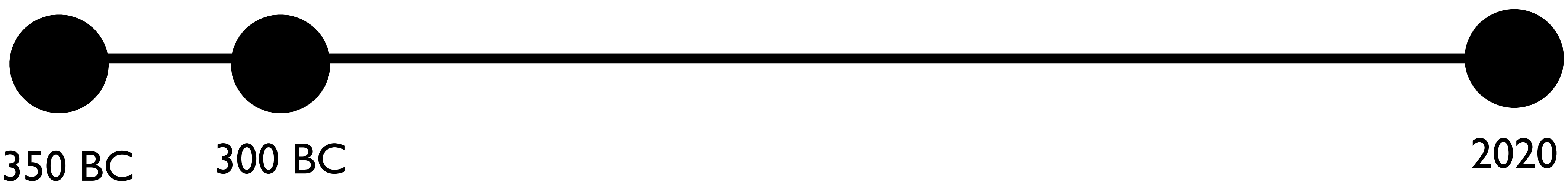
Historically speaking
(recall) …

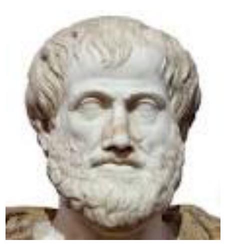
350 BC

2020

Euclid

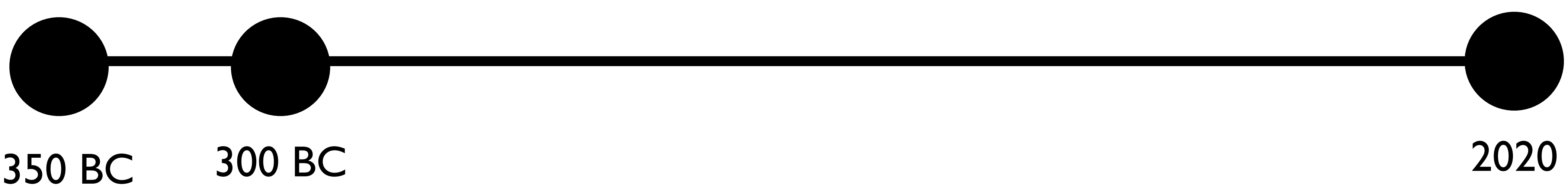

350 BC
300 BC
2020
Euclid





“I don’t believe in magic! Why exactly
is that so convincing? What the heck
is he doing?!? I know! …”

Euclid



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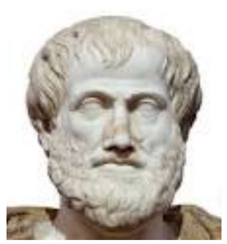

Organon

"I don't believe in magic! Why exactly is that so convincing? What the heck is he doing?!? I know! …"

"He's using syllogisms!"

E.g.,

All As are Bs.
All Bs are Cs.

All As are Cs.

350 BC

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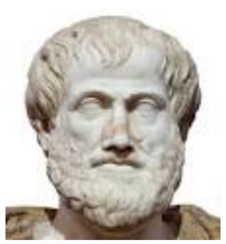

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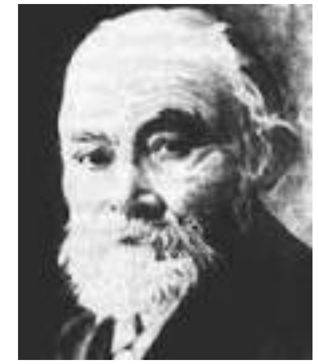
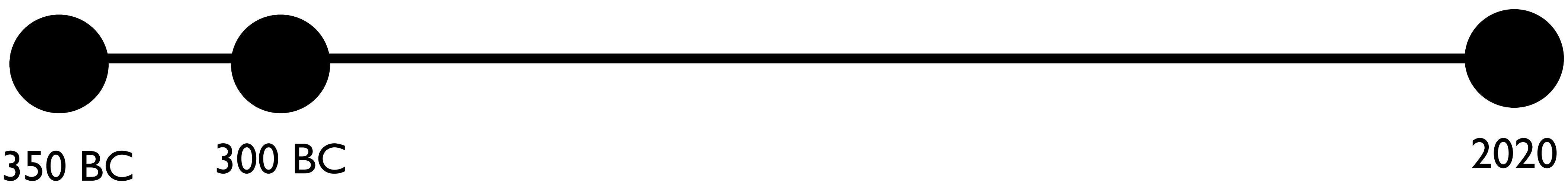


"No. Euclid's proofs are compelling because they are informal versions of proofs in something I've invented: first-order logic (= FOL = $\mathscr{L}_1$)."



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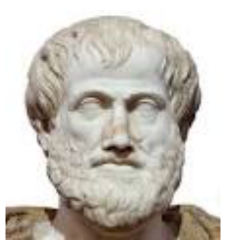

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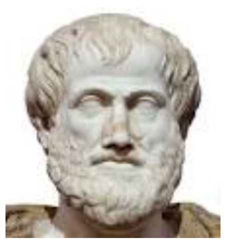

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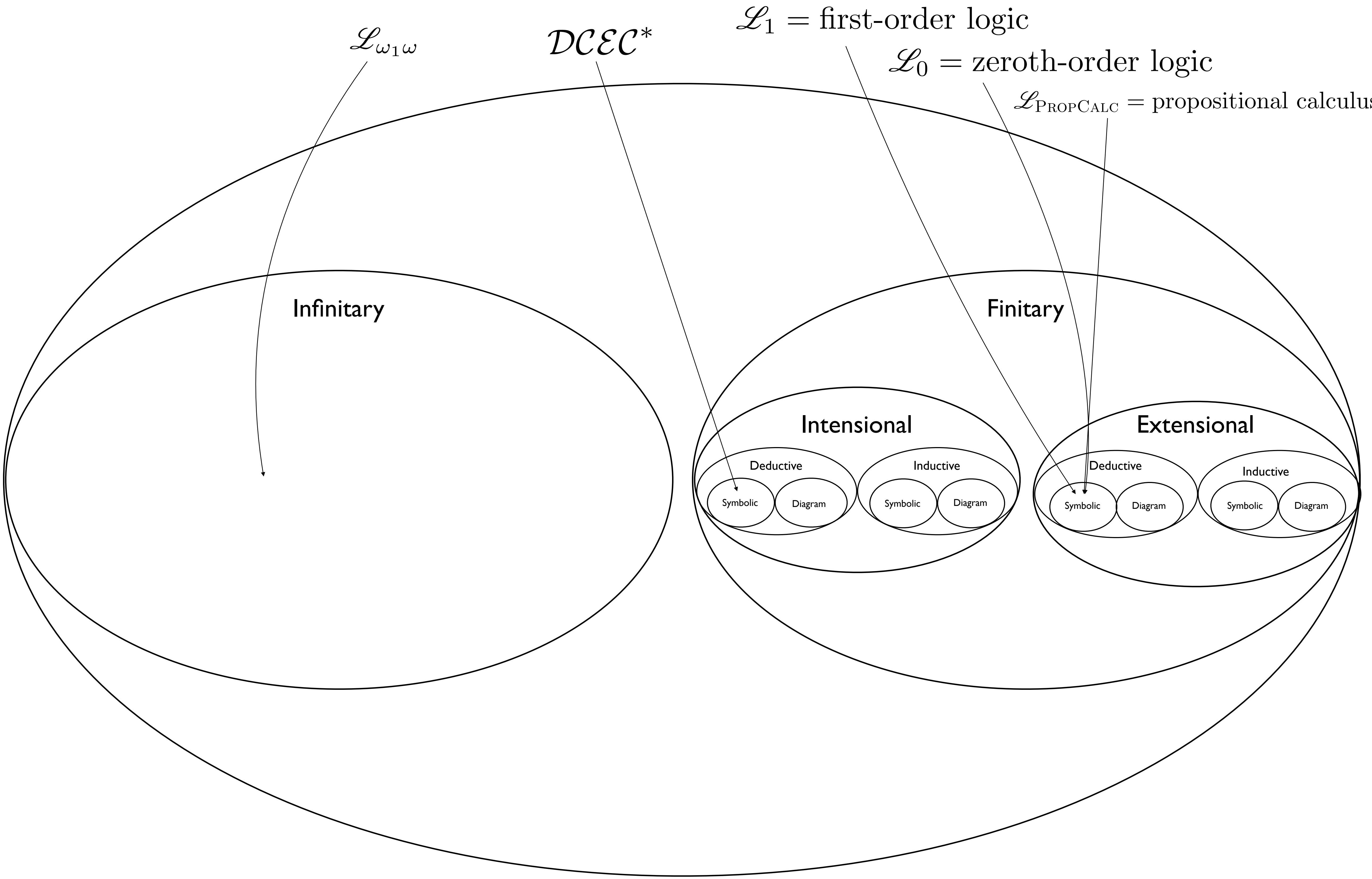
The Universe of Logics



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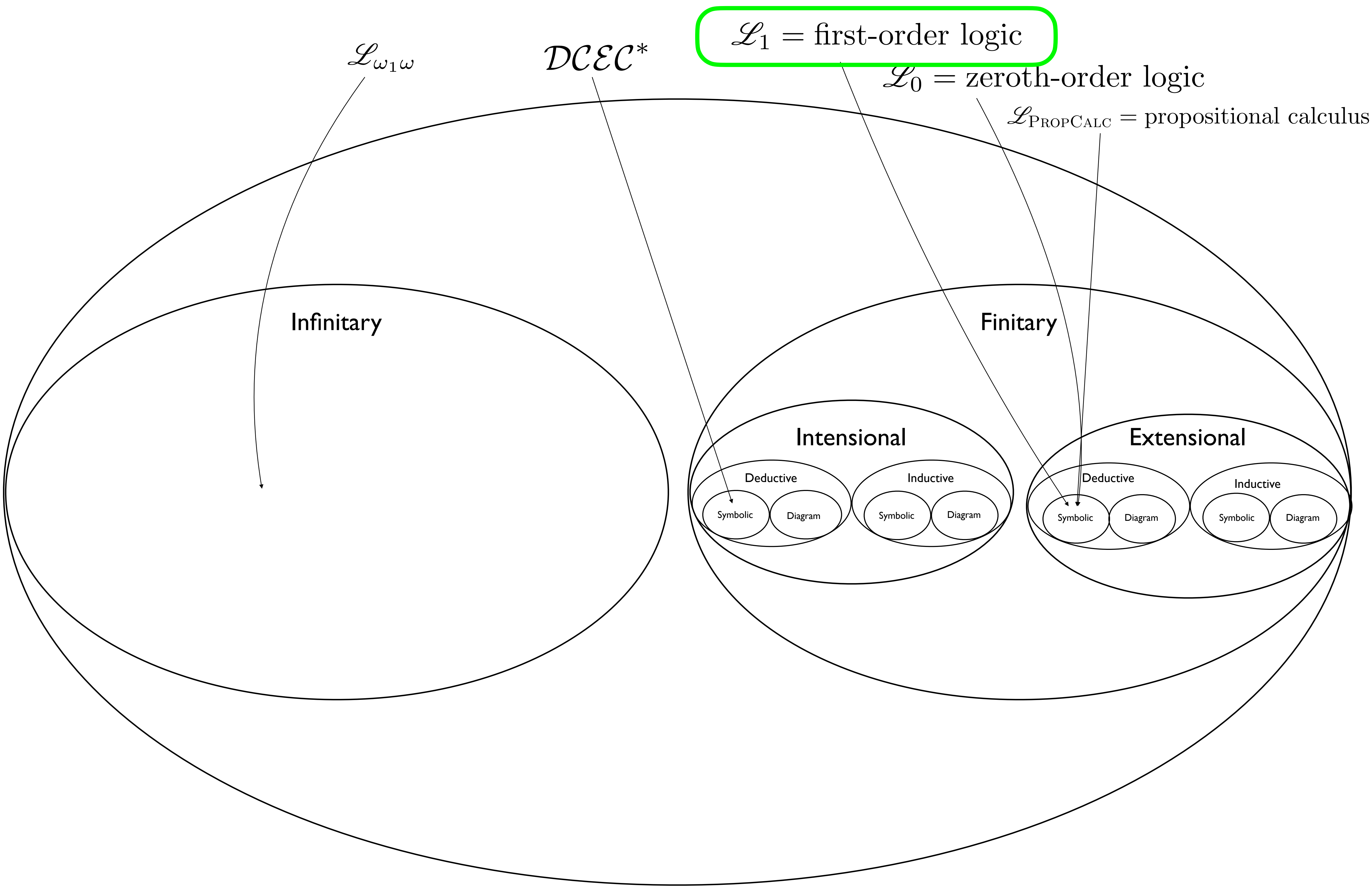

First Two New (Easy!!) Inference Rules in FOL

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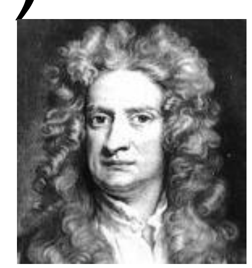
  - My apologies to:    

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Scott's Version of Gödel's Proof, Verified by AI

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$\boxed{\mathscr{L}_3 + \text{ modal logic } \mathbf{S5}}$

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First Two New (Easy!!) Inference Rules in FOL

- `universal elimination`
  - If everything is an *P*, then the particular

A1 Either a property or its negation is positive, but not both: $\forall\phi[P(\neg\phi) \leftrightarrow \neg P(\phi)]$

A2 A property necessarily implied by a positive property is positive: $\forall\phi\forall\psi[(P(\phi) \wedge \Box\forall x[\phi(x) \rightarrow \psi(x)]) \rightarrow P(\psi)]$

T1 Positive properties are possibly exemplified: $\forall\varphi[P(\varphi) \rightarrow \Diamond\exists x\varphi(x)]$

D1 A *God-like* being possesses all positive properties: $G(x) \leftrightarrow \forall\phi[P(\phi) \rightarrow \phi(x)]$

A3 The property of being God-like is positive: $P(G)$

C Possibly, God exists: $\Diamond\exists x G(x)$

A4 Positive properties are necessarily positive: $\forall\phi[P(\phi) \rightarrow \Box\, P(\phi)]$

D2 An *essence* of an individual is a property possessed by it and necessarily implying any of its properties: $\phi\ ess.\ x \leftrightarrow \phi(x) \wedge \forall\psi(\psi(x) \rightarrow \Box\forall y(\phi(y) \rightarrow \psi(y)))$

T2 Being God-like is an essence of any God-like being: $\forall x[G(x) \rightarrow G\ ess.\ x]$

D3 *Necessary existence* of an individual is the necessary exemplification of all its essences: $NE(x) \leftrightarrow \forall\phi[\phi\ ess.\ x \rightarrow \Box\exists y\phi(y)]$

A5 Necessary existence is a positive property: $P(NE)$

T3 Necessarily, God exists: $\Box\exists x G(x)$

Scott's Version of Gödel's Proof, Verified by AI

$\boxed{\mathscr{L}_3 + \text{ modal logic } \mathbf{S5}}$

Benighted “Understanding” of Logic

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Facts and Possibilities: A Model-Based Theory of Sentential Reasoning

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[b]*School of Psychology and Institute of Neuroscience, Trinity College Dublin, University of Dublin*
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Abstract

This article presents a fundamental advance in the theory of mental models as an explanation of reasoning about facts, possibilities, and probabilities. It postulates that the meanings of compound assertions, such as conditionals (*if*) and disjunctions (*or*), unlike those in logic, refer to conjunctions of epistemic possibilities that hold in default of information to the contrary. Various factors such as general knowledge can modulate these interpretations. New information can always override sentential inferences; that is, reasoning in daily life is defeasible (or nonmonotonic). The theory is a dual process one: It distinguishes between intuitive inferences (based on system 1) and deliberative inferences (based on system 2). The article describes a computer implementation of the theory, including its two systems of reasoning, and it shows how the program simulates crucial predictions that evidence corroborates. It concludes with a discussion of how the theory contrasts with those based on logic or on probabilities.

Keywords: Deduction; Logic; Mental models; Nonmonotonicity; Reasoning; Possibility

1. Introduction

People reason about facts, possibilities, and probabilities. Psychologists have carried out many studies of factual inferences, such as:

1. If the card is an ace then it is a heart.
   The card is an ace.
   Therefore, the card is a heart.

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seem true a priori and those that are contingent is "an unempirical dogma of empiricism." Not anymore. The empirical studies we have described show that individuals innocent of philosophical niceties judged that assertions can be true (or false) a priori as a result of their meaning.

In logic, if a material conditional is false then its *if*-clause is true. So a very short proof for the existence of God is sound in logic:

38. It is not the case that if God exists then atheism is correct.
    Therefore, God exists.

Its premise is true, and it implies both that God exists and that atheism is not correct. It therefore follows from this conjunction that God exists. In the model theory, a conditional's meaning is not a material implication, not a conditional probability, not a set of possible worlds, and not an inferential relation. It is instead a conjunction of possibilities, each of which is assumed in default of information to the contrary. And so the falsity of a conditional does not imply that its *if*-clause is true, which renders the "proof" in (38) invalid. Individuals judge that the following assertion is false:

39. If Sonia has pneumonia then she is healthy.

But its falsity does not imply that Sonia has pneumonia, and indeed individuals judge that it is possible that Sonia does not have pneumonia (Quelhas et al., 2016). Only one case is impossible:

`Sonia has pneumonia  Sonia is healthy`

That is why (39) is false. The modulation algorithm we described mirrors these evaluations.

Yet a complex sort of modulation is at present beyond the program. As Byrne (1989) showed, individuals draw their own conclusion from premises, such as:

42. If she meets her friend then she will go to a play.
    She meets her friend.

They infer that she will go to a play. But when the premises have a further conditional of the following sort added to them:

41. If she has enough money then she will go to a play.

reasoners tend not to make the inference (see also Byrne, Espino, & Santamaria, 1999). The additional premise reminds them of a necessary condition for going to a play: One needs money to pay for the tickets. But no premise has established this condition, and so they balk at the inference. The inference is complex, and the modulation algorithm has yet to capture it.

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seem true a priori and those that are contingent is "an unempirical dogma of empiricism." Not anymore. The empirical studies we have described show that individuals innocent of philosophical niceties judged that assertions can be true (or false) a priori as a result of their meaning.

In logic, if a material conditional is false then its *if*-clause is true. So a very short proof for the existence of God is sound in logic:

38. It is not the case that if God exists then atheism is correct.
    Therefore, God exists.

Its premise is true, and it implies both that God exists and that atheism is not correct. It therefore follows from this conjunction that God exists. In the model theory, a conditional's meaning is not a material implication, not a conditional probability, not a set of possible worlds, and not an inferential relation. It is instead a conjunction of possibilities, each of which is assumed in default of information to the contrary. And so the falsity of a conditional does not imply that its *if*-clause is true, which renders the "proof" in (38) invalid. Individuals judge that the following assertion is false:

39. If Sonia has pneumonia then she is healthy.

But its falsity does not imply that Sonia has pneumonia, and indeed individuals judge that it is possible that Sonia does not have pneumonia (Quelhas et al., 2016). Only one case is impossible:

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Sonia has pneumonia  Sonia is healthy
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That is why (39) is false. The modulation algorithm we described mirrors these evaluations.

Yet a complex sort of modulation is at present beyond the program. As Byrne (1989) showed, individuals draw their own conclusion from premises, such as:

Den rasjonelle delen av menneskesinnet er basert på logikk.