FOL II: universal intro

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Intro to Logic
3/11/2021

What *is* Logic?

- The key to becoming rational.
- "The science of reasoning." — so the not-unreasonable slogan goes.
- The only invincible subject there is.
- The basis for the formal sciences (from mathematics to game theory to decision theory to probability calculi to axiomatic physics ….) — and hence the basis for disciplines based on the formal sciences (e.g., engineering, computer science).
- The way of escape from shallow content and context to pure, immaterial, and immortal form and structure (which is why the exotic, imaginary, and seemingly non-sensical is so pedagogically useful).
- The most challenging subject there is.
- One of the chief differentiators between dogs and monkeys versus you (let alone bears and you); and mindless machines (like Deep Blue & Watson) versus you.
- A key to riches.
- The key to divining the meaning of life (and other such big questions).
- The better way to program computers; and fundamentally the *only* way to *reliably* program computers.
- One of two fundamental approaches to studying minds, and replicating/simulating minds in machines…
- The thing many creatures of fiction have mastered — have you (as a New Yorker)?…

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Re Test 1 Solutions …

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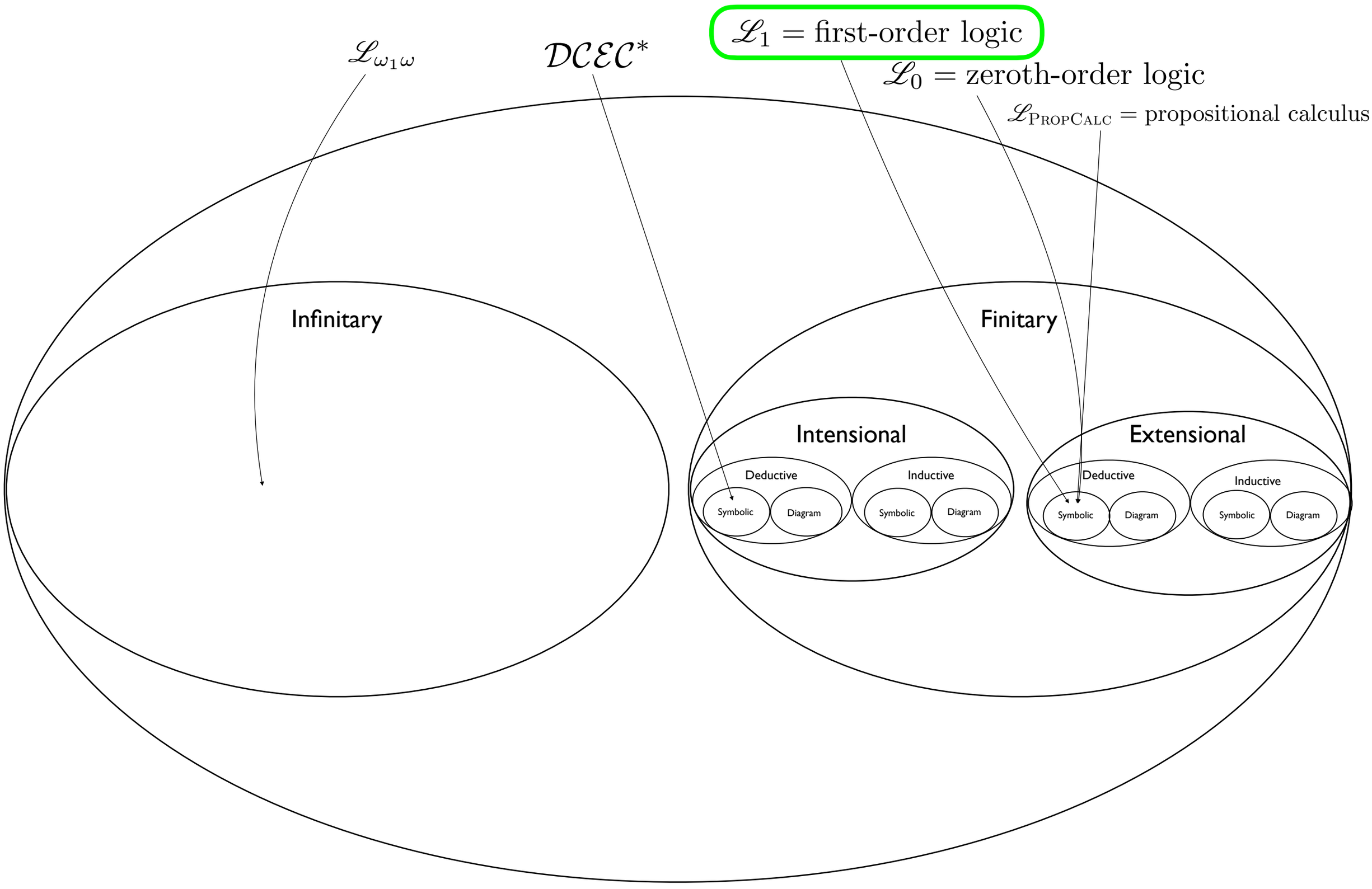
The Universe of Logics



Next New (*Not*-So-Easy!) Inference Rule in FOL

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- `universal introduction`

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  - If something *a* is an *R*, and the constant/name *a* is *genuinely arbitrary*, then we can deduce that everything is an *R*.

The Inference Schema

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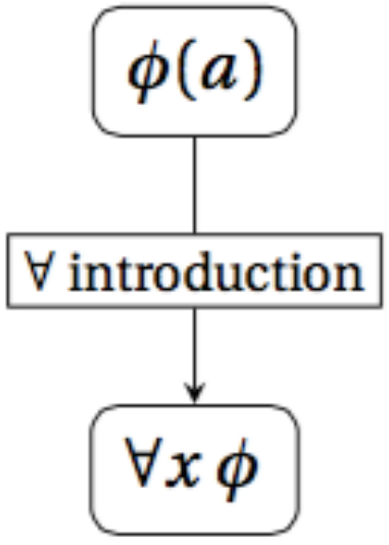

provided that a does not appear free in any in-scope assumption of ϕ, and that no occurrence of a appear in the inferred $\forall x\,\phi$ (3.16)

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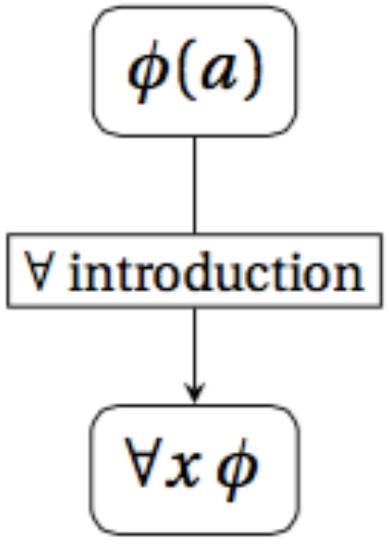

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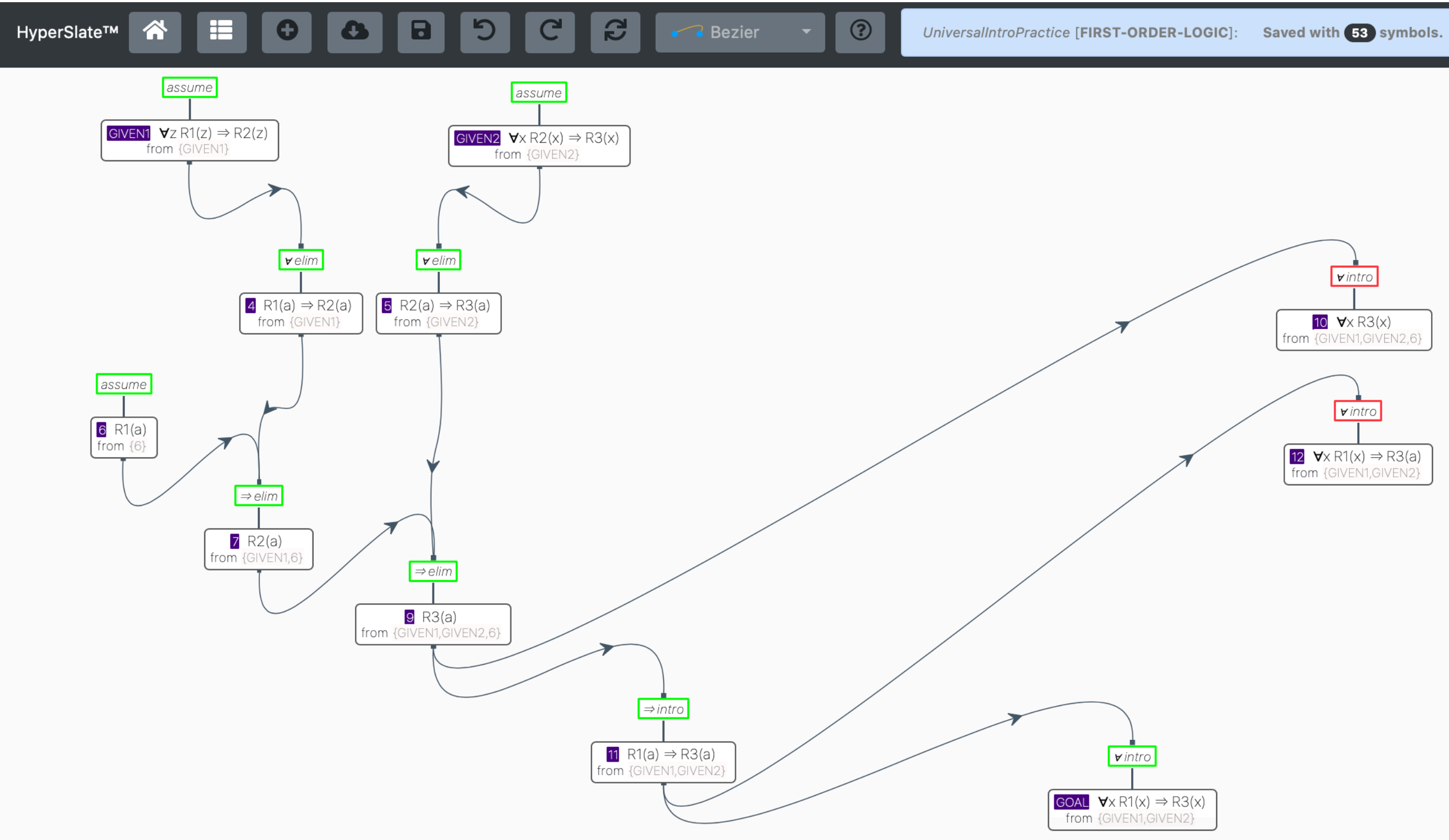
(Why the provisos?)

universal intro Example/Tutorial



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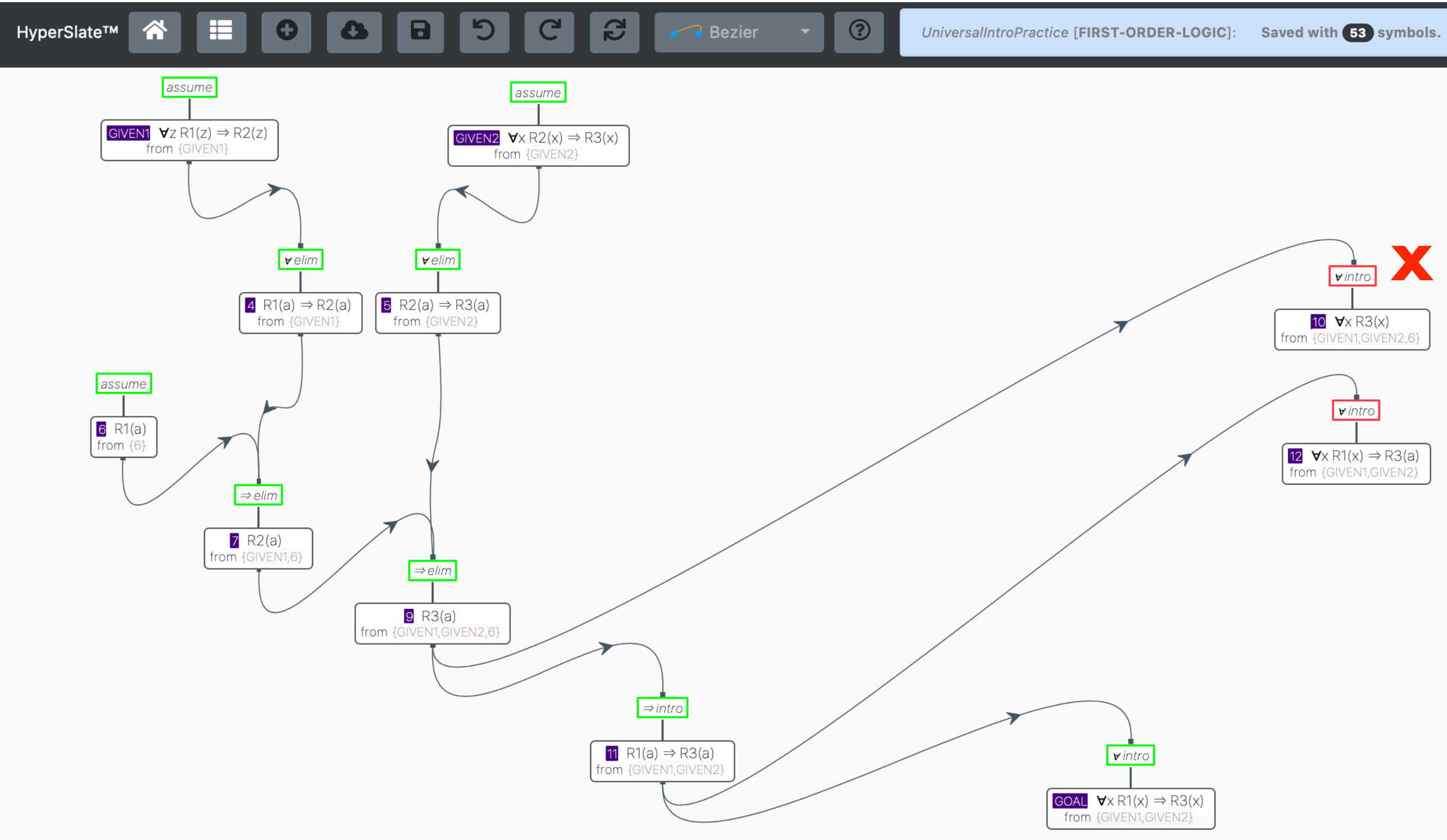

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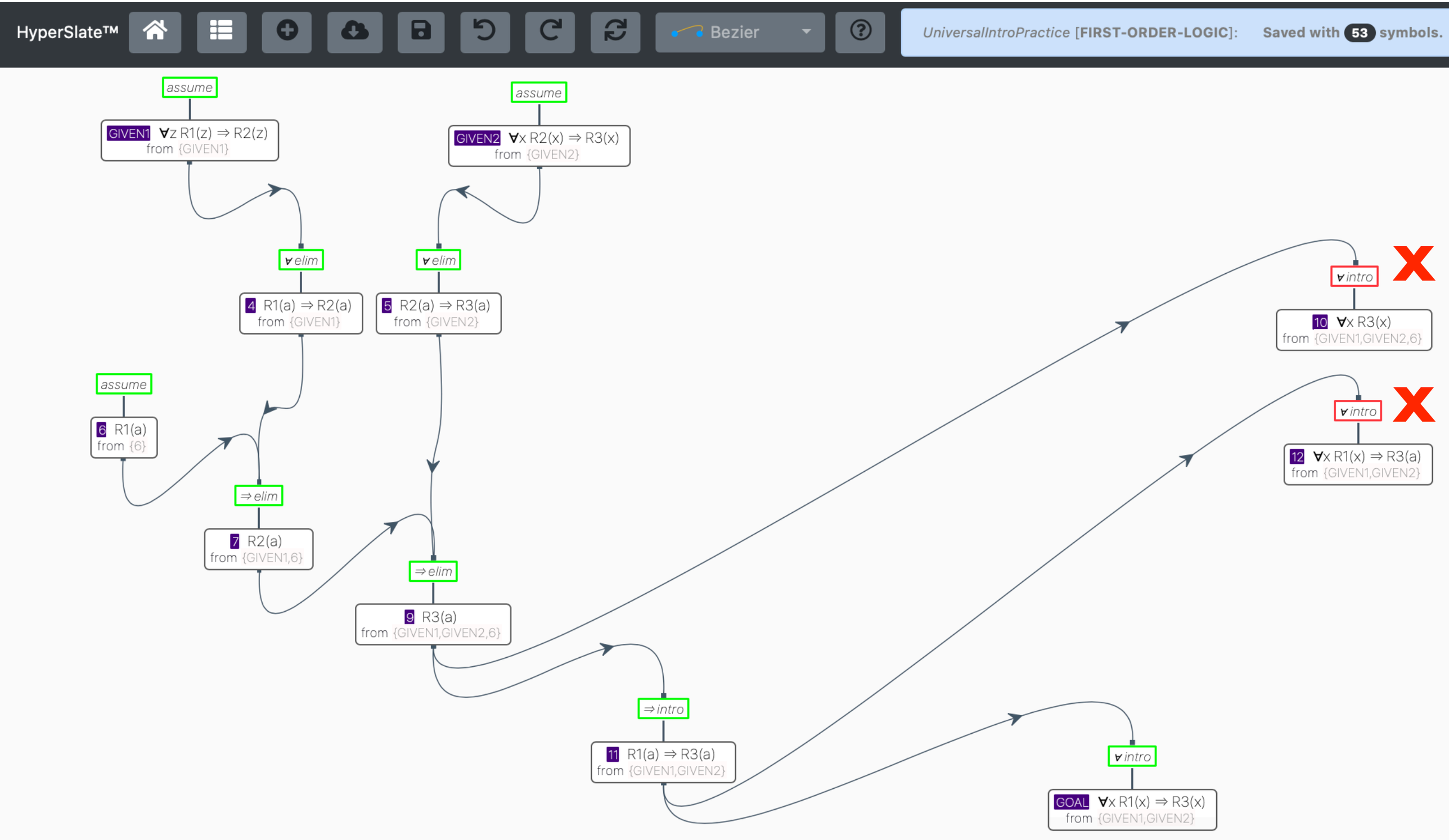

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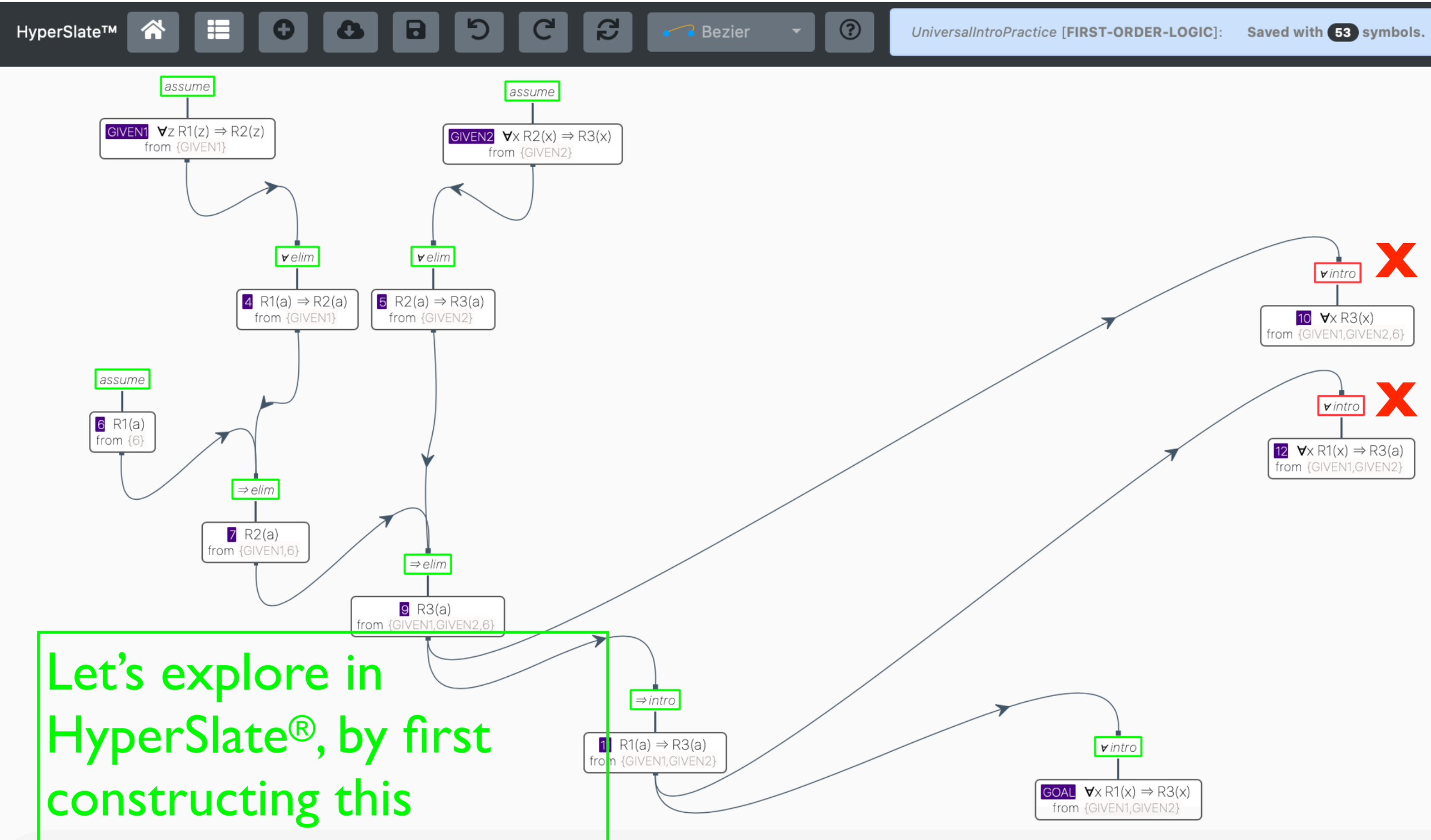

Let's explore in HyperSlate®, by first constructing this example from scratch …

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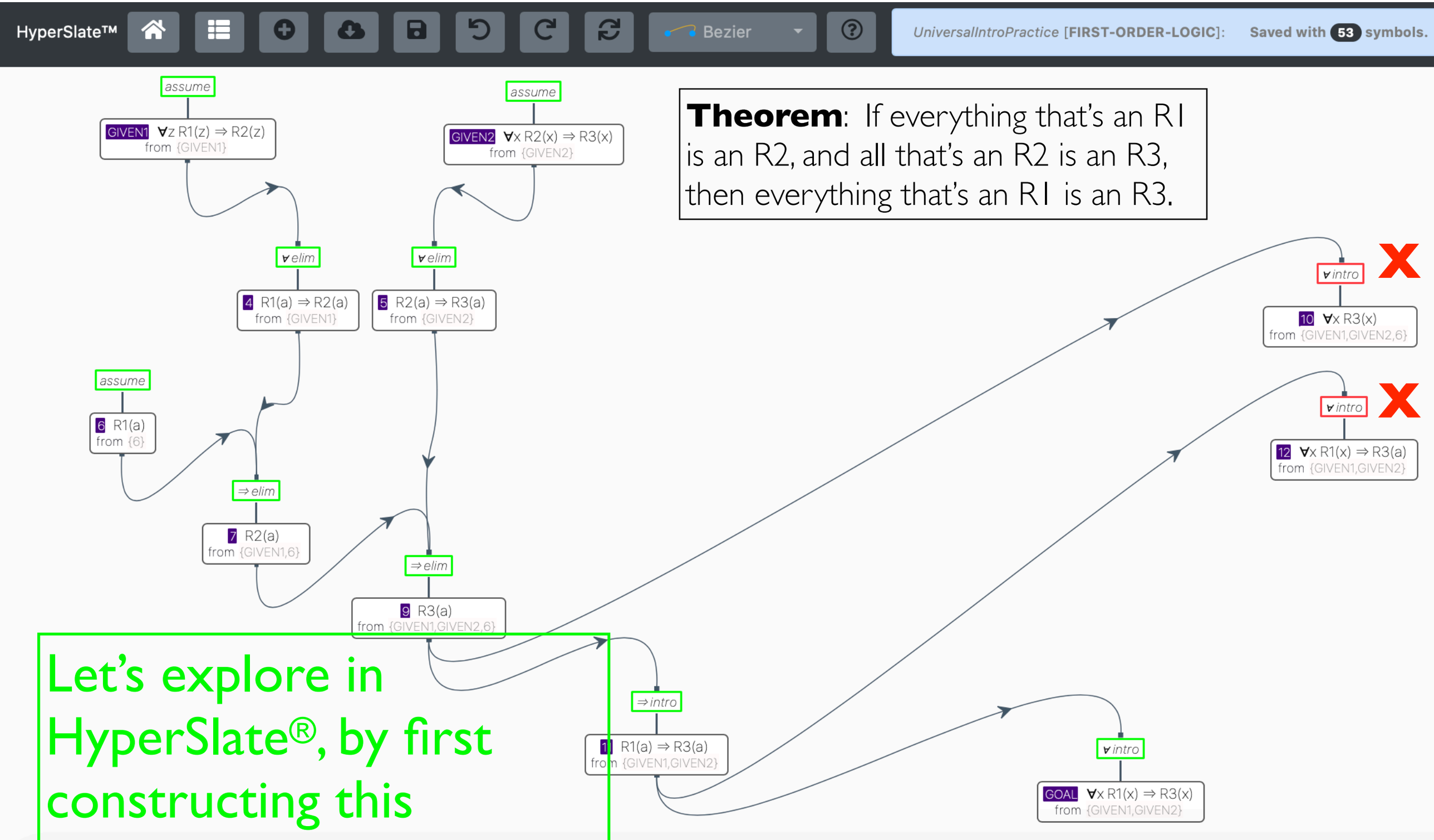

Theorem: If everything that's an R1 is an R2, and all that's an R2 is an R3, then everything that's an R1 is an R3.

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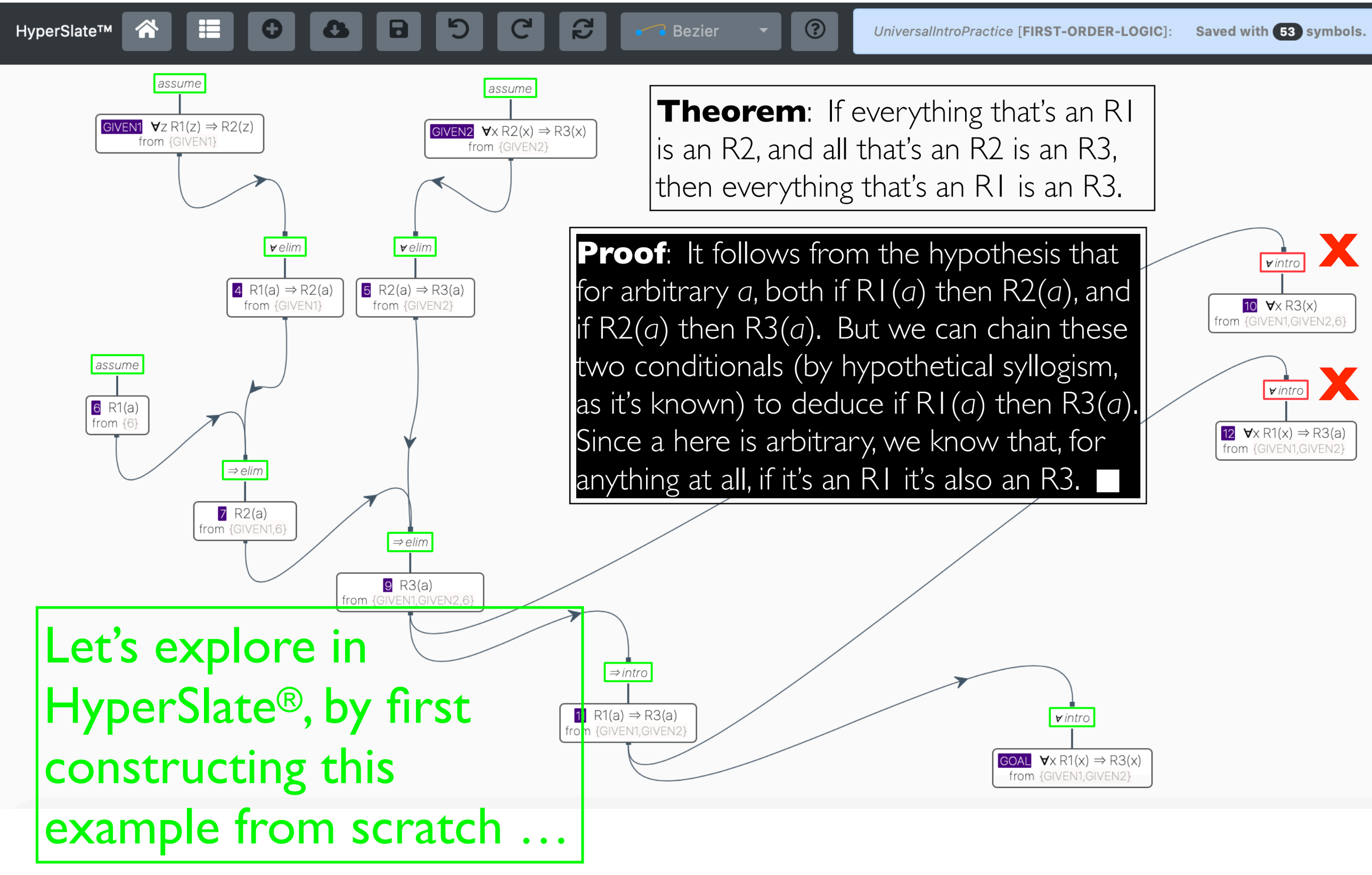

Theorem: If everything that's an R1 is an R2, and all that's an R2 is an R3, then everything that's an R1 is an R3.

Proof: It follows from the hypothesis that for arbitrary *a*, both if R1(*a*) then R2(*a*), and if R2(*a*) then R3(*a*). But we can chain these two conditionals (by hypothetical syllogism, as it's known) to deduce if R1(*a*) then R3(*a*). Since a here is arbitrary, we know that, for anything at all, if it's an R1 it's also an R3. ■

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Suggested Practice Problems in HyperSlate®!

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$$\{\forall \mathtt{x}(\mathtt{R}(\mathtt{x}) \leftrightarrow \mathtt{S}(\mathtt{x})), \forall \mathtt{x}\mathtt{R}(\mathtt{x})\} \vdash \forall \mathtt{x}\mathtt{S}(\mathtt{x})\} \textbf{ ?}$$

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Hvis du forstår det, kan du bevise det.