

Exhortation; Truth Trees; FOL IV: Layered Quantification and Measuring Intelligence Using This Phenomenon

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Troy, New York 12180 USA

Intro to (Formal) Logic
3/18/2021



Exhortation ...

Make sure you're up-to-date today-ish,
fully, on HyperGrader's current (Required
= Homework) Problems, due May 2.

FOL problems are forthcoming.

New Required Problem: FreqTHEN2 ...

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Truth Trees vs. Truth Tables

Truth Trees vs. Truth Tables



Truth Trees vs. Truth Tables



Truth Trees vs. Truth Tables



Violent breakage between tabular calculation and proof construction.

Truth Trees vs. ~~Truth Tables~~



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LAMA[®]'s hypergraphs/HyperLogic[®] achieves seamless unification of proofs and trees.

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First very simple: truth-tree for *modus ponens* ...

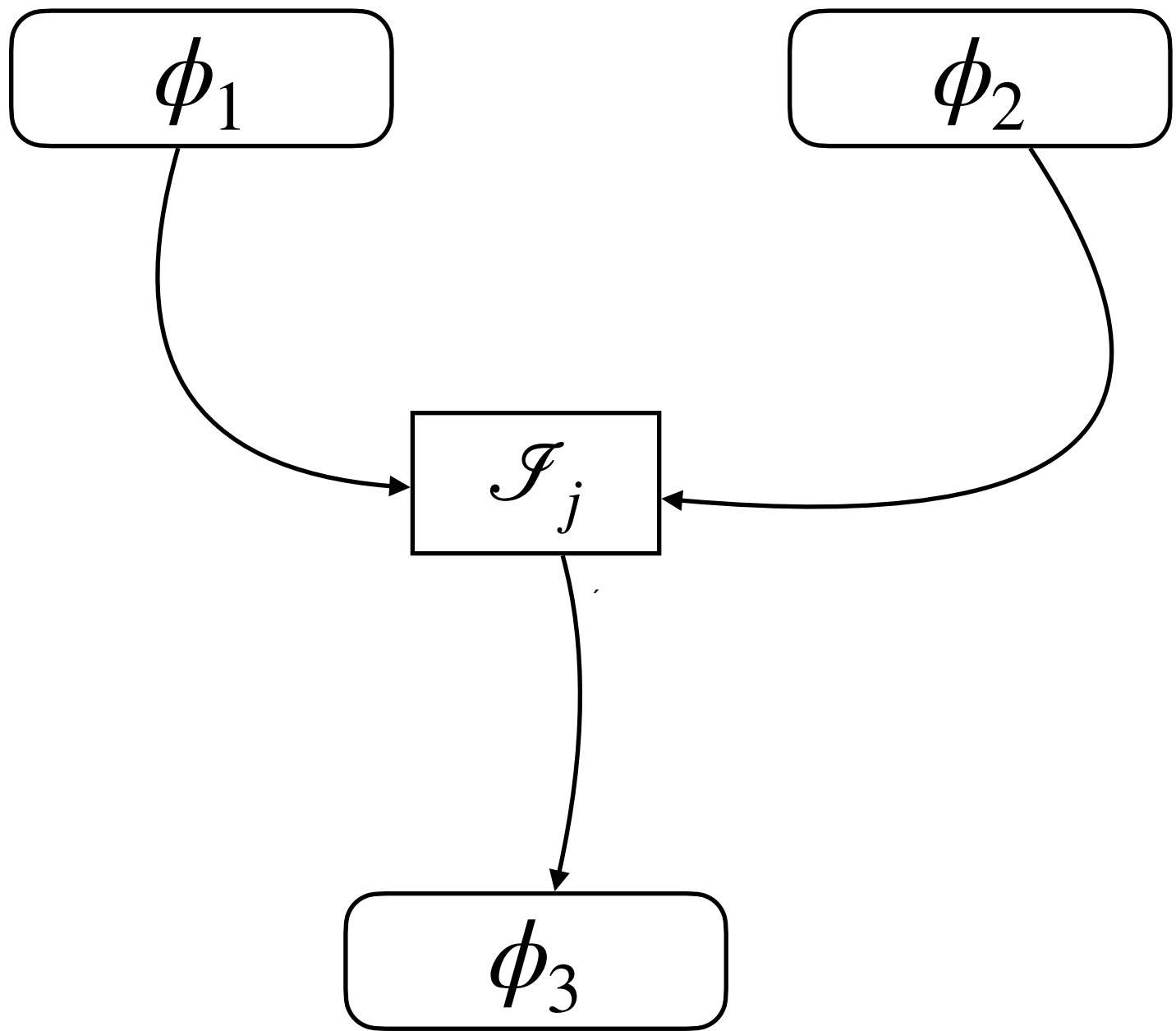
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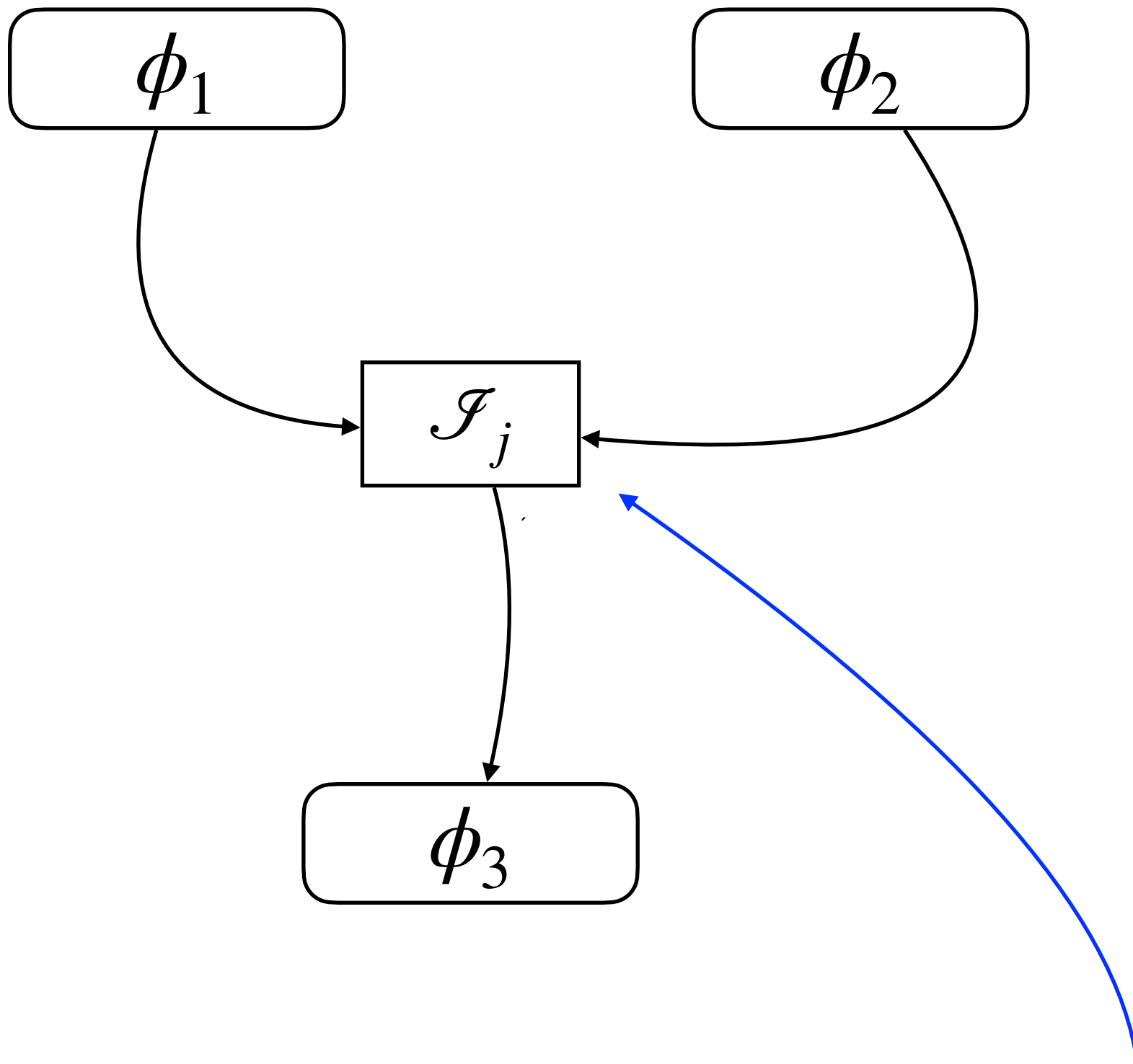


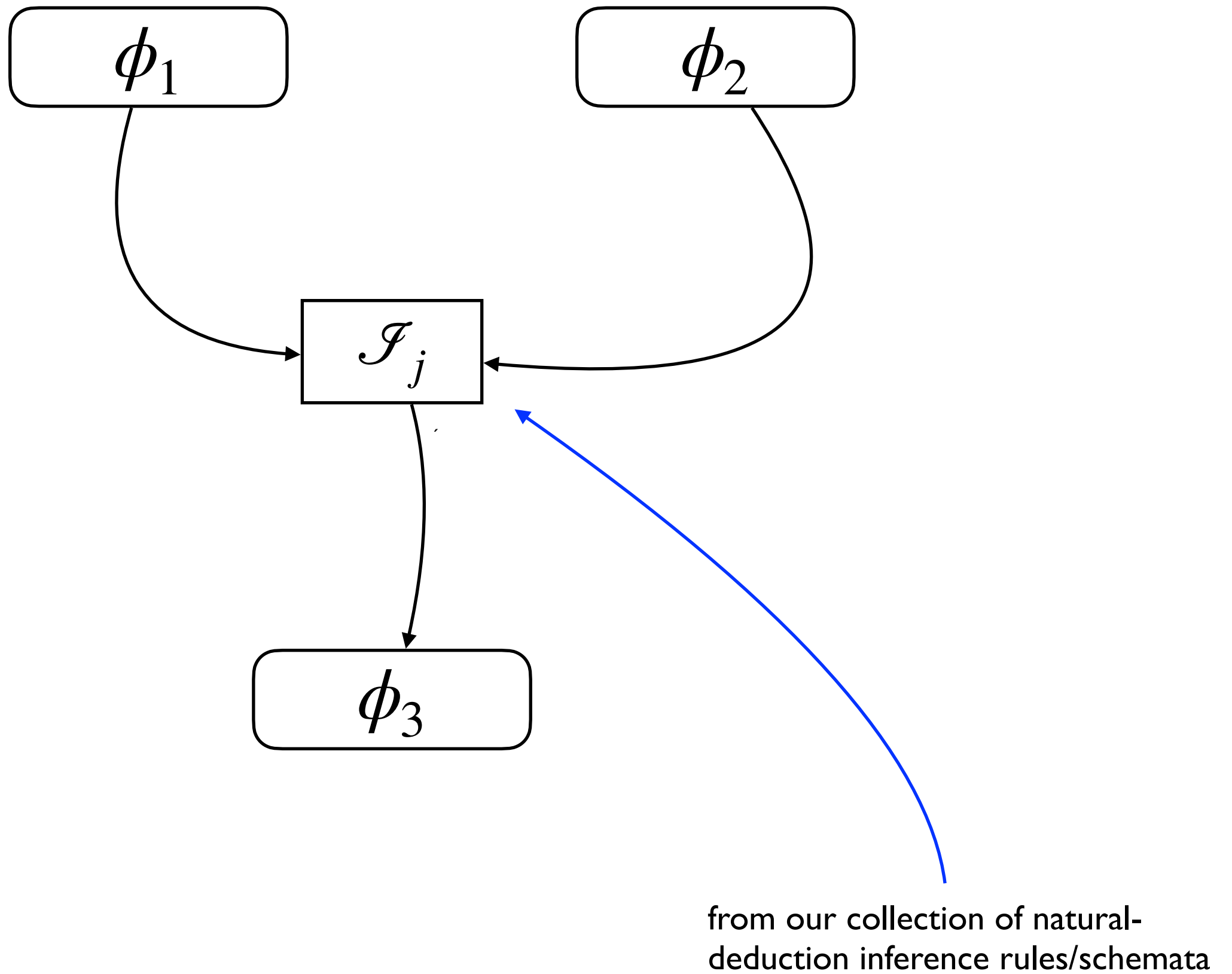
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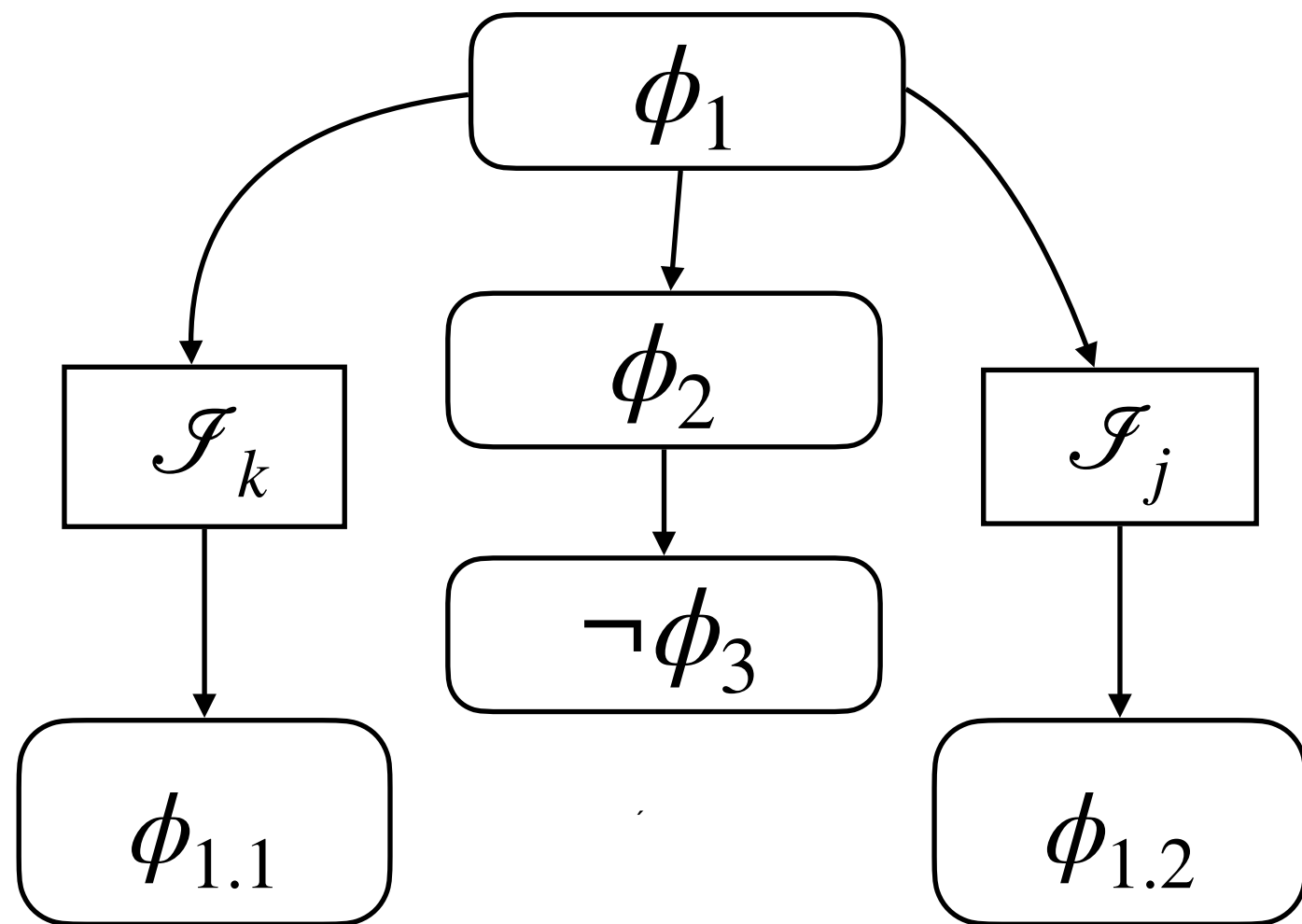
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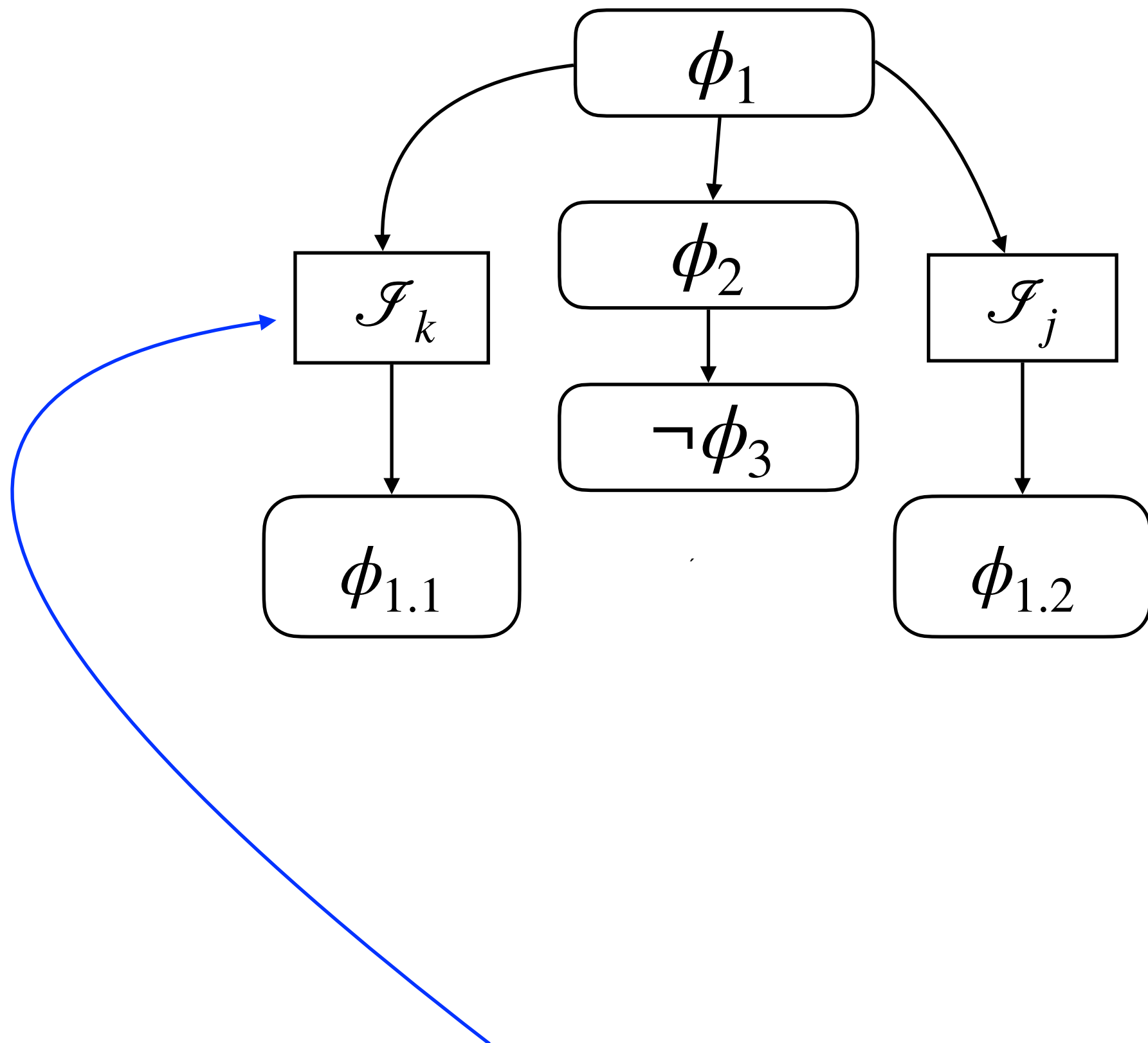
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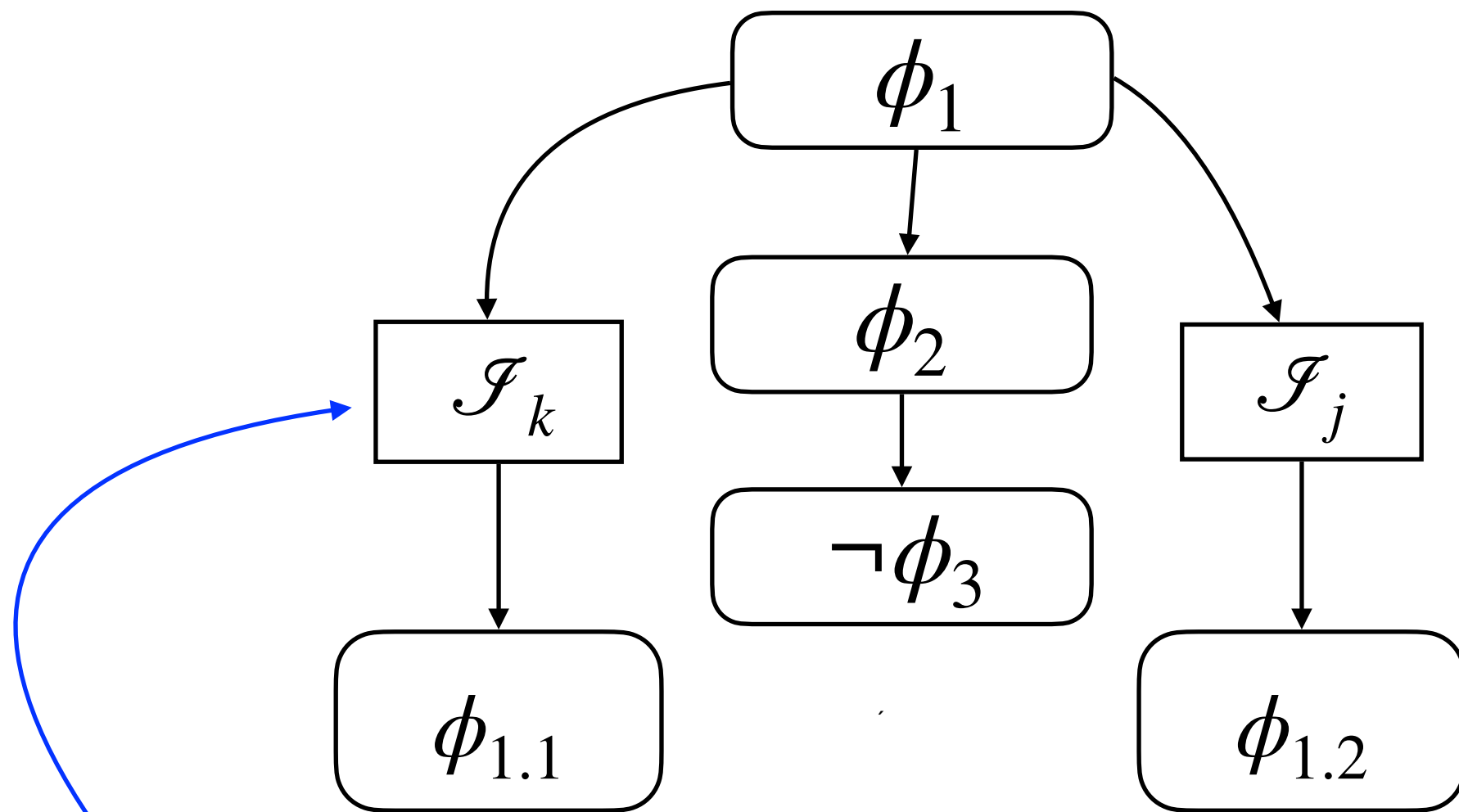




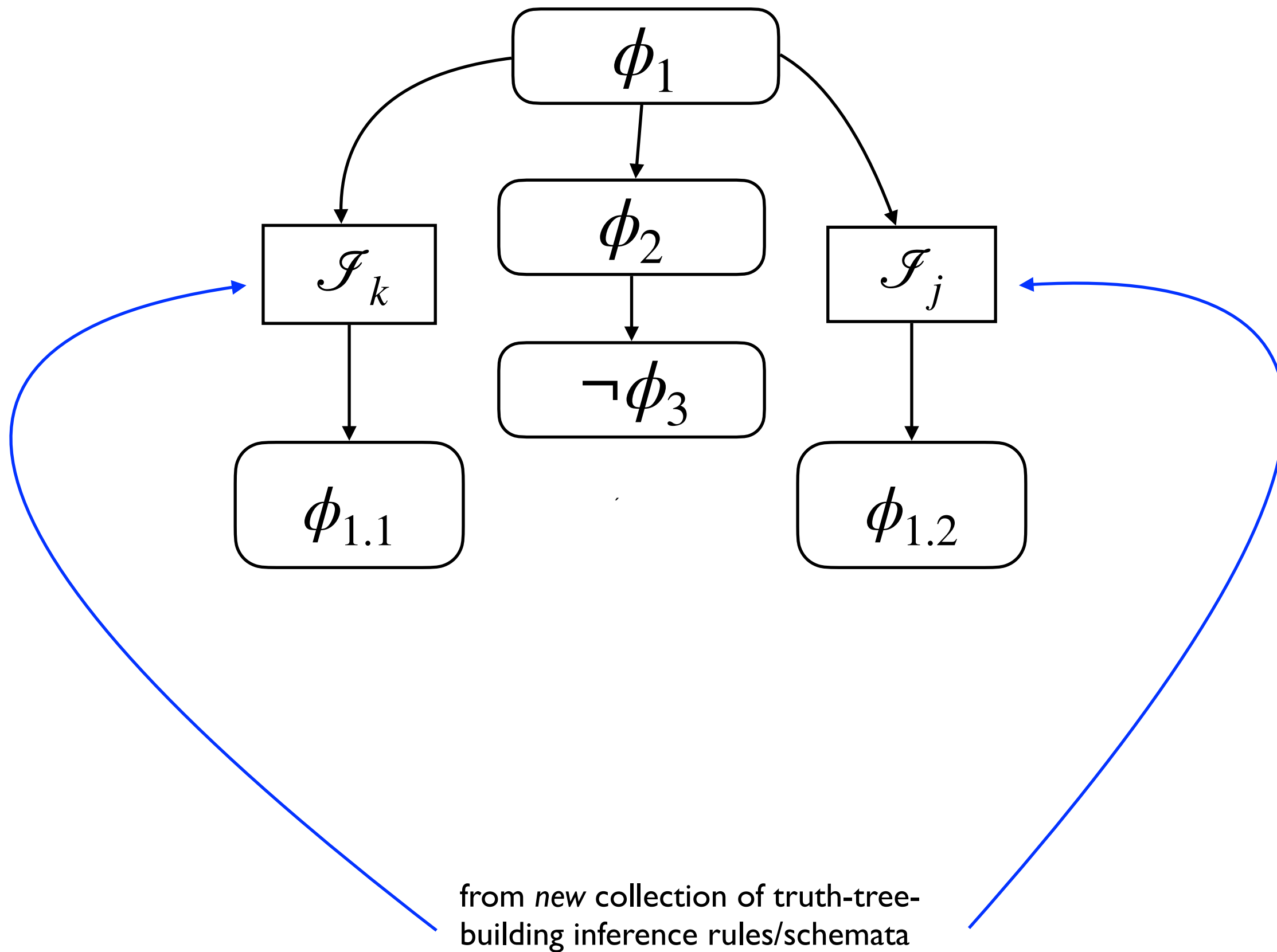








from *new* collection of truth-tree-
building inference rules/schemata



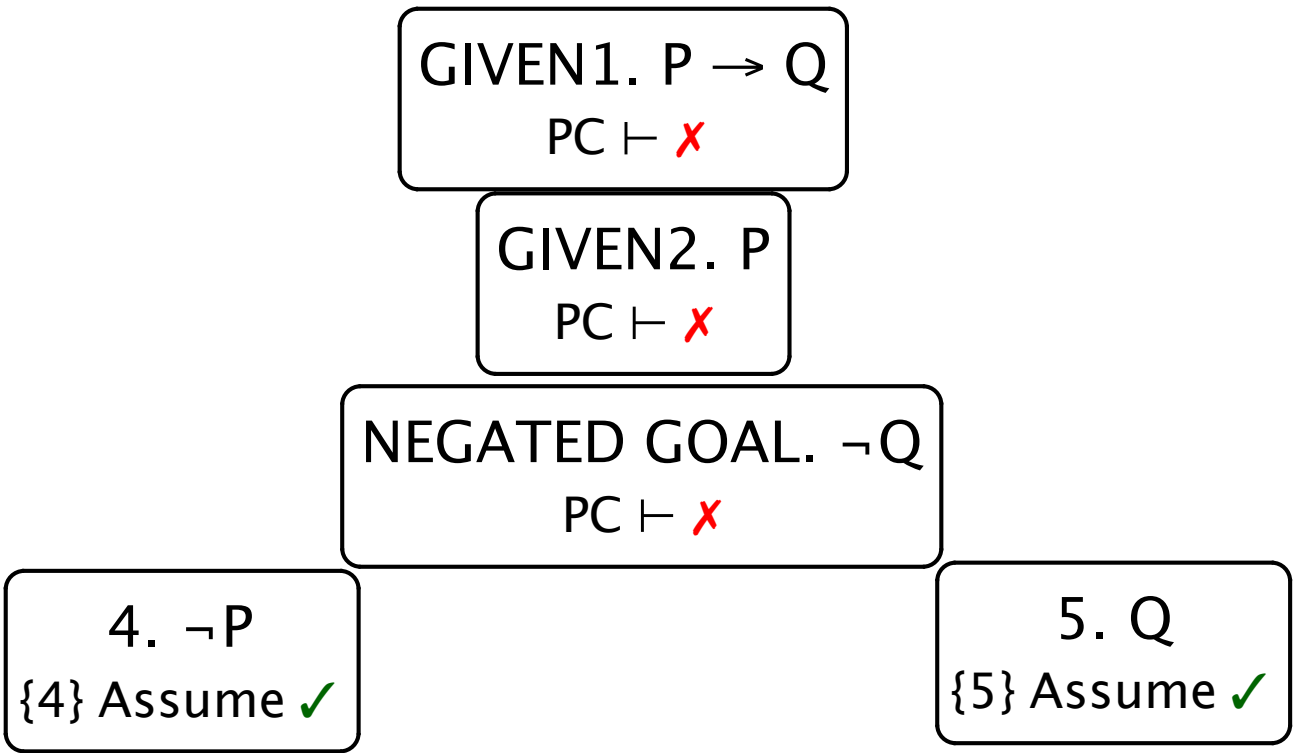
$\{P \rightarrow Q, P\} \vdash Q$

GIVEN1. $P \rightarrow Q$
PC $\vdash$ ~~X~~

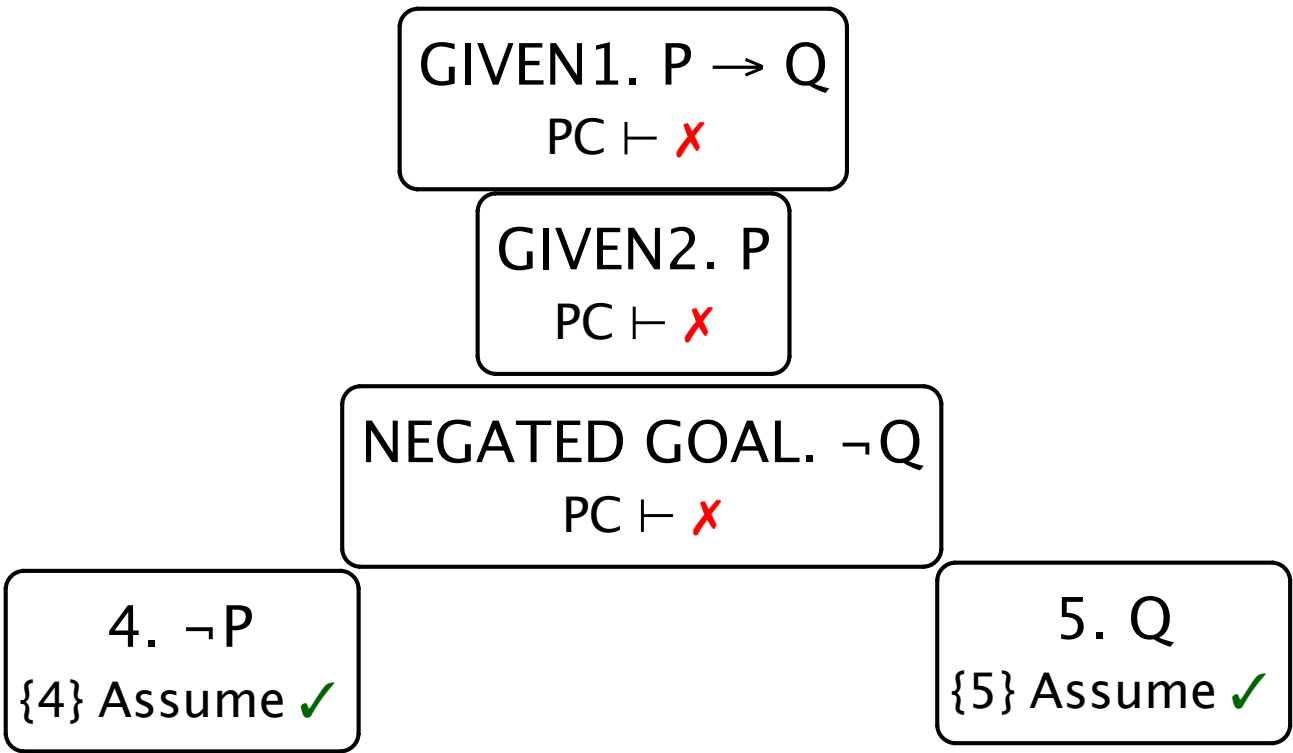
GIVEN2. P
PC $\vdash$ ~~X~~

NEGATED GOAL. $\neg Q$
PC $\vdash$ ~~X~~

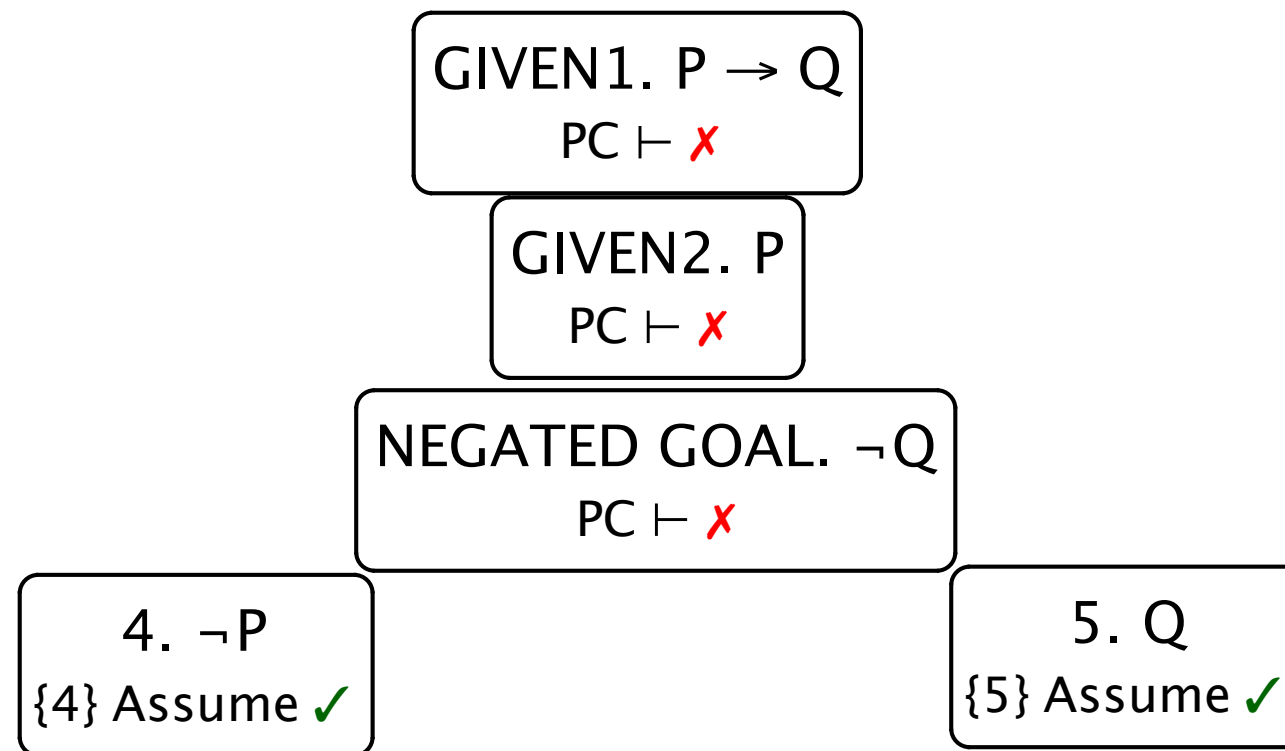
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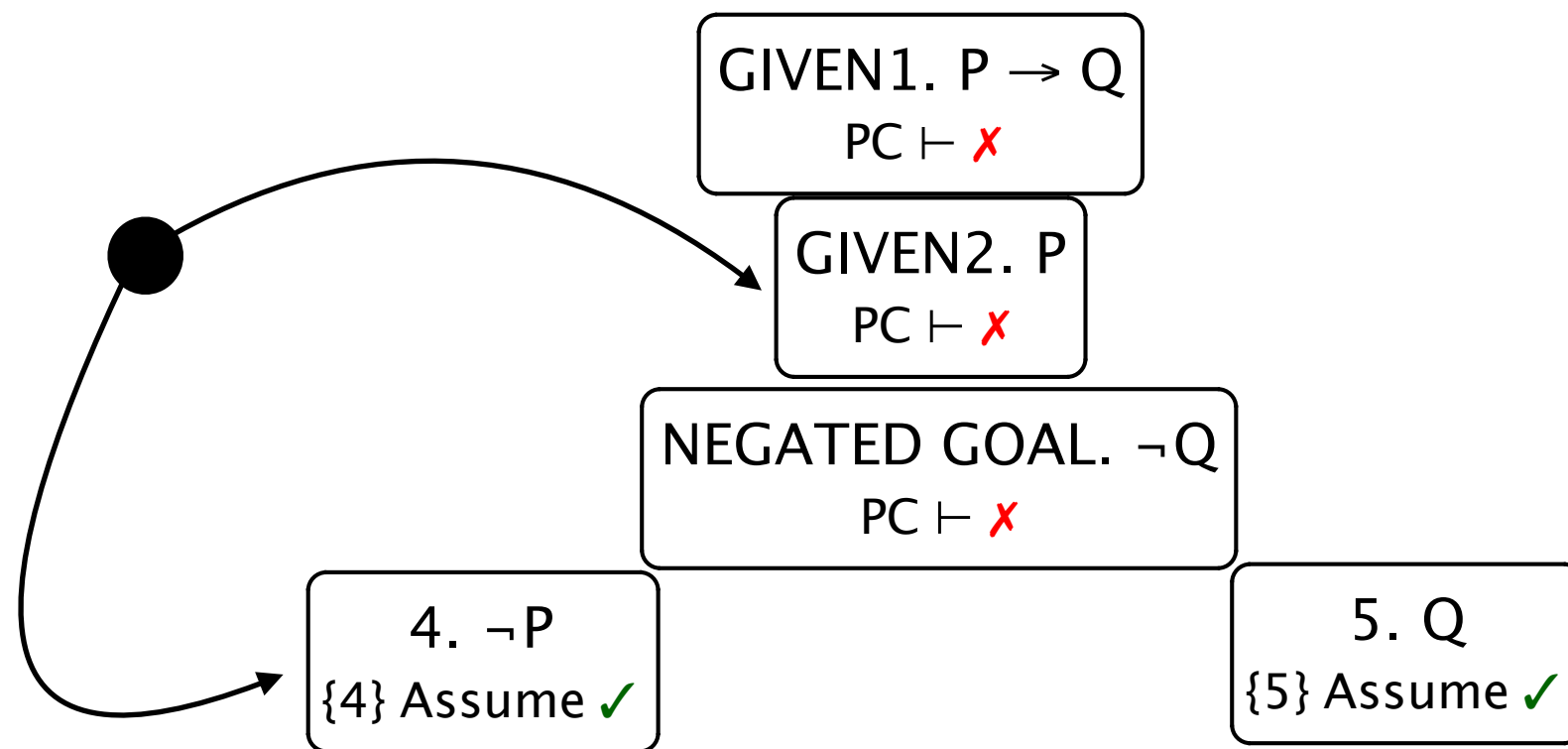


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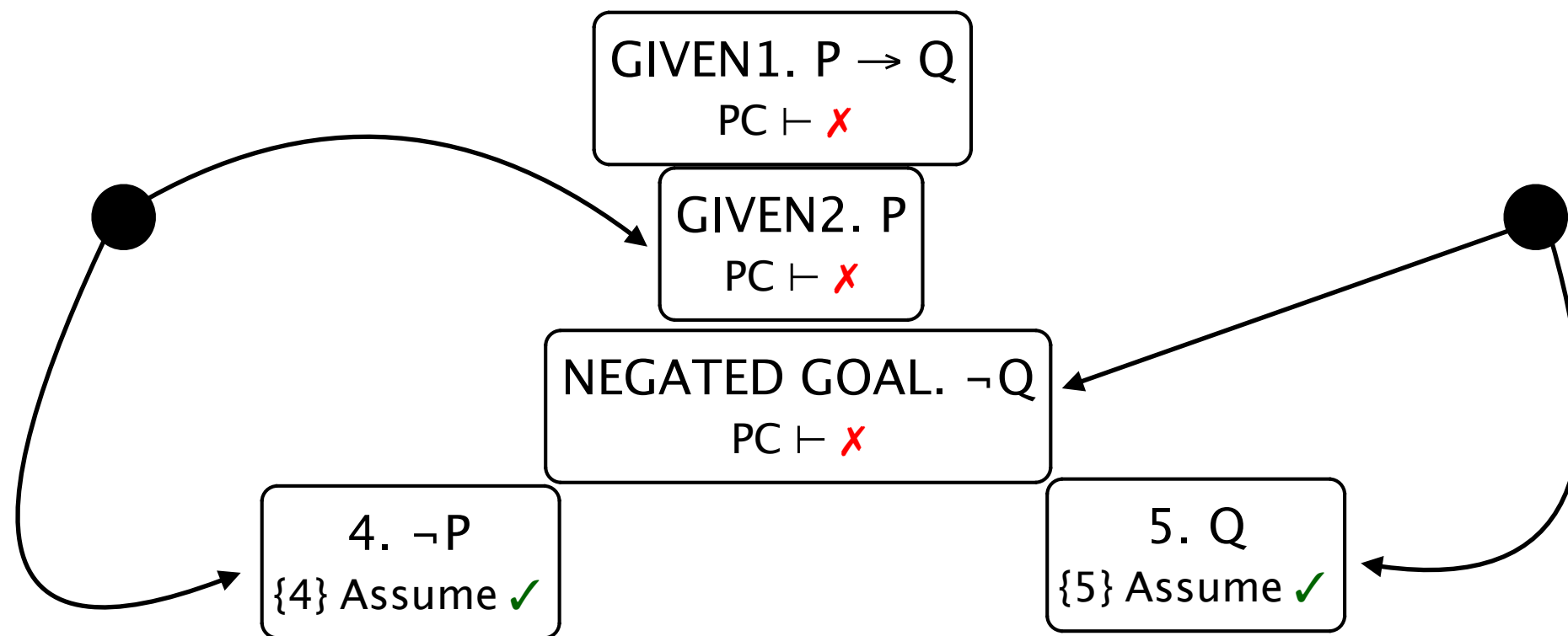
Either way, a contradiction!

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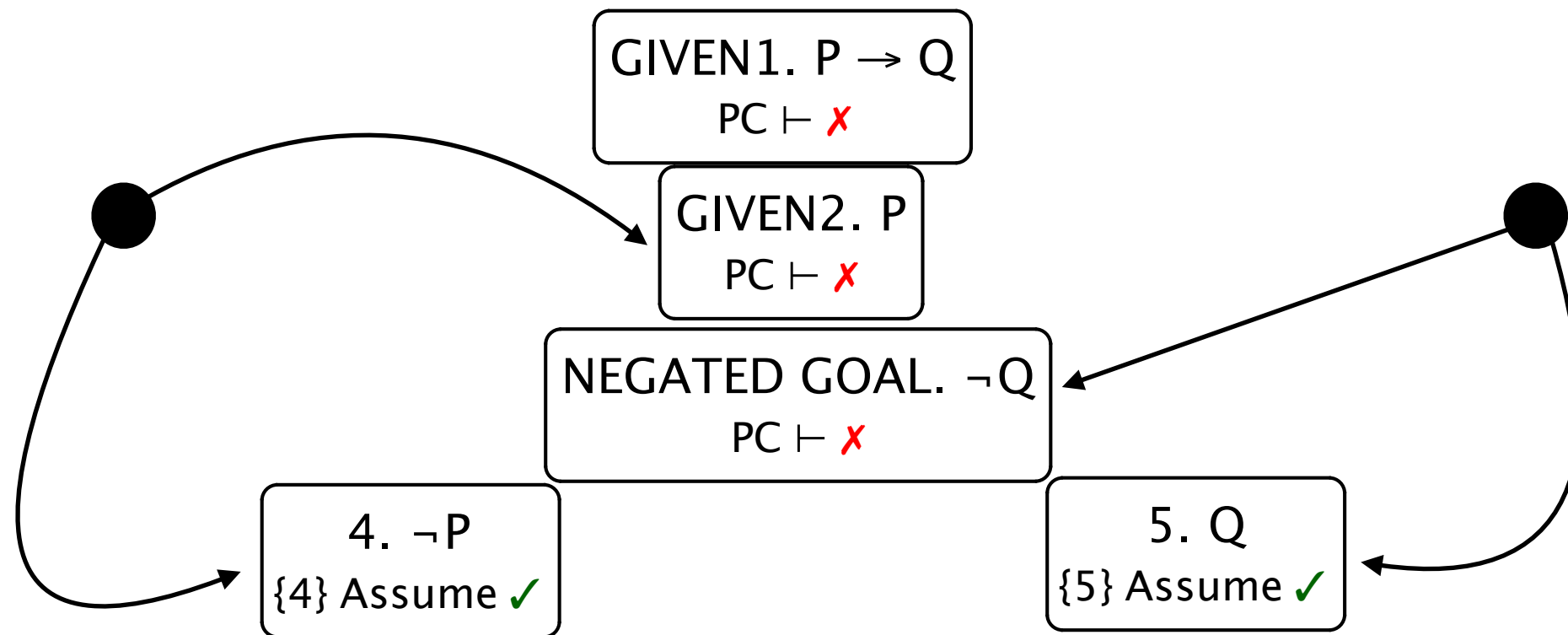
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Either way, a contradiction!

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Either way, a contradiction!

Therefore the entailment holds!

Slightly Harder Truth Tree

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$$\vdash (P \rightarrow (Q \rightarrow R)) \rightarrow ((P \rightarrow Q) \rightarrow (P \rightarrow R))$$

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(This is the axiom THEN-2 in Frege's (brutal) axiomatization of the propositional calculus.)



Frege

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Frege

https://en.wikipedia.org/wiki/Frege%27s_propositional_calculus

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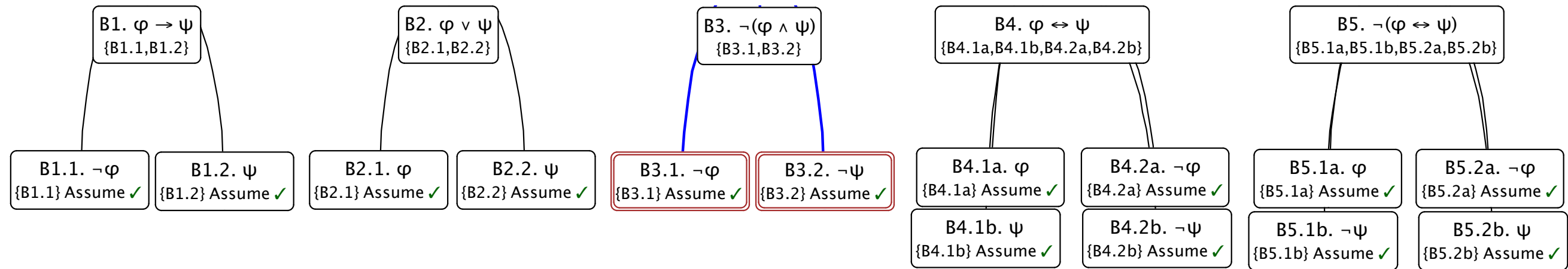


Frege

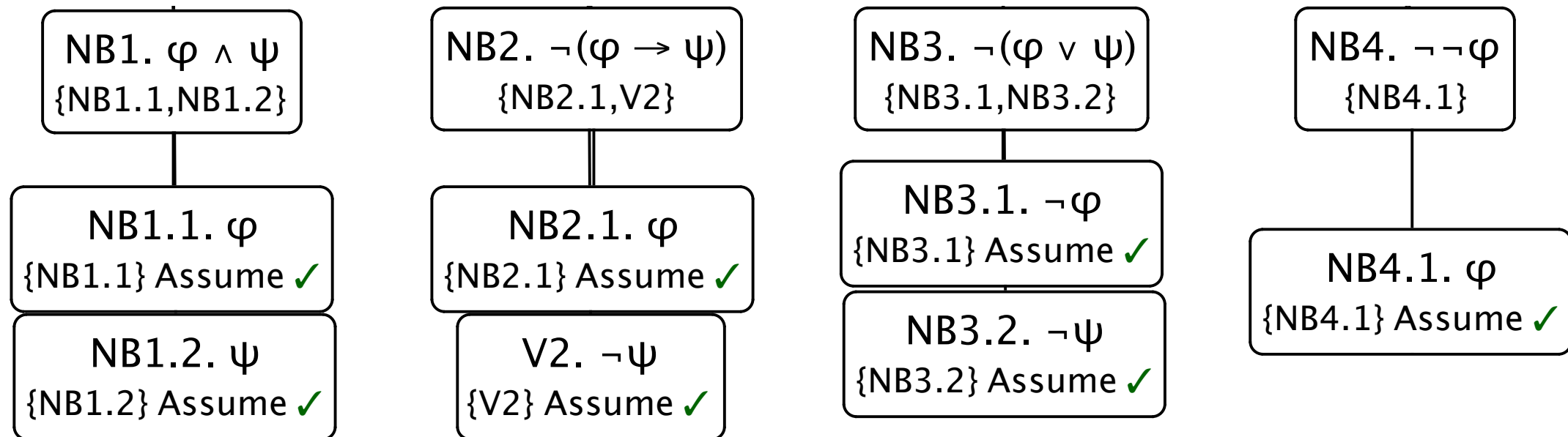
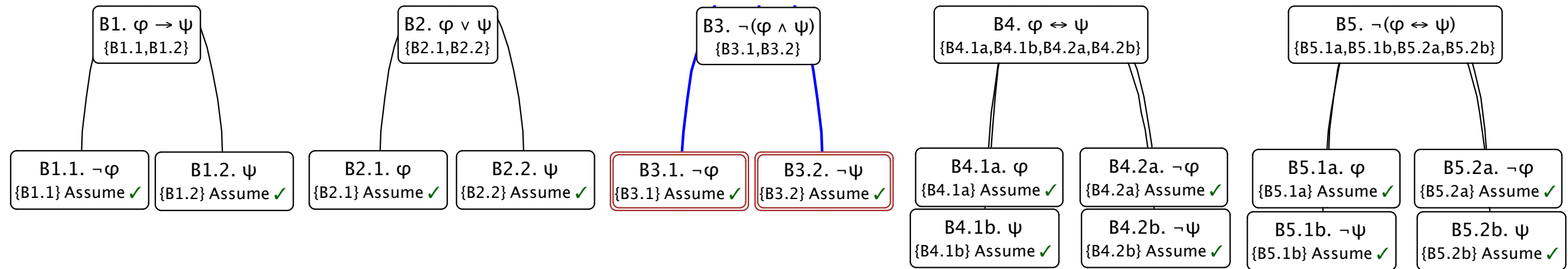
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The Rules of the Game!

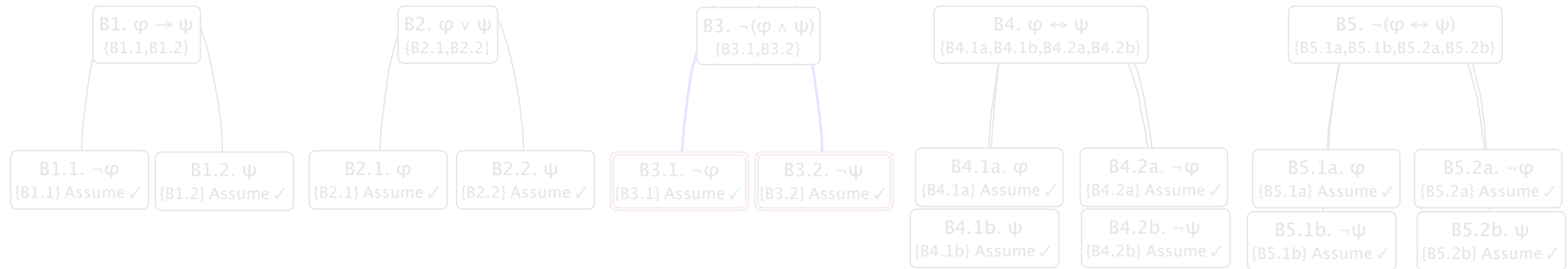
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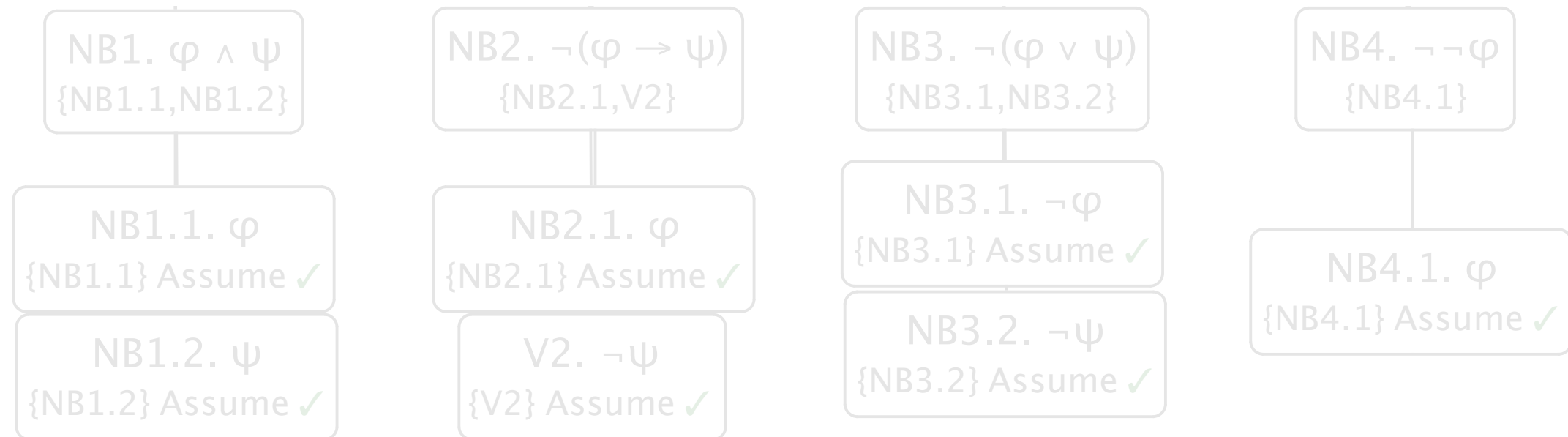
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The Rules of the Game!



Questions?



Theorem?

Let ϕ be a theorem in the propositional calculus $= \mathcal{L}_{\text{PC}}$.
Then the truth-tree algorithm will lead to no open branches.

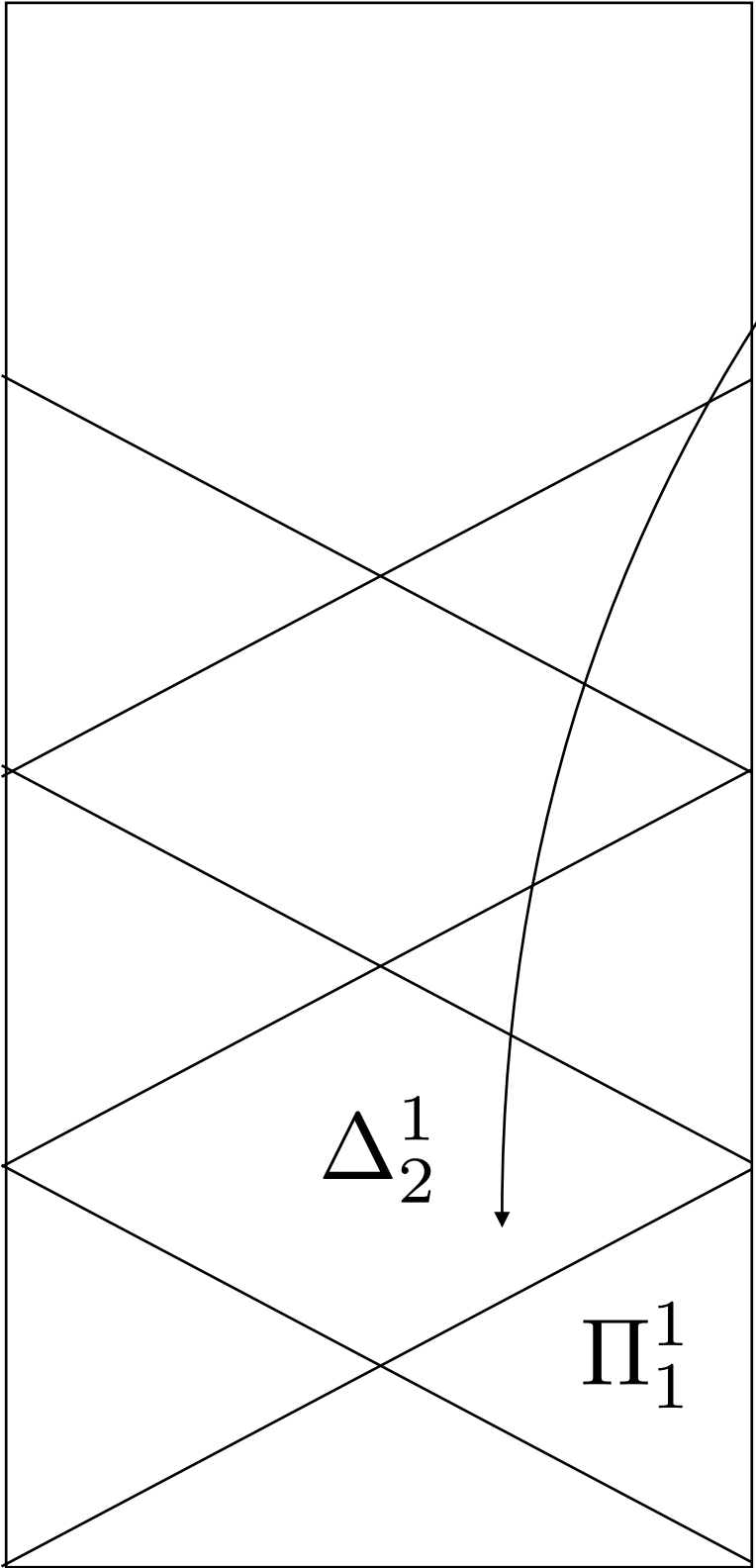
On Measuring The Intelligence of Agents

...

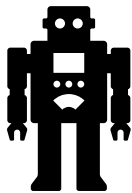
using quantification

$\mathcal{EM}$

$\mathcal{A}^n\mathcal{H}$ (Analytic Hierarchy)



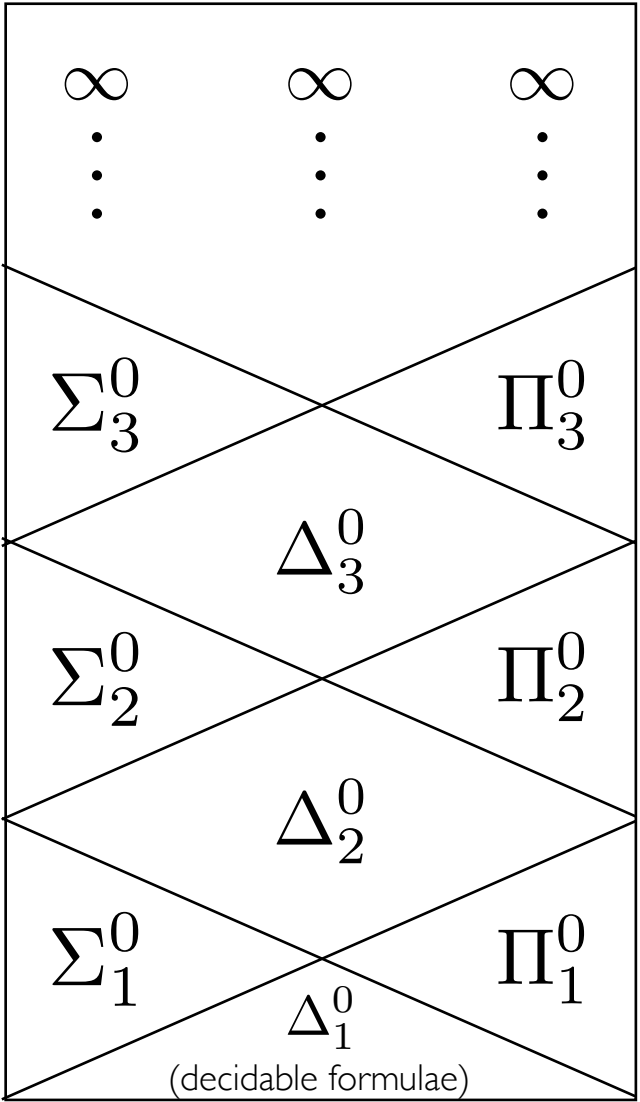
CogSci and AI need to say more about where AI falls/can fall in the landscape.



Infinite Time Turing Machines (ITTMs)

Human Persons
(according to Bringsjord)

$\mathcal{A}^r\mathcal{H}$ (Arithmetic Hierarchy)



Human Brains
(according to Granger)



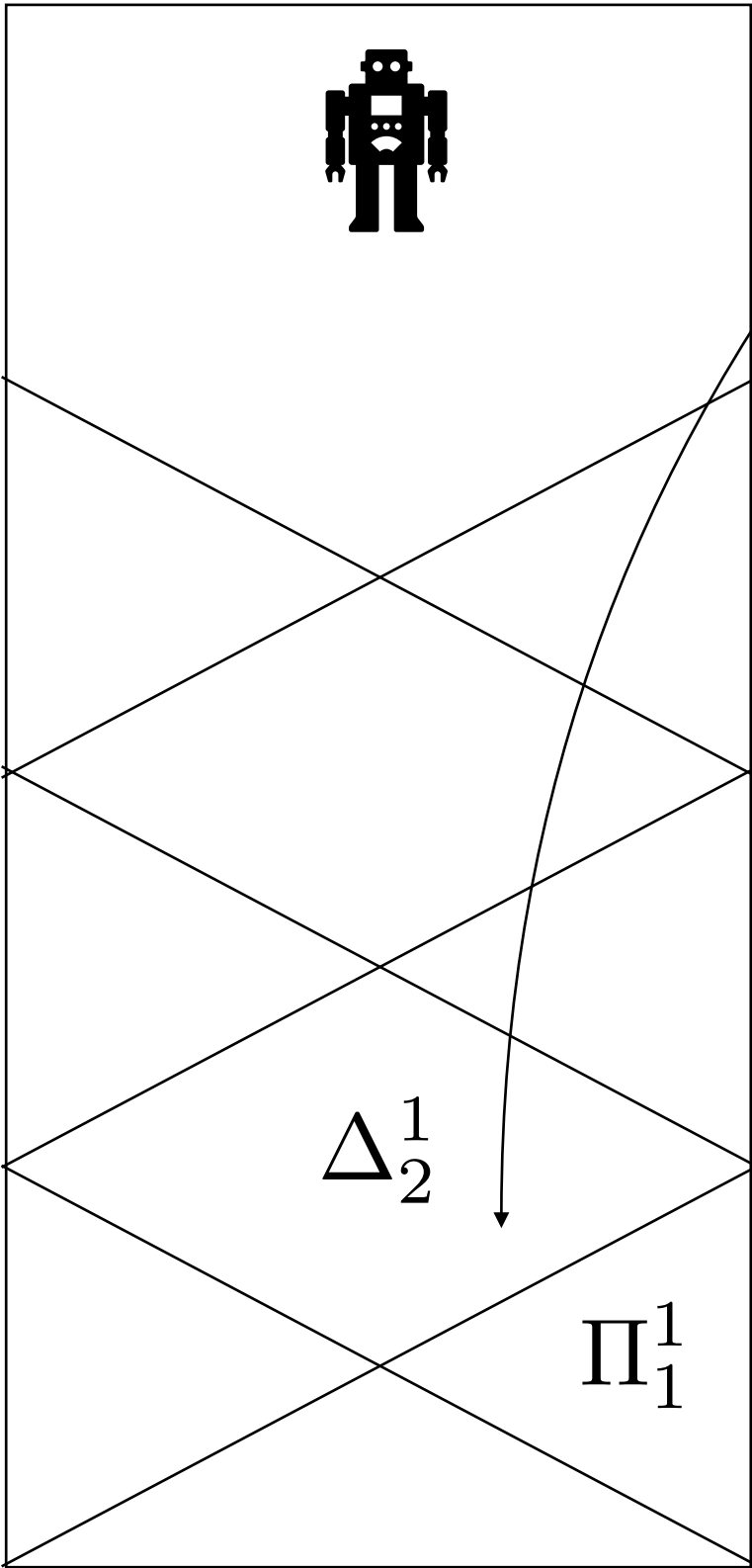
$\mathcal{CH}$ (Chomsky Hierarchy)

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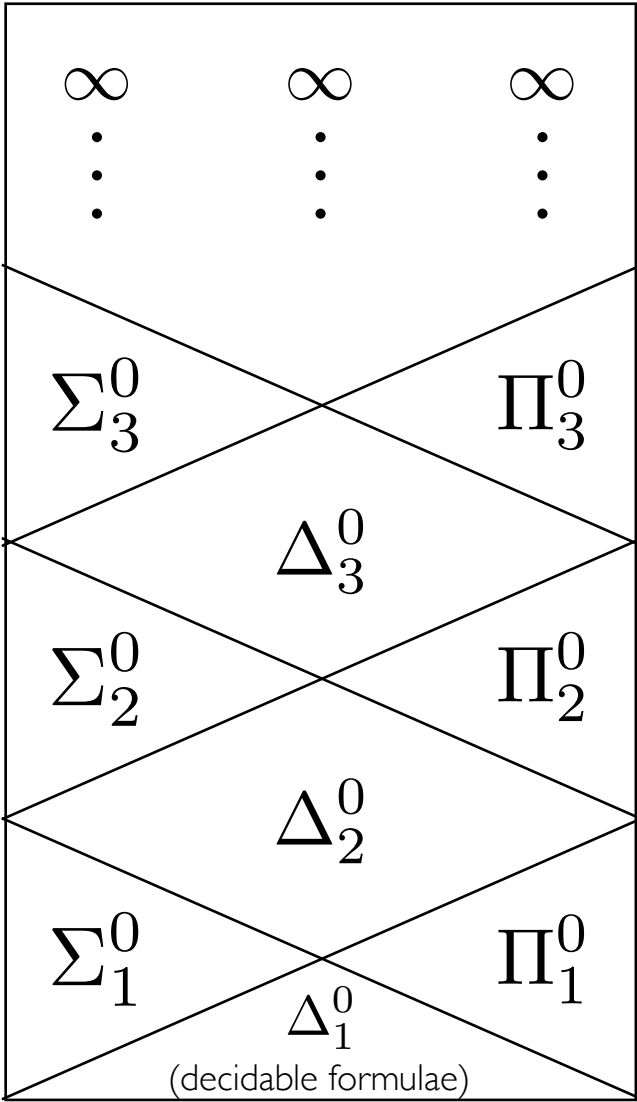


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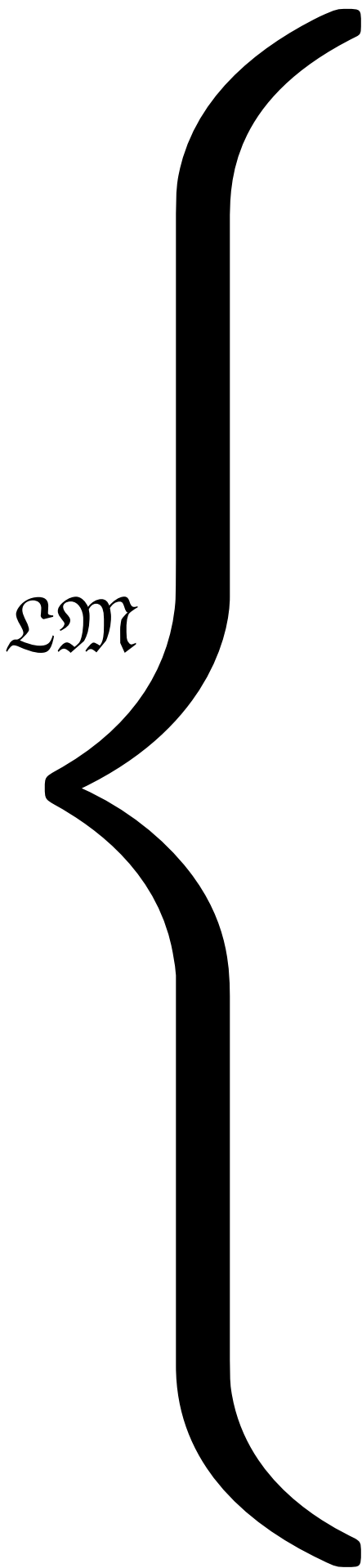
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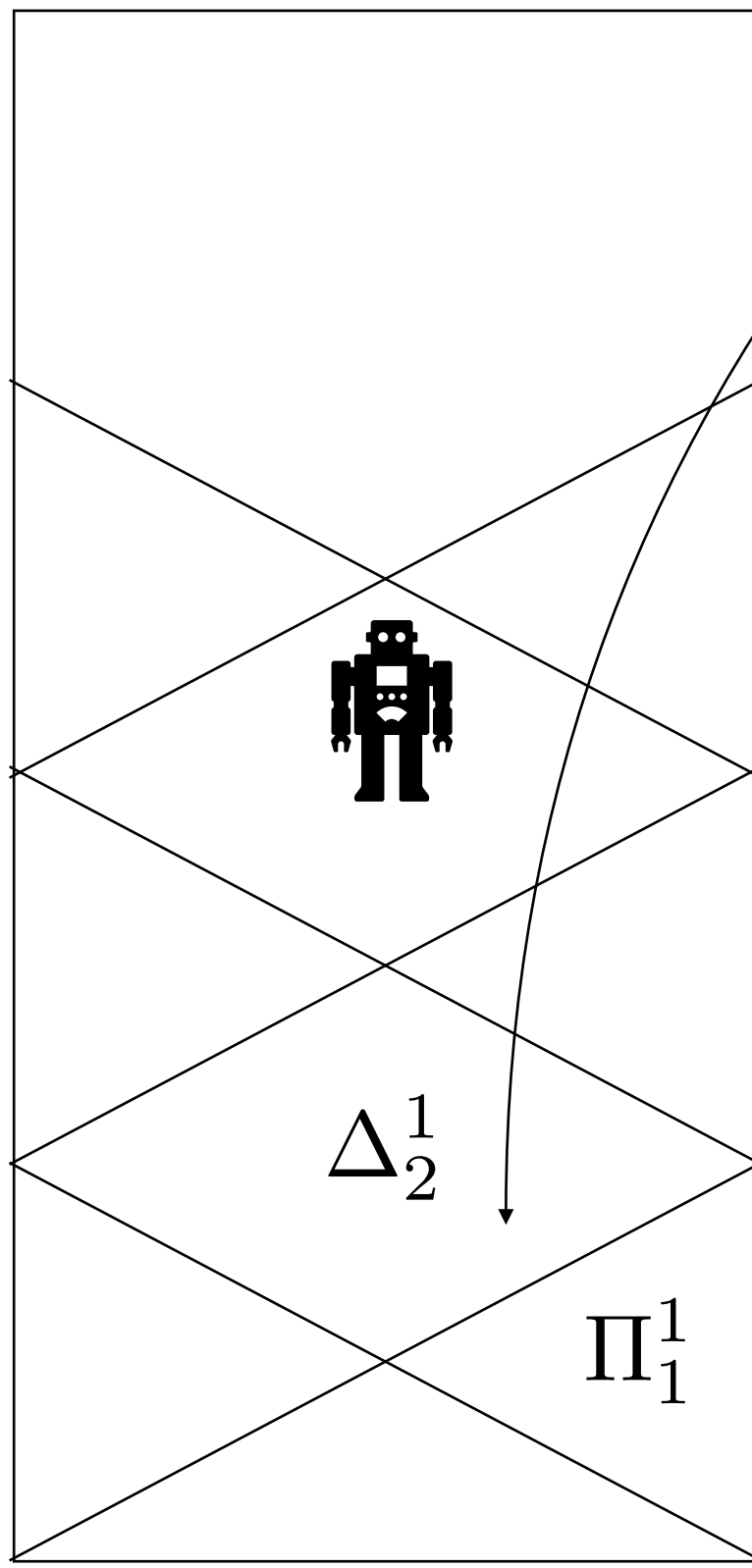
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$\mathfrak{M}$

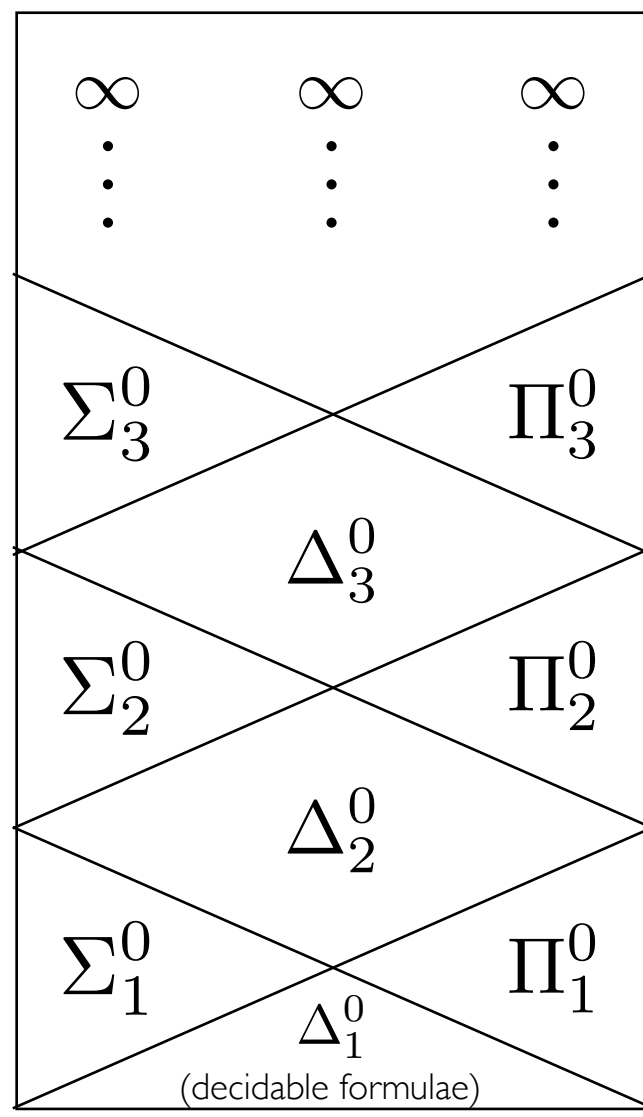
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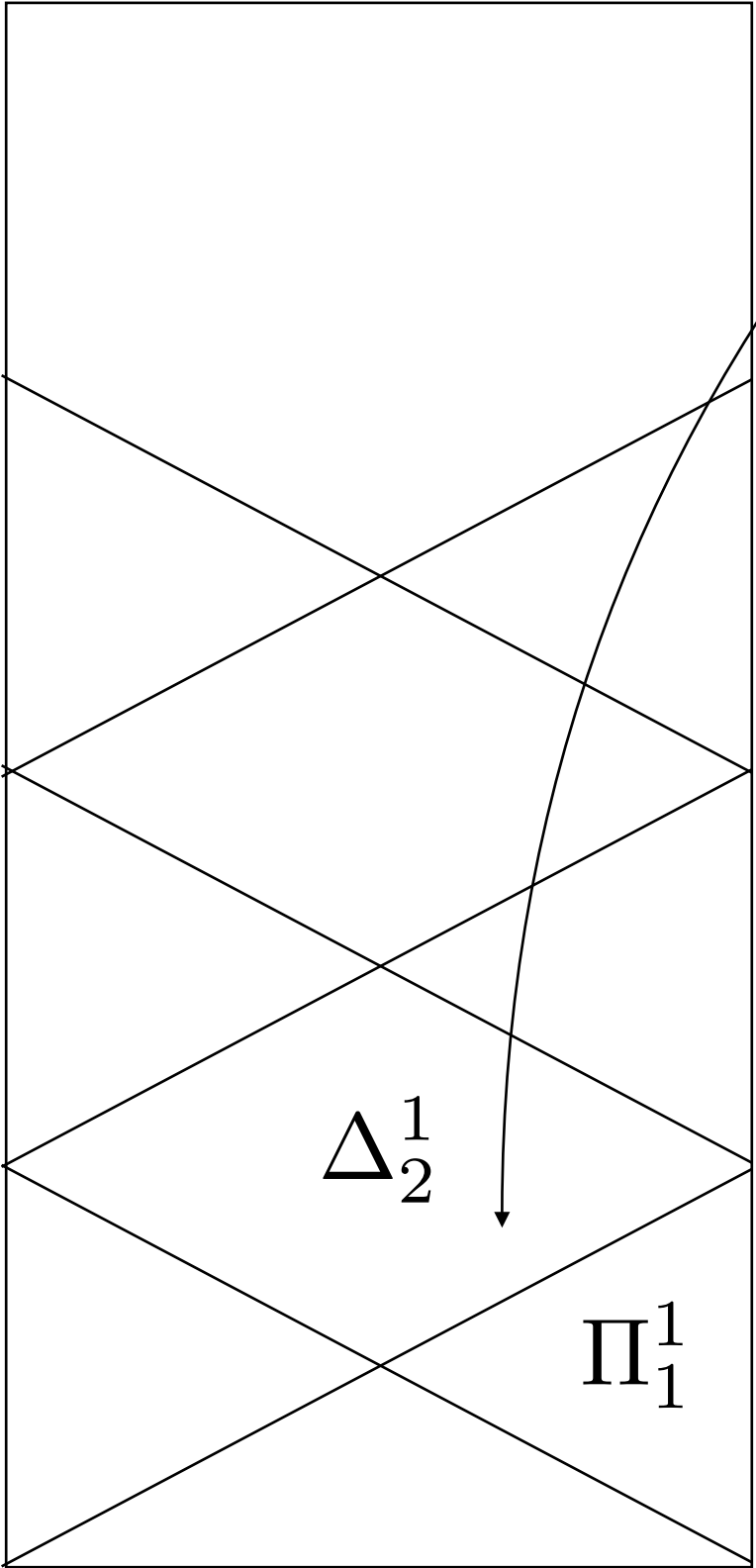


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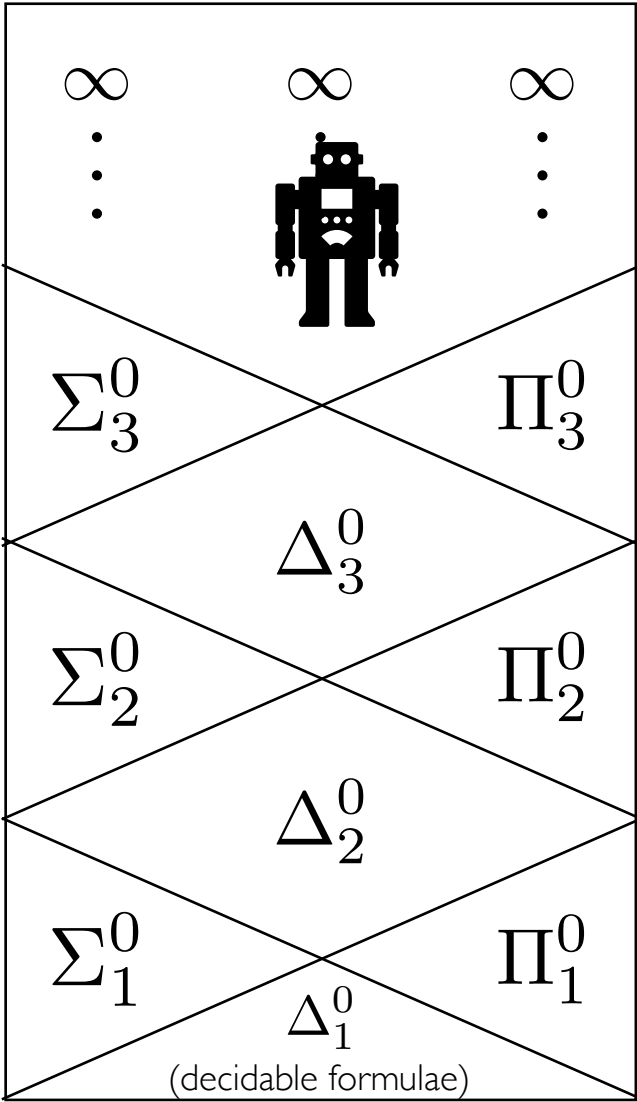


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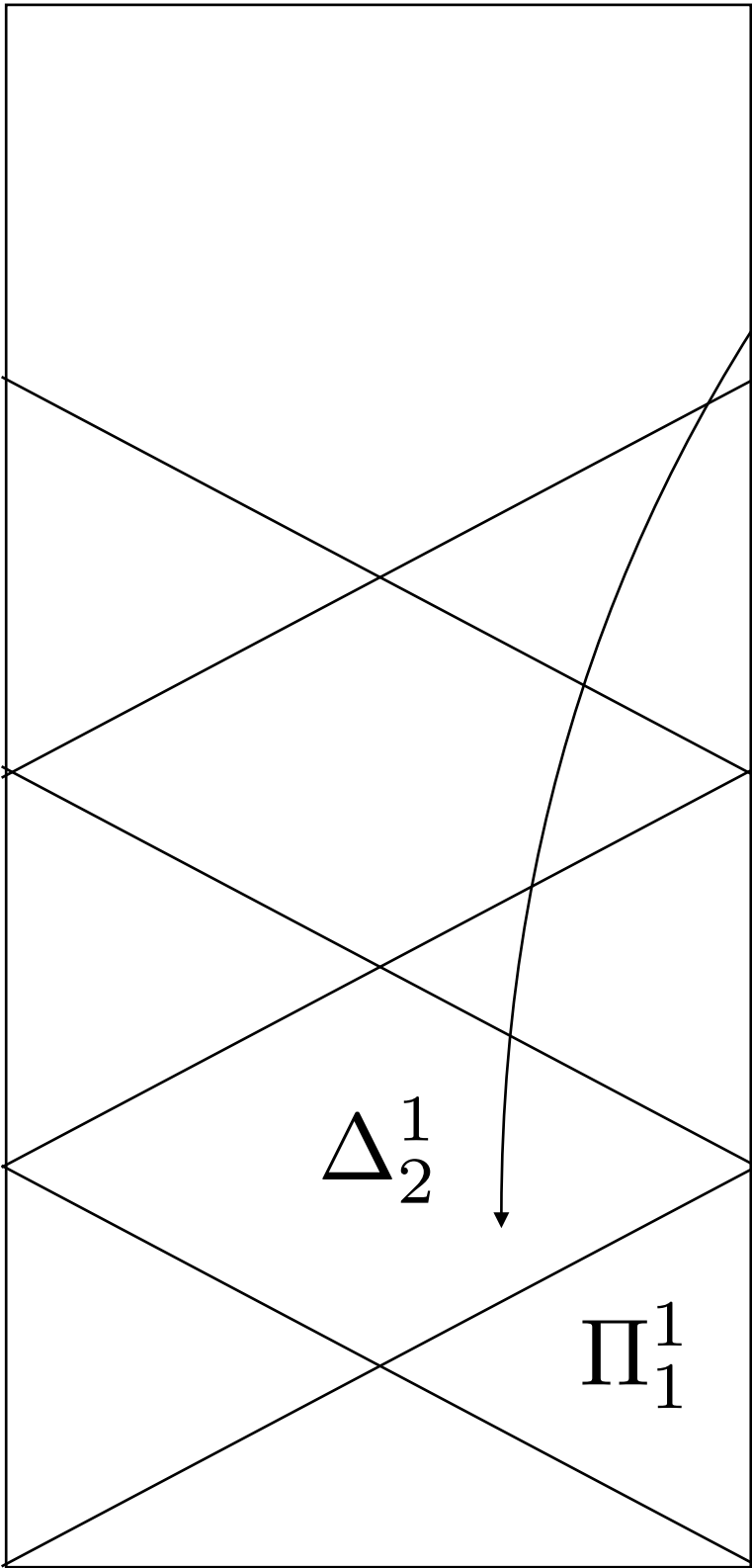
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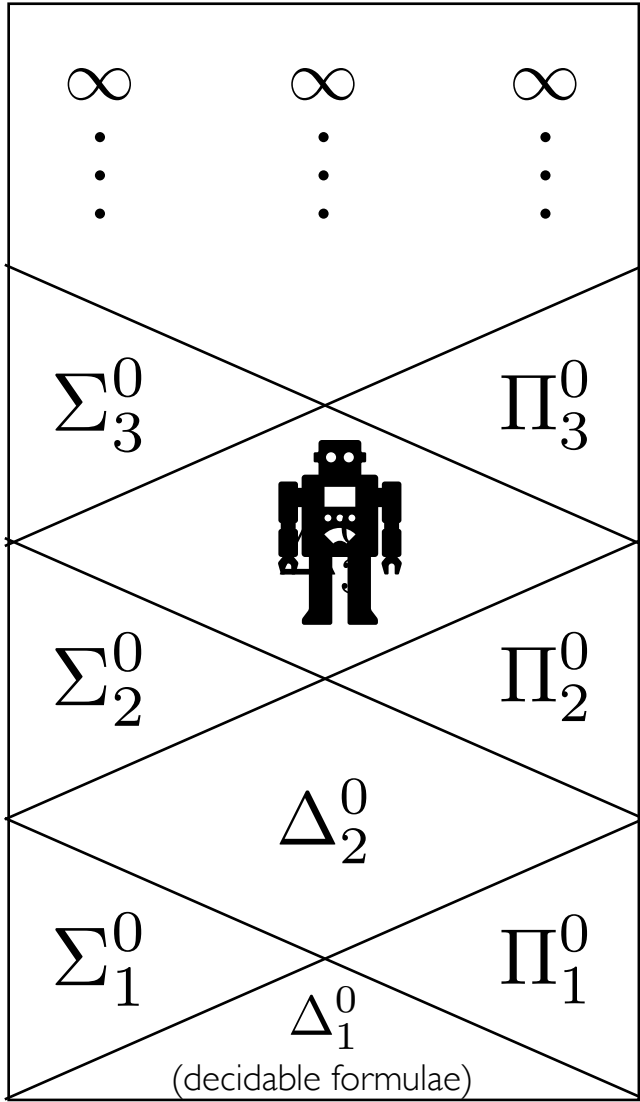
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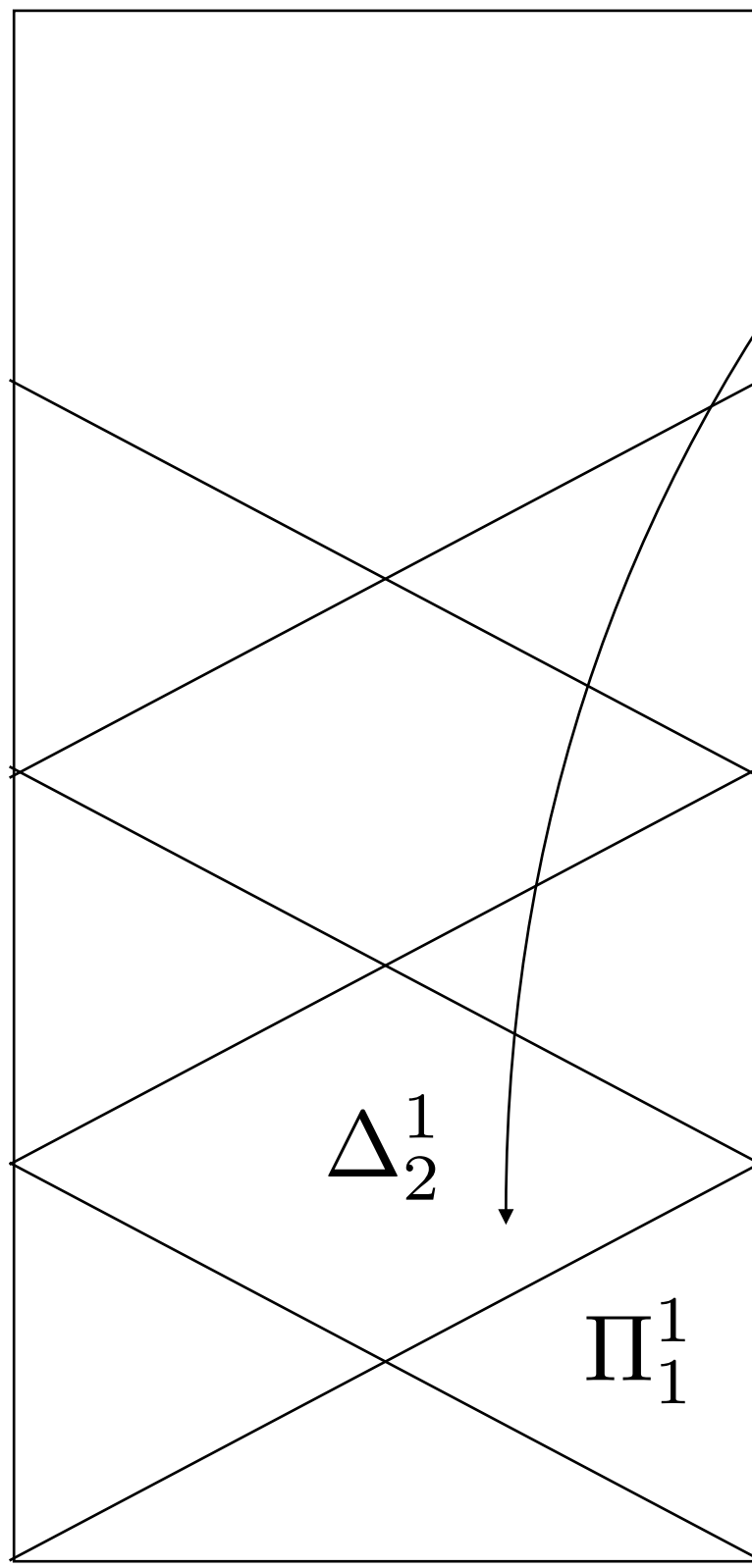
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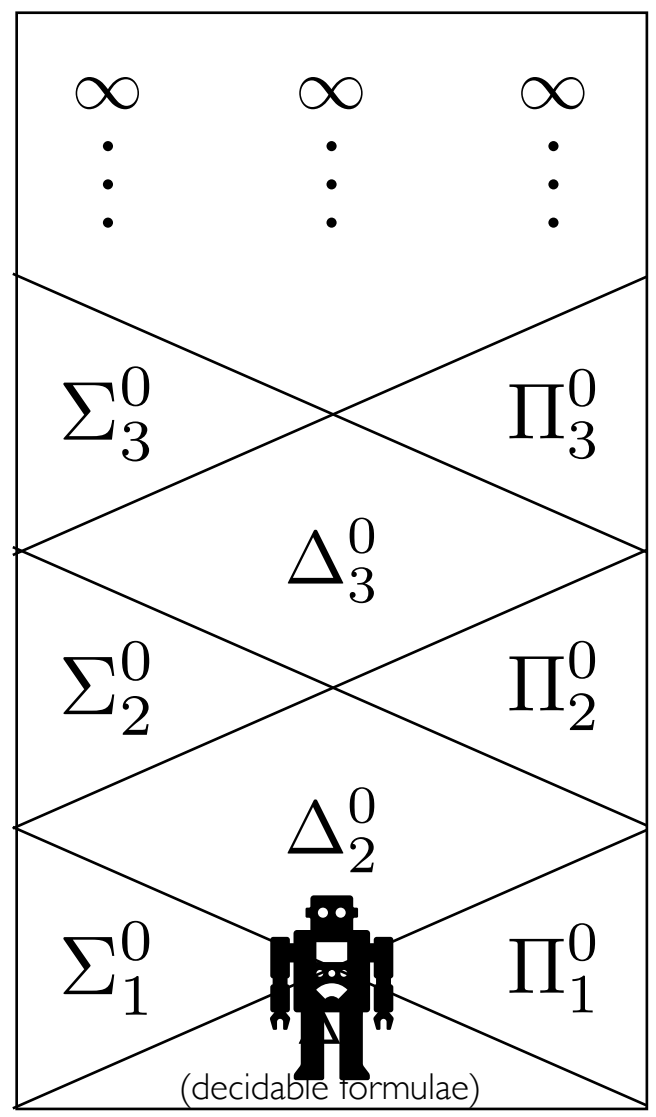
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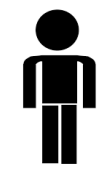
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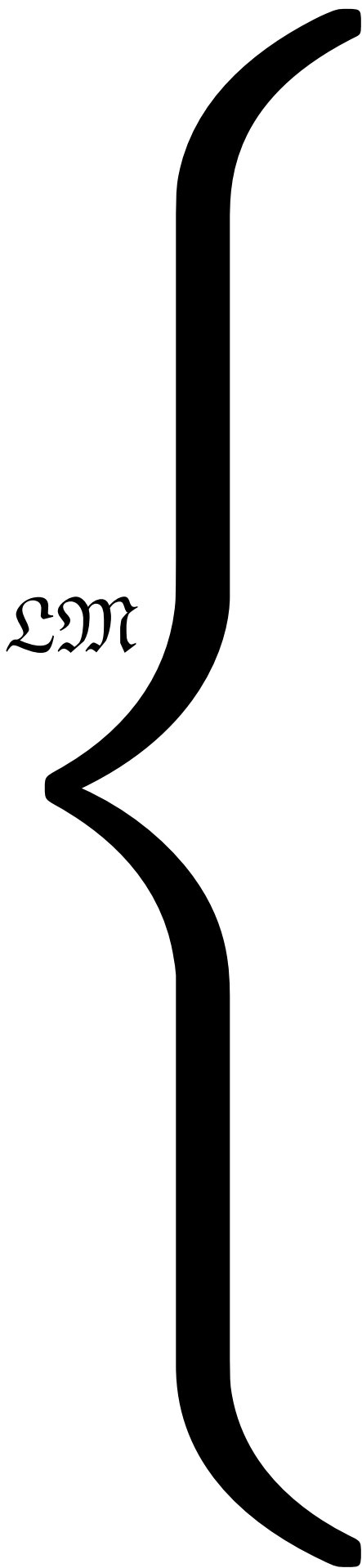
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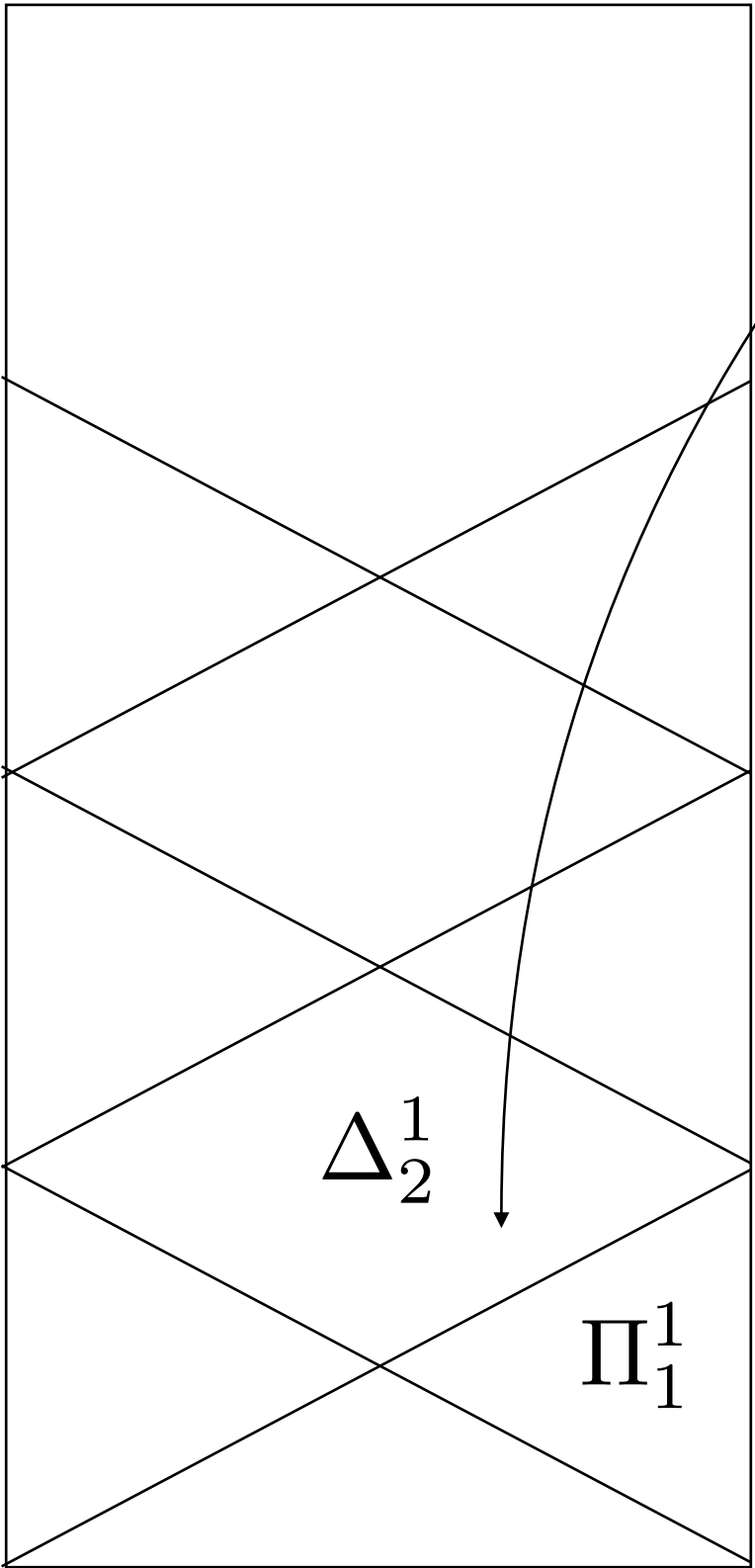
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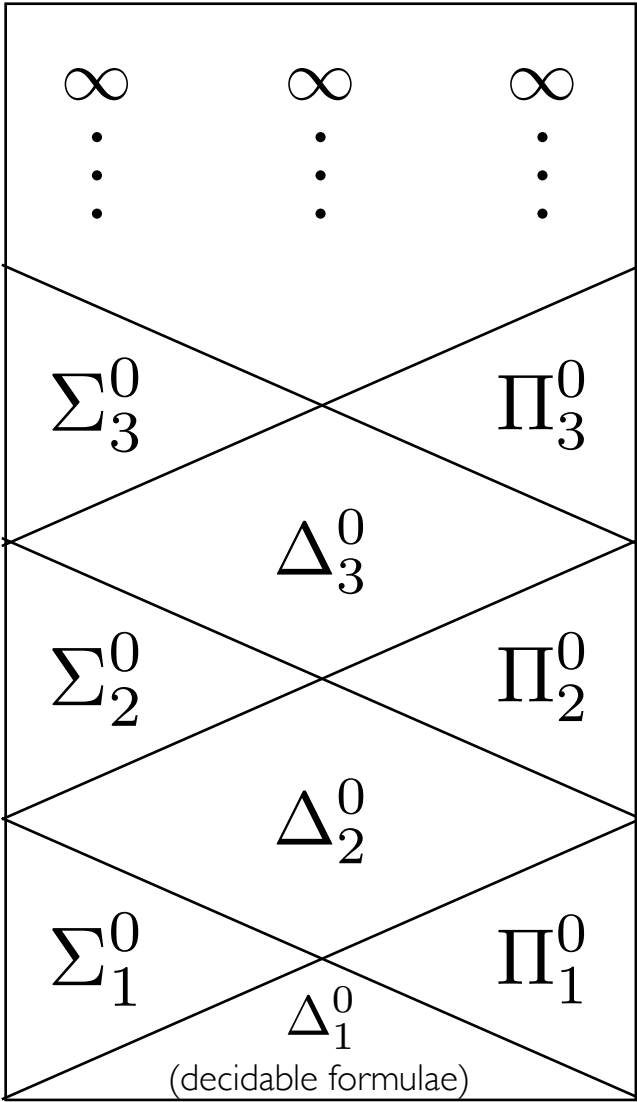
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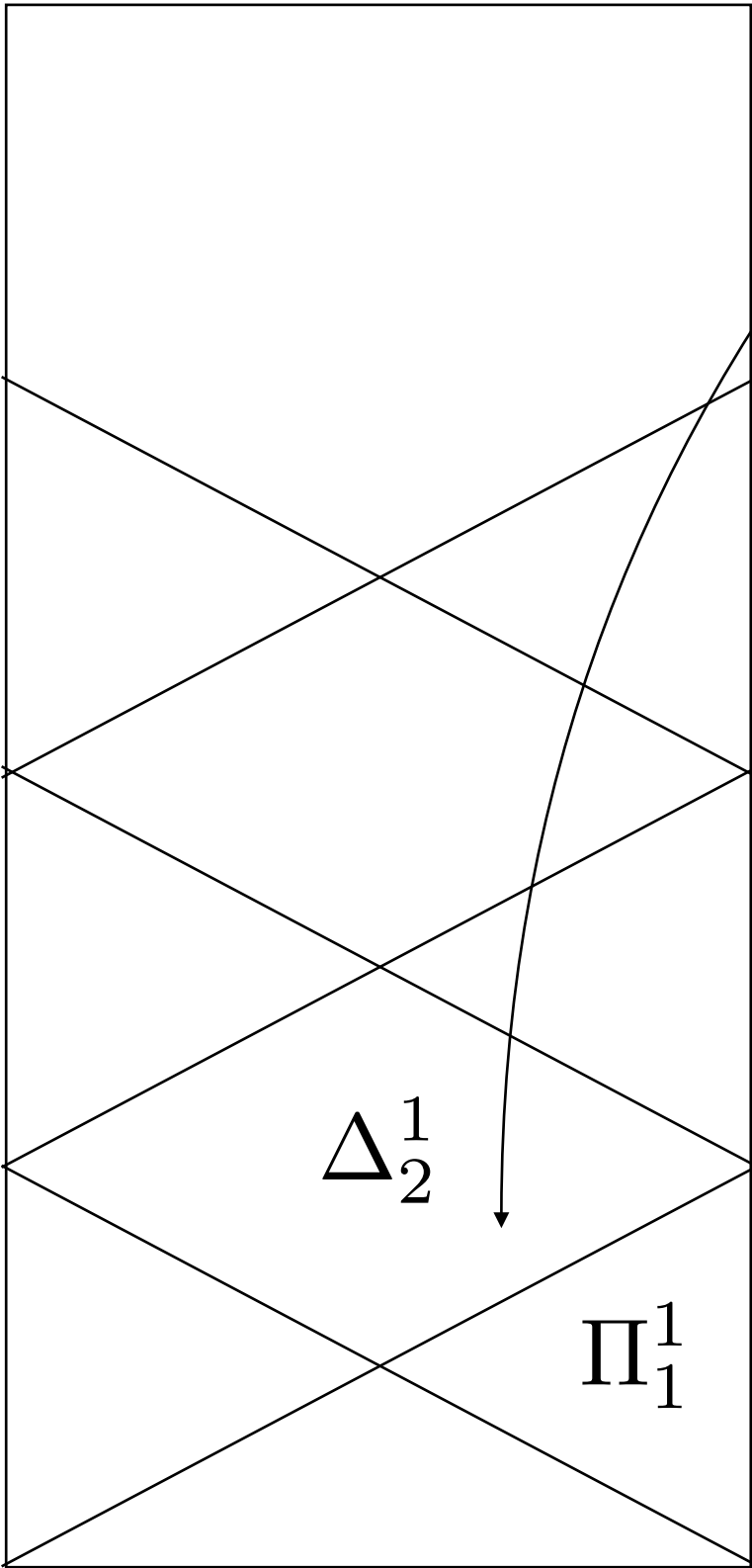
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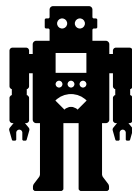


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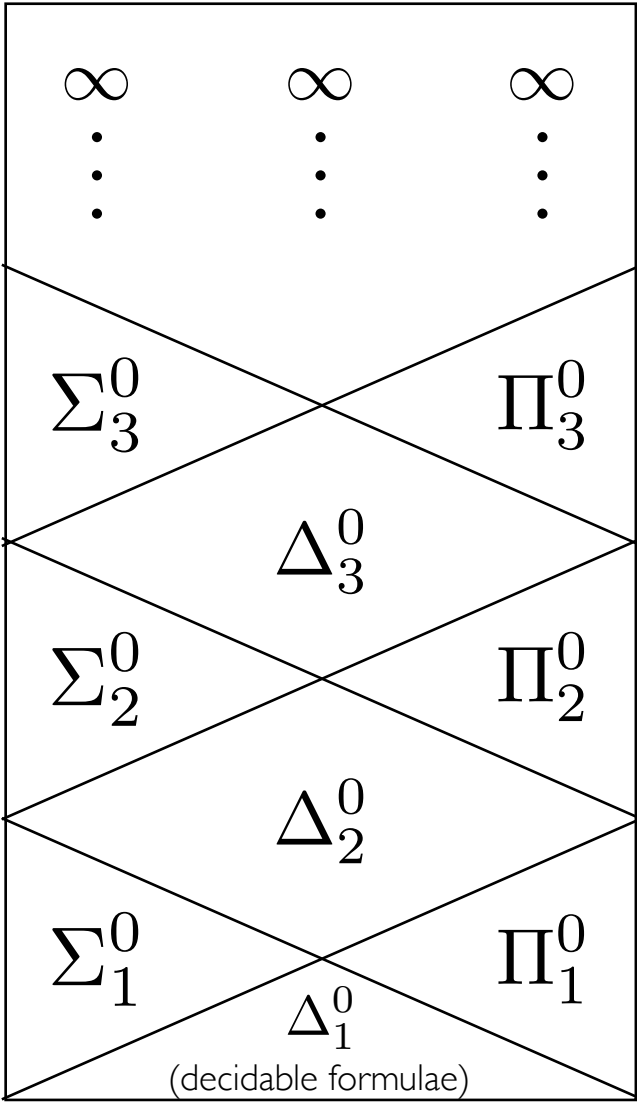


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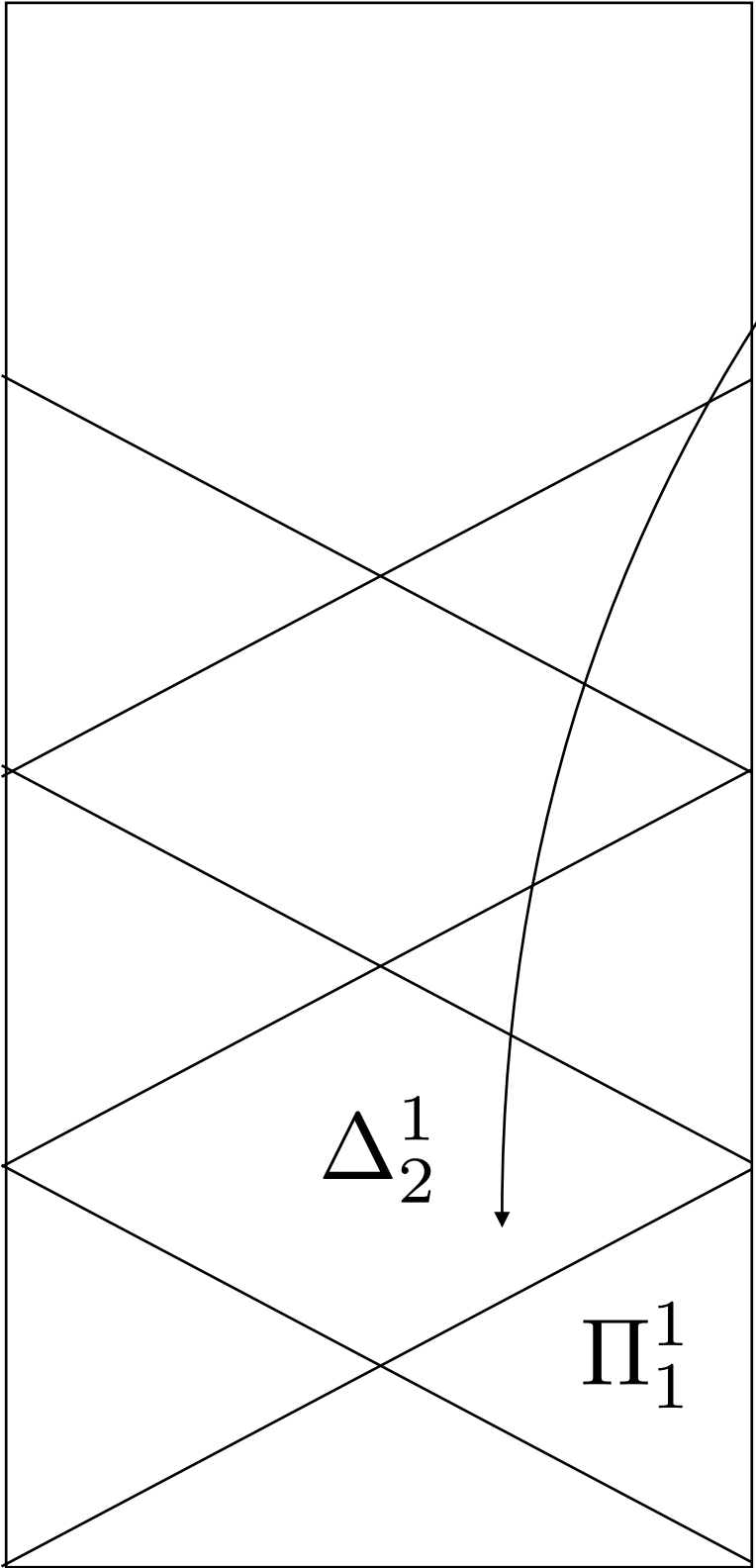
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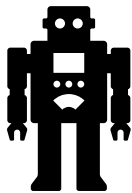


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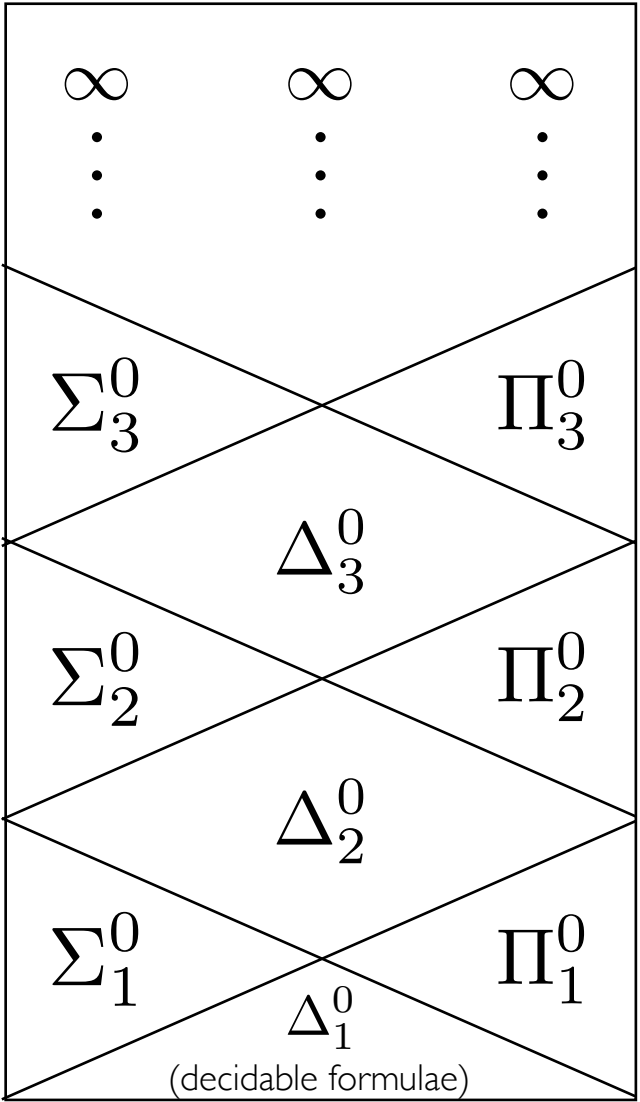
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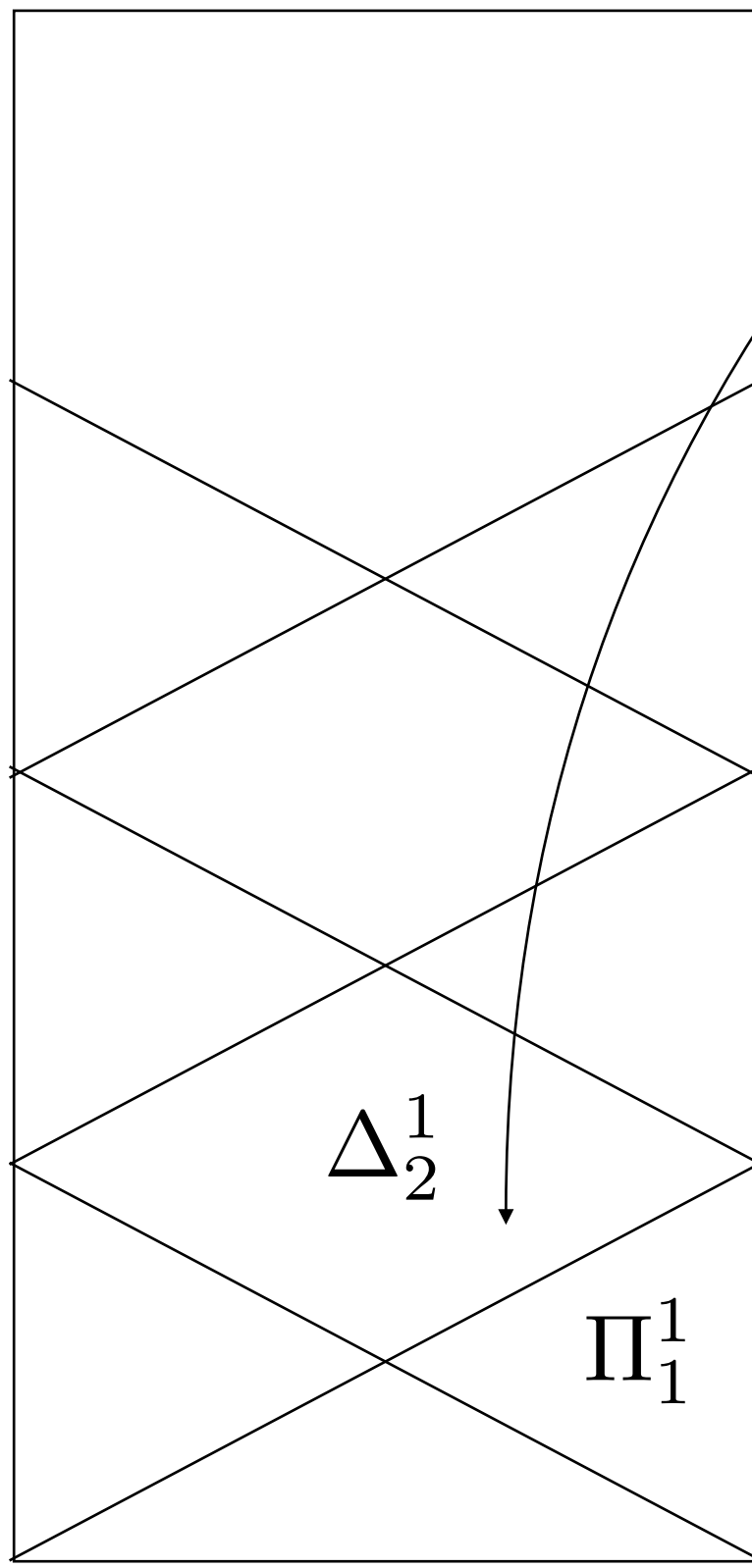
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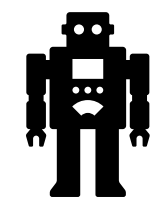


LM ?

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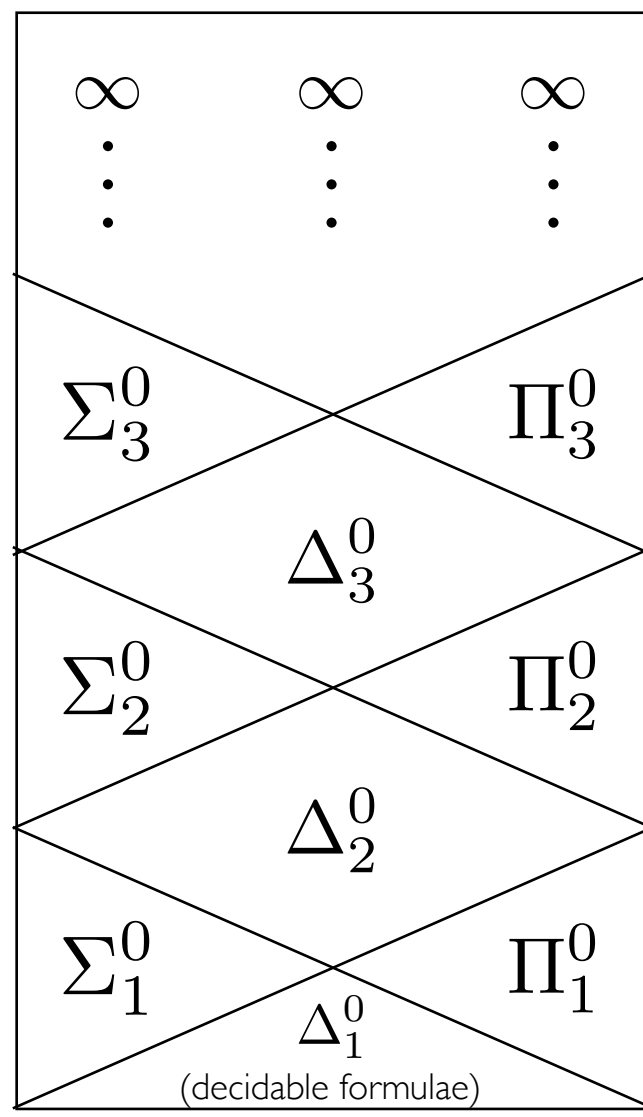


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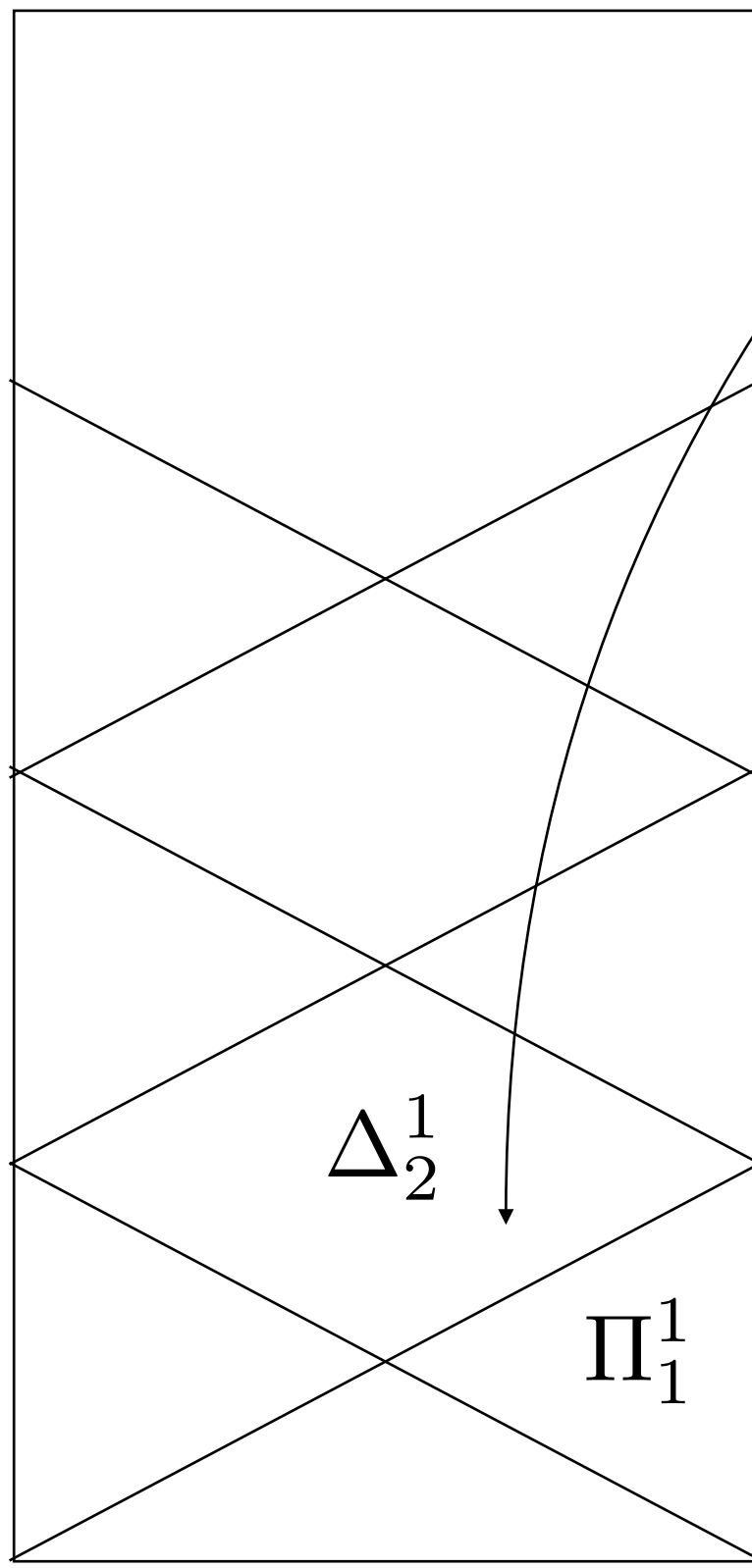
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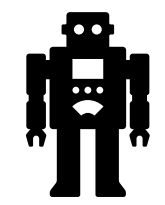


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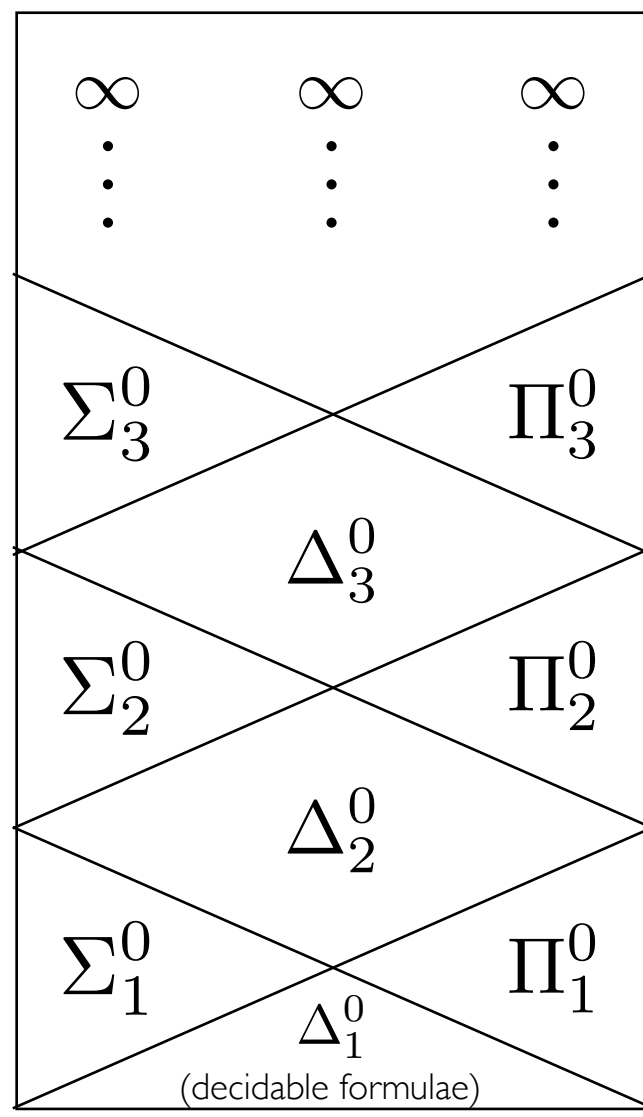


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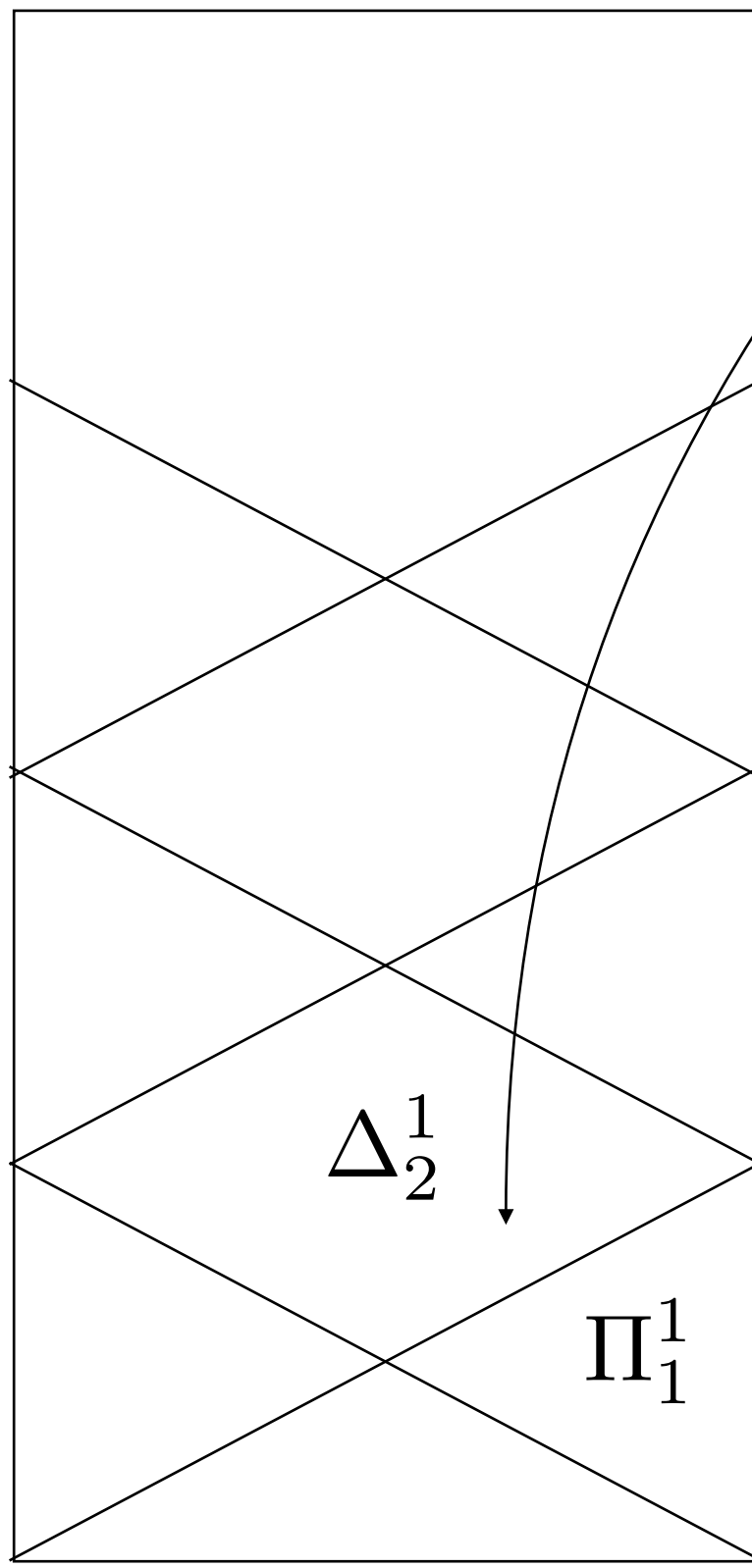


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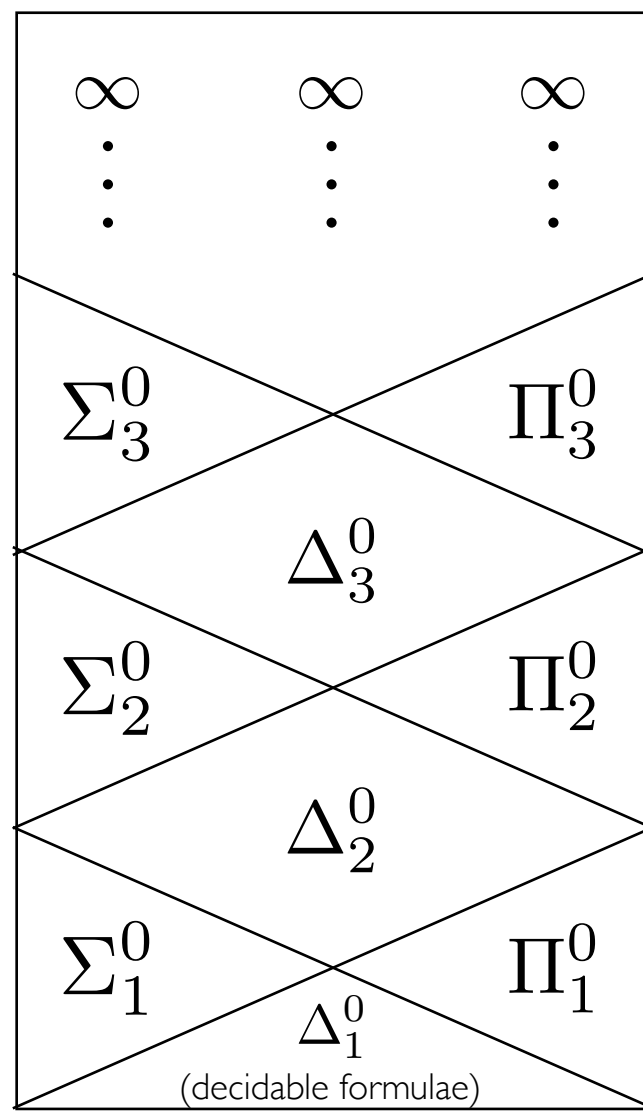
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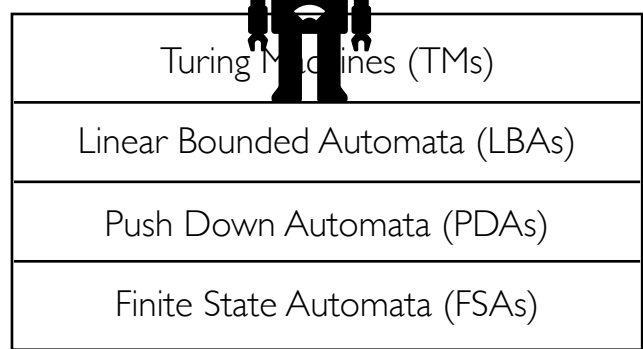


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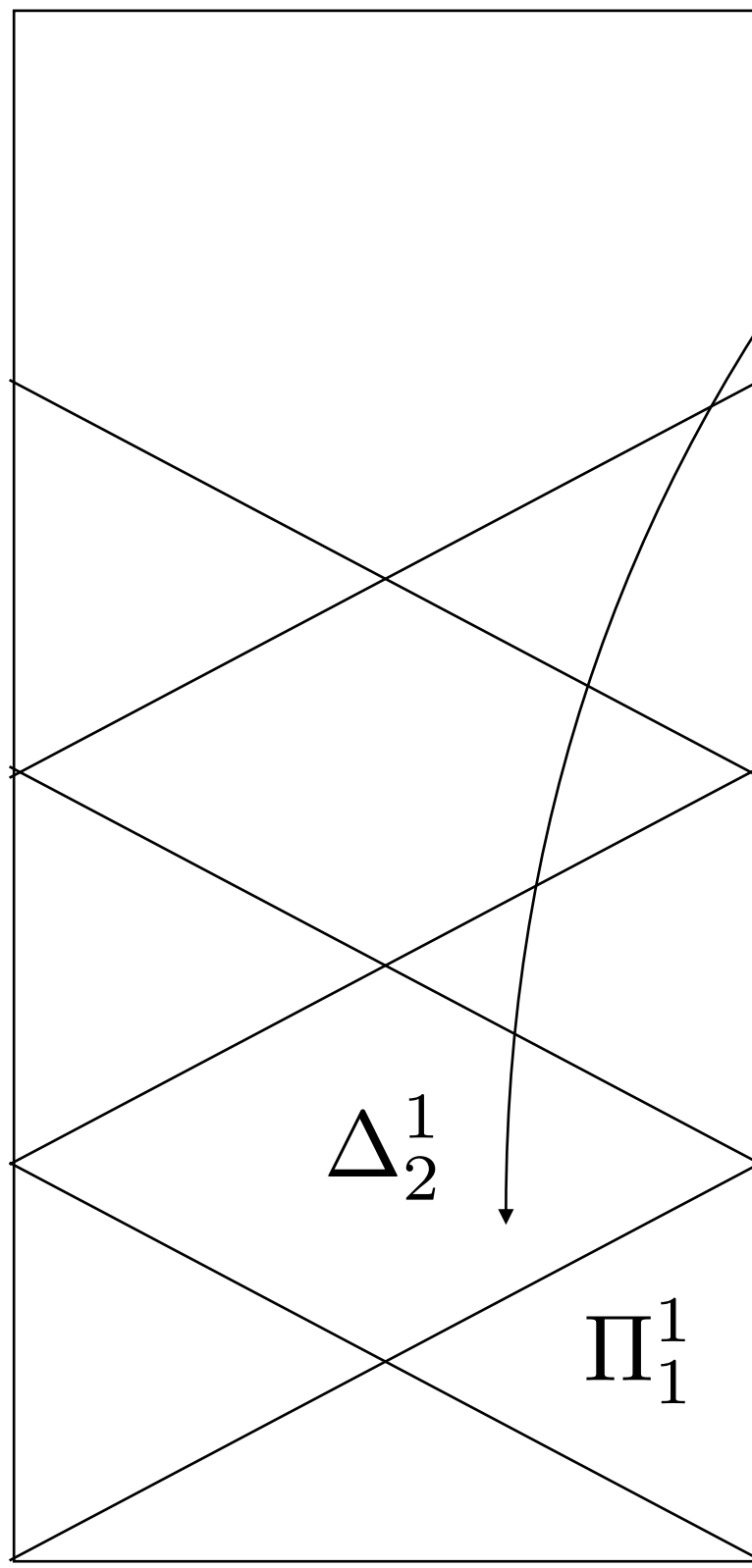


$\mathcal{CH}$ (Computational Hierarchy)



?
 $\mathcal{EM}$

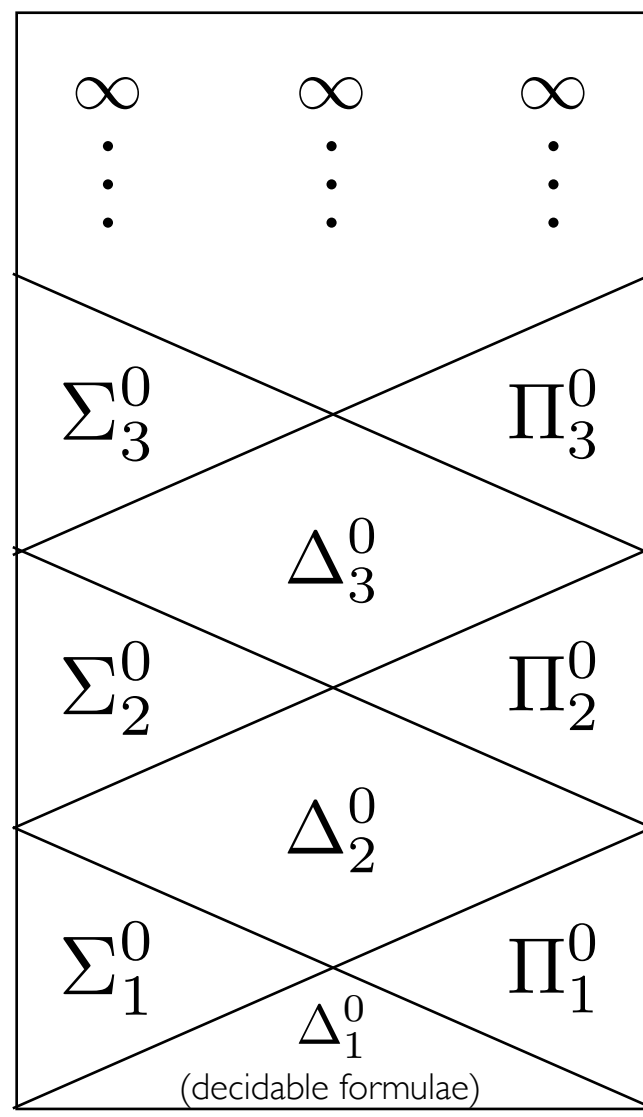
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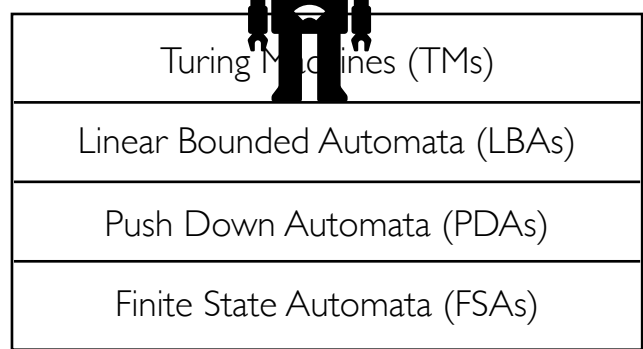


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$\mathcal{CH}$ (Chomsky Hierarchy)



Measuring Intelligence & AI/The Singularity

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The Singularity (superhuman machine intelligence) is near!!

Measuring Intelligence & AI/The Singularity



The Singularity (superhuman machine intelligence) is near!!

Is that so? And how are you measuring intelligence, pray tell?

Measuring Intelligence & AI/The Singularity

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Measuring Intelligence & AI/The Singularity

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Polynomial Hierarchy

Measuring Intelligence & AI/The Singularity

Polynomial Hierarchy

$$P \subseteq NP \subseteq PSPACE = NPSPACE \subseteq EXPTIME \subseteq NEXPTIME \subseteq EXPSPACE$$

Measuring Intelligence & AI/The Singularity

Checkers:Chinook



Polynomial Hierarchy

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Measuring Intelligence & AI/The Singularity

Go:AlphaGo



Polynomial Hierarchy

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Go:AlphaGo



Checkers:Chinook



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Measuring Intelligence & AI/The Singularity

Jeopardy! -



Polynomial Hierarchy

Go:AlphaGo



Checkers:Chinook



$P \subseteq NP \subseteq PSPACE = NPSPACE \subseteq EXPTIME \subseteq NEXPTIME \subseteq EXPSPACE$

Measuring Intelligence & AI/The Singularity

Polynomial Hierarchy

Jeopardy! -



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Arithmetical Hierarchy



Polynomial Hierarchy

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Arithmetical Hierarchy



$\vdots$
 Π_2
 Σ_2
 Π_1
 Σ_1
 Σ_0

Polynomial Hierarchy

Jeopardy! -



Go:AlphaGo



Checkers:Chinook



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Measuring Intelligence & AI/The Singularity

Arithmetical Hierarchy

“Hey, do these two Java programs compute the very same function?”



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Analytical Hierarchy

Arithmetical Hierarchy

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Analytical Hierarchy

Arithmetical Hierarchy

This, all of this, is derived from consideration of first-order logic and second-order logic, with an emphasis on *quantification* and proof.

“Hey, do these two Java programs compute the very same function?”



$\vdots$
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 Σ_2
 Π_1
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Polynomial Hierarchy

Jeopardy! -



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An “Advanced” Topic for Measuring Intelligence ...

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- FOL formulae that (only) enforce domain size:

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ϕ_n

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ϕ_n domain of at least n things

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ϕ_n

domain of at least n things

$\exists x \forall y (y = x)$

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ϕ_n

For now, let's
settle for a focus
on quantification.
Then ...

Measuring AI Intelligence via (in part) Logic:Quantification

Toby Walsh: “The Singularity May Never Be Near”
(<http://arxiv.org/pdf/1602.06462v1.pdf>)

Measuring AI Intelligence via (in part) Logic:Quantification

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“I will not tackle here head on what we mean by measuring the intelligence of machines (or of humans). I will simply suppose there is such a property as intelligence, that it can be measured and compared, and that the technological singularity is when this measure increases exponentially fast in an appropriate and reasonable scale.” (p. 1)

But logico-mathematical definitions of intelligence for animals, humans, machines, aliens, gods ... *are* possible; recall our consideration of the *Entscheidungsproblem*. We can specifically challenge today's AI on the basis of simple quantification and simple deduction ...

First, need some numerical quantifiers:

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$$\exists x \forall y (y = x \wedge \phi(x)) \text{ will be } \exists^{=1} x \phi(x)$$

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$\exists x \exists y \exists z (x \neq y \wedge y \neq z \wedge x \neq z \wedge \phi(x, y, z))$ will be $\exists^{\geq 3} x \phi(x)$

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How do we define formulae of this type: $\exists^{=k} x \psi(x)$

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Okay, now AI:

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Okay, now AI:

At least seven kenspeckle blookers are red.

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How do we define formulae of this type: $\exists^{=k} x \psi(x)$

How do we define formulae of this type: $\exists^{\leq n} x \psi(x)$

$\vdots$

Okay, now AI:

At least seven kenspeckle blookers are red.

Given this, is it true that there are two red blookers? Why, exactly?

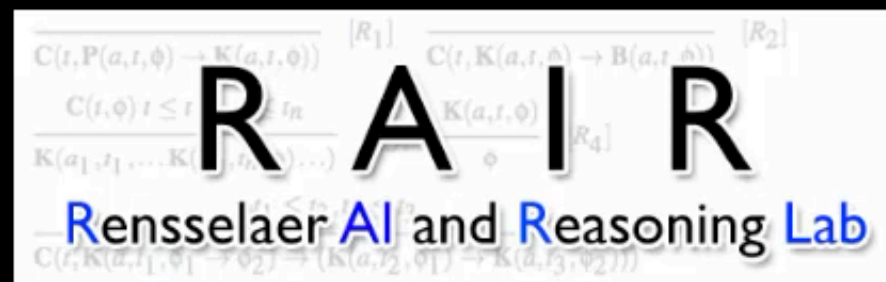
$$\begin{aligned}
& \forall x \forall y \forall z \{ [x \neq y \wedge y \neq z \wedge x \neq z \wedge Cx \wedge Cy \wedge Cz \wedge \\
& \hspace{25em} Tz' \wedge \\
& \exists w_1 \exists w_2 (w_1 \neq w_2 \wedge Aw_1 \wedge Aw_2 \wedge Gz'w_1 \wedge Gz'w_2) \wedge \\
& \forall u_1 \forall u_2 \forall u_3 ((Gz'u_1 \wedge Gz'u_2 \wedge Gz'u_3 \wedge C^b u_1 \wedge C^b u_2 \wedge C^b u_3) \rightarrow \\
& \hspace{10em} \forall v ((Gz'v \wedge C^b v) \rightarrow (v = u_1 \vee v = u_2 \vee v = u_3)))] \\
& \hspace{25em} \rightarrow \\
& \hspace{15em} (Gxz' \wedge Gyz' \wedge Gzz') \}
\end{aligned}$$

Every three cylinders glower at any triangular prism that glowers at at least two arches and at at most three cubes.

$$\forall x \forall y \forall z \forall z' \left\{ \left[\begin{array}{c} x \neq y \wedge y \neq z \wedge x \neq z \\ \wedge \\ Cx \wedge Cy \wedge Cz \\ \wedge \\ Tz' \\ \wedge \\ \exists w_1 \exists w_2 (w_1 \neq w_2 \wedge Aw_1 \wedge Aw_2 \wedge Gz'w_1 \wedge Gz'w_2) \\ \wedge \\ \forall u_1 \forall u_2 \forall u_3 \left(\begin{array}{c} [Gz'u_1 \wedge Gz'u_2 \wedge Gz'u_3 \wedge C^b u_1 \wedge C^b u_2 \wedge C^b u_3] \\ \rightarrow \\ \forall v [(Gz'v \wedge C^b v) \rightarrow (v = u_1 \vee v = u_2 \vee v = u_3)] \end{array} \right) \\ \rightarrow \\ (Gxz' \wedge Gyz' \wedge Gzz') \end{array} \right] \right\}$$

Intelligent Artificial Multi-Agent

Tentacular AI™ AI
at Work in Problem-Solving in VQ⁺AJV



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