Propositional Calculus II:

Two more Rules of Inference/Inference Schemata

(`conditional elim` = *modus ponens;*
proof by cases = *disjunction elimination*),
Applying Them to Additional Motivating Problems

Selmer Bringsjord

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Rensselaer Polytechnic Institute (RPI)
Troy, New York 12180 USA

Intro to Logic
2/11/2021

R A I R
Rensselaer AI and Reasoning Lab

Logistics …

- Distribution to students eg in other countries, according to Follett, started as planned (see syllabus) last week after class.

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- Bookstore says it has been good to go for in-person pickup since Feb 8, as in syllabus.

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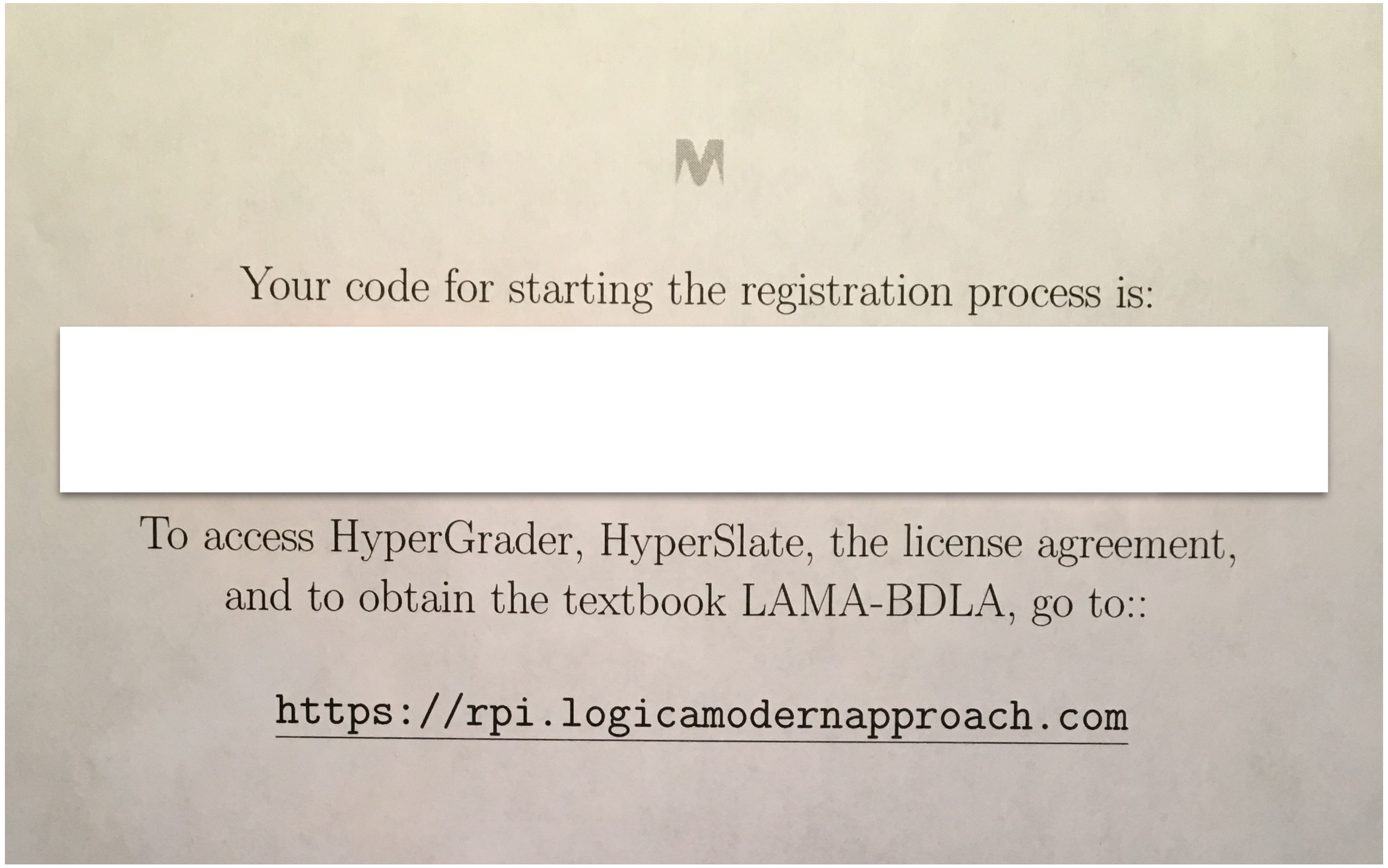
    - “Students with processed orders can pick them up at the back of the store.”

- Distribution to students eg in other countries, according to Follett, started as planned (see syllabus) last week after class.
- Bookstore says it has been good to go for in-person pickup since Feb 8, as in syllabus.
  - "Students with processed orders can pick them up at the back of the store."
  - "Students waiting until we open can go to the front registers."

The Starting Code to Purchase in Bookstore



Your code for starting the registration process is:

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https://rpi.logicamodernapproach.com

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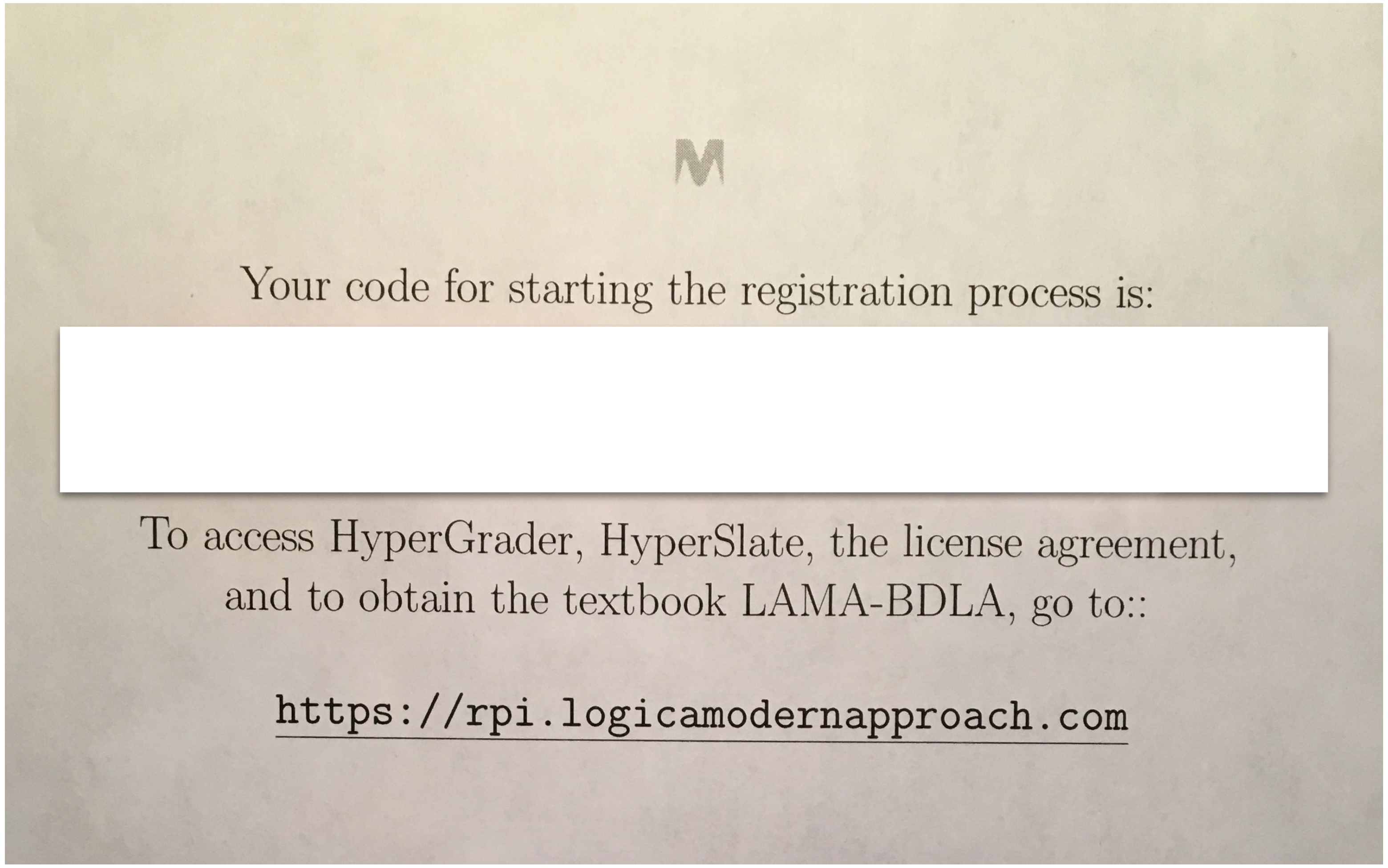
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Once seal broken on envelope, no return. Remember from first class, any reservations, opt for "Stanford" paradigm, with its software instead of LAMA® paradigm!

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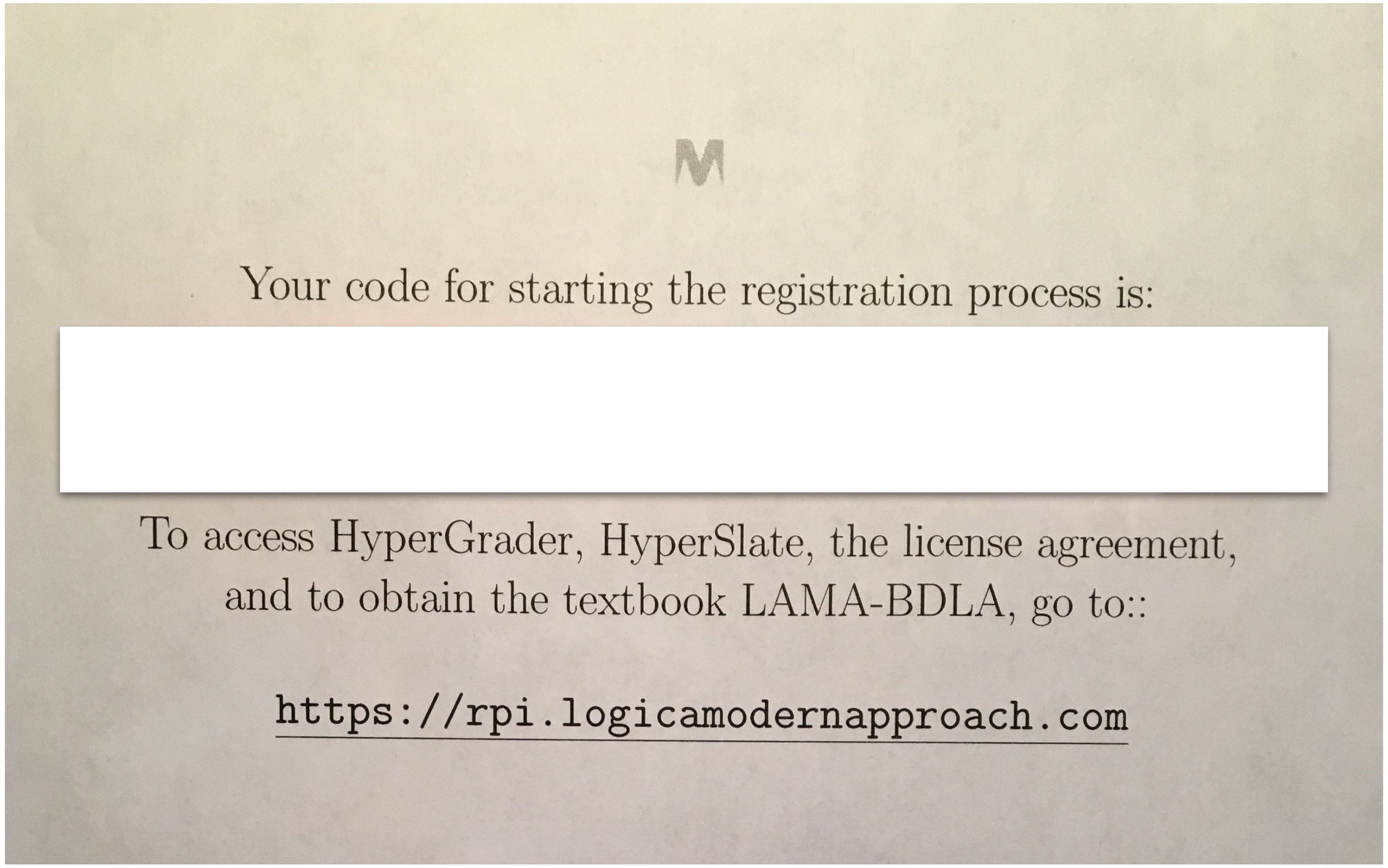
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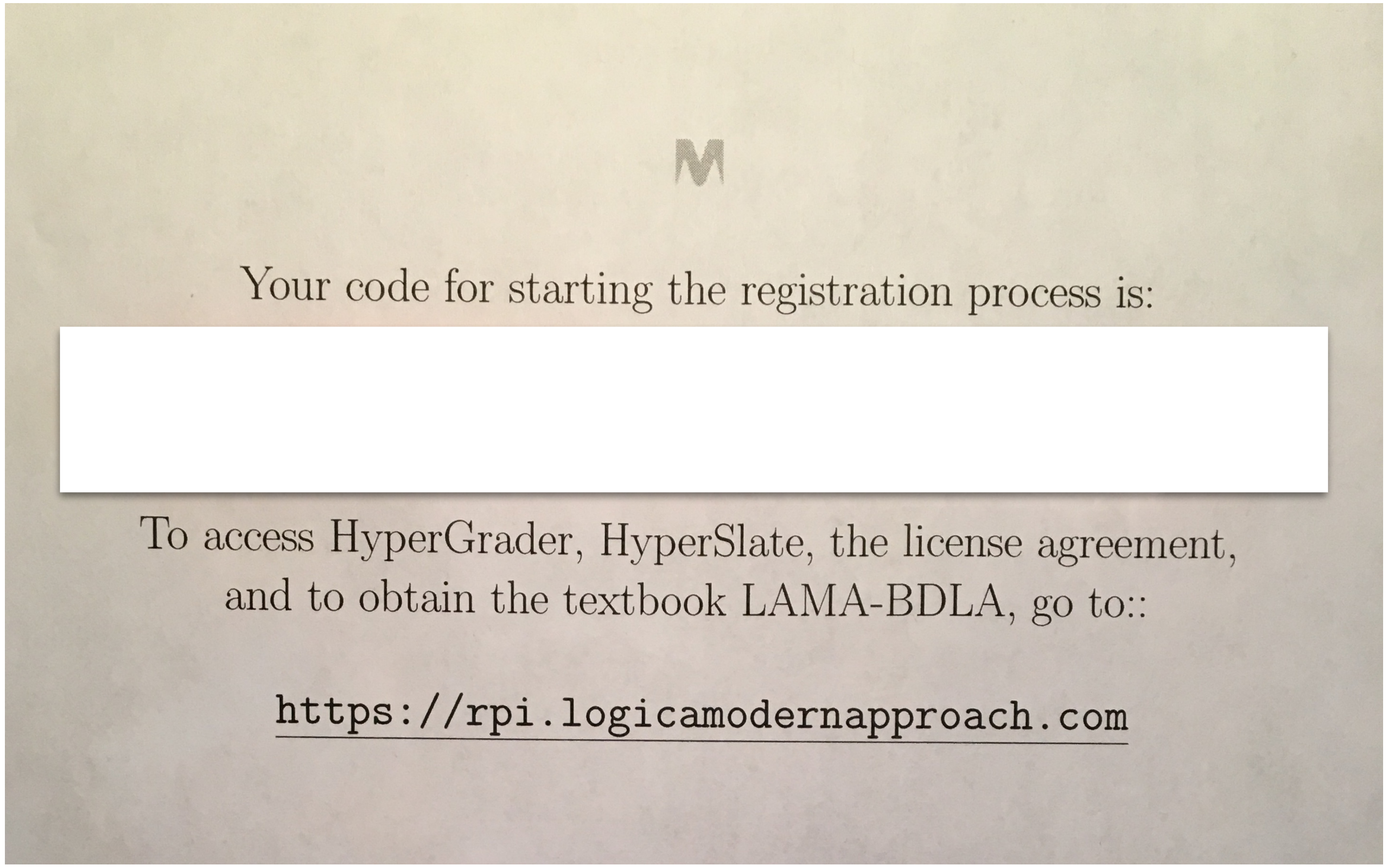

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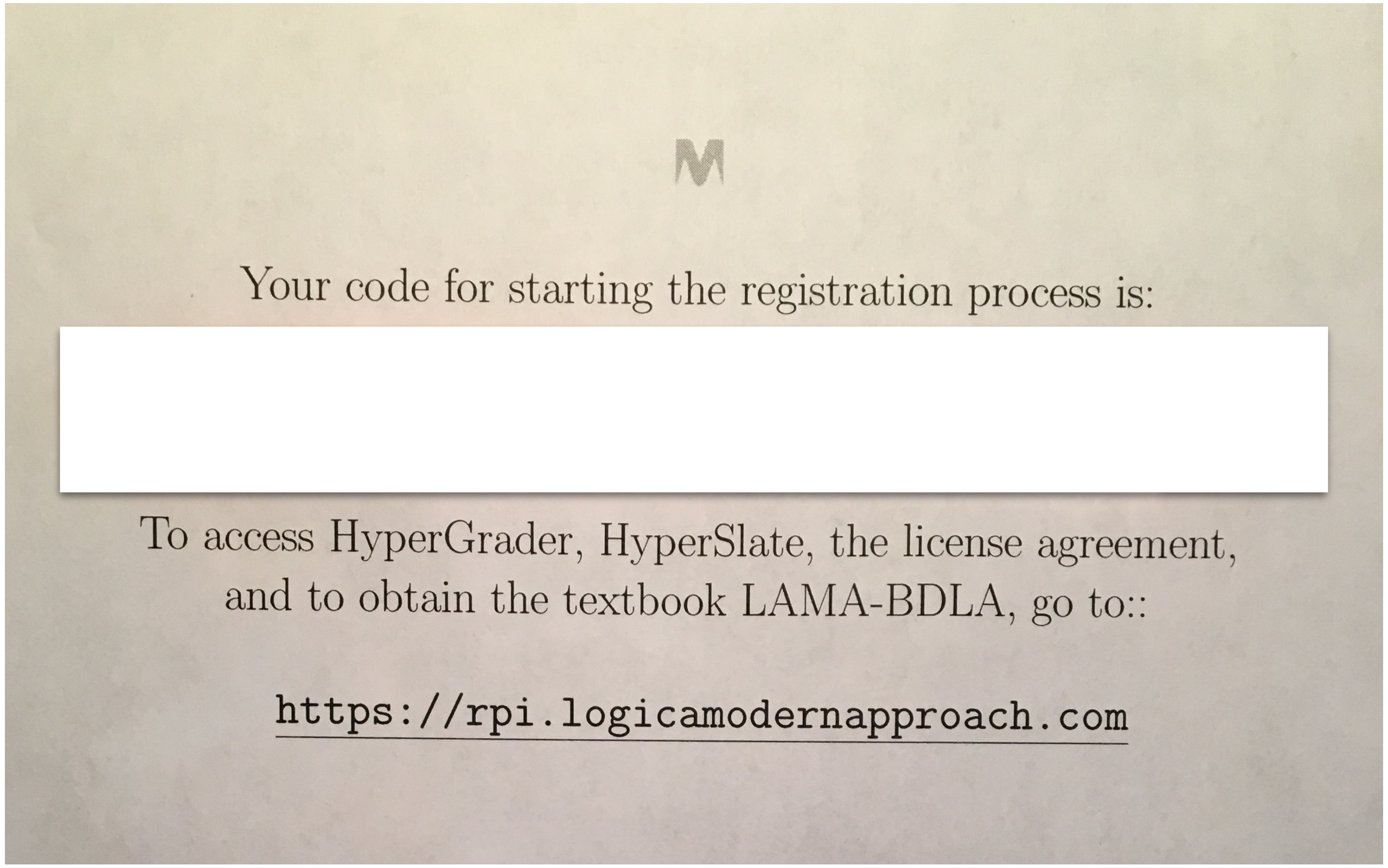

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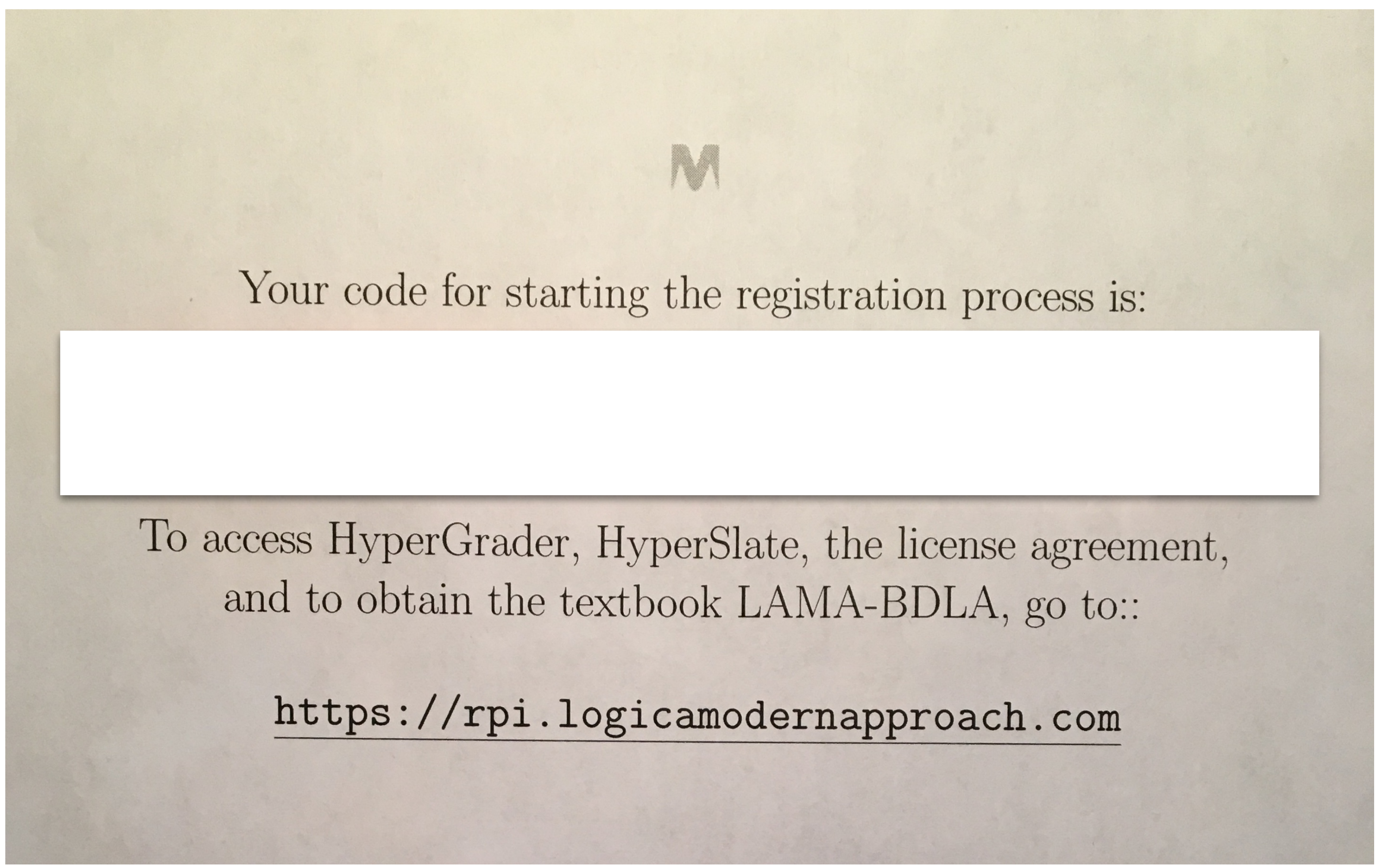
Watch that the link emailed to you doesn't end up being classified as spam.

The Starting Code Purchased in Bookstore Should By Now've Been/Soon Be Used to Register & Subsequently Sign In

First prop. calc. (Exercise) Problem: `switching_conjuncts_fine`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E-Housekeeping Pts, Redux

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E-Housekeeping Pts, Redux

- Must input your RIN. (This is your ID at your university.)
- Make sure OS fully up-to-date.
- Make sure browser fully up-to-date.
- Chrome best (but I use Safari).
- Always work in the same browser window with multiple tabs; must do this with email and HyperGrader® & HyperSlate®.

Introduction to (Formal) Logic (and AI)

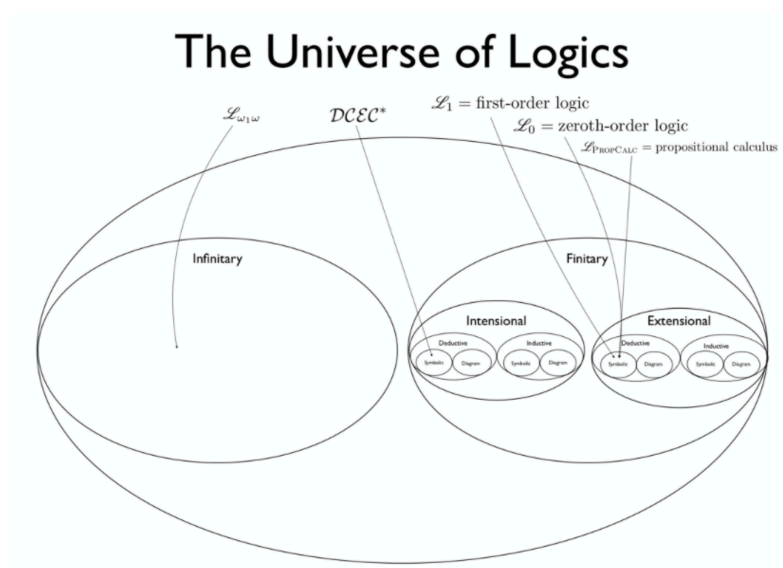
Spring 2021 edition of IFLAI1

Selmer Bringsjord

Table of Contents

A fully online course, thanks to singular AI technology.

with Naveen Sundar G.
∧ KB Foushée ∧ Joshua Taylor ∧ …



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RAIR
Rensselaer AI and Reasoning Lab

Last time we introduced and and lauded the power of **oracles**, and questions … and now … picking up where we left off …

“NYS 3” Revisited

Given the statements

$\neg\neg c$

$c \rightarrow a$

$\neg a \vee b$

$b \rightarrow d$

$\neg(d \vee e)$

which one of the following statements must also be true?

$\neg c$

e

h

$\neg a$

all of the above

"NYS 3" Revisited

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“NYS 3” Revisited

Given the statements

$\neg\neg c$

$c \rightarrow a$

$\neg a \vee b$

$b \rightarrow d$

$\neg(d \vee e)$

After last class, should have explored if you are registered … Show in HyperSlate® as I did that each of the first four options can be proved using the PC entailment (= provability) oracle.

which one of the following statements must also be true?

$\neg c$

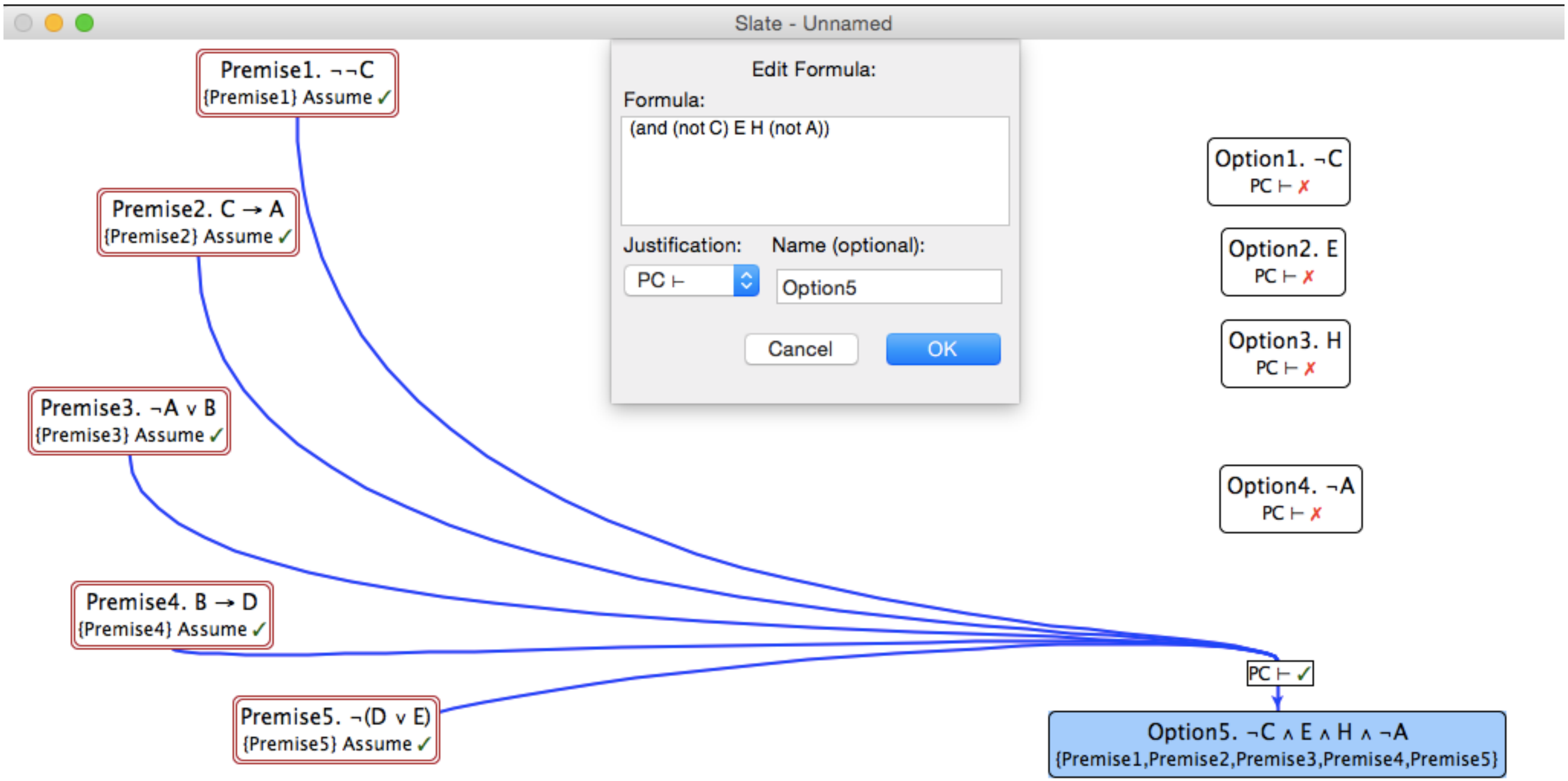
e

h

$\neg a$

all of the above

(and (not C) E H (not A))



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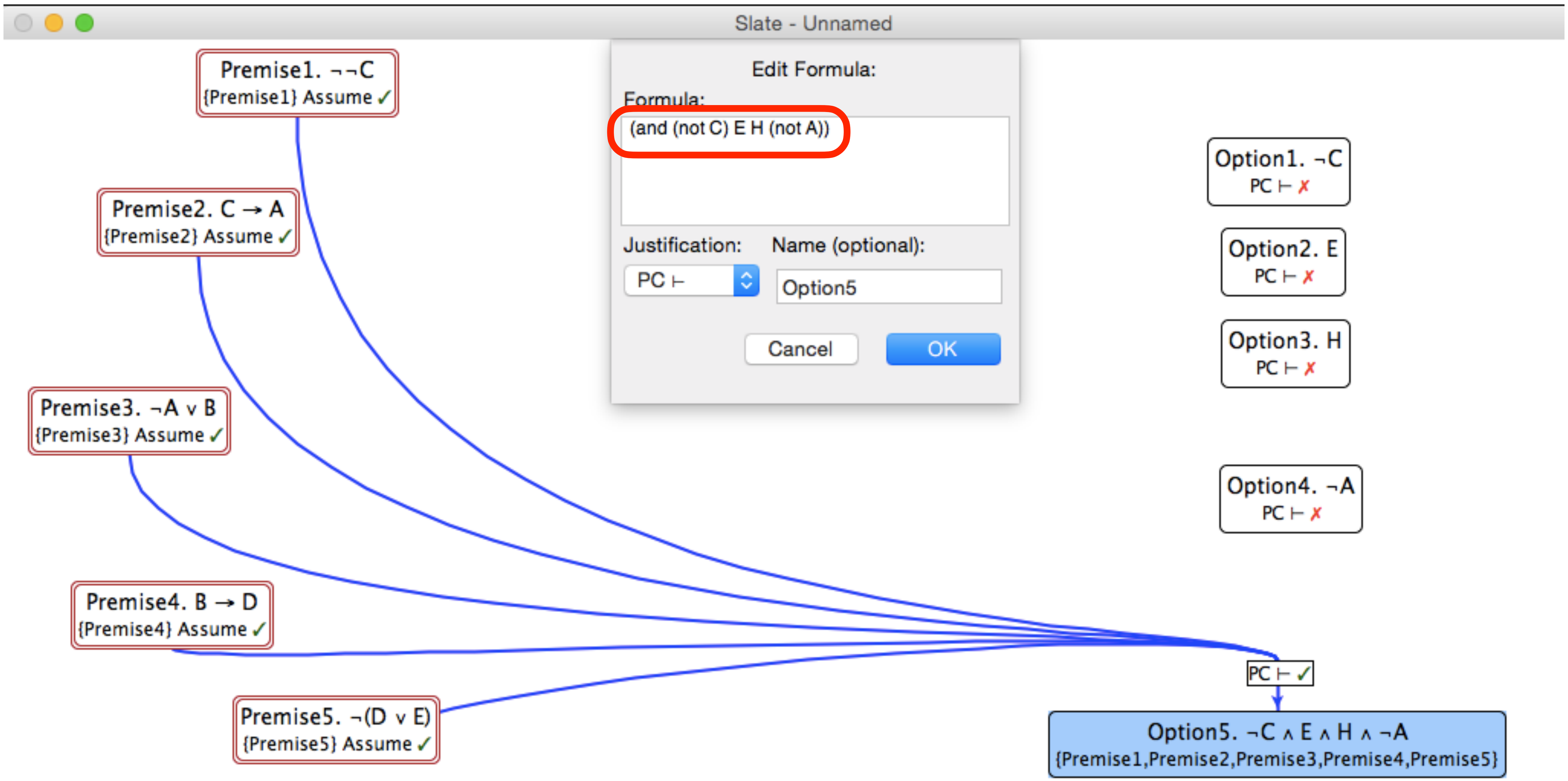

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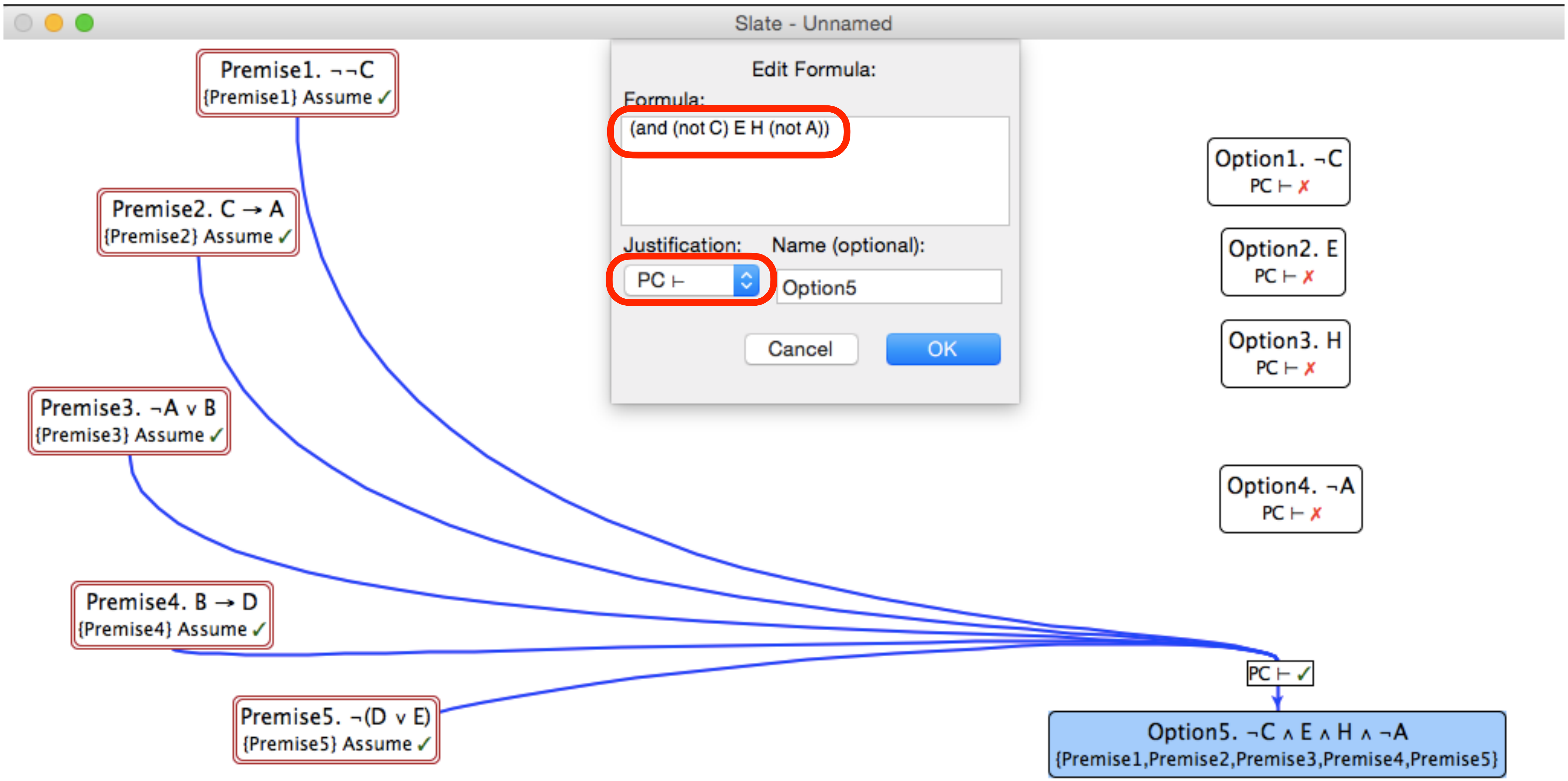

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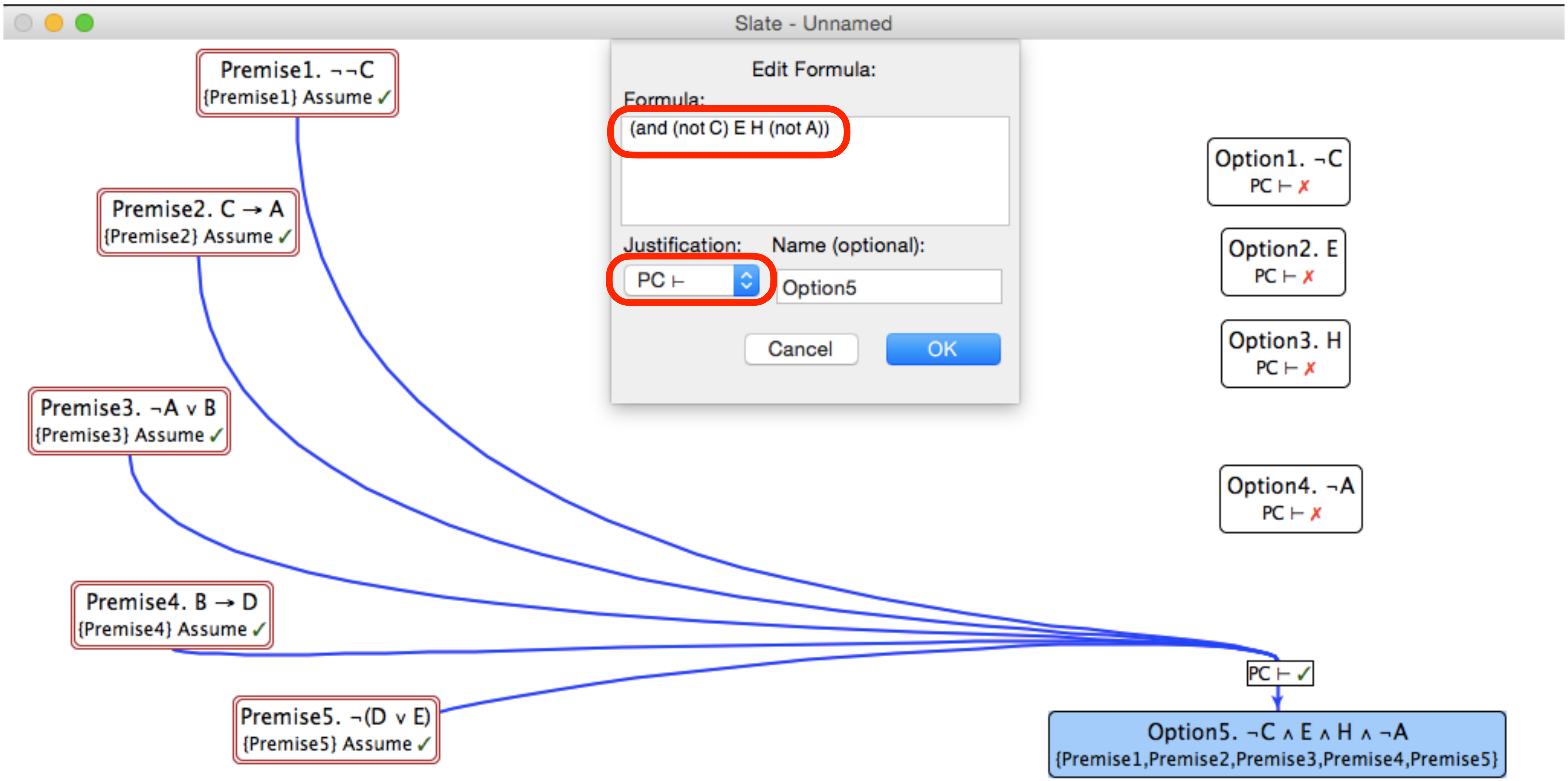

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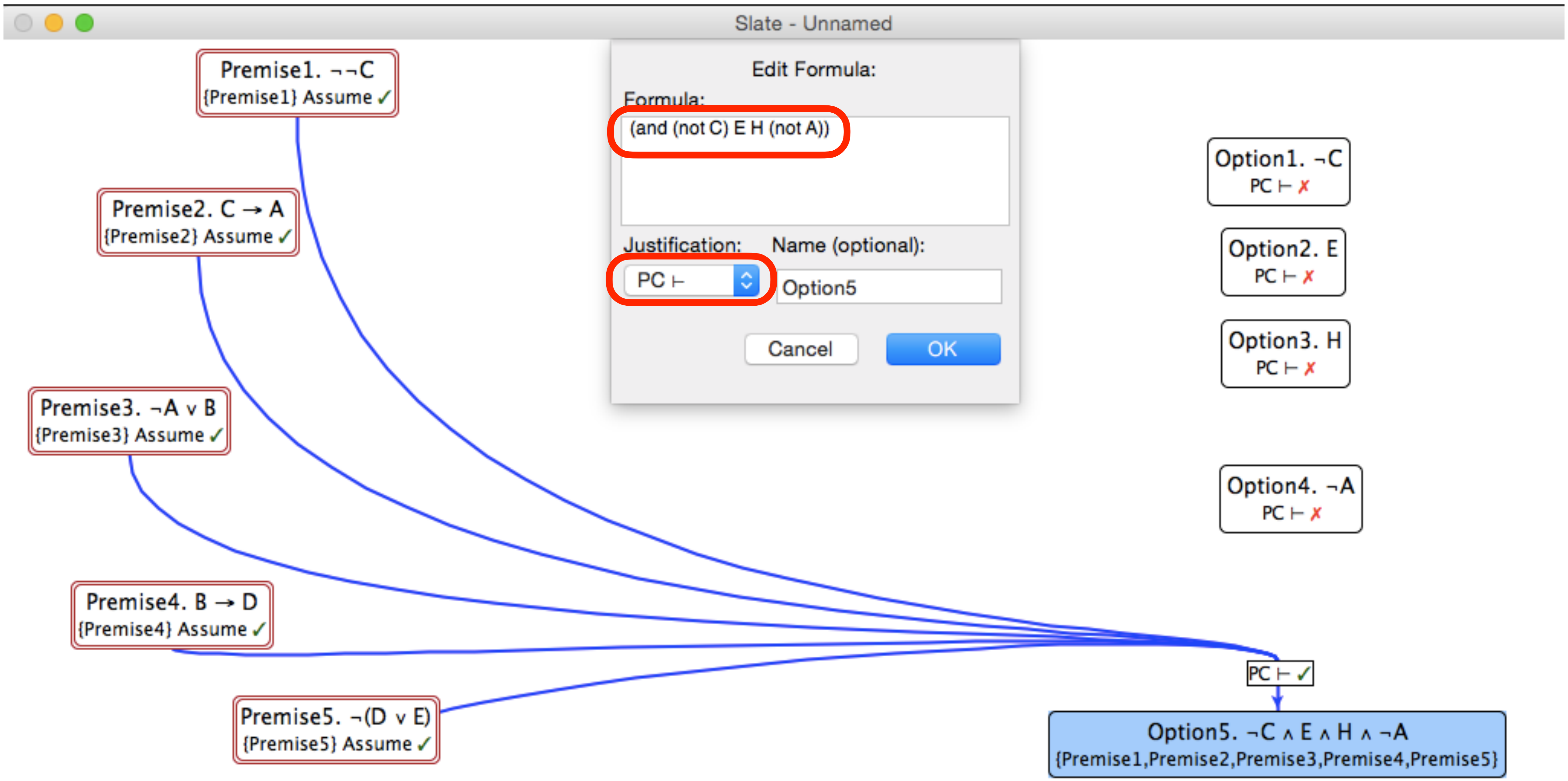

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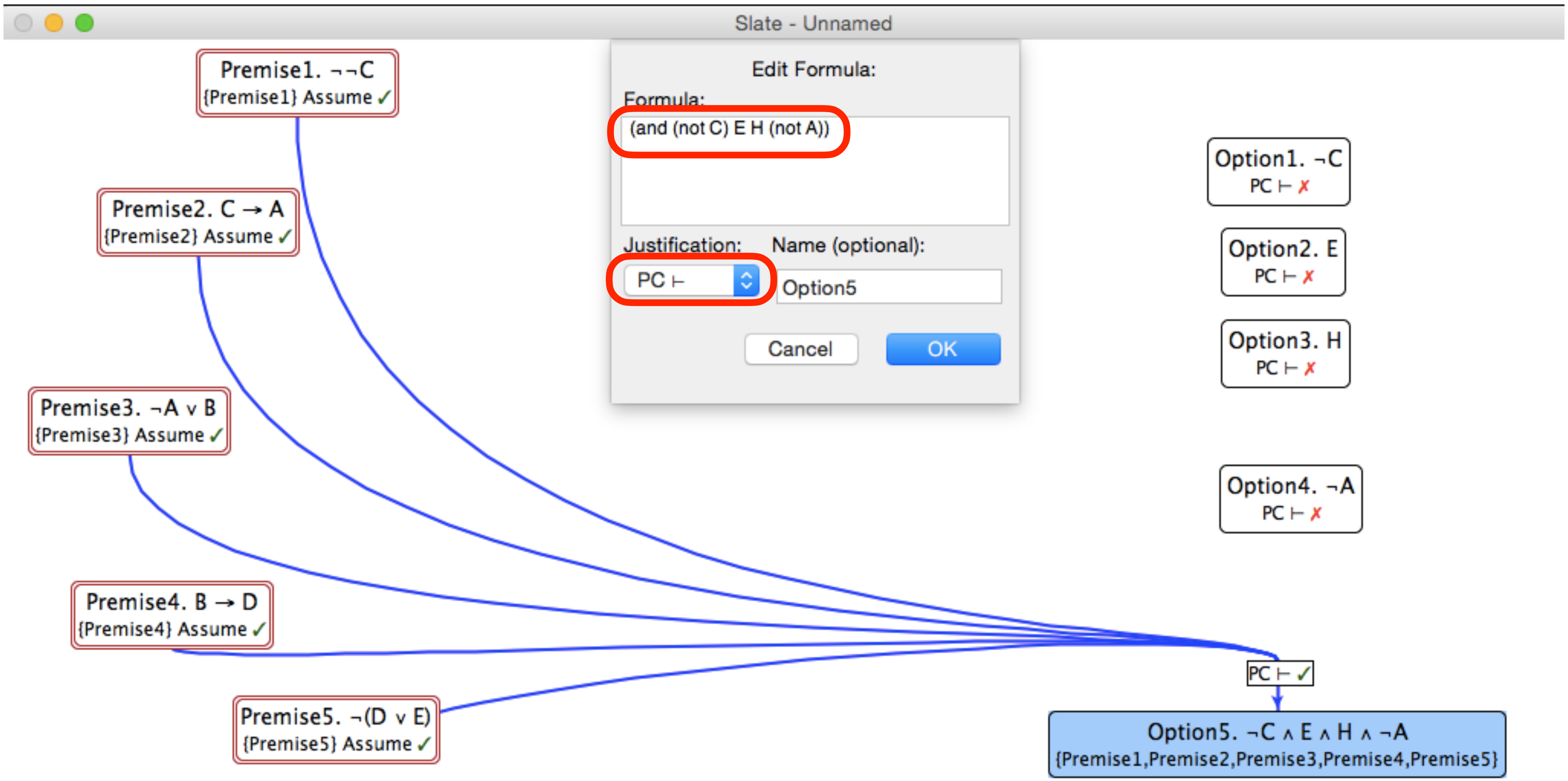

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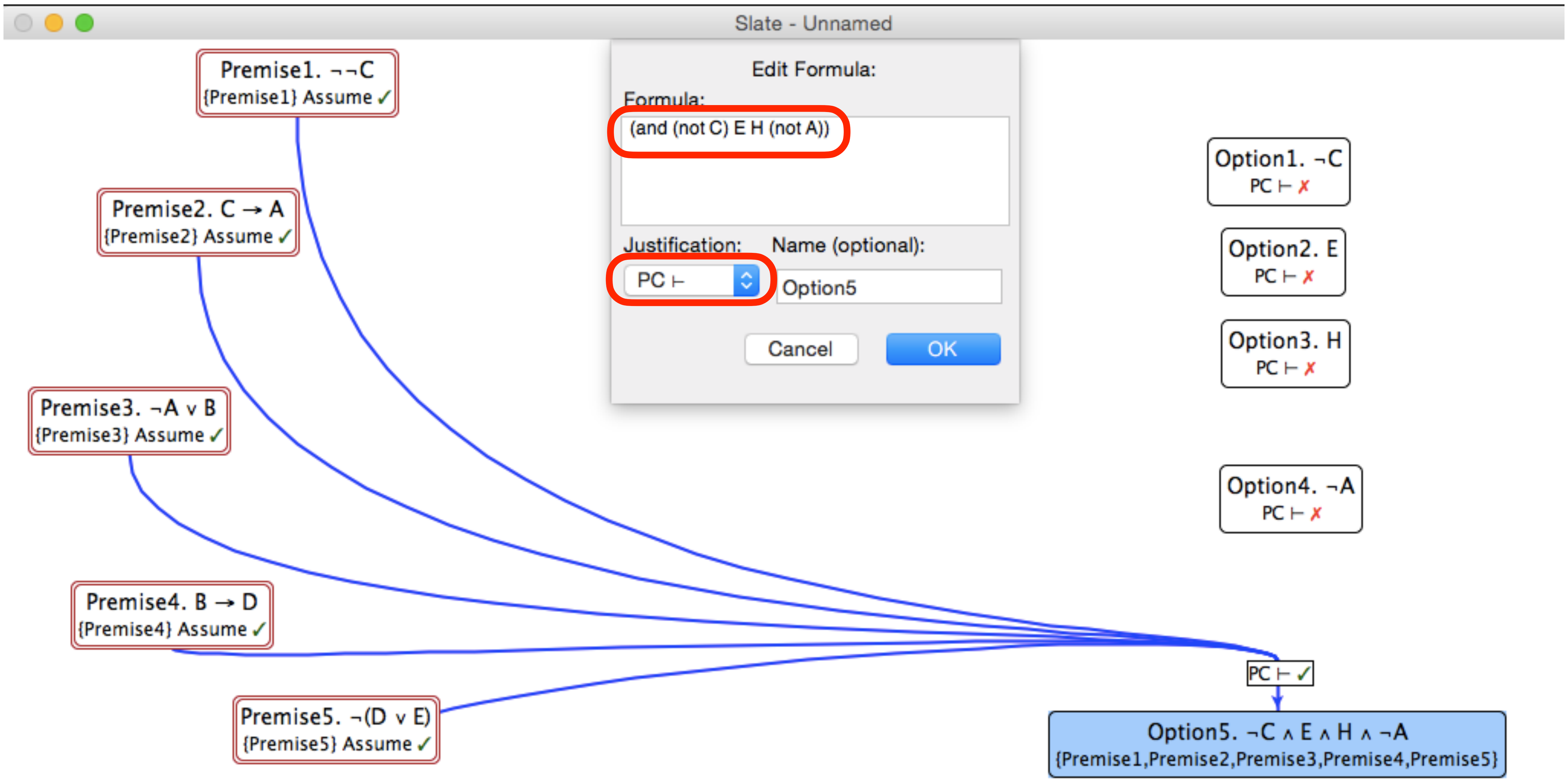

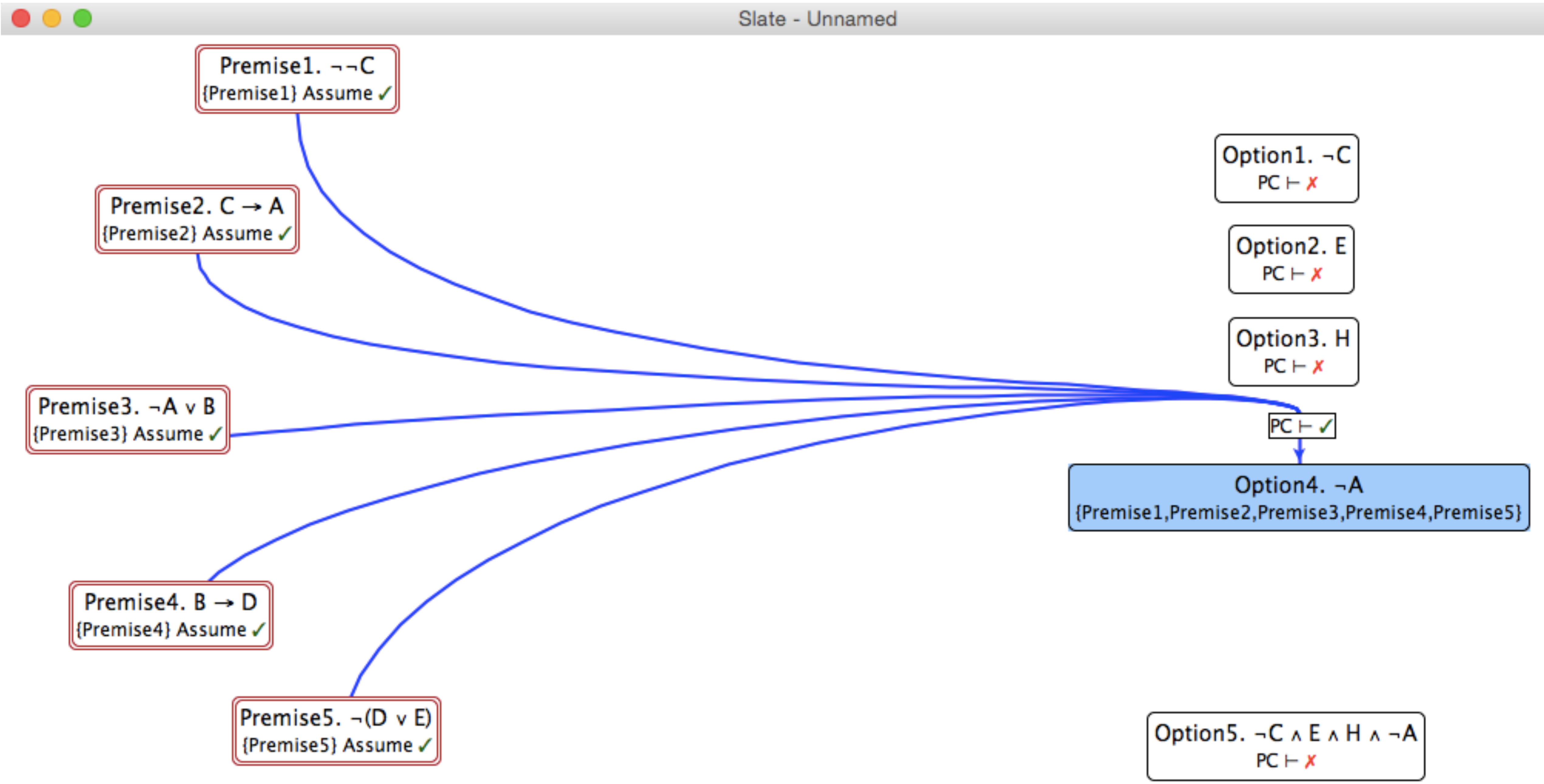

Slate - Unnamed
Premise1. ¬¬C
{Premise1} Assume ✓
Premise2. C → A
{Premise2} Assume ✓
Premise3. ¬A ∨ B
{Premise3} Assume ✓
Premise4. B → D
{Premise4} Assume ✓
Premise5. ¬(D ∨ E)
{Premise5} Assume ✓
Option1. ¬C
PC ⊢ ✗
Option2. E
PC ⊢ ✗
Option3. H
PC ⊢ ✗
PC ⊢ ✓
Option4. ¬A
{Premise1,Premise2,Premise3,Premise4,Premise5}
Option5. ¬C ∧ E ∧ H ∧ ¬A
PC ⊢ ✗

Proof Plan …

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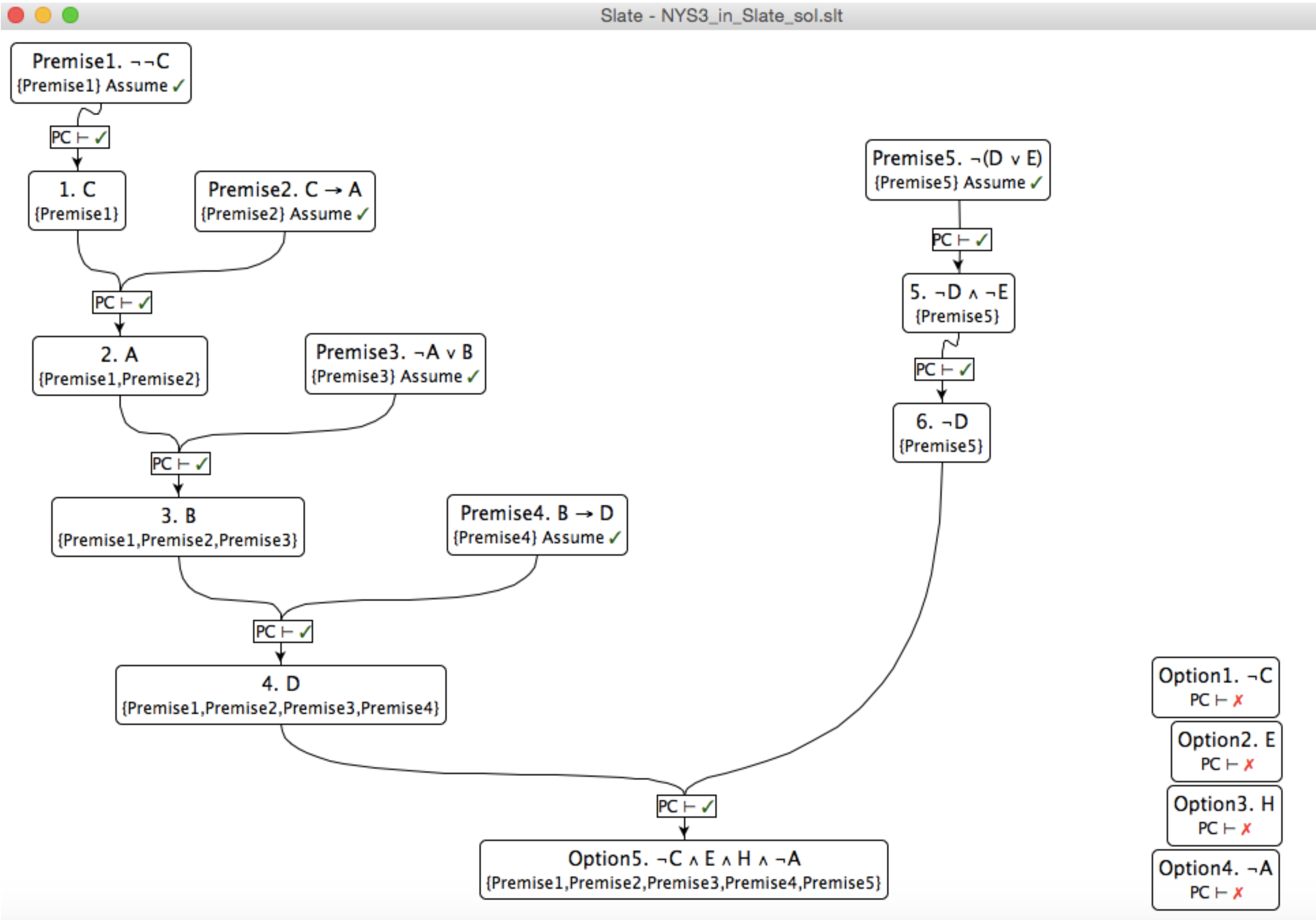
Proof Plan …



Slate - NYS3_in_Slate_sol.slt
Premise1. ¬¬C
{Premise1} Assume ✓
PC ⊢ ✓
1. C
{Premise1}
Premise2. C → A
{Premise2} Assume ✓
PC ⊢ ✓
2. A
{Premise1,Premise2}
Premise3. ¬A ∨ B
{Premise3} Assume ✓
PC ⊢ ✓
3. B
{Premise1,Premise2,Premise3}
Premise4. B → D
{Premise4} Assume ✓
PC ⊢ ✓
4. D
{Premise1,Premise2,Premise3,Premise4}
Premise5. ¬(D ∨ E)
{Premise5} Assume ✓
PC ⊢ ✓
5. ¬D ∧ ¬E
{Premise5}
PC ⊢ ✓
6. ¬D
{Premise5}
PC ⊢ ✓
Option5. ¬C ∧ E ∧ H ∧ ¬A
{Premise1,Premise2,Premise3,Premise4,Premise5}
Option1. ¬C
PC ⊢ ✗
Option2. E
PC ⊢ ✗
Option3. H
PC ⊢ ✗
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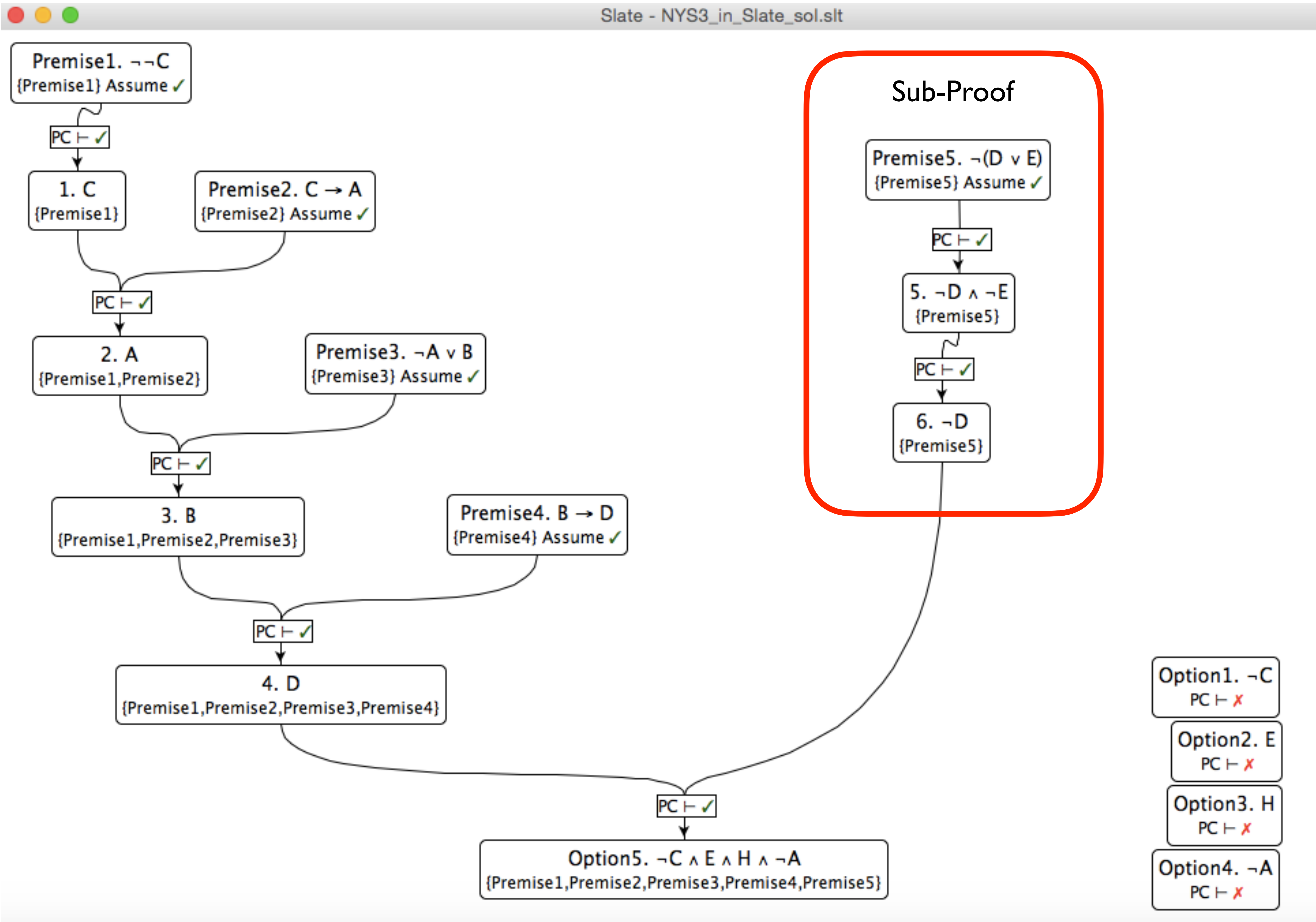
PC ⊢ ✗

Proof Plan …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Premise3. ¬A ∨ B
{Premise3} Assume ✓
PC ⊢ ✓
3. B
{Premise1,Premise2,Premise3}
Premise4. B → D
{Premise4} Assume ✓
PC ⊢ ✓
4. D
{Premise1,Premise2,Premise3,Premise4}
Sub-Proof
Premise5. ¬(D ∨ E)
{Premise5} Assume ✓
PC ⊢ ✓
5. ¬D ∧ ¬E
{Premise5}
PC ⊢ ✓
6. ¬D
{Premise5}
PC ⊢ ✓
Option5. ¬C ∧ E ∧ H ∧ ¬A
{Premise1,Premise2,Premise3,Premise4,Premise5}
Option1. ¬C
PC ⊢ ✗
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PC ⊢ ✗
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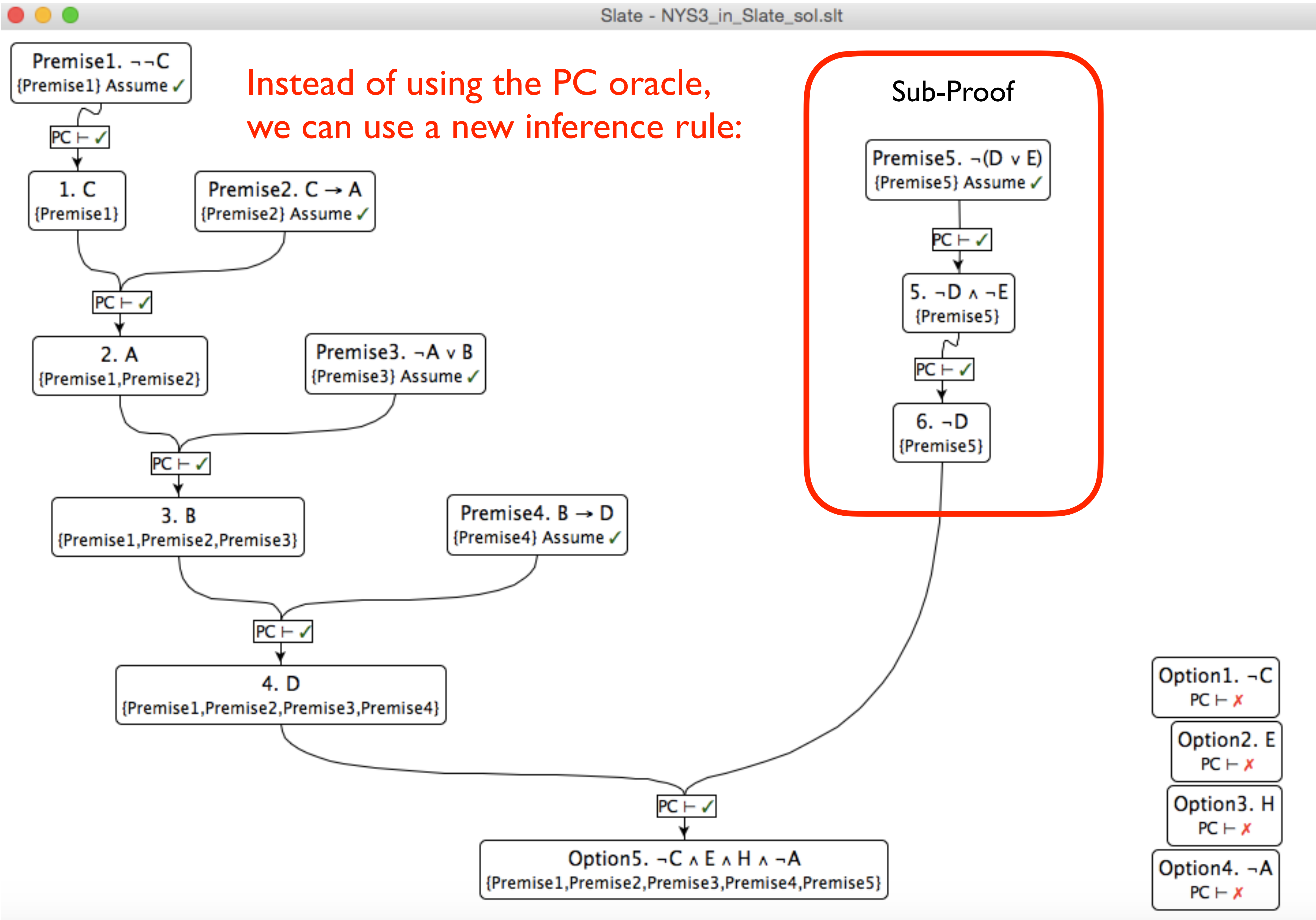
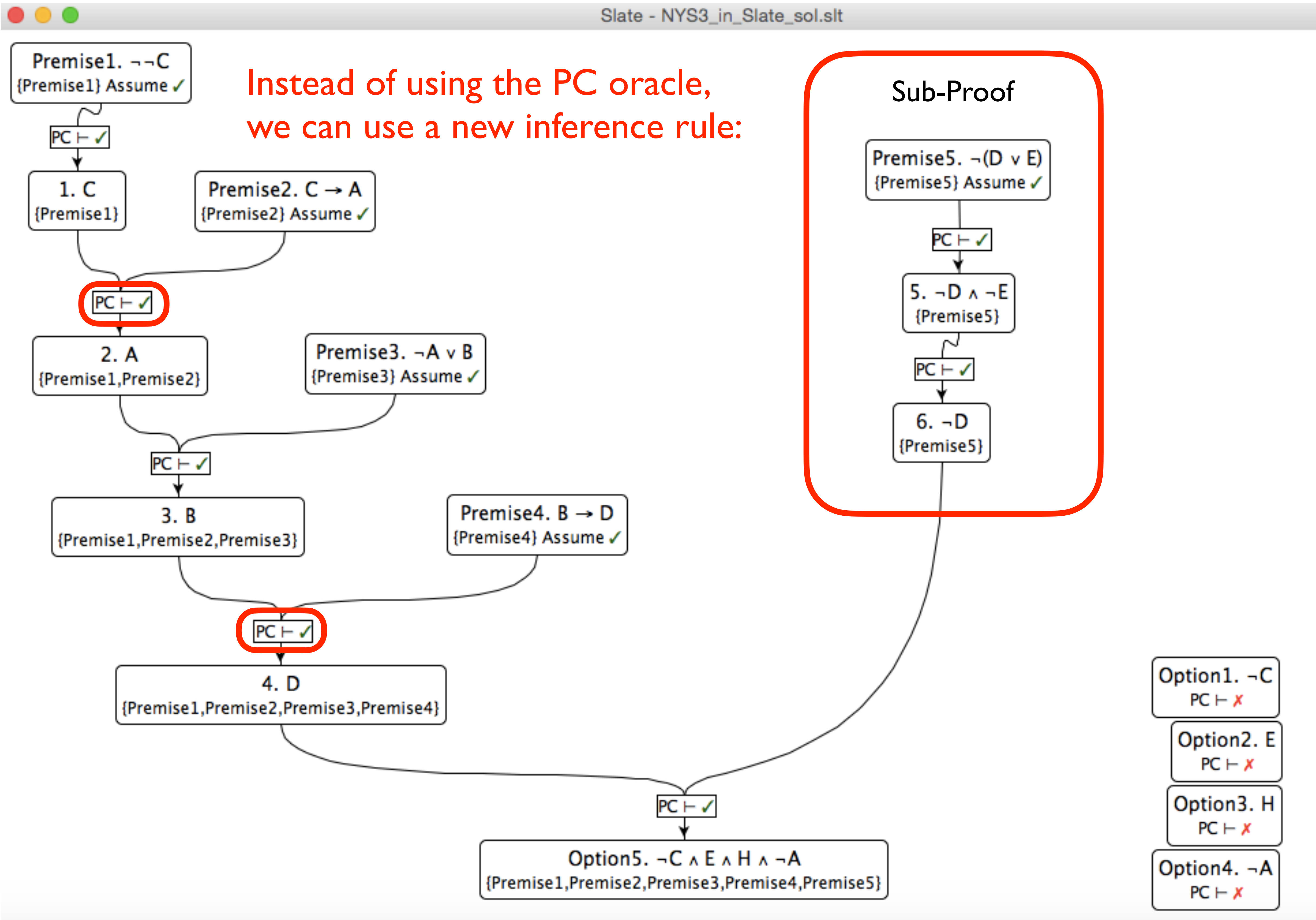
PC ⊢ ✗
Option4. ¬A
PC ⊢ ✗

Proof Plan …



Slate - NYS3_in_Slate_sol.slt
Premise1. ¬¬C
{Premise1} Assume ✓
PC ⊢ ✓
1. C
{Premise1}
Premise2. C → A
{Premise2} Assume ✓
PC ⊢ ✓
2. A
{Premise1,Premise2}
Premise3. ¬A ∨ B
{Premise3} Assume ✓
PC ⊢ ✓
3. B
{Premise1,Premise2,Premise3}
Premise4. B → D
{Premise4} Assume ✓
PC ⊢ ✓
4. D
{Premise1,Premise2,Premise3,Premise4}
Instead of using the PC oracle,
we can use a new inference rule:
Sub-Proof
Premise5. ¬(D ∨ E)
{Premise5} Assume ✓
PC ⊢ ✓
5. ¬D ∧ ¬E
{Premise5}
PC ⊢ ✓
6. ¬D
{Premise5}
PC ⊢ ✓
Option5. ¬C ∧ E ∧ H ∧ ¬A
{Premise1,Premise2,Premise3,Premise4,Premise5}
Option1. ¬C
PC ⊢ ✗
Option2. E
PC ⊢ ✗
Option3. H
PC ⊢ ✗
Option4. ¬A
PC ⊢ ✗

Proof Plan …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Premise4. B → D
{Premise4} Assume ✓
PC ⊢ ✓
4. D
{Premise1,Premise2,Premise3,Premise4}
Sub-Proof
Premise5. ¬(D ∨ E)
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PC ⊢ ✓
5. ¬D ∧ ¬E
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PC ⊢ ✓
6. ¬D
{Premise5}
PC ⊢ ✓
Option5. ¬C ∧ E ∧ H ∧ ¬A
{Premise1,Premise2,Premise3,Premise4,Premise5}
Option1. ¬C
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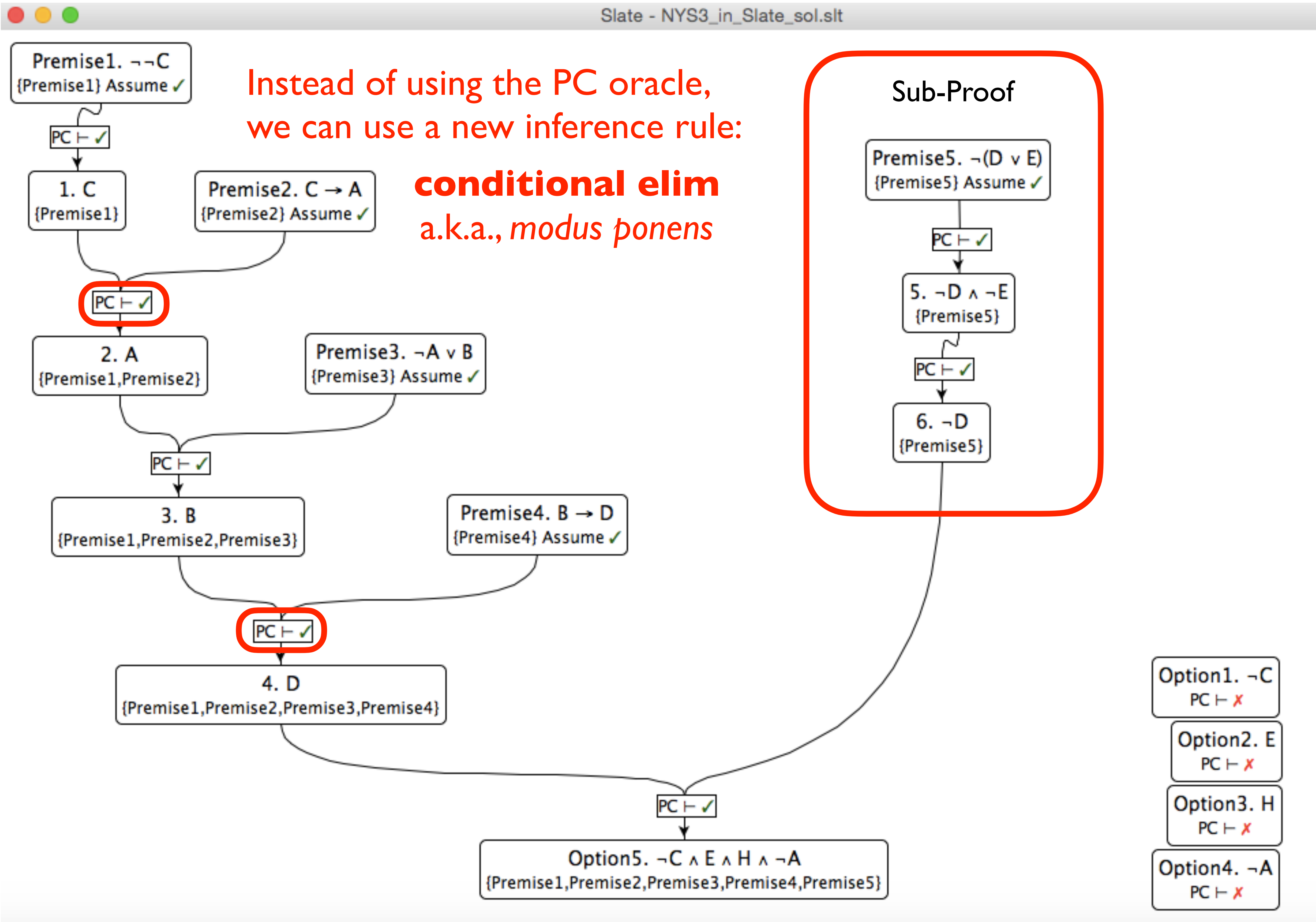
Option2. E
PC ⊢ ✗
Option3. H
PC ⊢ ✗
Option4. ¬A
PC ⊢ ✗

Proof Plan …



Slate - NYS3_in_Slate_sol.slt
Premise1. ¬¬C
{Premise1} Assume ✓
PC ⊢ ✓
1. C
{Premise1}
Premise2. C → A
{Premise2} Assume ✓
Instead of using the PC oracle,
we can use a new inference rule:
conditional elim
a.k.a., *modus ponens*
PC ⊢ ✓
2. A
{Premise1,Premise2}
Premise3. ¬A ∨ B
{Premise3} Assume ✓
PC ⊢ ✓
3. B
{Premise1,Premise2,Premise3}
Premise4. B → D
{Premise4} Assume ✓
PC ⊢ ✓
4. D
{Premise1,Premise2,Premise3,Premise4}
Sub-Proof
Premise5. ¬(D ∨ E)
{Premise5} Assume ✓
PC ⊢ ✓
5. ¬D ∧ ¬E
{Premise5}
PC ⊢ ✓
6. ¬D
{Premise5}
PC ⊢ ✓
Option5. ¬C ∧ E ∧ H ∧ ¬A
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Option1. ¬C
PC ⊢ ✗
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Option3. H
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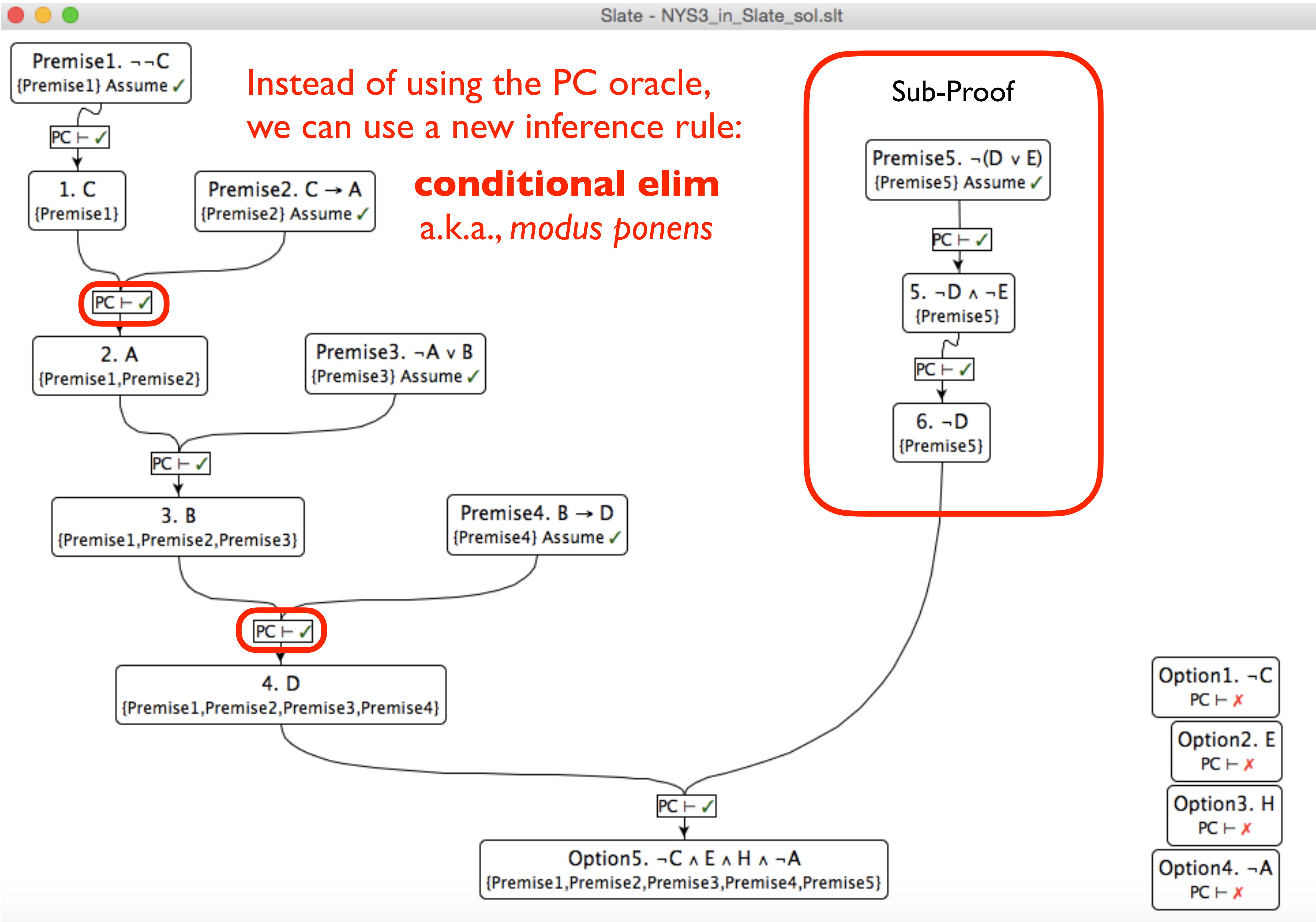
Proof Plan …



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Instead of using the PC oracle, we can use a new inference rule:
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Premise1. ¬¬C {Premise1} Assume ✓
PC ⊢ ✓
1. C {Premise1}
Premise2. C → A {Premise2} Assume ✓
2. A {Premise1,Premise2}
Premise3. ¬A ∨ B {Premise3} Assume ✓
3. B {Premise1,Premise2,Premise3}
Premise4. B → D {Premise4} Assume ✓
4. D {Premise1,Premise2,Premise3,Premise4}
Premise5. ¬(D ∨ E) {Premise5} Assume ✓
5. ¬D ∧ ¬E {Premise5}
6. ¬D {Premise5}
Option5. ¬C ∧ E ∧ H ∧ ¬A {Premise1,Premise2,Premise3,Premise4,Premise5}
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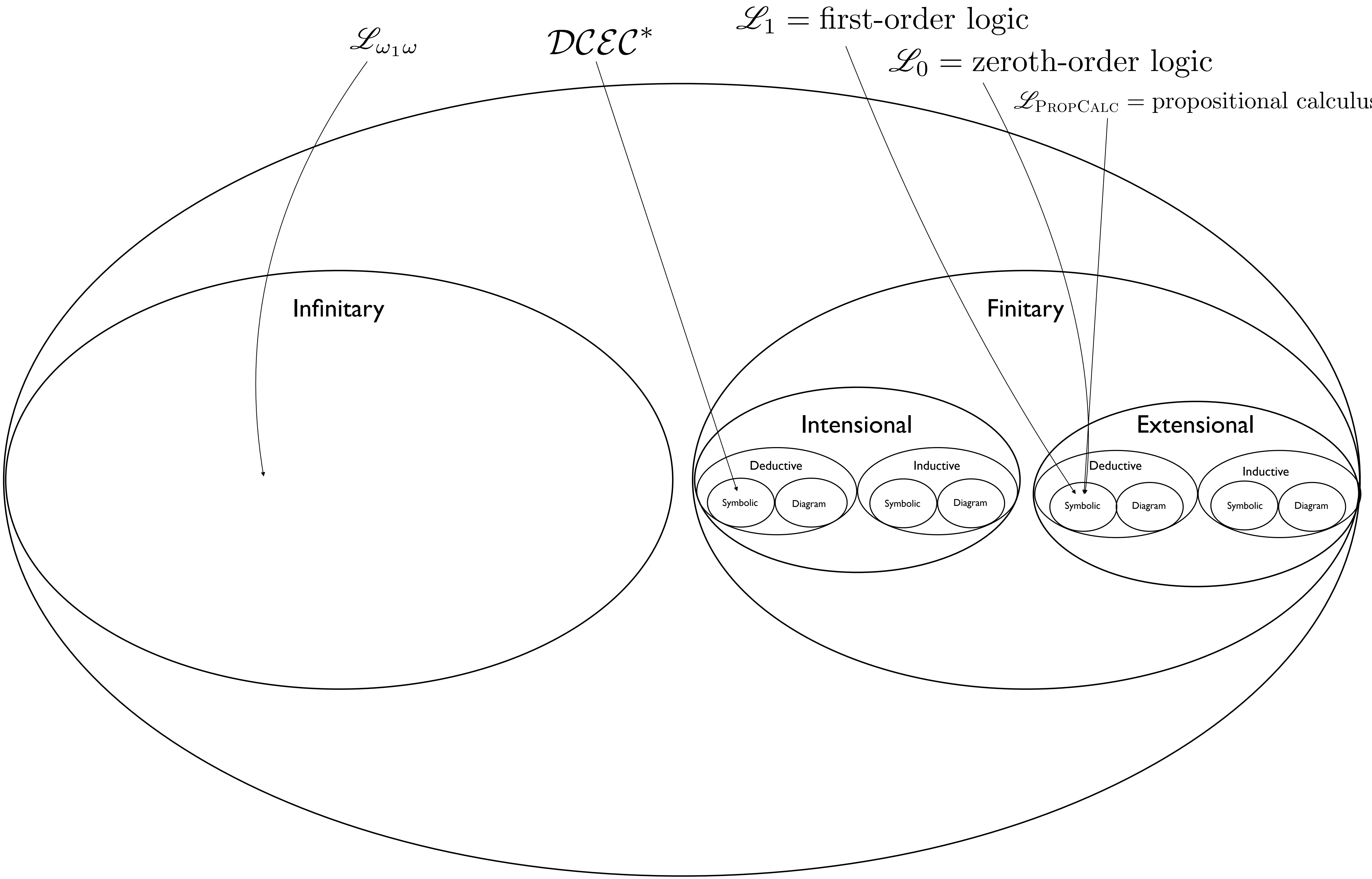
Option2. E PC ⊢ ✗
Option3. H PC ⊢ ✗
Option4. ¬A PC ⊢ ✗

The Universe of Logics



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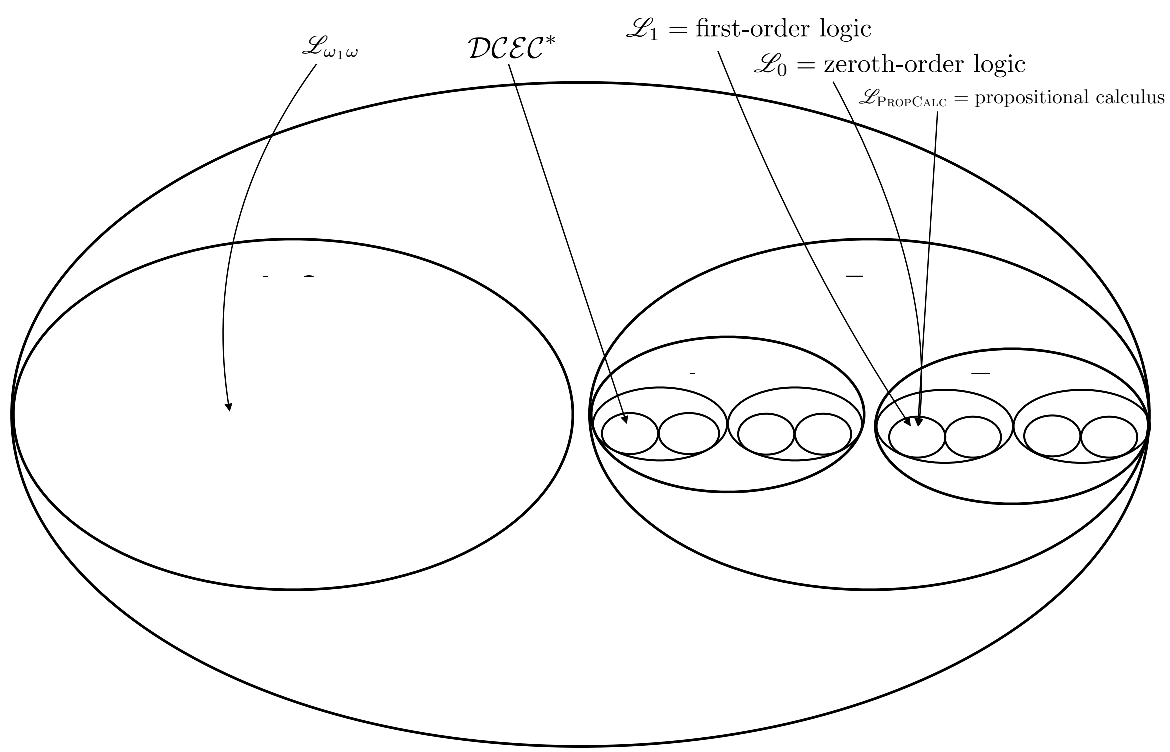
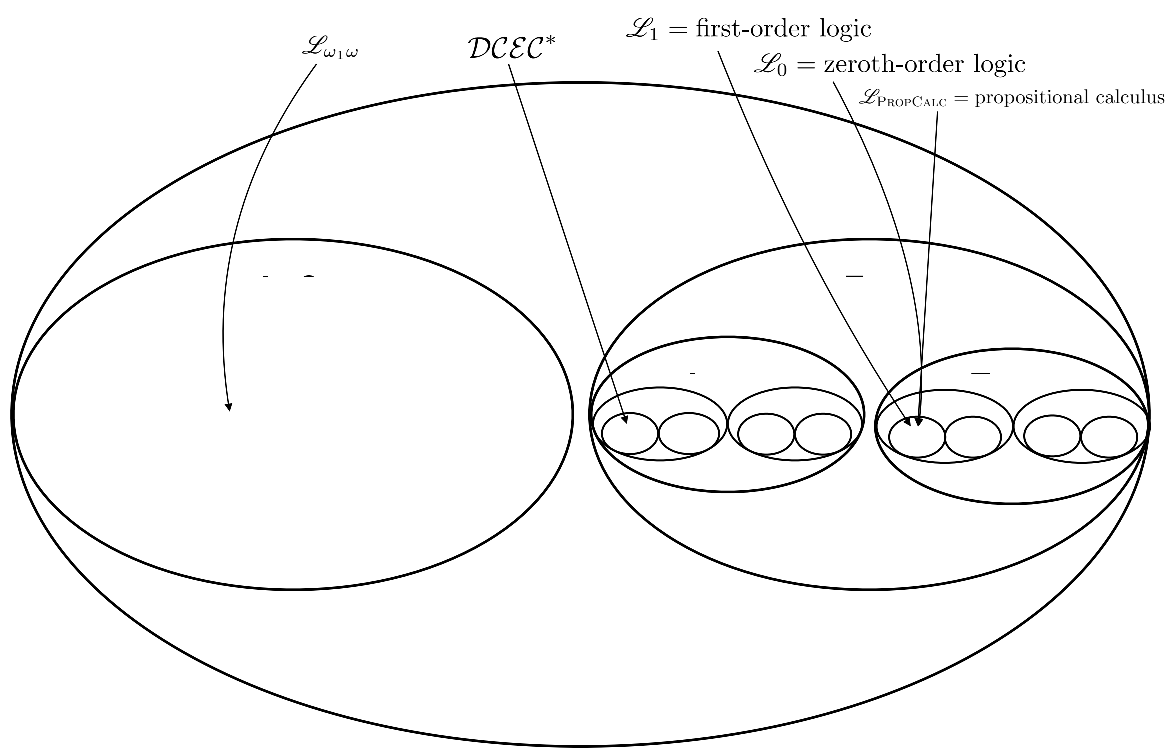
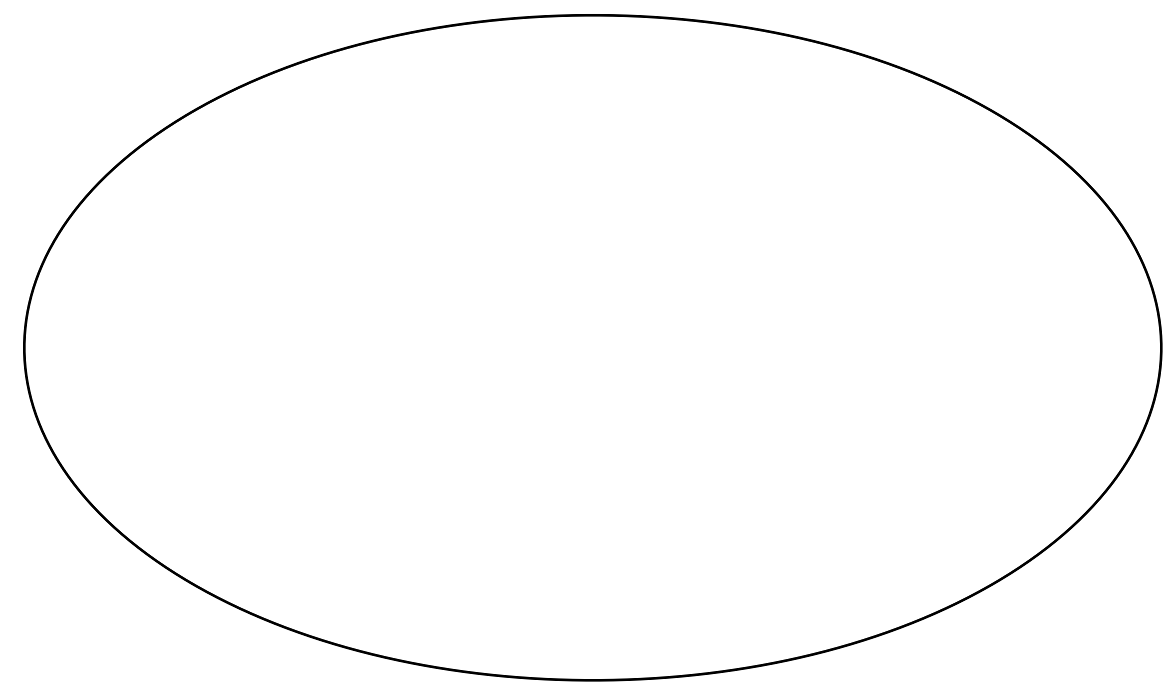


The Universe of Logics

The Physical Universe





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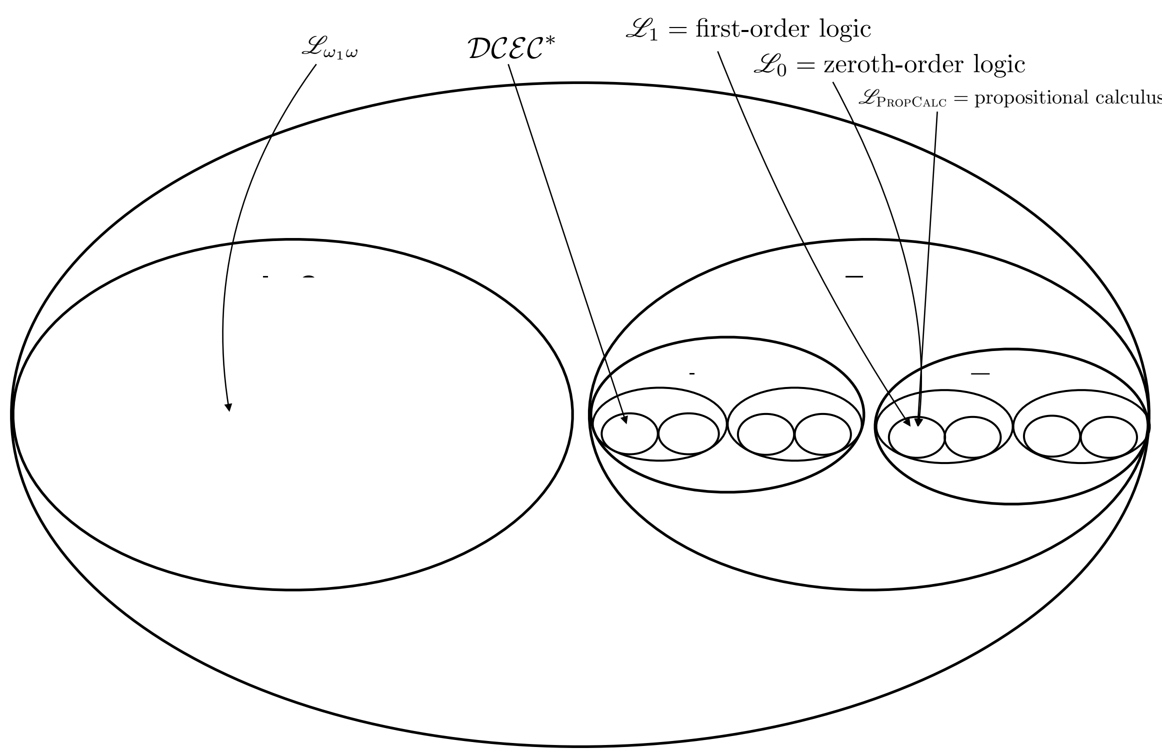

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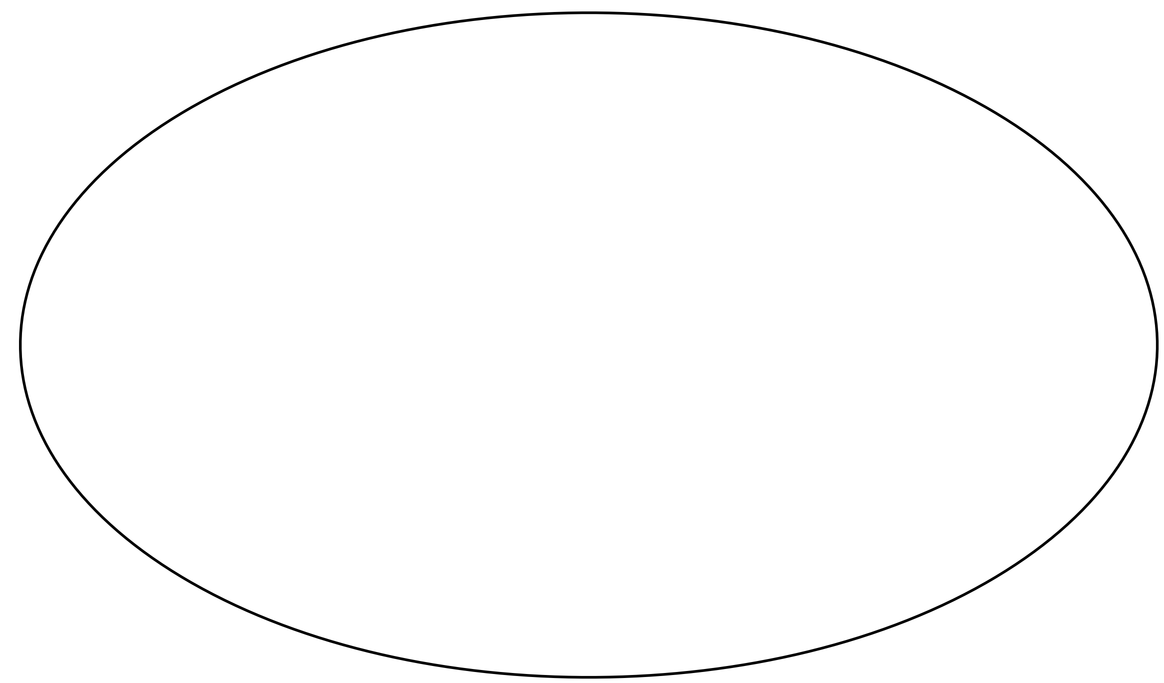
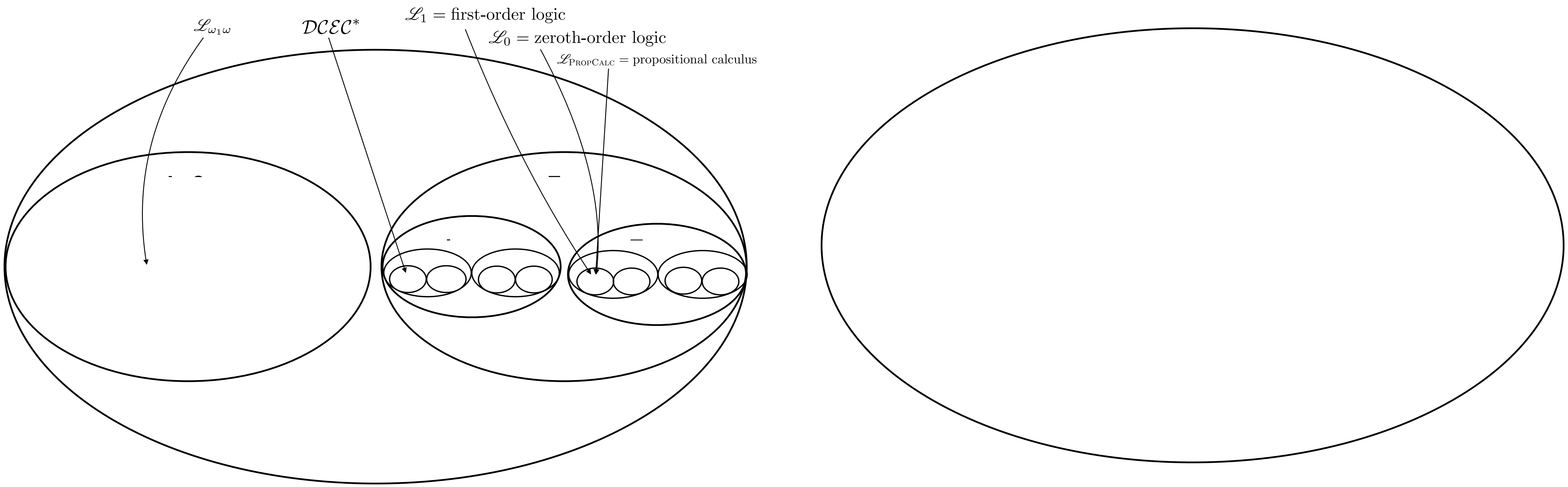


Non-Physical

The Universe of Logics

The Physical Universe



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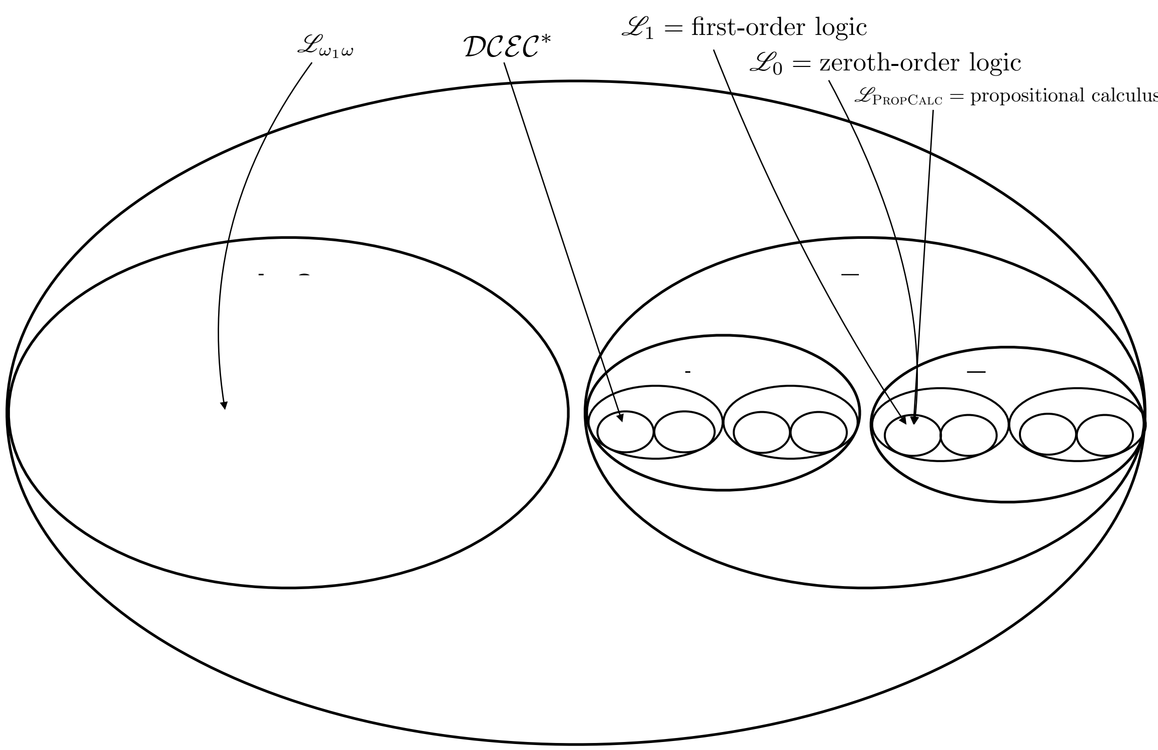
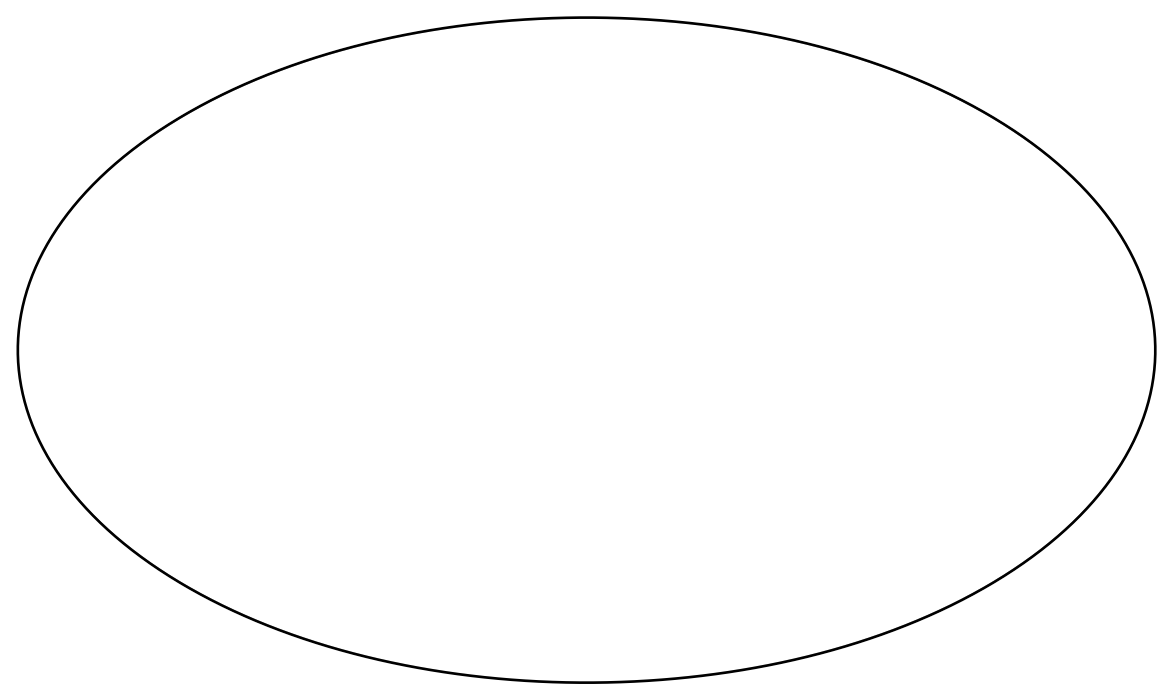
$\mathbb{N}$

The Universe of Logics



The Physical Universe



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$\mathbb{R}$

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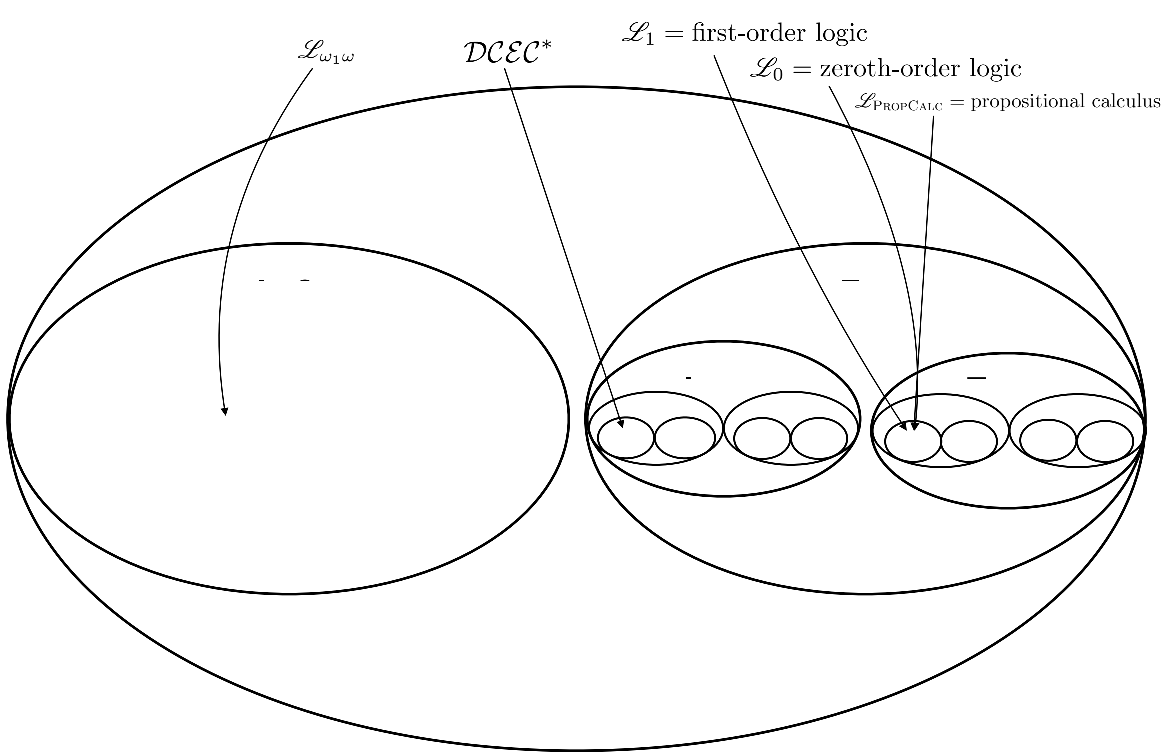
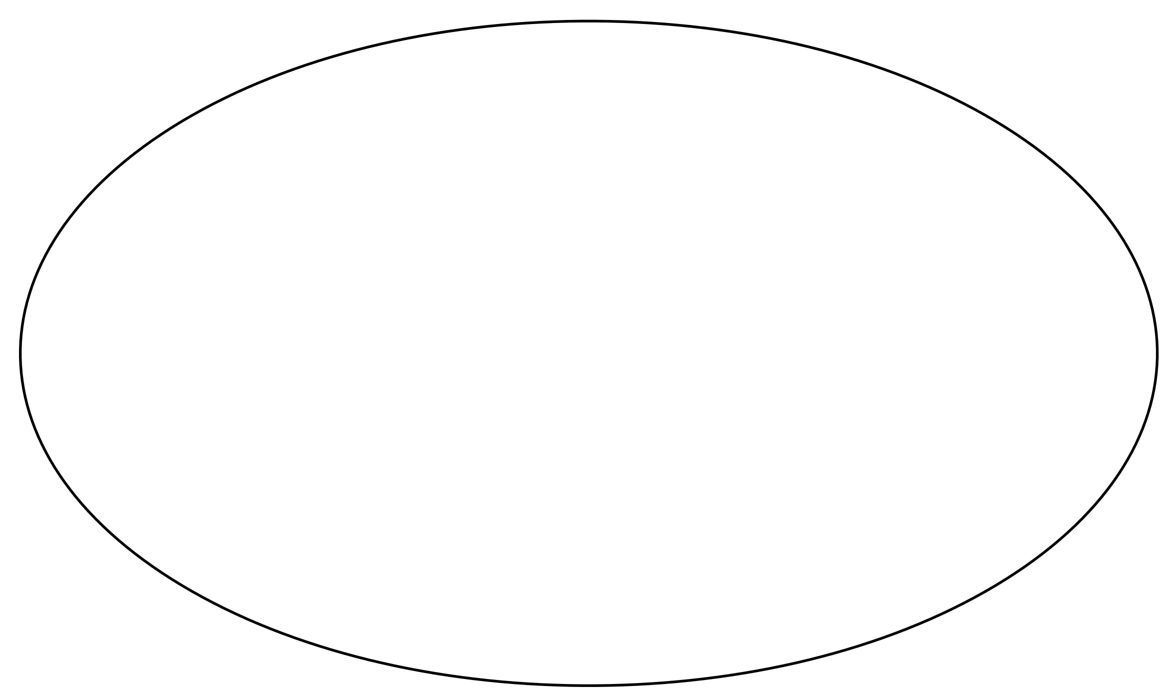
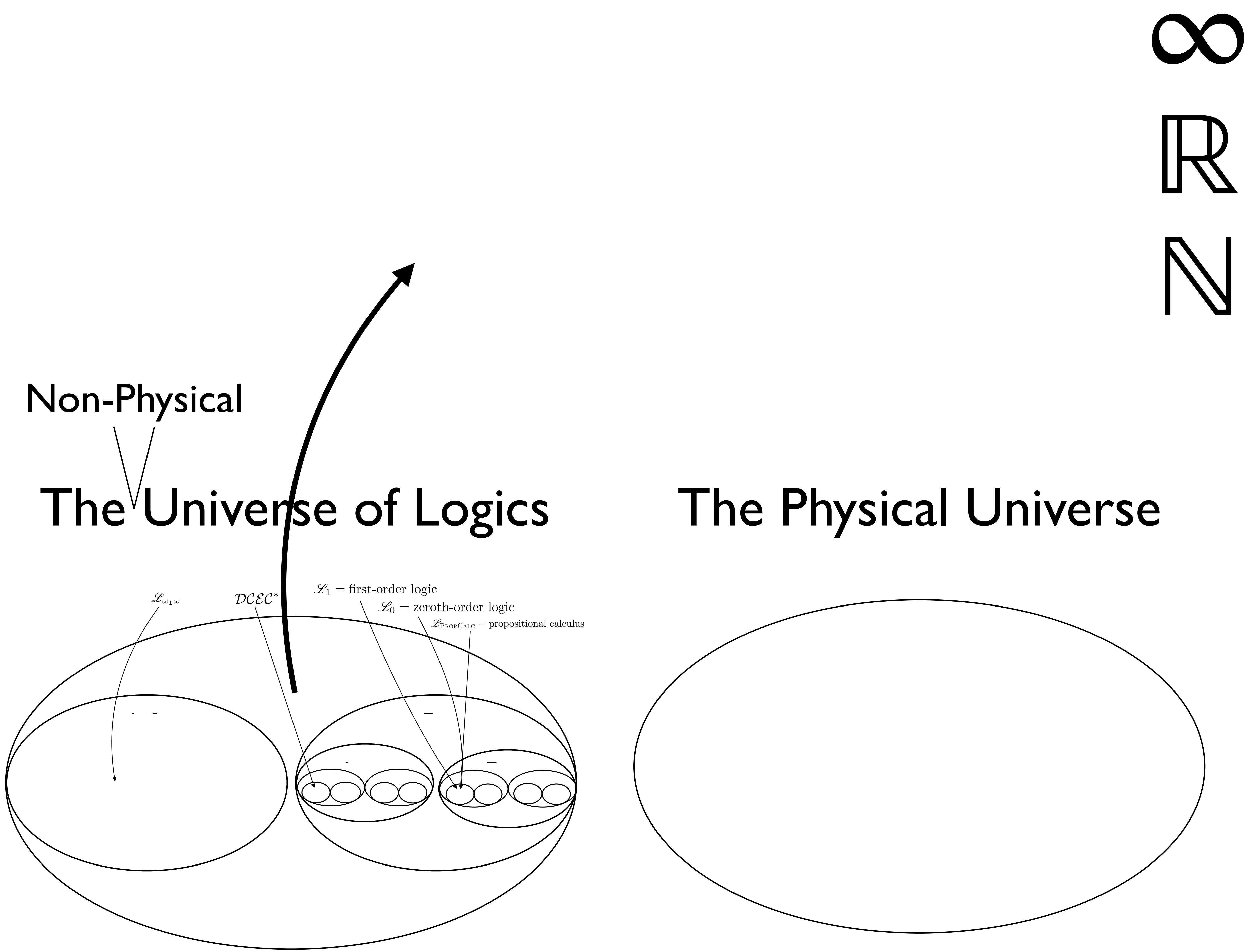
Non-Physical

The Universe of Logics

The Physical Universe







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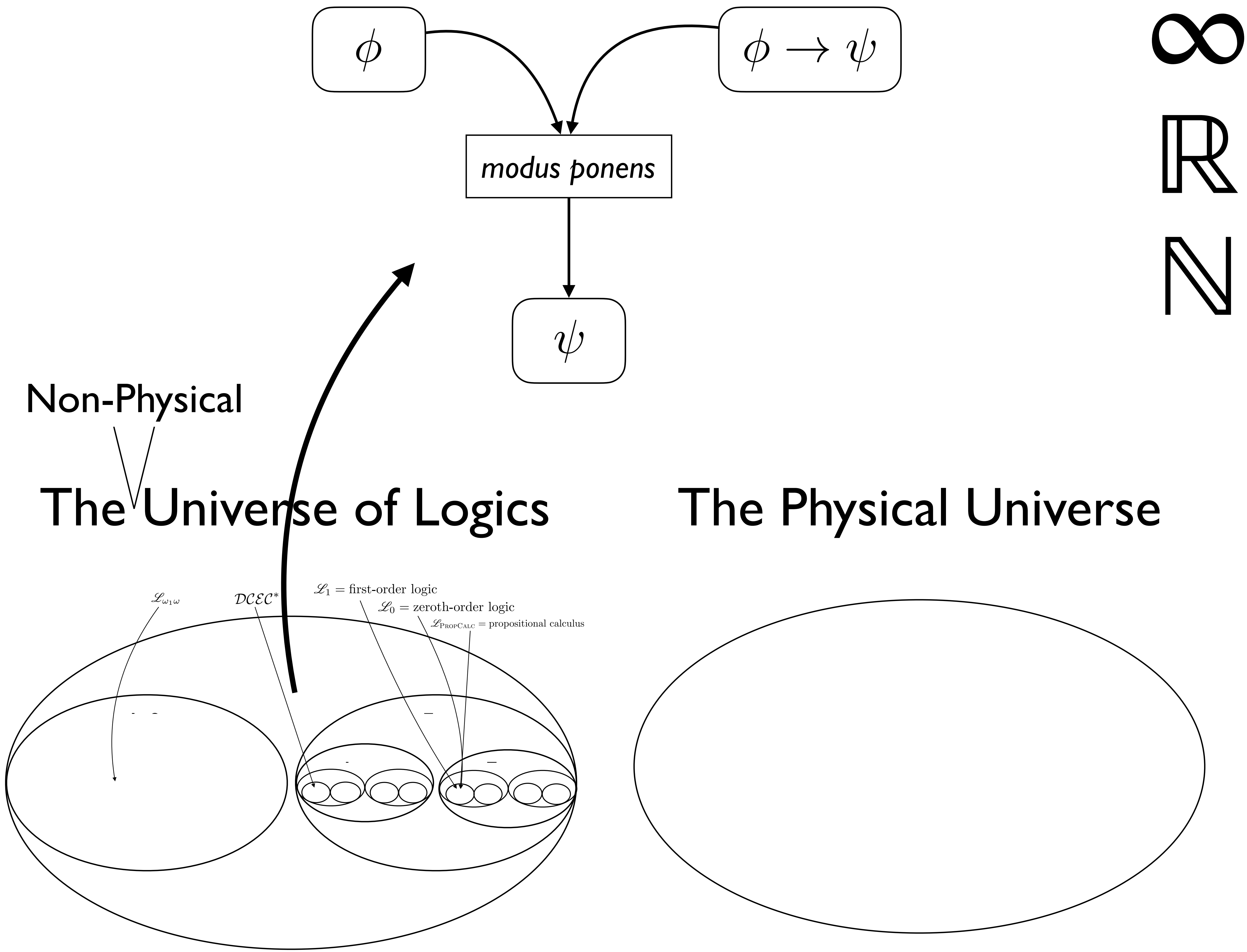
$\mathbb{R}$
$\mathbb{N}$
Non-Physical
The Universe of Logics
The Physical Universe
$\mathscr{L}_{\omega_1\omega}$
$\mathcal{DCEC}^*$
$\mathscr{L}_1$ = first-order logic
$\mathscr{L}_0$ = zeroth-order logic
$\mathscr{L}_{\text{PropCalc}}$ = propositional calculus



ϕ
$\phi \rightarrow \psi$
modus ponens
ψ
∞
$\mathbb{R}$
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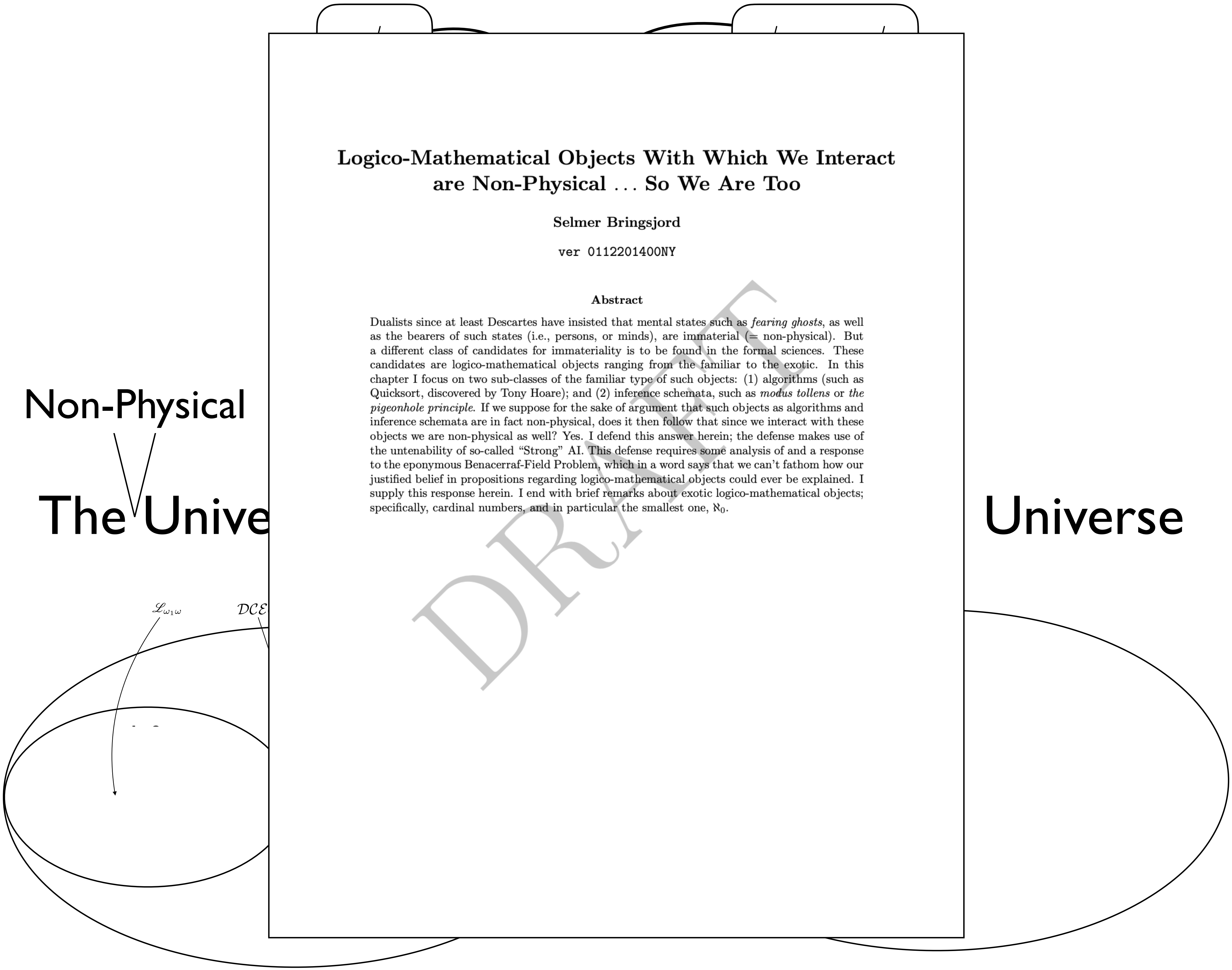

Logico-Mathematical Objects With Which We Interact are Non-Physical ... So We Are Too

Selmer Bringsjord

ver 0112201400NY

Abstract

Dualists since at least Descartes have insisted that mental states such as *fearing ghosts*, as well as the bearers of such states (i.e., persons, or minds), are immaterial (= non-physical). But a different class of candidates for immateriality is to be found in the formal sciences. These candidates are logico-mathematical objects ranging from the familiar to the exotic. In this chapter I focus on two sub-classes of the familiar type of such objects: (1) algorithms (such as Quicksort, discovered by Tony Hoare); and (2) inference schemata, such as *modus tollens* or *the pigeonhole principle*. If we suppose for the sake of argument that such objects as algorithms and inference schemata are in fact non-physical, does it then follow that since we interact with these objects we are non-physical as well? Yes. I defend this answer herein; the defense makes use of the untenability of so-called "Strong" AI. This defense requires some analysis of and a response to the eponymous Benacerraf-Field Problem, which in a word says that we can't fathom how our justified belief in propositions regarding logico-mathematical objects could ever be explained. I supply this response herein. I end with brief remarks about exotic logico-mathematical objects; specifically, cardinal numbers, and in particular the smallest one, $\aleph_0$.

Contents

1 Introduction

Chimpanzees, the chair in which I presently sit, and the chunk of aged cheddar cheese and wine before me on the table; these things are physical, clearly. Are there any non-physical things? Even those who would answer this question with an adamantine negative, if reflective, will agree that perhaps the best candidates for this category are not my mental states had while enjoying such cheddar with fine Carménère (states which dualists since Descartes have long insisted are non-physical, since they are bearers of so-called "qualia"), but instead logico-mathematical objects with which plenty of humans are acquainted. These immaterial objects range from the familiar to the exotic, and are the targets of study in the formal) sciences.[1] I focus herein on two familiar and elementary classes of such logico-mathematical objects: (1) algorithms (such as Quicksort, discovered by Tony Hoare); and (2) inference schemata that form the foundation of the formal sciences (such as *modus tollens*, that from two declarative propositions 'if ϕ then ψ' and 'not-ψ' one can deduce 'not-ϕ).' Inference schemata form this foundation because the formal sciences are theorem-driven, theorems are obtained by proofs, and proofs are sequences of propositions linked by inferences that are sanctioned by such schemata (though often the schemata employed are left implicit and not called out by name).

The overarching structure of my case for our being immaterial will have two steps. In Step 1 I adapt and render prior reasoning from James Ross (1992) in order to show that that such objects as algorithms and inference schemata and are non-physical (= immaterial). Then, in Step 2, I show that we interact with these objects in a certain crucial way: viz., we *understand* that we frequently validly implement them. I then argue that such understanding entails that we must ourselves be non-physical. Of course, inevitably some will want to resist my ultimate conclusion. Accordingly, I consider and rebut some objections, including one based on the eponymous Benacerraf-Field Problem, which in a word says that we can't fathom how our justified belief in propositions regarding logico-mathematical objects could ever be explained. When I wrap up the paper, I point out that in point of fact algorithms and inference schemata are actually in the same category of "formal objects," and briefly point to some much more exotic formal objects that are likewise immaterial, and with which we also interact. Here I specifically point to cardinal numbers, and to keep things brief and simple, the smallest cardinal number: $\aleph_0$.

2 Logico-mathematical Objects, in General

Some readers may find the phrase 'logico-mathematical object' to be a bit of a mouthful, and perhaps even pedantic. Actually, the idea is quite straightforward, and the objects in question are encountered and reasoned over by even very young schoolchildren, who usually continue in this regard for many years, and are along the way introduced to more and more such objects of increasing complexity. One of the first such logico-mathematical objects young children come across is $\mathbb{N}$: the set of all natural numbers

$$\{0, 1, 2, \ldots\}.$$

This object is often called "the number line," and of course before this object is introduced, the young mind will have been introduced to the numbers 0, 1, 2, and so forth, and often to the arithemtic functions of addition and subtraction. In public education in the U.S. State in which I reside, New York, Grade-4 mathematics instruction introduces students to a new logico-mathematical

[1] Pure mathematics, mathematical/theoretical physics, formal logic, decision theory, etc.

DRAFT

Next problem
(King-Ace) …

King-Ace 2

Suppose that the following premise is true:

If there is a king in the hand, then there is an ace in the hand; or if there isn't a king in the hand, then there is an ace; but not both of these if-then statements are true.

What can you infer from this premise?

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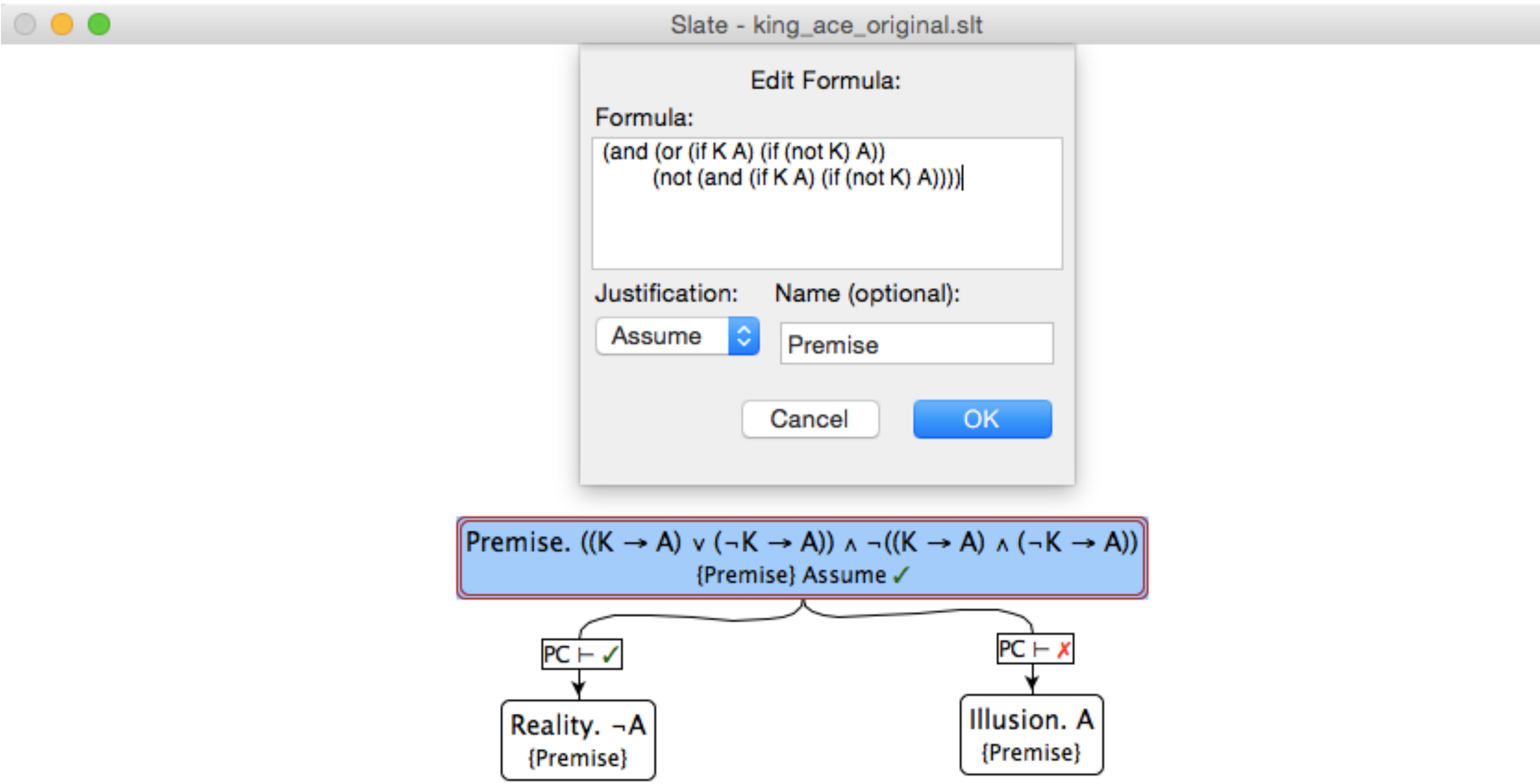
King-Ace Solved

Proposition: There is *not* an ace in the hand.

Proof: We know that at least one of the if-thens (i.e., at least one of the **conditionals**) is false. So we have two cases to consider, viz., that *K* => *A* is false, and that ¬*K* => *A* is false. Take first the first case; accordingly, suppose that *K* => *A* is false. Then it follows that *K* is true (since when a conditional is false, its antecedent holds but its consequent doesn't), and *A* is false. Now consider the second case, which consists in ¬*K* => *A* being false. Here, in a direct parallel, we know ¬*K* and, once again, ¬*A*. In both of our two cases, which are exhaustive, there is no ace in the hand. The proposition is established. **QED**

Study the S-expression



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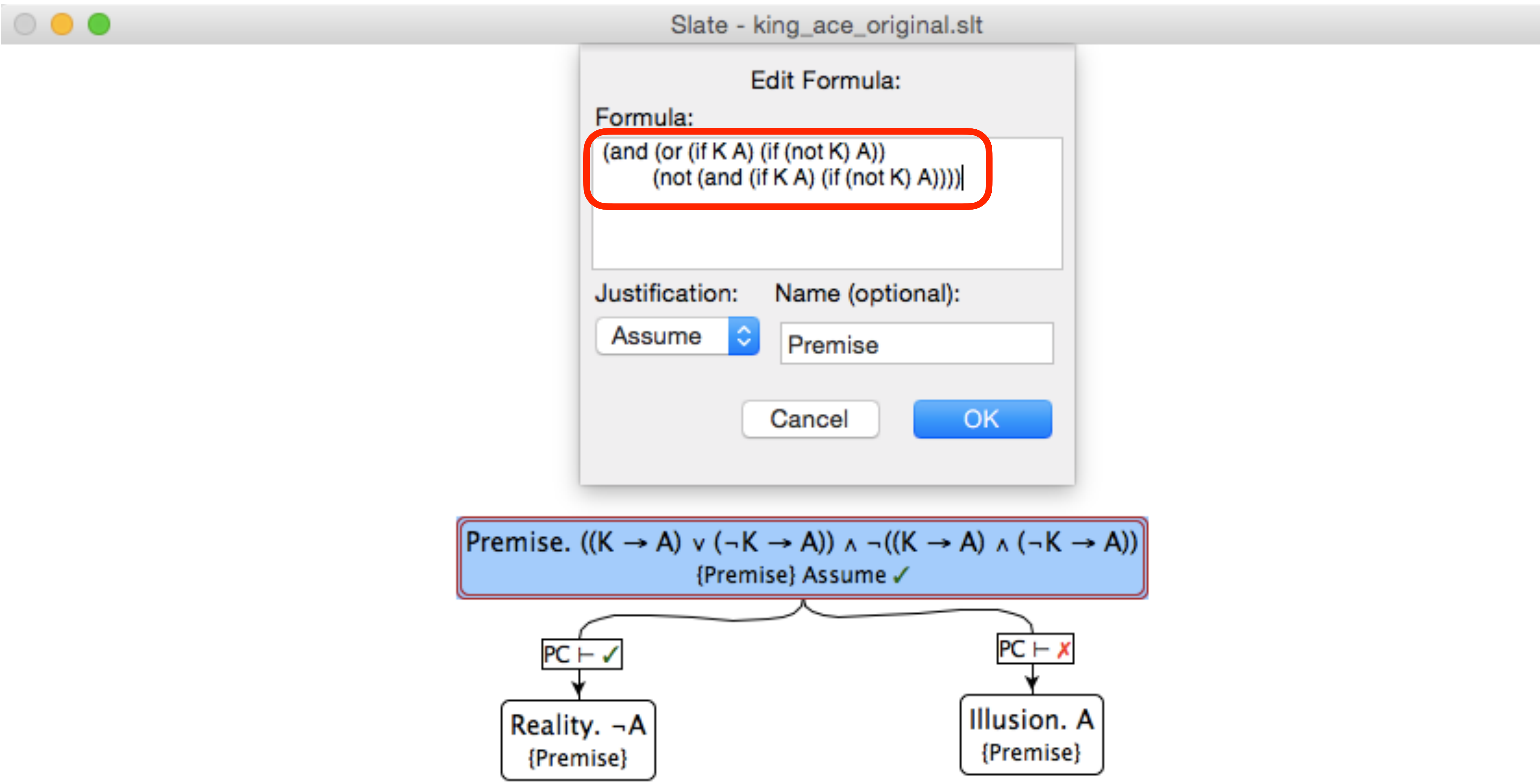

We need another rule of inference to crack this problem … …

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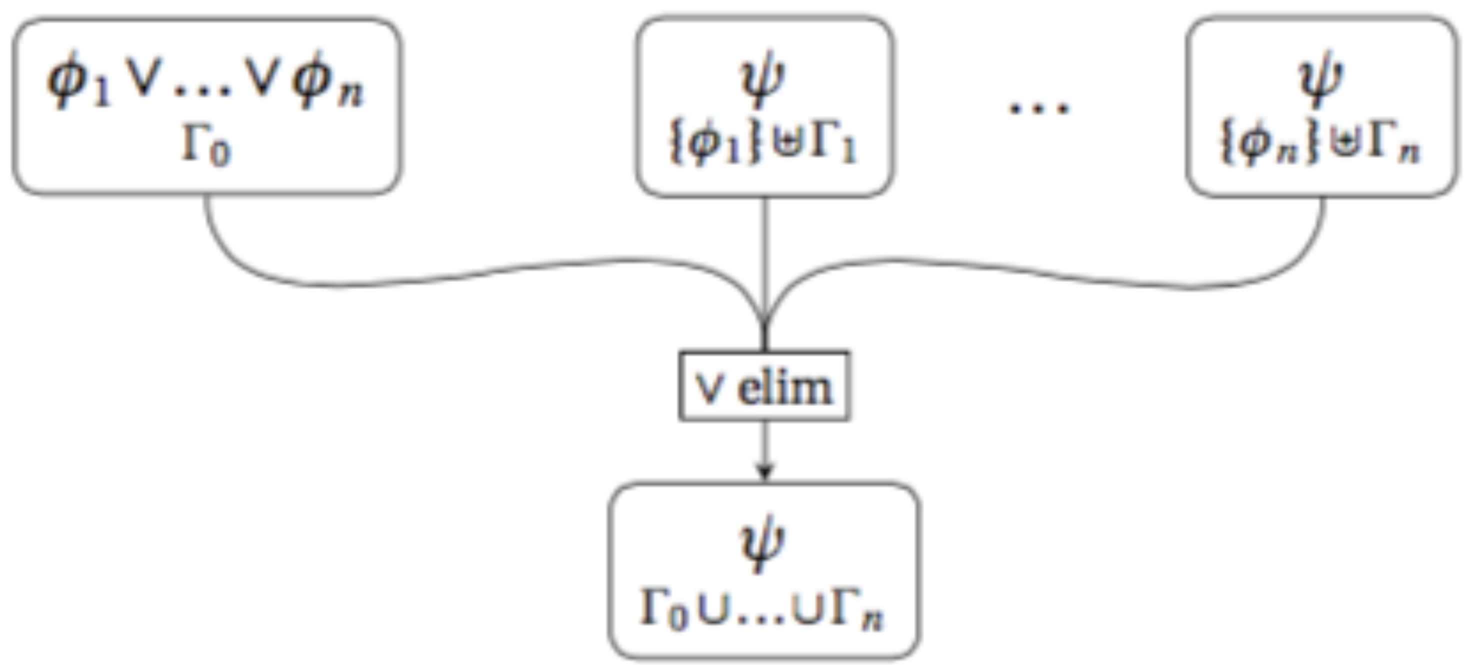
`disjunction elimination`

From ~ p. 54 in LAMA-BDLA

from each ϕ_i, then we may conclude ψ. That is, if we can, for each ϕ_i, assume ϕ_i and show that ψ follows, then we may conclude ψ from the disjunction $\phi_1 \vee \ldots \vee \phi_n$ and the derivations of ψ. There is one more subtle point, however. In the days-of-the-week example above, the conclusion that Susan has class on a weekday should not be in the scope of both the assumptions that she has class on Monday and that she has class on Tuesday; these assumptions are *discharged*. Disjunction elimination discharges each assumption ϕ_i from the line of reasoning that corresponds to that case.



(2.25)

The various Γ_i on the premises of disjunction elimination might make this rule seem more complicated than it really is. Their presence makes it clear that the only assumptions discharged from each line of reasoning is the assumption corresponding to that particular case.

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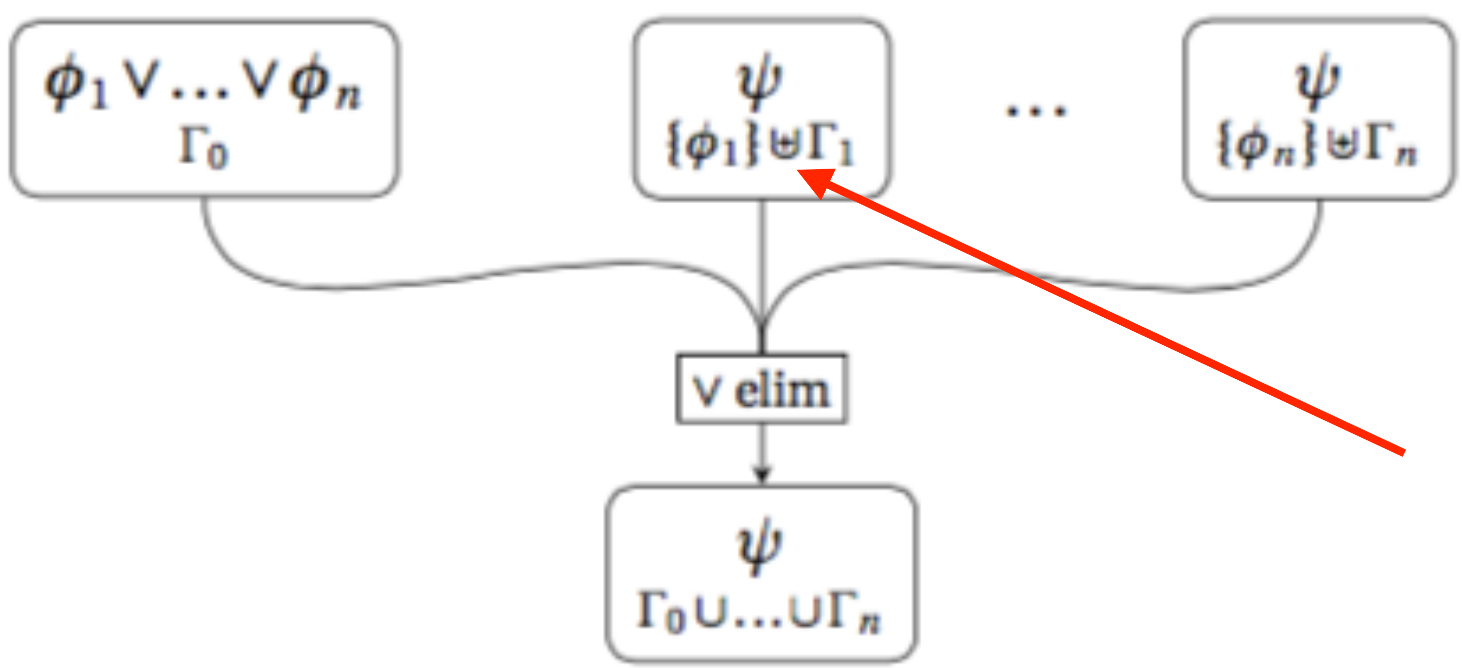

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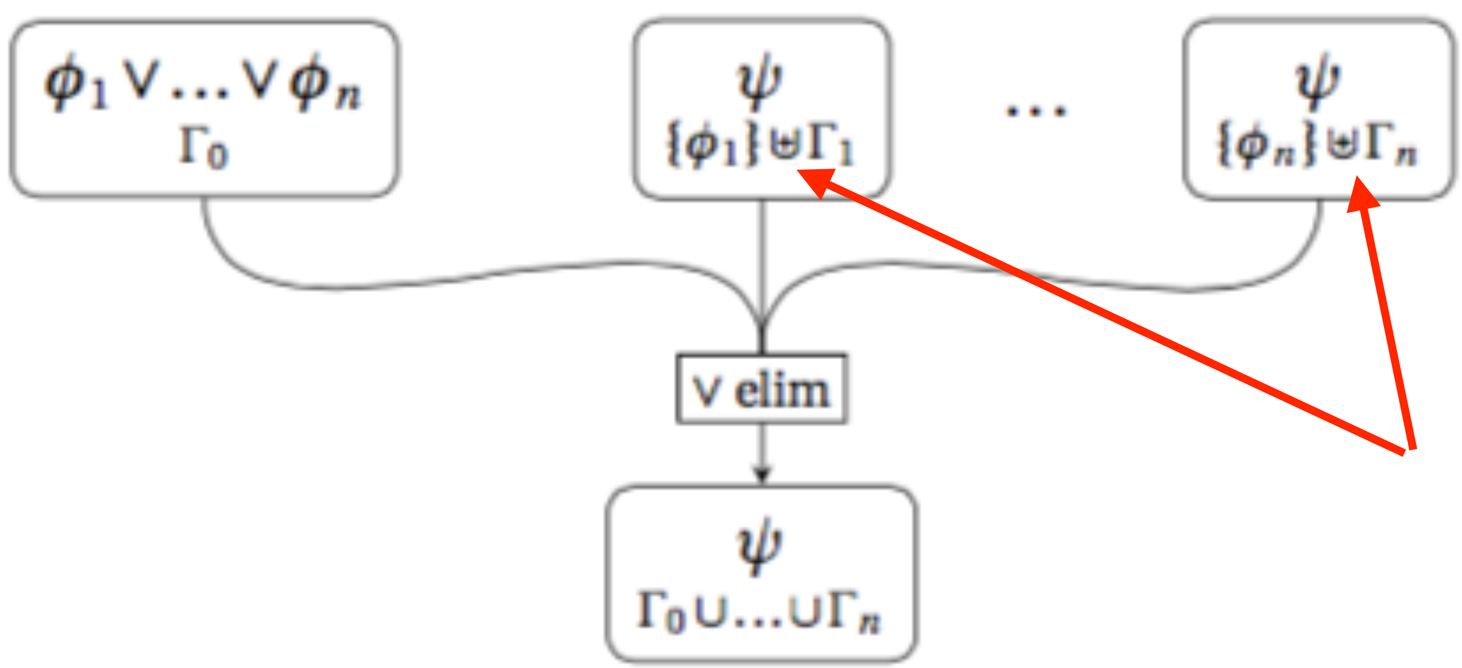

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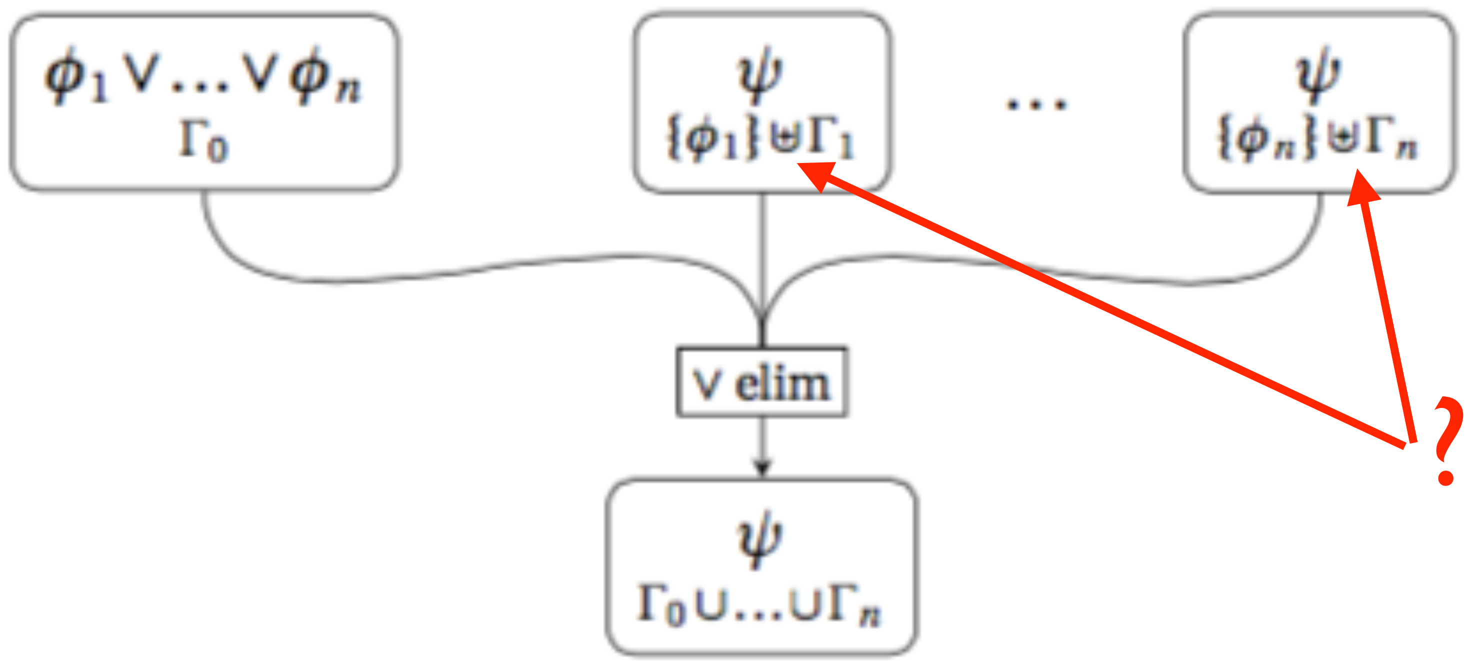

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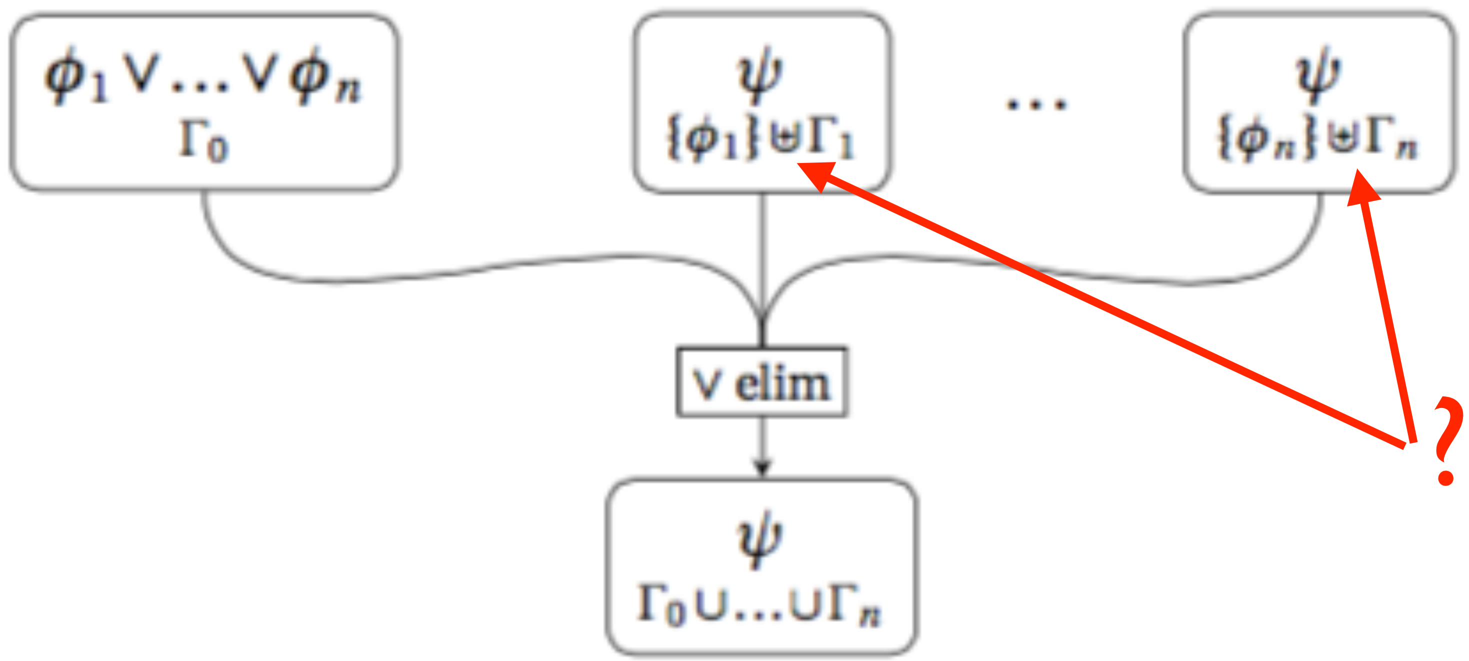

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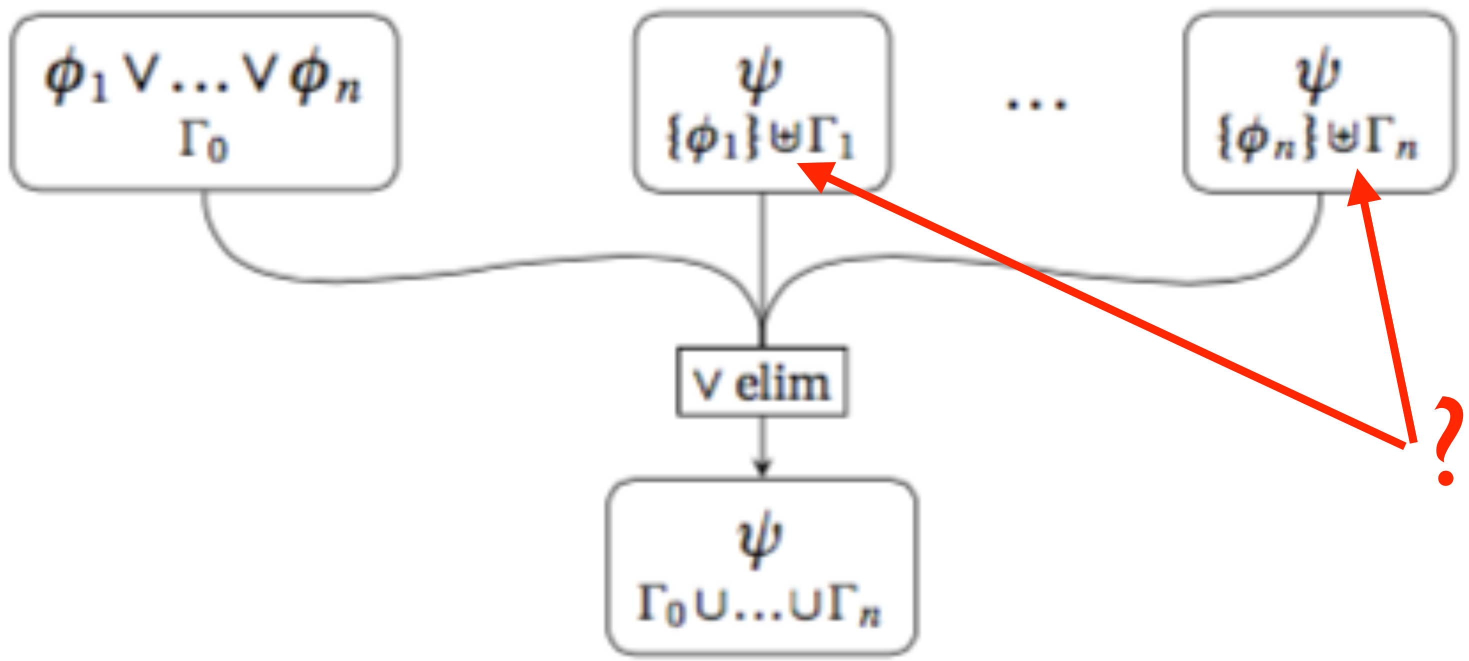

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there i…
statem…

Coming Exercise (on HyperGrader®):
Finish the proof in HyperSlate® —
with no remaining use of an oracle.
But COVID-19 will be allowed to slow
everything down.

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Det er en ære å lære formell logikk!