

# **How'd We Arrive Here?**

(Selmer's Leibnizian Whirlwind History of Logic,  
with Discussion of "The Singularity")

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Intro to Logic  
2/4/2021



**Last time ...**



A criminal genius nearly a match for Sherlock Holmes (Do you recognize the Dr?) has built a massive hydrogen bomb, and life on Earth is hanging in the balance, hinging on whether you make the logical prediction. Dr M gives you a sporting chance to: make the right prediction, snip or not snip accordingly, and prove that you're right ...





A criminal genius nearly a  
match for Sherlock Holmes  
(Do you recognize the Dr?)





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If one of the following assertions is true then so is the other:

(1) If the red wire runs to the bomb, then the blue wire runs to the bomb; and, if the blue wire runs to the bomb, then the red wire runs to the bomb.

(2) The red wire runs to the bomb.

Given this perfectly reliable clue from Dr Moriarty, if either wire is more likely to run to the bomb, that wire *does* run to the bomb, and the bomb is ticking, with only a minute left! If both are equiprobable, neither runs to the bomb, and you are powerless. Make your prediction as to what will happen when a wire is snipped, and then make your selected snip by clicking on the wire you want to snip! Or leave well enough alone!



Red more likely.

Blue more likely.

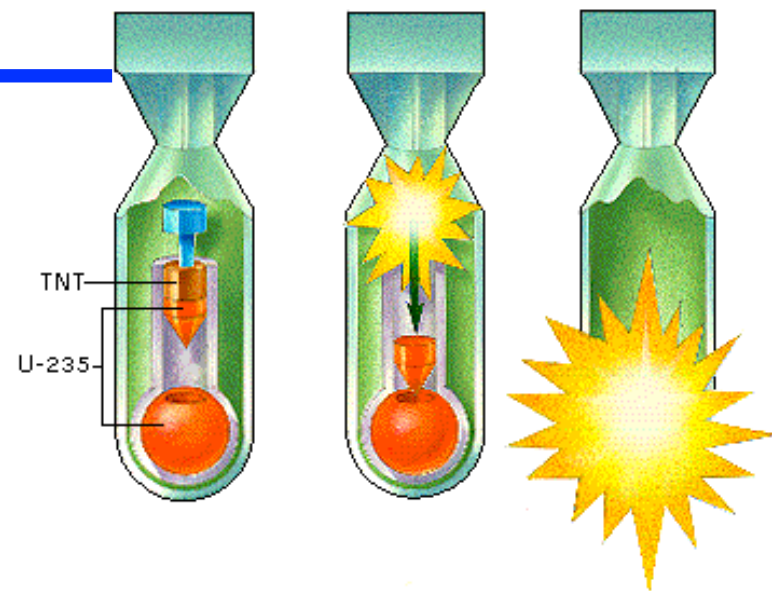
Equiprobable.

\_\_\_\_\_

\_\_\_\_\_

Snip

---



Life  
on  
Earth  
has  
ended

•

advance one more  
slide to see a proof  
that you indeed made  
an irrational  
decision...

**Proposition:** The blue wire is more likely!

**Proof:** (1) can be treated as a biconditional, obviously ( $R \iff B$ ).

There are two top-level cases to consider: (1) and (2) are both true; or both are false. In the case where they are both true, it's trivial to deduce both R and B. So far, then, R and B are equiprobable. What happens in the case where (1) and (2) are both false? We immediately have  $\sim R$  from the denial of (2). But a biconditional is true just in case both sides are true, or both sides are false; so we have two sub-cases to consider.

Consider first the case where R is true and B is false. We have an immediate contradiction in this sub-case, so both R and B can both be deduced here, and we have not yet departed from equiprobable. So what about the case where R is false and B is true? The falsity of R is not new information (we already have that from the denial of (2)), but we can still derive B. Hence the blue wire is more likely. **QED**

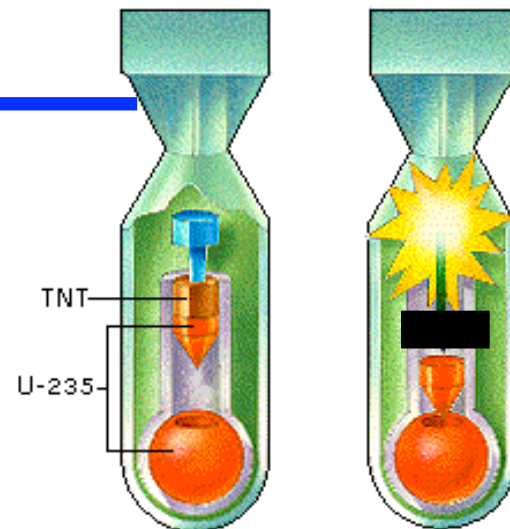
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---

Snip

---



Life on  
Earth  
is  
saved!

*if you can now hand Dr  
M a proof that your  
decision was the rational  
one!*

Advance one more slide  
to see a proof from  
Bringsjord that yours  
had better match up to  
...

**Proposition:** The blue wire is more likely!

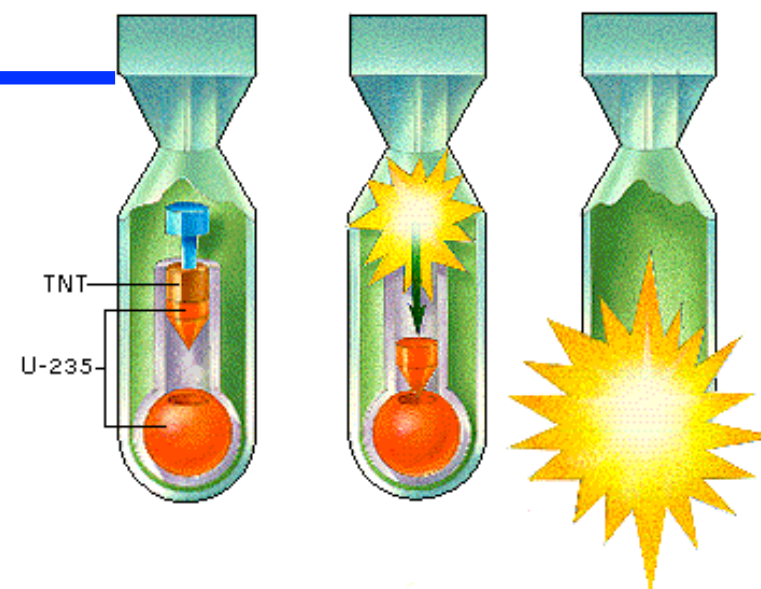
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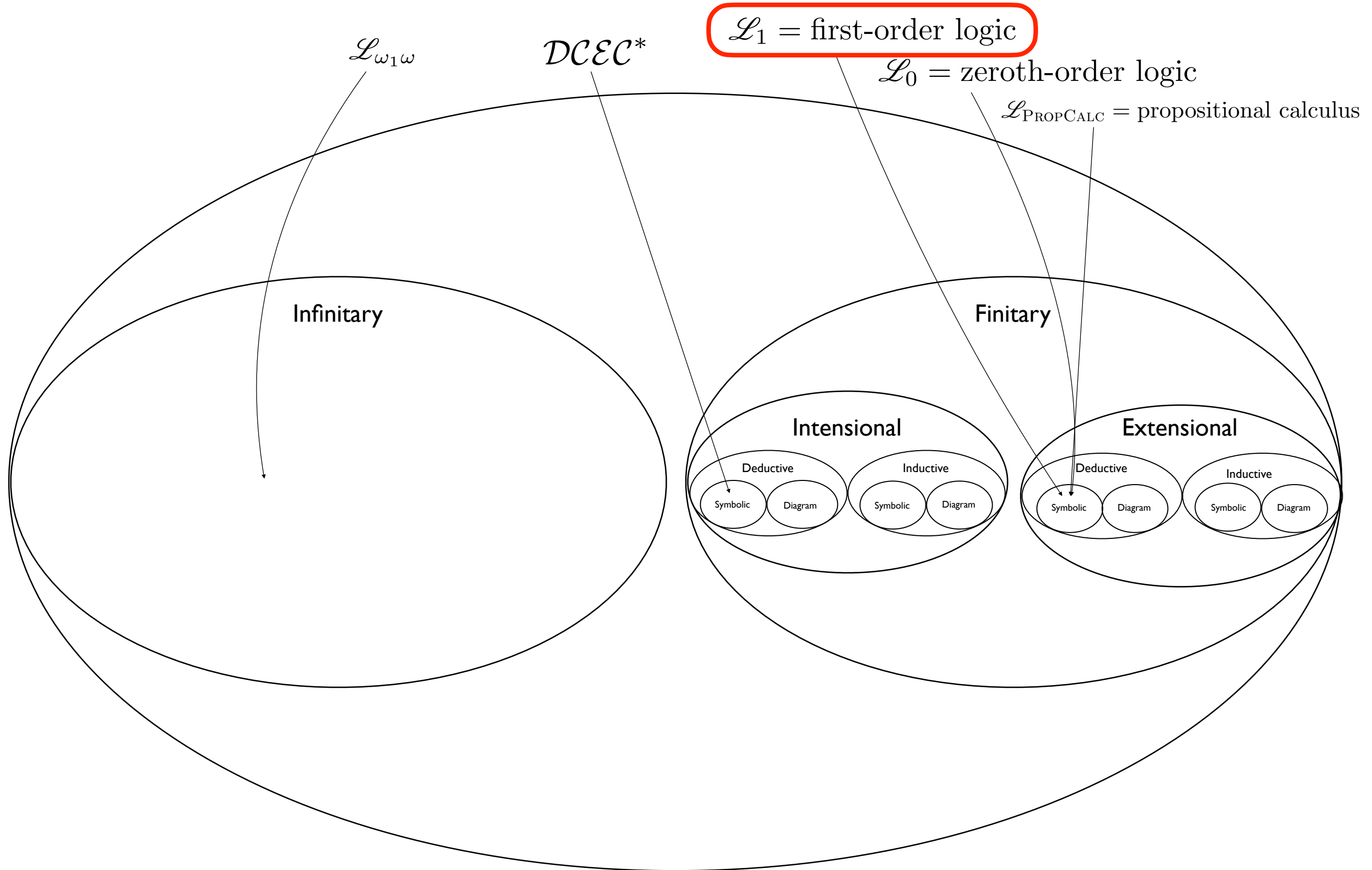
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STOP

# The Universe of Logics



# Special Llamas Disjunction

There's a thing such that it's both a llama and a non-llama;  
or  
there's a thing such that if it's a llama, everything is a llama;  
or  
there's a thing such that every llama is a non-llama.

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Is this disjunction TRUE, FALSE, or UNKNOWN?

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Is this disjunction **TRUE**, FALSE, or UNKNOWN?

Supply a formal proof!



# abstract-and-valid inference schemata

## Background Claim

$\mathcal{R}$  Humans, at least neurobiologically normal ones, are fundamentally rational, where rationality is constituted by certain logico-mathematically based reasoning and decision-making in response to real-world stimuli, including stimuli given in the form of focused tests; but mere animals are not fundamentally rational, since, *contra* Darwin, their minds are fundamentally qualitatively inferior to the human mind. As to whether computing machines/robots are fundamentally rational, the answer is “No.” For starters, if  $x$  can’t read, write, and create,  $x$  can’t be rational; computing machines/robots can neither read nor write nor create; ergo, they aren’t fundamentally rational.

# abstract-and-valid inference schemata

## quantification

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To infinity and beyond! — routinely

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HS<sup>®</sup>

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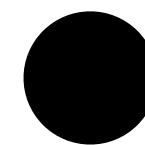
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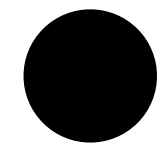
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And now the whirlwind  
history ...





2021

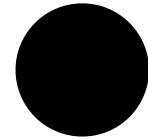


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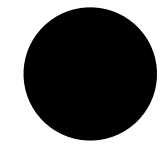
# $\mathcal{DCEC}^*$

| Syntax                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | Rules of Inference                                                                                                                                                                                   |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $S ::=$<br>Object   Agent   Self $\sqsubseteq$ Agent   ActionType   Action $\sqsubseteq$ Event  <br>Moment   Boolean   Fluent   Numeric                                                                                                                                                                                                                                                                                                                                                         | $\frac{}{C(t, \mathbf{P}(a, t, \phi) \rightarrow \mathbf{K}(a, t, \phi))} [R_1] \quad \frac{}{C(t, \mathbf{K}(a, t, \phi) \rightarrow \mathbf{B}(a, t, \phi))} [R_2]$                                |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | $\frac{C(t, \phi) \ t \leq t_1 \dots t \leq t_n}{\mathbf{K}(a_1, t_1, \dots \mathbf{K}(a_n, t_n, \phi) \dots)} [R_3] \quad \frac{\mathbf{K}(a, t, \phi)}{\phi} [R_4]$                                |
| $action : \text{Agent} \times \text{ActionType} \rightarrow \text{Action}$<br>$initially : \text{Fluent} \rightarrow \text{Boolean}$<br>$holds : \text{Fluent} \times \text{Moment} \rightarrow \text{Boolean}$<br>$happens : \text{Event} \times \text{Moment} \rightarrow \text{Boolean}$<br>$clipped : \text{Moment} \times \text{Fluent} \times \text{Moment} \rightarrow \text{Boolean}$                                                                                                   | $\frac{}{C(t, \mathbf{K}(a, t_1, \phi_1 \rightarrow \phi_2)) \rightarrow \mathbf{K}(a, t_2, \phi_1) \rightarrow \mathbf{K}(a, t_3, \phi_2)} [R_5]$                                                   |
| $f ::=$<br>$initiates : \text{Event} \times \text{Fluent} \times \text{Moment} \rightarrow \text{Boolean}$<br>$terminates : \text{Event} \times \text{Fluent} \times \text{Moment} \rightarrow \text{Boolean}$<br>$prior : \text{Moment} \times \text{Moment} \rightarrow \text{Boolean}$<br>$interval : \text{Moment} \times \text{Boolean}$<br>$* : \text{Agent} \rightarrow \text{Self}$<br>$payoff : \text{Agent} \times \text{ActionType} \times \text{Moment} \rightarrow \text{Numeric}$ | $\frac{}{C(t, \mathbf{B}(a, t_1, \phi_1 \rightarrow \phi_2)) \rightarrow \mathbf{B}(a, t_2, \phi_1) \rightarrow \mathbf{B}(a, t_3, \phi_2)} [R_6]$                                                   |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | $\frac{}{C(t, C(t_1, \phi_1 \rightarrow \phi_2)) \rightarrow C(t_2, \phi_1) \rightarrow C(t_3, \phi_2)} [R_7]$                                                                                       |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | $\frac{}{C(t, \forall x. \phi \rightarrow \phi[x \mapsto t])} [R_8] \quad \frac{}{C(t, \phi_1 \leftrightarrow \phi_2 \rightarrow \neg \phi_2 \rightarrow \neg \phi_1)} [R_9]$                        |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | $\frac{}{C(t, [\phi_1 \wedge \dots \wedge \phi_n \rightarrow \phi] \rightarrow [\phi_1 \rightarrow \dots \rightarrow \phi_n \rightarrow \psi])} [R_{10}]$                                            |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | $\frac{\mathbf{B}(a, t, \phi) \ \phi \rightarrow \psi}{\mathbf{B}(a, t, \psi)} [R_{11a}] \quad \frac{\mathbf{B}(a, t, \phi) \ \mathbf{B}(a, t, \psi)}{\mathbf{B}(a, t, \psi \wedge \phi)} [R_{11b}]$ |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | $\frac{\mathbf{S}(s, h, t, \phi)}{\mathbf{B}(h, t, \mathbf{B}(s, t, \phi))} [R_{12}]$                                                                                                                |
| $t ::= x : S \mid c : S \mid f(t_1, \dots, t_n)$                                                                                                                                                                                                                                                                                                                                                                                                                                                | $\frac{\mathbf{I}(a, t, happens(action(a^*, \alpha), t'))}{\mathbf{P}(a, t, happens(action(a^*, \alpha), t))} [R_{13}]$                                                                              |
| $t : \text{Boolean} \mid \neg \phi \mid \phi \wedge \psi \mid \phi \vee \psi \mid$<br>$\mathbf{P}(a, t, \phi) \mid \mathbf{K}(a, t, \phi) \mid \mathbf{C}(t, \phi) \mid \mathbf{S}(a, b, t, \phi) \mid \mathbf{S}(a, t, \phi)$<br>$\phi ::=$<br>$\mathbf{B}(a, t, \phi) \mid \mathbf{D}(a, t, holds(f, t')) \mid \mathbf{I}(a, t, happens(action(a^*, \alpha), t'))$<br>$\mathbf{O}(a, t, \phi, happens(action(a^*, \alpha), t'))$                                                              | $\frac{\mathbf{B}(a, t, \phi) \ \mathbf{B}(a, t, \mathbf{O}(a^*, t, \phi, happens(action(a^*, \alpha), t')))}{\mathbf{O}(a, t, \phi, happens(action(a^*, \alpha), t'))} [R_{14}]$                    |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | $\frac{\phi \leftrightarrow \psi}{\mathbf{O}(a, t, \phi, \gamma) \leftrightarrow \mathbf{O}(a, t, \psi, \gamma)} [R_{15}]$                                                                           |



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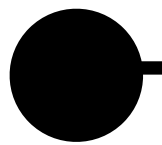
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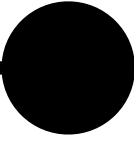


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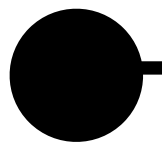


350 BC



2021

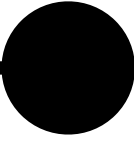
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350 BC



Euclid



2021

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# Euclidean “Magic”

**Theorem:** There are infinitely many primes.

**Proof:** We take an indirect route. Let  $\Pi = p_1 = 2, p_2 = 3, p_3 = 5, \dots, p_k$  be a finite, exhaustive consecutive sequence of prime numbers. Next, let  $\mathbf{M}_\Pi$  be  $p_1 \times p_2 \times \dots \times p_k$ , and set  $\mathbf{M}'_\Pi$  to  $\mathbf{M}_\Pi + 1$ . Either  $\mathbf{M}'_\Pi$  is prime, or not; we thus have two (exhaustive) cases to consider.

- C1 Suppose  $\mathbf{M}'_\Pi$  is prime. In this case we immediately have a prime number beyond any in  $\Pi$  — contradiction!
- C2 Suppose on the other hand that  $\mathbf{M}'_\Pi$  is *not* prime. Then some prime  $p$  divides  $\mathbf{M}'_\Pi$ . (Why?) Now,  $p$  itself is either in  $\Pi$ , or not; we hence have two sub-cases. Supposing that  $p$  is in  $\Pi$  entails that  $p$  divides  $\mathbf{M}_\Pi$ . But we are operating under the supposition that  $p$  divides  $\mathbf{M}'_\Pi$  as well. This implies that  $p$  divides 1, which is absurd (a contradiction). Hence the prime  $p$  is outside  $\Pi$ .

Hence for *any* such list  $\Pi$ , there is a prime outside the list. That is, there are infinitely many primes. **QED**

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**Proof:** We take an indirect route. Let  $\Pi = p_1 = 2, p_2 = 3, p_3 = 5, \dots, p_k$  be a finite, exhaustive consecutive sequence of prime numbers. Next, let  $M_\Pi$  be  $p_1 \times p_2 \times \dots \times p_k$ , and set  $M'_\Pi$  to  $M_\Pi + 1$ . Either  $M'_\Pi$  is prime, or not; we thus have two (exhaustive) cases to consider.

- C1 Suppose  $M'_\Pi$  is prime. In this case we immediately have a prime number beyond any in  $\Pi$  — contradiction!
- C2 Suppose on the other hand that  $M'_\Pi$  is *not* prime. Then some prime  $p$  divides  $M'_\Pi$ . (Why?) Now,  $p$  itself is either in  $\Pi$ , or not; we hence have two sub-cases. Supposing that  $p$  is in  $\Pi$  entails that  $p$  divides  $M_\Pi$ . But we are operating under the supposition that  $p$  divides  $M'_\Pi$  as well. This implies that  $p$  divides 1, which is absurd (a contradiction). Hence the prime  $p$  is outside  $\Pi$ .

Hence for *any* such list  $\Pi$ , there is a prime outside the list. That is, there are infinitely many primes. **QED**

# Euclidean “Magic”

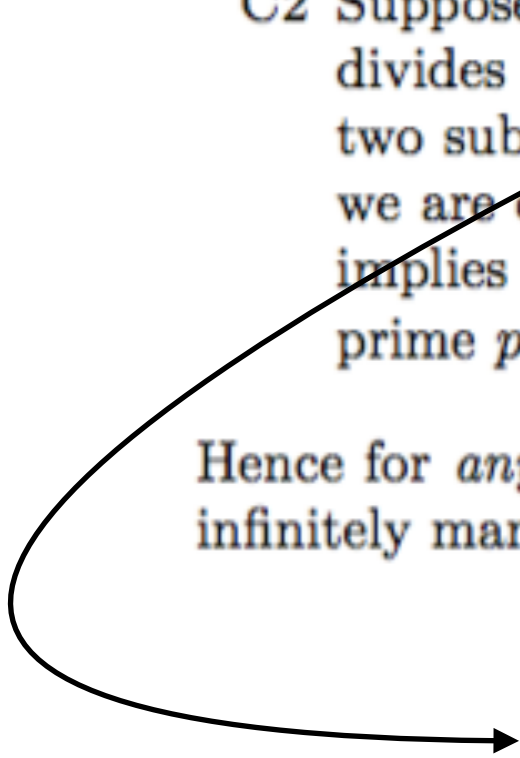
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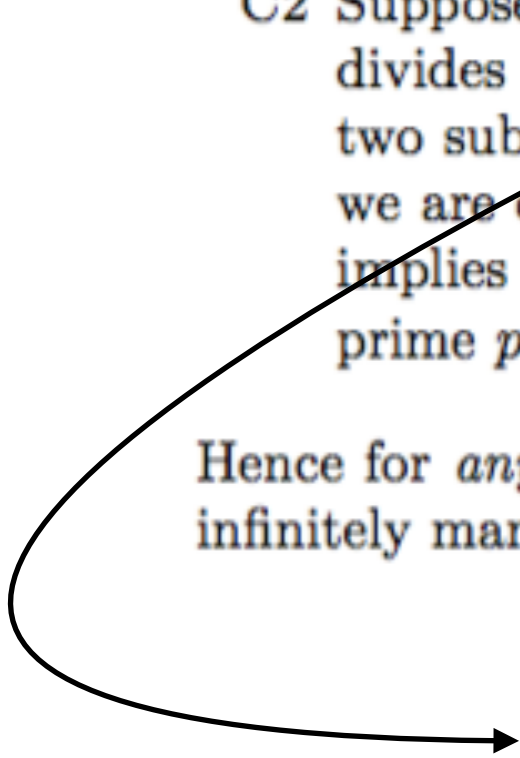
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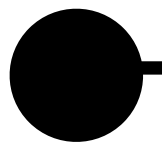
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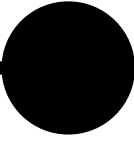
The Fundamental Theorem of Arithmetic



350 BC

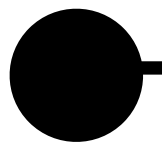


Euclid



2021

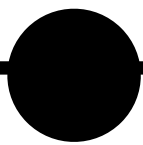
*Intro to (Formal) Logic (& AI) @ RPI*



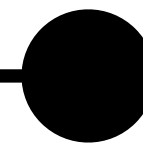
350 BC



Euclid

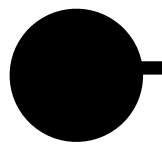


300 BC



2021

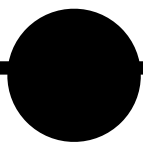
*Intro to (Formal) Logic (& AI) @ RPI*



350 BC



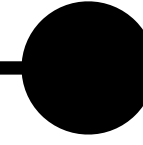
Euclid



300 BC

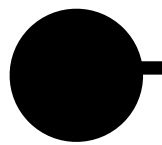


I don't believe in magic! Why exactly is that so convincing? What exactly is he doing?!?



2021

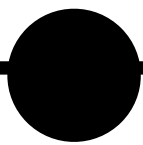
*Intro to (Formal) Logic (& AI) @ RPI*



350 BC



Euclid

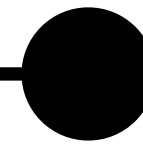


300 BC



*Organon*

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2021

*Intro to (Formal) Logic (& AI) @ RPI*

He's using syllogisms!

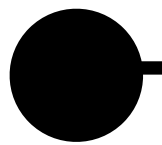
E.g.,

All As are Bs.

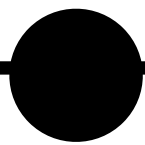
All Bs are Cs.

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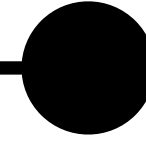
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350 BC



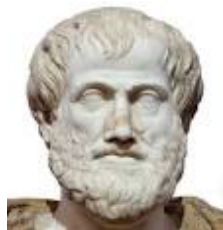
300 BC



2021



Euclid



*Organon*

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*Intro to (Formal) Logic (& AI) @ RPI*

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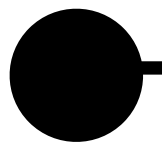
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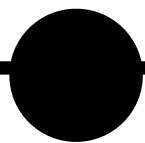
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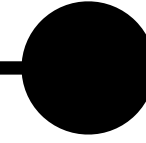
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350 BC



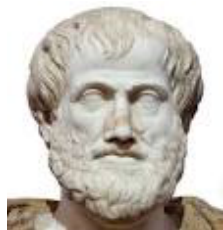
300 BC



2021



Euclid



*Organon*

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*Intro to (Formal) Logic (& AI) @ RPI*

Balderdash!

He's using syllogisms!



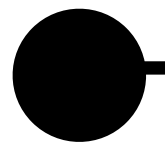
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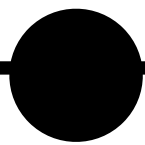
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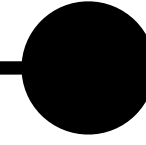
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350 BC



300 BC



2021



Euclid

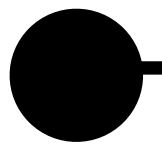


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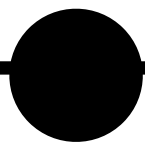
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*Intro to (Formal) Logic (& AI) @ RPI*

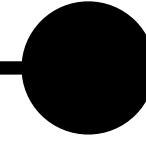
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350 BC



300 BC



2021



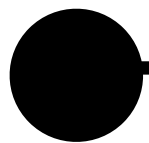
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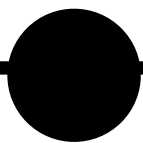
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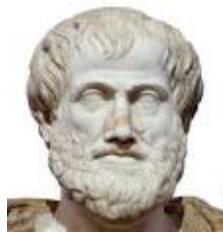
350 BC



Euclid

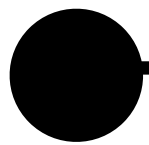


300 BC



*Organon*

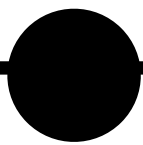
*Intro to (Formal) Logic (& AI) @ RPI*



350 BC



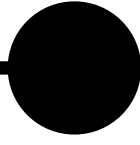
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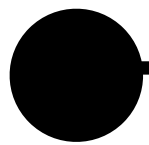


*Organon*



1666

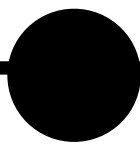
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350 BC



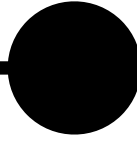
Euclid



300 BC



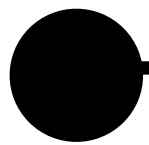
*Organon*



1666



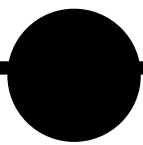
Leibniz



350 BC



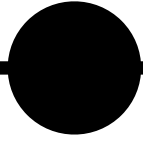
Euclid



300 BC



*Organon*



1666

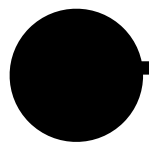
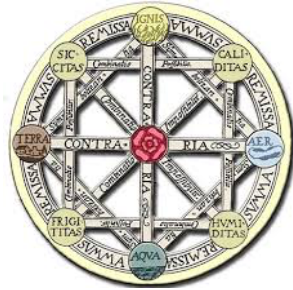


Leibniz

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*Intro to (Formal) Logic (& AI) @ RPI*

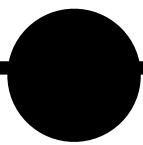
# “Universal Computational Logic”



350 BC



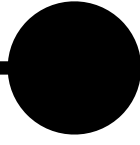
Euclid



300 BC



*Organon*



1666

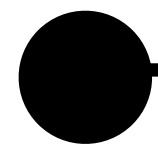
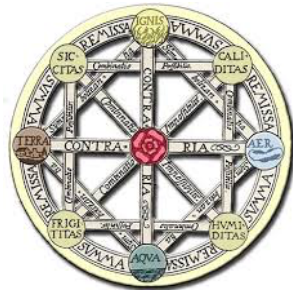


Leibniz



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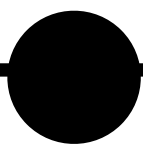
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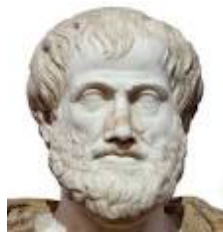
350 BC



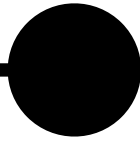
Euclid



300 BC



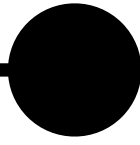
*Organon*



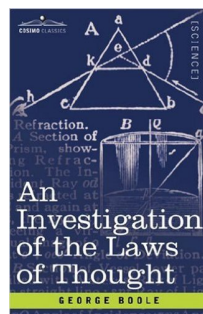
1666



Leibniz

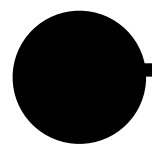
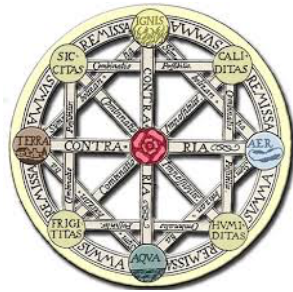


1854



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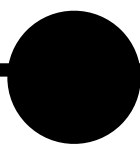
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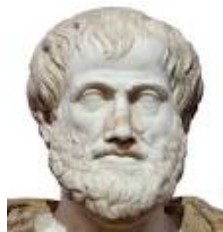
350 BC



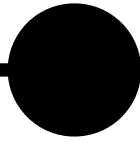
Euclid



300 BC



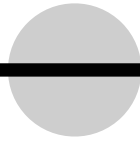
*Organon*



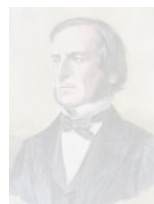
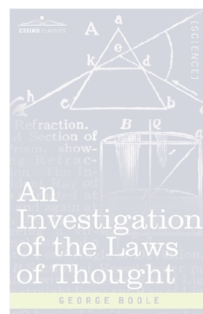
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Leibniz

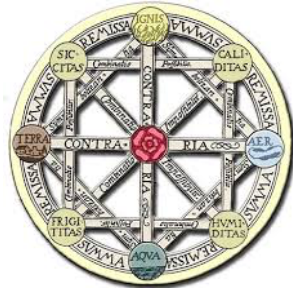


1854

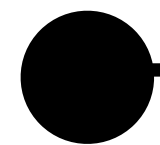


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# “Universal Computational Logic”



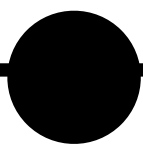
## Logic Theorist (birth of modern logicist AI)



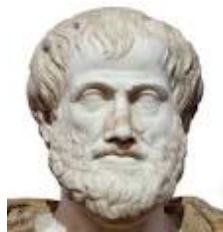
350 BC



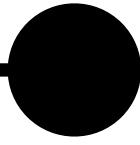
Euclid



300 BC



*Organon*

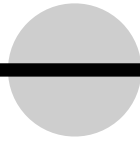


1666

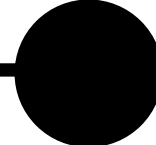
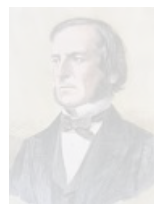
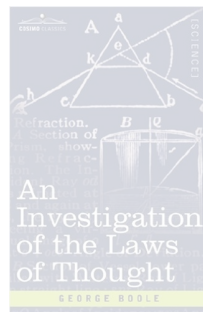


Leibniz

*∫*



1854



1956



Simon

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# “Astonishing” Logic Theorist Proof @ Dawn of AI

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|   |                                                    |                      |
|---|----------------------------------------------------|----------------------|
| 1 | $(\phi \vee \phi) \rightarrow \phi$                | axiom                |
| 2 | $(\neg\phi \vee \neg\phi) \rightarrow \neg\phi$    | substitution         |
| 3 | $(\phi \rightarrow \neg\phi) \rightarrow \neg\phi$ | a “replacement rule” |
| 4 | $(A \rightarrow \neg A) \rightarrow \neg A$        | substitution         |

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At dawn of AI: 10 seconds.

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At dawn of AI: 10 seconds.

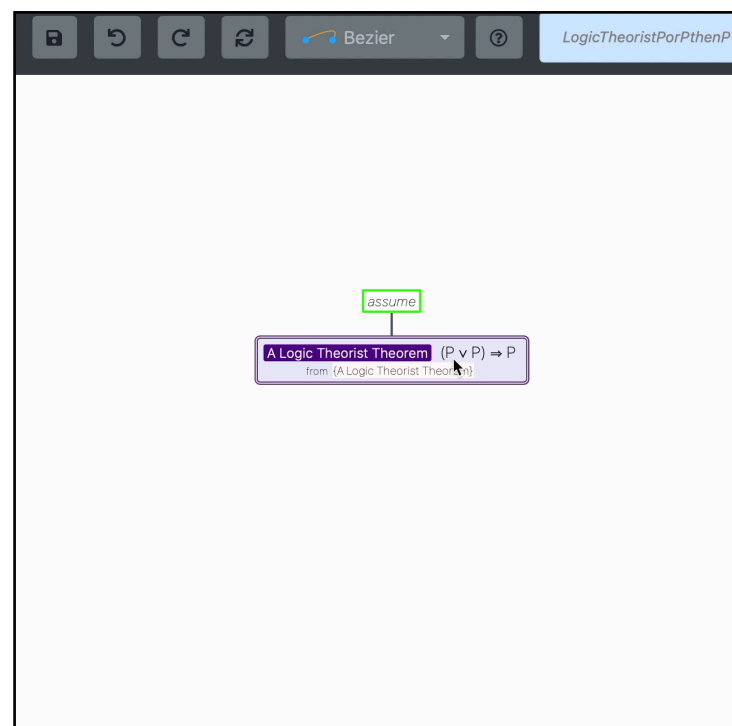
AI of today: vanishingly small amount of time (in eg HS<sup>®</sup>).

# “Astonishing” Logic Theorist Proof @ Dawn of AI

|   |                                                    |                      |
|---|----------------------------------------------------|----------------------|
| 1 | $(\phi \vee \phi) \rightarrow \phi$                | axiom                |
| 2 | $(\neg\phi \vee \neg\phi) \rightarrow \neg\phi$    | substitution         |
| 3 | $(\phi \rightarrow \neg\phi) \rightarrow \neg\phi$ | a “replacement rule” |
| 4 | $(A \rightarrow \neg A) \rightarrow \neg A$        | substitution         |

At dawn of AI: 10 seconds.

AI of today: vanishingly small amount of time (in eg HS<sup>®</sup>).

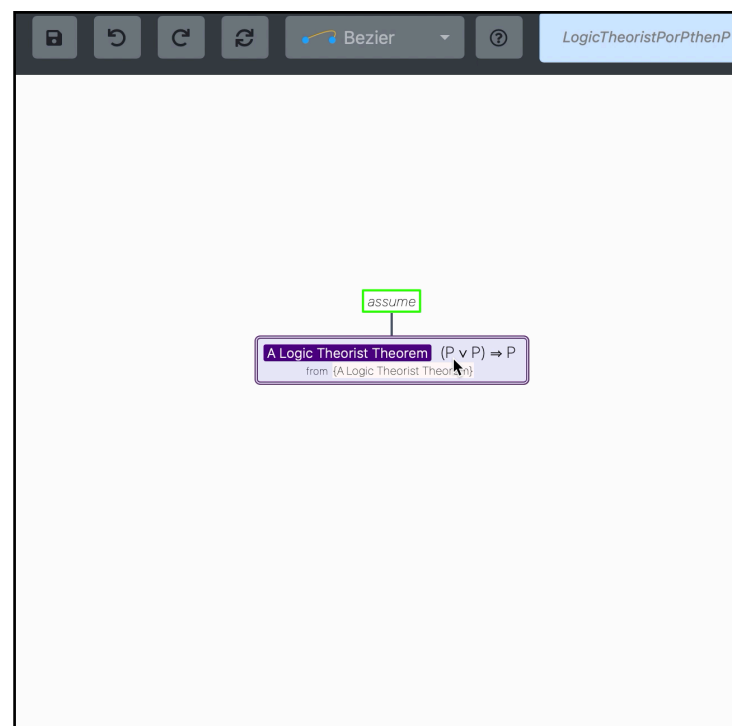


# “Astonishing” Logic Theorist Proof @ Dawn of AI

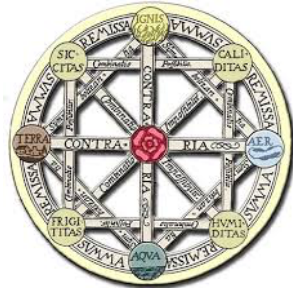
|   |                                                    |                      |
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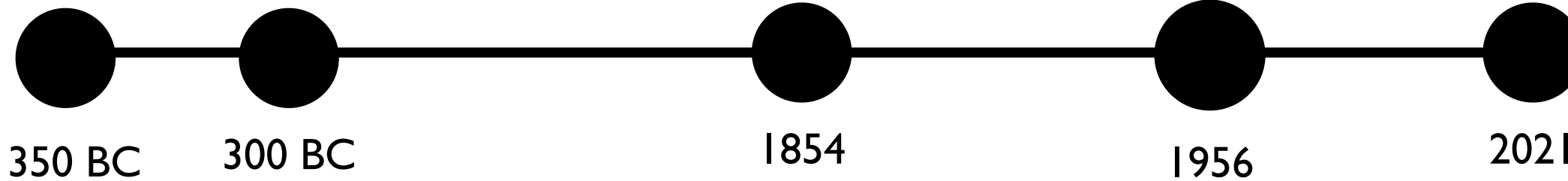
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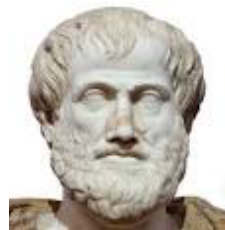
# “Universal Computational Logic”



## Logic Theorist (birth of modern logicist AI)



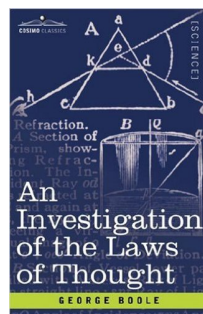
Euclid



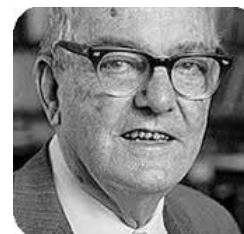
Organon



Leibniz



1854



Simon

1956

2021

*Intro to (Formal) Logic (& AI) @ RPI*

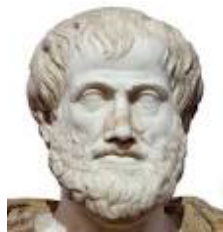


## 300 BC

1666

1854

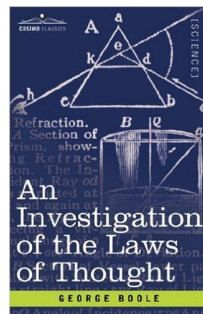
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# Euclid

# Organon

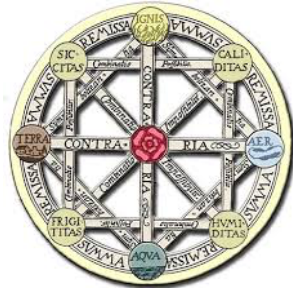
# Leibniz



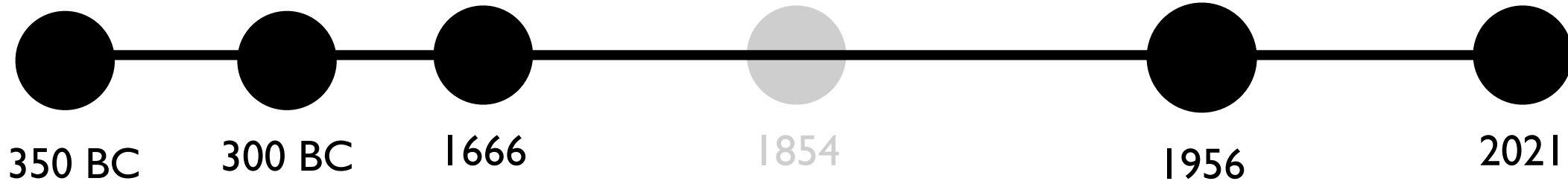
# Simon

# Intro to (Formal) Logic (& AI) @ RPI

# “Universal Computational Logic”



## Logic Theorist (birth of modern logicist AI)



350 BC

300 BC

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1956

2021



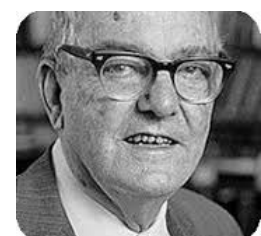
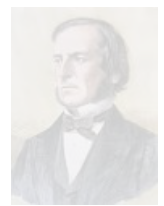
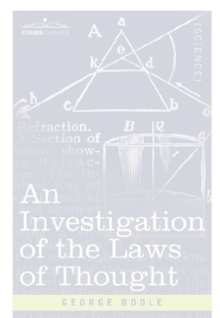
Euclid



*Organon*



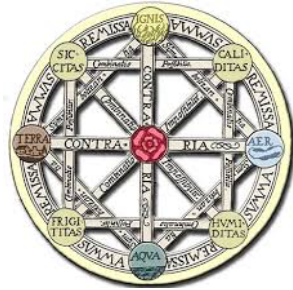
Leibniz



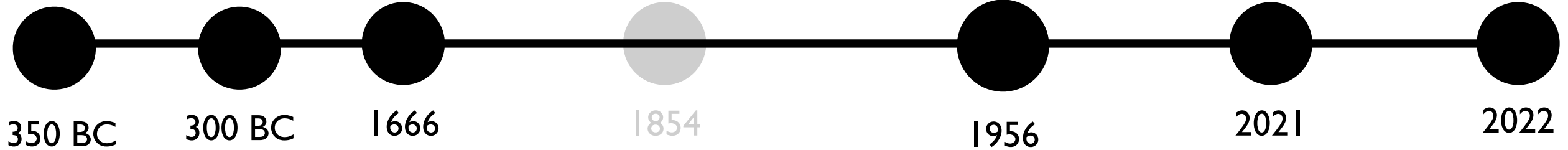
Simon

*Intro to (Formal) Logic (& AI) @ RPI*

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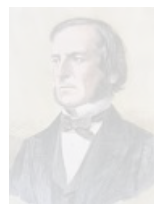
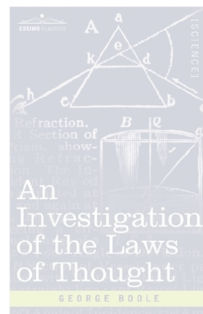


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Leibniz

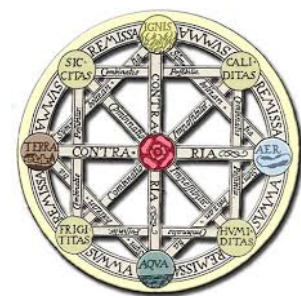
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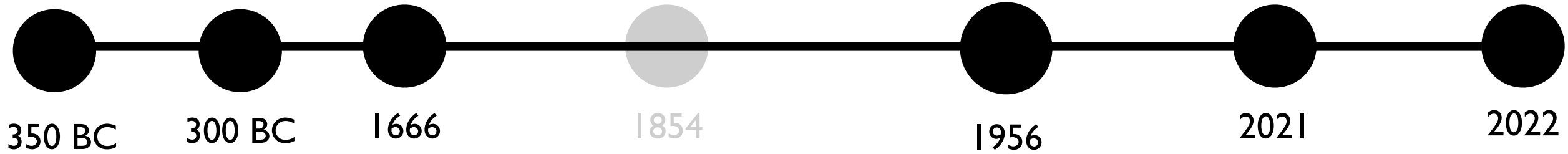
Simon

*Intro to (Formal) Logic (& AI) @ RPI*

“Universal  
Computational  
Logic”



Logic Theorist  
(birth of modern logicist AI)



Euclid

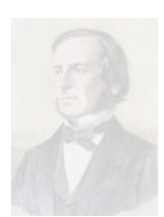
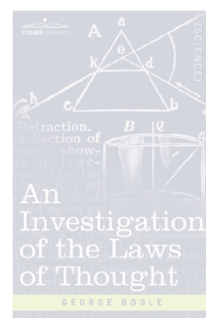


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Leibniz

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1854



Simon

1956

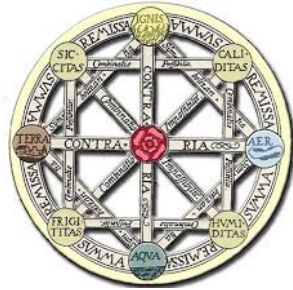
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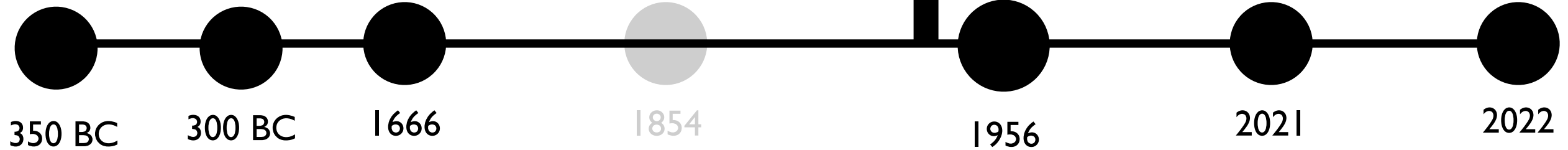
*Intro to (Formal) Logic (& AI) @ RPI*

# Entscheidungsproblem

“Universal  
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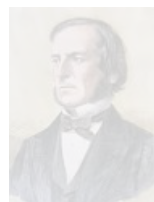
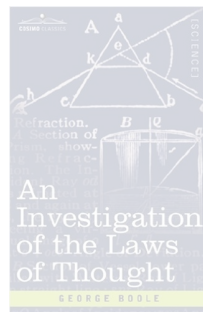


*Organon*



Leibniz

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Simon

*Intro to (Formal) Logic (& AI) @ RPI*

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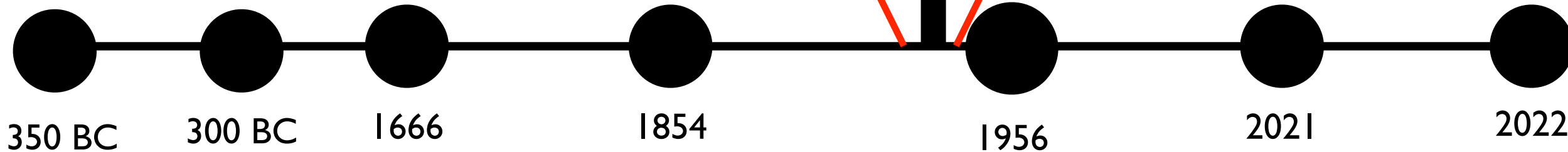
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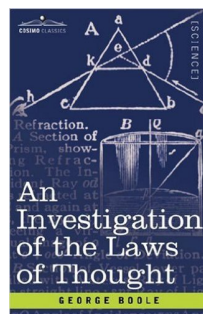


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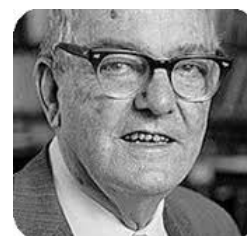


Leibniz

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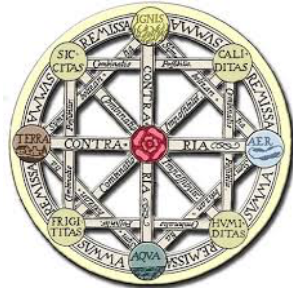
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*Intro to (Formal) Logic (& AI) @ RPI*

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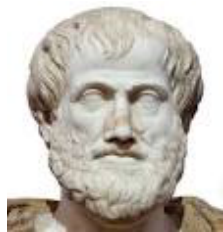
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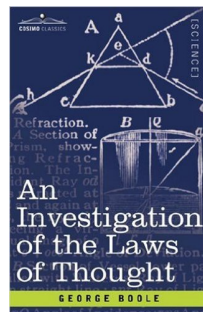


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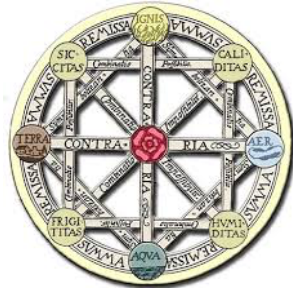
Simon

*Intro to (Formal) Logic (& AI) @ RPI*

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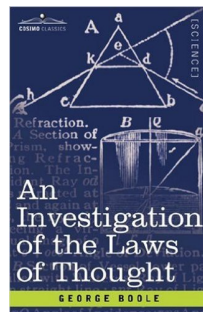


*Organon*



Leibniz

$\int$



Simon



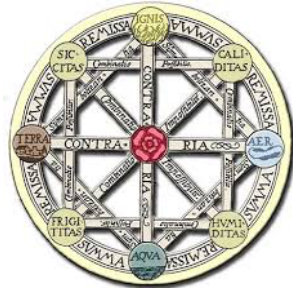
Frege

*Intro to (Formal) Logic (& AI) @ RPI*

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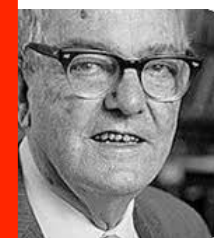
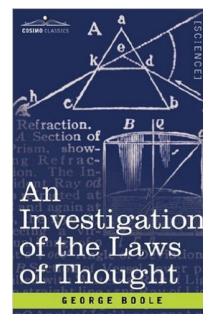


*Organon*



Leibniz

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Simon



Frege

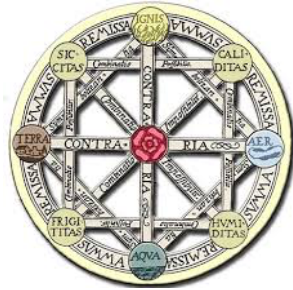
Exceeds Leibniz & de-mystifies  
Euclid: the “compellingness” of  
these proofs consists in their  
being, at bottom, formal proofs  
in first-order logic (FOL).

*Intro to (Formal) Logic (& AI) @ RPI*

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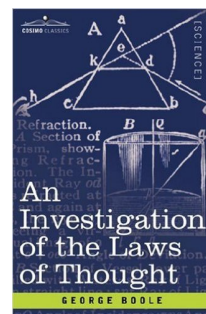


*Organon*



Leibniz

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Frege

Exceeds Leibniz & de-mystifies Euclid: the “compellingness” of these proofs consists in their being, at bottom, formal proofs in first-order logic (FOL).



Church



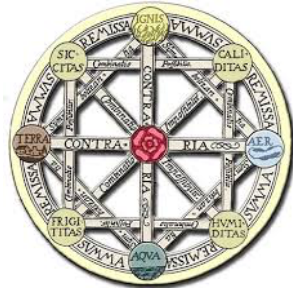
Simon

*Intro to (Formal) Logic (& AI) @ RPI*

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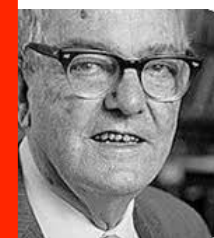
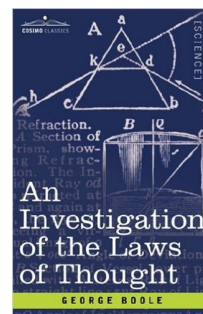


*Organon*



Leibniz

$\int$



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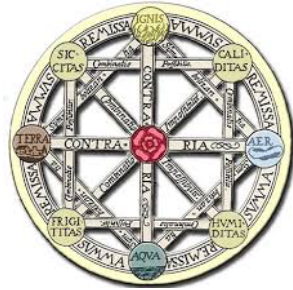
Turing

*Intro to (Formal) Logic (& AI) @ RPI*

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# Entscheidungsproblem

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Logic Theorist  
(birth of modern logicist AI)

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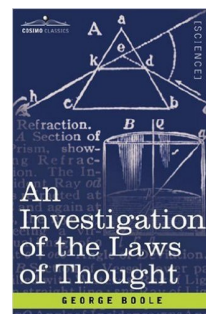


*Organon*



Leibniz

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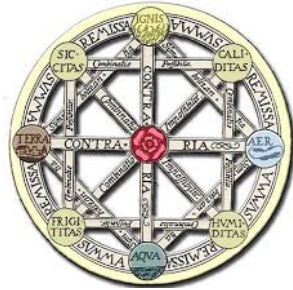
Post

*Intro to (Formal) Logic (& AI) @ RPI*

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# Entscheidungsproblem

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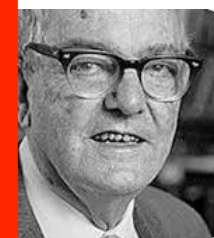
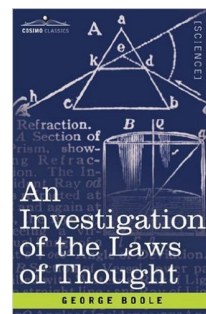


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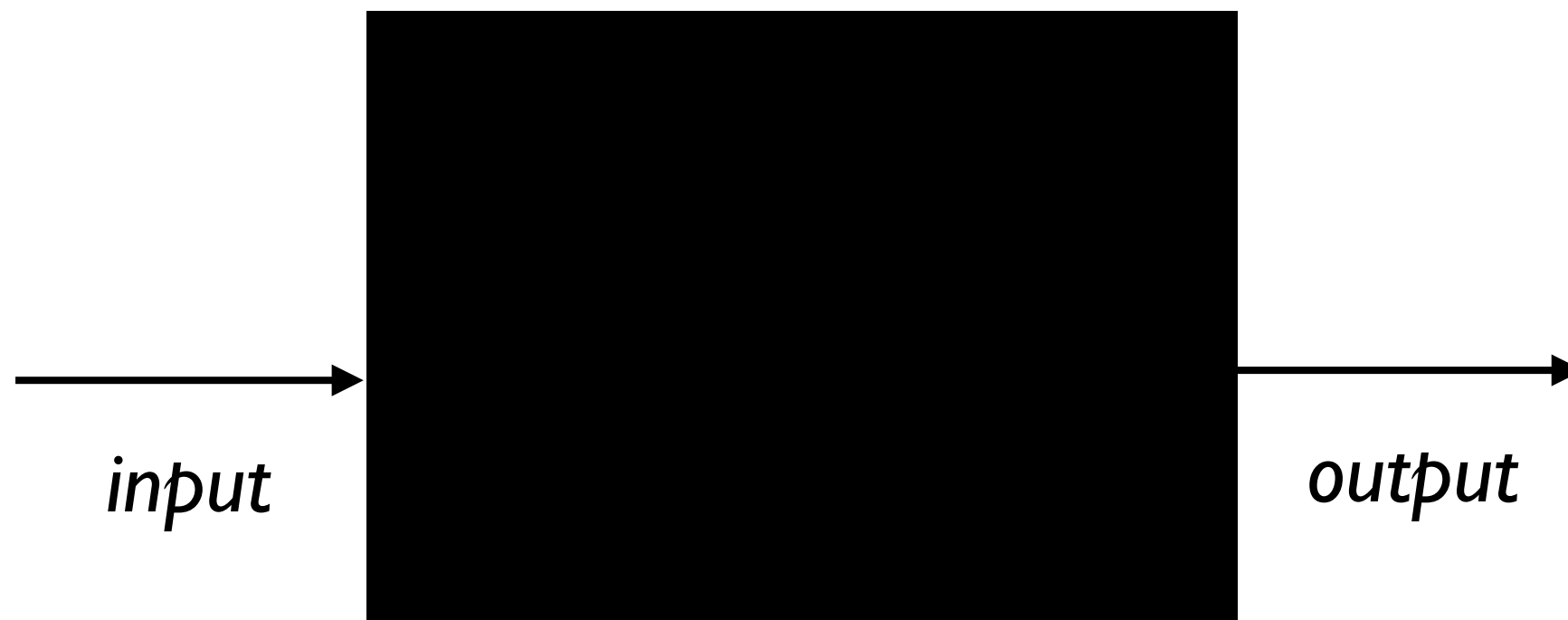
Post

Intro to (Formal) Logic (& AI) @ RPI

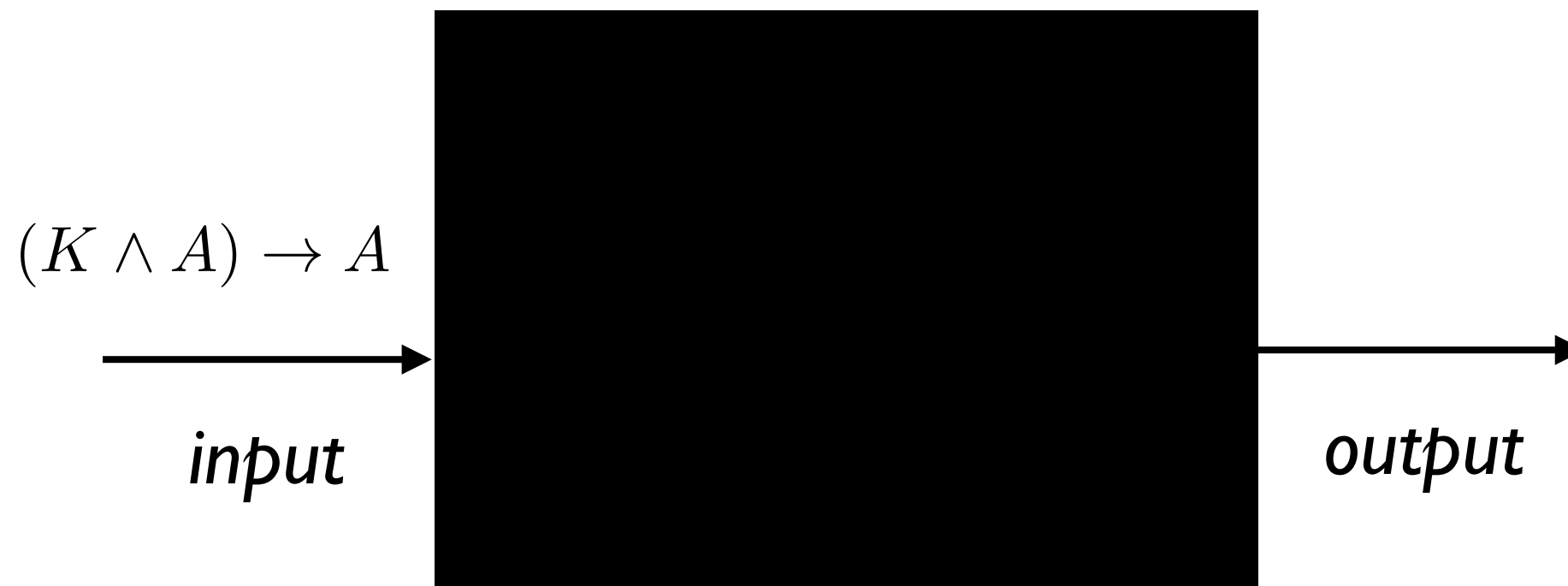
Here’s what a computer is, and given that, sorry, the *Entscheidungsproblem* can’t be solved by such a machine!

The Singularity?

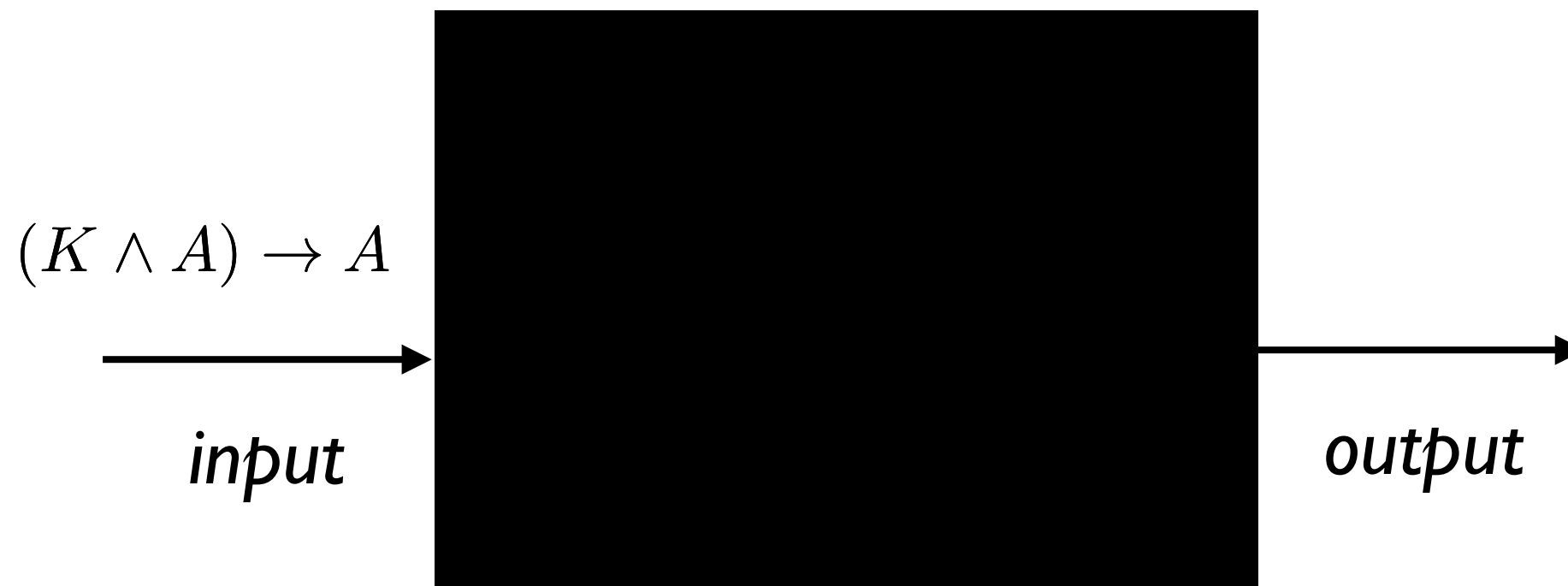
First, the Theoremhood Decision Problem  
( $\text{THEOREM}_{\text{PC}}$ )  
for the Propositional Calculus



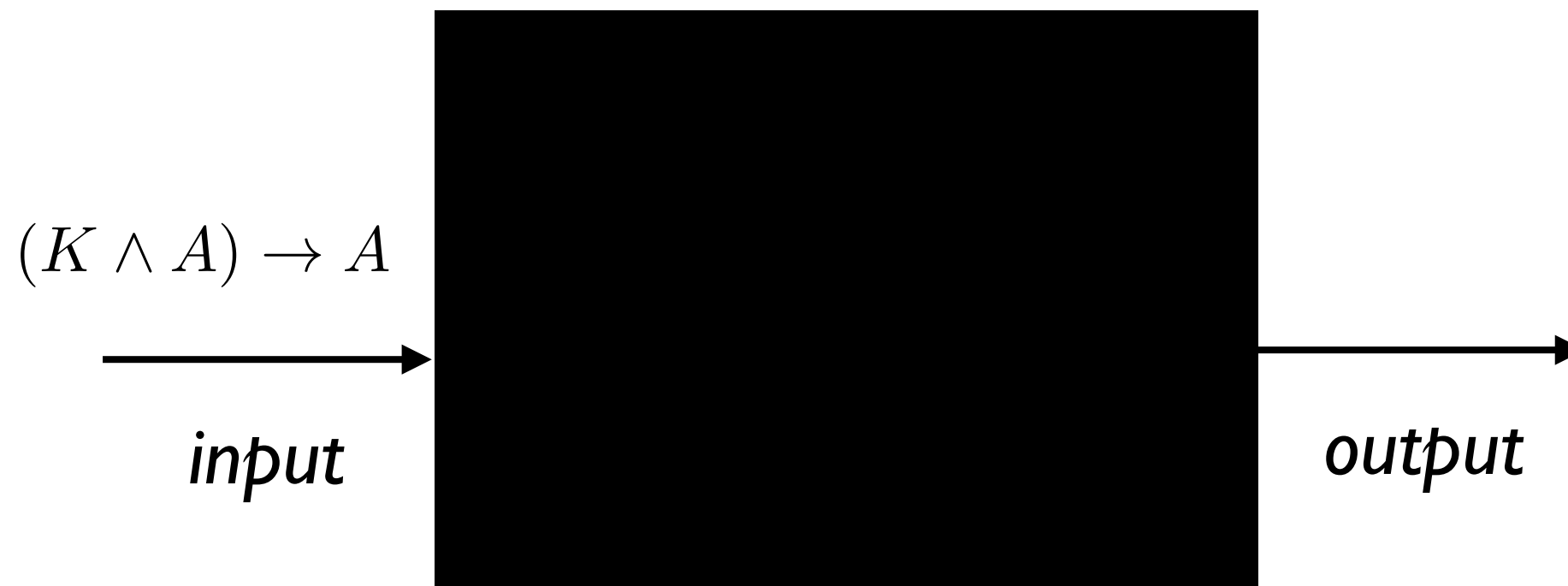
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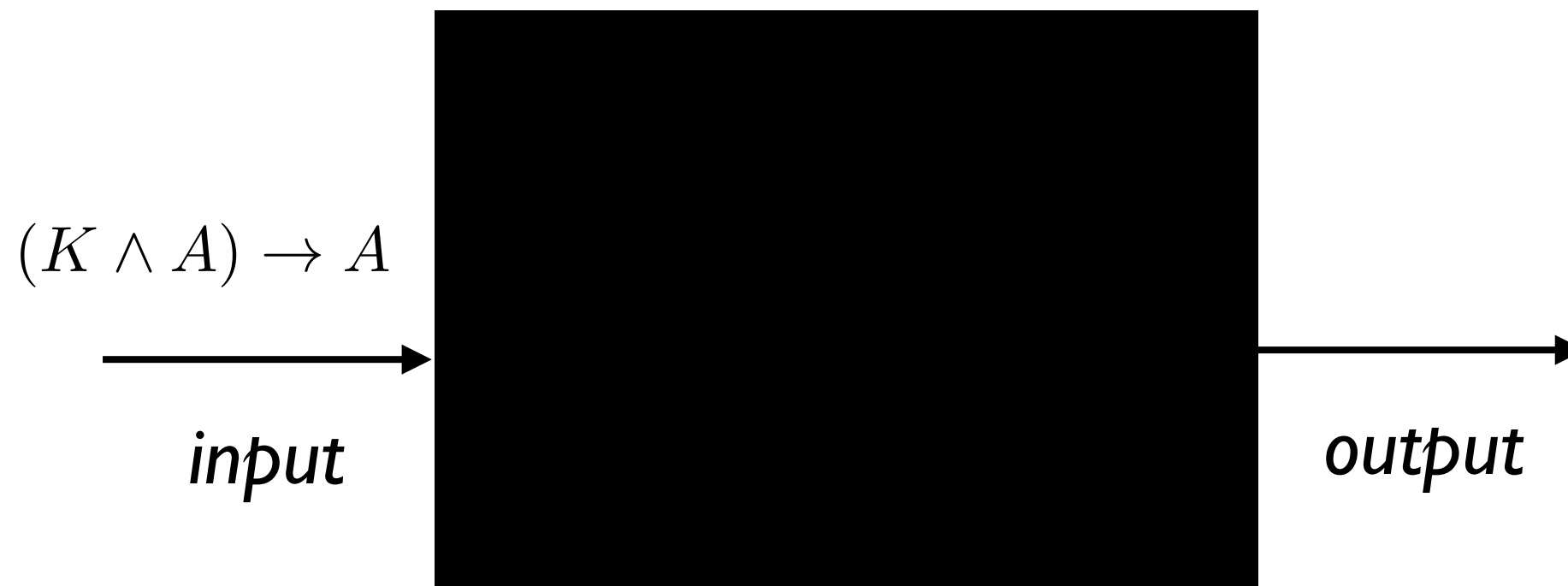
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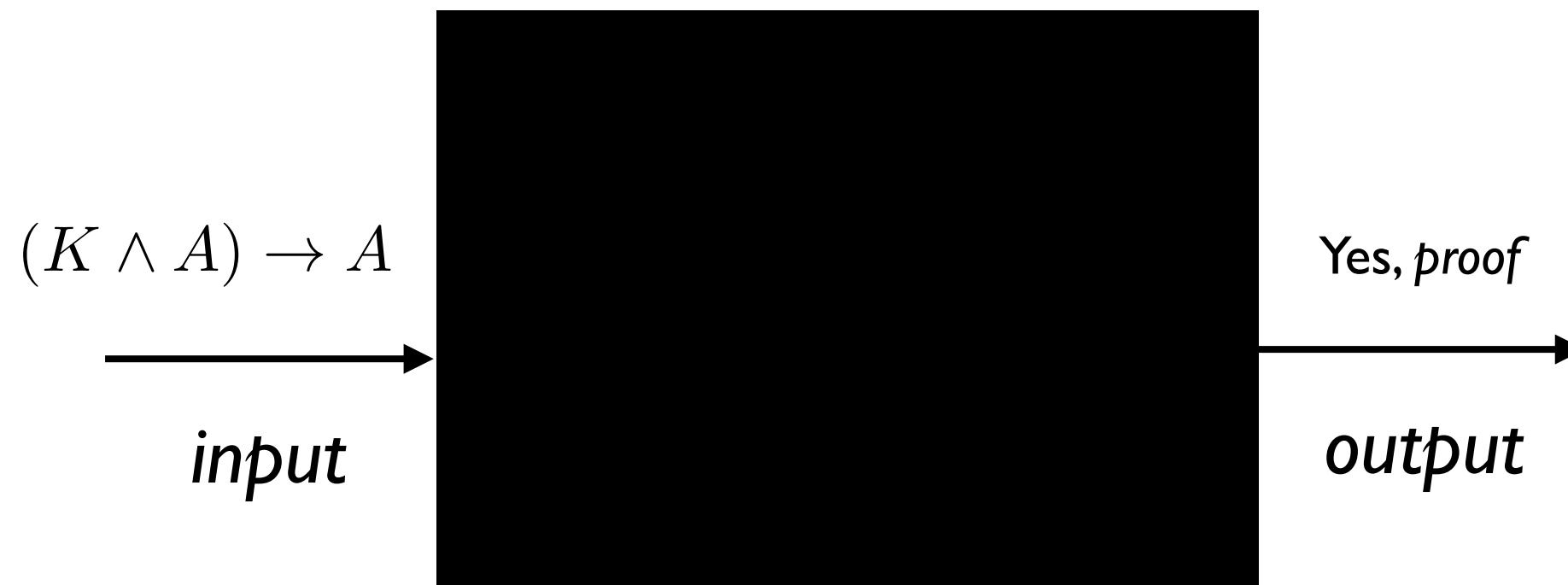
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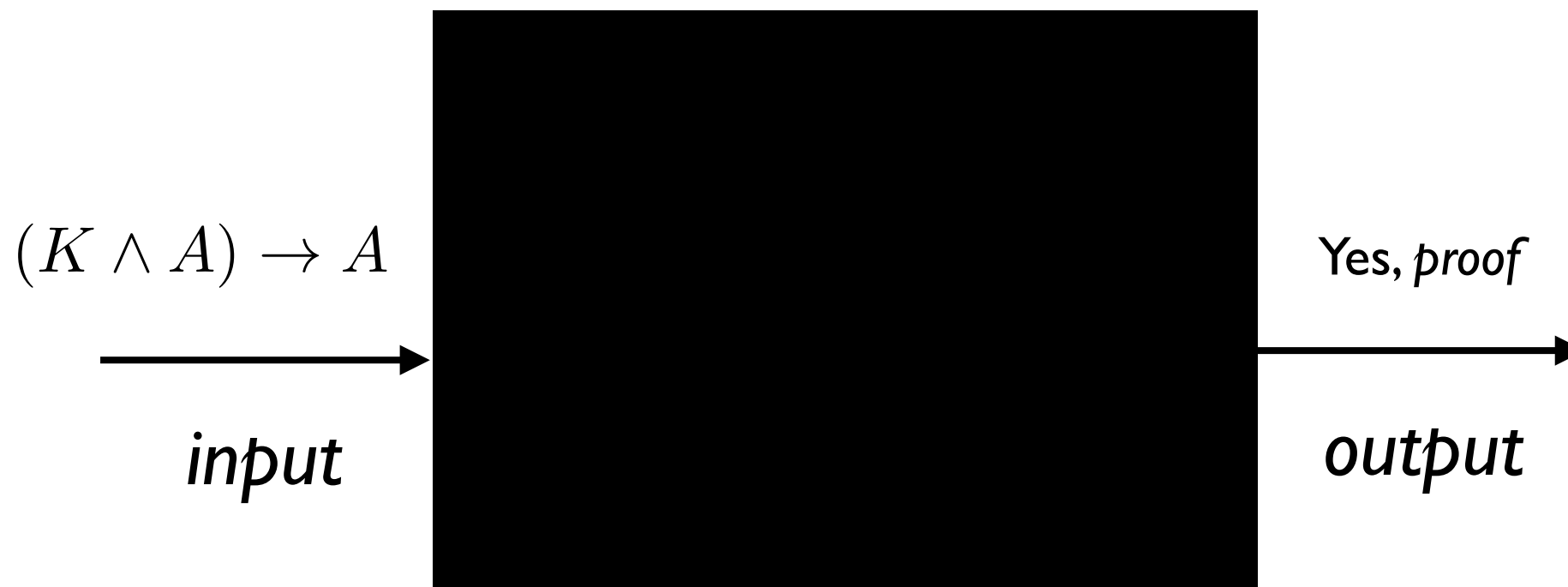
First, the Theoremhood Decision Problem  
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# First, the Theoremhood Decision Problem ( $\text{THEOREM}_{\text{PC}}$ ) for the Propositional Calculus

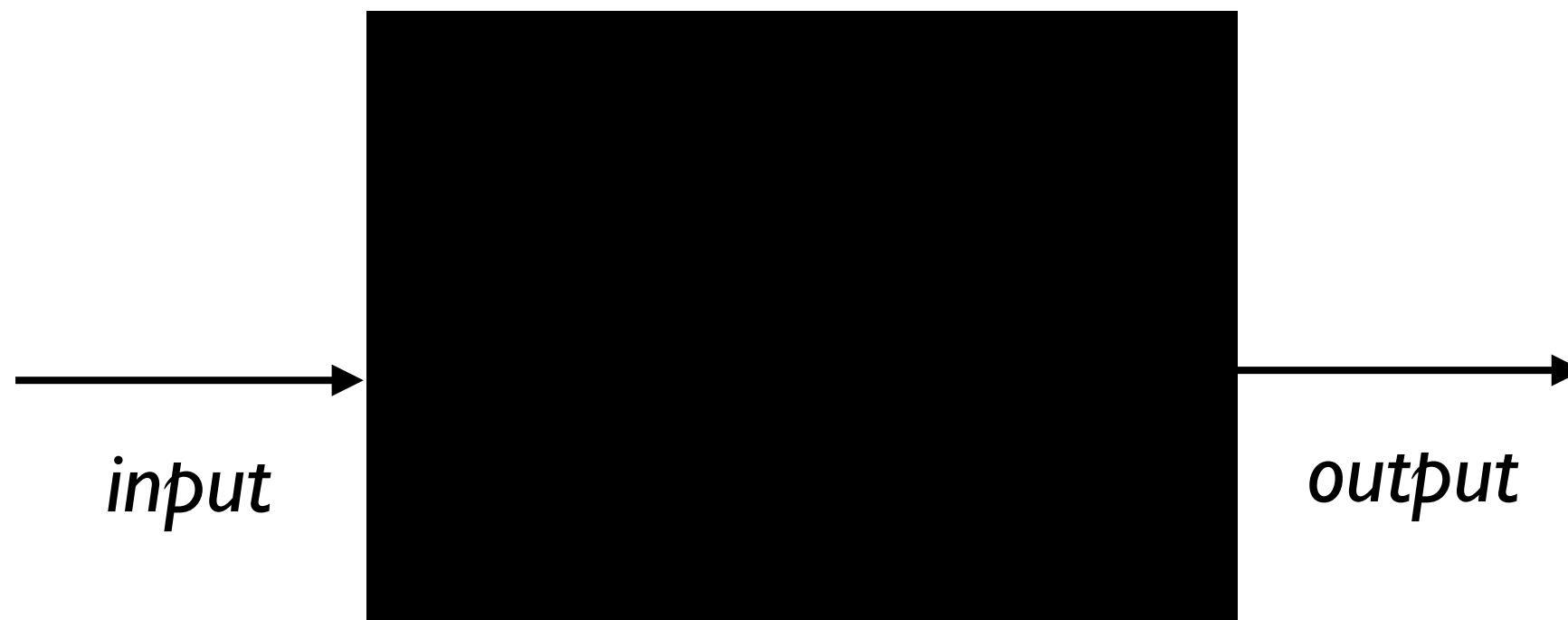


First, the Theoremhood Decision Problem  
( $\text{THEOREM}_{\text{PC}}$ )  
for the Propositional Calculus



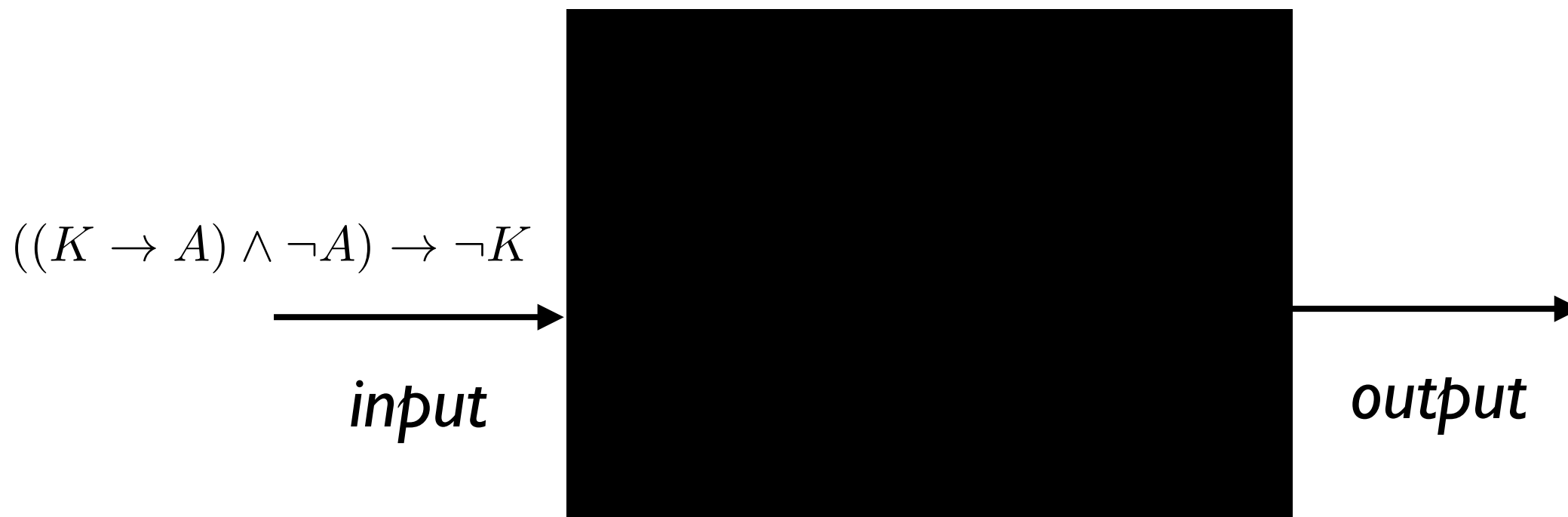
**Hard!! — for apparently no polynomial-time algorithm for this!**

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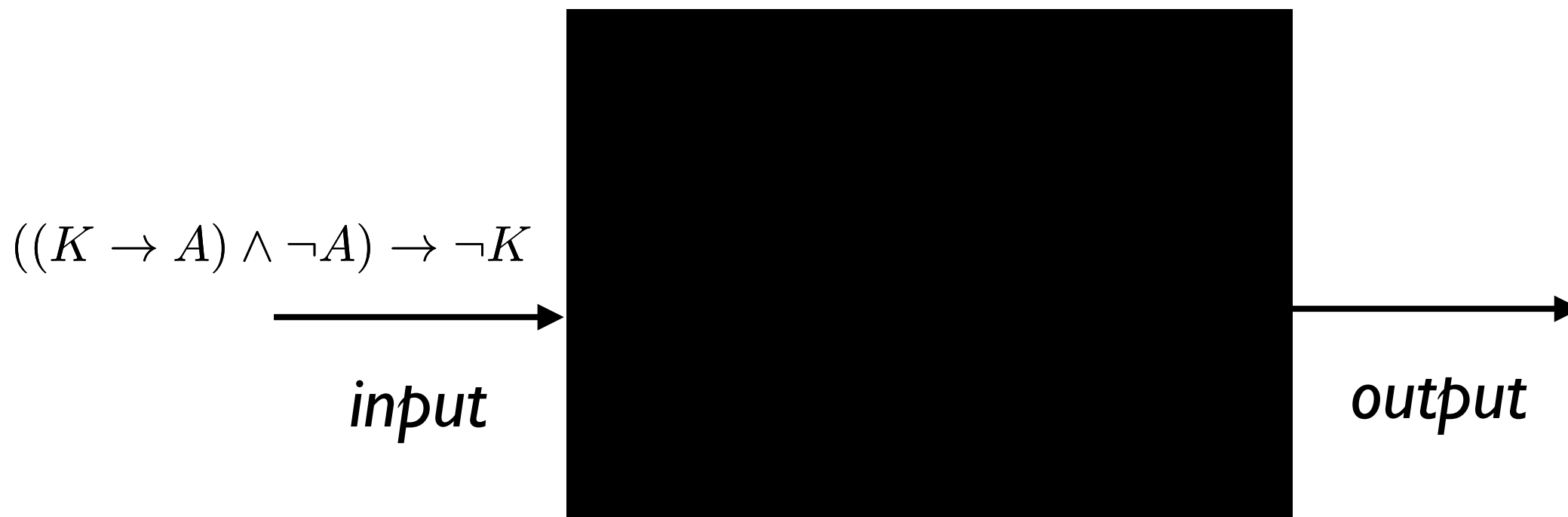
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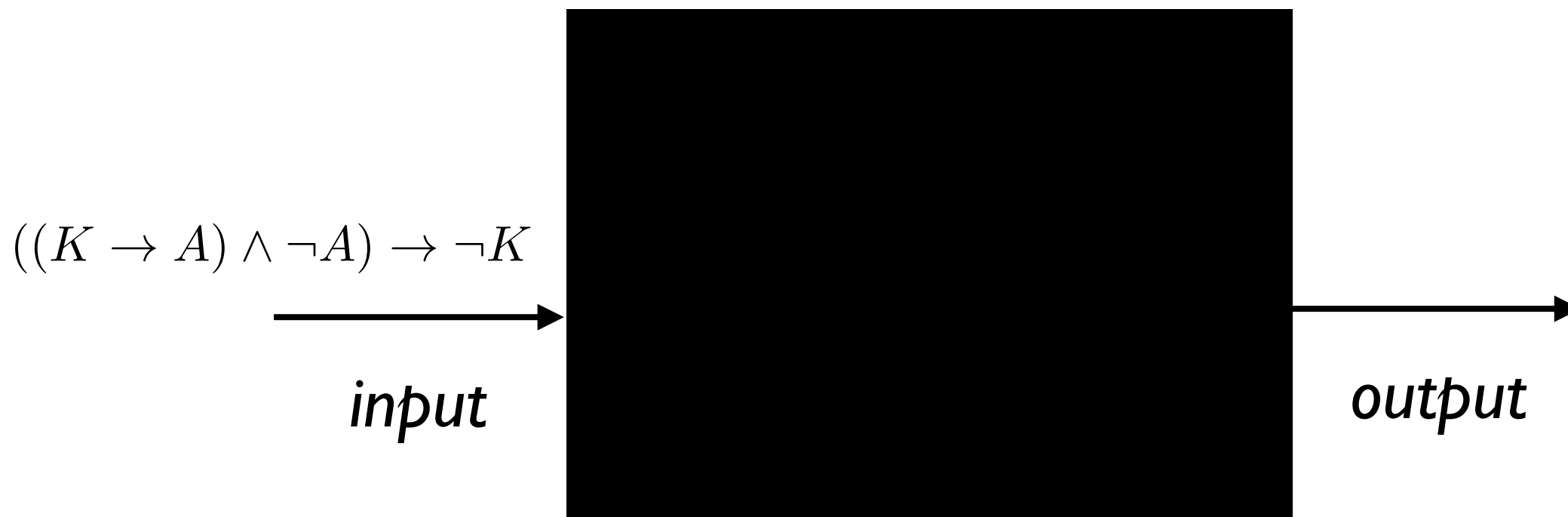
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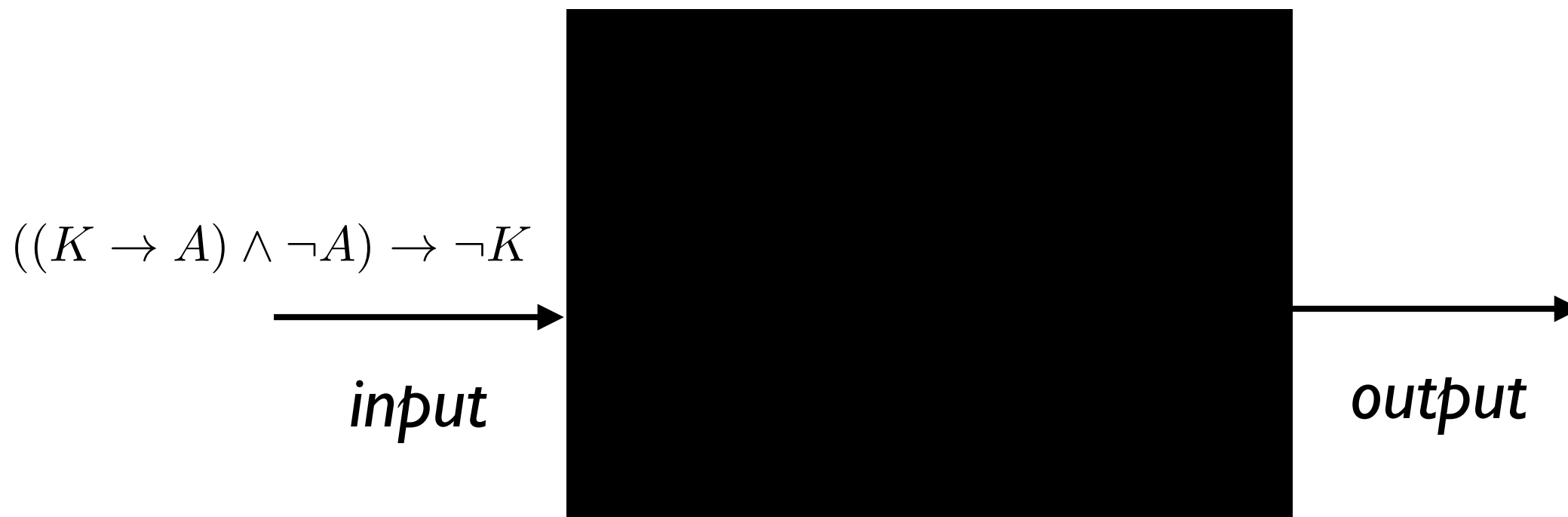
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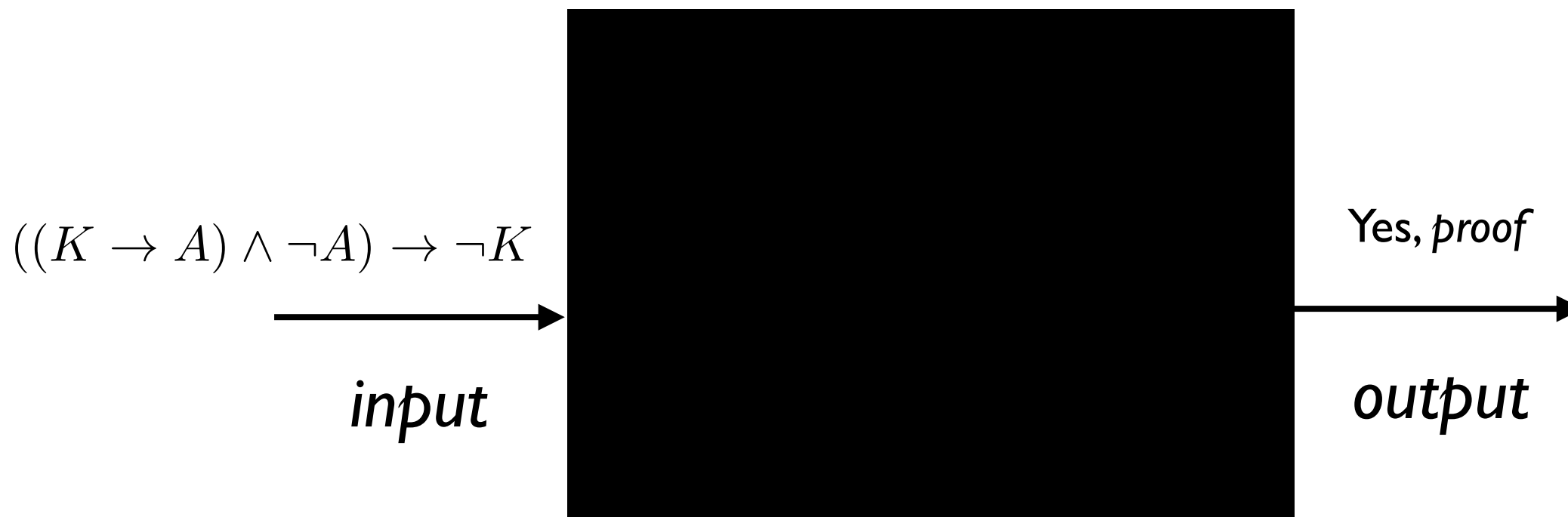
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First, the Theoremhood Decision Problem  
(THEOREM<sub>PC</sub>)  
for the Propositional Calculus



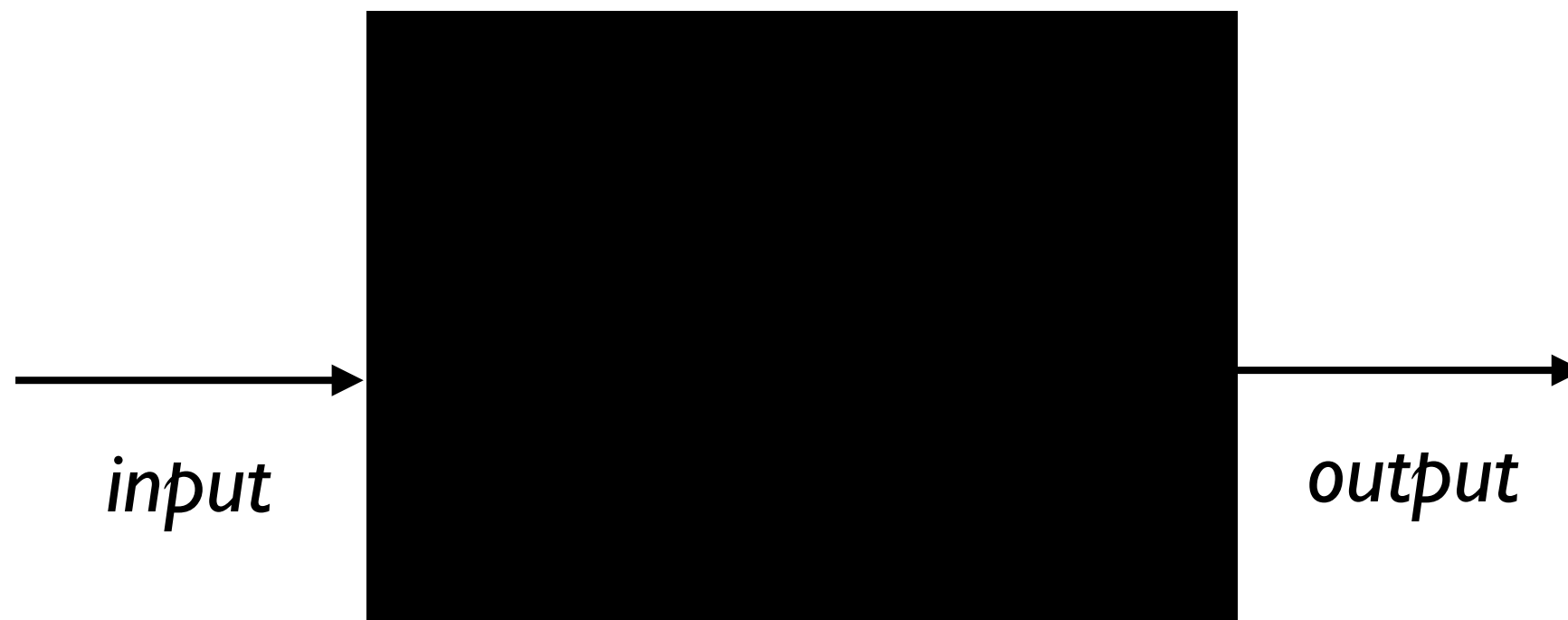
**Hard!! — for apparently no polynomial-time algorithm for this!**

# First, the Theoremhood Decision Problem (THEOREM<sub>PC</sub>) for the Propositional Calculus

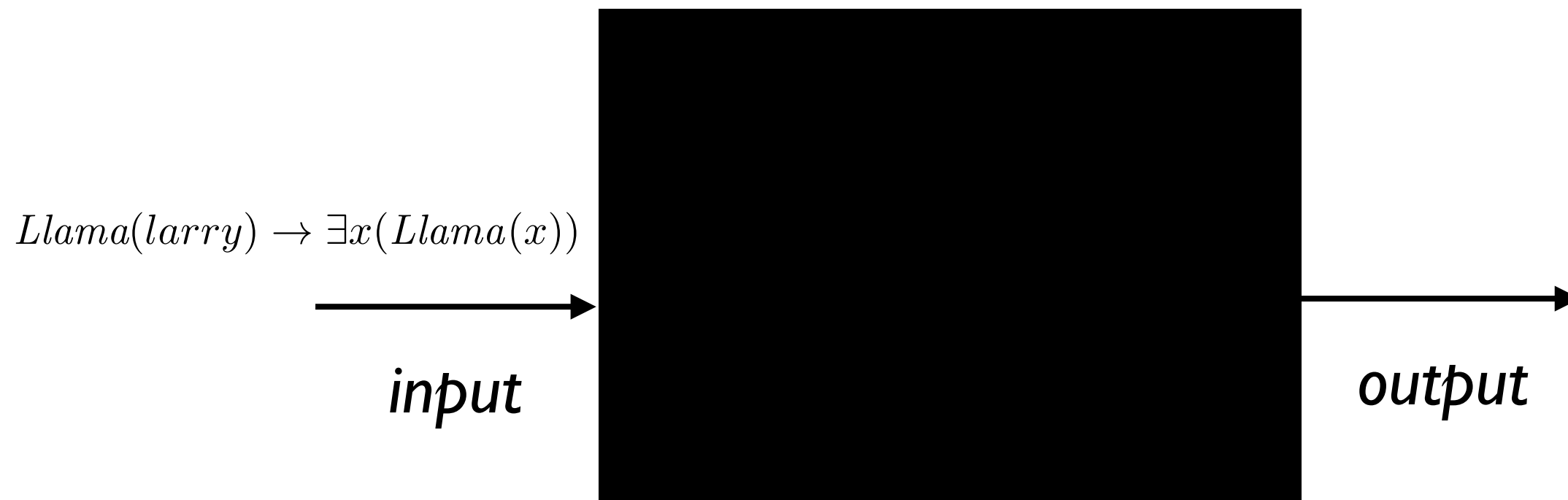


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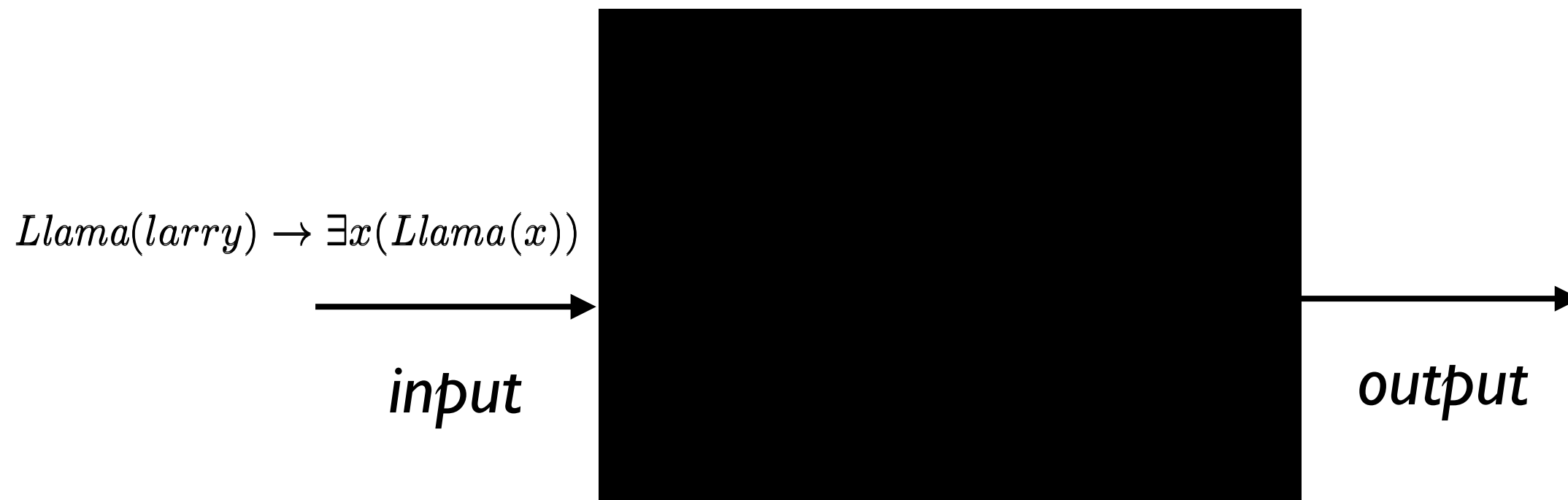
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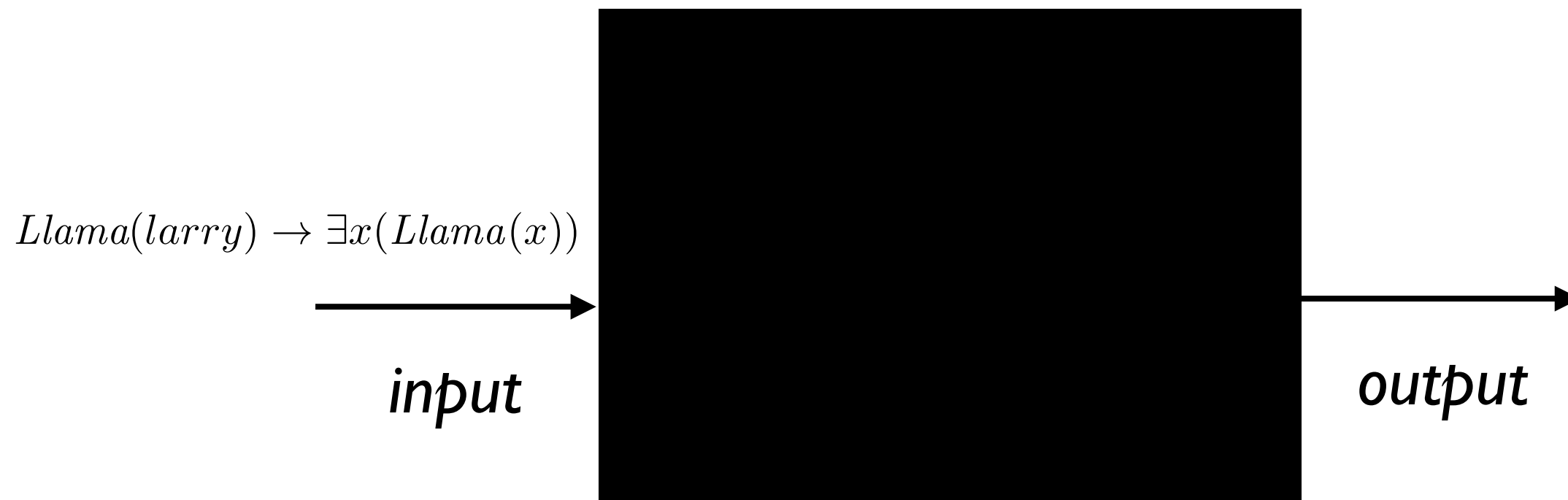
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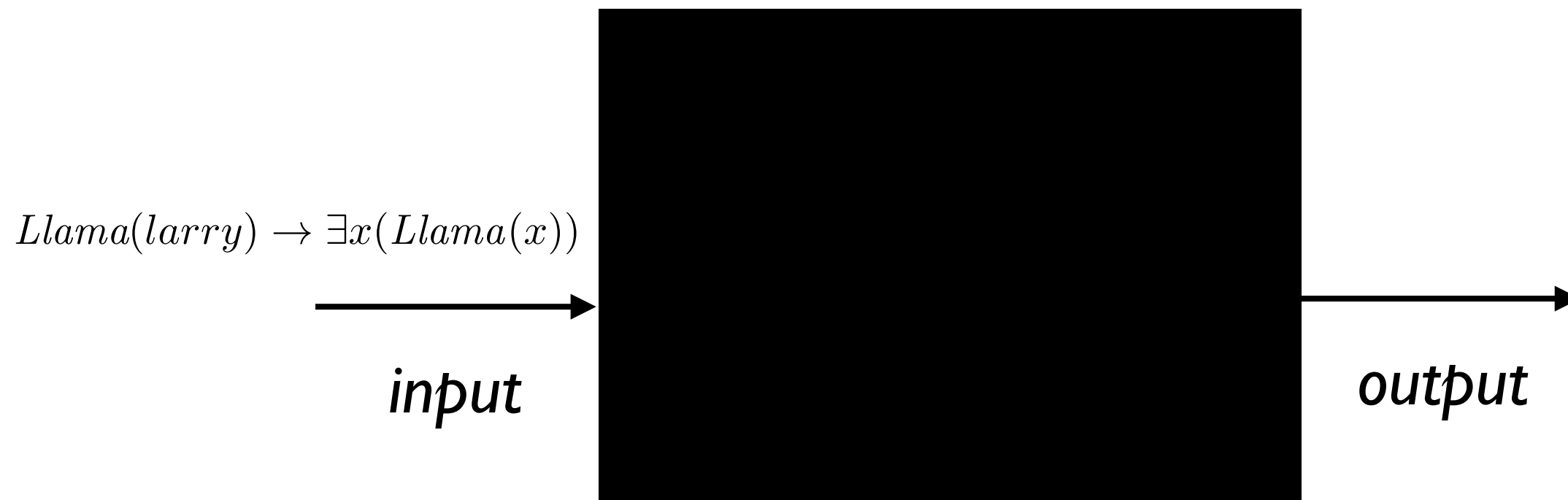
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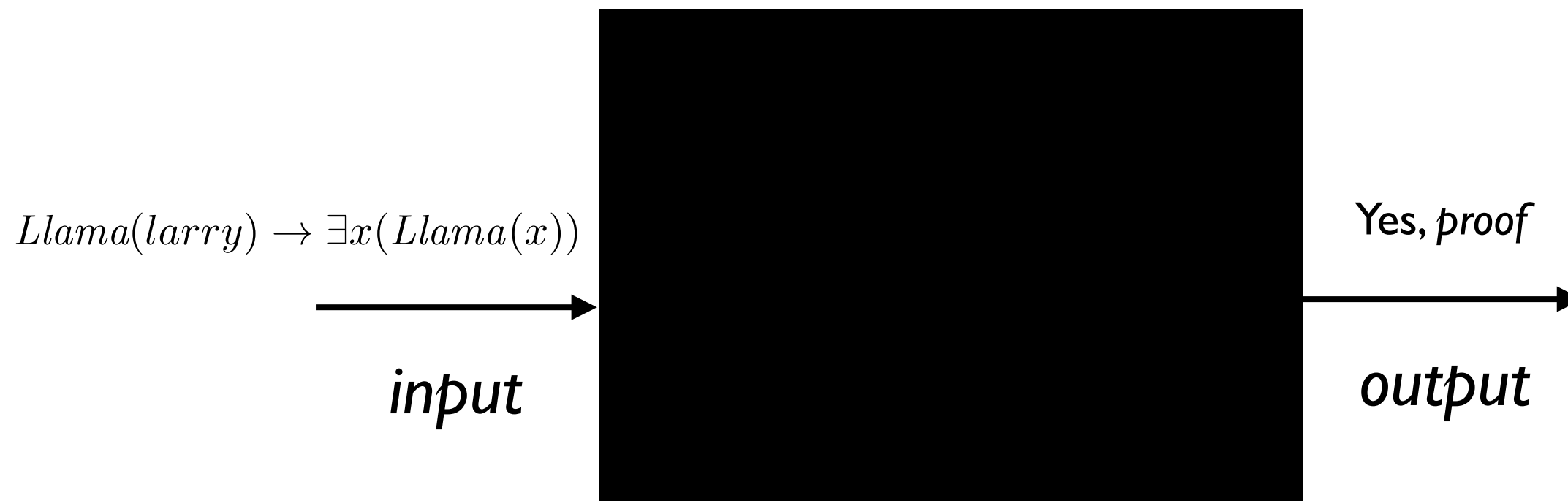
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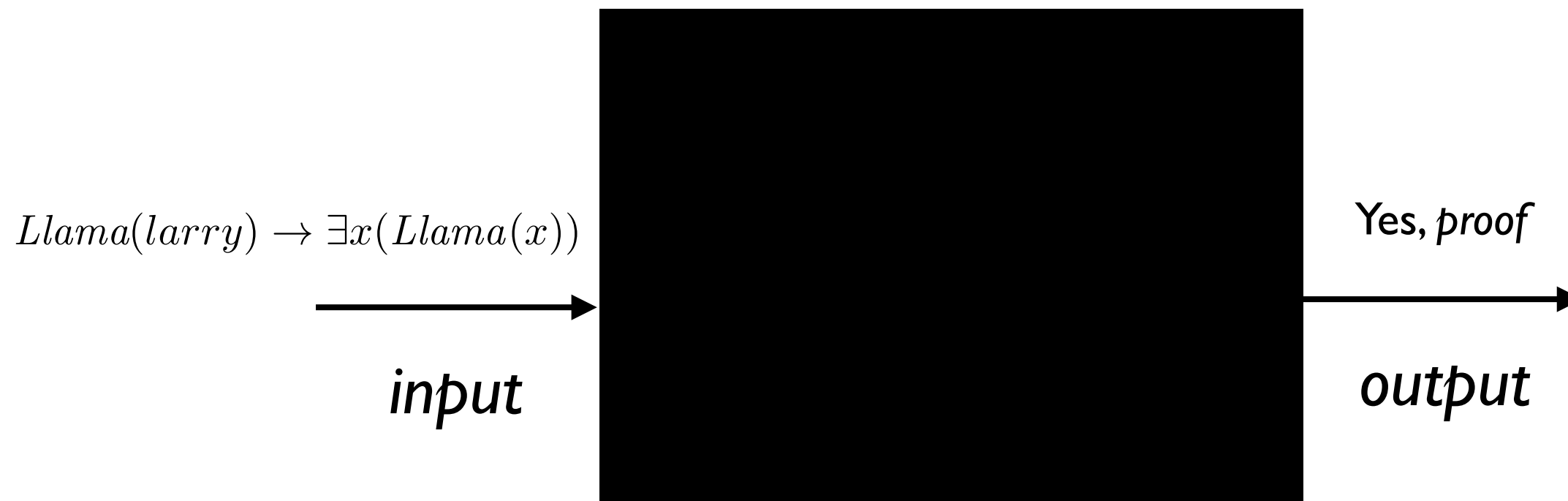
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And now, the Theoremhood Decision Problem,  
i.e., the *Entscheidungsproblem*,  
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for First-Order Logic (FOL)



**Not just hard: *impossible* for a (and this needed to be *invented* in the course of clarifying and solving the problem) standard computing machine.**

# Applying this to ...

## The Singularity Question

# Applying this to ...

## The Singularity Question

$\mathcal{A}$ :

**Premise 1**    There will be AI (created by HI and such that  $AI = HI$ ).

**Premise 2**    If there is AI, there will be  $AI^+$  (created by AI).

**Premise 3**    If there is  $AI^+$ , there will be  $AI^{++}$  (created by  $AI^+$ ).

$\therefore$     **S**            There will be  $AI^{++}$  (=  $\mathcal{S}$  will occur).

(Good-Chalmers Argument)

(Kurzweil is an “extrapolationist.”)

# Applying this to ...

## The Singularity Question

So, these super-smart machines that will be built by human-level-smart machines, they can't *possibly* be smart enough to solve the *Entscheidungsproblem*. Hence they'll be just (recursively) faster at solving problems we can routinely solve? What's so super-smart about *that*?



LAMA-BDLA

**LAMA-BDLA**

# LAMA-BDLA

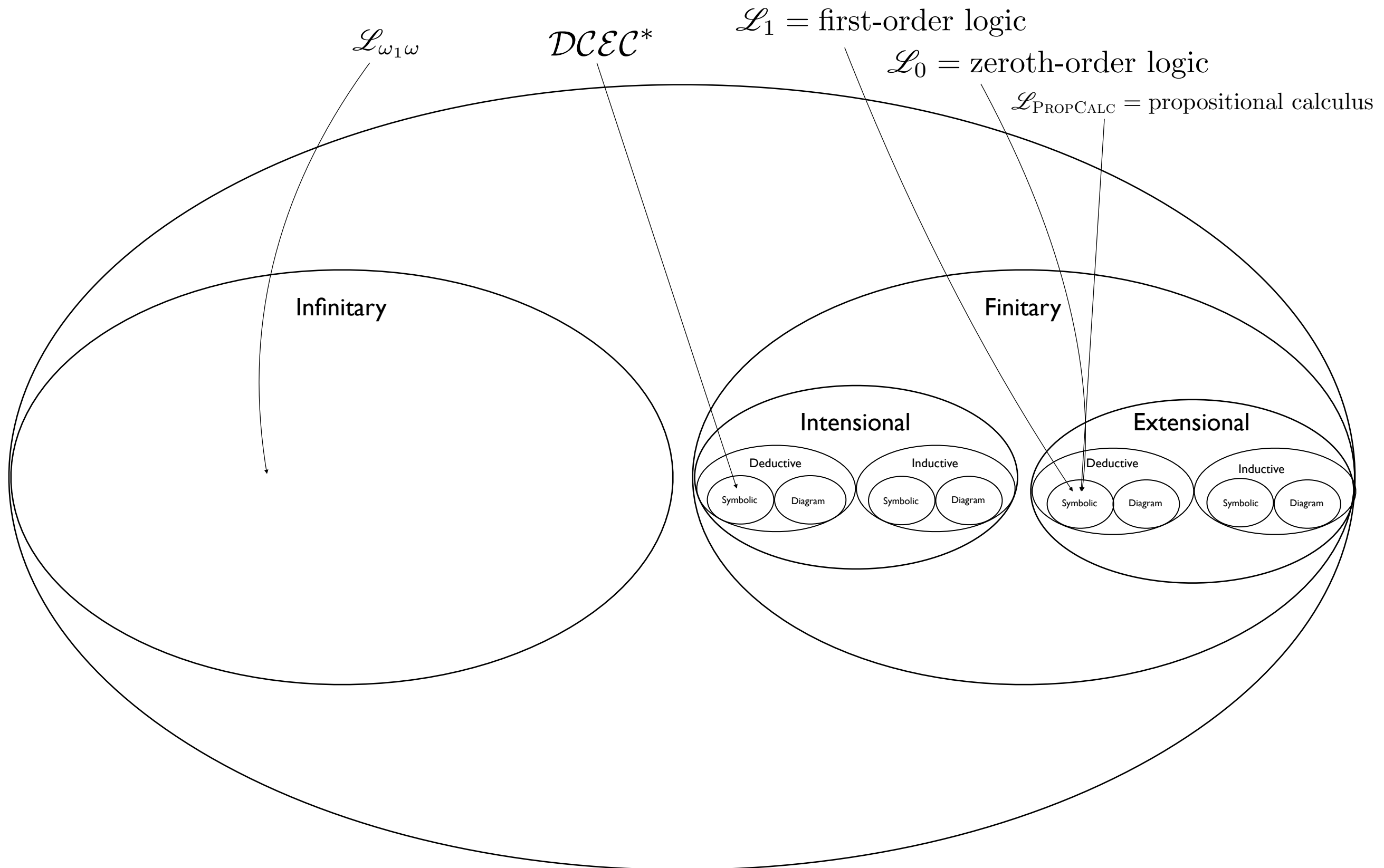


**LAMA-BDLA**

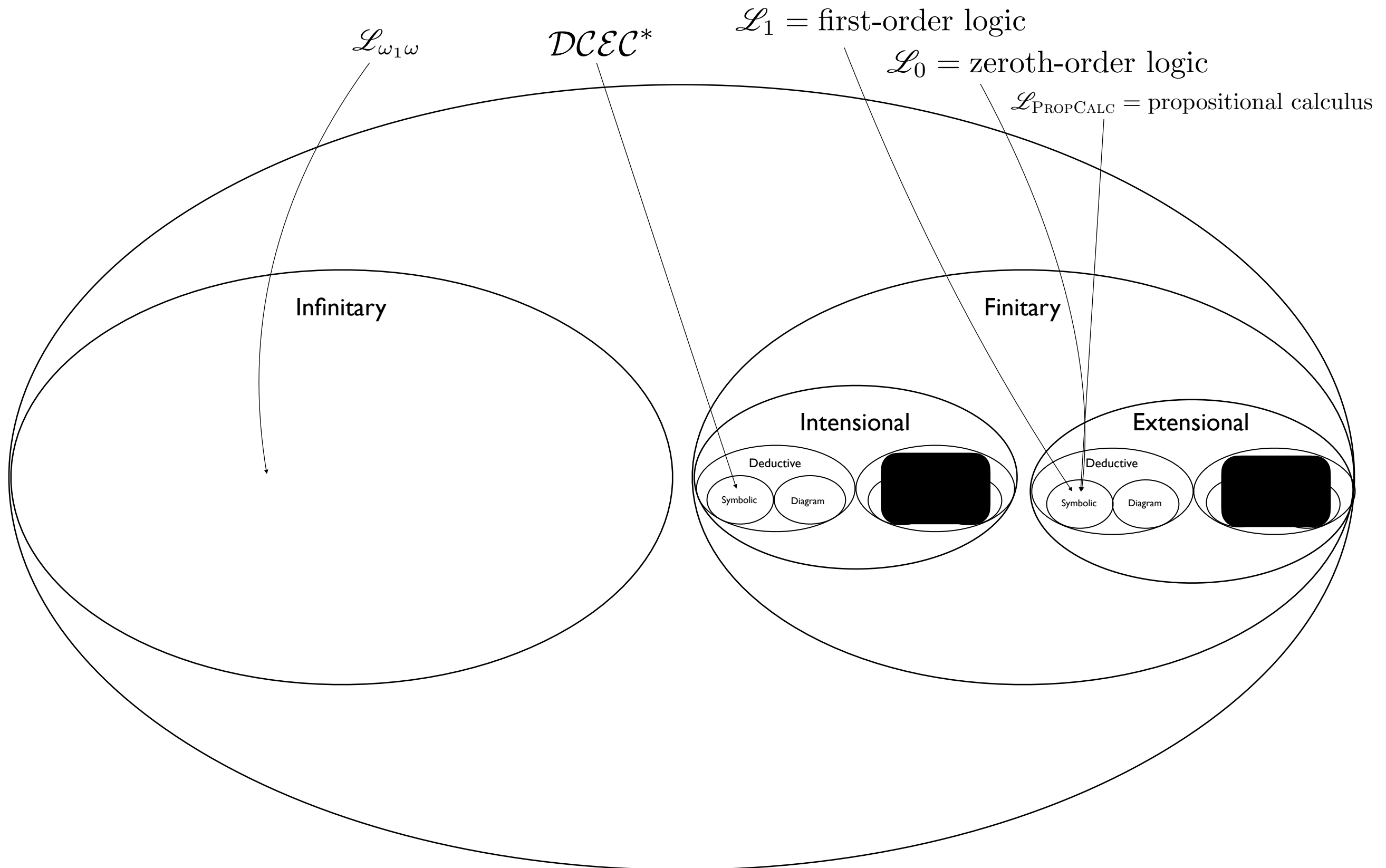


**LAMA-BIL, a bit.**

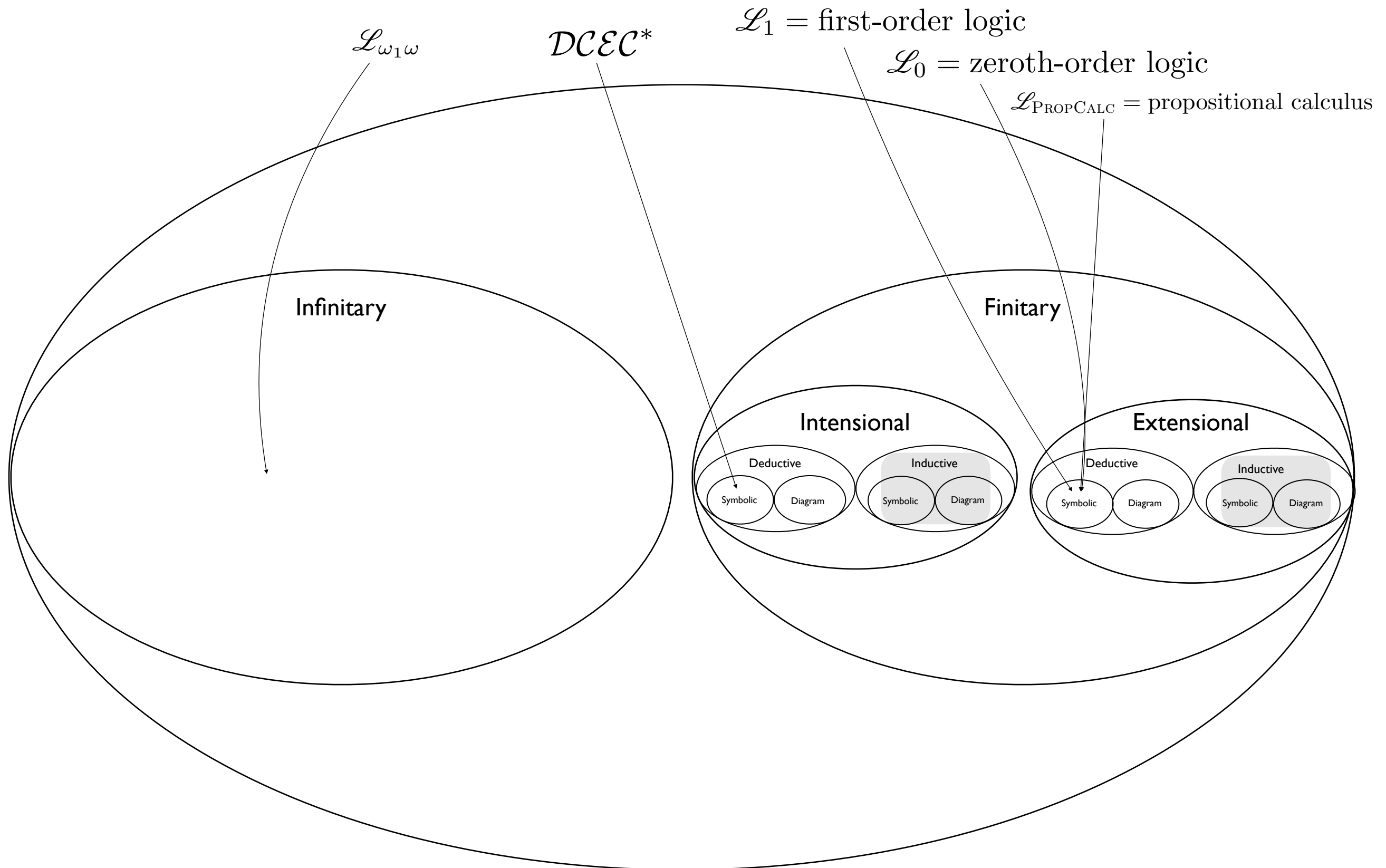
# The Universe of Logics



# The Universe of Logics



# The Universe of Logics





# The Monty Hall Problem



\$1M





# The Monty Hall Problem



\$1M





# The Monty Hall Problem



\$1M





# The Monty Hall Problem



\$1M





# The Monty Hall Problem



\$1M





# The Monty Hall Problem





# The Monty Hall Problem



\$1M



# The Monty Hall Problem



\$1M



# MHP Defined

Jones has come to a game show, and finds himself thereon selected to play a game on national TV with the show's suave host, Full Monty. Jones is told correctly by Full that hidden behind one of three closed, opaque doors facing the two of them is \$1,000,000, while behind each of the other two is a feculent, obstreperous llama whose value on the open market is charitably pegged at \$1. Full reminds Jones that this is a game, and a fair one, and that if Jones ends up selecting the door with \$1M behind it, all that money will indeed be his. (Jones' net worth has nearly been exhausted by his expenditures in traveling to the show.) Full also reminds Jones that he (= Full) knows what's behind each door, fixed in place until the game ends.

Full asks Jones to select which door he wants the contents of. Jones says, "Door 1." Full then says: "Hm. Okay. Part of this game is my revealing at this point what's behind one of the doors you didn't choose. So ... let me show you what's behind Door 3." Door 3 opens to reveal a very unsavory llama. Full now to Jones: "Do you want to switch to Door 2, or stay with Door 1? You'll get what's behind the door of your choice, and our game will end." Full looks briefly into the camera, directly.

(PI.1) What should Jones do if he's rational?

(PI.2) Prove that your answer is correct. (Diagrammatic proofs are allowed.)

(PI.3) A quantitative hedge fund manager with a PhD in finance from Harvard zipped this email off to Full before Jones made his decision re. switching or not: "Switching would be a royal waste of time (and time is money!). Jones hasn't a doggone clue what's behind Door 1 or Door 2, and it's obviously a 50/50 chance to win whether he stands firm or switches. So the chap shouldn't switch!" Is the fund manager right? Prove that your diagnosis is correct.

(PI.4) Can these answers and proofs be exclusively Bayesian in nature?

# The Switching Policy Rational!

**Proof:** Our overarching technique will be proof by cases.

We denote the possible cases for initial distribution using a simple notation, according to which for example 'LLM' means that, there is a lama behind Door 1, a llama behind Door 2, and the million dollars behind Door 3. With this notation in hand, our three starting cases are: Case 1: MLL; Case 2: LML; Case 3: LLM. There are only three top-level cases for distribution. The odds of picking at the start the million-dollar door is  $1/3$ , obviously — for each case. Hence we know that the odds of a HOLD policy winning is  $1/3$ .

Now we proceed in a proof by sub-cases under the three cases above, to show that the overall odds of a SWITCH policy is greater than  $1/3$ . Each sub-case is simply based on what the initial choice by Jones is, under one of the three main cases. Here we go:

Suppose Case 3, LLM, holds, and that [this (Case 3.1) is the first of three sub-cases under Case 3] Jones picks Door 1. Then FM must reveal Door 2 to reveal a llama. Switching to Door 3 wins, guaranteed. In sub-case 3.2 suppose that J's choice Door 2. Then FM will reveal Door 1. Again, switching to Door 3 wins, guaranteed. In the final sub-case, J initially selects Door 3 under Case 3; this is sub-case 3.3. Here, FM shows either Door 1 or Door 2 (as itself a random choice). This time switching loses, guaranteed. Hence, in two of the sub-cases out of three ( $2/3$ ), winning is guaranteed (*prob* of 1). An exactly parallel result can be deduced for Case 2 and Case 1; i.e., in each of these two, in two of the three ( $2/3$ ) sub-cases winning is 1. Hence the odds of winning by following the switching policy is  $2/3$ , which is greater than  $1/3$ . Hence it's rational to be a switcher. **QED**

# Logistics ...

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M

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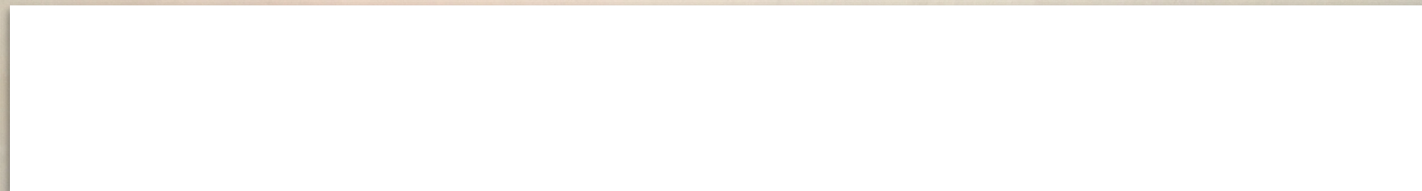
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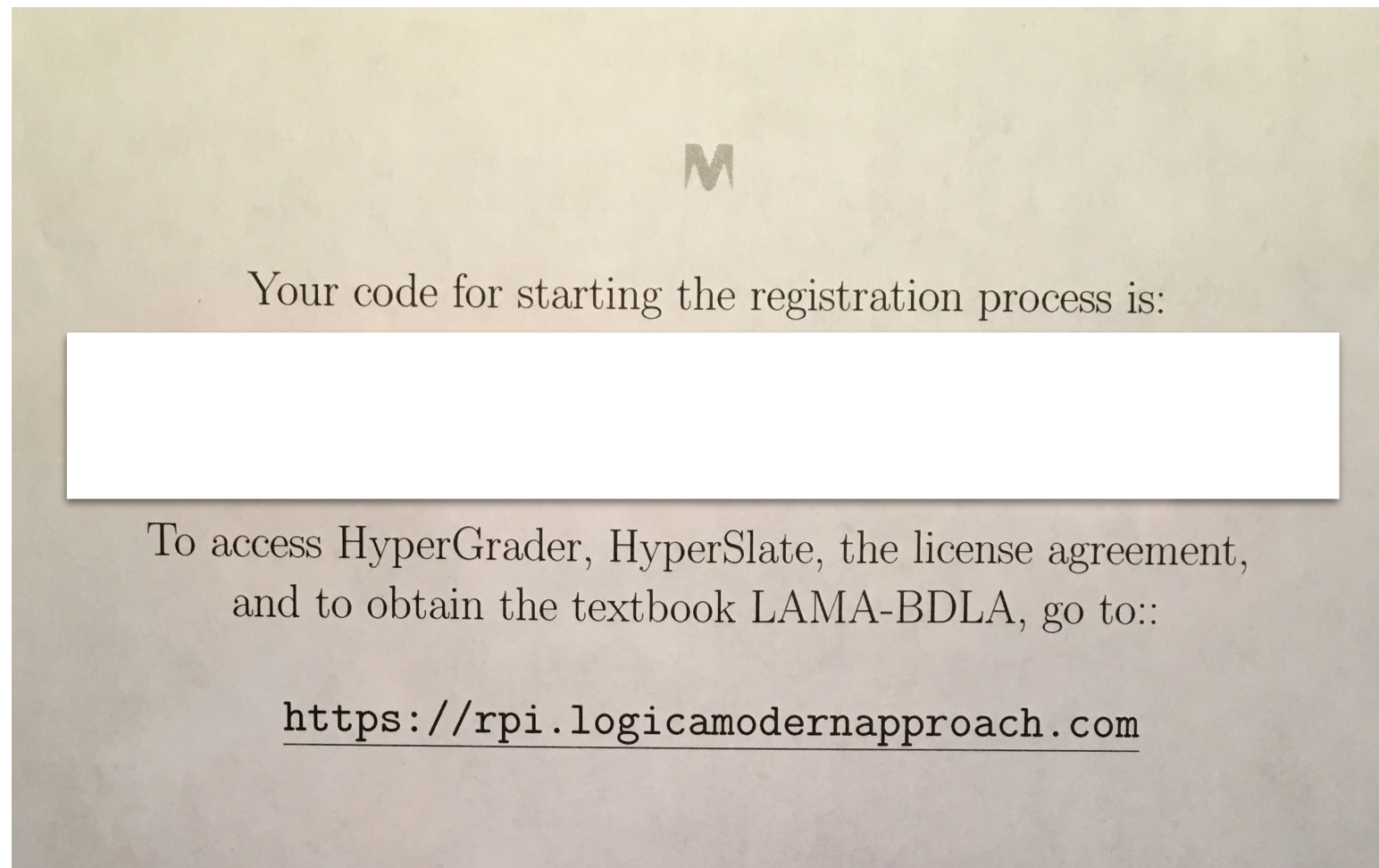


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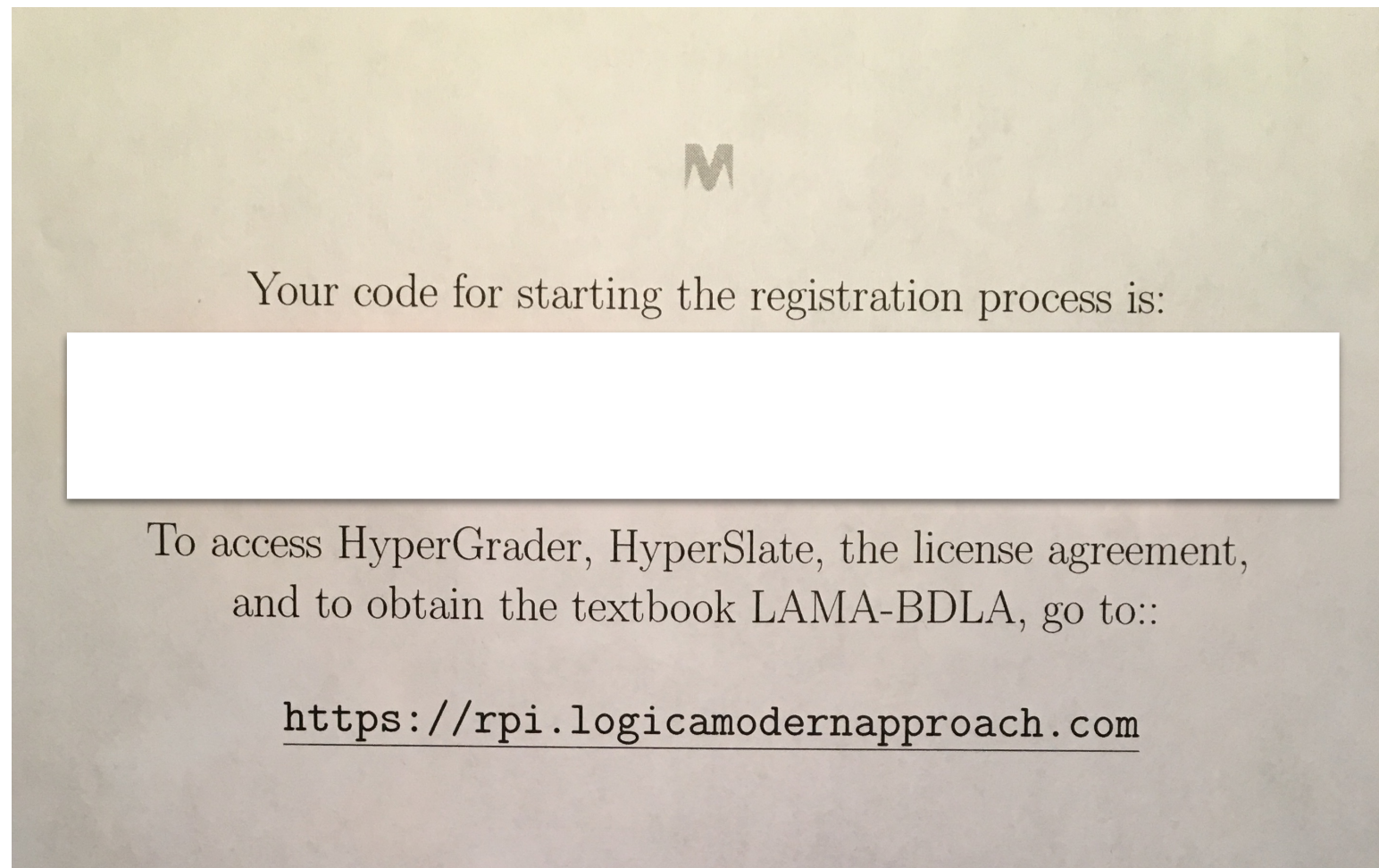
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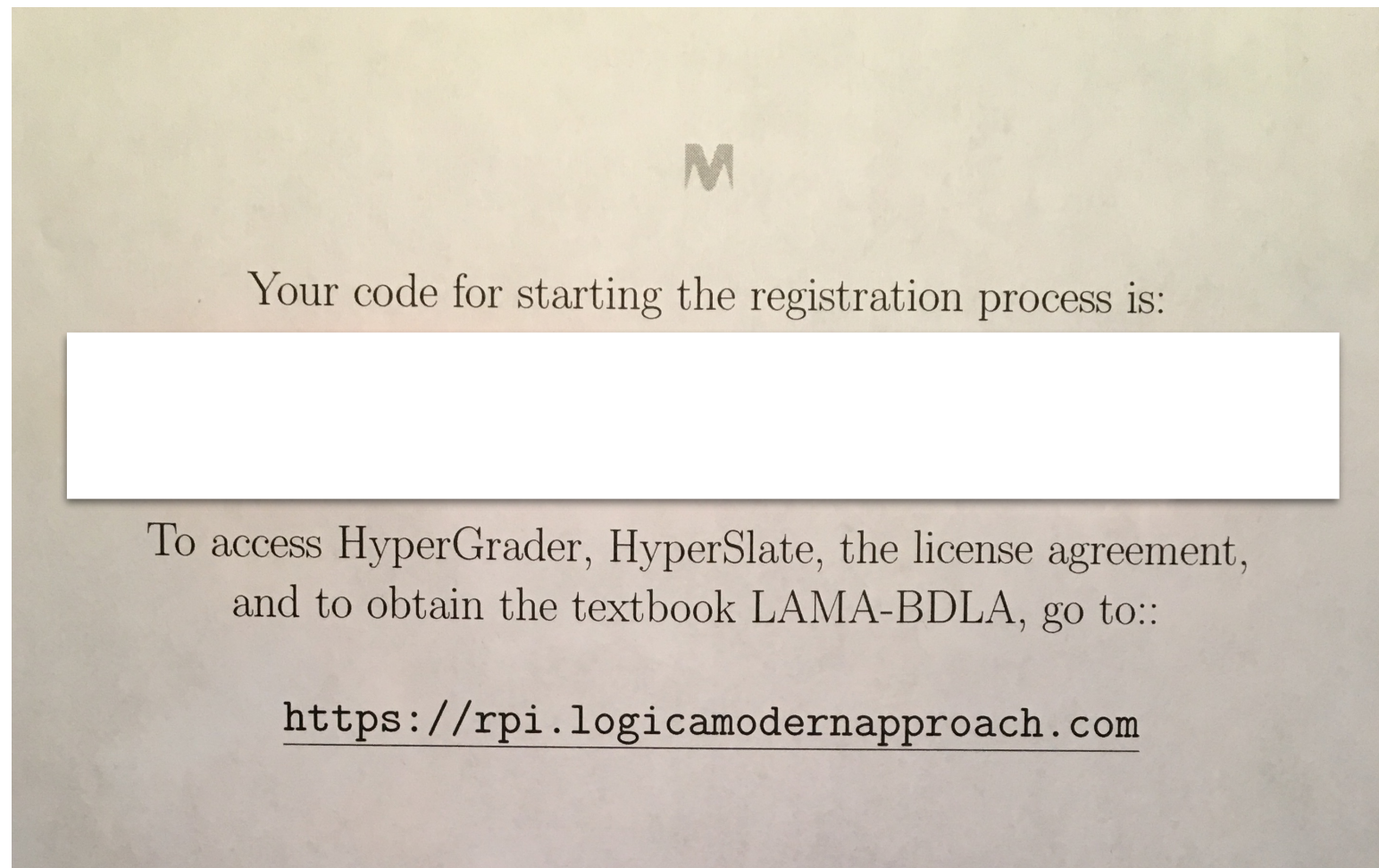


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# Introduction to (Formal) Logic (and AI)

Spring 2021 edition of IFLAI1

Selmer Bringsjord

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A fully online course, thanks to singular AI technology.

with Naveen Sundar G.  
^ KB Foush   ^ Joshua Taylor ^ ...

