Introduction to (Formal) Logic (and AI)

(IFLAI, pronounced "eye" • "fly")

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1 General Orientation

This course is an accelerated, advanced introduction, within the LAMA® paradigm,[1] to deductive formal logic (with at least some brief but informative pointers to both *in*ductive and heterogeneous formal logic), and to a substantive degree logic-based AI.[2] The phrase we use to describe what the student is principally introduced to in this class is: *beginning deductive logic, advanced* (BDLA). AI plays a significant role in advancing learning in the class; and the class includes an introduction to logicist aspects of AI and computer programming. After this class, the student can proceed to the intermediate level in formal deductive logic, and — with a deeper understanding and better prepared to flourish — to various areas within the *formal sciences*, which are all based on formal logic. The formal sciences include e.g. theoretical computer science (e.g., computability theory, complexity theory, rigorous coverage of programming and programming languages), mathematics in its traditional branches (analysis, topology, algebra, etc), decision theory, game theory, set theory, probability theory, mathematical statistics, etc. (and of course formal logic itself). The system HyperSlate®, an important system in the class, can be used productively, by the way, in all these disciplines.

We have referred above to "the LAMA® paradigm." What is that? This question will be answered in more detail later, but we do say here that while the LAMA® paradigm is based upon a number of pedagogical principles, first and foremost among them is what can be labelled the Driving Dictum:[3]

If you can't prove it, you don't get it.

Turning back to the nature of formal logic, it can accurately be said that it's the science and engineering of reasoning,[4] but even this supremely general slogan fails to convey the flexibility and enormity of the field. For example, a vast part of classical mathematics can be deductively derived from a small set of formulae (e.g., **ZFC** set theory, which you'll be hearing more about, and indeed

[1]'LAMA®' is an acronym for 'Logic: A Modern Approach," and is pronounced to rhyme with 'llama' in contemporary English, the name of the exotic and sure-footed camelid whose binomial name is *Lama glama*, and has in fact been referred to in the past by the single-l 'lama.'

[2]Sometimes 'symbolic' is used in place of 'formal,' but that's a bad practice, since — as students in this class will soon see — formal logic includes the representation of and systematic reasoning over *pictorial* information, and such information is decidedly *not* symbolic. For a discussion of the stark difference between the pictorial vs. the symbolic, and presentation of a formal logic that enables representation of and reasoning over both, see (Arkoudas & Bringsjord 2009), which directly informs Chapter 8 of the LAMA-BDLAHGHS textbook.

[3]It's profitable to ponder a variant of this dictum, applicable in venues [e.g. legal hearings, courtrooms, reports by analysts in various domains that are not exclusively formal (e.g. fundamental investing, intelligence, etc.)] in which reasoning is not only deductive, but inductive, viz. "If you can't show by explicit argument that it's likelihood reaches a sufficient level, you don't get it."

[4]Warning: Increasingly, the term 'reasoning' is used by some who don't *really* do anything related to reasoning, as traditionally understood, to nonetheless label what they do. Fortunately, it's easy to verify that some reasoning is that which is covered by formal logic: *If the reasoning is explicit, links declarative statements or declarative formulae together via explicit, abstract reasoning schemata or rules of inference (giving rise to at least explicit arguments, and often proofs), is surveyable and inspectable, and ultimately machine-checkable, then the reasoning in question is what formal logic is the science and engineering of.* In order to characterize *in*formal logic, one can remove from the previous sentence the requirements that the links must conform to explicit reasoning schemata or rules of inference, and machine-checkability. It follows that so-called informal logic would revolve around arguments, but not proofs. An excellent overview of informal logic, which will be completely ignored in this class and its LAMA-BDLAHGHS textbook, is provided in "Informal Logic" in the Stanford Encyclopedia of Philosophy. In this article, it's made clear that, yes, informal logic concentrates on the nature and uses of argument.

experimenting with in the HyperSlate® system) expressed in the formal logic known as 'first-order logic' (= FOL = $\mathscr{L}_1$, which you'll *also* be hearing more about), and, as we shall see and discuss in class, computer science emerged from and is in large part based upon logic (for cogent coverage of this emergence, see Glymour 1992, Halpern, Harper, Immerman, Kolaitis, Vardi & Vianu 2001). Logic is indeed the foundation for *all* at once rational-and-rigorous intellectual pursuits. (If you can find a counter-example, i.e. such a pursuit that doesn't directly and crucially partake of logic, S Bringsjord would be very interested to see it.)

2 Assistance to Bringsjord

The TA for this course is Sergei Bugrov; email address: `sergei.iflai1@gmail.com`. Sergei will hold office hours on Thurs 6–8p, online only. Some guest lectures may be provided by researchers working in the RAIR Lab, a logic-based AI lab.

3 Prerequisites

There are no formal prerequisites. However, as said above, this course introduces *formal* logic, and does so in an accelerated, advanced way. This implies that — for want of a better phrase — students are expected to have a degree of logico-mathematical maturity. You have this on the assumption that you understood the math you were supposed to learn in order to make it where you are.[5] For example, to get to where you are now, you were supposed to have learned the technique of *indirect proof* (= proof by contradiction = *reductio ad absurdum*). An example of the list of concepts and techniques you are assumed to be familiar with from high-school geometry can be found in the common-core-connected (Bass & Johnson 2012). An example of the list of concepts and techniques you are assumed to be familiar with from high-school Algebra 2 can be found in the common-core-connected (Bellman, Bragg & Handlin 2012). (Note in particular that this Algebra 2 textbook has extensive coverage of proof by contradiction.) It's recommended that during the first two weeks of the class, students review their high-school coverage of formal logic, which includes at minimum the rudiments of the propositional calculus = $\mathscr{L}_{\mathrm{PC}}$.[6]

4 Textbook/Courseware

Students will purchase the inseparable and symbiotic triadic combination published by Motalen:

- the e-textbook *Logic: A Modern Approach; Beginning Deductive Logic, Advanced via HyperGrader® & HyperSlate®* (LAMA-BDLAHGHS);

[5] If you happen to be a student reading this as one wanting to be introduced to formal logic, from outside RPI, please examine your own case realistically. If you are not in command of the traditional high-school-level content for algebra, geometry, trigonometry, and at least some (differential and integral) calculus, you will need to go with a standard, non-advanced introduction to logic in the LAMA® paradigm, or in some other paradigm. Specifically, if in the LAMA® paradigm, you will need the LAMA-BDL textbook, *not* LAMA-BDL**A**. The 'A' in 'LAMA-BDL**A**' is for 'Advanced.' Check which textbook you have!

[6] Sometimes referred to as 'sentential logic' or 'zeroth-order logic.' (For us, zeroth-order logic, $\mathscr{L}_0$, includes relation symbols and function symbols, as well as identity; these things are not part of the propositional calculus.) If you are at all confused about how these terms were used before reaching the present course, please discuss asap with the instructor or TAs.

- access to and use of the HyperGrader® AI system (for, among other things, assessing student work); and
- access to and use of the HyperSlate® AI system (for, among other things, engineering proofs in collaboration with AI);

All three items will be available after purchase in the RPI Bookstore of an envelope with a personalized starting code for registration. Logistics of the purchase, and the contents of the envelope that purchase will secure, will be encapsulated in the first class meeting, Jan 25 2021, and then gone over in more detail on Feb 4 2020, after which the envelopes in question will soon be on sale from and in the Rensselaer Bookstore. (Due to COVID-19, students may not be allowed to go physically to the Bookstore until at least Monday Feb 8.) The first use in earnest of HyperSlate® and HyperGrader® will happen, at the earliest, during class on Feb 8 2021, so by the start of class on that day students should attempt to have LAMA-BDLAHGHS, and be able to open both HyperSlate® and HyperGrader® in a browser on their laptops in class. Updates to LAMA-BDLAHGHS, and additional exercises, will be provided by listings on HyperGrader® (and sometimes by email) through the course of the semester. You will need to manage many electronic files in the course of this course, and e-housekeeping and e-orderliness are of paramount importance. You will specifically need to assemble a library of completed and partially completed proofs/arguments/truth-trees etc. in HyperSlate® so that you can use them as building blocks in harder proofs; in other words, building up your own "logical library" in the cloud will be crucial.

Please note that HyperSlate® and HyperGrader® are copyrighted, trademarked software based on Pat. Pend. methods: copying and/or reverse-engineering and/or distributing this software to others is strictly prohibited. You will need to submit online a signed version of a License Agreement. This agreement will also reference the textbook, which is copyrighted as well, and since it's an ebook, cannot be copied or distributed or resold in any way.

In addition, pre-publication chapters of Bringsjord's *Gödel's Great Theorems*, forthcoming from Oxford University Press, will be made available (and as per publisher rules cannot be copied or shared); and occasionally papers may be assigned as reading. Two background ones, indeed, are hereby assigned: (Bringsjord, Taylor, Shilliday, Clark & Arkoudas 2008, Bringsjord 2008).

Finally, slide decks used in class will contain crucial additional content above and beyond LAMA-BDLAHGHS and HyperSlate® and HyperGrader® content, and will be available on the web site for our course for study. Along with slide decks, video and audio tutorials and mini-lectures will be provided as well.

5 Schedule

The progression of class meetings is divided into seven parts: first a motivation/history stretch I, during which we show that the logically untrained have great trouble reasoning (and hence living) well, and set an historical context for modern formal logic and AI, and then six additional parts II–VII. In the first of these remaining parts, II, we'll focus on the **propositional calculus** ($= \mathscr{L}_{\mathrm{PC}}$) and **zero-order logic** (ZOL $= \mathscr{L}_0$). We will also introduce *Pure General Logic Programming* (PGLP), and the new programming language, Hyperlog, that makes PGLP concrete.

In Part III we shall focu on **first-order logic** ($=$ FOL $= \mathscr{L}_1$), with substantive study of **second-order logic** ($=$ SOL $= \mathscr{L}_2$) and beyond. Proofs will be constructed in the AI-infused

HyperSlate® system; and in IV we'll cover **modal logic**, in the form, specifically, of four closely related modal logics: **T**, **S4**, **D** (= **SDL**), and **S5**, with the emphasis on **SDL** as a candidate formalism for AI/machine ethics — a candidate that fails. Once we understand the reasons for this failure, we will look at a very expressive quantified modal logic that has been used with considerable success in AI ethics: $\mathcal{DCEC}^*$. Emphasis will be on learning how to construct hypergraphical proofs in each system. Part V of the course looks at formal axiom systems, or as they are often called in mathematical logic, **theories**. Part VI of the course looks at formal *in*ductive logic, and to a degree at logics for reasoning over visual content (e.g., diagrams). The seventh (VII) and final part of the course is a synoptic look at some of the astonishing work of the greatest logician: Kurt Gödel. Part VII will include private, non-copyable distribution of a pre-publication version of *Gödel's Great Theorems*, forthcoming from Oxford University Press. Distribution of this content outside of students in the class is prohibited (by the policies of the Press itself).

A more fine-grained schedule now follows.[7]

[7] Note that the Rensselaer Academic Calendar is available here.

5.1 Why Study Logic?; Its History (I)

- **Jan 25**: *General Orientation to the LAMA® Paradigm, Logistics, Mechanics.* The syllabus is reviewed in detail, and discussed. It's made clear to students that, in this class, there is a very definite, comprehensive, theoretical position on computational formal logic and the teaching thereof; this position corresponds to the affirmation of the LAMA® (= Logic: A Modern Approach) paradigm, and that in lockstep with this position the tightly integrated trio of the

  1. LAMA-BDLAHGHS textbook,
  2. HyperSlate® AI-infused proof-construction system, and
  3. HyperGrader® system for (among other things) automated assessment of proofs and management of points earned on the leaderboards,

  are used. Students wishing to learn under the venerable "Stanford" paradigm are strongly encouraged to drop this LAMA®-based course and take PHIL 2140 in its alternating spot (i.e., Fall semester, annually).

- **Jan 28**: *Motivating Puzzles, Problems, Paradoxes, $\mathcal{R}$, $\mathcal{H}$, Part I.* Here we among other things tackle problems which, if solvable before further learning, obviate taking the course. We also discuss Bringsjord's "elevated" view of the human mind as potentially near-perfectly rational, and specifically capable of systematic and productive reasoning about the infinite.

- **Feb 1**: *Motivating Puzzles, Problems, Paradoxes, $\mathcal{R}$, $\mathcal{H}$, Part II.* A continuation of Part I; the problems in question get harder!

- **Feb 4**: *Whirlwind History and Overview of Formal Logic (in intimate connection with computer science and AI), From Euclid to today's Cutting-Edge Computational Logic in AI and Automated Reasoning.* In one class meeting we surf the timeline of *all* of formal logic, from Euclid to the present. A particular emphasis is placed on Leibniz, the inventor of modern formal logic. Aristotle is cast as the inventor of formal logic in its original form (syllogistic deduction). The crucial timepoint of the discovery of the unsolvability of the *Entscheidungsproblem* by Turing-level computers figures prominently, and supports a skeptical position on The Singularity.

  The class ends with instructions for purchase and initial use of a personalized code from the Bookstore, which will enable students to obtain LAMA-BDLAHGHS and for both both HyperSlate® and HyperGrader®. Codes, in laser-tagged, sealed envelopes, should be on sale fairly soon after this class meeting (but not before!).

5.2 Propositional Calculus ($\mathscr{L}_{\mathbf{pc}}$) & "Pure" Predicate Calc. ($\mathscr{L}_0$) (II)

- **Feb 8**: *Review from High School: Variables & Connectives; Propositional Calculus I.* This meeting will tie up any loose ends on the history side of things. Students by this point should be registered and have HyperSlate® running in a browser laptops, and have signed and accepted their LA. This is the start of coverage of the propositional calculus, $\mathscr{L}_{\mathrm{PC}}$. We see AI in action in HyperSlate®.

- **Feb 11**: *Propositional Calculus II: The Formal Language, First Rules of Inference/Inference Schemata, and Immaterialism.* Application to some of the original problems used to motivate the course (meetings Jan 28 & Feb 4). Simple proofs settle these problems. The view that formal logic, in particular some of the rudiments of the propositional calculus, exists in an immaterial world, a view that can be defended with help from the late James Ross (1992), is presented. This view is extended to a conception of all of computer science based on formal logic/logic machines.

- **Feb 15**: No Class (President's Day)

- **Feb 18**: *Propositional Calculus III: Remaining Rules of Inference/Inference Schemata; Propositional Trees.* Here we discuss the "harder" inference rules/schemata; e.g. proof-by-cases = disjunction elimination. More substantive proofs achieved. In addition, hypergraphical indirect proof (= proof by contradiction = *reductio ad absurdum*) is introduced in earnest. We also introduce propositional truth trees, explain their superiority over (infernal) truth tables, and show

how these trees can be easily constructed in HyperSlate®.

- **Feb 22**: *PGLP (Pure General Logic Programming) at the Level of $\mathscr{L}_{\text{PC}}$, and Hyperlog.* Some harder proofs obtained. By this class meeting students will be comfortable using HyperSlate® in conjunction with HyperGrader®. Demonstrations will be given. Coverage here of resolution, and PGLP/Hyperlog at the level of the propositional calculus.
- **Feb 25**: *The Pure Predicate Calculus; Metalogic: Soundness and Completeness of $\mathscr{L}_{\text{PC}}$ and $\mathscr{L}_0$.* This is zeroth-order logic, or $\mathscr{L}_0$, for us. What kind of logic do we get if we add to the propositional calculus machinery for relation symbols, function symbols, and identity (=)? The result is $\mathscr{L}_0$, and we explore some problems and proofs in this logic. We end with proof-sketches of both soundness and completeness of both the propositional calculus and zero-order logic.
- **Mar 1**: **Presentation/Discussion of Test #1**.

5.3 Extensional Logics in General (e.g. First-Order Logic (FOL = $\mathscr{L}_1$, & Second-Order Logic (SOL = $\mathscr{L}_2$); Some (Extensional) Paradoxes; ZFC (III)

- **Mar 4**: *Extensional Logic; The Need for Quantification, and the Centrality Thereof in Human Thought and Communication.* We use our picture of the entire, vast universe of logics to establish a context in which to then zero in on *extensional* logics, and then discuss the crucial need for having quantifiers like ∀ and ∃, and the centrality of quantifiers to human cognition, which in this regard is discontinuous with the cognition of nonhuman animals.
- **Mar 8**: *New Inference Schemata in $\mathscr{L}_1$ = FOL, I.* We here introduce, discuss, and employ `existential intro` and `universal elim` in their hypergraphical form; these are the two "easy" new inference rules/schemata of $\mathscr{L}_1$ =. But easy as they might be, do they suffice to enable us to prove that God exists? ...
- **Mar 11**: *The Two New (Harder) Inference Schemata in $\mathscr{L}_1$ = FOL, II.* We introduce hypergraphical `universal intro`, and also introduce `existential elim`, and related matters. Some further discussion of games that require and supposedly cultivate logical reasoning is also carried out.
- **Mar 15**: *Quantificational Trees; FOl, IV (Numerical Quantification, Intro to CVQ$^+$A$^+$JV); $\mathscr{L}_2$ = second-order logic = SOL, $\mathscr{L}_3$ = third-order logic = TOL, and Beyond.* We look at trees with quantification, and see how to build and test them with help from AI in HyperSlate®. We also introduce an advanced, interesting form of Visual Question Answering (VQA) that contemporary ML can't handle. Finally, we discuss logic-based hierarchies (the Polynomial and Arithmetic) for measuring the level of intelligence in intelligent agents, whether natural (like us), artificial, or extraterrestrial/alien.
- **Mar 18**: *Some (Extensional) Paradoxes, and the Rebuilding Thereafter via **ZFC*** The Liar; Russell's Paradox; Richard's Paradox; and perhaps some coverage of the more complicated Skolem's Paradox. Then, the solution: **ZFC**. We see how a computational version of **ZFC** is available for exploration in HyperSlate®.
- **Mar 22**: **Presentation/Discussion of Test #2.**

5.4 More Theories (= Axiom Systems); Beyond FOL (IV)

- **Mar 25**: *Theories of Arithmetic I.* **PA**, and its simpler parts (e.g. so-called "Baby Arithmetic"). We here make what I hope are some interesting connections to AI-based math education in the early grades.
- **Mar 29**: *SOL, TOL, and Beyond; Second-Order Theories of Arithmetic II; The Reduction of Mathematics to Formal Logic* One issue dealt with here is whether mathematics itself can be reduced to formal logic applied to some particular axiom systems, as long as we move to SOL = $\mathscr{L}_2$. We draw on the "reverse mathematics" work of Shapiro (1991) and Simpson (2010).

5.5 Intensional Logic; Deontic Logic and Killer Robots/Vehicles (V)

- **Apr 1**: *On to Intensional Logic; Modal Logic: What and Why.* This is a general introduction to the crucial difference between *extensional* logic versus *intensional* logic. The logics $\mathscr{L}_{\text{PC}}$, $\mathscr{L}_0$, $\mathscr{L}_1$, $\mathscr{L}_2$, $\mathscr{L}_3$ are all extensional. Now we move to the intensional category, which includes modal logics. Five modal logics are introduced, rapidly for now: **K**, **T** = **M**, **D**, **S4**, and **S5**.

- **Apr 5**: *The System* **D** = **SDL**. After a quick peek ahead at the PAID Problem and "The Four Steps" that — we claim — can solve it, we proceed to consider deontic logic **D** = **SDL**. This is the basic system of logic intended to capture central categories in ethics (e.g., obligation, permissibiity, etc.). It will turn out that **SDL** is in need of major improvement, if not outright replacement, because (for starters) of two paradoxes: Chisholm's Paradox and the Free-Choice Permission Paradox. We explore these paradoxes in HyperSlate®.

- **Apr 8**: *The Threat of "Killer" Robots; Logic Can Save Us; Here's How.* Here again is presented the "PAID" problem: artificial agents (e.g. robots) that are **p**owerful, **a**utonomous, and **i**ntelligent, are **d**angerous (if not capable of **d**estroying us). We also discuss the dangers arising from using logic-less AI to engineer self-driving vehicles. After taking note of the fact that *Star Trek* (original) teaches us that logic can save us, this class introduces an engineered quantified multi-operator modal logic, $\mathcal{DCEC}^*$, developed at Rensselaer, and explains how use of the computational version thereof, implemented, can be used to enable an AI/robot to adjudicate thorny ethical dilemmas. We explore $\mathcal{DCEC}^*$ in HyperSlate®.

5.6 Beginning Heterogenous Logic & Beginning *In*ductive Logic (BIL): Glimpses (VI)

- **Apr 12**: *Whirlwind History & Overview Beginning (Formal) Inductive Logic (LAMA-BIL) in the LAMA® Paradigm.* A solution to the Lottery Paradox is provided, and recent work in the RAIR Lab devoted to solving the St Petersburg Paradox will also be covered. So-called "Pure Inductive Logic," the modern version of inductive logic stemming from R. Carnap,[8] is encapsulated, and distinguished from inductive logic in the LAMA® paradigm.

5.7 Gödel (VII)

- **Apr 15**: *Introduction to & Overview of <u>Gödel's Great Theorems</u>; Gödel's Completeness Theorem.* We seek here to understand the brilliant core of Gödel's CT, from his doctoral dissertation.

- **Apr 19**: *Gödel's First Incompleteness Theorem* The main trick here (the taking of which is in line with the dictum that "there's no free lunch," is to take as a given the Fixed-Point Theorem, and to use it as the powerful pivot for not only G1 here, but later for G2 and GST.

- **Apr 22**: *Gödel's Second Incompleteness Theorem.* Given certain assumptions about the power of our proof methods, along with Selmer's assumption that mathematics must include Peano Arithmetic (**PA**), we can prove that we can't prove that mathematics is free of contradiction!

- **Apr 26**: *Gödel's Greatest Theorem*! Pretty much everyone outside cognoscenti on Gödel's *oeuvre* believes that Gödel's greatest achievement is his incompleteness result (it's actually 'results' in the plural, of course). Nothing could be further from the truth, as we see.

- **Apr 29**: *Gödel's Time-Travel Theorem, and Gödel's God Theorem.* Gödel proved that travel backwards in time is mathematically possible; the proof was a present he gave to Einstein. We give an intuitive version of the proof based on visualizations stemming from the famous, must-read fantasy book *Flatland* (Abbott 2006). We also consider Gödel's remarkable attempt to prove that God exists.

- **May 3**: *In the Light of Gödel, Will AI Ever Match (or Exceed) Human Intelligence?; Loose-Ends Wrap-up; Presentation/Discussion of Test #3.*

[8] And definitively presented in (Paris & Vencovská 2015).

6 Grading

Grades are based in part on three tests. Each of these tests will call for timed use of HyperSlate® in conjunction with HyperGrader®; they will be sometimes discussed in class on the relevant day in the Schedule (§5); and then released online in HyperGrader® later that same day ($\approx$ 12 noon Pacific Time) with a countdown timer (no work is allowed beyond zero time left). The three tests are weighted 10%, 15%, and 25%, respectively.

In addition, grades are based on a series of *required*, self-paced homework problems to be done in the HyperSlate® system, and verified by HyperGrader®. This are called 'Required' problems in HyperGrader®. Every problem in the collection must be certified 100% correct by HyperGrader® in order to pass the course, and a grade of 'A' is earned for the series when it's completed, which is 40% of the final grade. All required homework assignments on HyperGrader® must be completed and submitted in order to receive a final grade.

The remaining 10% of one's grade is based on performance on "pop" problems released online in HyperGrader®, to be solved before the countdown timer for the problem hits zero.

7 Some Learning Outcomes

There are four desired outcomes. *One*: Students will be able to carry out formal proofs and disproofs, within the HyperSlate® system and its workspaces, at the level of the propositional and predicate calculi, and propositional modal logic (the aforementioned systems **T**, **S4**, **D**, and **S5**). *Two*: Students will be able to translate suitable reasoning in English into interconnected formulae in the languages of these four calculi, and assess this reasoning by determining if the desired structures are present in the formulae and relationships between them. *Three*, students will be able to carry out informal proofs. *Four*, students will demonstrate significant understanding of the advanced topics covered.

8 Academic Honesty

Student-teacher relationships are built on mutual respect and trust. Students must be able to trust that their teachers have made responsible decisions about the structure and content of the course, and that they're conscientiously making their best effort to help students learn. Teachers must be able to trust that students do their work conscientiously and honestly, making their best effort to learn. Acts that violate this mutual respect and trust undermine the educational process; they counteract and contradict our very reason for being at Rensselaer and will not be tolerated. Any student who engages in any form of academic dishonesty will receive an F in this course and will be reported to the Dean of Students for further disciplinary action. (The *Rensselaer Handbook* defines various forms of Academic Dishonesty and procedures for responding to them. All of these forms are violations of trust between students and teachers. Please familiarize yourself with this portion of the handbook.) In particular, all solutions submitted to HyperGrader® for course credit under a student id are to be the work of the student associated with that id alone, and not in any way copied or based directly upon the work of anyone else.

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