

On to *Intensional* Logics

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Questions re Theories (ZFC &/
or PA)? Brief hands-on session
on of **PA** via **Q** in
HyperSlate® ...

On those *extensional*
logics ... questions?

Climbing the k -order Ladder

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a is a llama, as is b , a likes b , and the father of a is a llama as well.

Climbing the k -order Ladder

$Llama(a) \wedge Llama(b) \wedge Likes(a, b) \wedge Llama(fatherOf(a))$

a is a llama, as is b , a likes b , and the father of a is a llama as well.

Climbing the k -order Ladder

ZOL $Llama(a) \wedge Llama(b) \wedge Likes(a, b) \wedge Llama(fatherOf(a))$

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Climbing the k -order Ladder

There's some thing which is a llama and likes b (which is also a llama), and whose father is a llama too.

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Climbing the k -order Ladder

$\exists x[Llama(x) \wedge Llama(b) \wedge Likes(x, b) \wedge Llama(fatherOf(x))]$

There's some thing which is a llama and likes b (which is also a llama), and whose father is a llama too.

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Climbing the k -order Ladder

Things x and y , along with the father of x , share a certain property (and x likes y).

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Climbing the k -order Ladder

$$\exists x \exists y \exists R [R(x) \wedge R(y) \wedge Likes(x, y) \wedge R(fatherOf(x))]$$

Things x and y , along with the father of x , share a certain property (and x likes y).

FOL $\exists x [Llama(x) \wedge Llama(b) \wedge Likes(x, b) \wedge Llama(fatherOf(x))]$

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Climbing the k -order Ladder

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Climbing the k -order Ladder

Things x and y , along with the father of x , share a certain property; and, x R^2 s y , where R^2 is a positive property.

SOL $\exists x \exists y \exists R [R(x) \wedge R(y) \wedge Likes(x, y) \wedge R(fatherOf(x))]$

Things x and y , along with the father of x , share a certain property (and x likes y).

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Climbing the k -order Ladder

$$\exists x, y \exists R, R^2 [R(x) \wedge R(y) \wedge R^2(x, y) \wedge \textit{Positive}(R^2) \wedge R(\textit{fatherOf}(x))]$$

Things x and y , along with the father of x , share a certain property; and, x R^2 s y , where R^2 is a positive property.

SOL
$$\exists x \exists y \exists R [R(x) \wedge R(y) \wedge \textit{Likes}(x, y) \wedge R(\textit{fatherOf}(x))]$$

Things x and y , along with the father of x , share a certain property (and x likes y).

FOL
$$\exists x [Llama(x) \wedge Llama(b) \wedge \textit{Likes}(x, b) \wedge Llama(\textit{fatherOf}(x))]$$

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ZOL
$$Llama(a) \wedge Llama(b) \wedge \textit{Likes}(a, b) \wedge Llama(\textit{fatherOf}(a))$$

a is a llama, as is b , a likes b , and the father of a is a llama as well.

Climbing the k -order Ladder

TOL $\exists x, y \exists R, R^2 [R(x) \wedge R(y) \wedge R^2(x, y) \wedge \text{Positive}(R^2) \wedge R(\text{fatherOf}(x))]$

Things x and y , along with the father of x , share a certain property; and, x R^2 s y , where R^2 is a positive property.

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Climbing the k -order Ladder

TOL $\exists x, y \exists R, R^2 [R(x) \wedge R(y) \wedge R^2(x, y) \wedge \text{Positive}(R^2) \wedge R(\text{fatherOf}(x))]$

\mathcal{L}_3 Things x and y , along with the father of x , share a certain property; and, x R^2 s y , where R^2 is a positive property.

SOL $\exists x \exists y \exists R [R(x) \wedge R(y) \wedge \text{Likes}(x, y) \wedge R(\text{fatherOf}(x))]$

\mathcal{L}_2 Things x and y , along with the father of x , share a certain property (and x likes y).

FOL $\exists x [Llama(x) \wedge Llama(b) \wedge \text{Likes}(x, b) \wedge Llama(\text{fatherOf}(x))]$

\mathcal{L}_1 There's some thing which is a llama and likes b (which is also a llama), and whose father is a llama too.

ZOL $Llama(a) \wedge Llama(b) \wedge \text{Likes}(a, b) \wedge Llama(\text{fatherOf}(a))$

\mathcal{L}_0 a is a llama, as is b , a likes b , and the father of a is a llama as well.

Climbing the k -order Ladder

⋮

TOL $\exists x, y \exists R, R^2 [R(x) \wedge R(y) \wedge R^2(x, y) \wedge \textit{Positive}(R^2) \wedge R(\textit{fatherOf}(x))]$

\mathcal{L}_3 Things x and y , along with the father of x , share a certain property; and, x R^2 s y , where R^2 is a positive property.

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\mathcal{L}_2 Things x and y , along with the father of x , share a certain property (and x likes y).

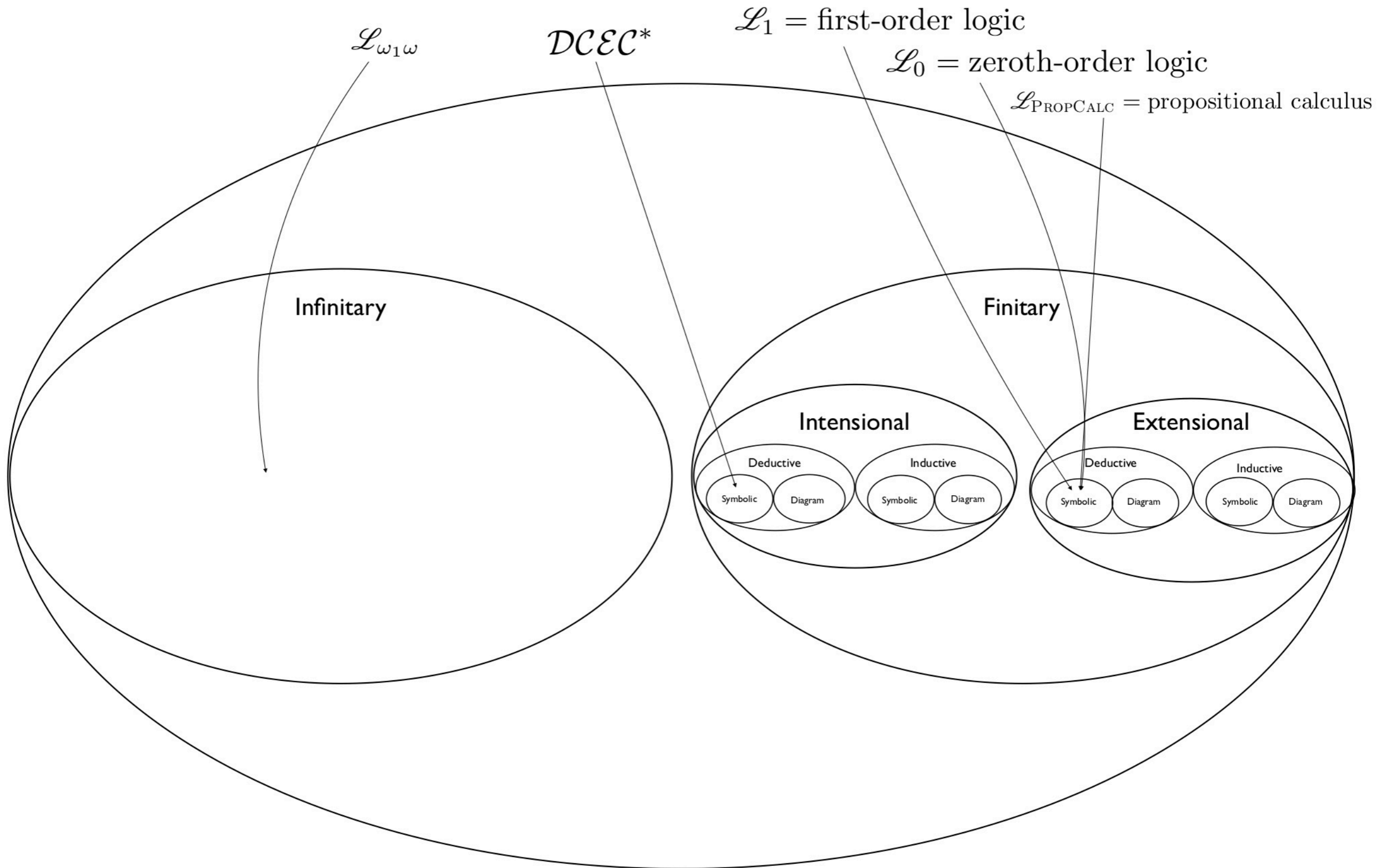
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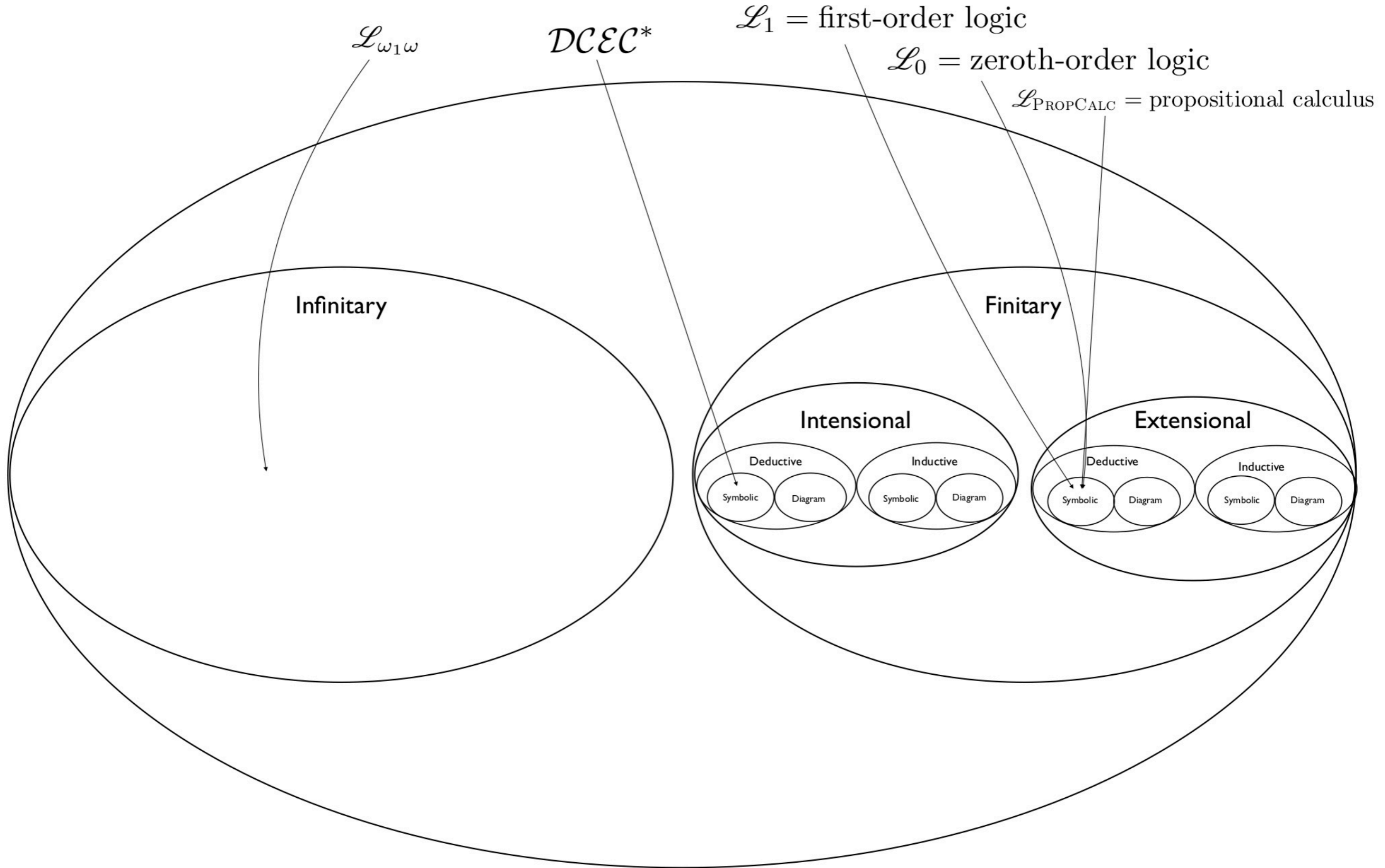
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The Universe of Logics



\mathcal{L}_3
 \mathcal{L}_2

The Universe of Logics



The Universe of Logics

\mathcal{L}_3

\mathcal{L}_2

$\mathcal{L}_{\omega_1\omega}$

\mathcal{DCEC}^*

$\mathcal{L}_1 =$ first-order logic

$\mathcal{L}_0 =$ zeroth-order logic

$\mathcal{L}_{\text{PROPCALC}} =$ propositional calculus

Infinitary

Finitary

Intensional

Extensional

Deductive

Inductive

Deductive

Inductive

Symbolic

Diagram

Symbolic

Diagram

Symbolic

Diagram

Symbolic

Diagram

Climbing the k -order Ladder

⋮

TOL $\exists x, y \exists R, R^2 [R(x) \wedge R(y) \wedge R^2(x, y) \wedge \textit{Positive}(R^2) \wedge R(\textit{fatherOf}(x))]$

\mathcal{L}_3 Things x and y , along with the father of x , share a certain property; and, x R^2 s y , where R^2 is a positive property.

SOL $\exists x \exists y \exists R [R(x) \wedge R(y) \wedge \textit{Likes}(x, y) \wedge R(\textit{fatherOf}(x))]$

\mathcal{L}_2 Things x and y , along with the father of x , share a certain property (and x likes y).

FOL $\exists x [Llama(x) \wedge Llama(b) \wedge \textit{Likes}(x, b) \wedge Llama(\textit{fatherOf}(x))]$

\mathcal{L}_1 There's some thing which is a llama and likes b (which is also a llama), and whose father is a llama too.

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\mathcal{L}_0 a is a llama, as is b , a likes b , and the father of a is a llama as well.

Climbing the k -order Ladder

⋮

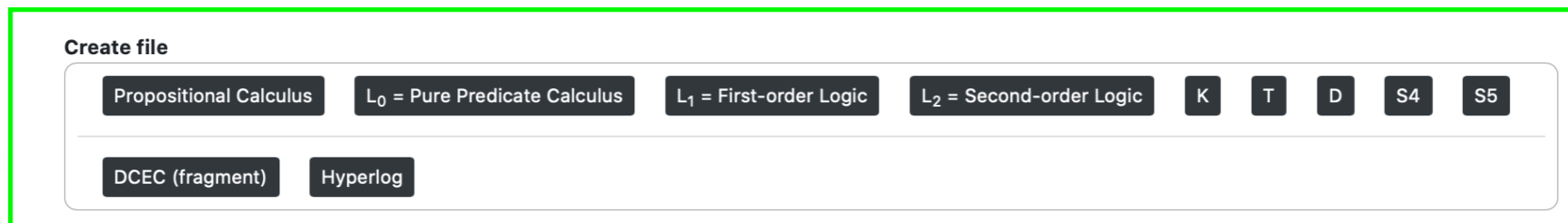
TOL $\exists x, y \exists R, R^2 [R(x) \wedge R(y) \wedge R^2(x, y) \wedge \text{Positive}(R^2) \wedge R(\text{fatherOf}(x))]$

\mathcal{L}_3

Things x and y , along with the father of x , share a certain property; and, x R^2 s y , where R^2 is a positive property.

SOL $\exists x, y \exists R [R(x) \wedge R(y) \wedge R(x, y) \wedge \text{Positive}(R) \wedge R(\text{fatherOf}(x))]$

\mathcal{L}_2



Things x and y , along with the father of x , share a certain property (and x likes y).

FOL $\exists x [Llama(x) \wedge Llama(b) \wedge Likes(x, b) \wedge Llama(\text{fatherOf}(x))]$

\mathcal{L}_1

There's some thing which is a llama and likes b (which is also a llama), and whose father is a llama too.

ZOL $Llama(a) \wedge Llama(b) \wedge Likes(a, b) \wedge Llama(\text{fatherOf}(a))$

\mathcal{L}_0

a is a llama, as is b , a likes b , and the father of a is a llama as well.

Climbing the k -order Ladder

⋮

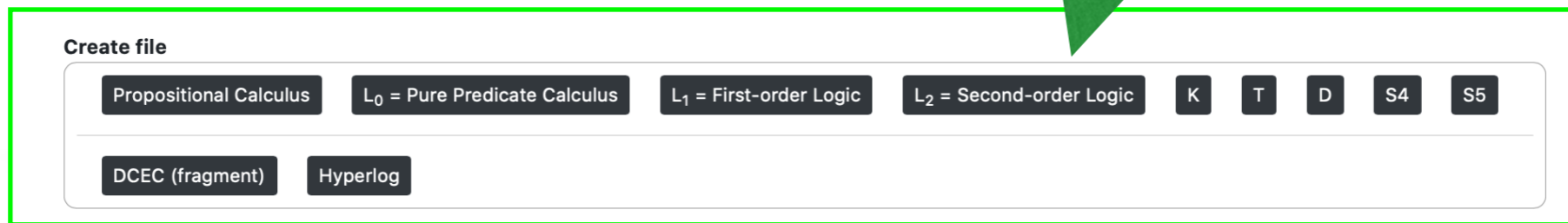
TOL

$\exists x, y \exists R, R^2 [R(x) \wedge R(y) \wedge R^2(x, y) \wedge \text{Positive}(R^2) \wedge R(\text{fatherOf}(x))]$

\mathcal{L}_3

Things x and y , along with the father of x , share a certain property; and, x R^2 s y , where R^2 is a positive property.

SOL



\mathcal{L}_2

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FOL

$\exists x [Llama(x) \wedge Llama(b) \wedge Likes(x, b) \wedge Llama(\text{fatherOf}(x))]$

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ZOL

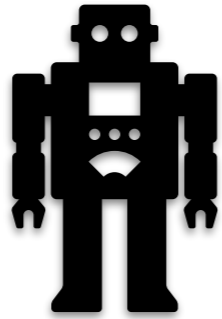
$Llama(a) \wedge Llama(b) \wedge Likes(a, b) \wedge Llama(\text{fatherOf}(a))$

\mathcal{L}_0

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**Blinky as portal to
intensional logics ...**

Blinky



1

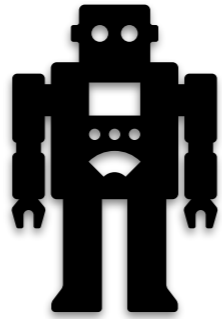


2



3

Blinky



1



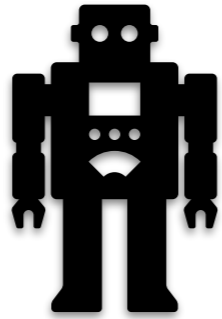
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3



Blinky



1

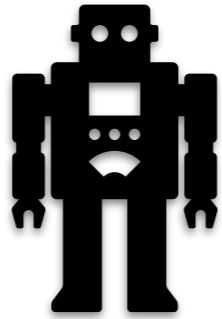


2



3

Blinky



1

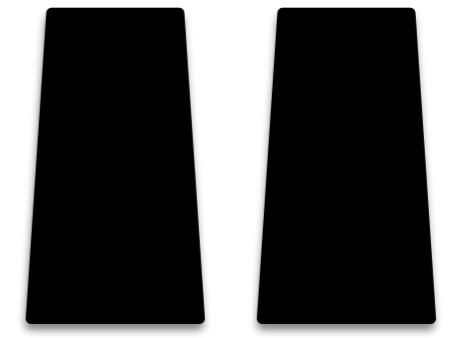
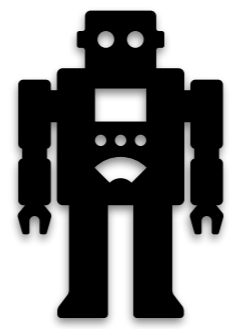


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3

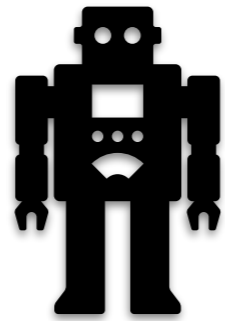
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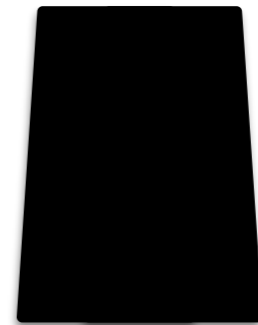
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Blinky



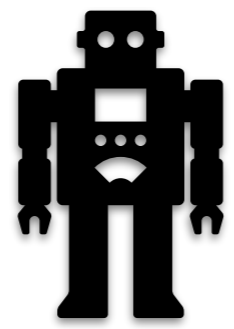
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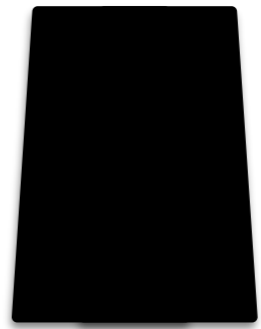
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3

Blinky



1

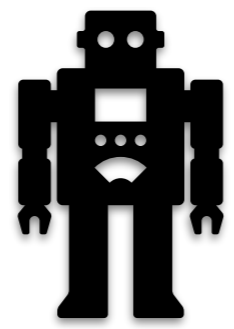


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3

Blinky



1

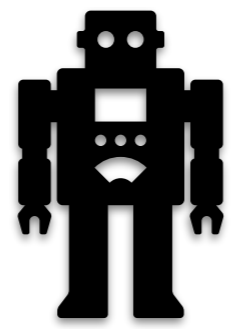


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3

Blinky



1

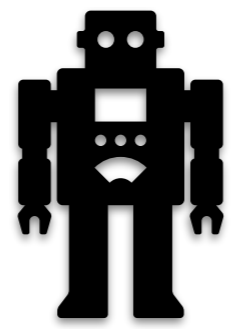


2



3

Blinky



1



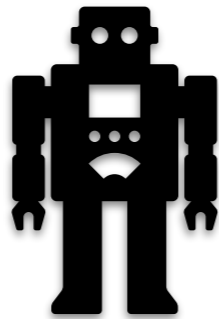
2



3

The ball is in the cup at location #1.

Blinky



1



2

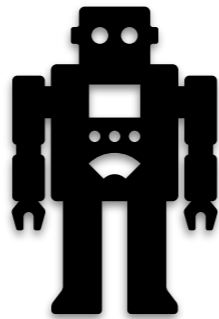


3

The ball is in the cup at location #1.

Loc(ball,1)

Blinky



1



2



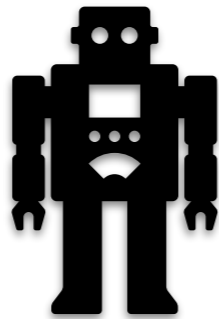
3

The ball is in the cup at location #1.

Loc(ball,1)

(Loc ball 1)

Blinky



1



2



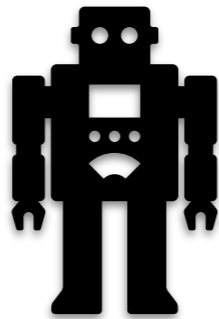
3

The ball is in the cup at location #1.

FALSE Loc(ball,1)

(Loc ball 1)

Blinky



1



2

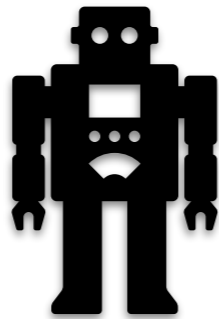


3

FALSE Loc(ball,1)

(Loc ball 1)

Blinky



1



2

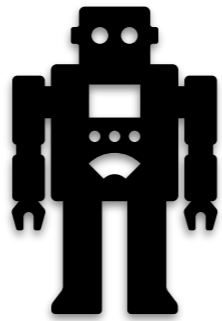


3

FALSE

(Loc ball 1)

Blinky



1



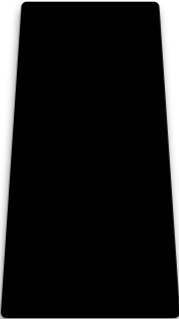
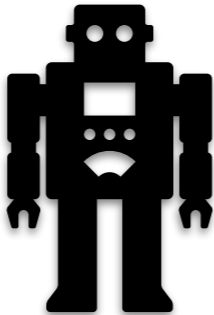
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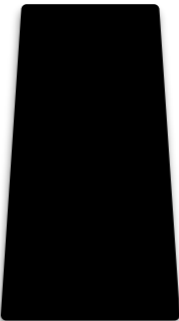
3

(Loc ball 1)

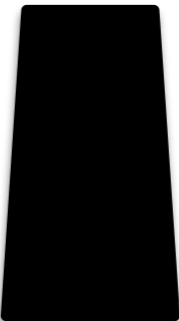
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1

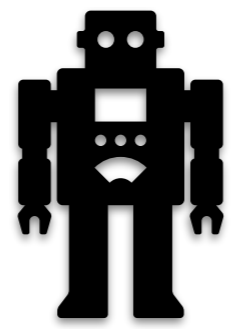


2



3

Blinky



1



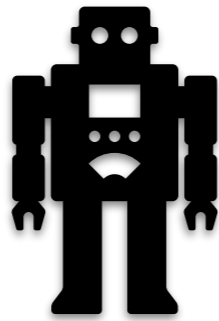
2



3

The ball is in the cup at location #1 or the ball is at location #3.

Blinky



1



2

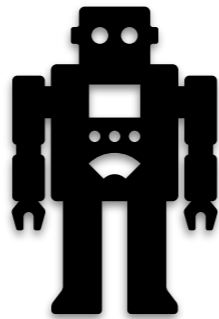


3

The ball is in the cup at location #1 or the ball is at location #3.

$\text{Loc}(\text{ball},1) \vee \text{Loc}(\text{ball},3)$

Blinky



1



2



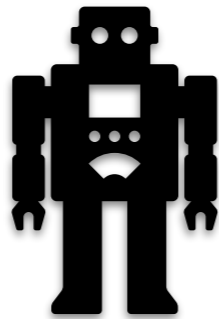
3

The ball is in the cup at location #1 or the ball is at location #3.

$\text{Loc}(\text{ball},1) \vee \text{Loc}(\text{ball},3)$

(or (Loc ball 1) (Loc ball 3))

Blinky



1



2



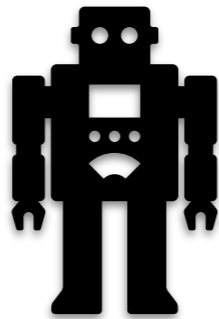
3

The ball is in the cup at location #1 or the ball is at location #3.

FALSE $\text{Loc}(\text{ball},1) \vee \text{Loc}(\text{ball},3)$

(or (Loc ball 1) (Loc ball 3))

Blinky



1



2

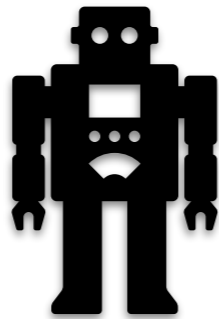


3

FALSE $\text{Loc}(\text{ball},1) \vee \text{Loc}(\text{ball},3)$

(or (Loc ball 1) (Loc ball 3))

Blinky



1



2

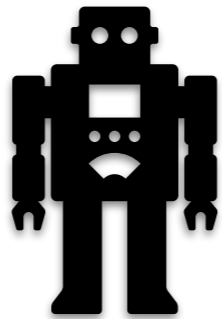


3

FALSE

(or (Loc ball 1) (Loc ball 3))

Blinky



1



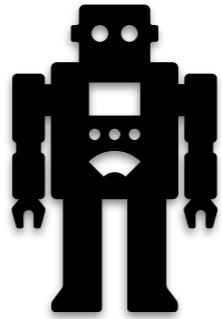
2



3

FALSE

Blinky



1

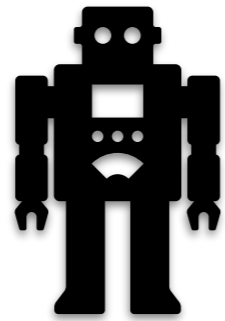


2



3

Blinky



1



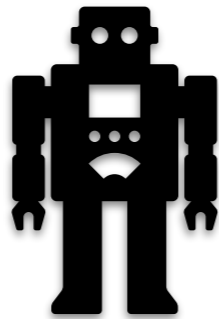
2



3

Blinky believes that the ball is in the cup at location #1.

Blinky



1



2

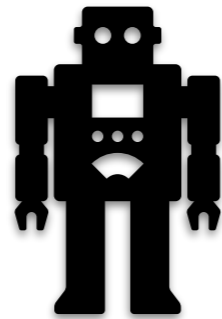


3

Blinky believes that the ball is in the cup at location #1.

$B(\text{blinky}, \text{Loc}(\text{ball}, 1))$

Blinky



1



2



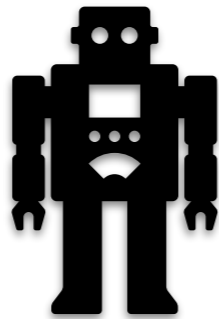
3

Blinky believes that the ball is in the cup at location #1.

$B(\text{blinky}, \text{Loc}(\text{ball}, 1))$

(Believes! blinky (Loc ball 1))

Blinky



1



2



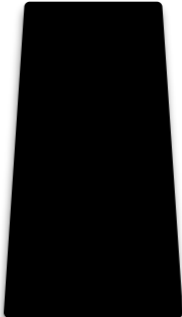
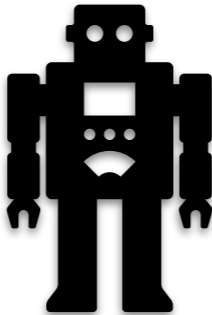
3

Blinky believes that the ball is in the cup at location #1.

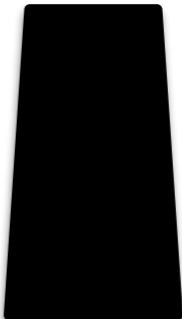
??? B(blinky, Loc(ball,1))

(Believes! blinky (Loc ball 1))

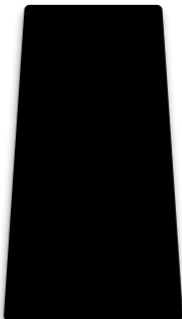
Blinky



1



2



3

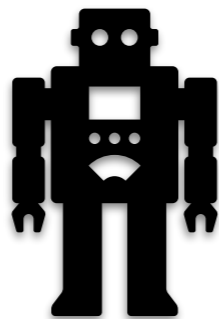
Blinky believes that the ball is in the cup at location #1.

???

$B(\text{blinky}, \text{Loc}(\text{ball}, 1))$

(Believes! blinky (Loc ball 1))

Blinky



1



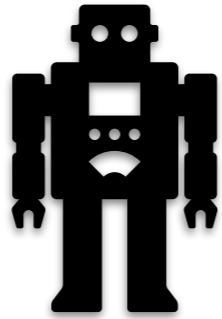
2



3

In extensional logics, what is denoted is conflated with meaning (the latter being naively compositional), but intensional attitudes like *believes*, *knows*, *hopes*, *fears*, etc cannot be represented and reasoned over smoothly (e.g. without fear of inconsistency rising up).

Blinky



1

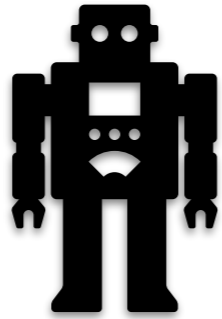


2



3

Blinky



1



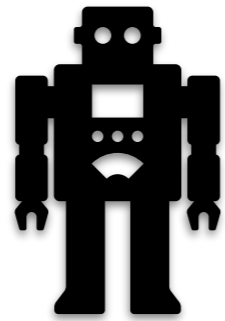
2



3



Blinky



1

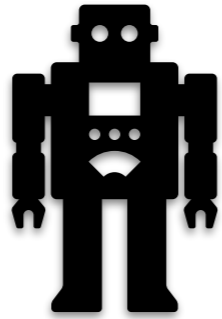


2



3

Blinky



1

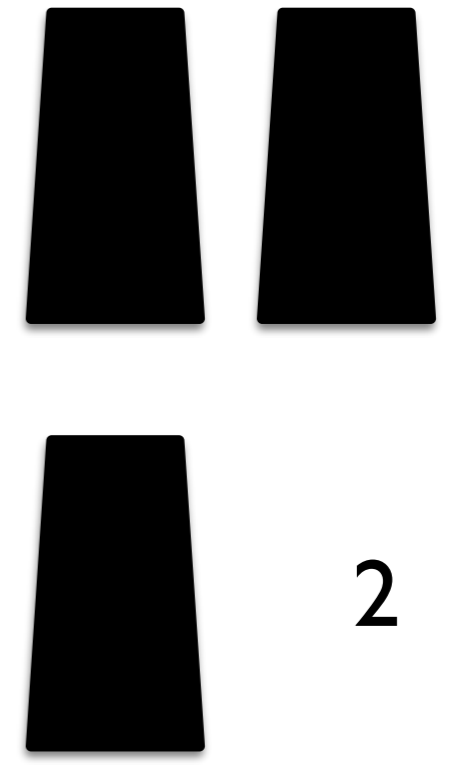
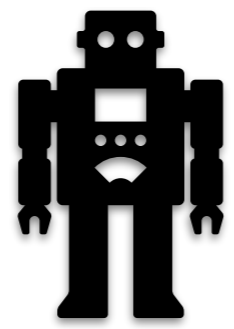


2



3

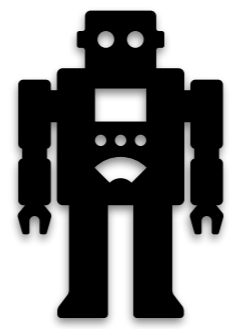
Blinky



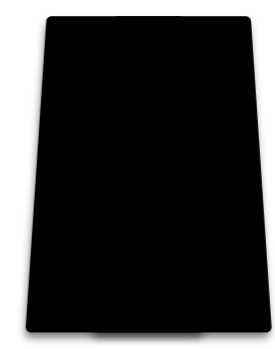
2

3

Blinky



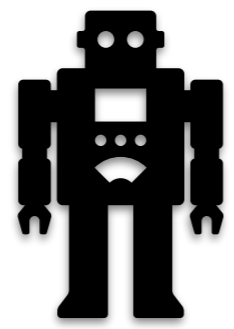
1



2

3

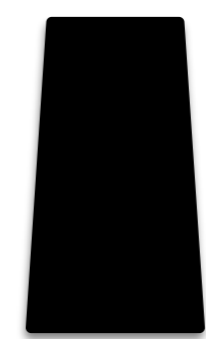
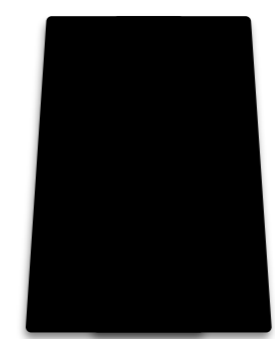
Blinky



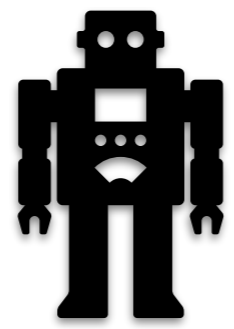
1

2

3



Blinky



1

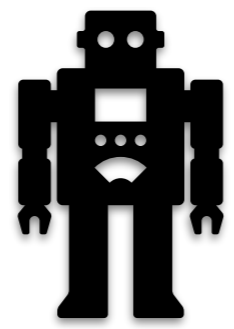


2



3

Blinky



1

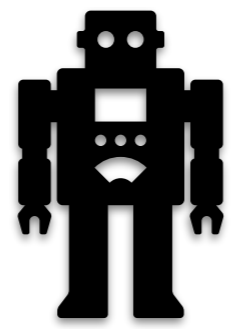


2



3

Blinky



1



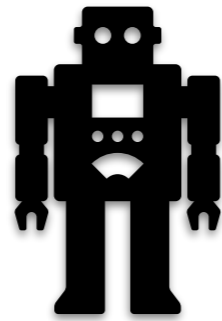
2



3

Blinky believes that the ball is in the cup at location #1.

Blinky



1



2

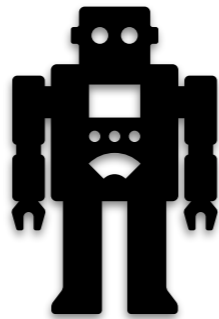


3

Blinky believes that the ball is in the cup at location #1.

$B(\text{blinky}, \text{loc-ball-1})$

Blinky



1



2



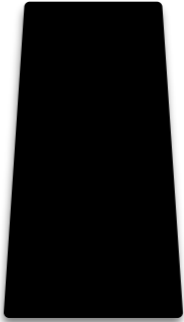
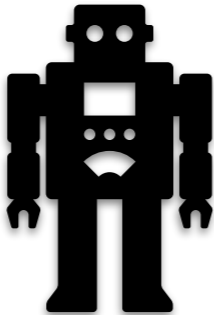
3

Blinky believes that the ball is in the cup at location #1.

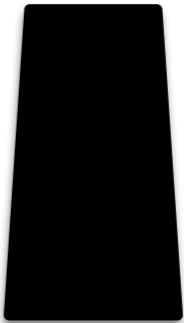
B(blinky, loc-ball-1)

(Believes! blinky loc-ball-1)

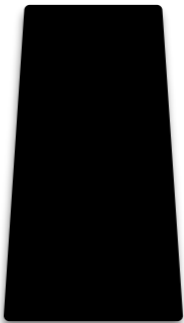
Blinky



1



2



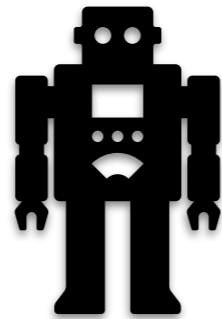
3

Blinky believes that the ball is in the cup at location #1.

$B(\text{blinky}, \text{loc-ball-1})$

(Believes! blinky loc-ball-1)

Blinky



1



2



3

In intensional logics, meaning and designation are separated, and compositionality is abandoned.

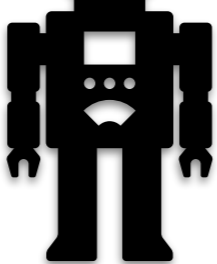
Blinky believes that the ball is in the cup at location #1.

$B(\text{blinky}, \text{loc-ball-1})$

(Believes! blinky loc-ball-1)



Blinky



1

2

3

In intensional logics, meaning and designation are separated, and compositionality is abandoned.

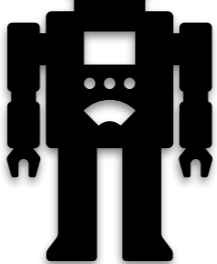
Blinky believes that the ball is in the cup at location #1.

$B(\text{blinky}, \text{loc-ball-1})$

(Believes! blinky loc-ball-1)



Blinky



1

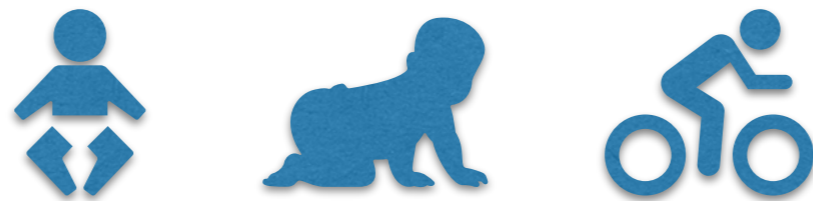
2

3

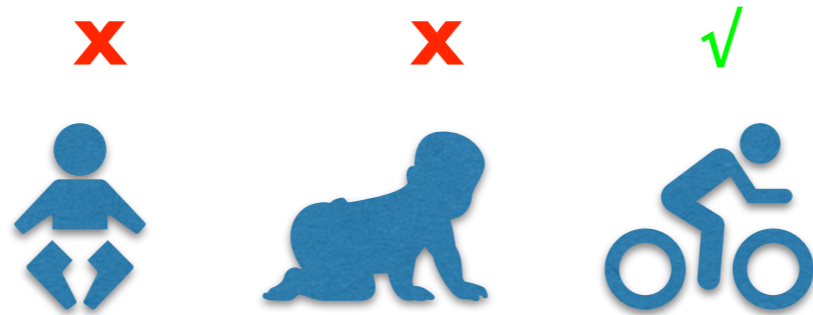
In intensional logics, meaning and designation are separated, and compositionality is abandoned.

**False Belief Task Demands
Intensional Logic ...**

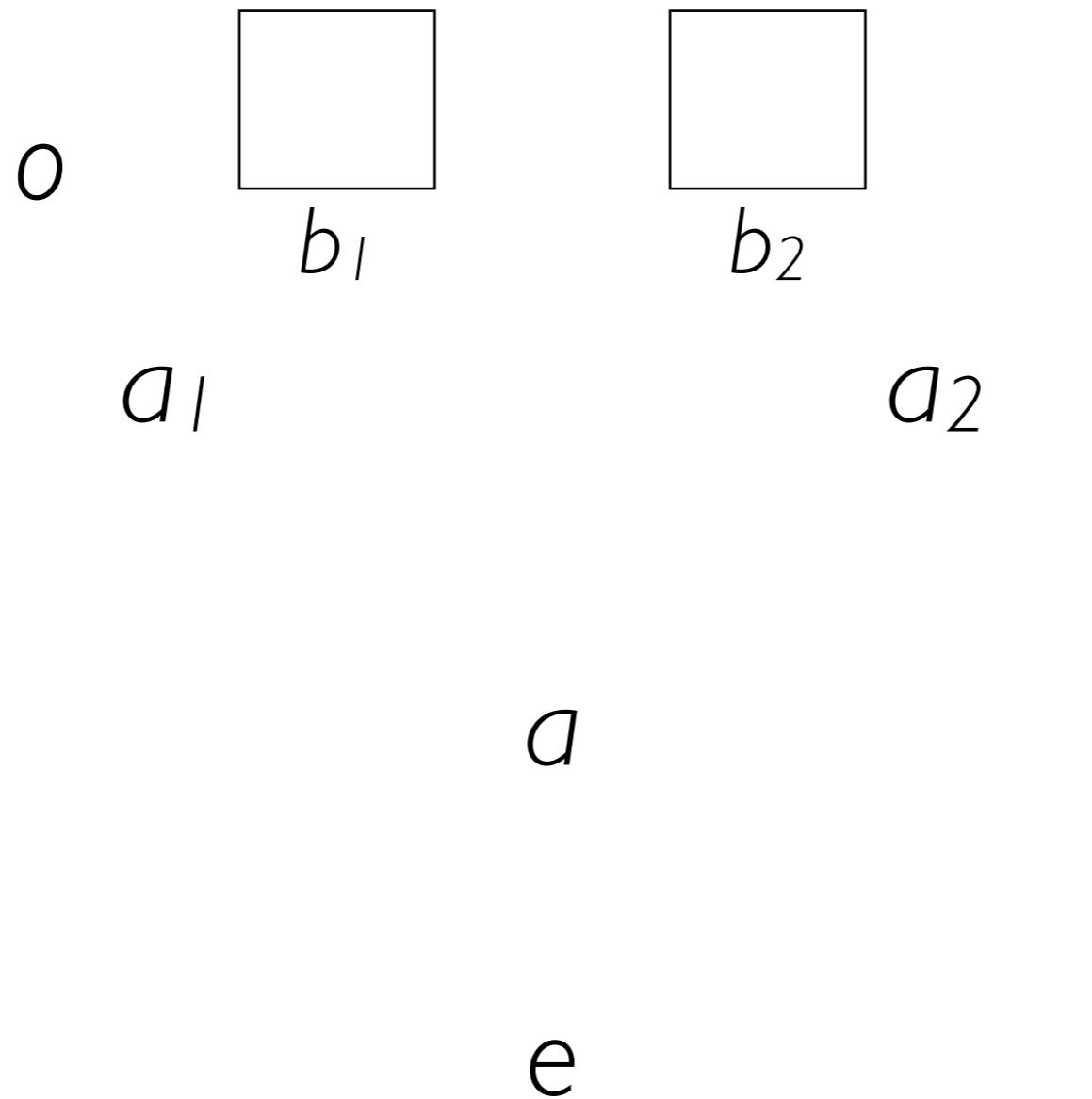
False Belief Task Demands Intensional Logic ...



False Belief Task Demands Intensional Logic ...

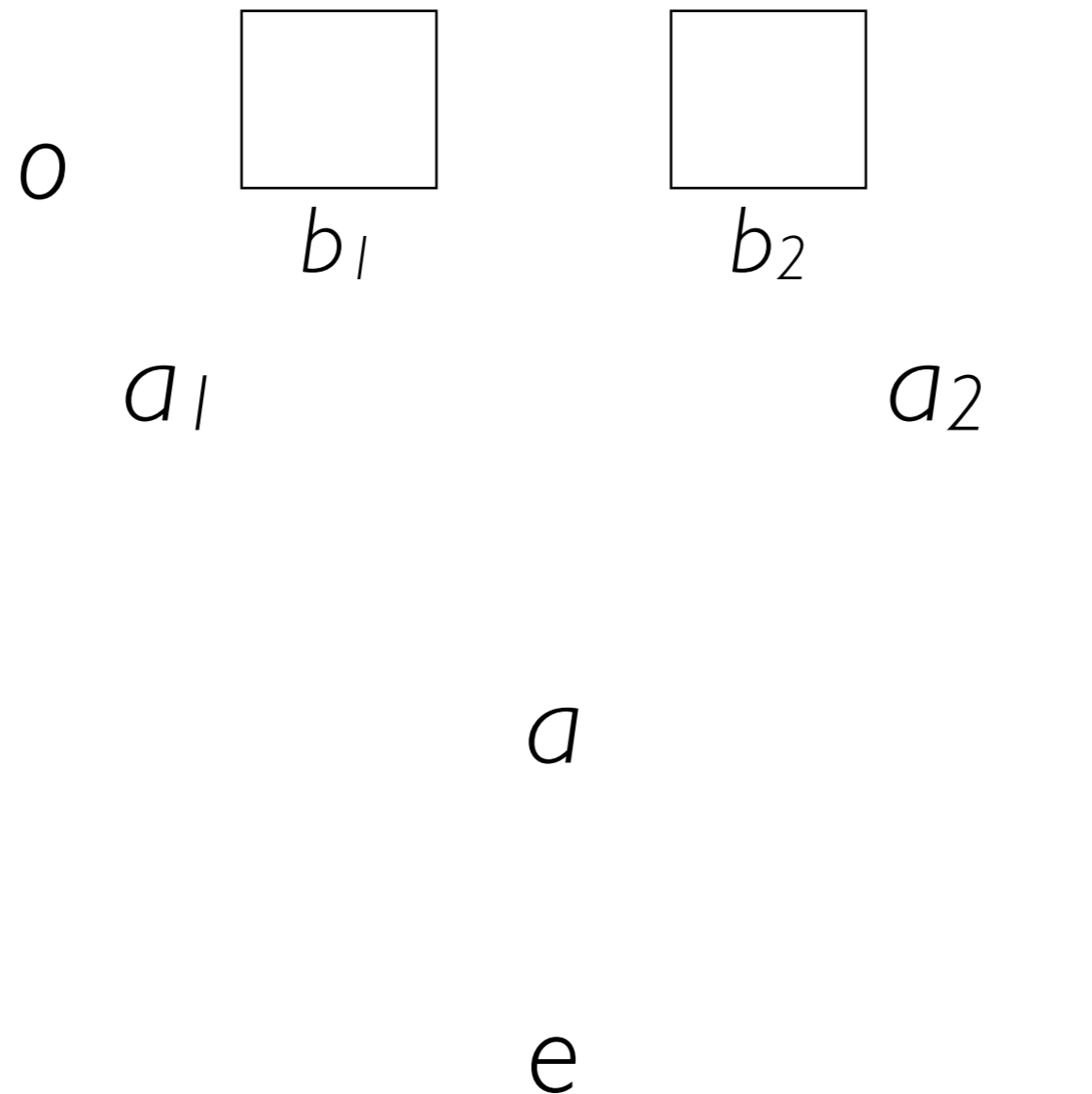


Framework for FBT^0_1



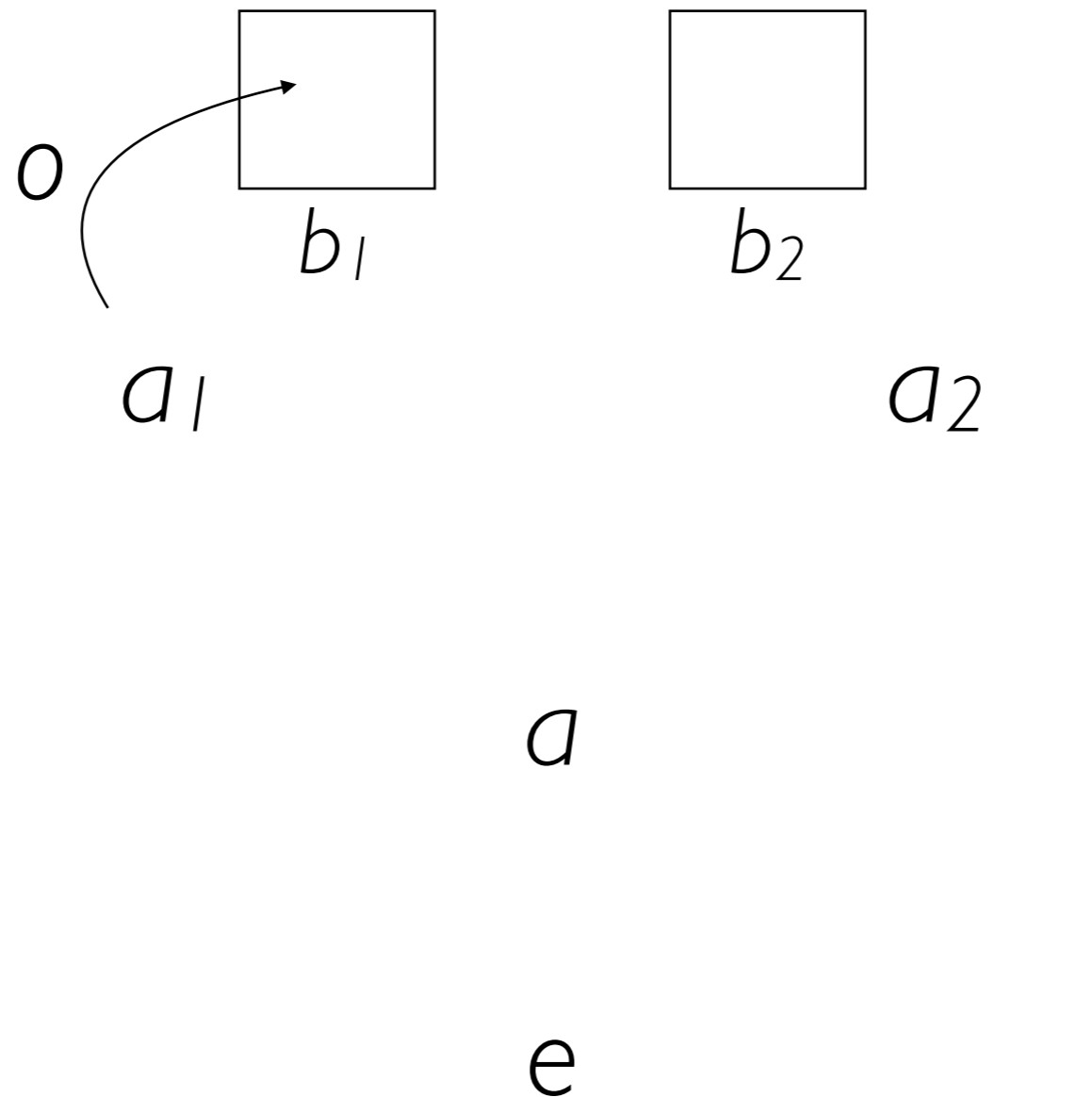
Framework for FBT^0_1

(five timepoints)



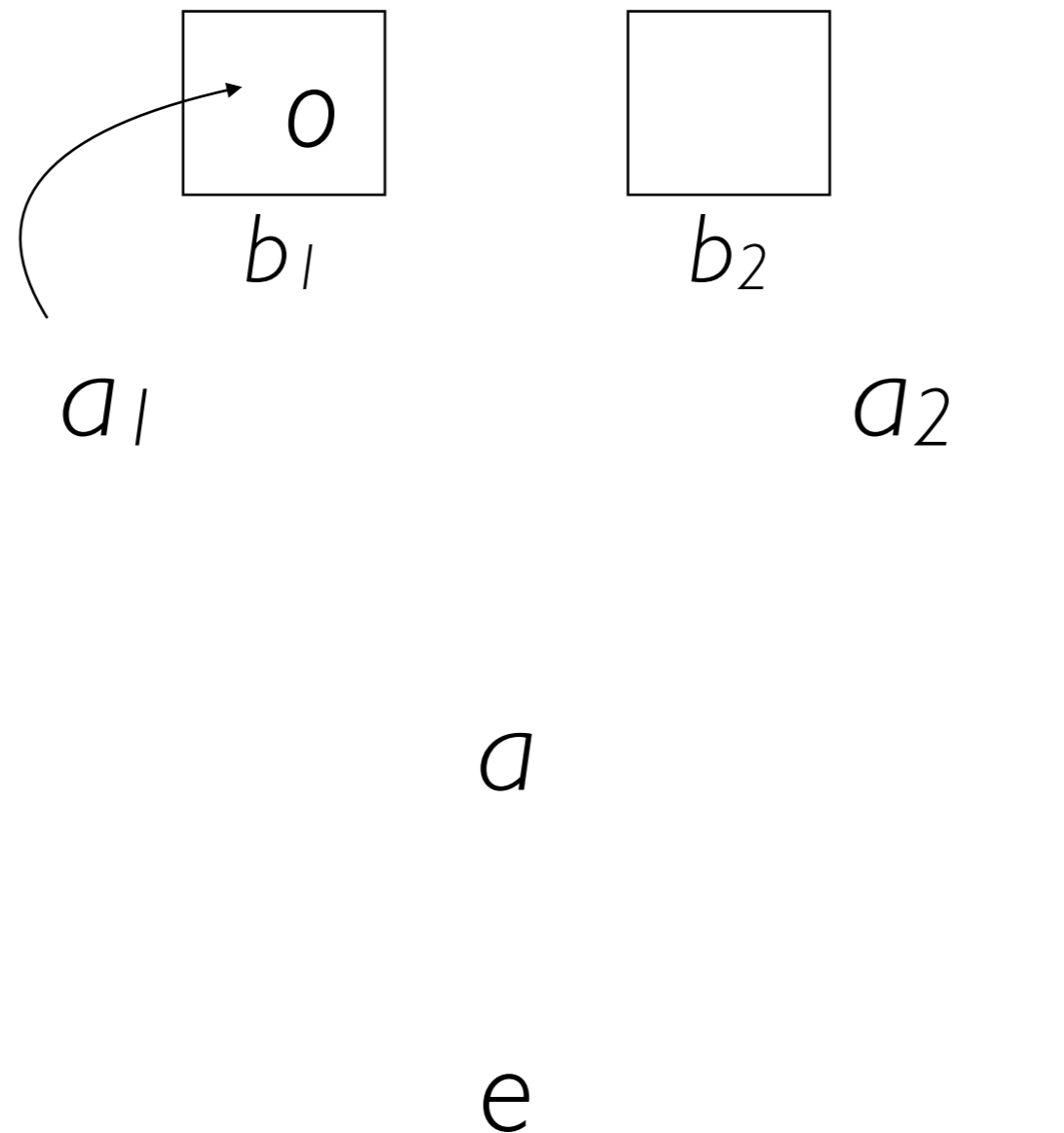
Framework for FBT^0_1

(five timepoints)



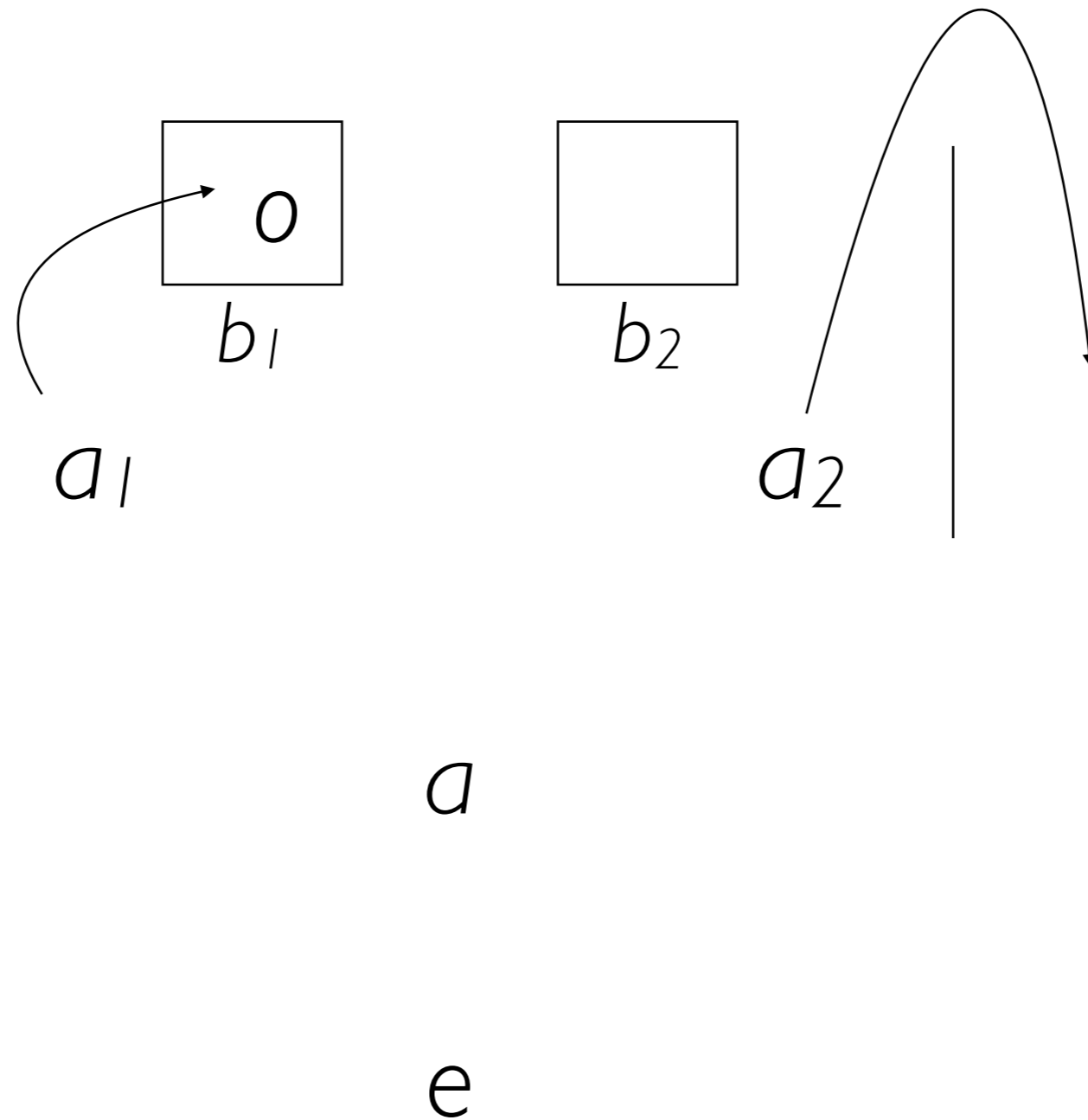
Framework for FBT^0_1

(five timepoints)



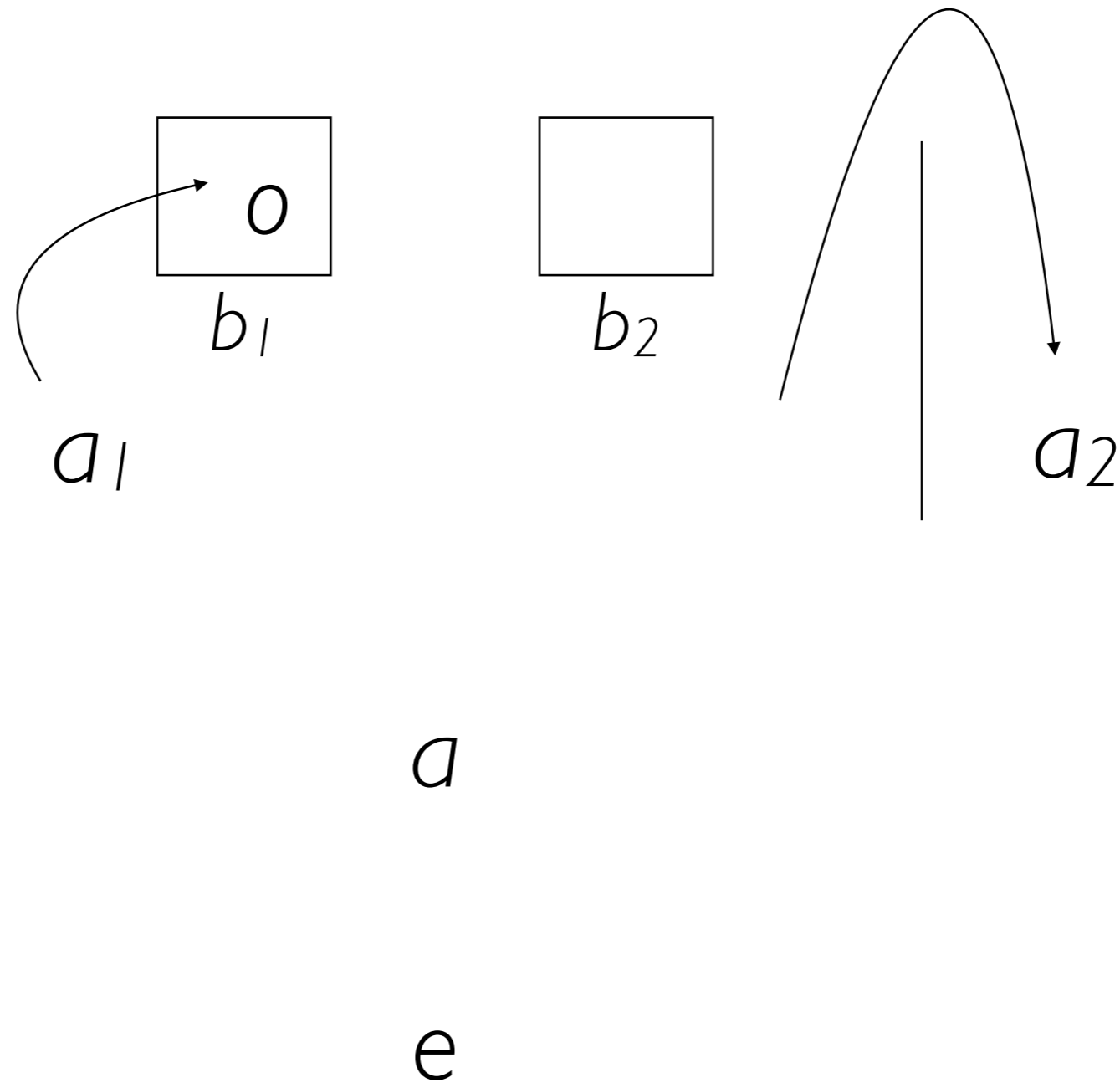
Framework for FBT^0_1

(five timepoints)



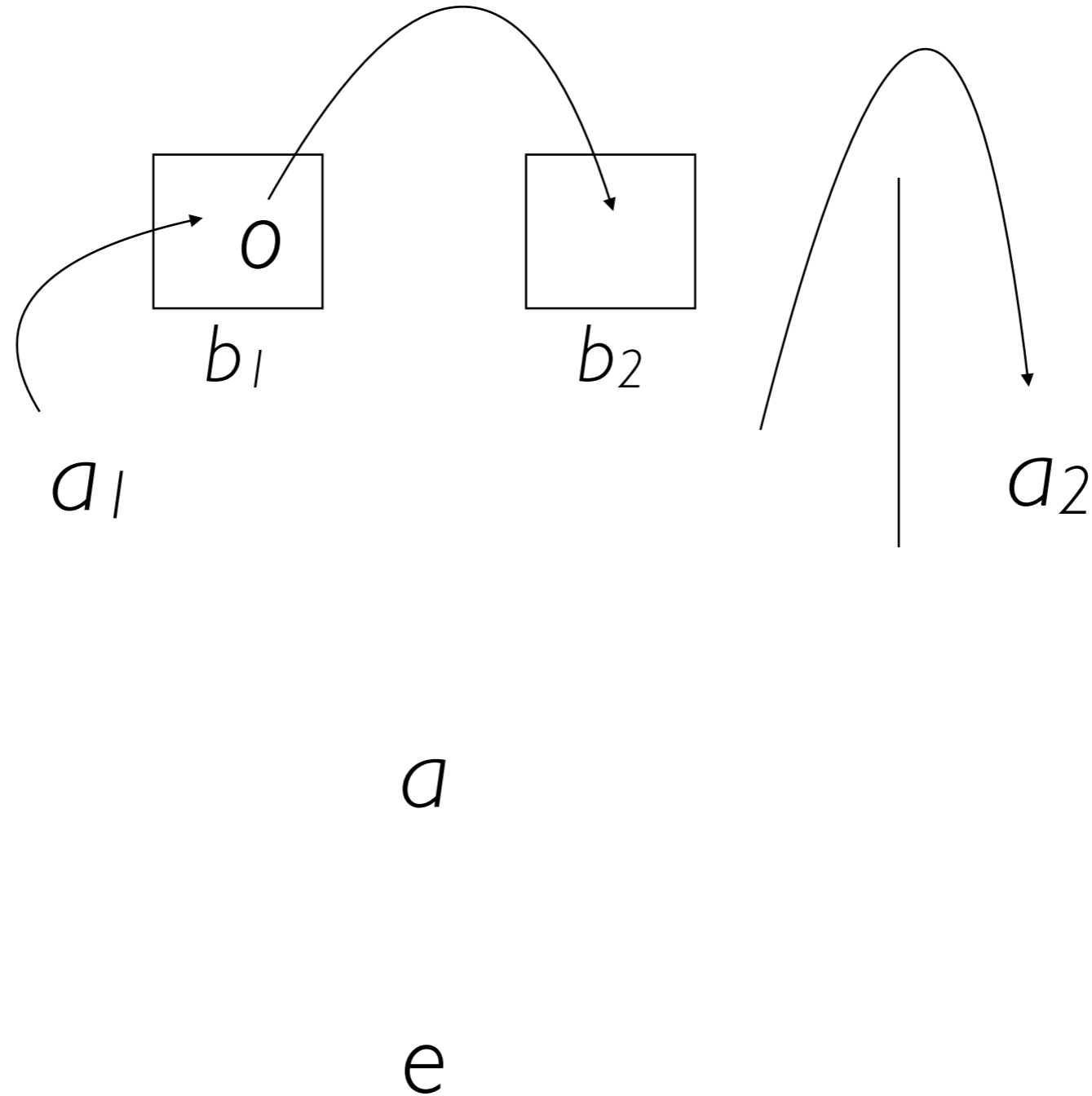
Framework for FBT^0_1

(five timepoints)



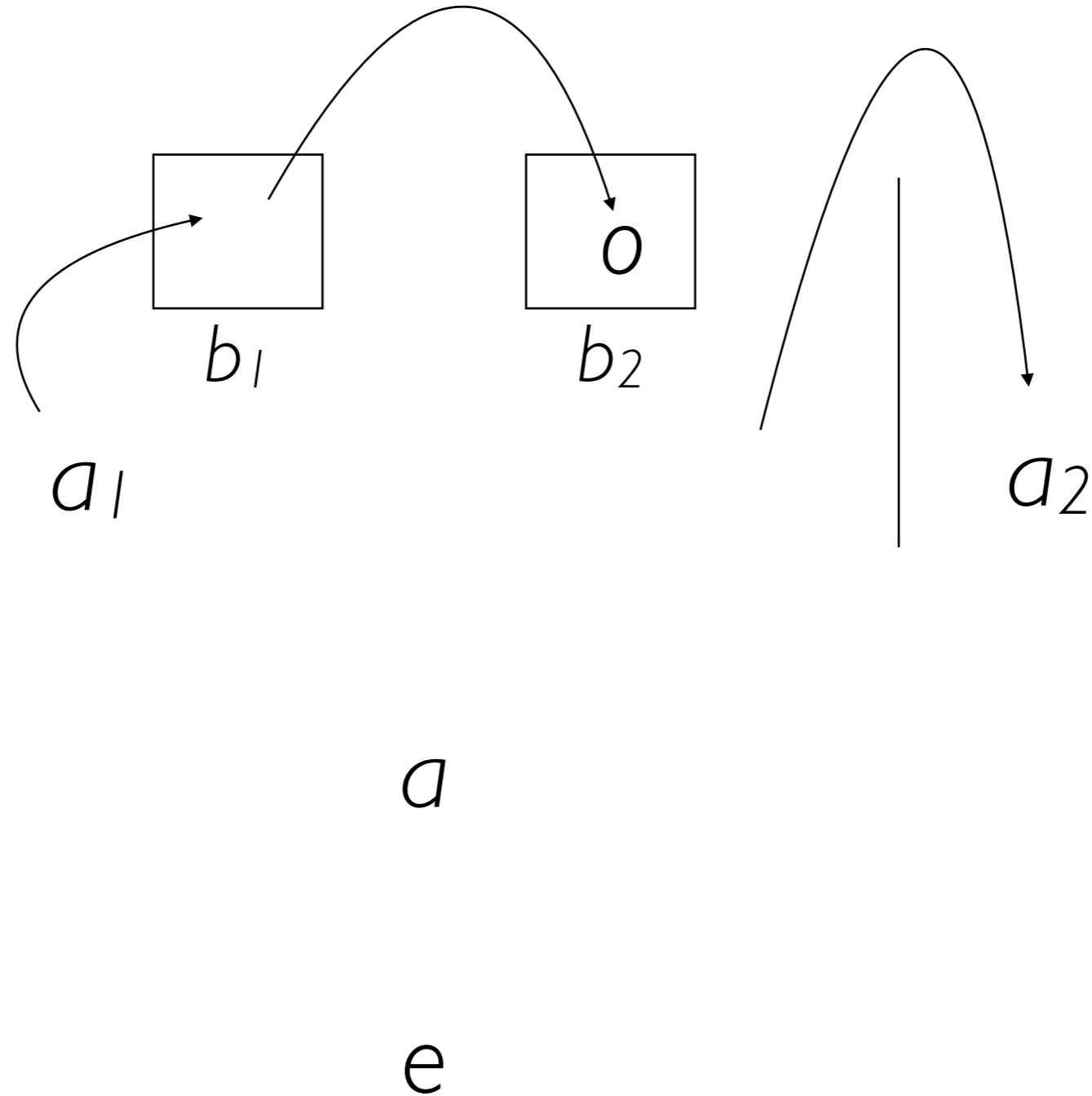
Framework for FBT^0_1

(five timepoints)



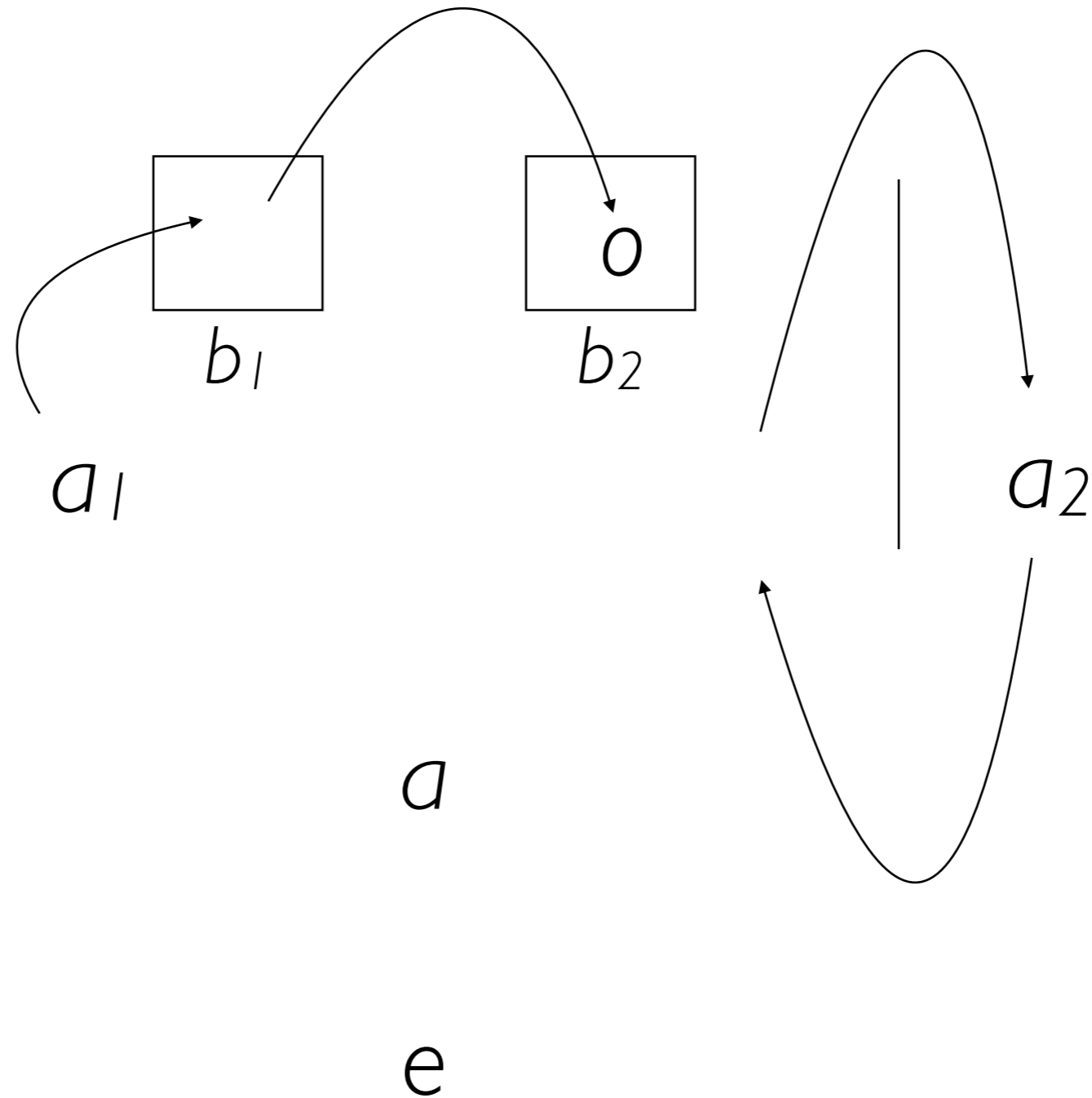
Framework for FBT^0_1

(five timepoints)



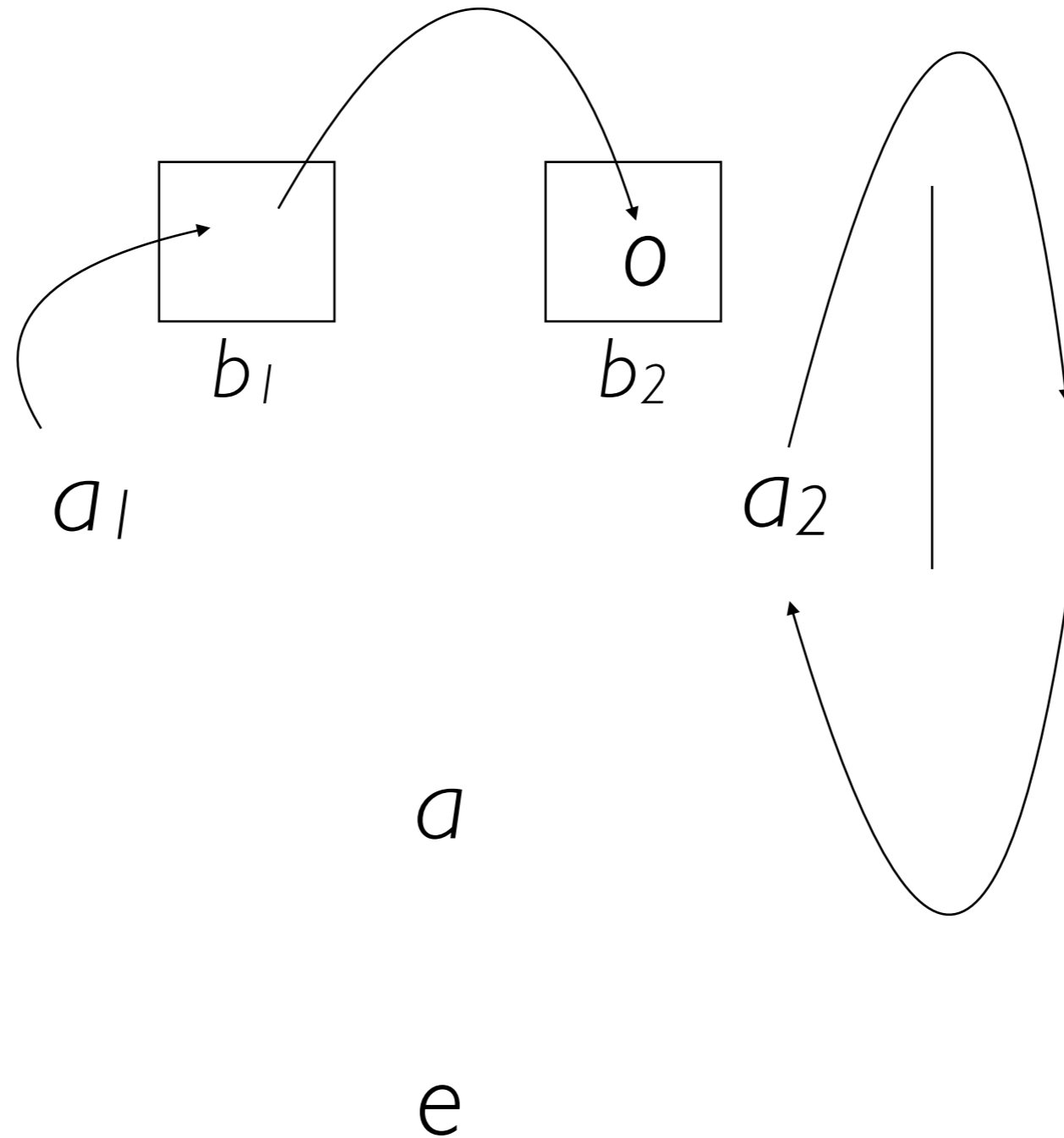
Framework for FBT^0_1

(five timepoints)

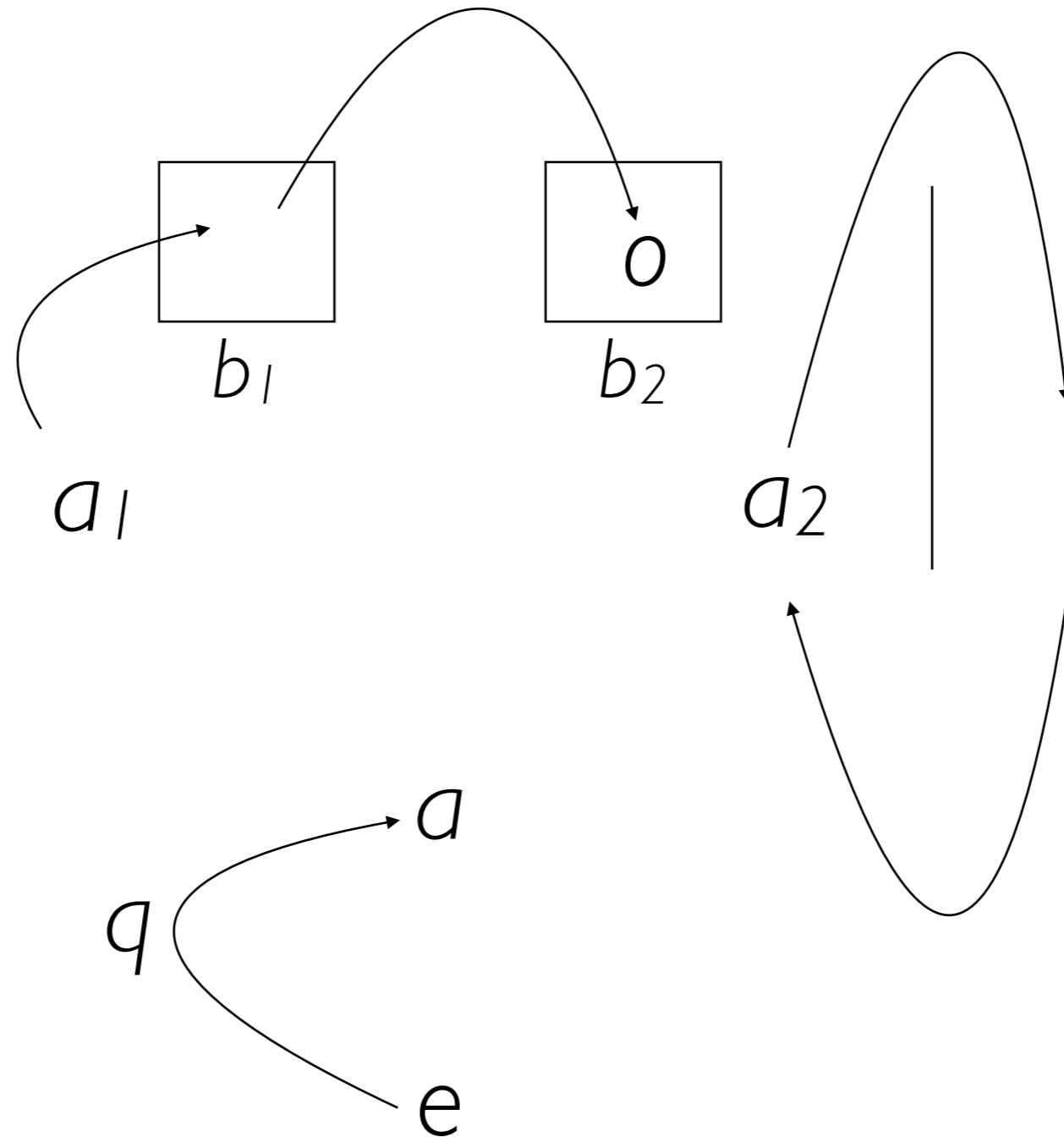


Framework for FBT^0_1

(five timepoints)

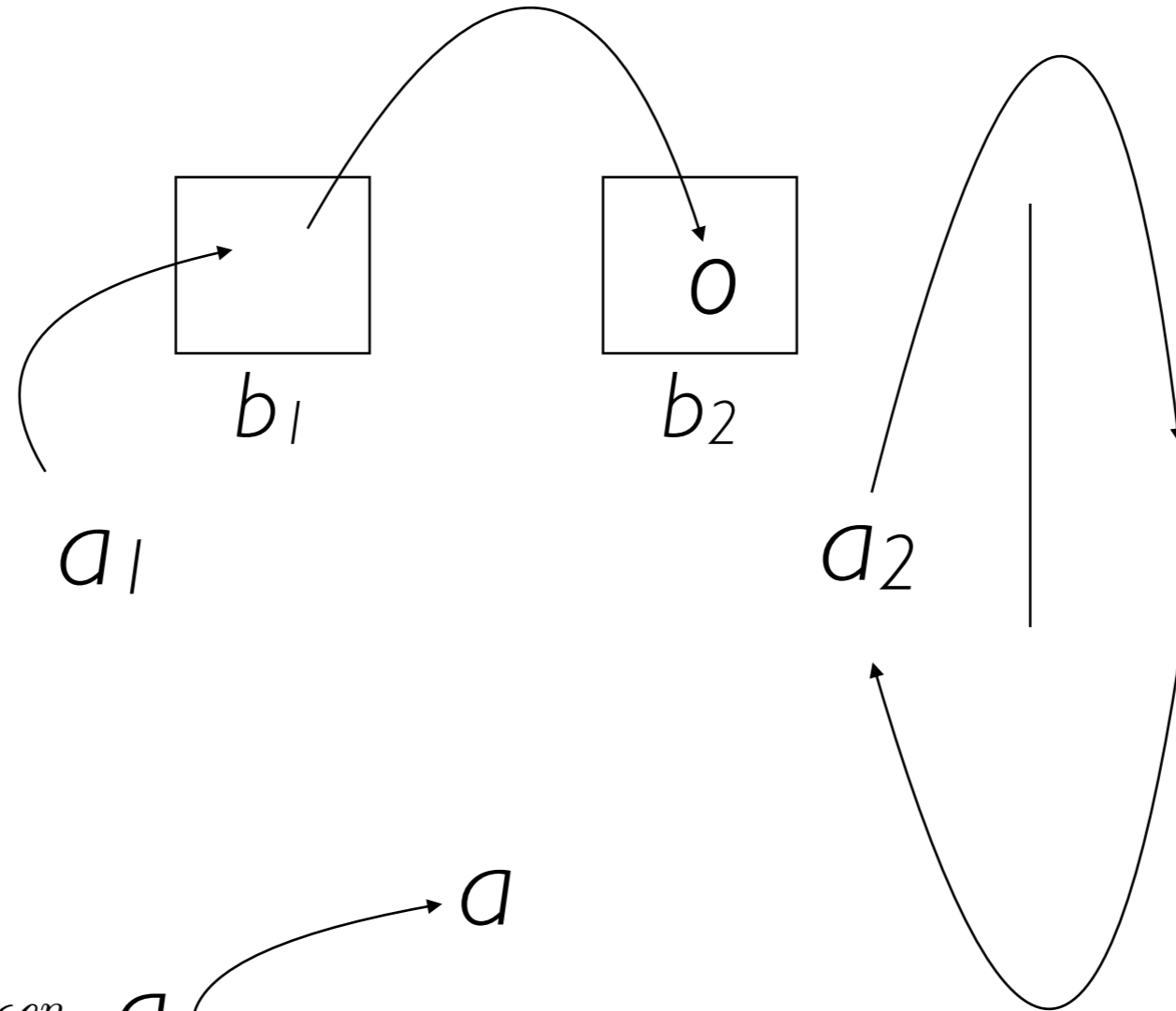


Framework for FBT^0_1 (five timepoints)

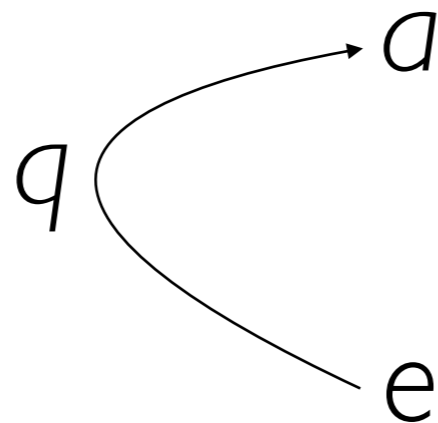


Framework for FBT^0_1

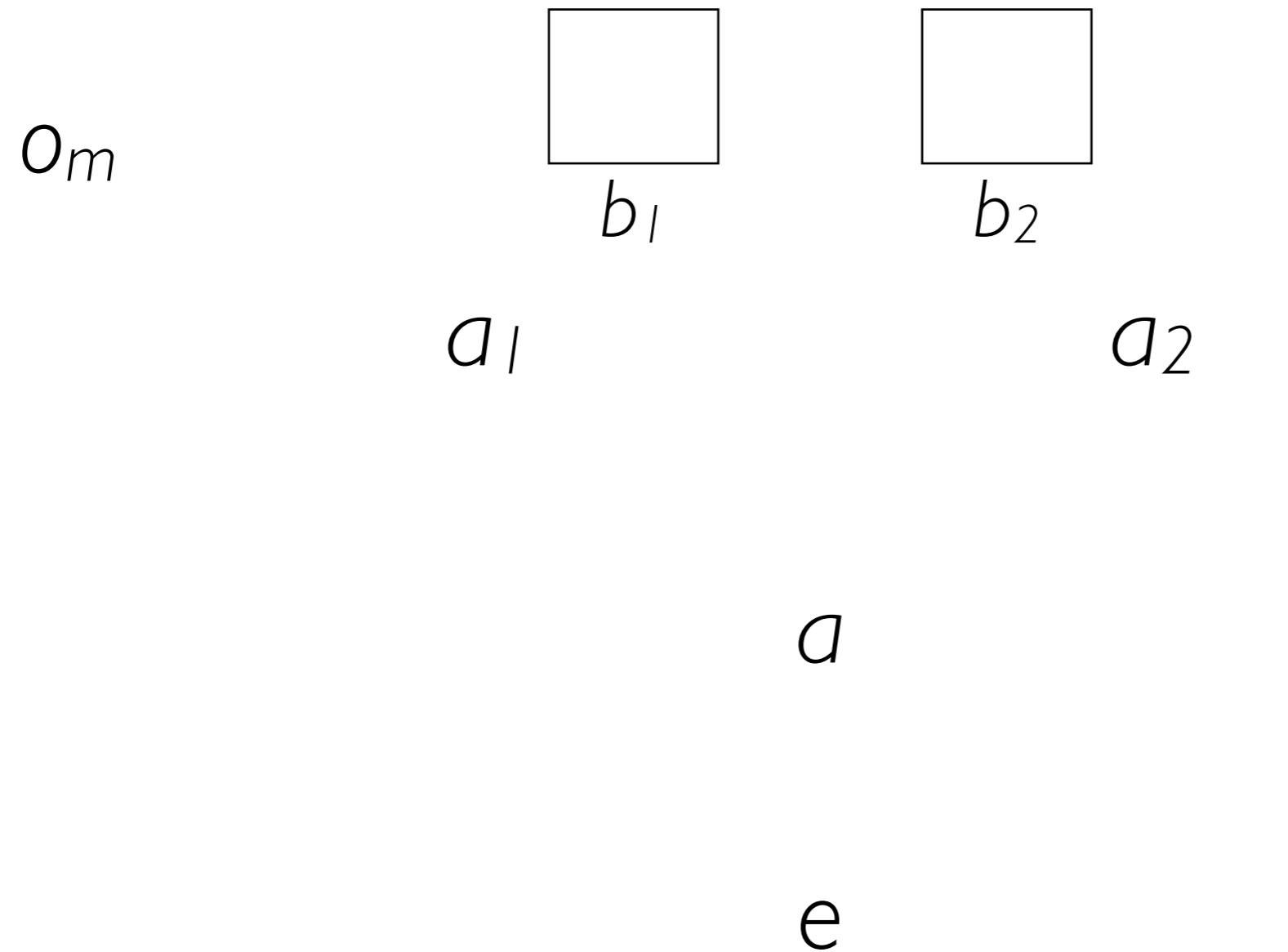
(five timepoints)



q a formula in modal \mathcal{L}^n

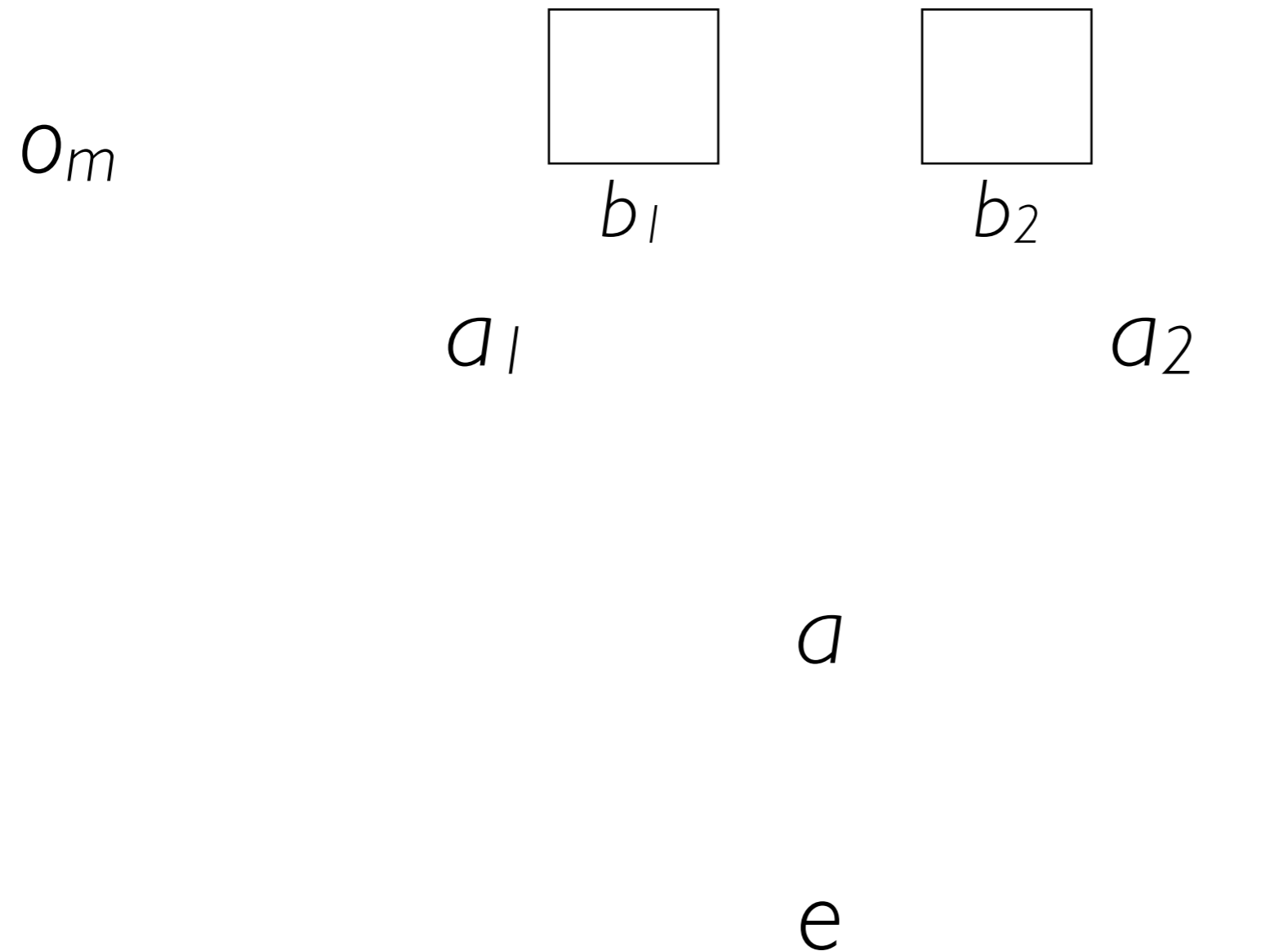


Framework for FBT₁



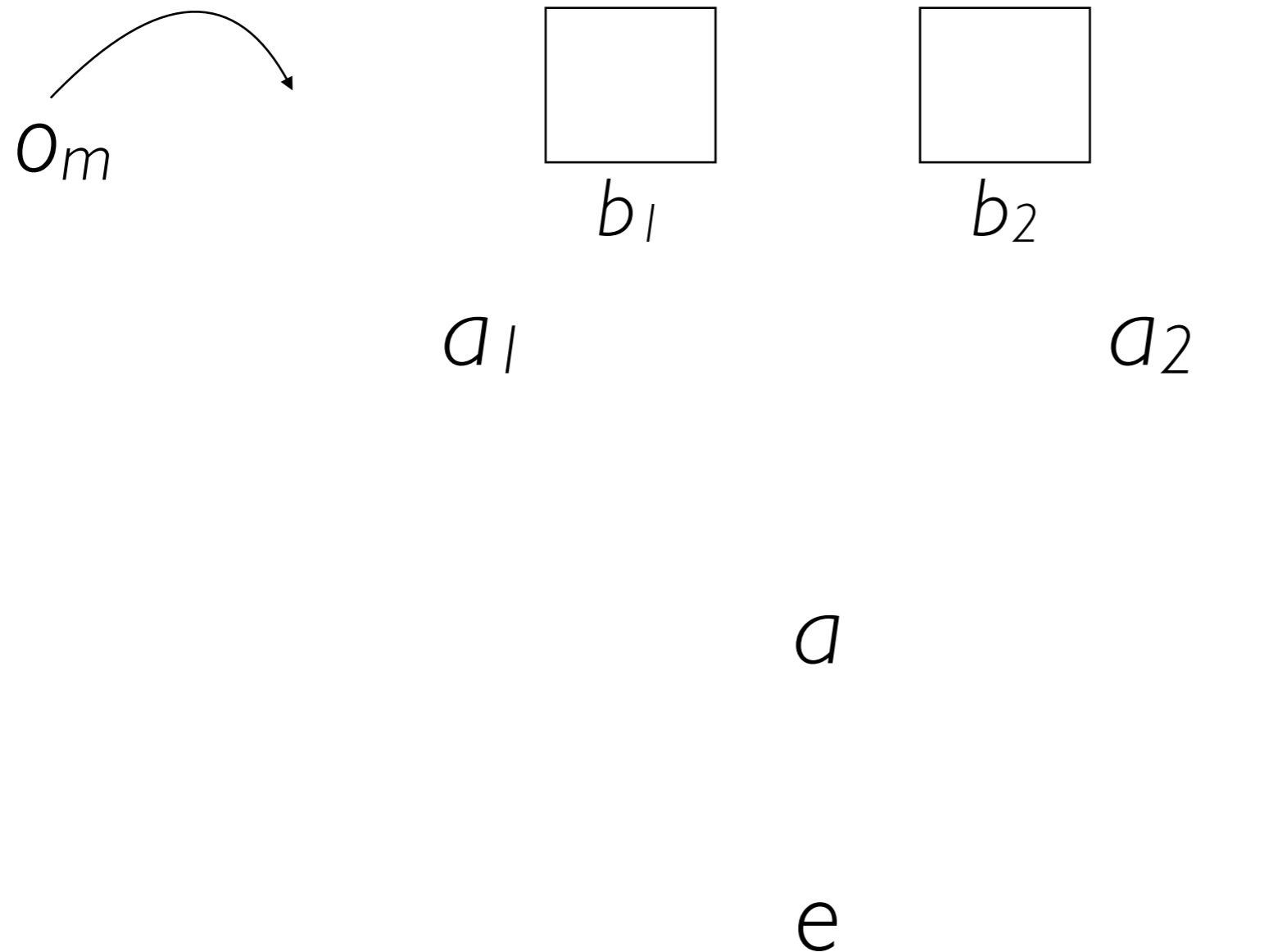
Framework for FBT₁

(six timepoints)



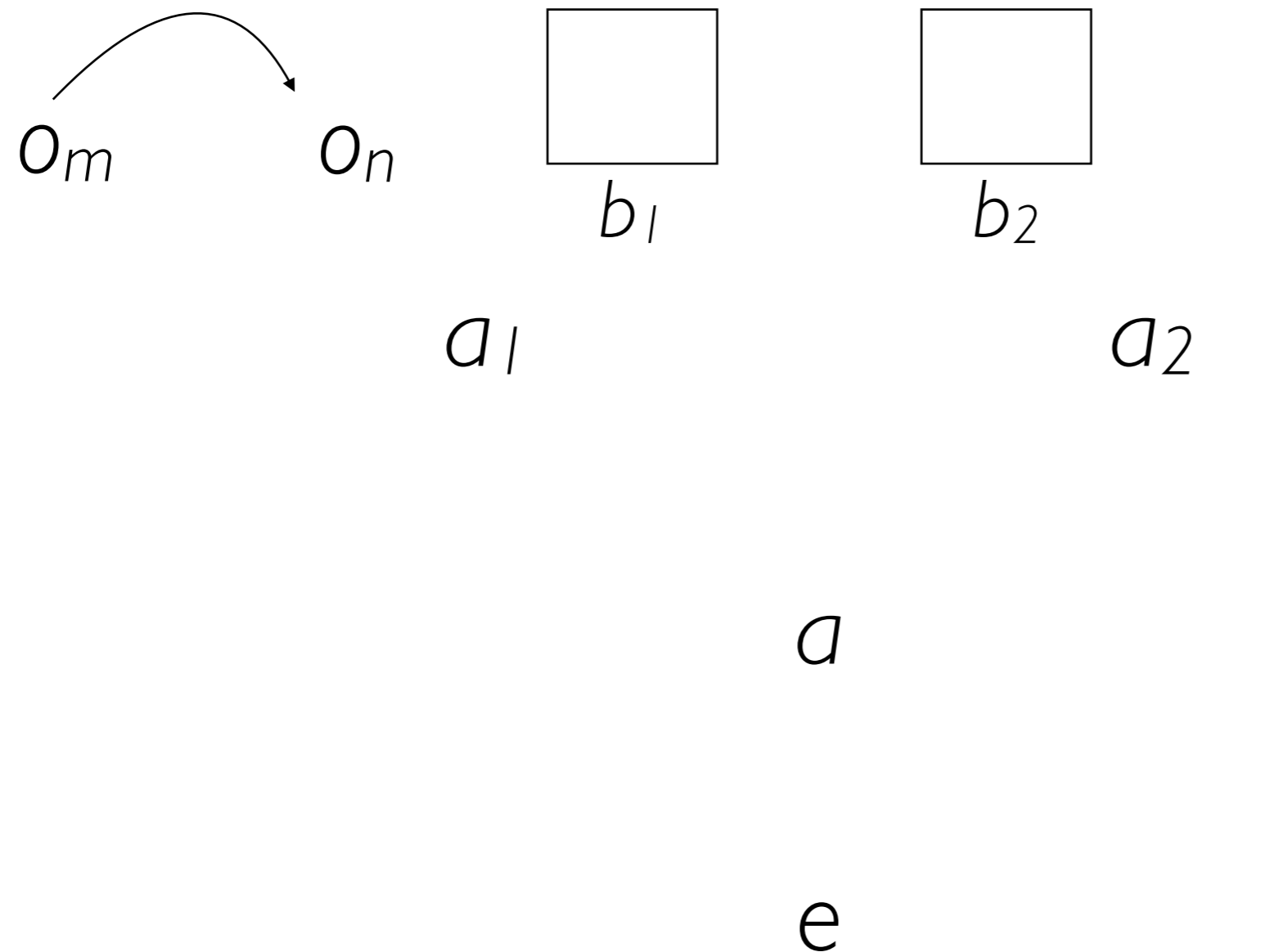
Framework for FBT₁

(six timepoints)



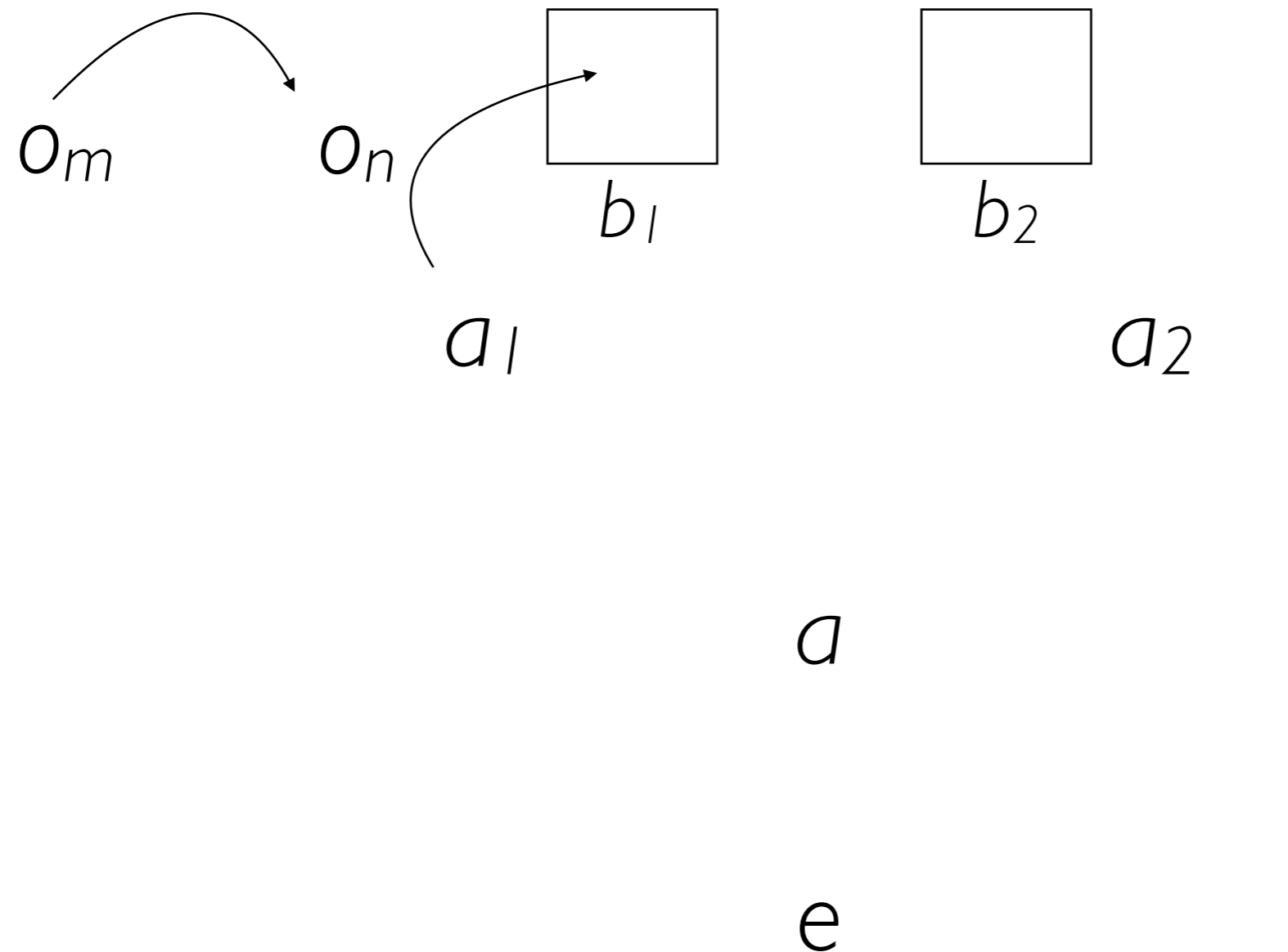
Framework for FBT₁

(six timepoints)



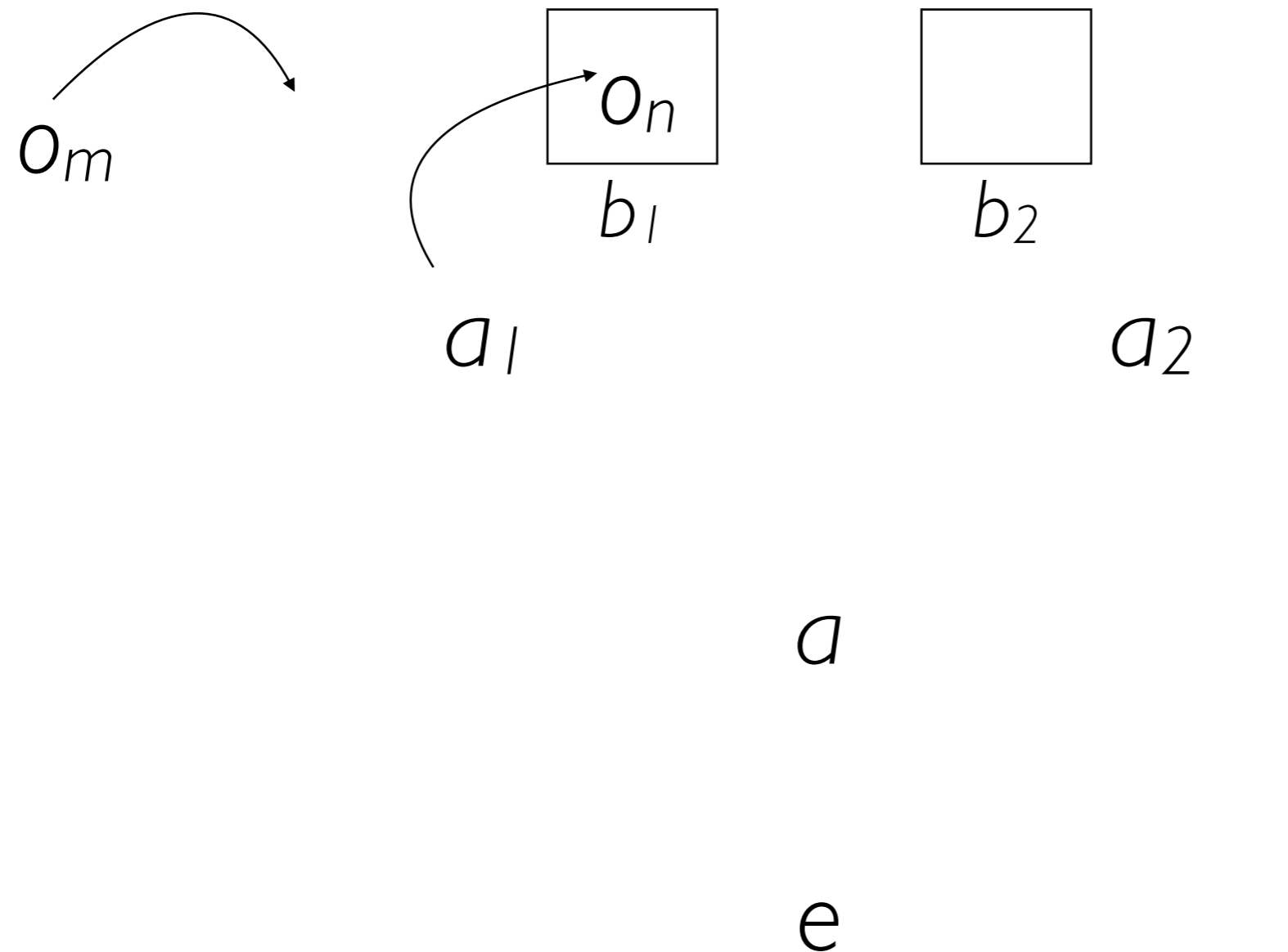
Framework for FBT₁

(six timepoints)



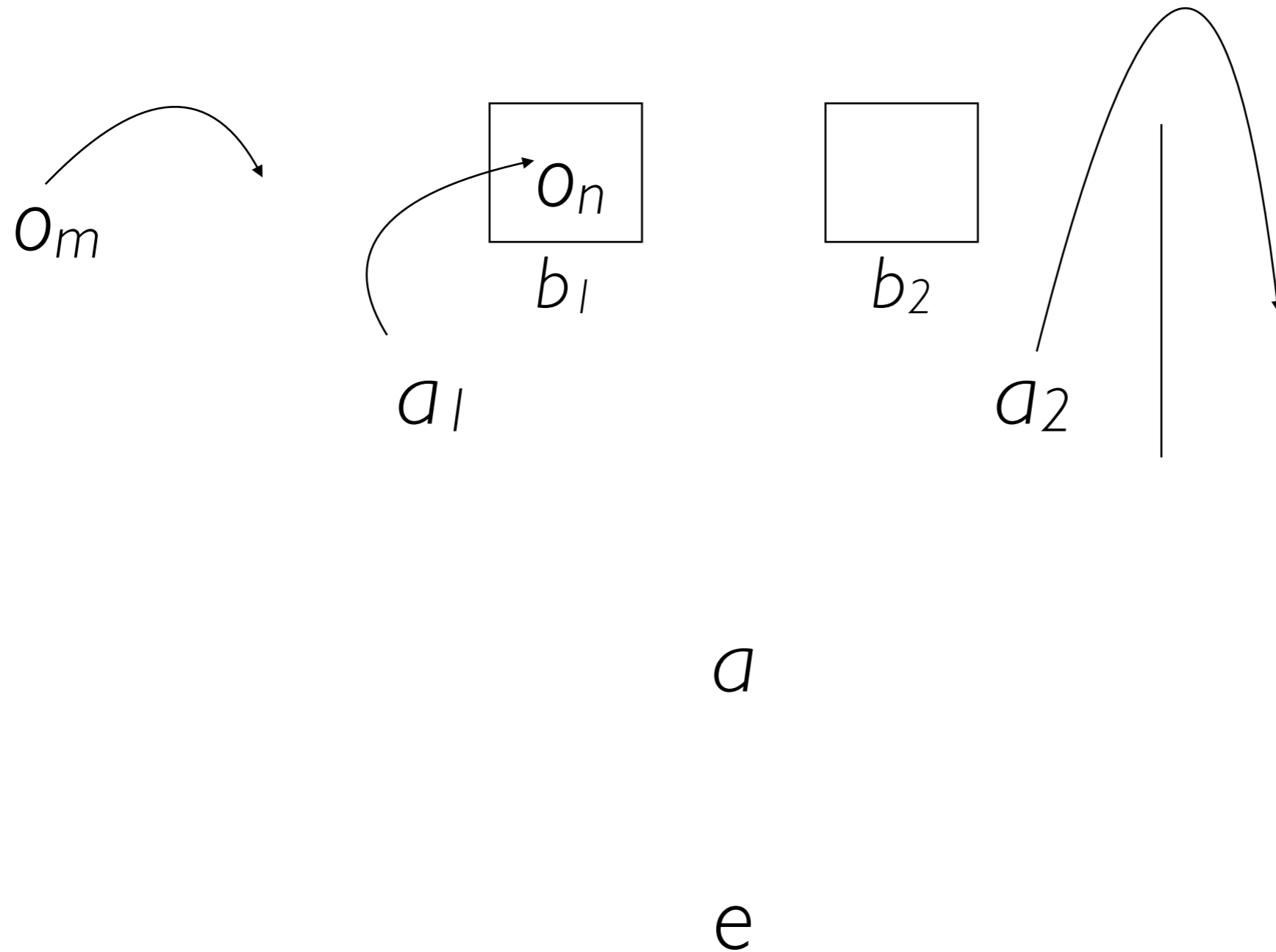
Framework for FBT₁

(six timepoints)



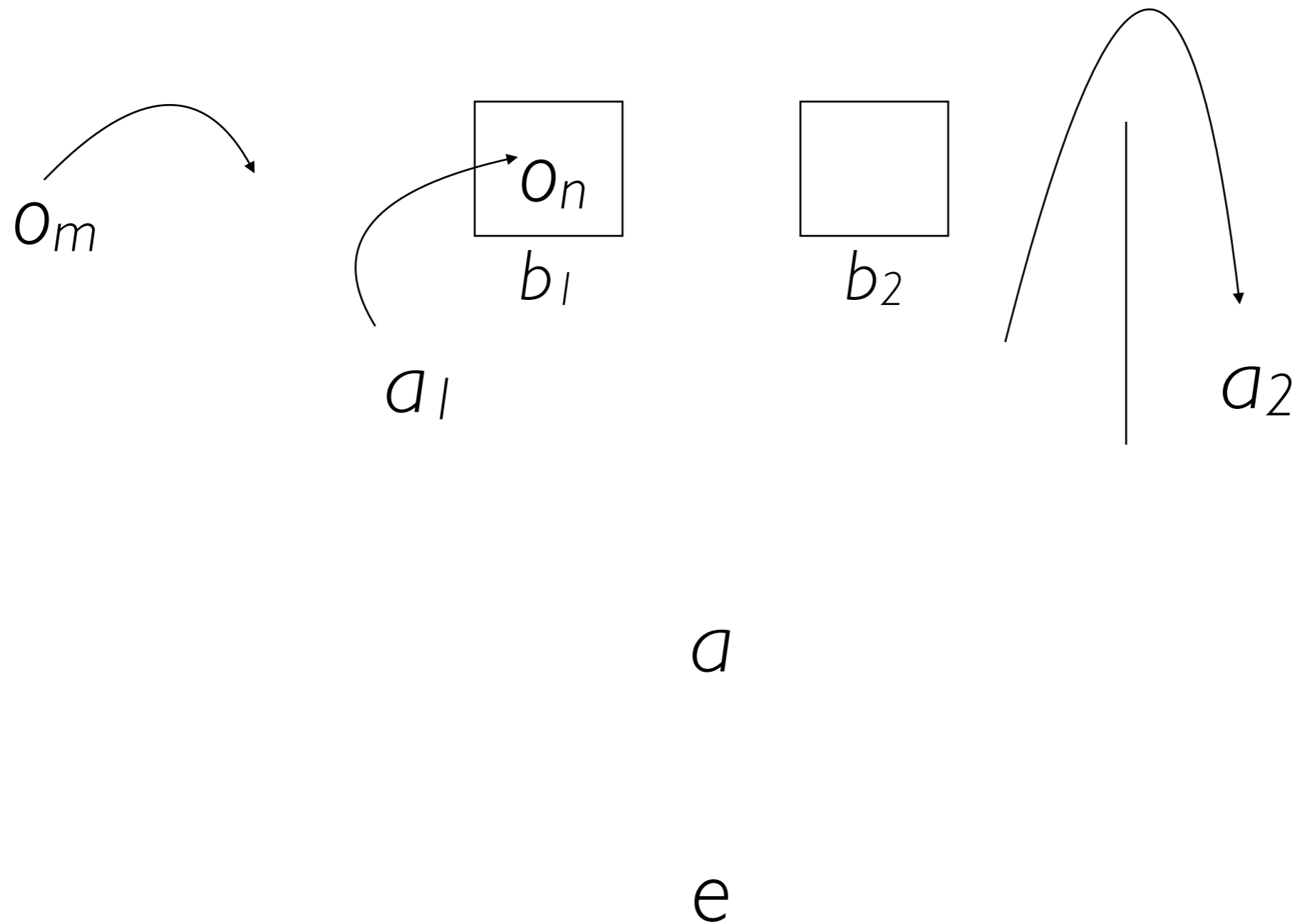
Framework for FBT₁

(six timepoints)



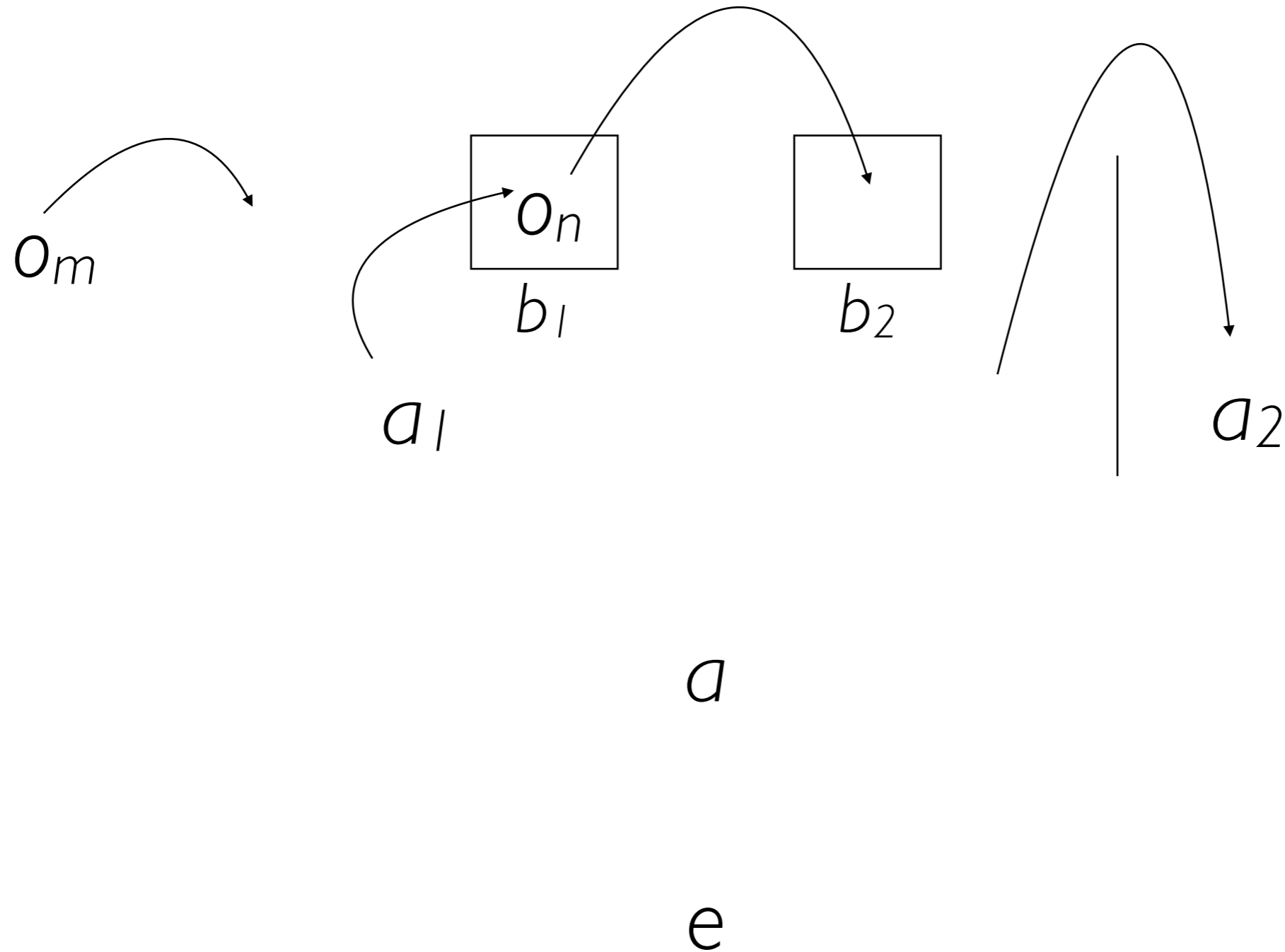
Framework for FBT₁

(six timepoints)



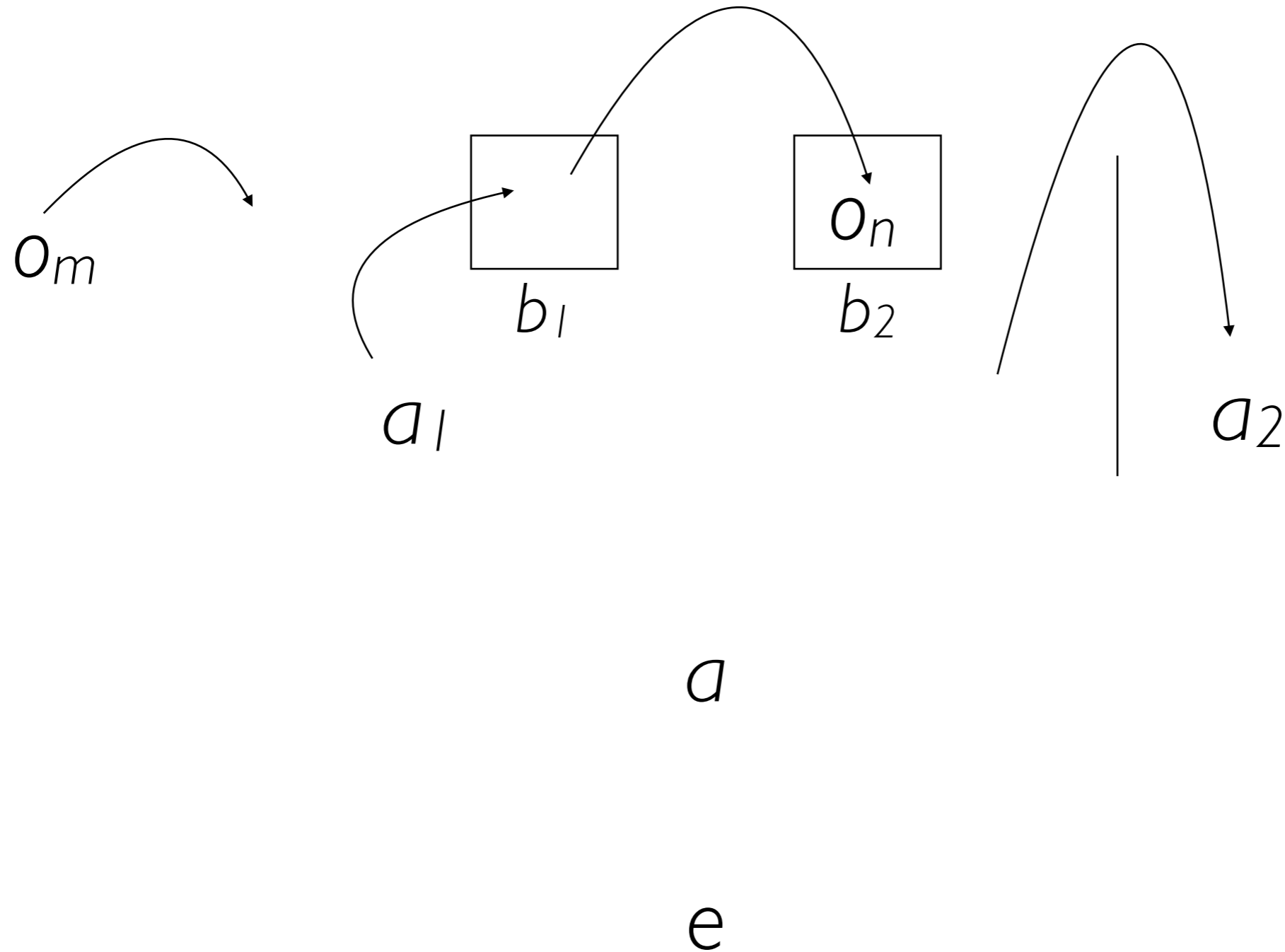
Framework for FBT₁

(six timepoints)



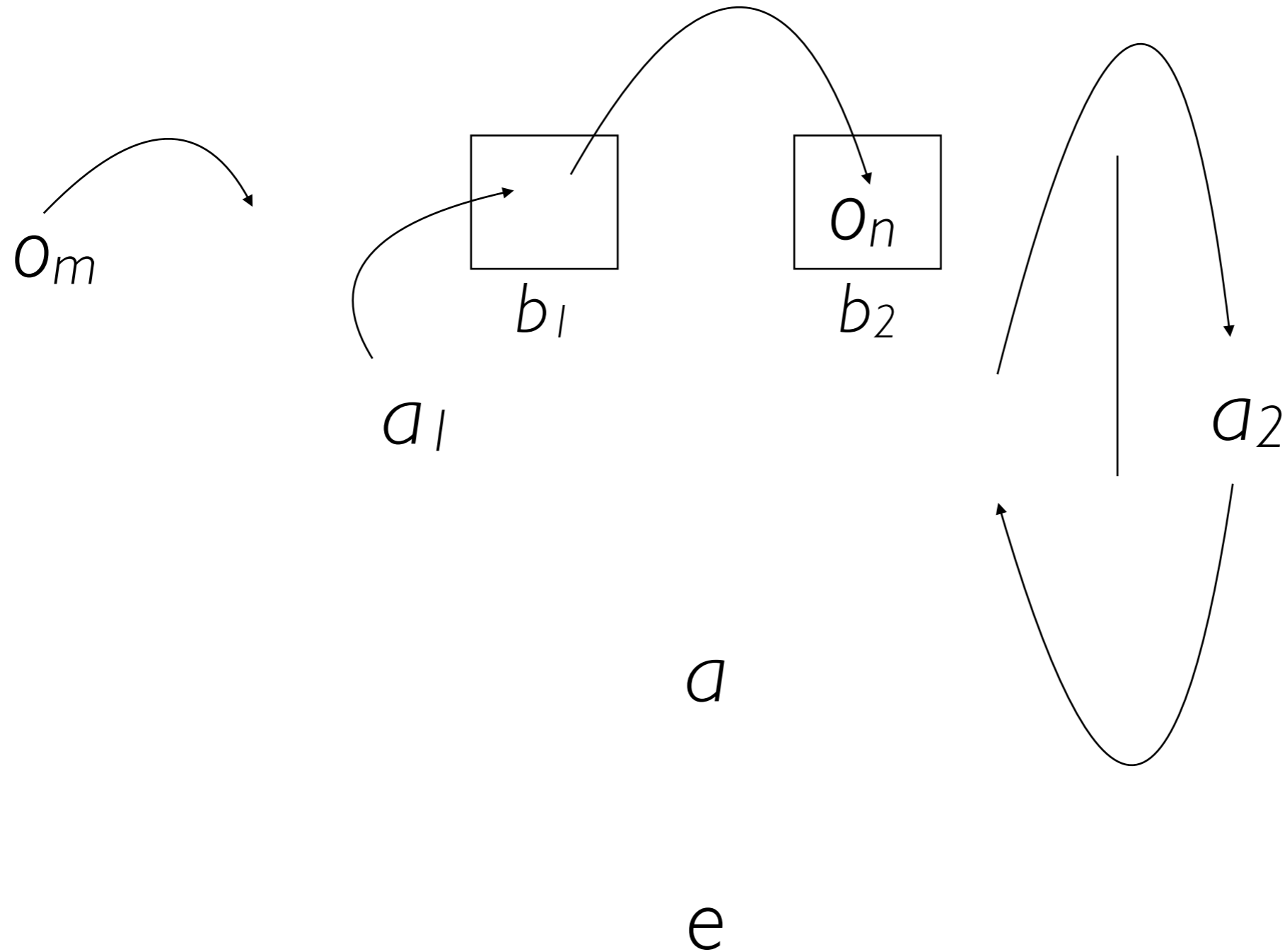
Framework for FBT₁

(six timepoints)



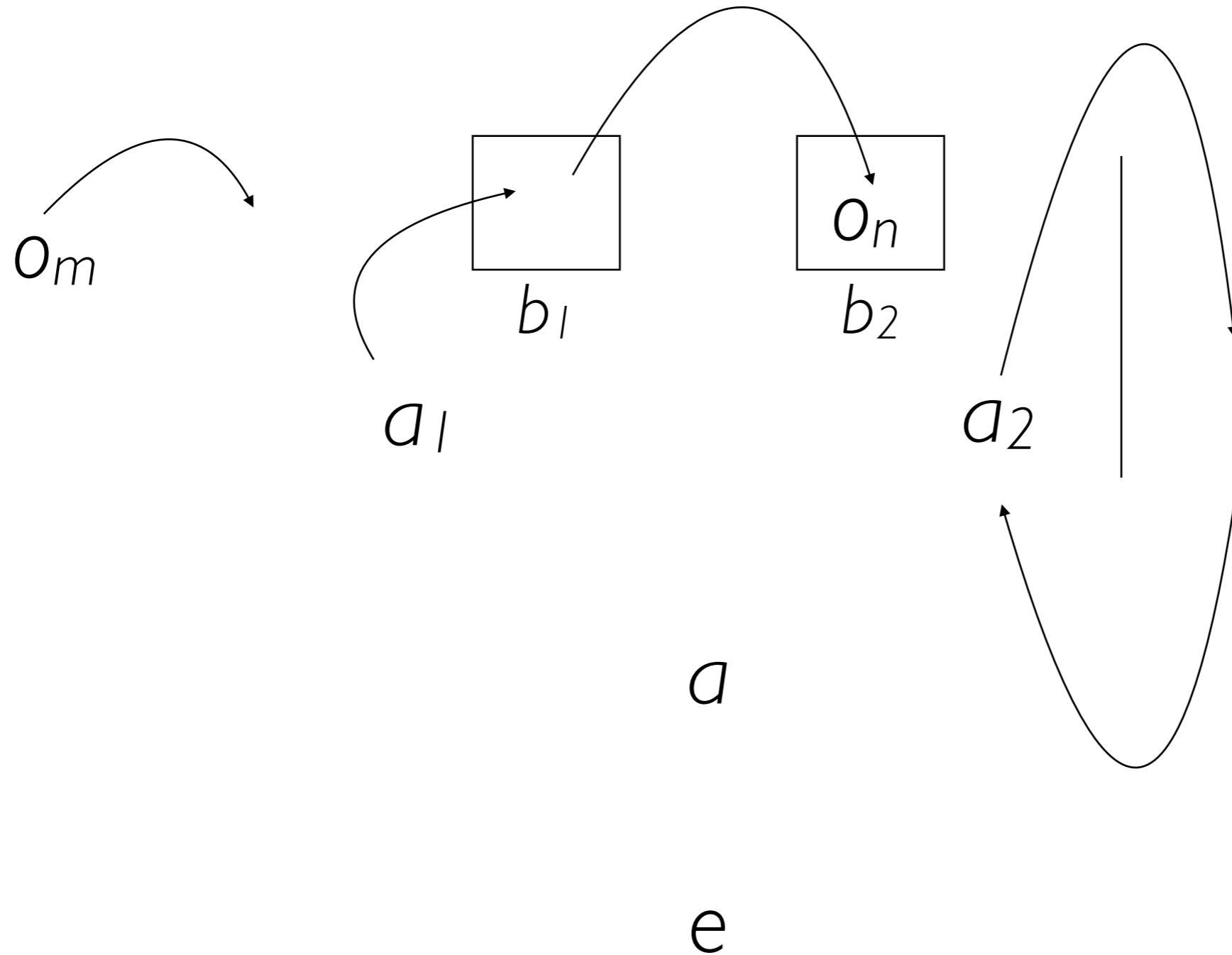
Framework for FBT₁

(six timepoints)



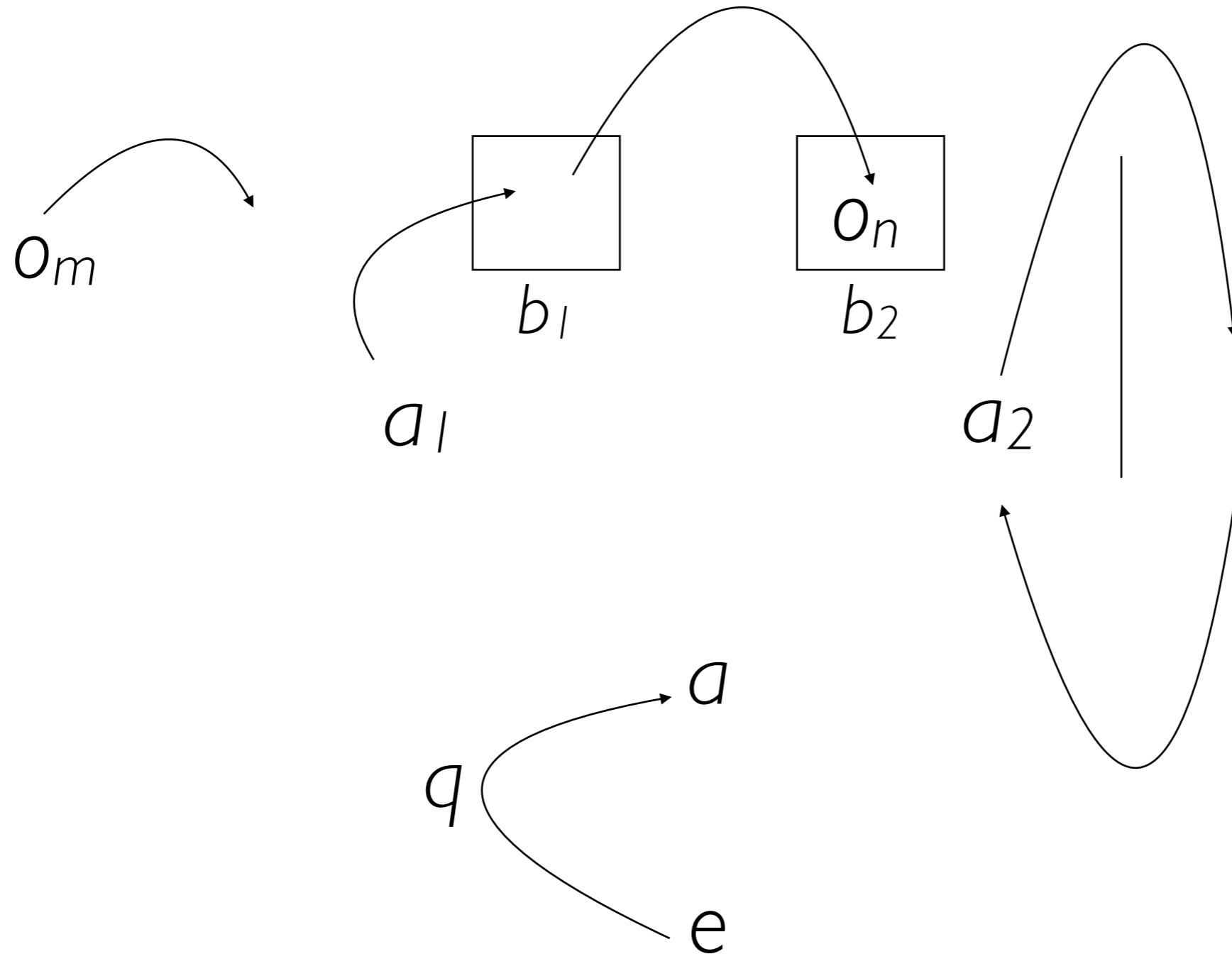
Framework for FBT₁

(six timepoints)



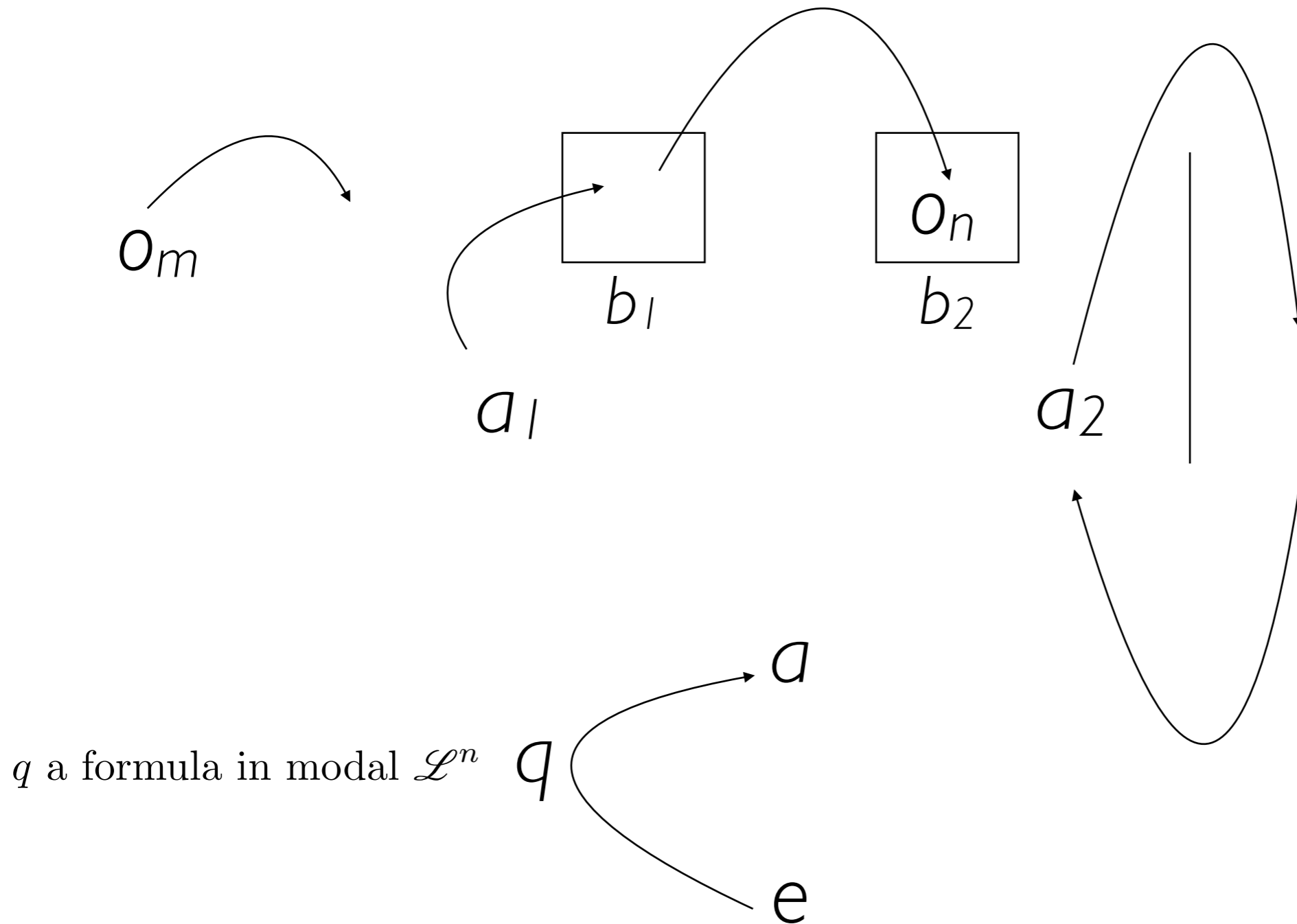
Framework for FBT₁

(six timepoints)



Framework for FBT^1_1

(six timepoints)



Done, a Decade Ago, Formally & Implementation/Simulation

Arkoudas, K. & Bringsjord, S.
(2009) “Propositional
Attitudes and Causation”
*International Journal of Software
and Informatics* **3.1**: 47–65.

http://kryten.mm.rpi.edu/PRICAI_w_sequentialcalc_041709.pdf

Propositional attitudes and causation

Konstantine Arkoudas and Selmer Bringsjord

Cognitive Science and Computer Science Departments, RPI
arkouk@rpi.edu, brings@rpi.edu

Abstract. Predicting and explaining the behavior of others in terms of mental states is indispensable for everyday life. It will be equally important for artificial agents. We present an inference system for representing and reasoning about mental states, and use it to provide a formal analysis of the false-belief task. The system allows for the representation of information about events, causation, and perceptual, doxastic, and epistemic states (vision, belief, and knowledge), incorporating ideas from the event calculus and multi-agent epistemic logic. Unlike previous AI formalisms, our focus here is on mechanized proofs and proof programmability, not on metamathematical results. Reasoning is performed via relatively cognitively plausible inference rules, and a degree of automation is achieved by general-purpose inference methods and by a syntactic embedding of the system in first-order logic.

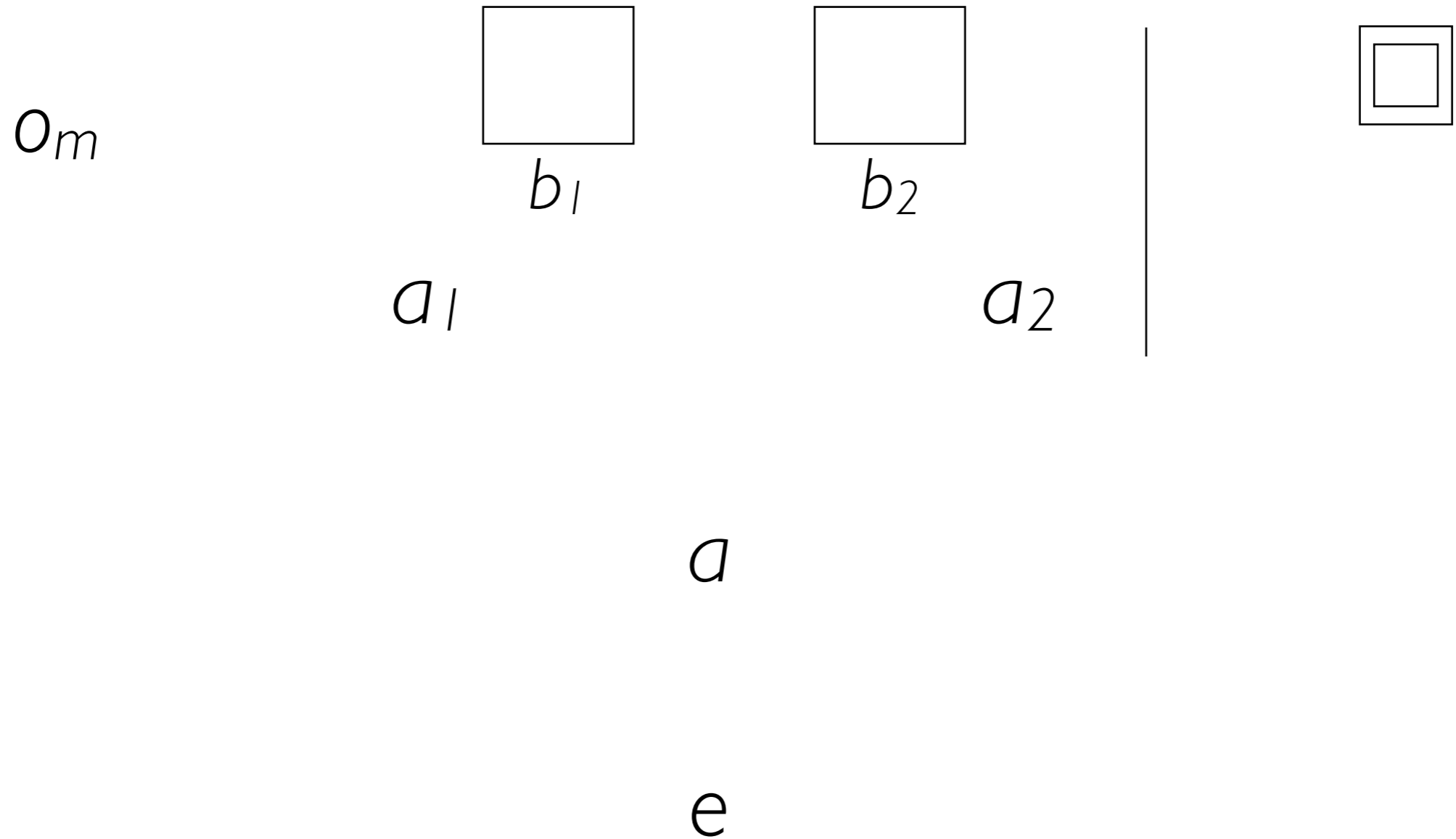
1 Introduction

Interpreting the behavior of other people is indispensable for everyday life. It is something that we do constantly, on a daily basis, and it helps us not only to make sense of human behavior, but also to predict it and—to a certain extent—to control it. How exactly do we manage that? That is not currently known, but many have argued that the ability to ascribe mental states to others and to reason about such mental states is a key component of our capacity to understand human behavior. In particular, all social transactions, from engaging in commerce and negotiating to making jokes and empathizing with other people’s pain or joy, appear to require at least a rudimentary grasp of common-sense psychology (CSP), i.e., a large body of truisms such as the following: When an agent a (1) wants to achieve a certain state of affairs p , and (2) believes that some action c can bring about p , and (3) a knows how to carry out c ; then, *ceteris paribus*,¹ a will carry out c ; when a sees that p , a knows that p ; when a fears that p and a discovers that p is the case, a is disappointed; and so on.

Artificial agents without a mastery of CSP would be severely handicapped in their interactions with humans. This could present problems not only for artificial agents trying to interpret human behavior, but also for artificial agents trying to interpret the behavior of one another. When a system exhibits a complex but rational behavior, and detailed knowledge of its internal structure is not

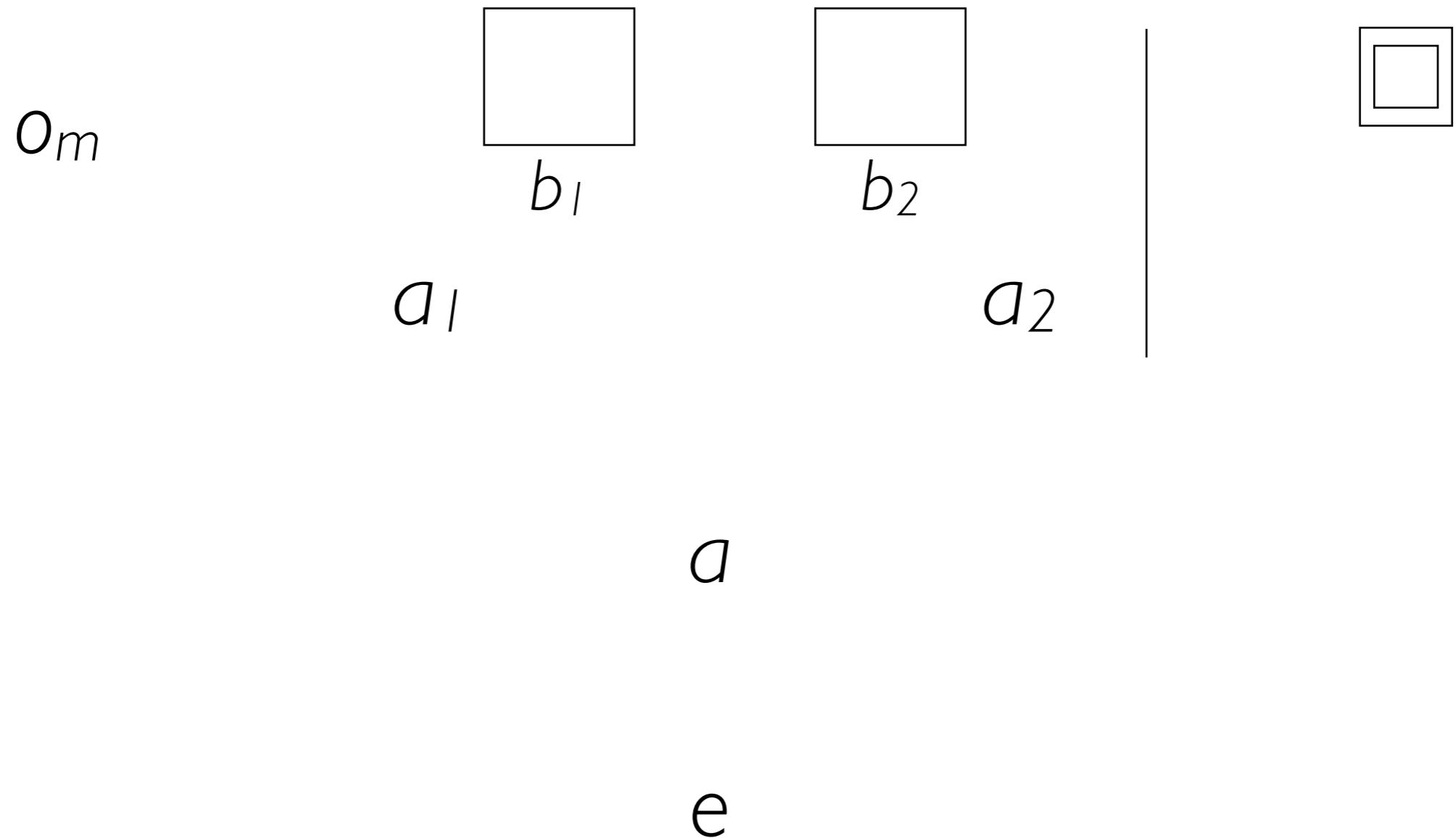
¹ Assuming that a is able to carry out c , that a has no conflicting desires that override his goal that p ; and so on.

Framework for FBT₂



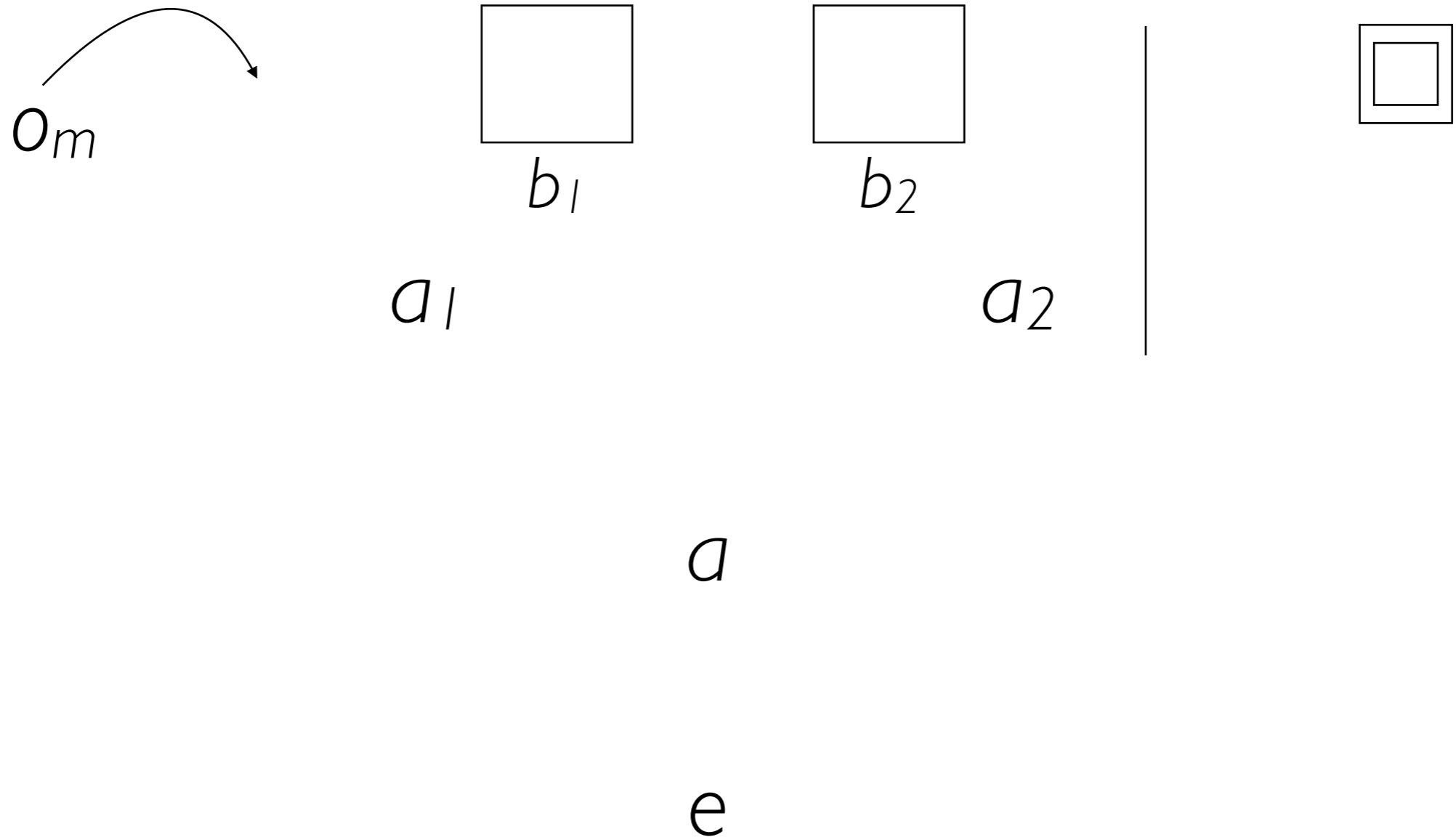
Framework for FBT^1_2

(seven timepoints)



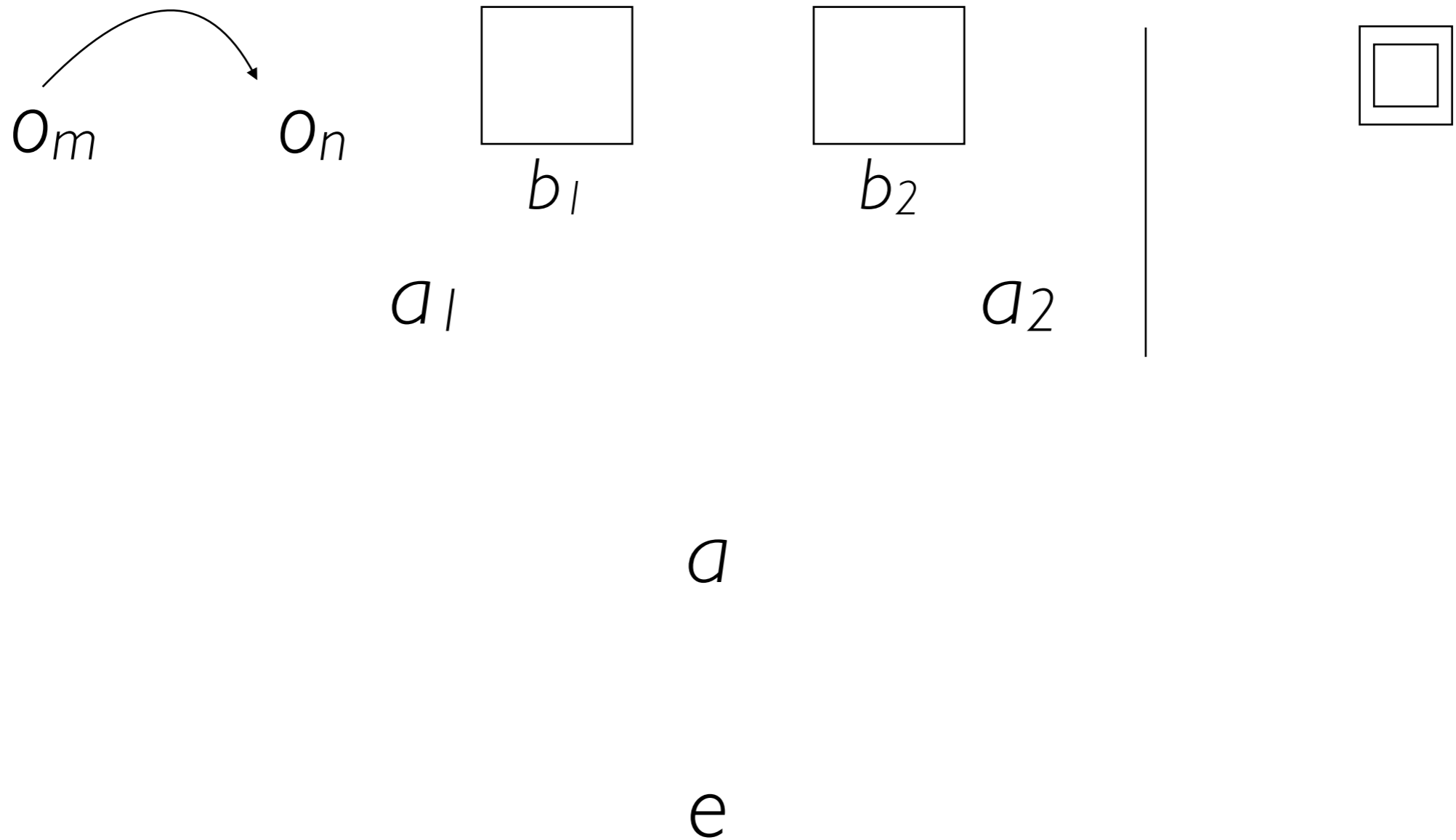
Framework for FBT^1_2

(seven timepoints)



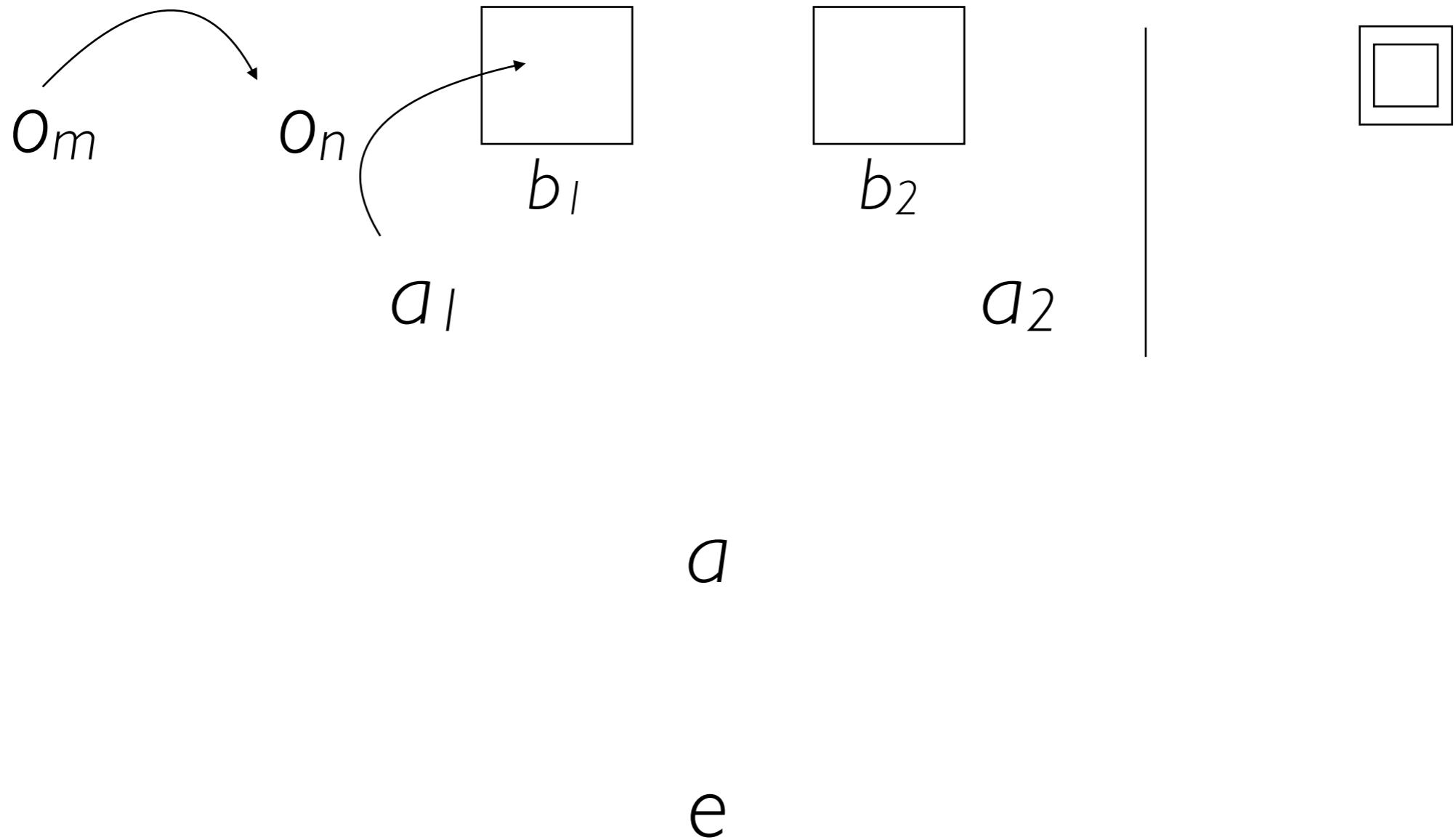
Framework for FBT₂

(seven timepoints)



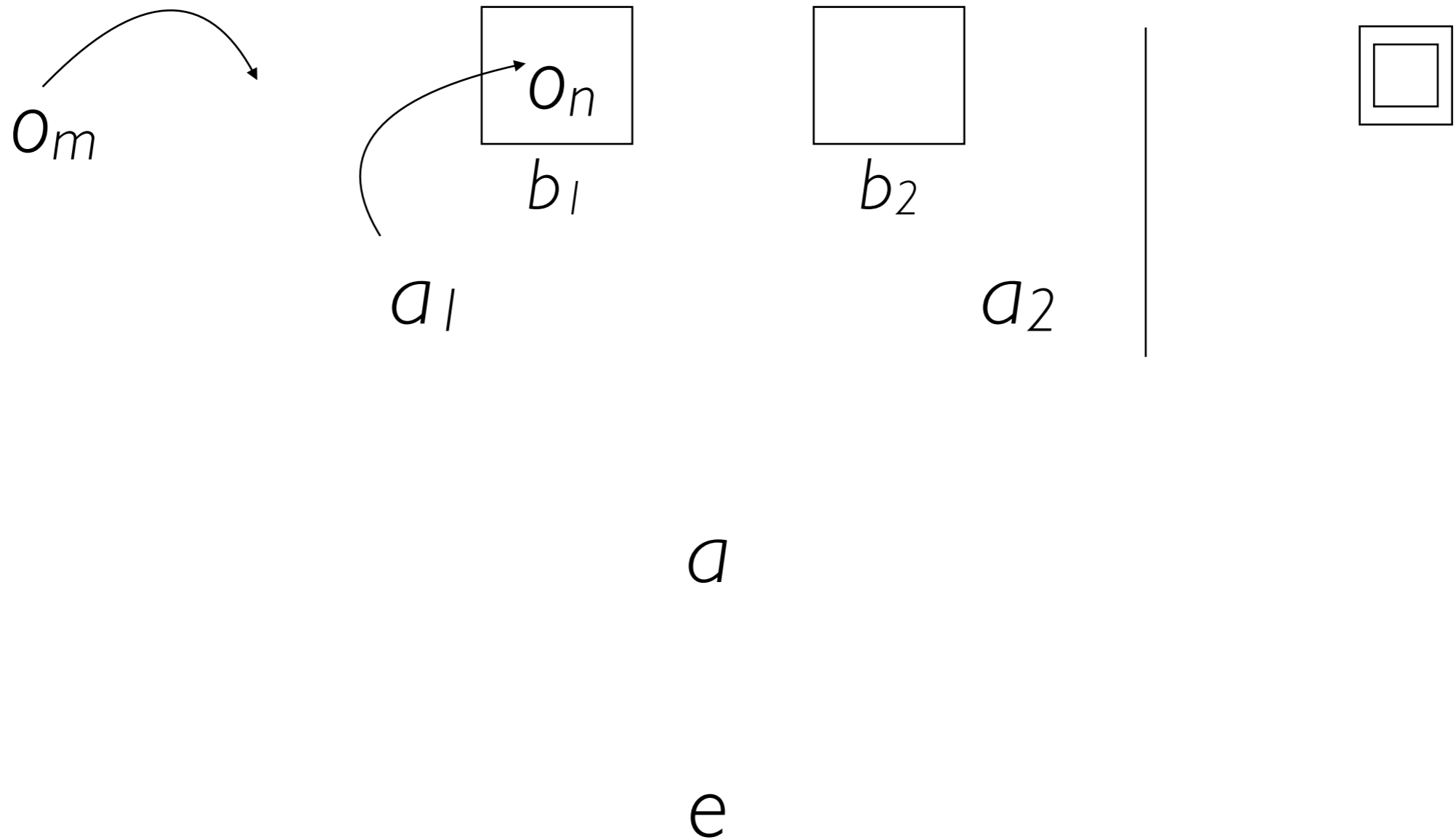
Framework for FBT^1_2

(seven timepoints)



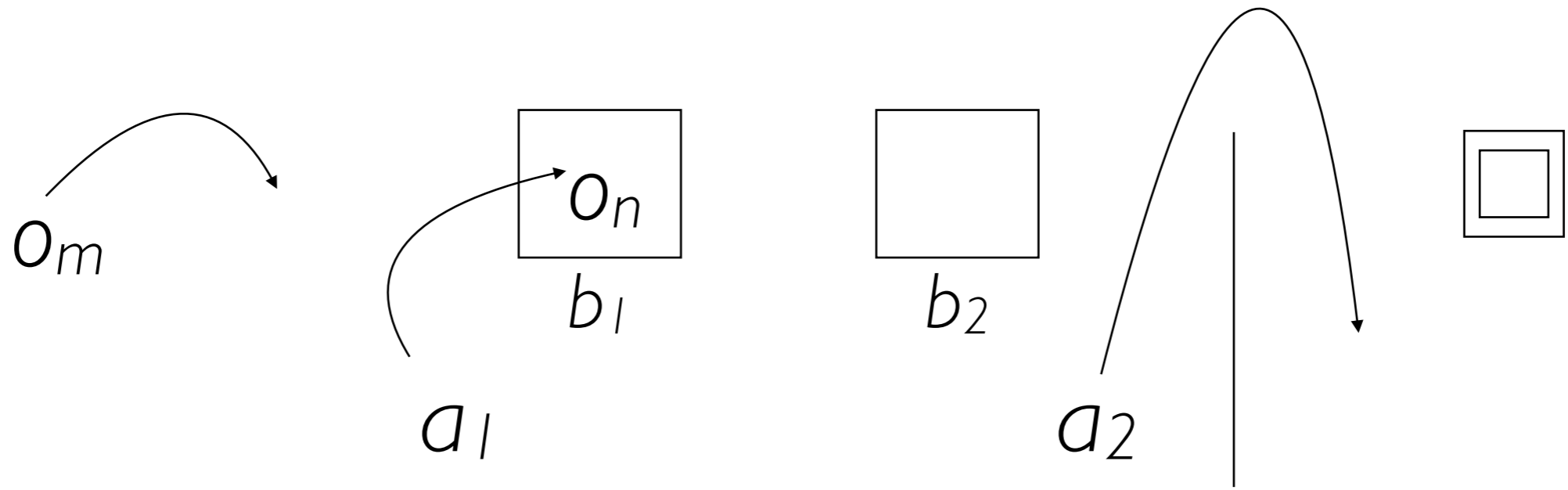
Framework for FBT^1_2

(seven timepoints)



Framework for FBT^1_2

(seven timepoints)

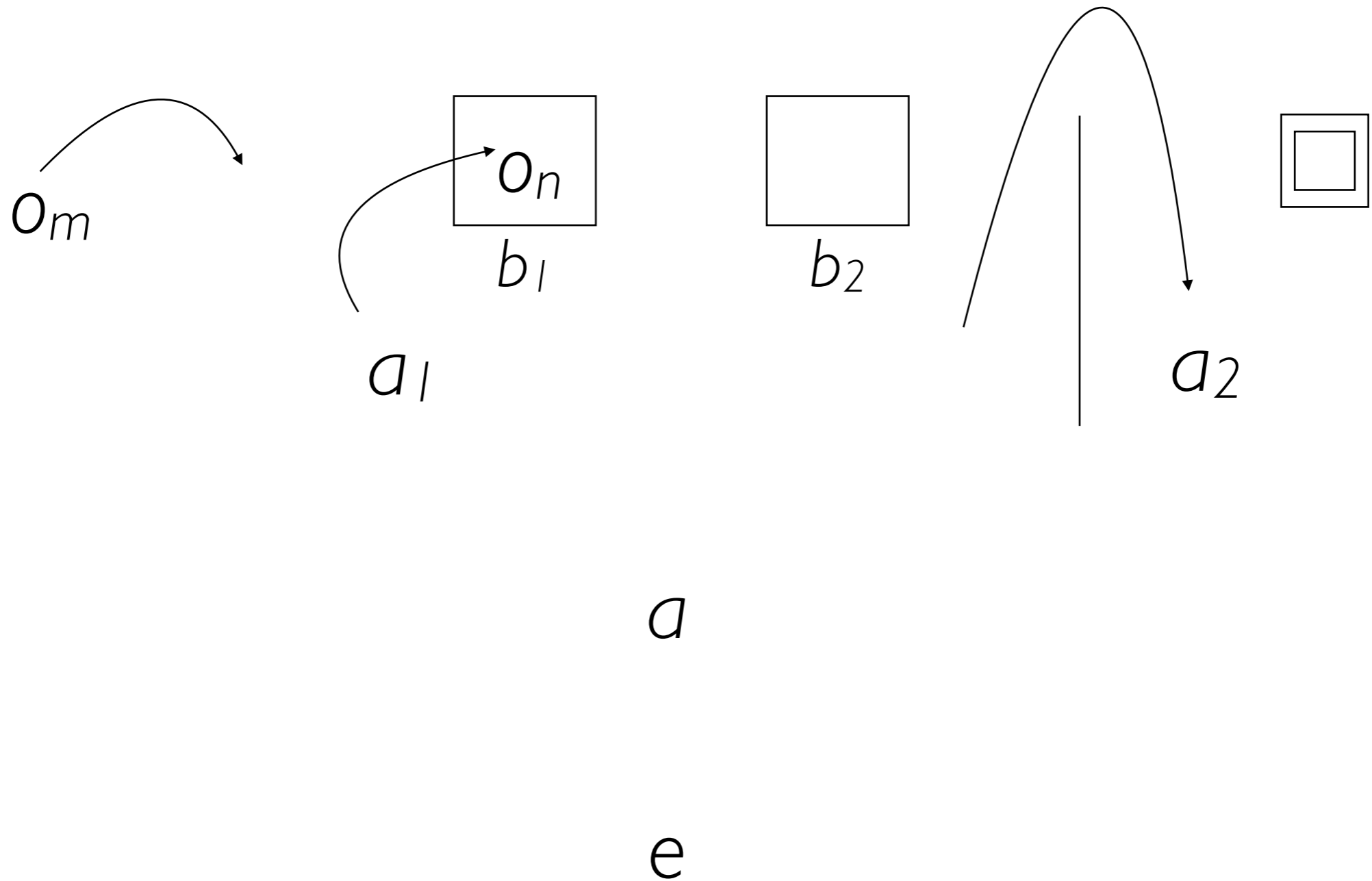


a

e

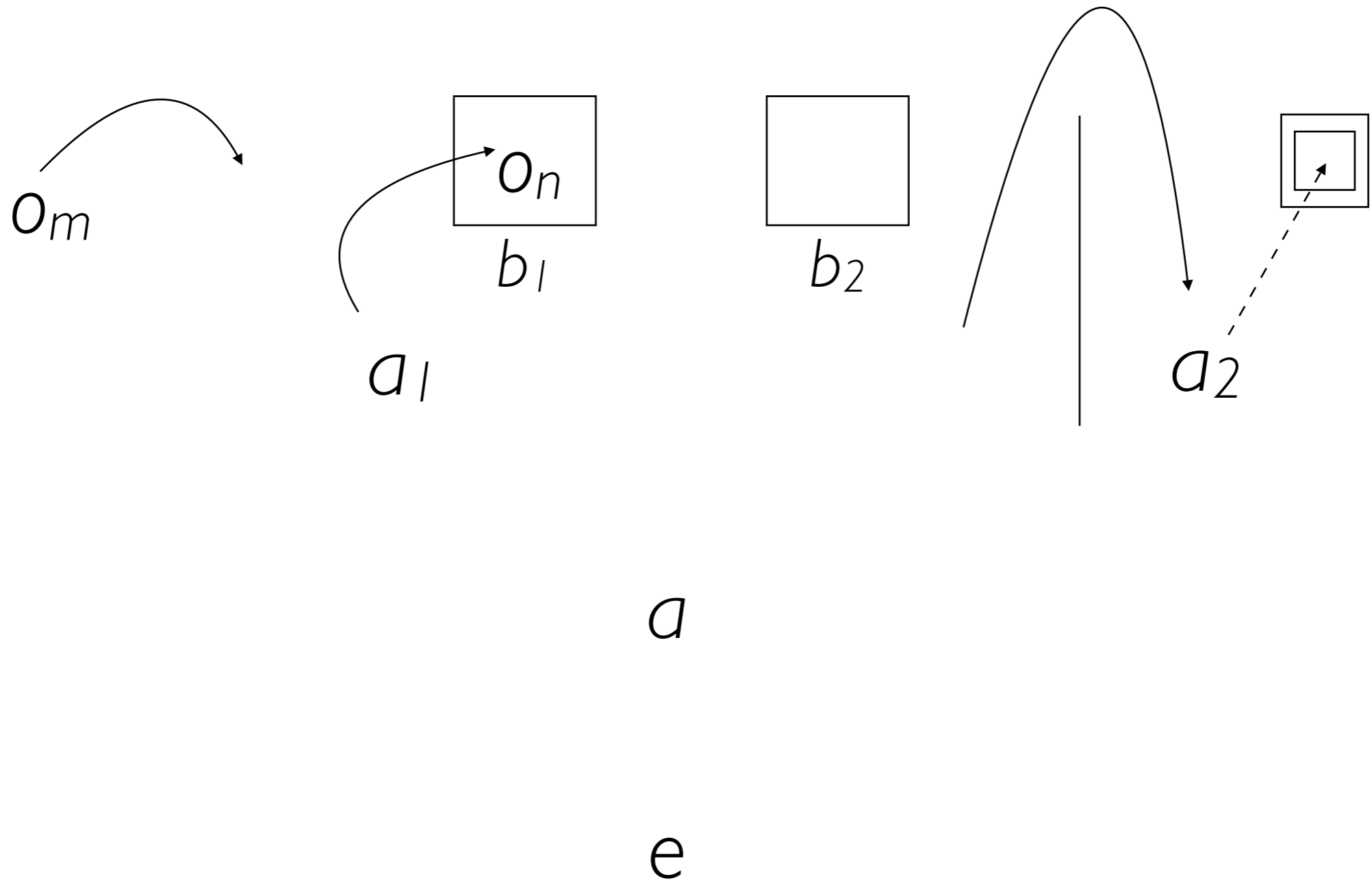
Framework for FBT^1_2

(seven timepoints)



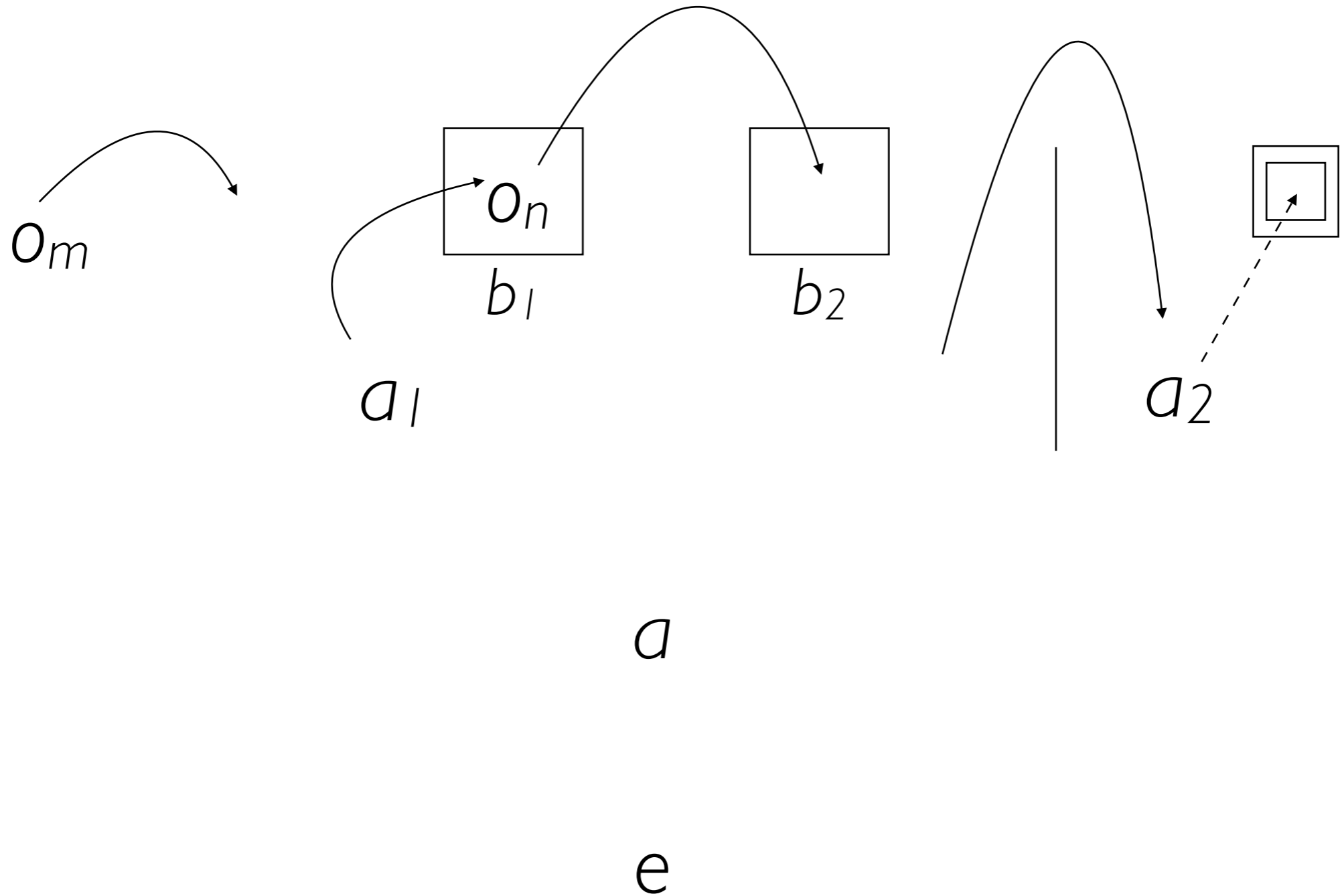
Framework for FBT^1_2

(seven timepoints)



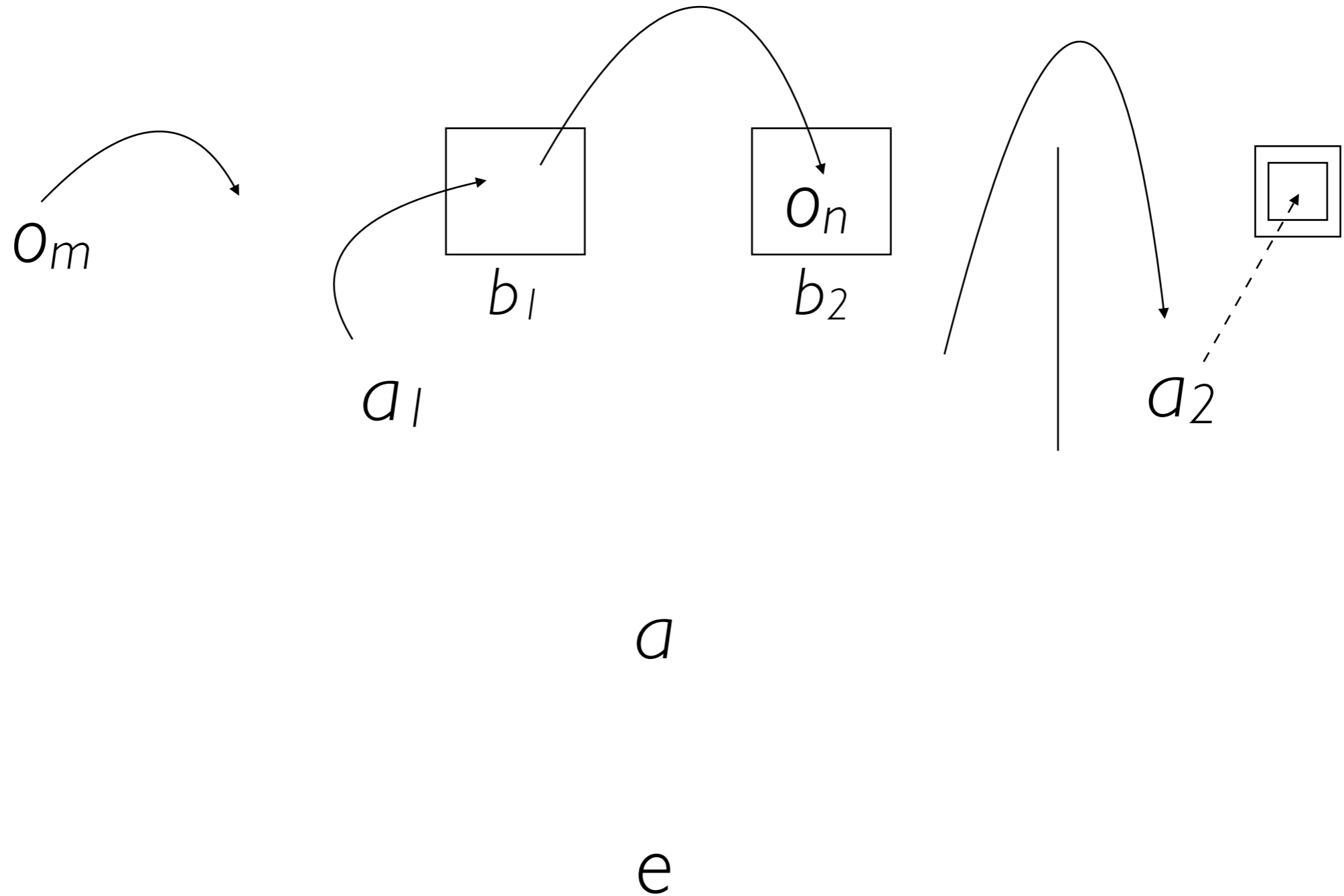
Framework for FBT^1_2

(seven timepoints)



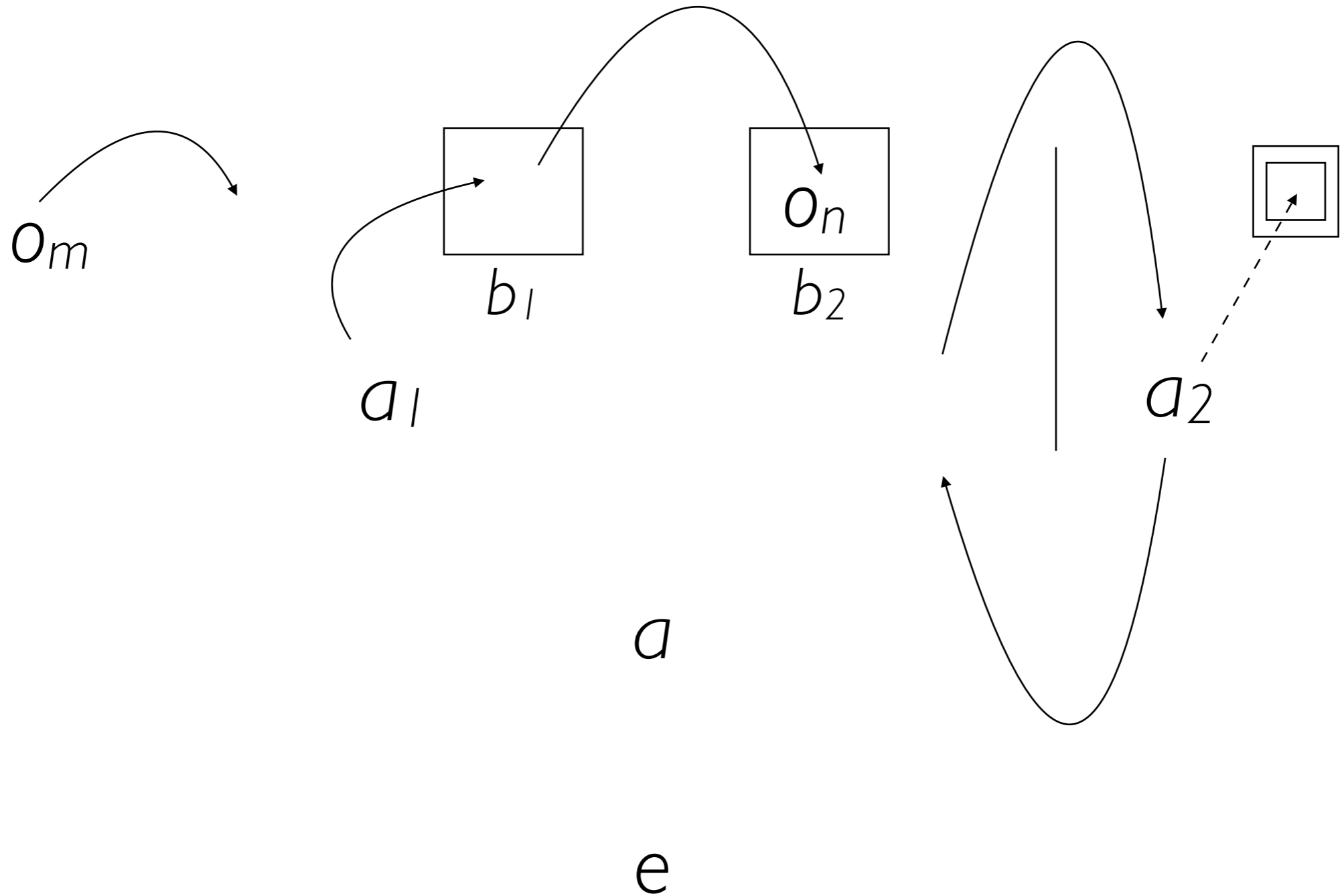
Framework for FBT^1_2

(seven timepoints)



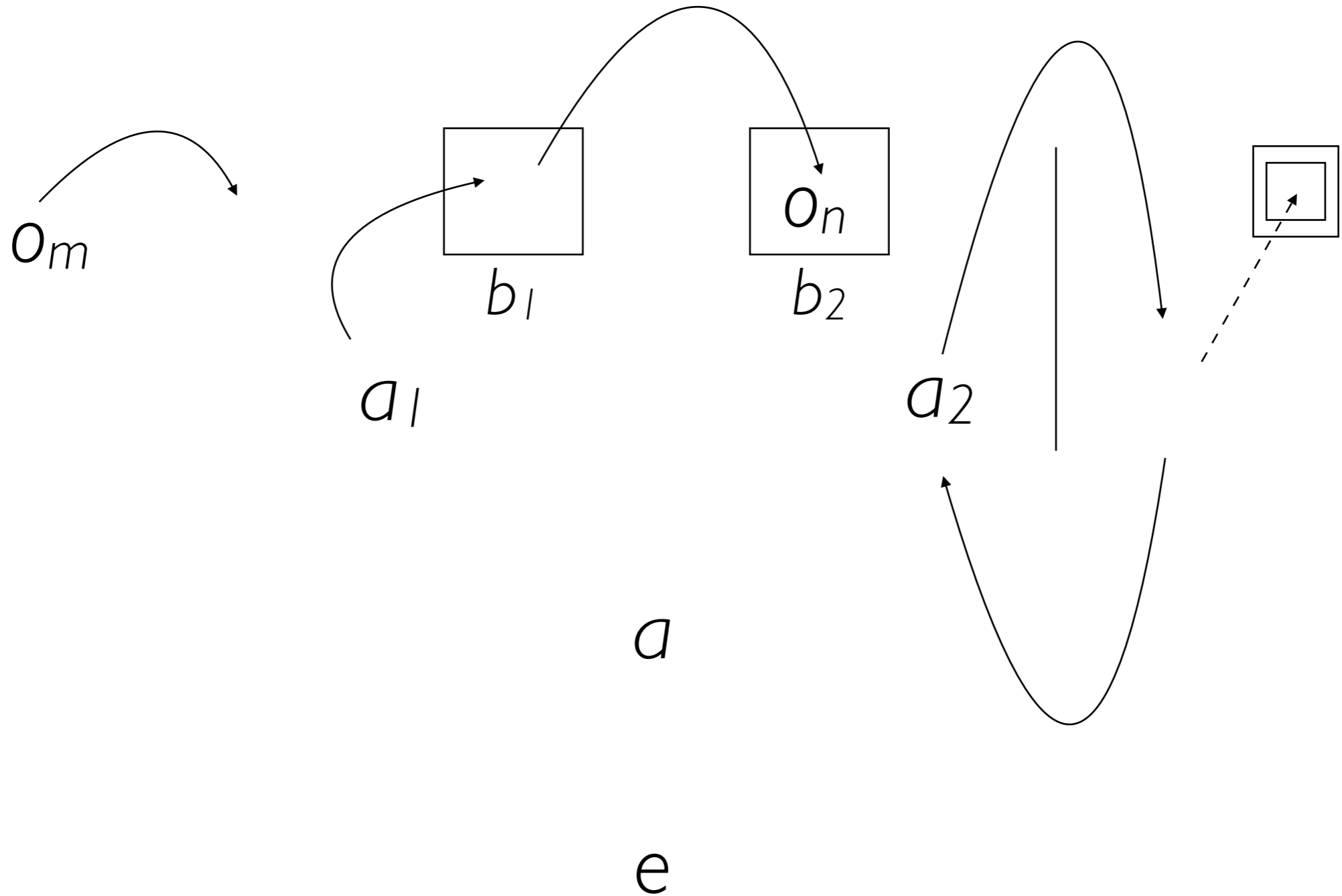
Framework for FBT^1_2

(seven timepoints)



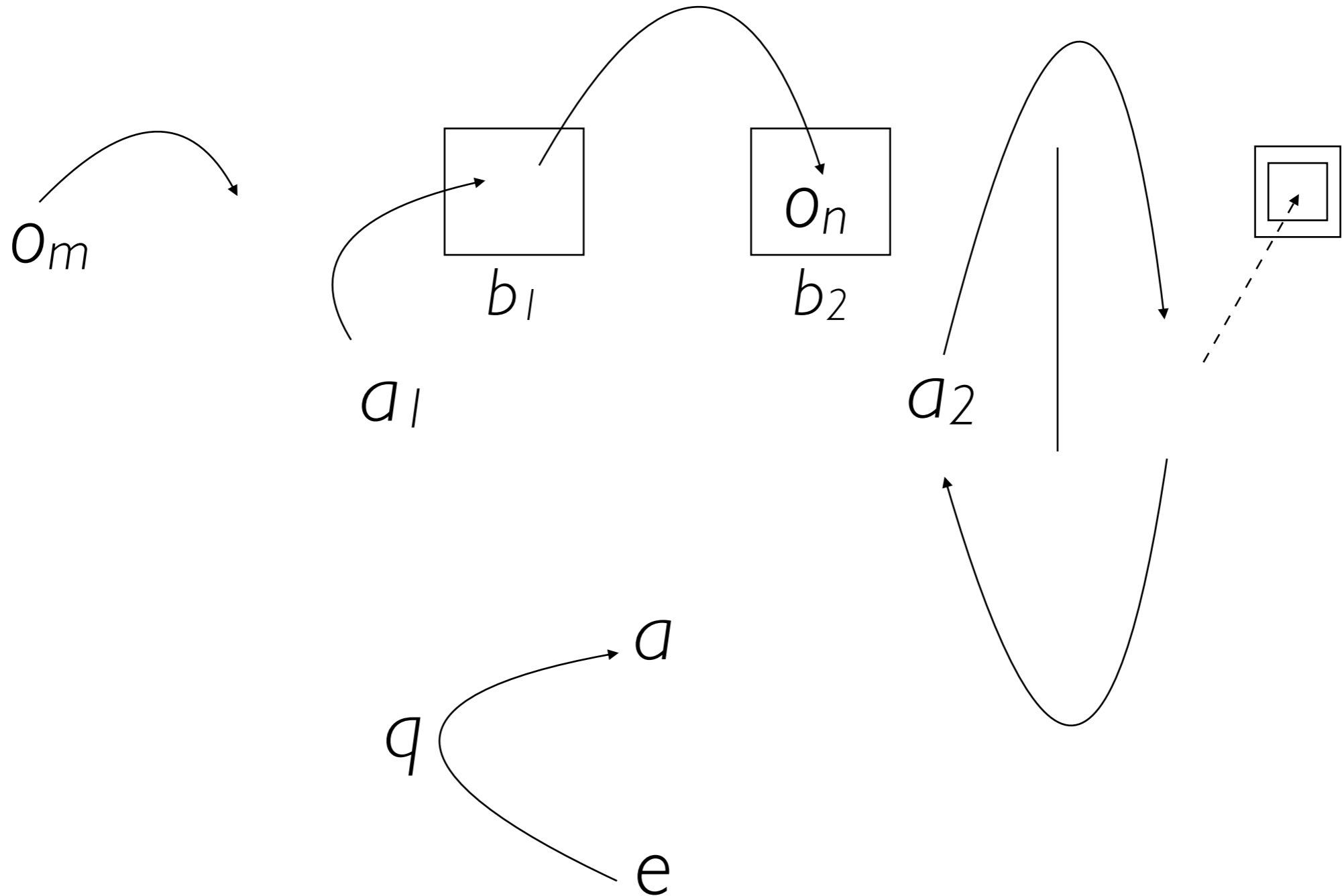
Framework for FBT^1_2

(seven timepoints)



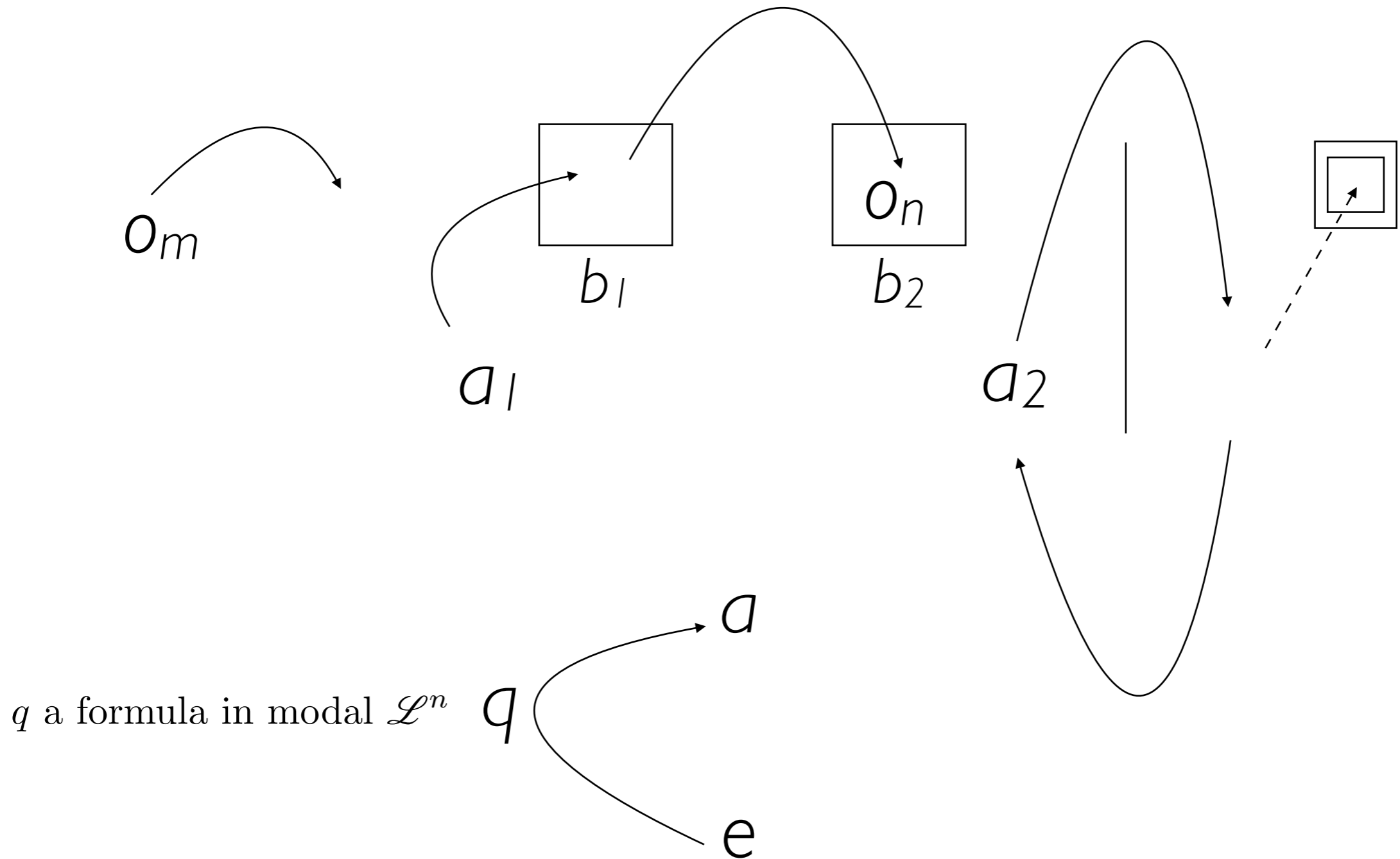
Framework for FBT^1_2

(seven timepoints)

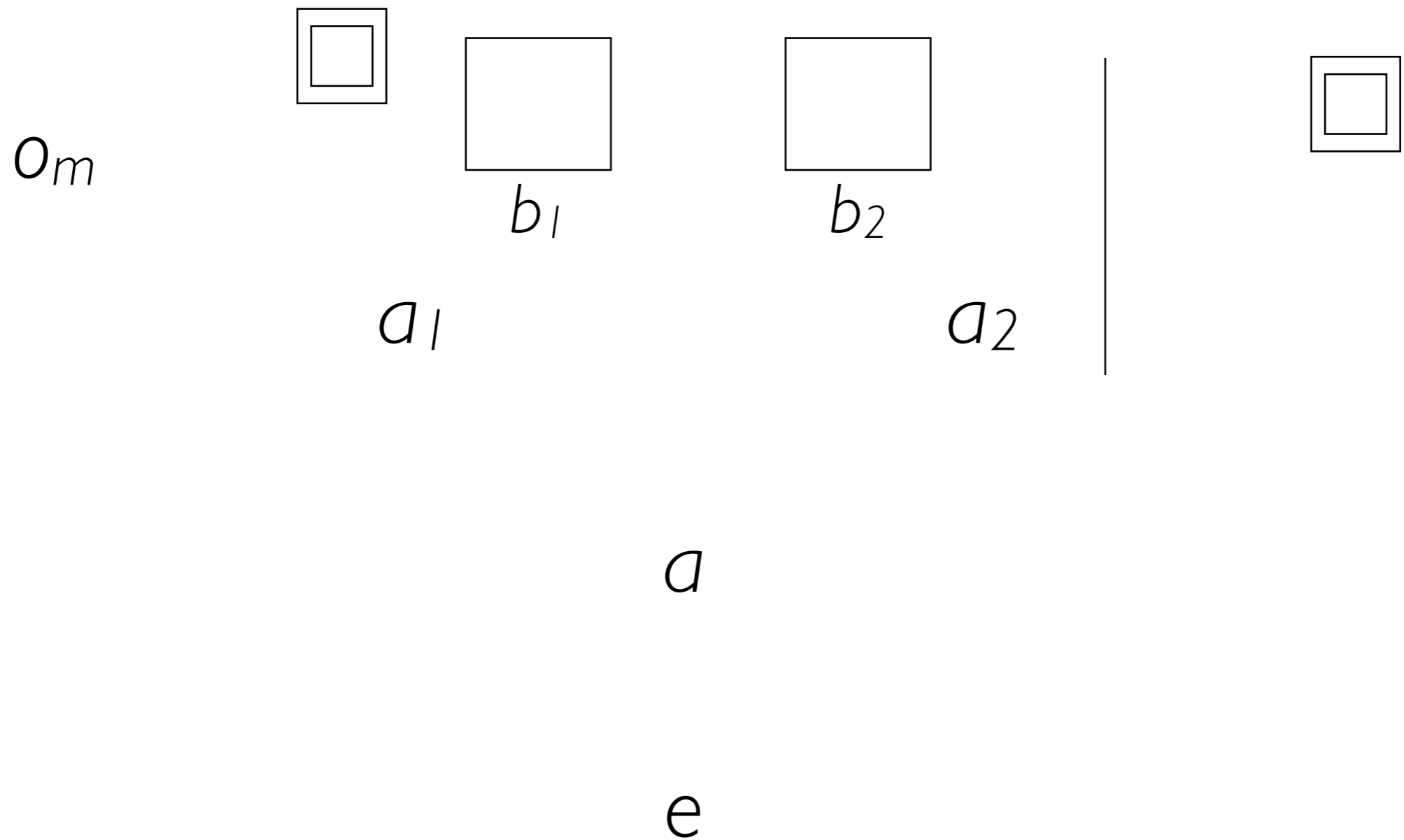


Framework for FBT^1_2

(seven timepoints)

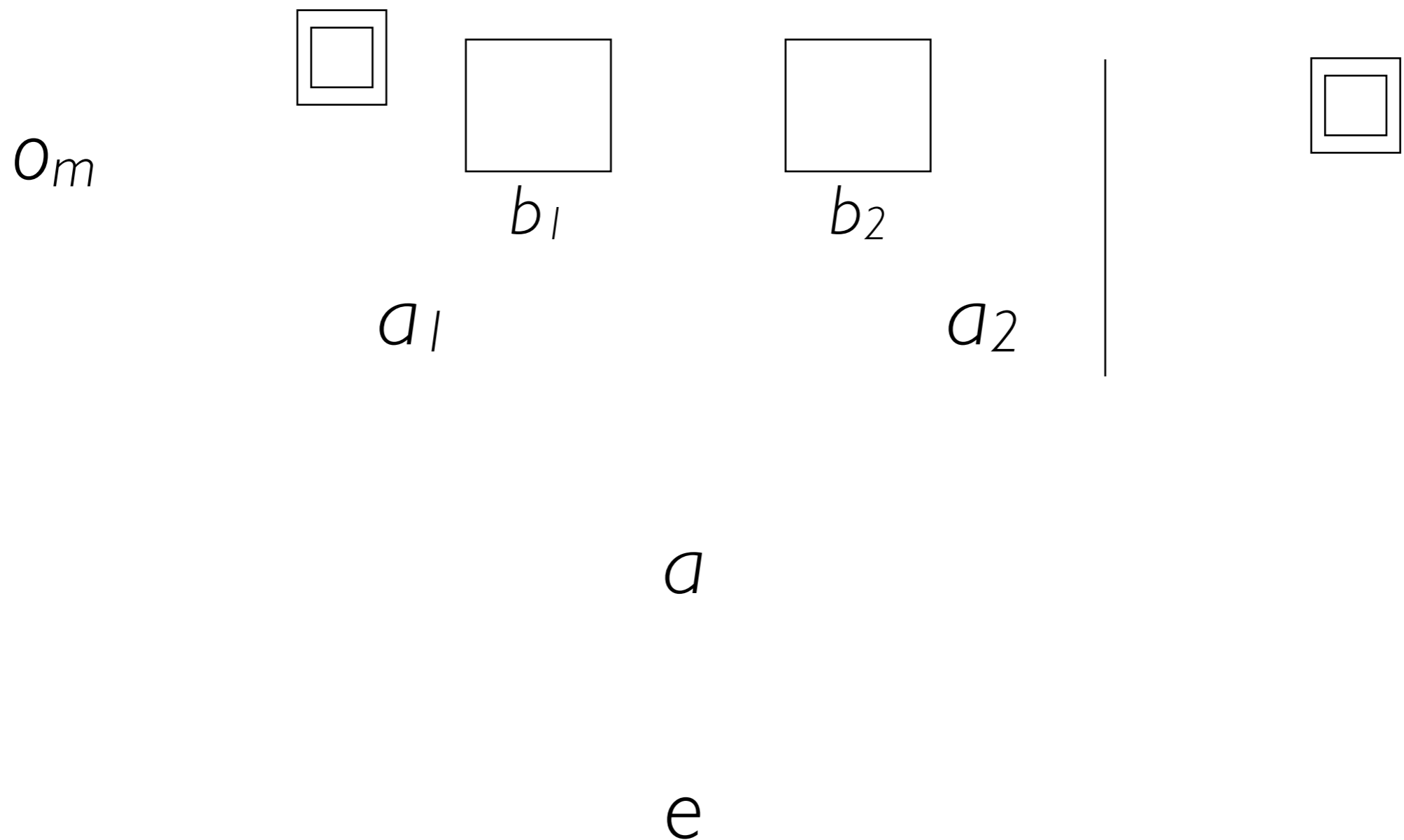


Framework for FBT^1_3



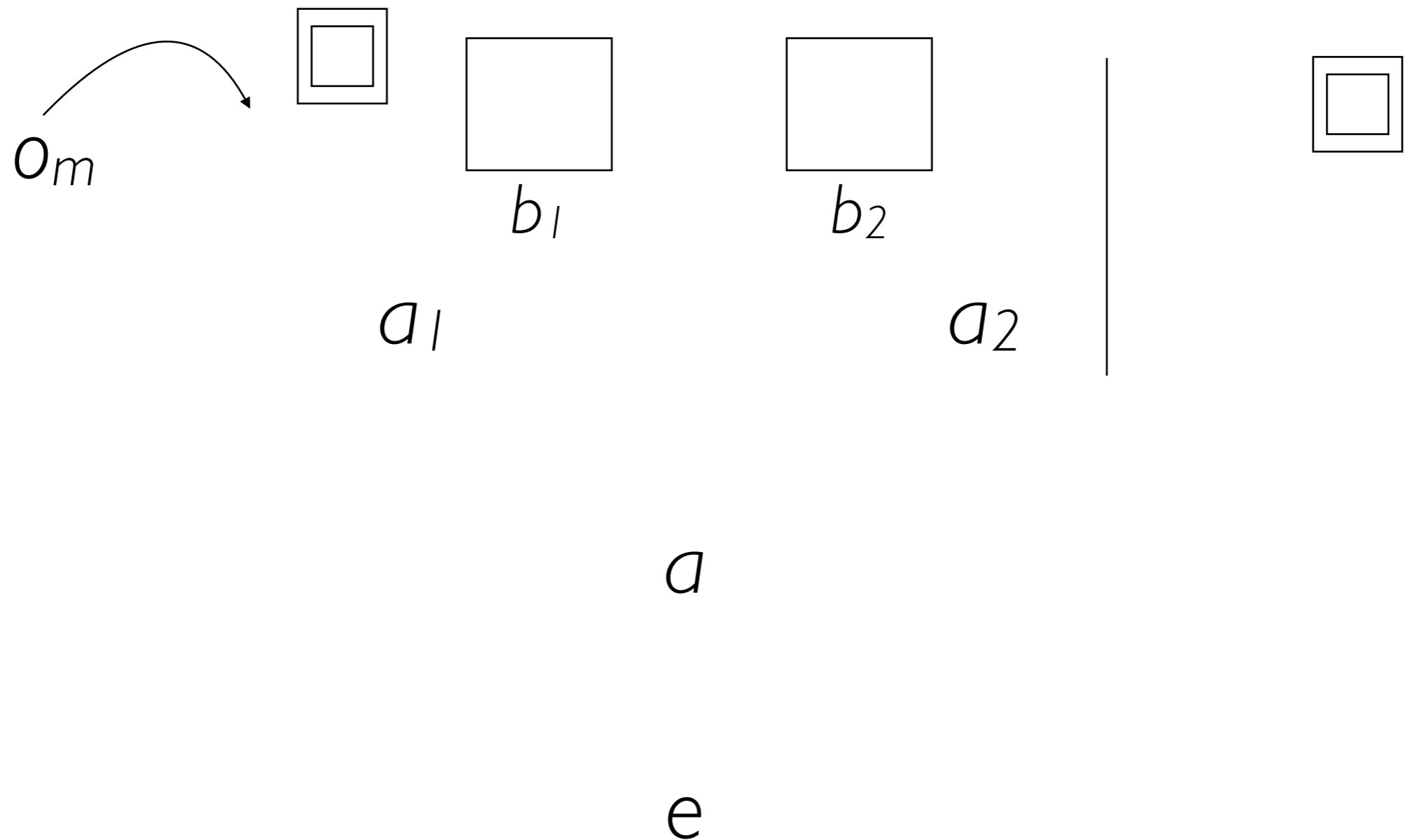
Framework for FBT^1_3

(eight timepoints)



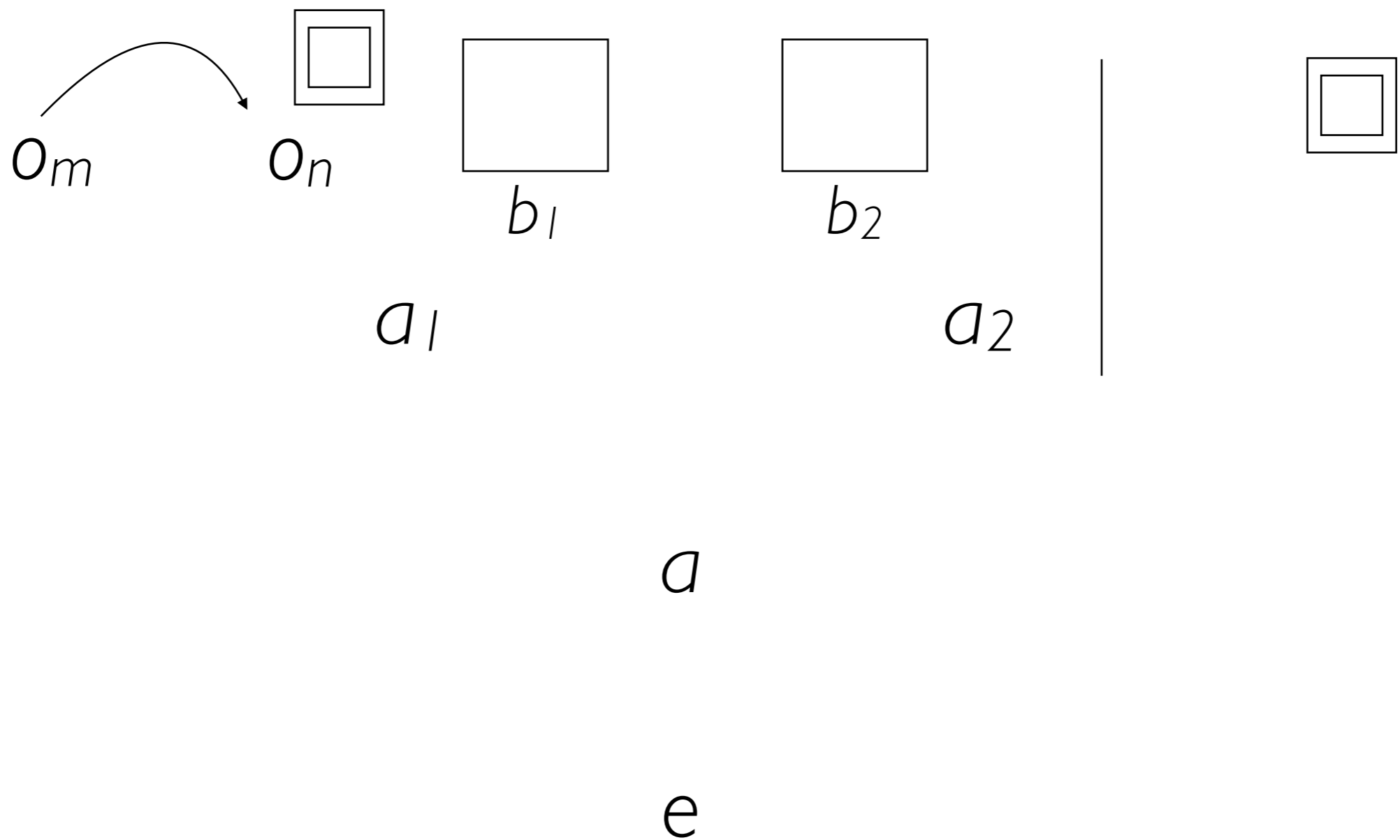
Framework for FBT^1_3

(eight timepoints)



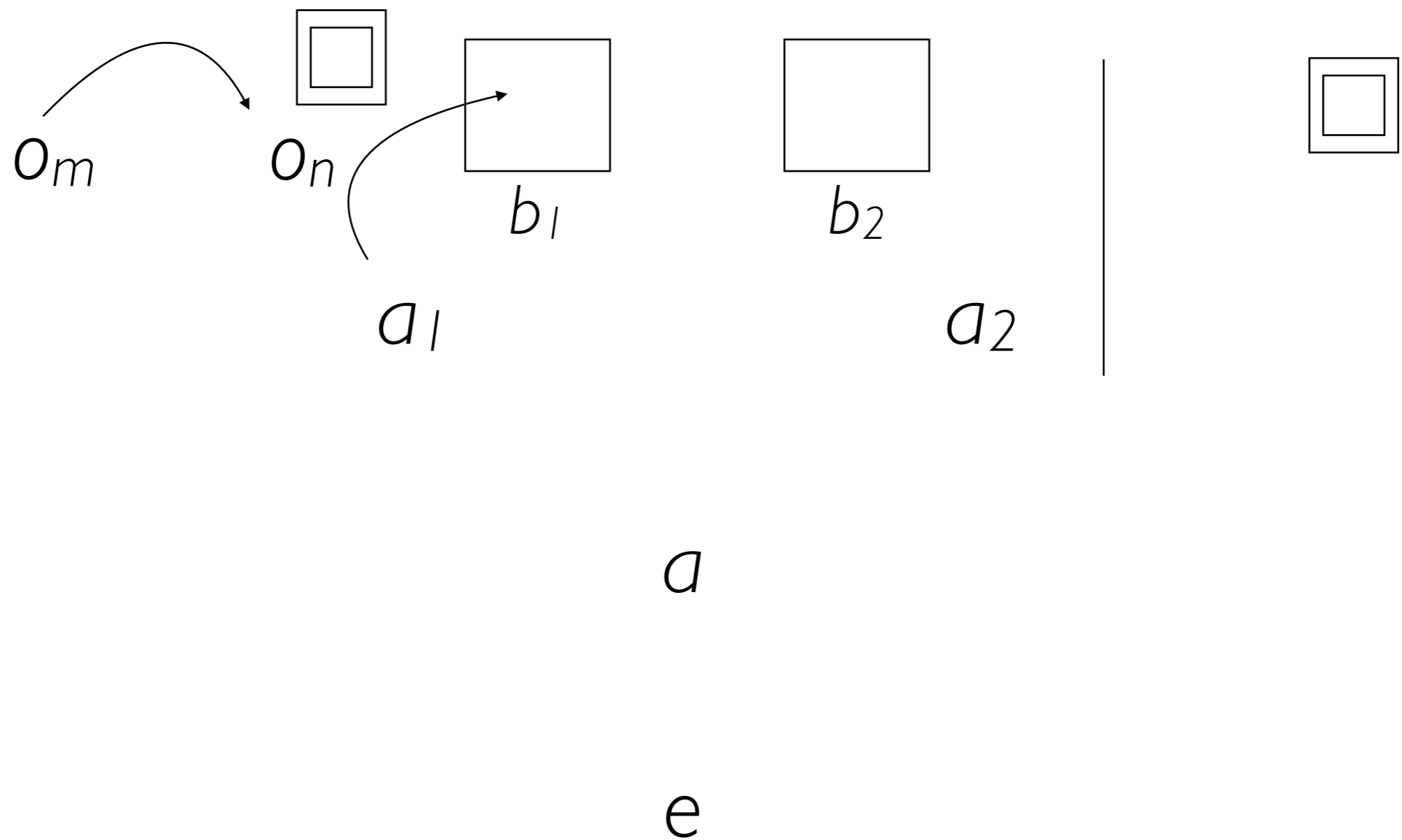
Framework for FBT^1_3

(eight timepoints)



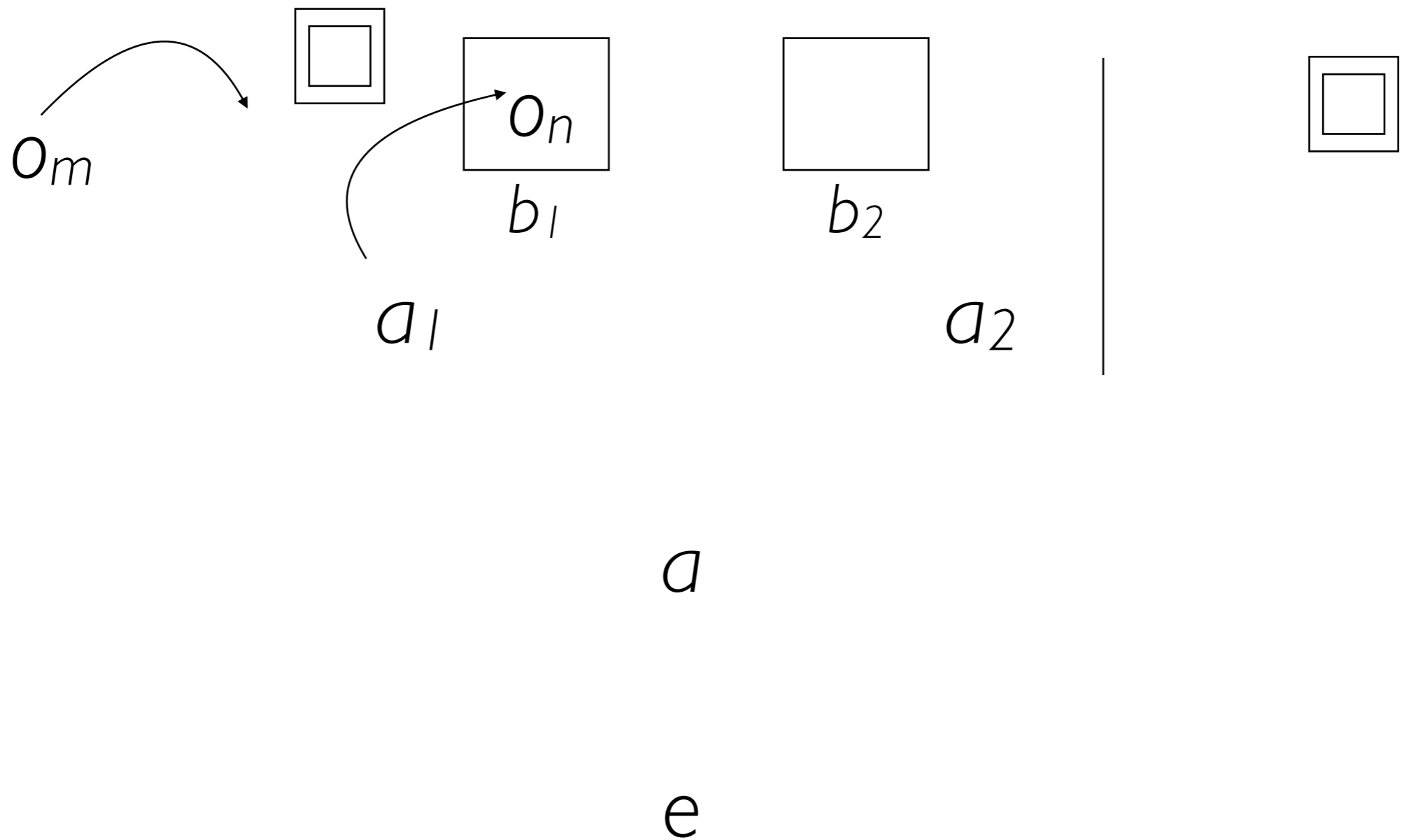
Framework for FBT^1_3

(eight timepoints)



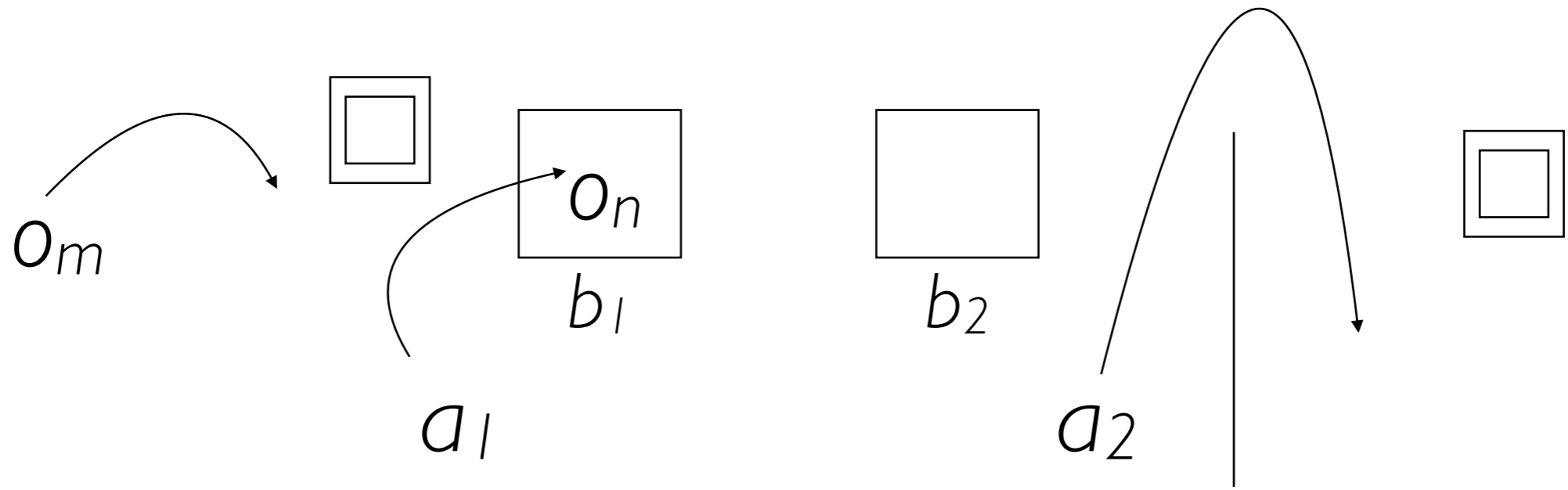
Framework for FBT^1_3

(eight timepoints)



Framework for FBT^1_3

(eight timepoints)

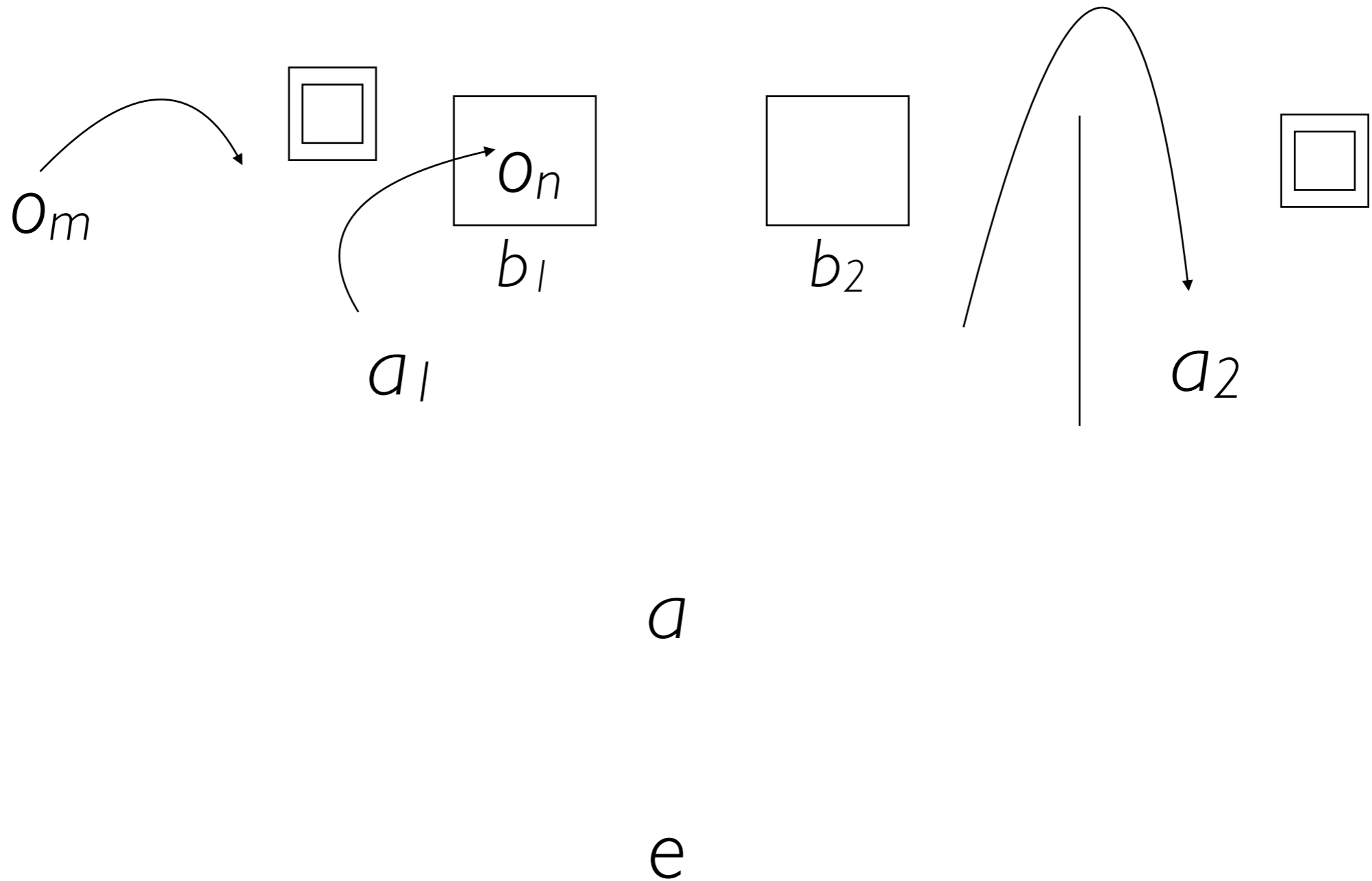


a

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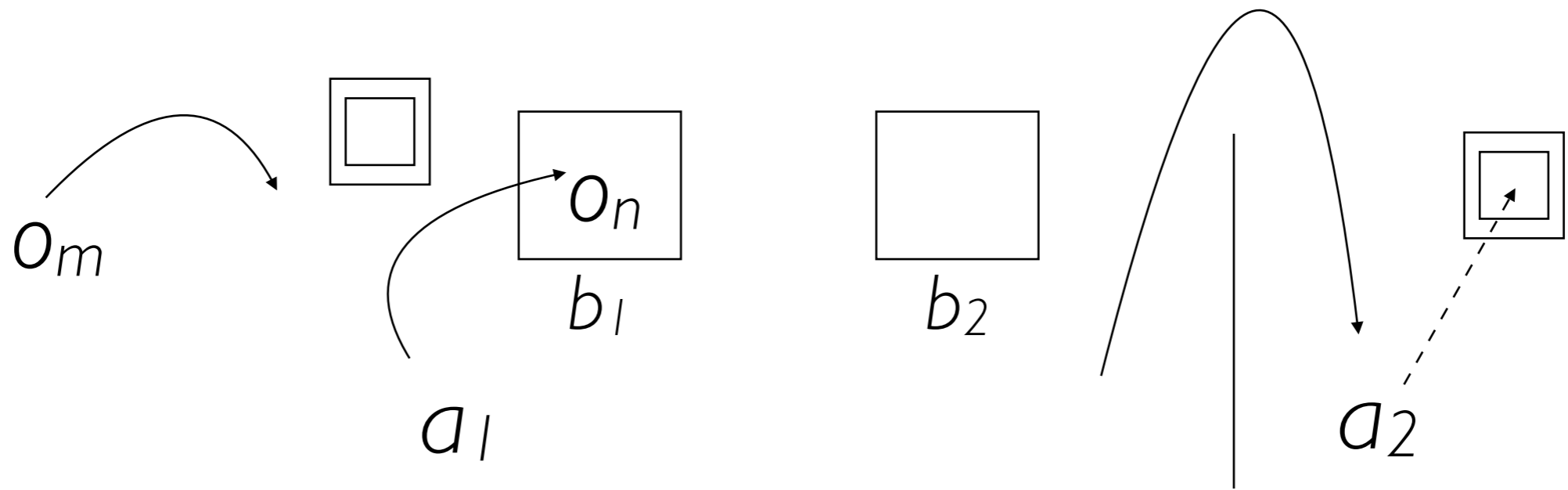
Framework for FBT^1_3

(eight timepoints)



Framework for FBT^1_3

(eight timepoints)

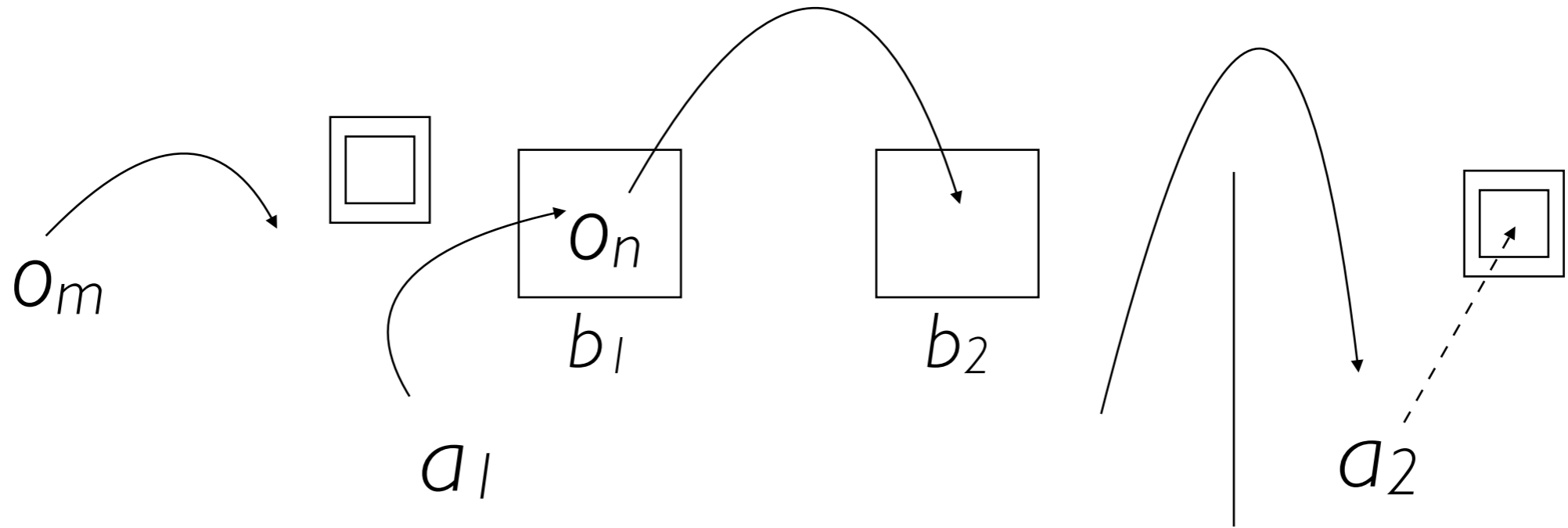


a

e

Framework for FBT^1_3

(eight timepoints)

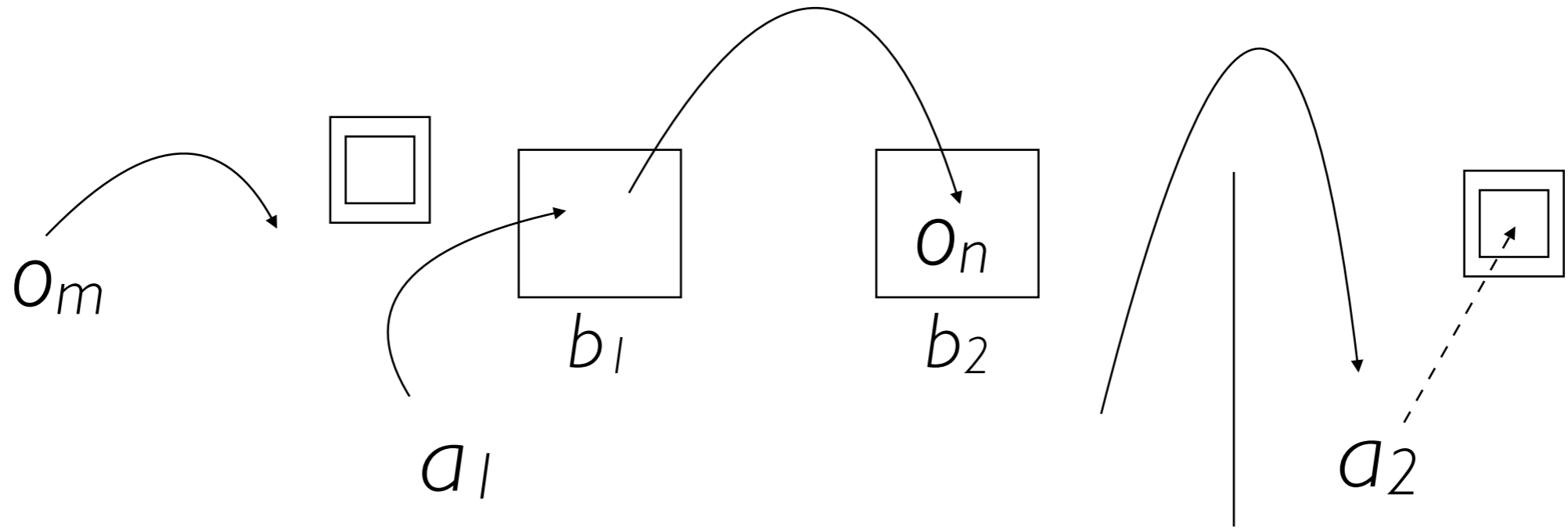


a

e

Framework for FBT^1_3

(eight timepoints)

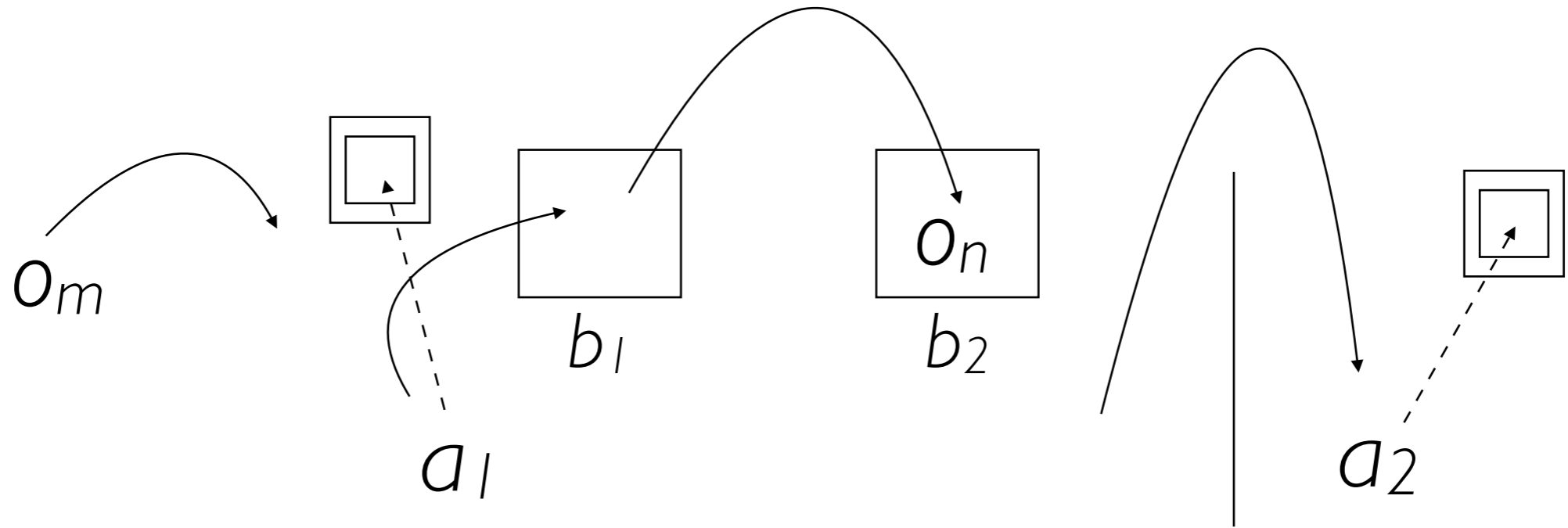


a

e

Framework for FBT^1_3

(eight timepoints)

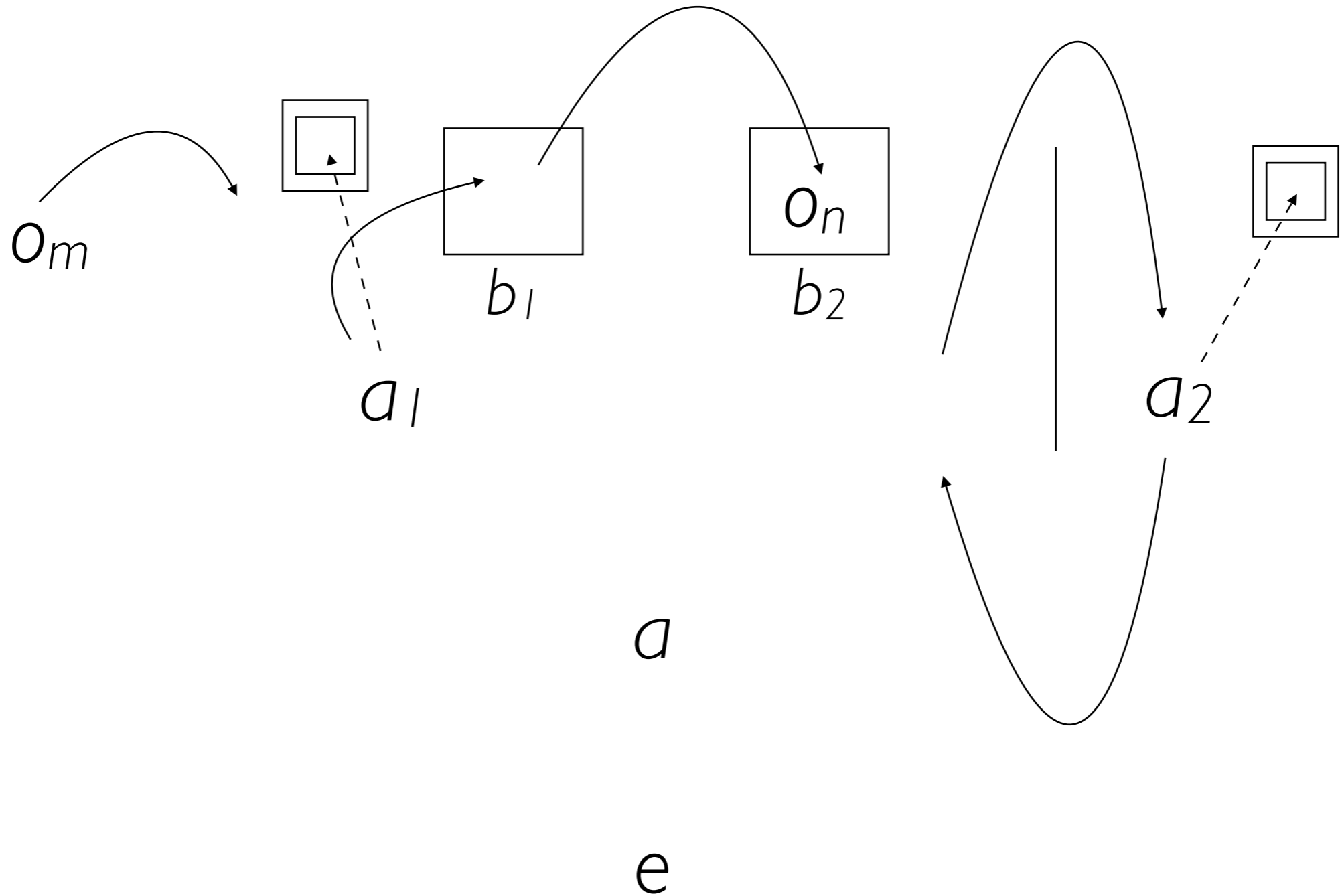


a

e

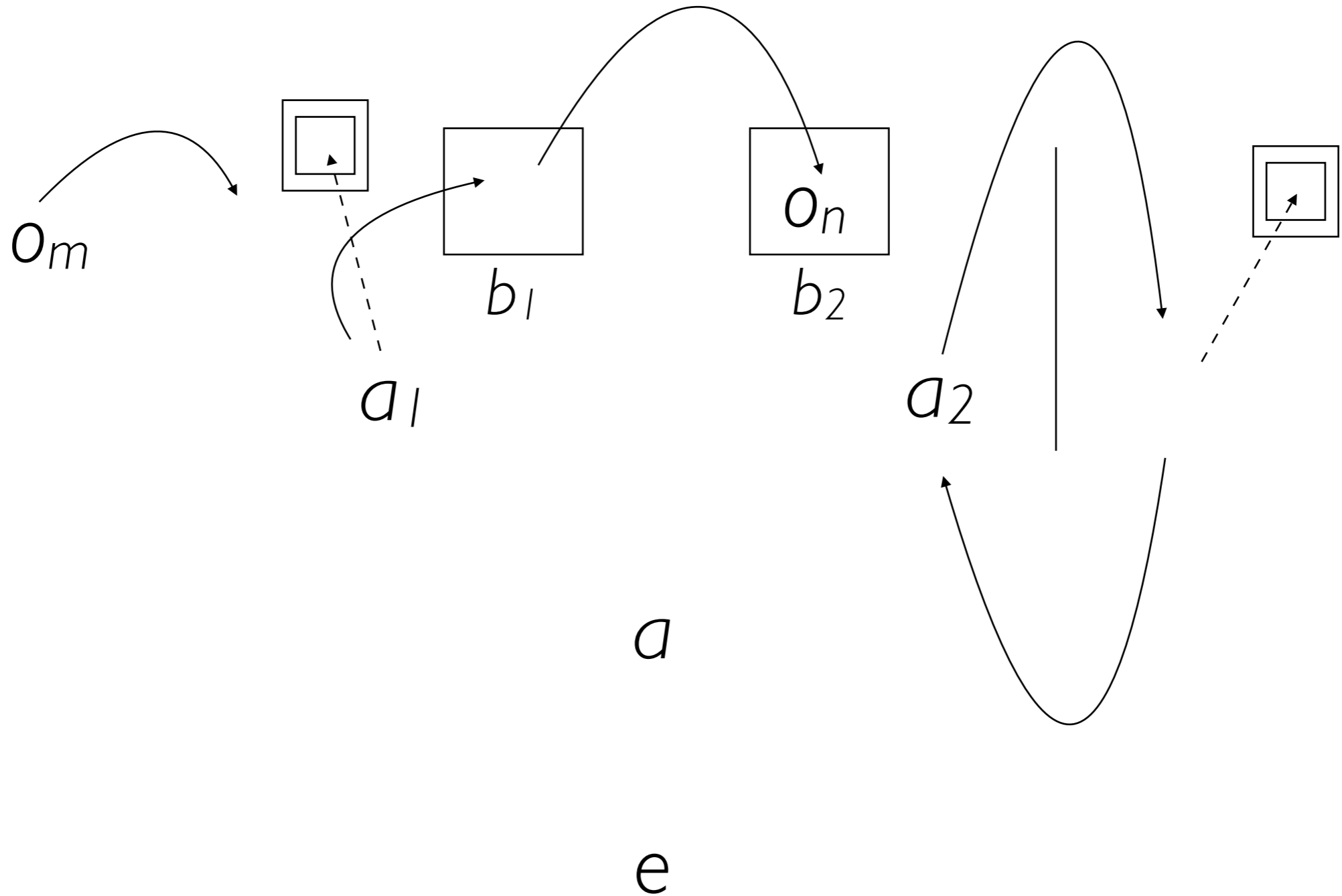
Framework for FBT^1_3

(eight timepoints)

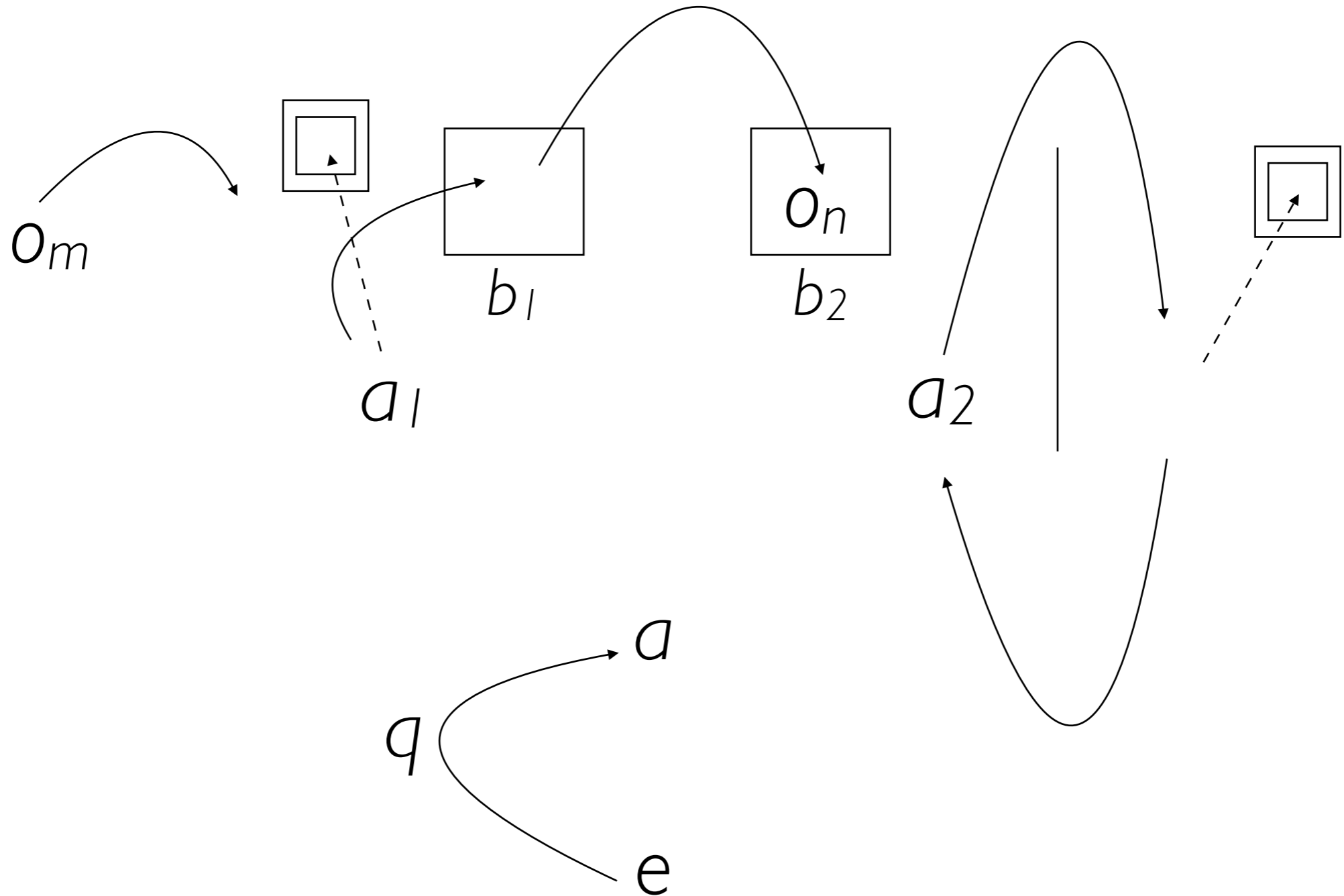


Framework for FBT^1_3

(eight timepoints)

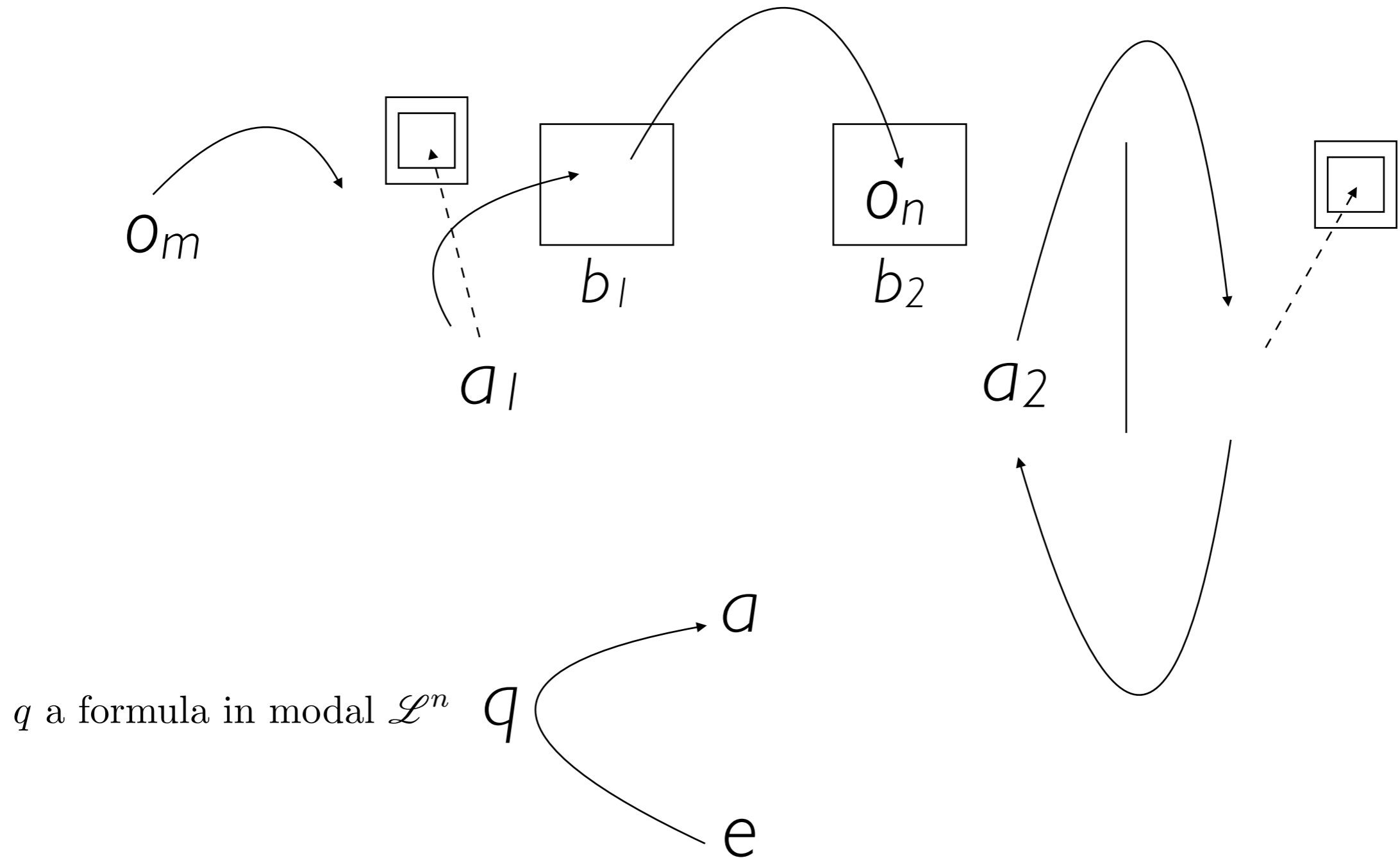


Framework for FBT^1_3 (eight timepoints)

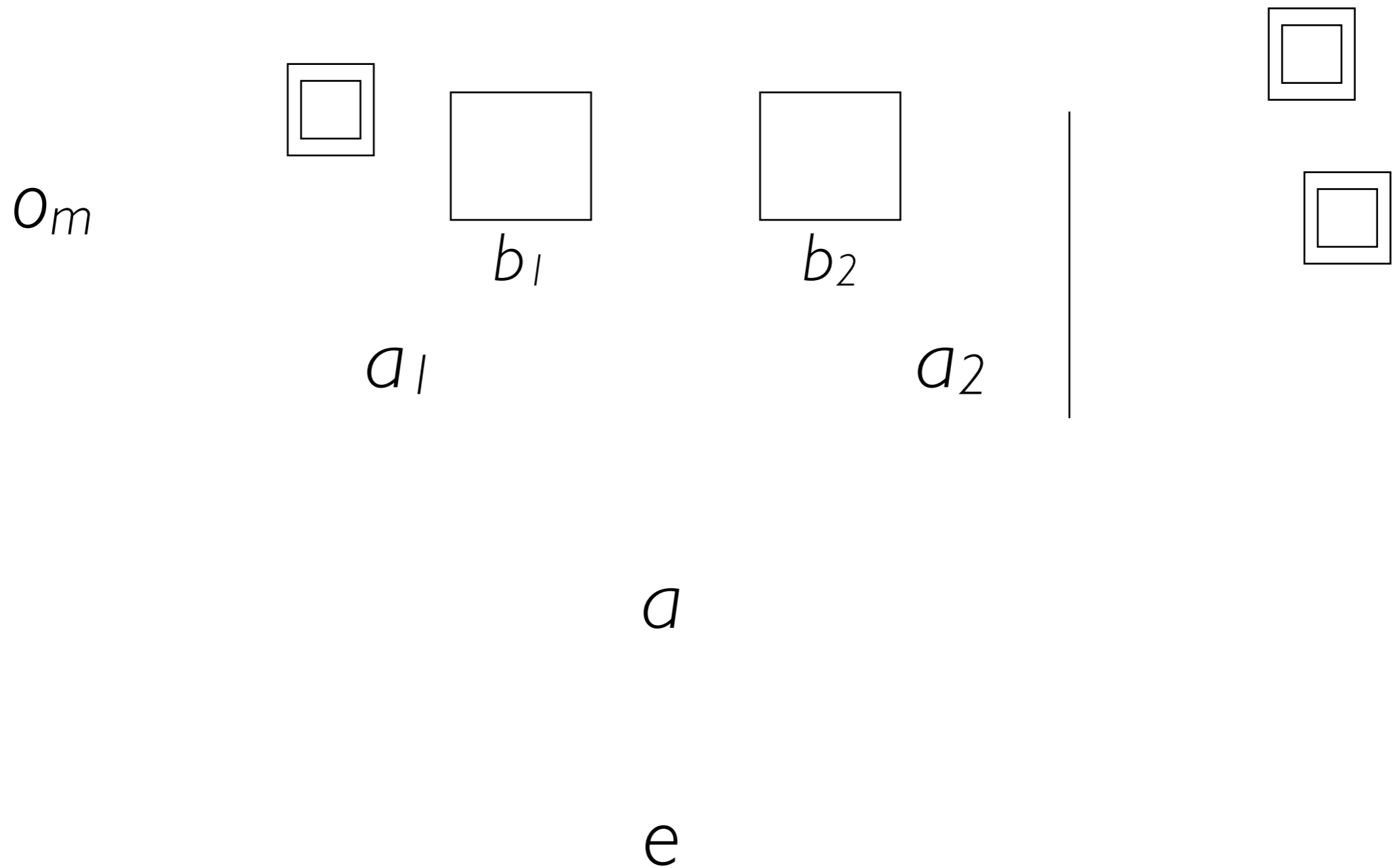


Framework for FBT^1_3

(eight timepoints)

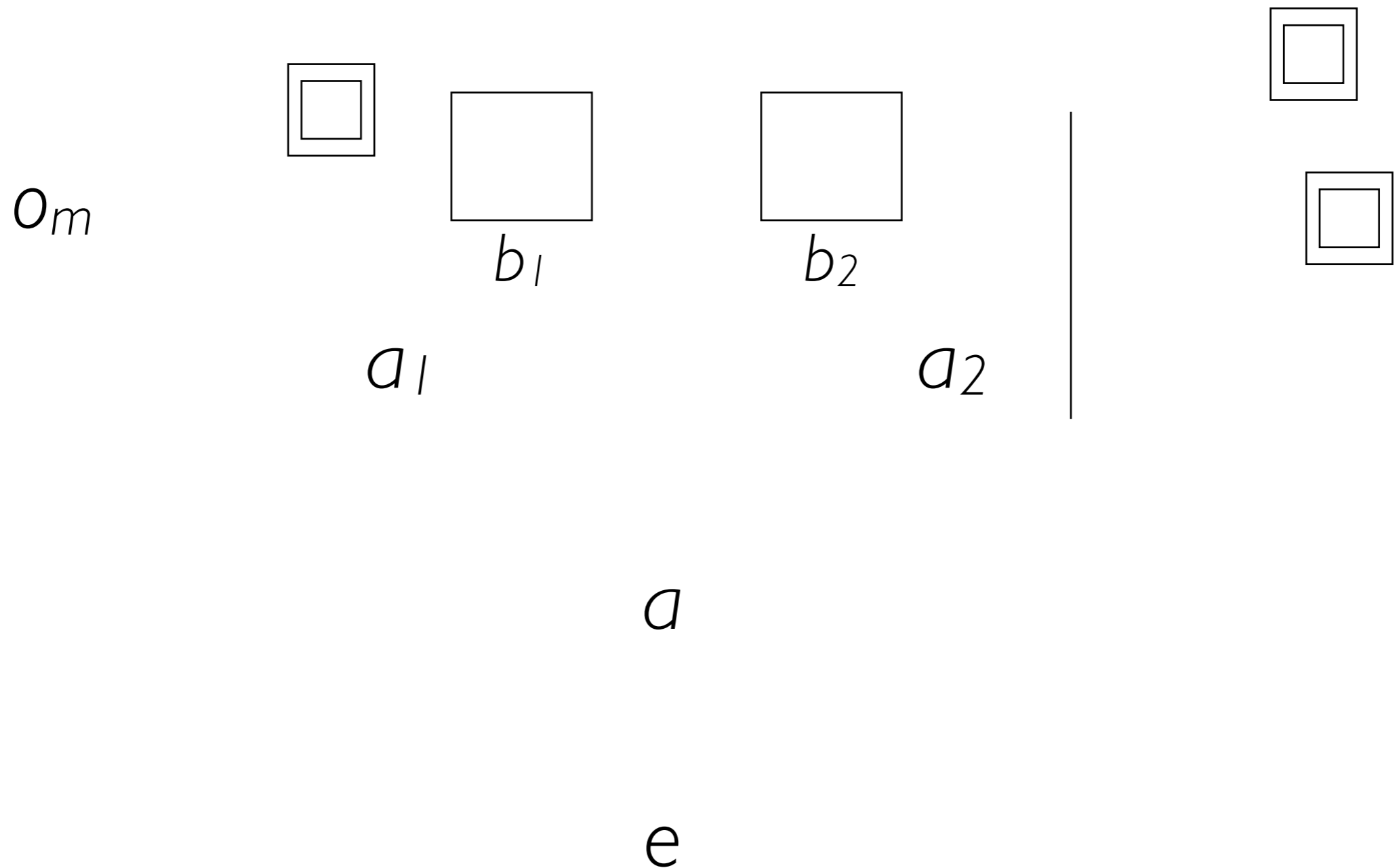


Framework for FBT^1_4



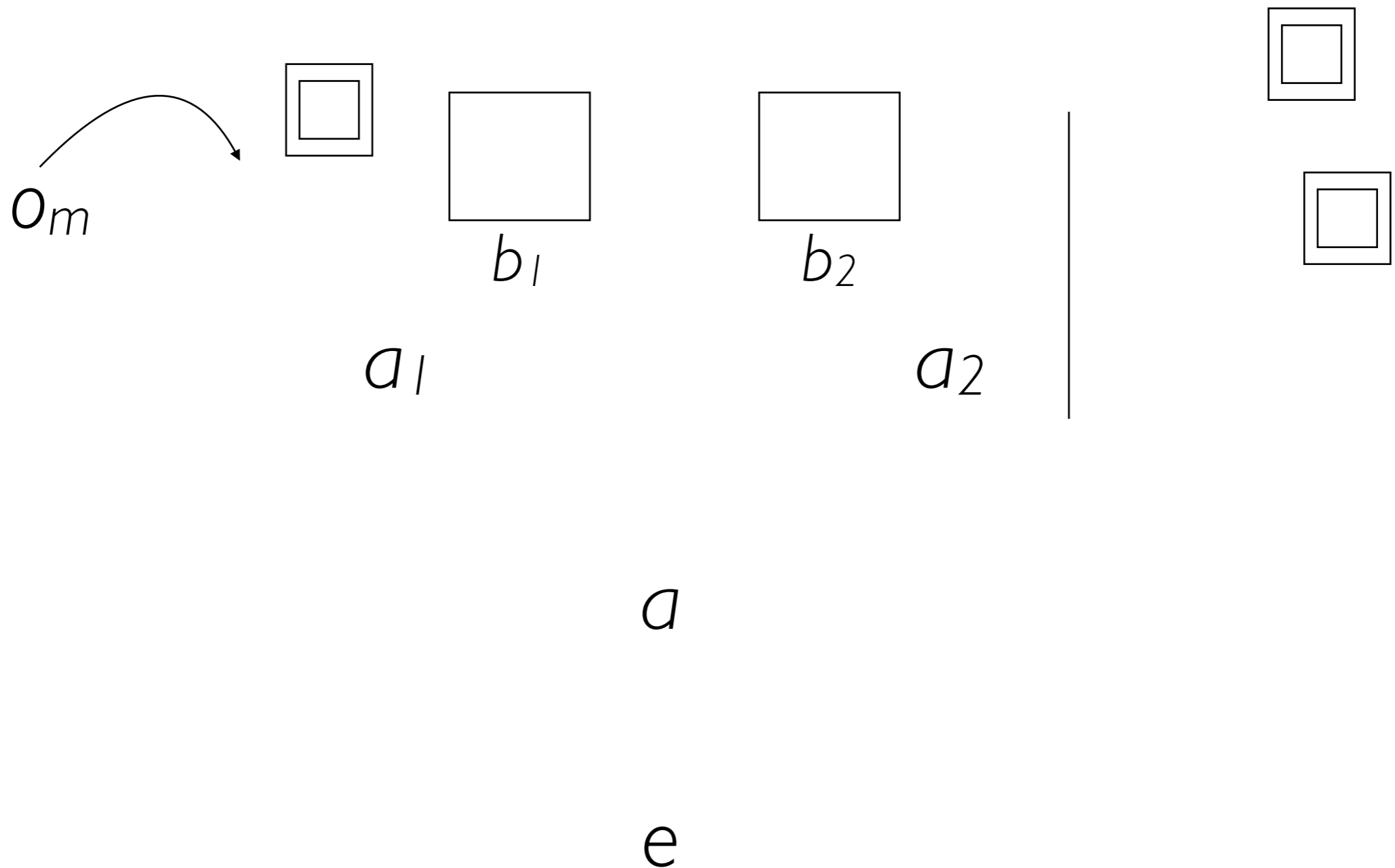
Framework for FBT^1_4

(nine timepoints)



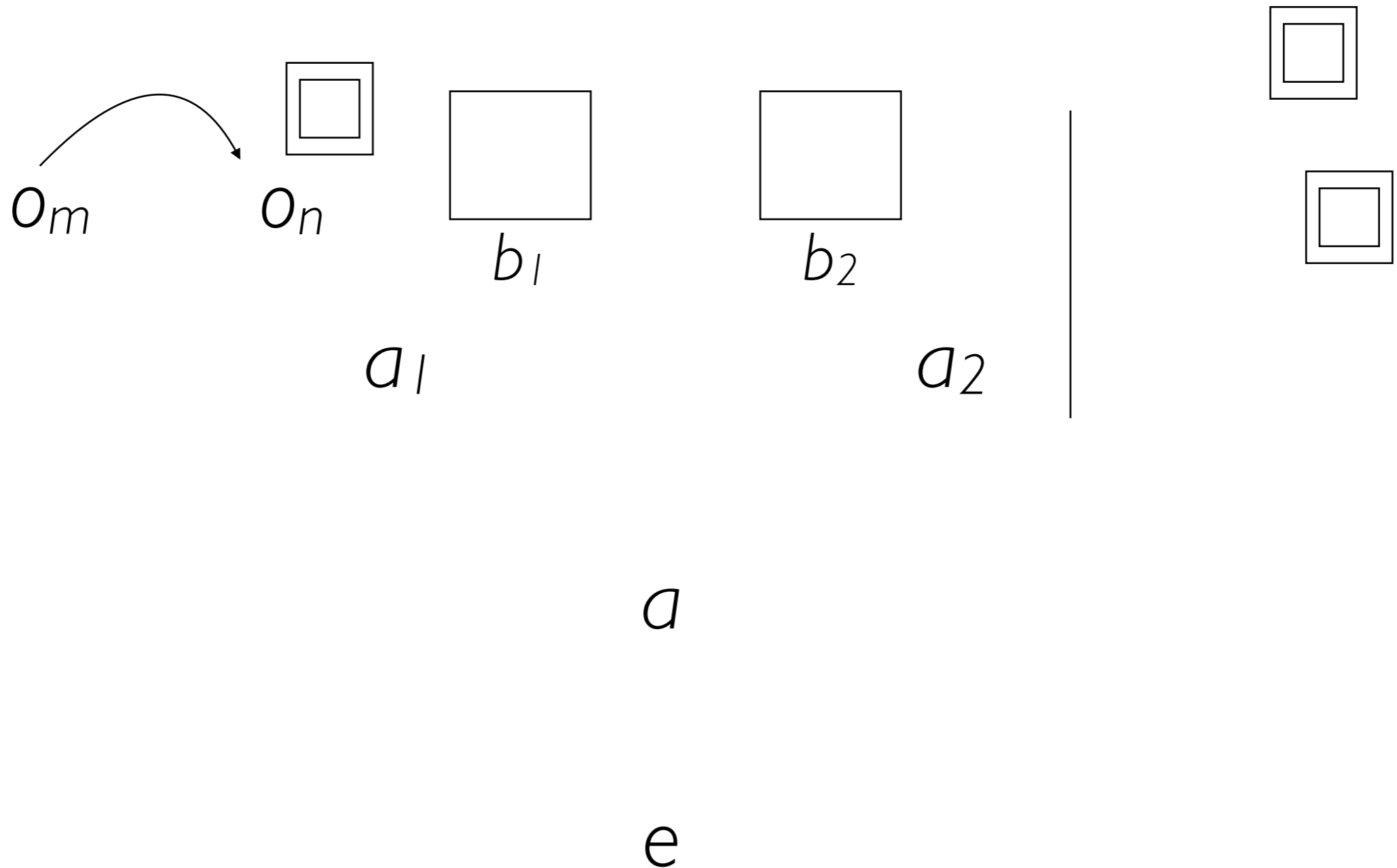
Framework for FBT^1_4

(nine timepoints)



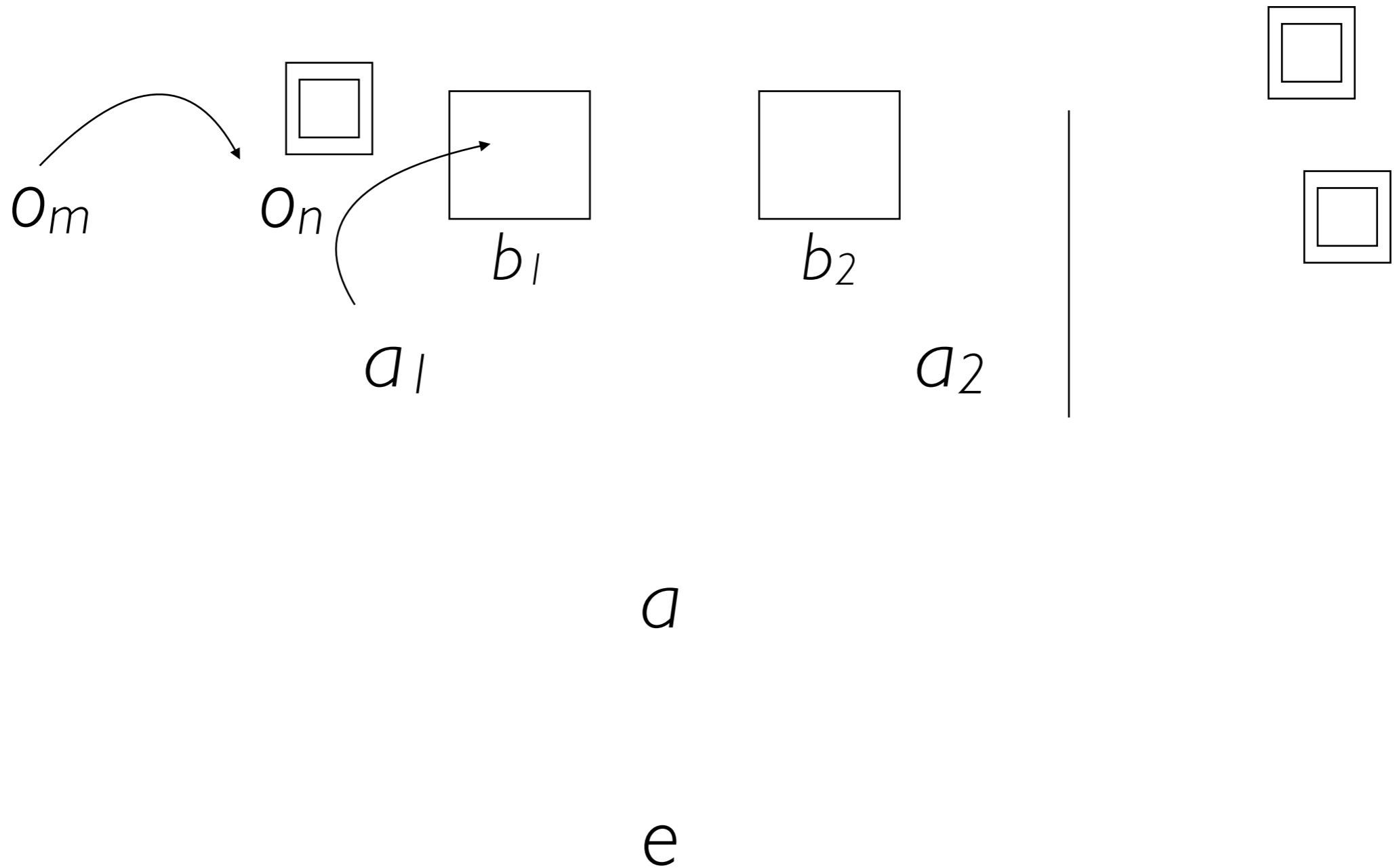
Framework for FBT^1_4

(nine timepoints)



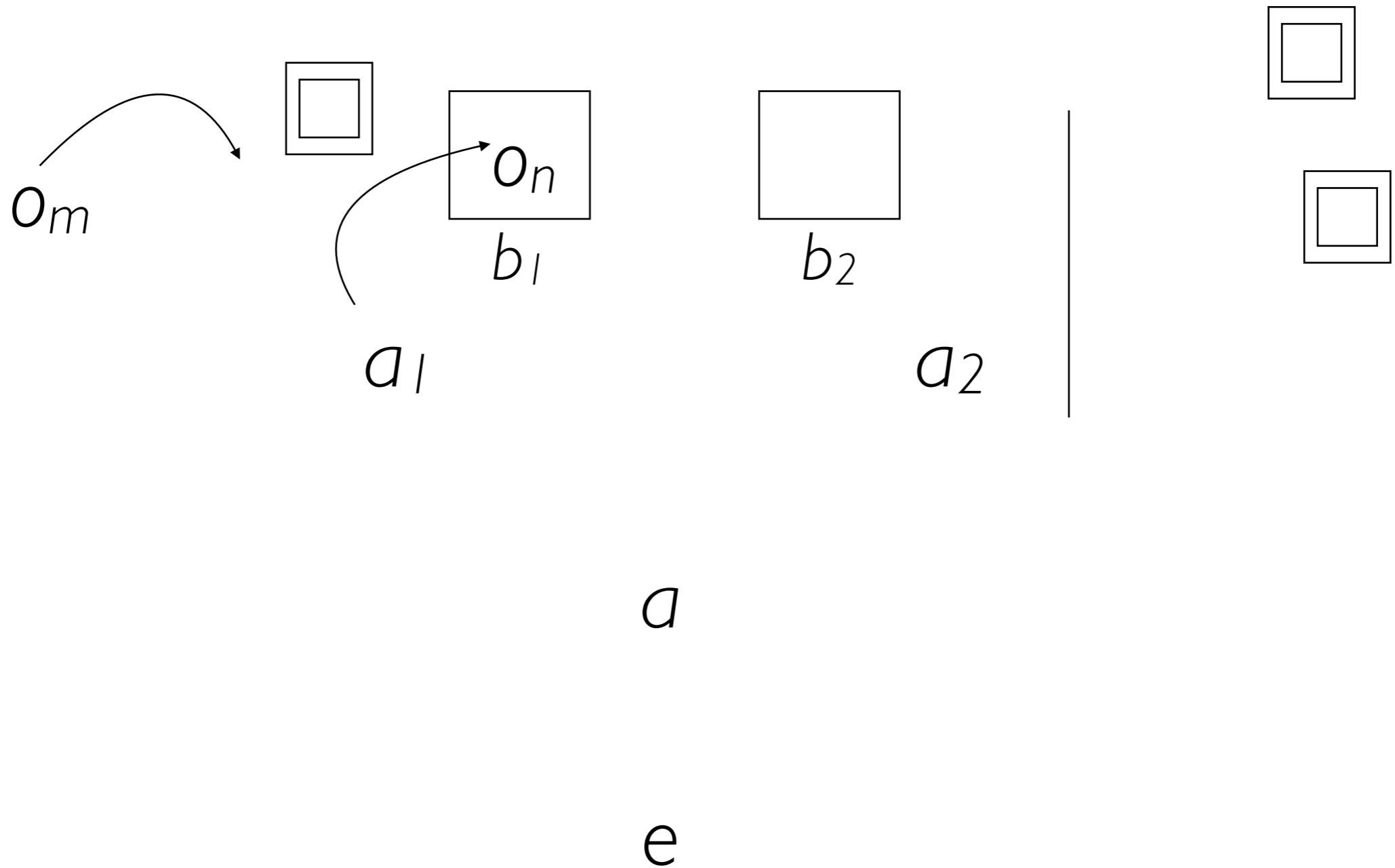
Framework for FBT^1_4

(nine timepoints)



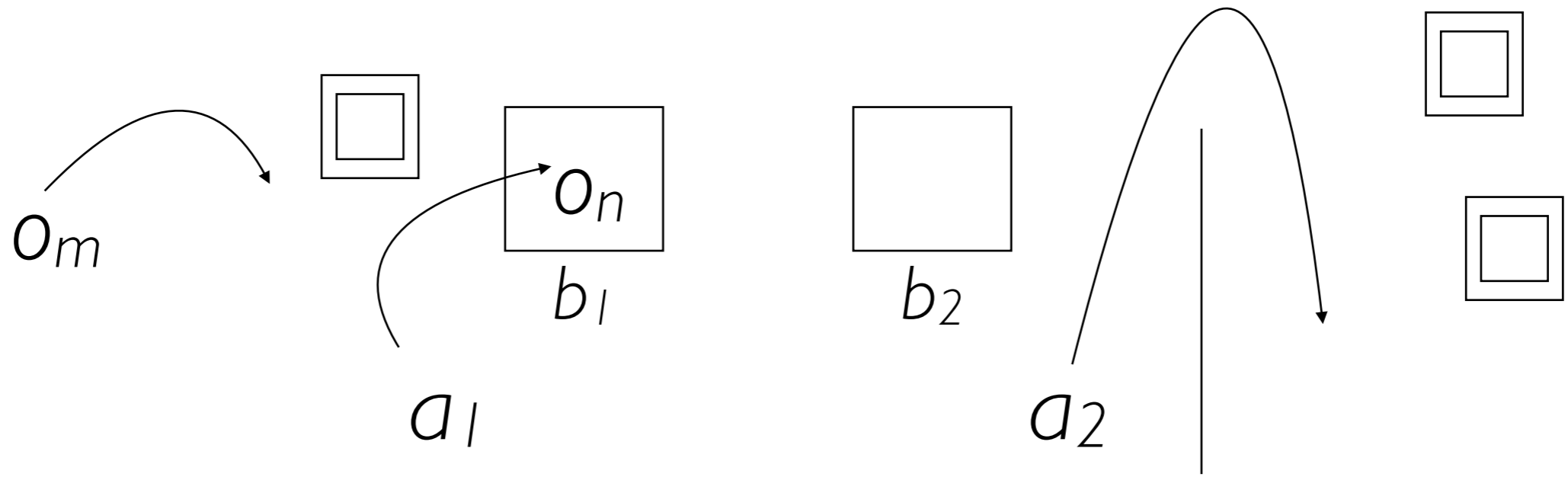
Framework for FBT^1_4

(nine timepoints)



Framework for FBT^I_4

(nine timepoints)

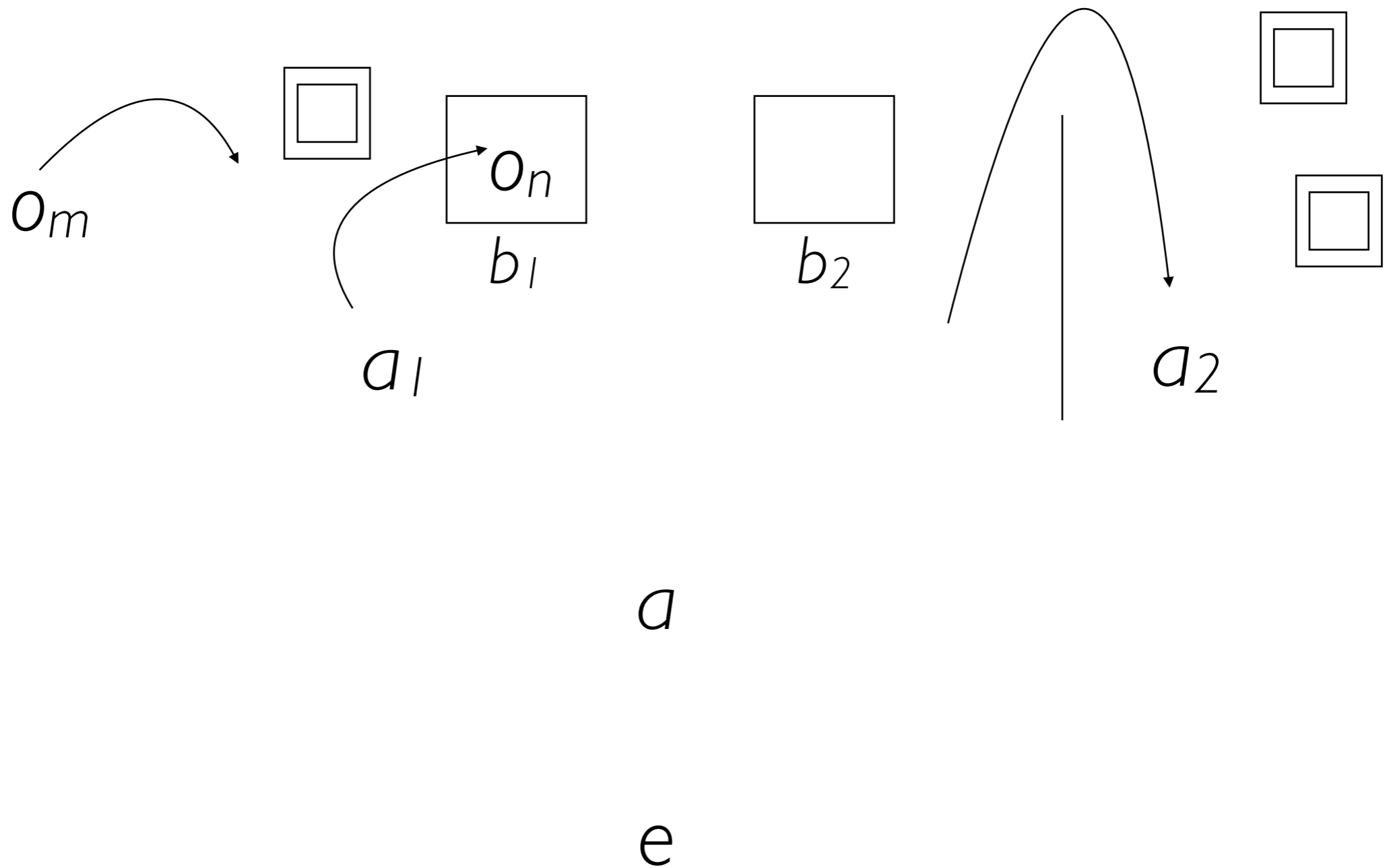


a

e

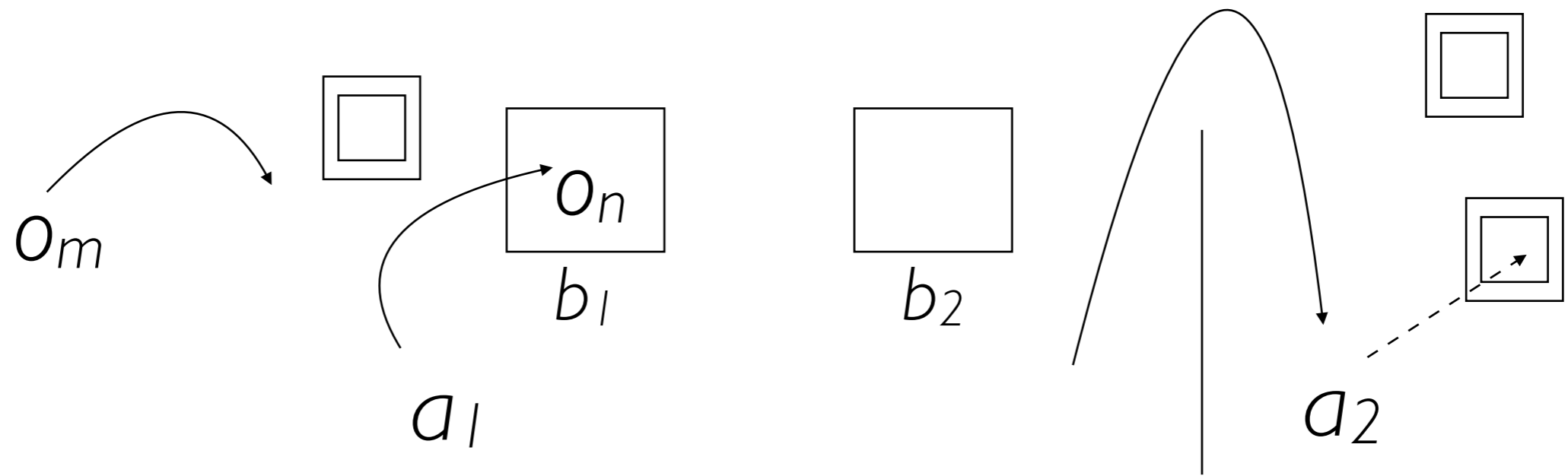
Framework for FBT^1_4

(nine timepoints)



Framework for FBT^1_4

(nine timepoints)

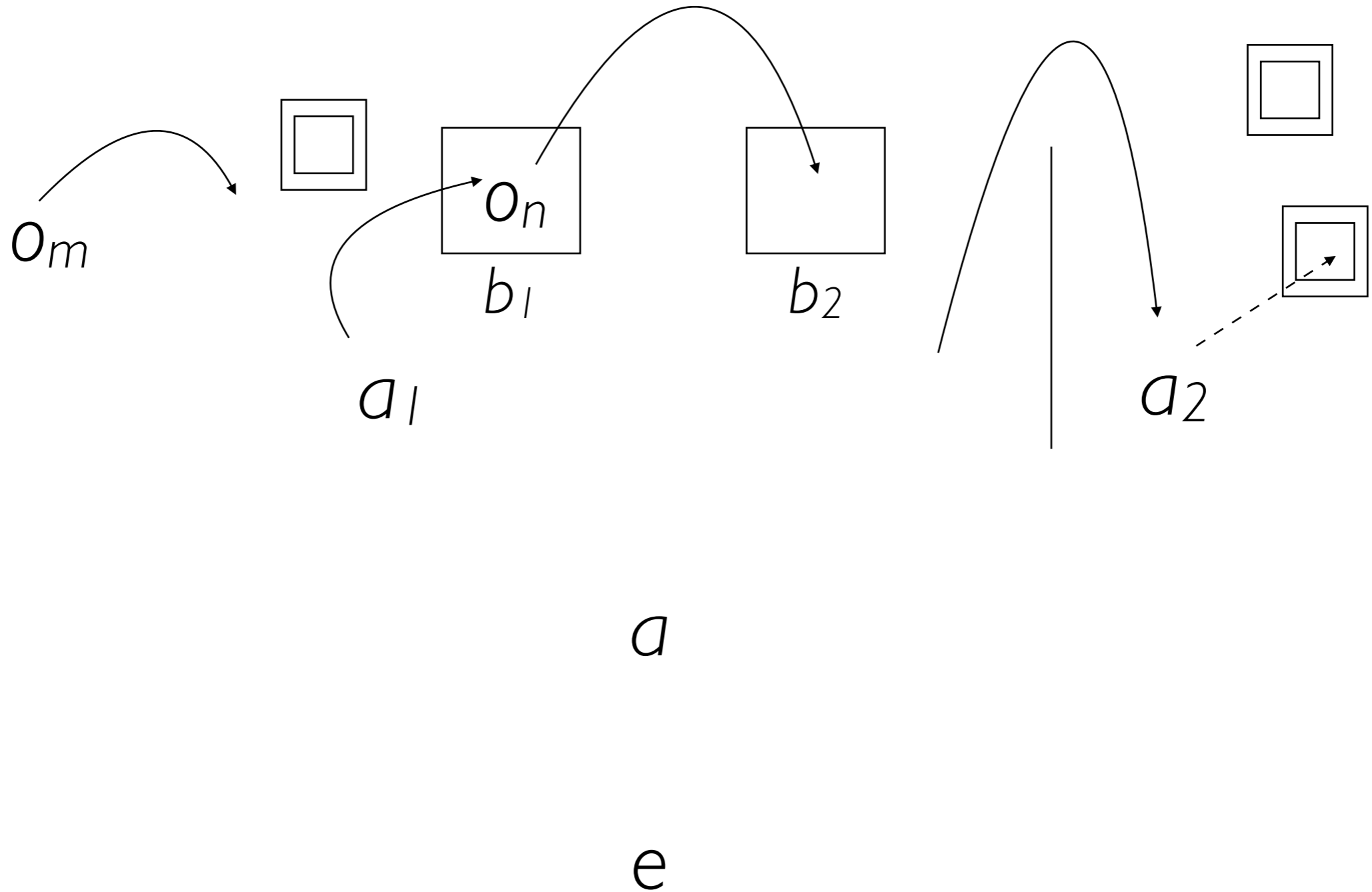


a

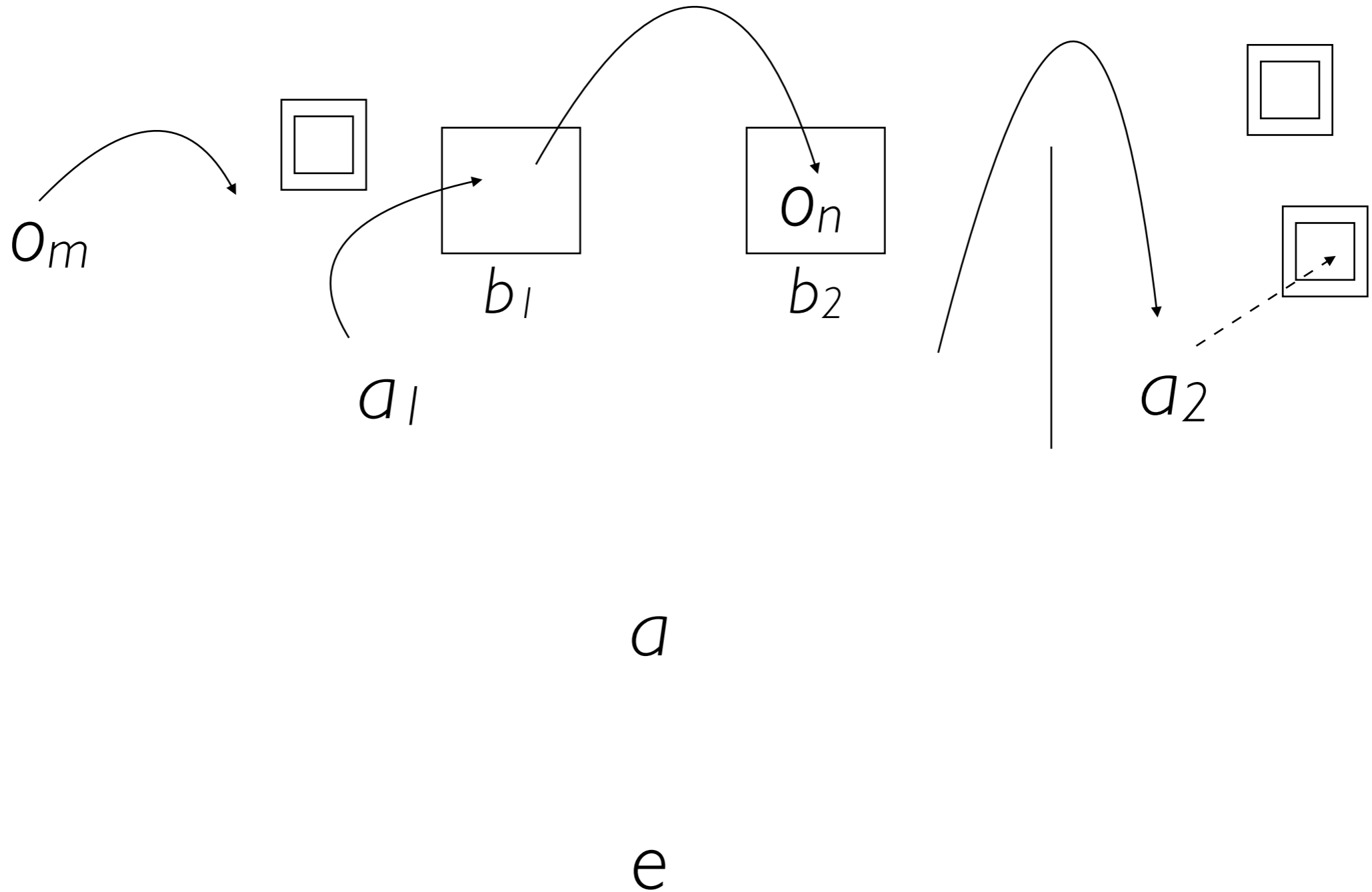
e

Framework for FBT^1_4

(nine timepoints)

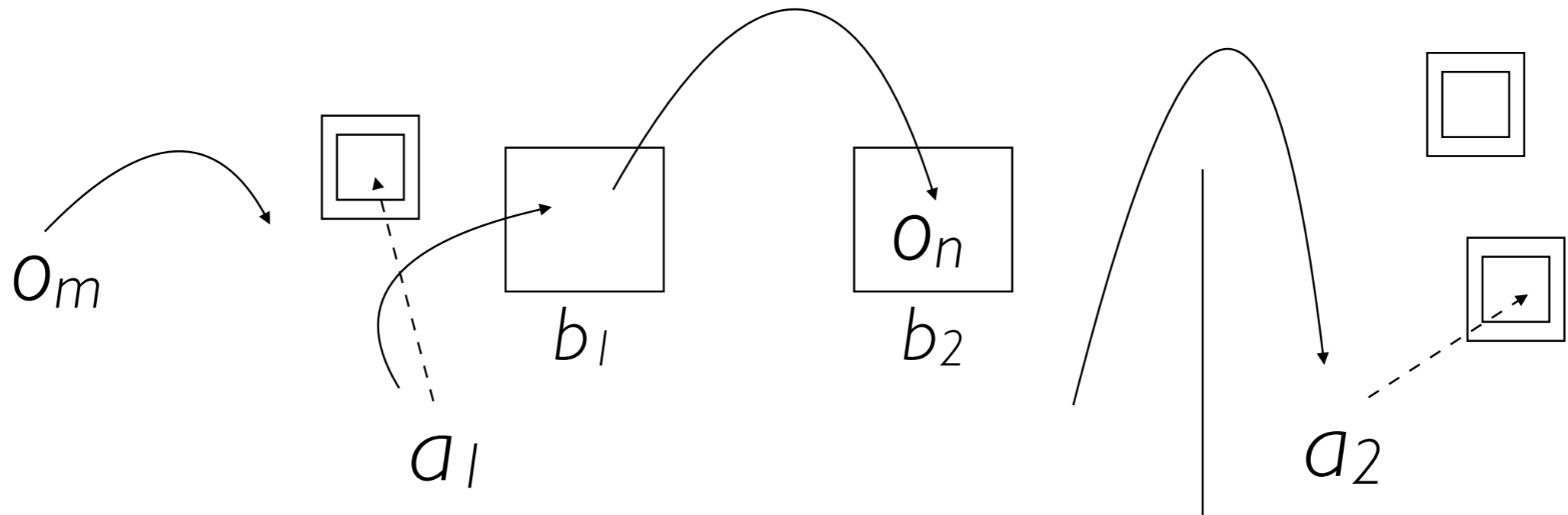


Framework for FBT^1_4 (nine timepoints)



Framework for FBT^1_4

(nine timepoints)

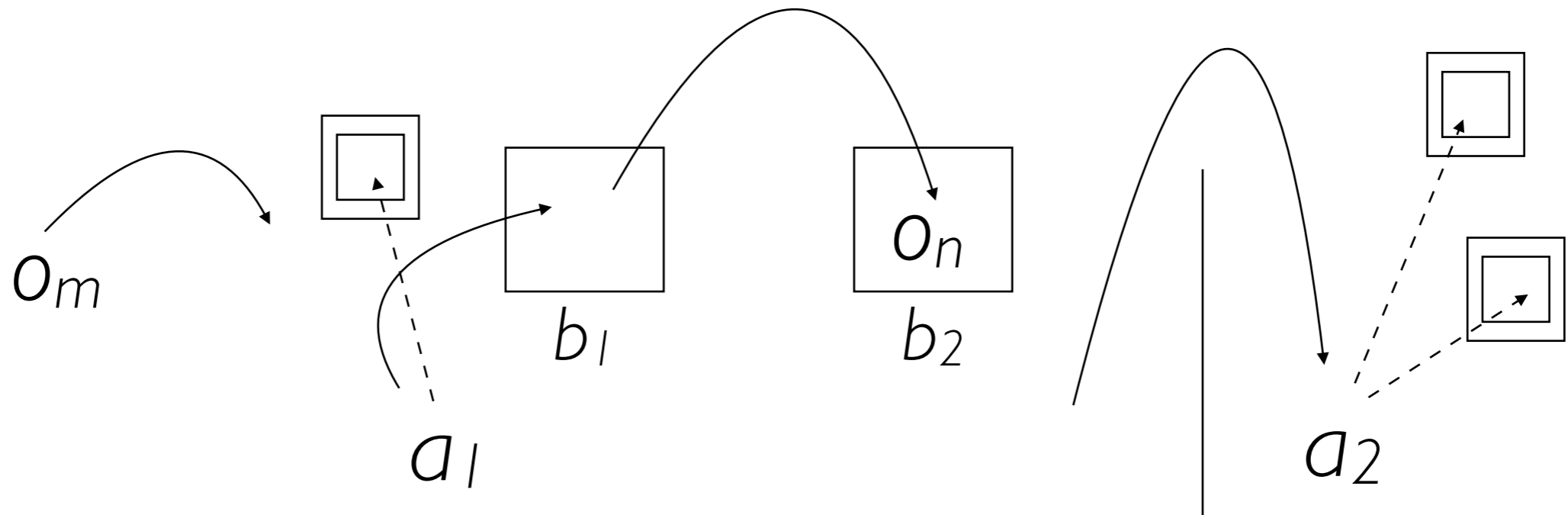


a

e

Framework for FBT^1_4

(nine timepoints)

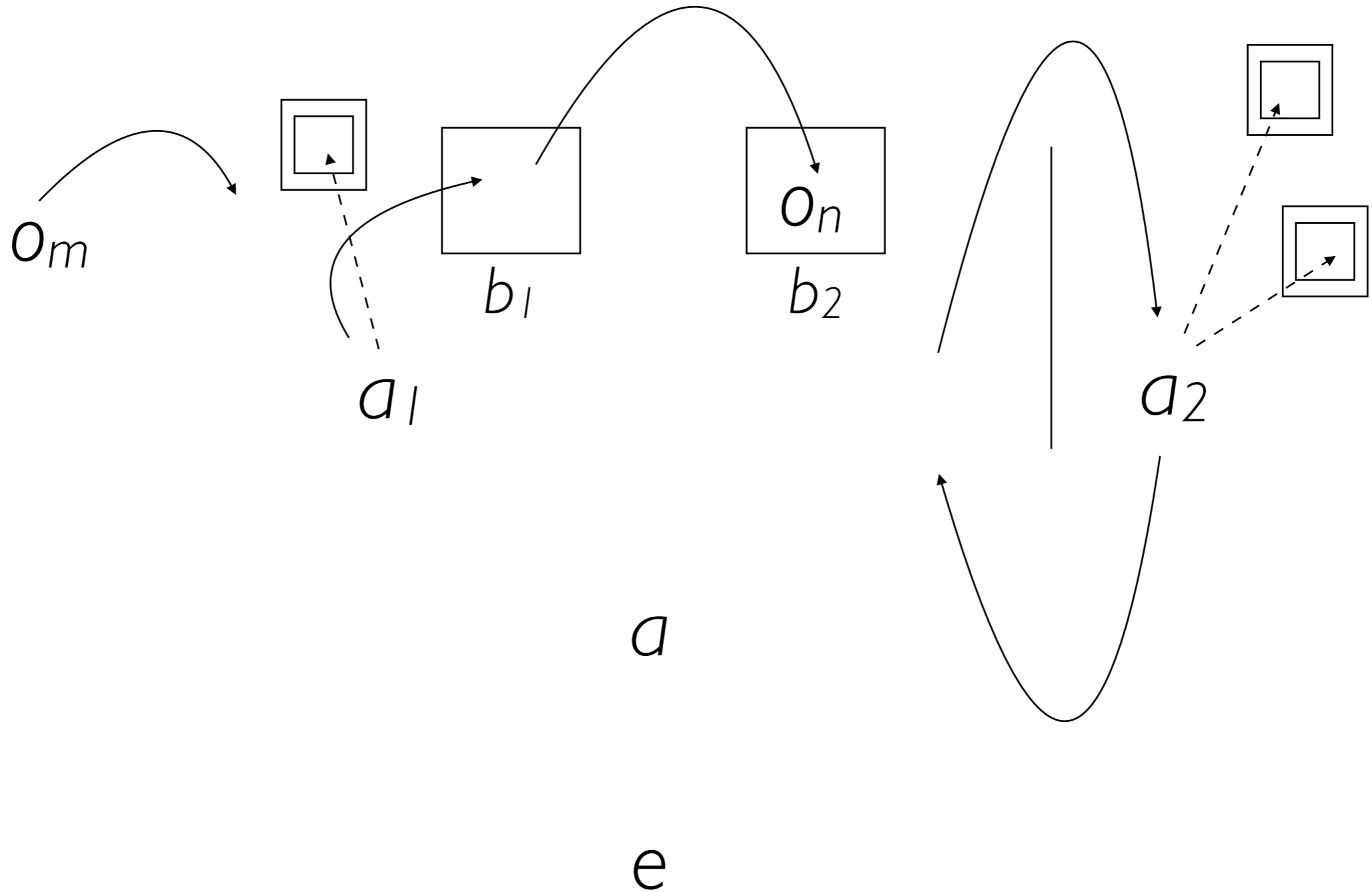


a

e

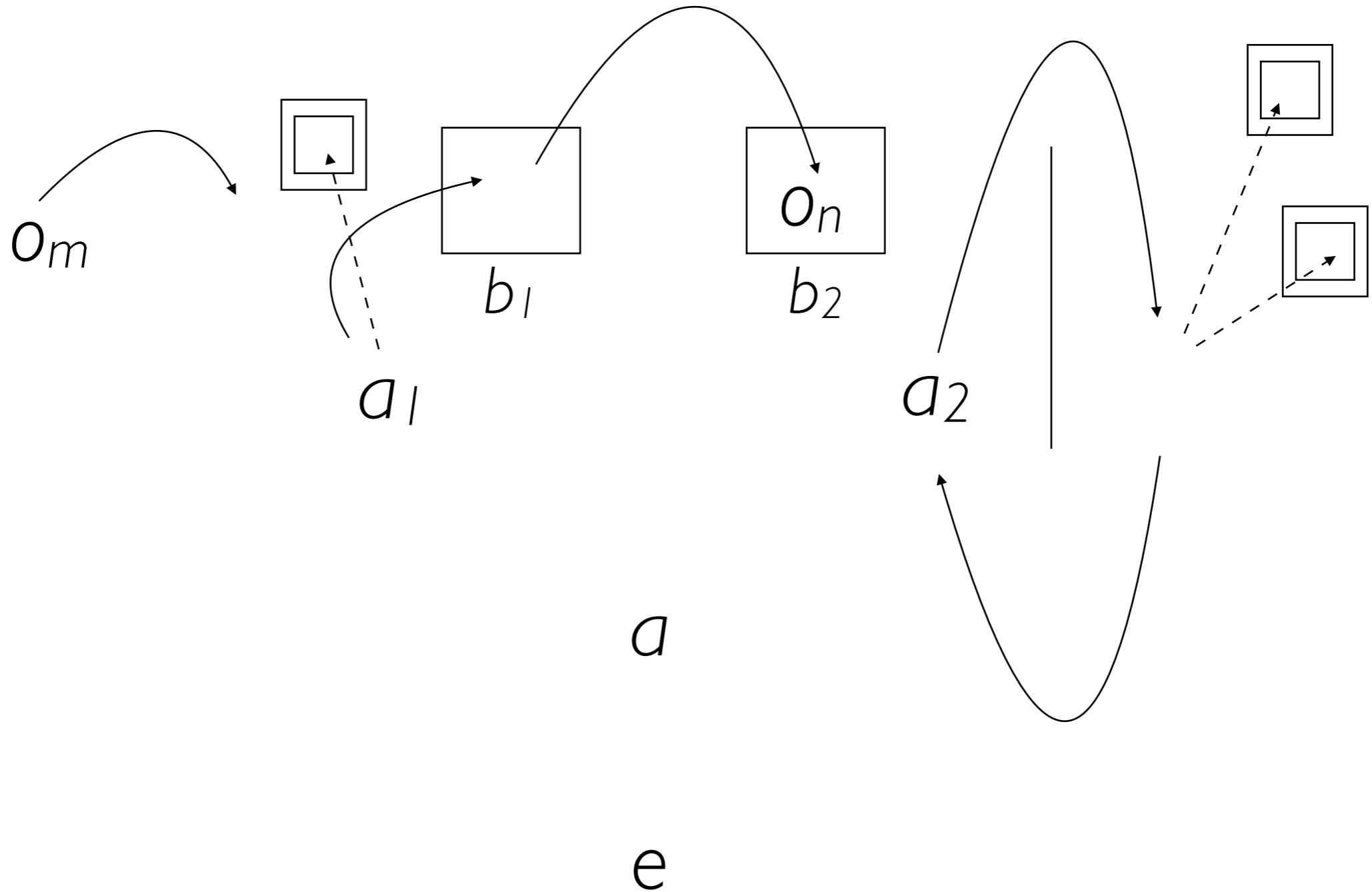
Framework for FBT^1_4

(nine timepoints)

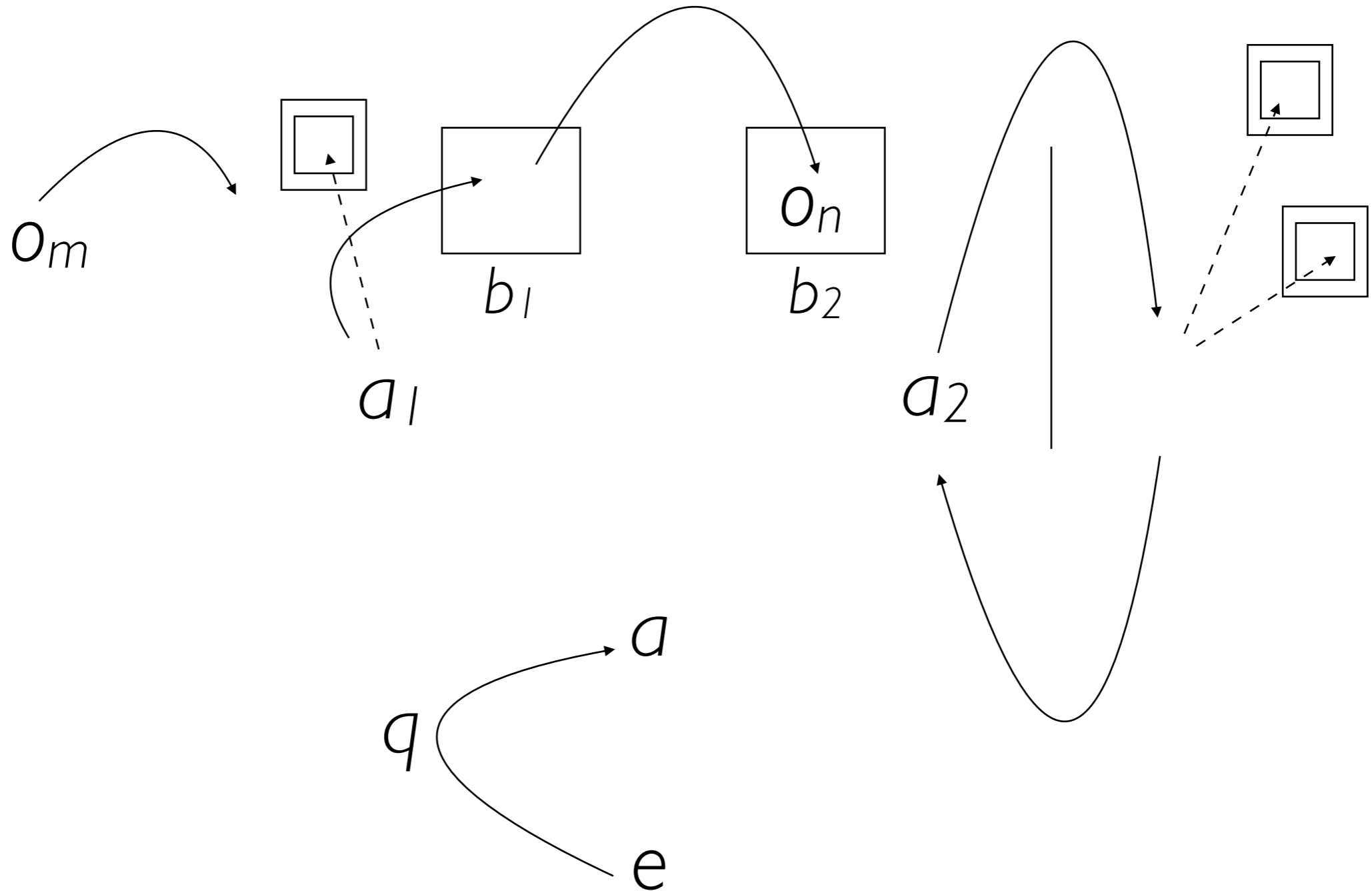


Framework for FBT^I_4

(nine timepoints)

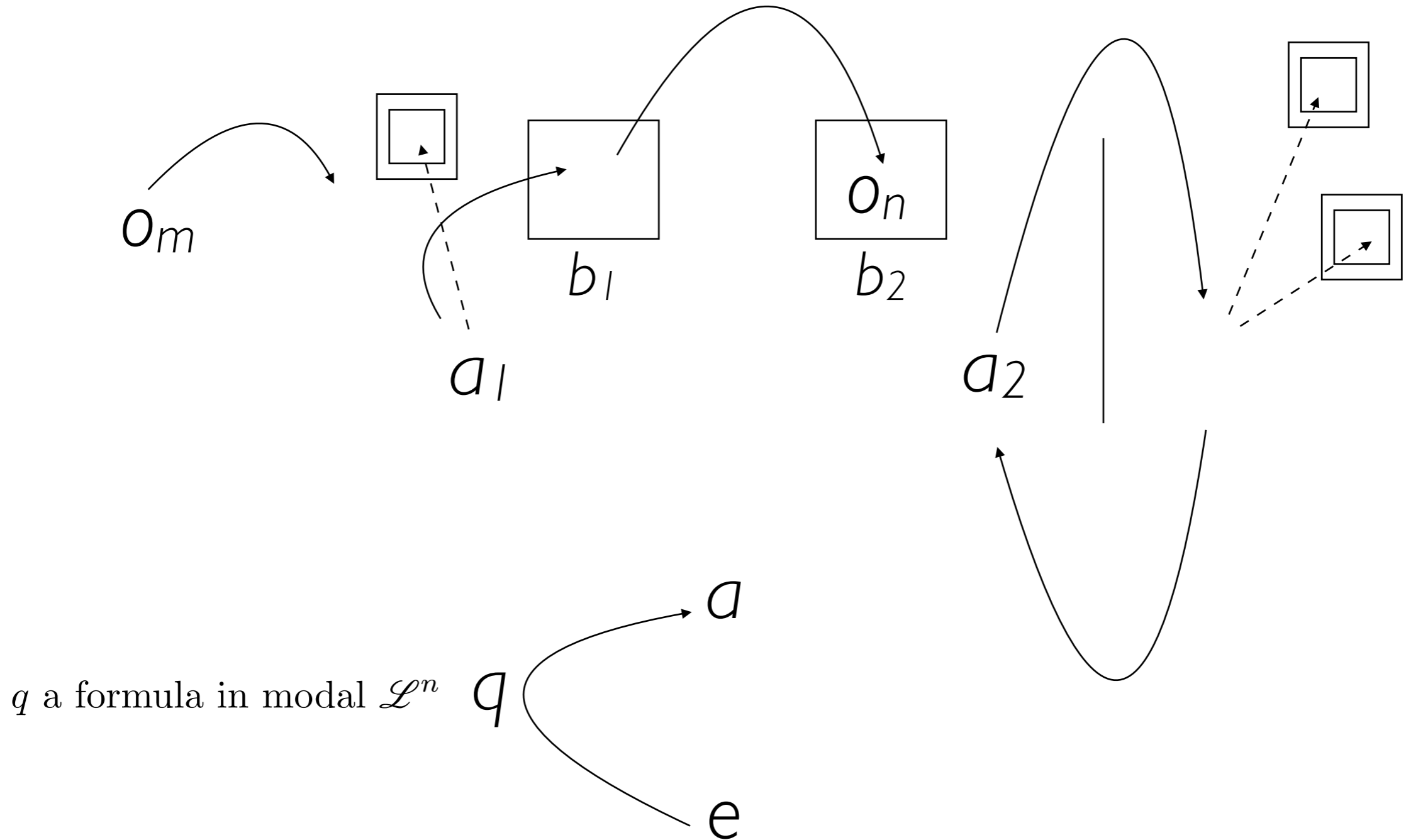


Framework for FBT^1_4 (nine timepoints)

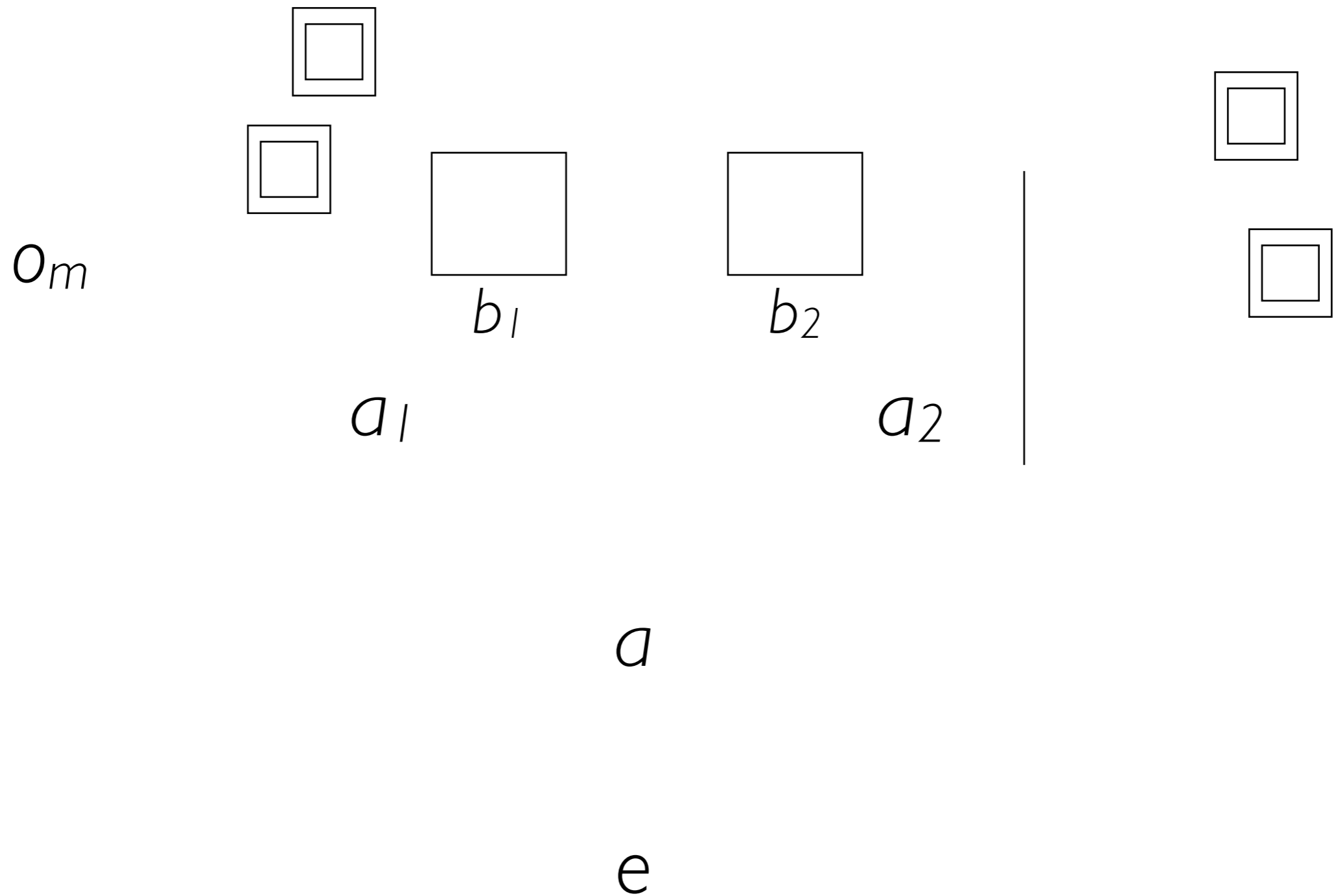


Framework for FBT^1_4

(nine timepoints)

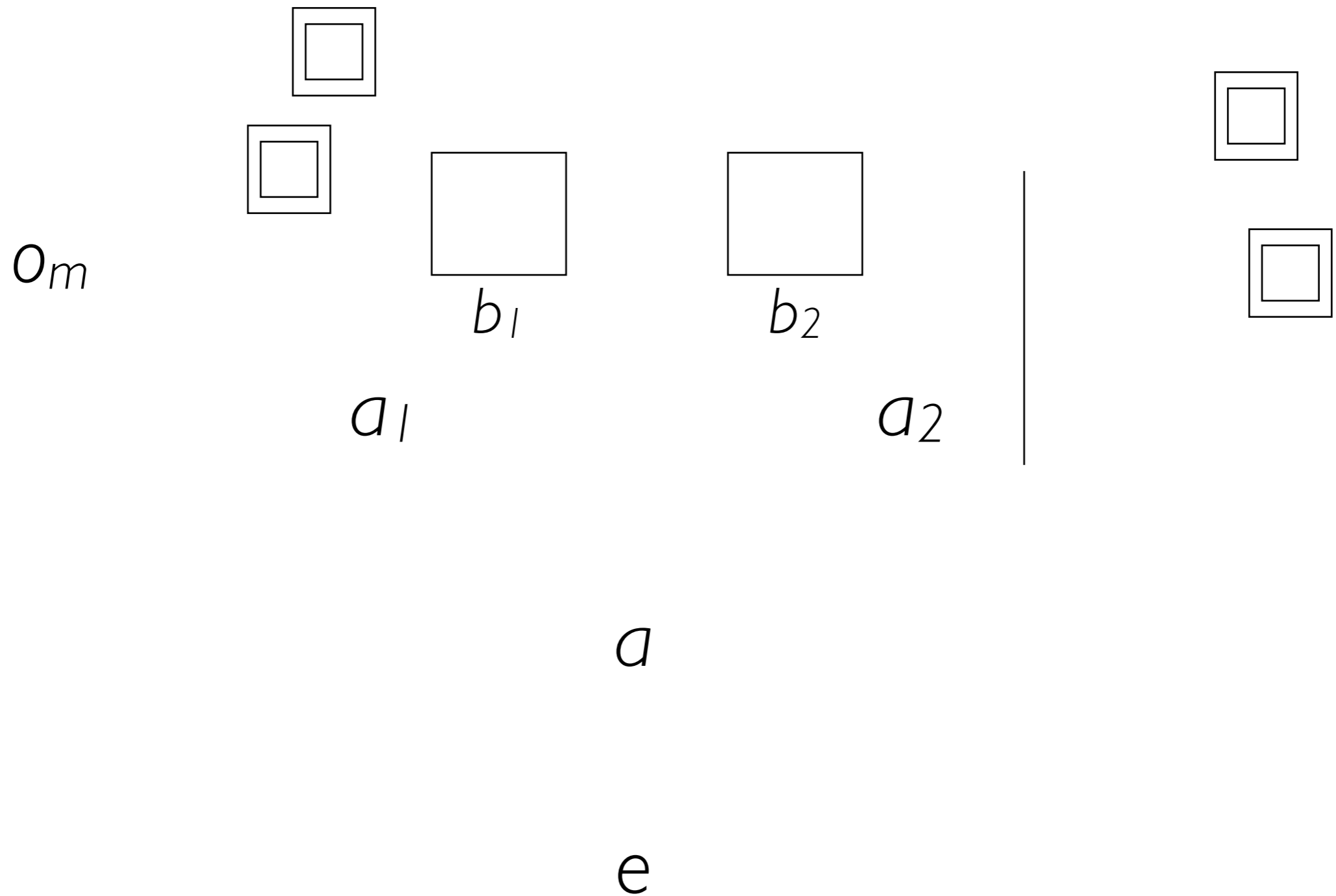


Framework for FBT^1_5



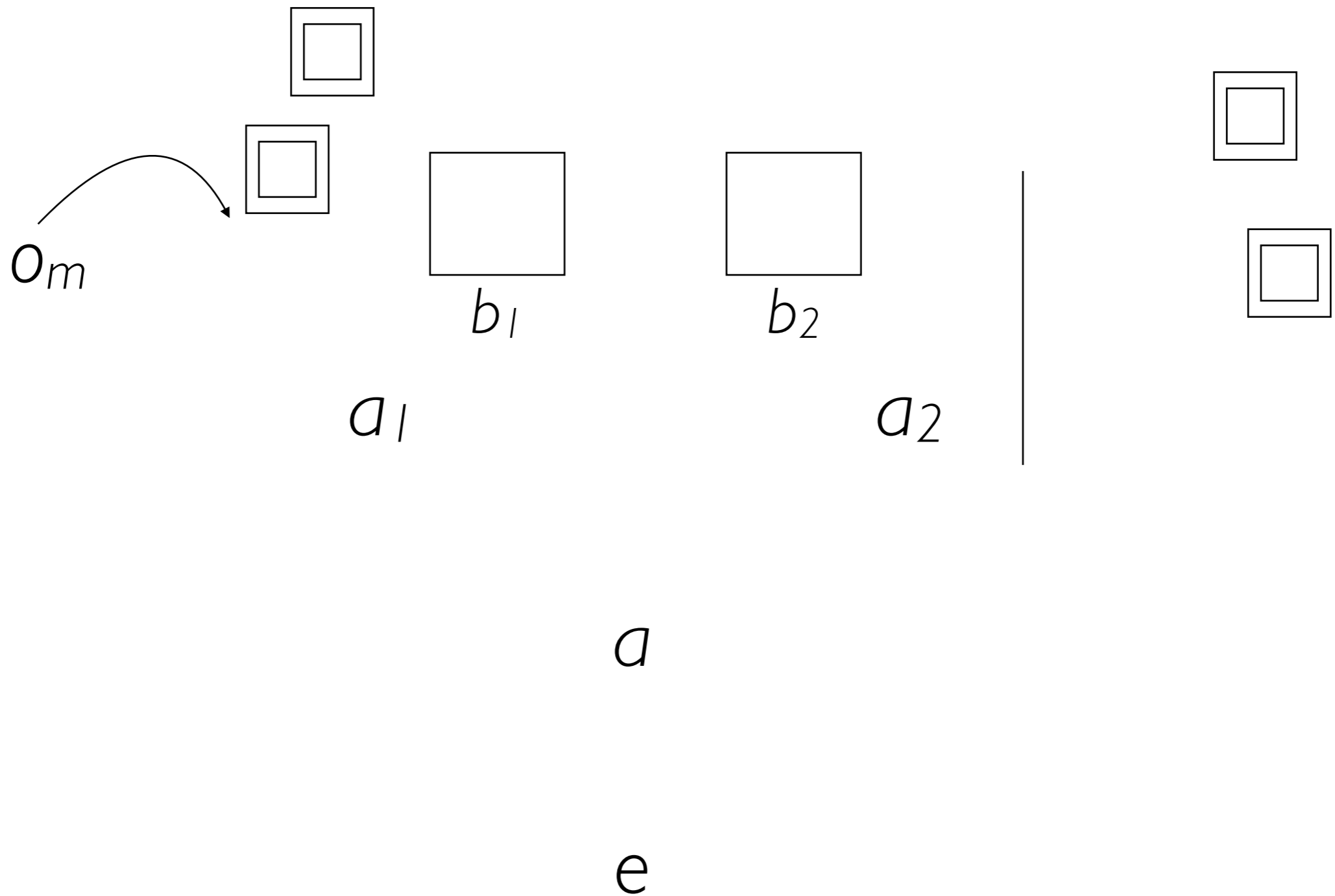
Framework for FBT¹⁵

(ten timepoints)



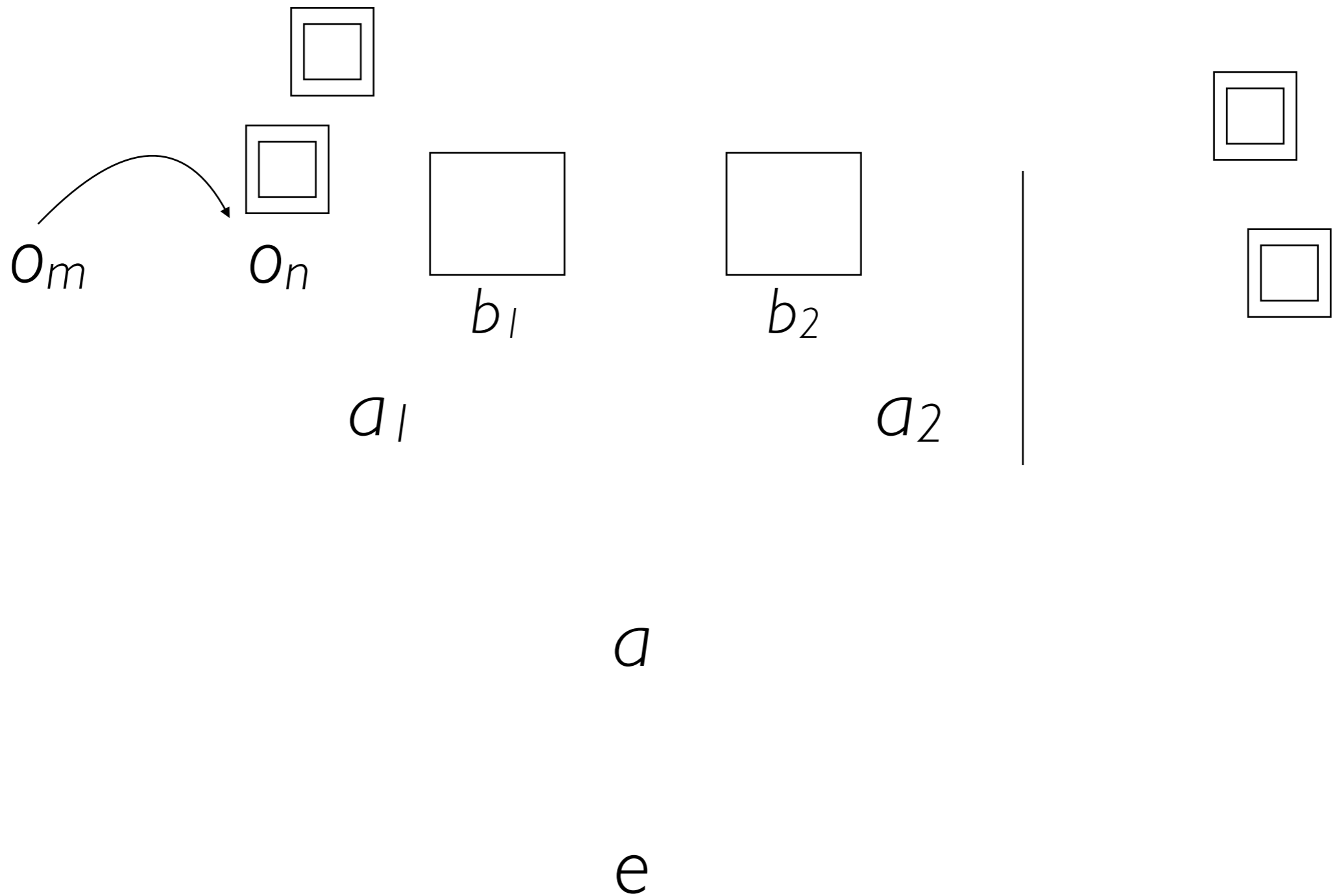
Framework for FBT¹⁵

(ten timepoints)



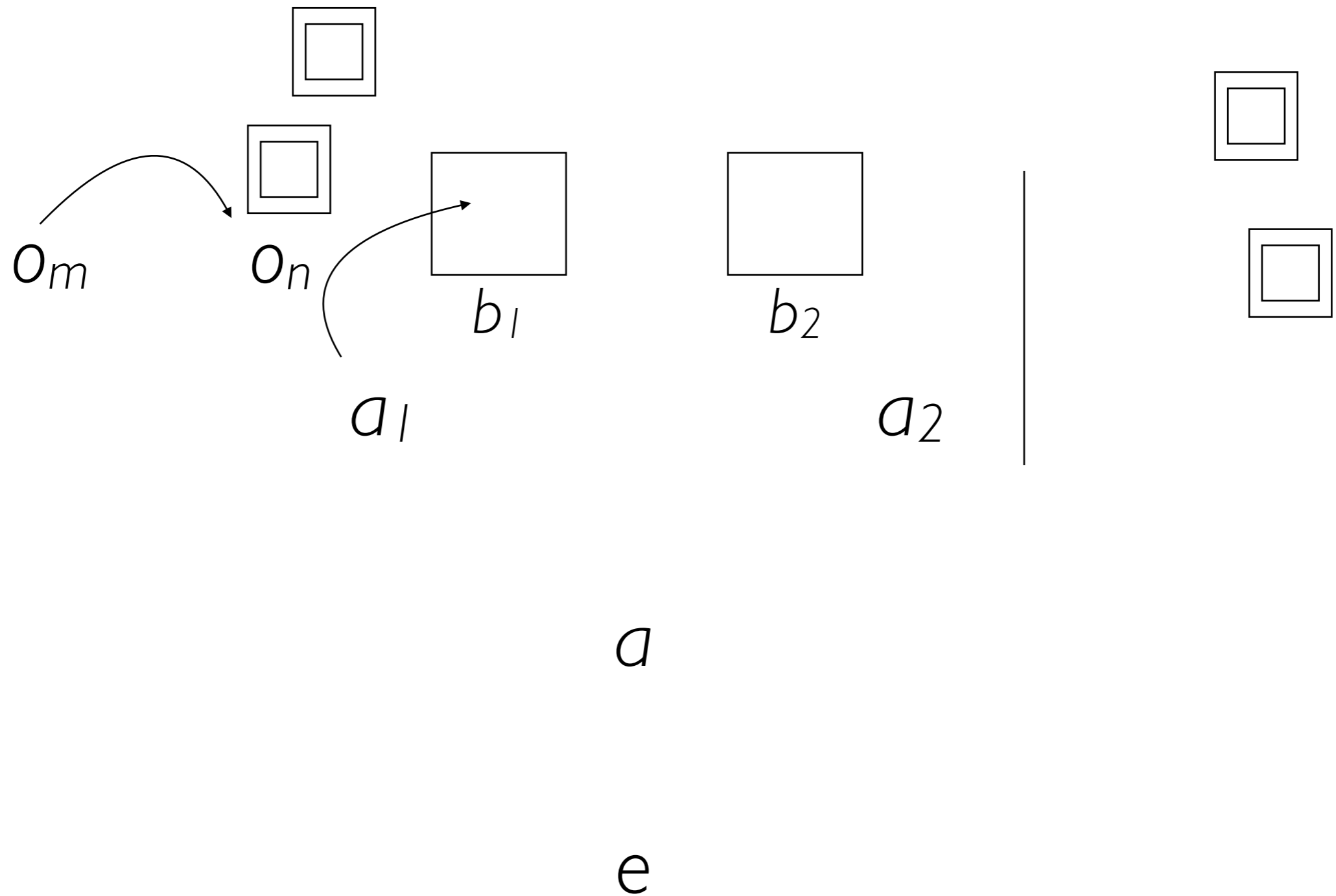
Framework for FBT¹⁵

(ten timepoints)



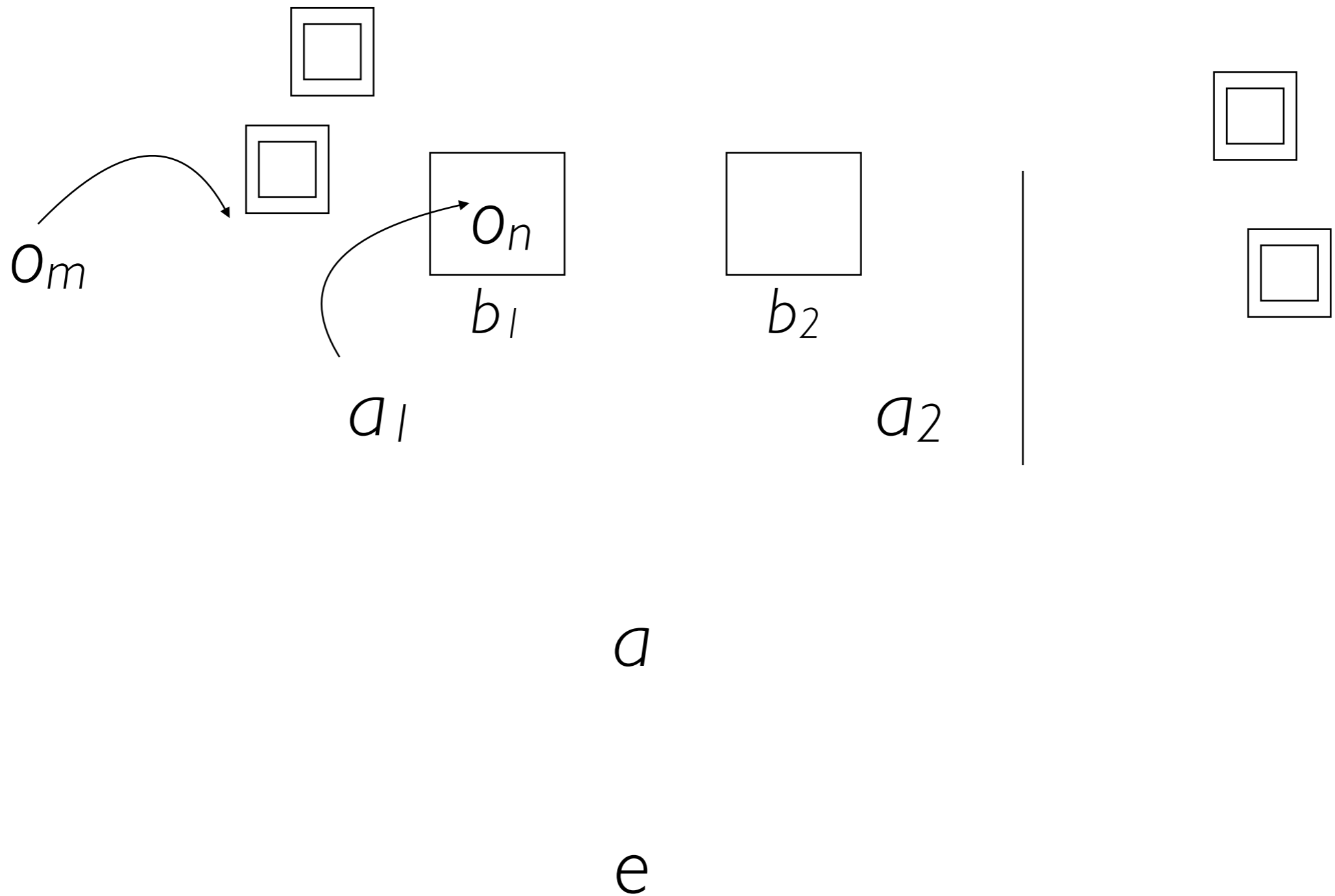
Framework for FBT¹⁵

(ten timepoints)



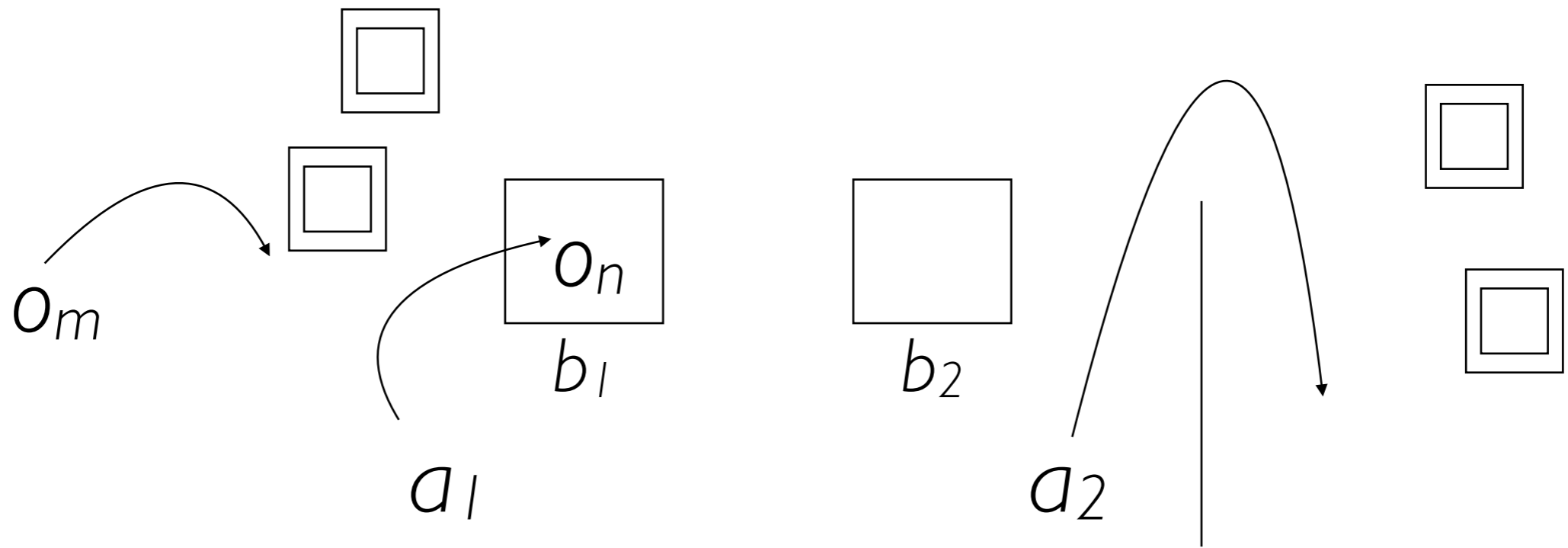
Framework for FBT¹⁵

(ten timepoints)



Framework for FBT^1_5

(ten timepoints)

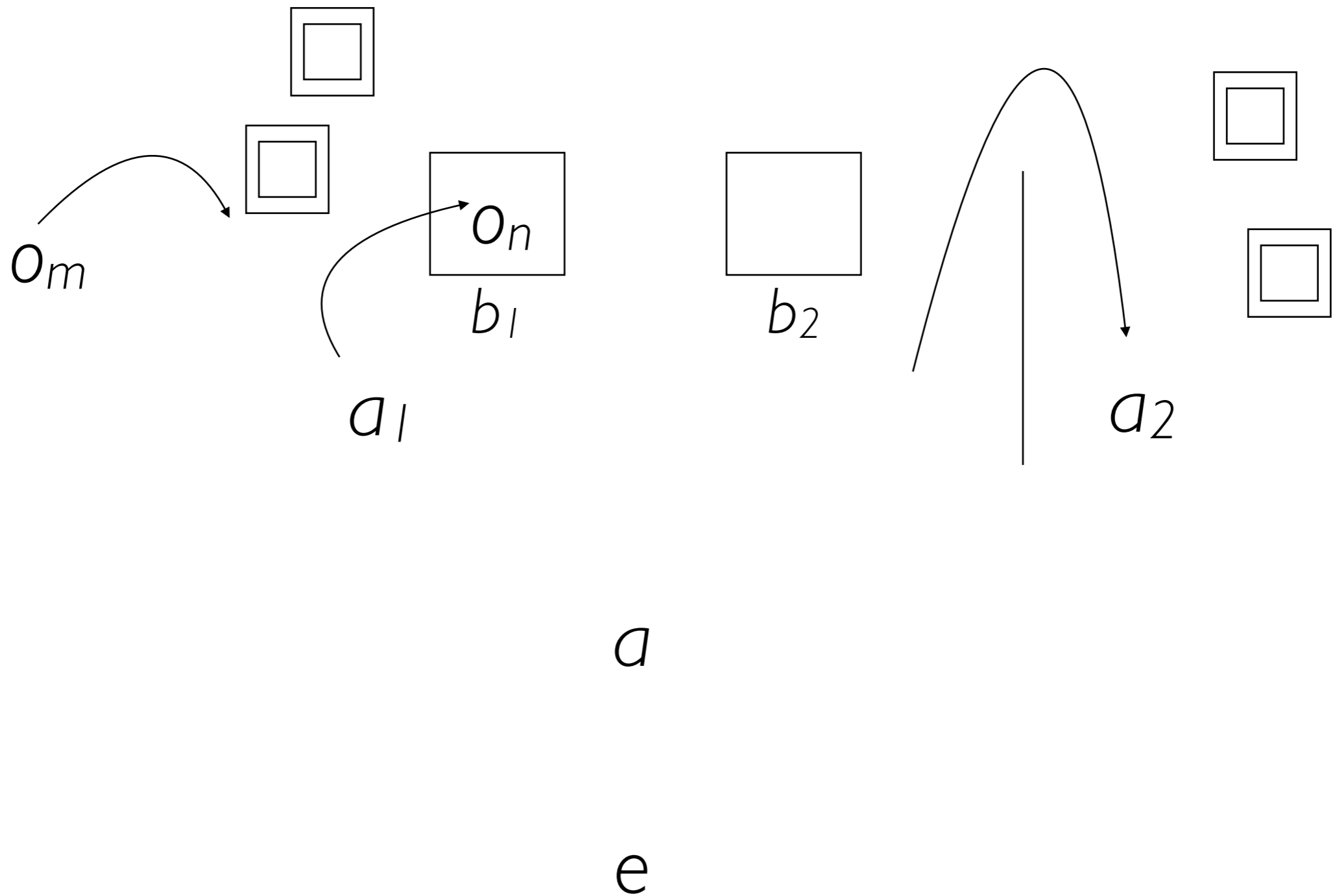


a

e

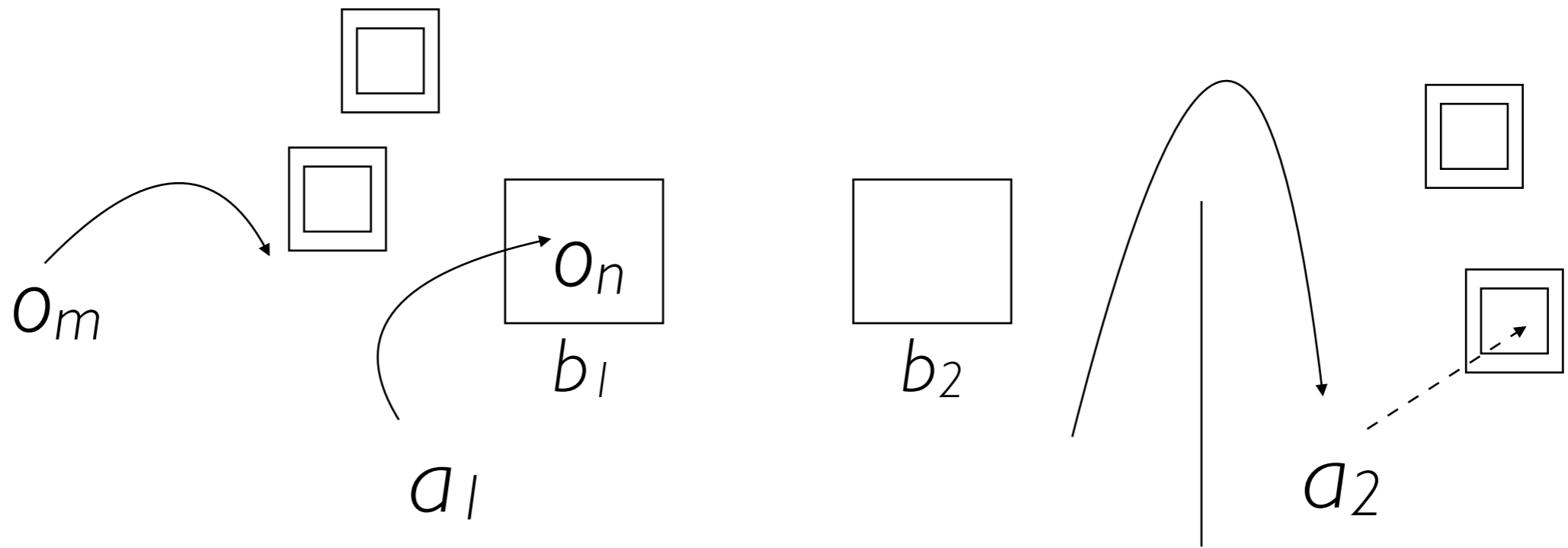
Framework for FBT^1_5

(ten timepoints)



Framework for FBT¹⁵

(ten timepoints)

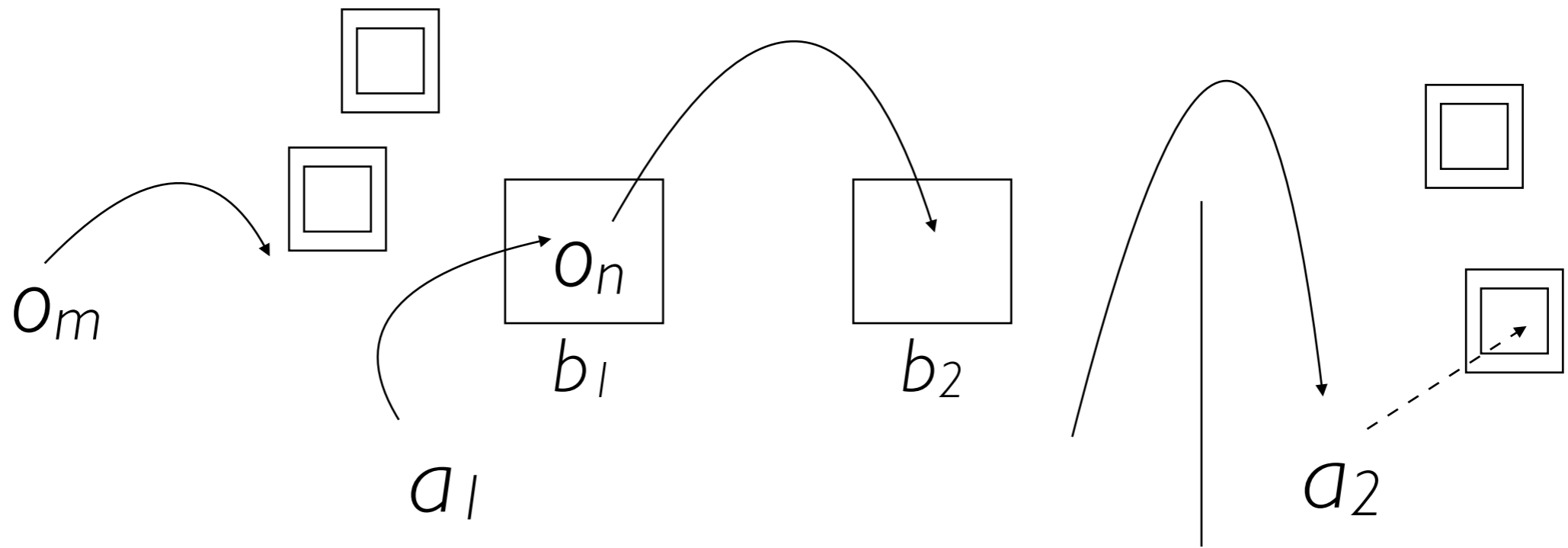


a

e

Framework for FBT¹⁵

(ten timepoints)

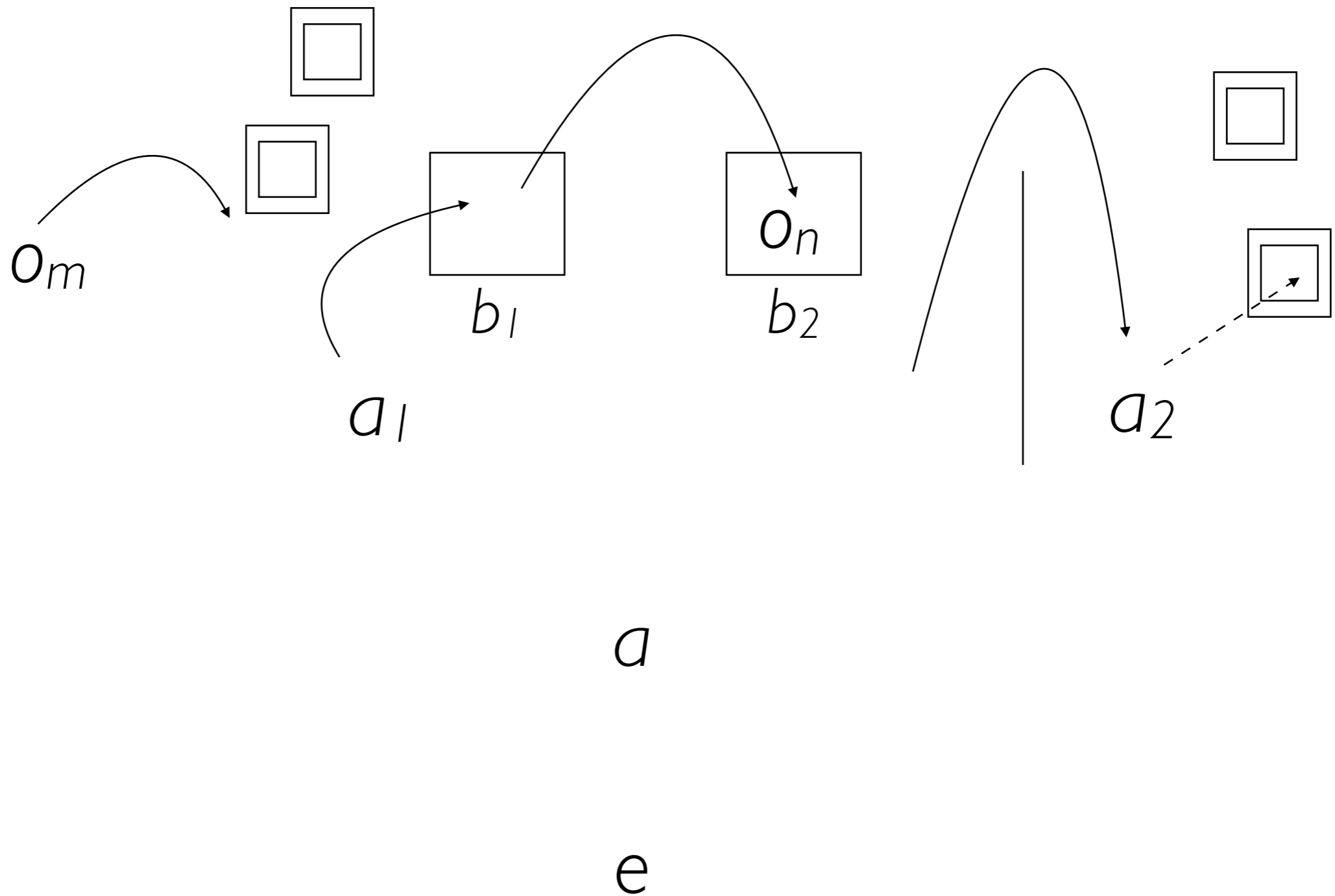


a

e

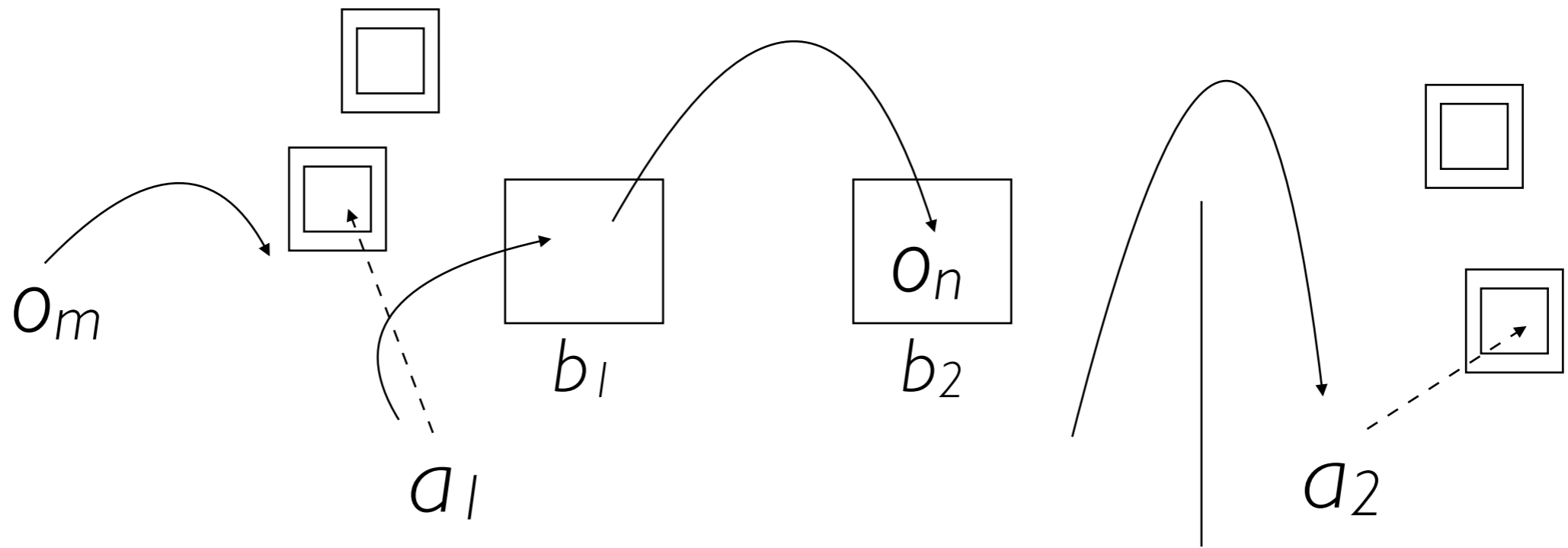
Framework for FBT¹₅

(ten timepoints)



Framework for FBT¹₅

(ten timepoints)

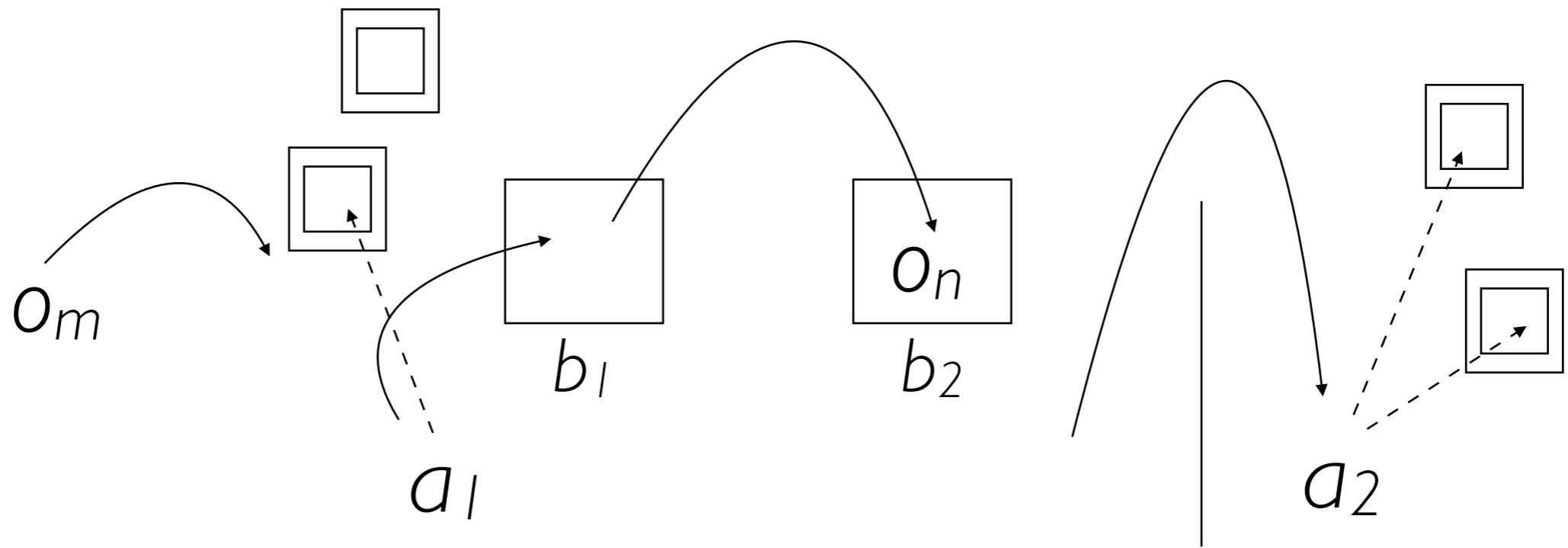


a

e

Framework for FBT^1_5

(ten timepoints)

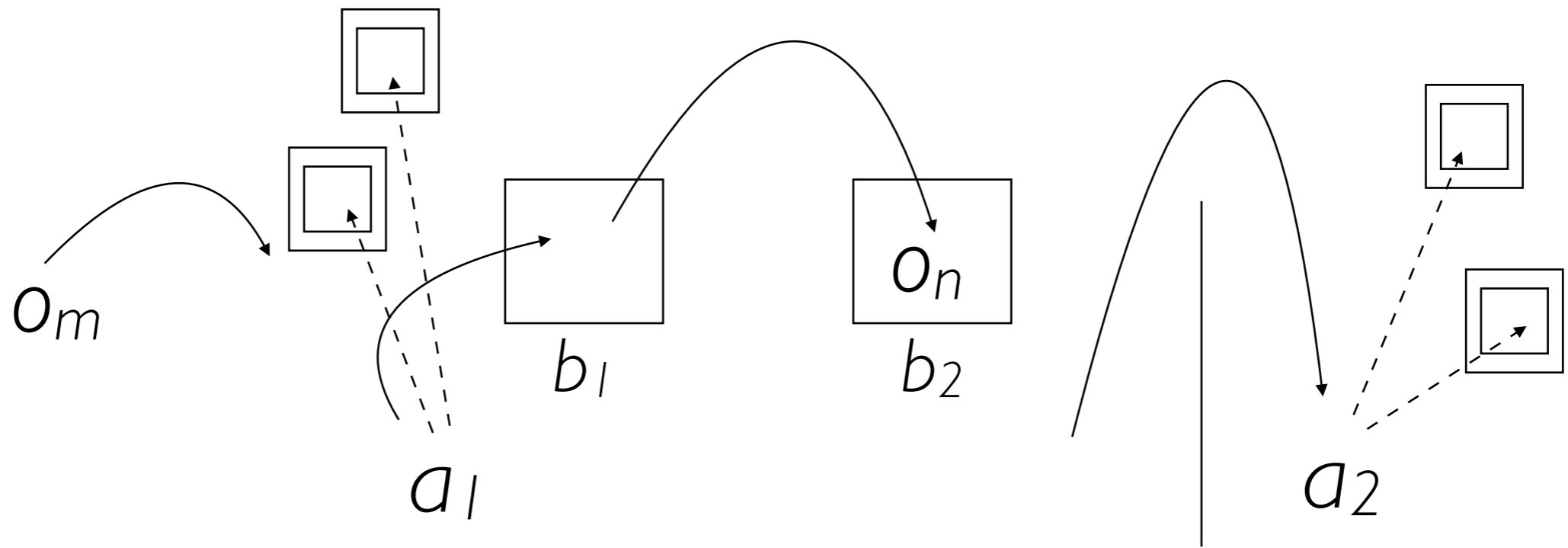


a

e

Framework for FBT¹₅

(ten timepoints)

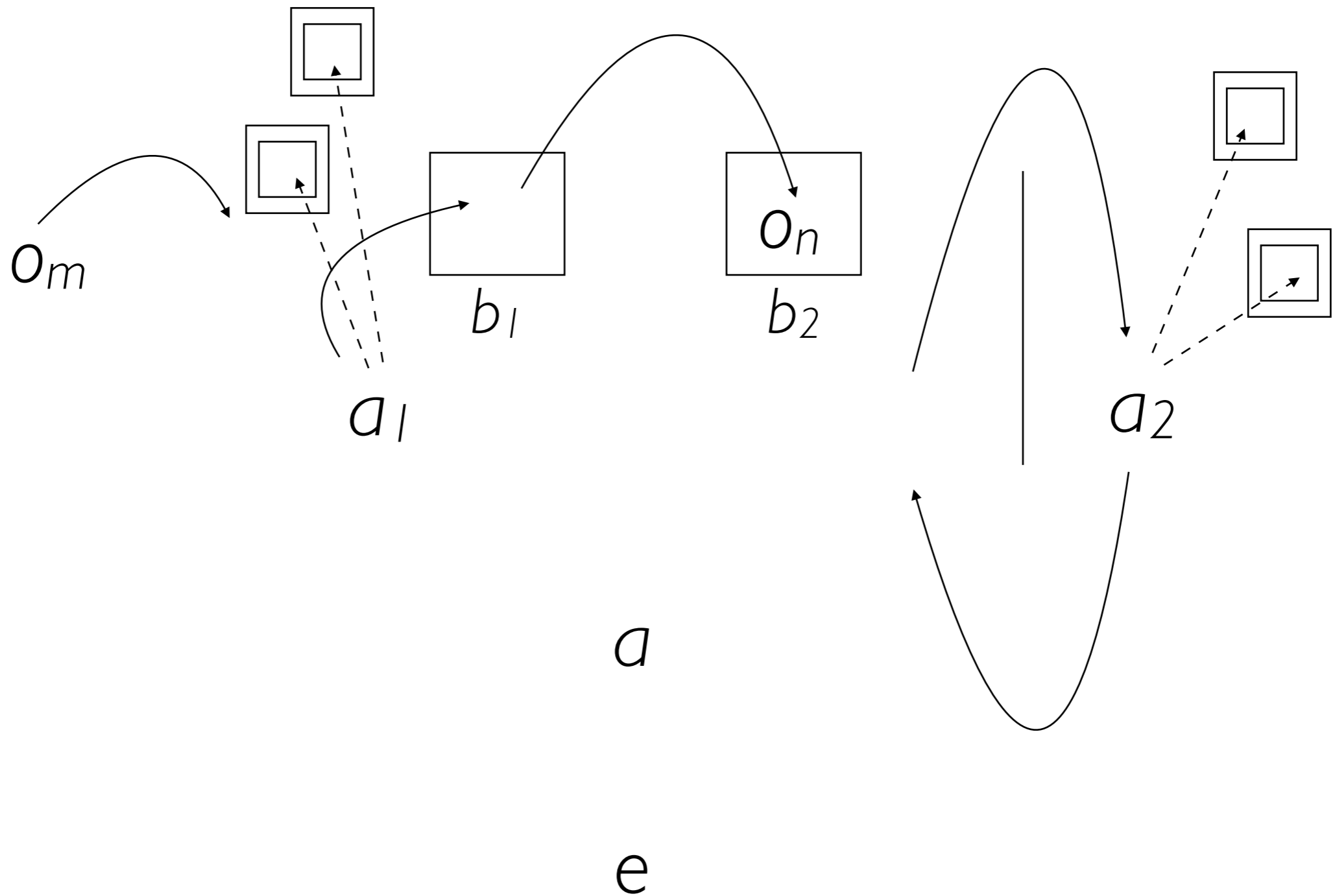


a

e

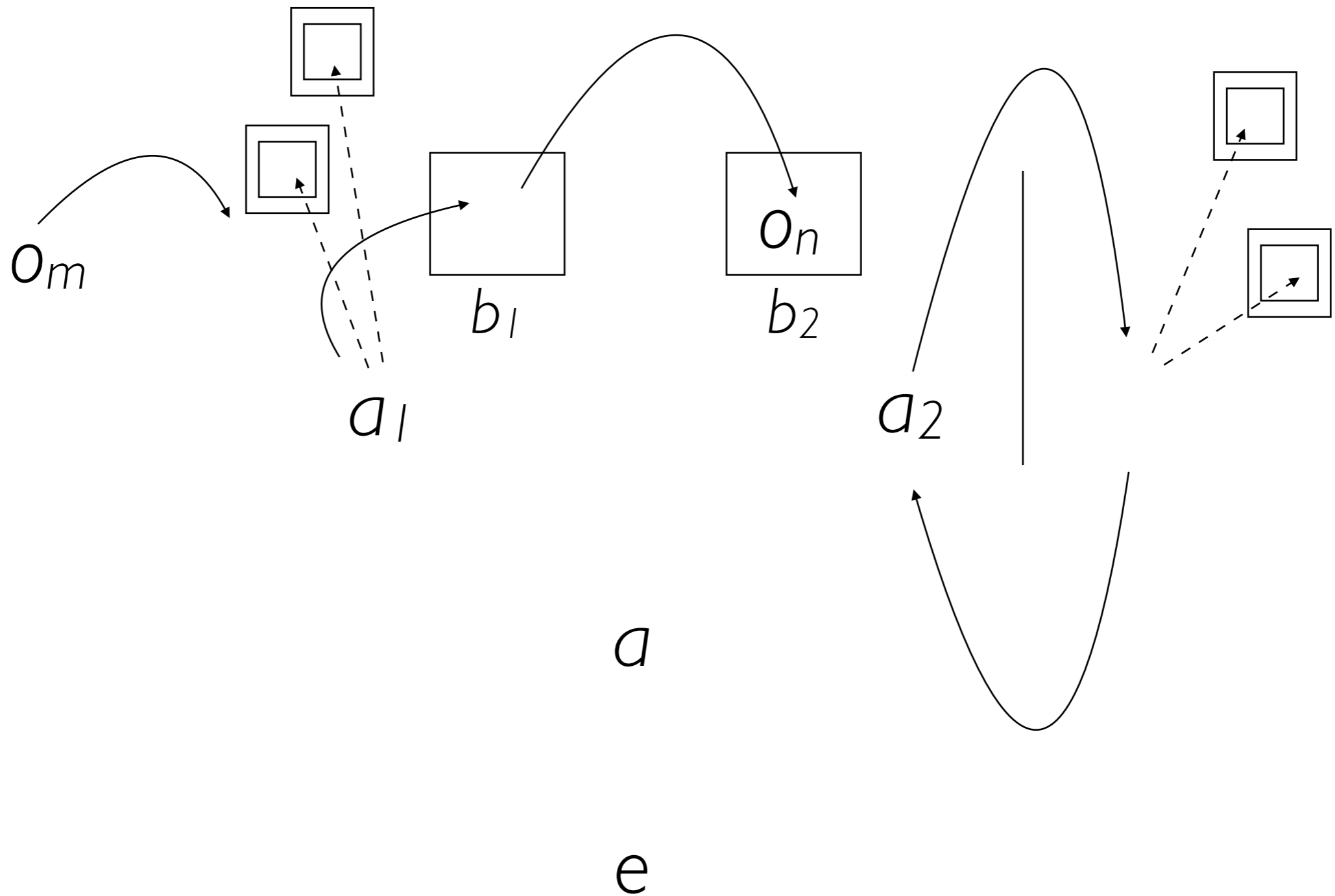
Framework for FBT¹₅

(ten timepoints)



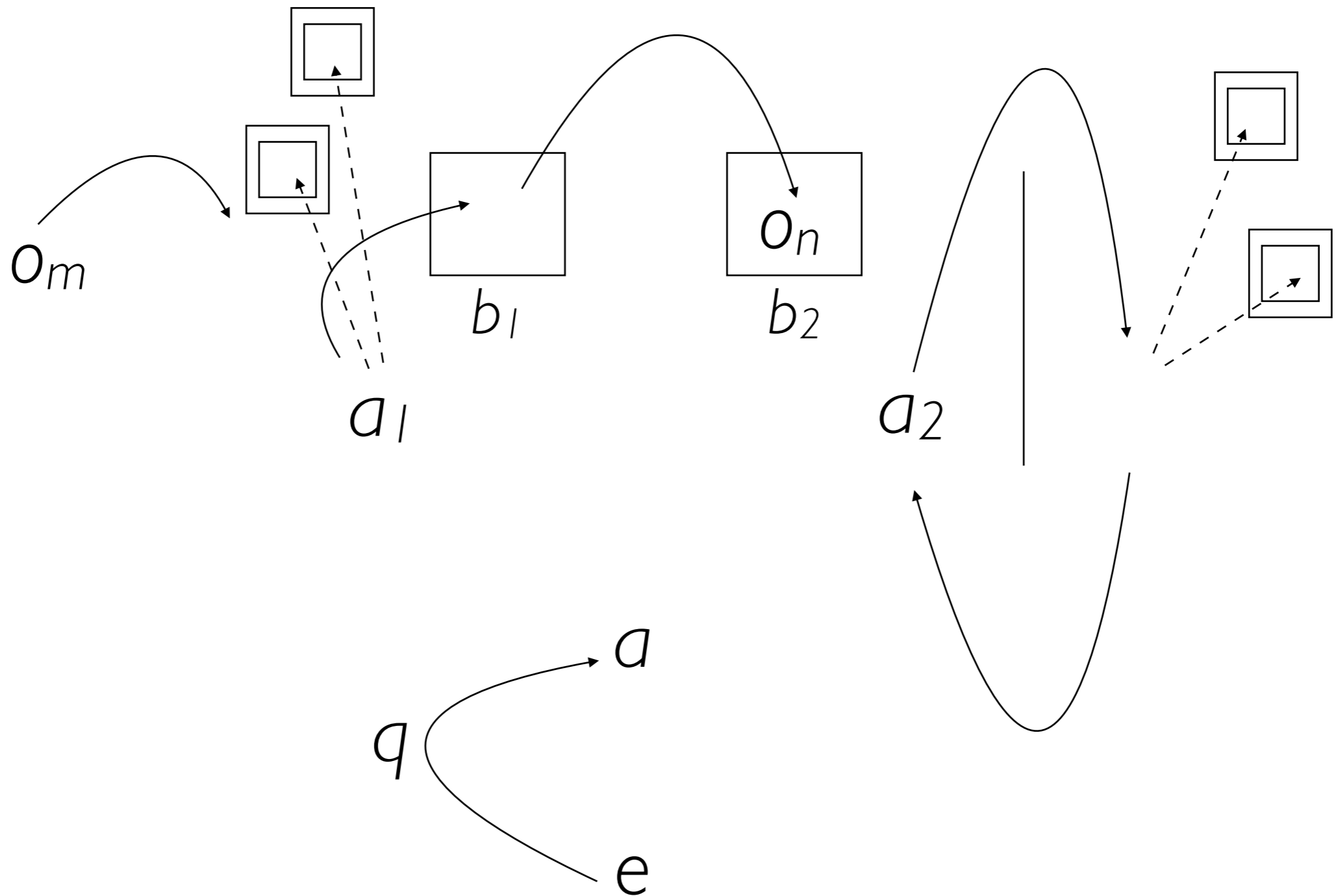
Framework for FBT¹₅

(ten timepoints)



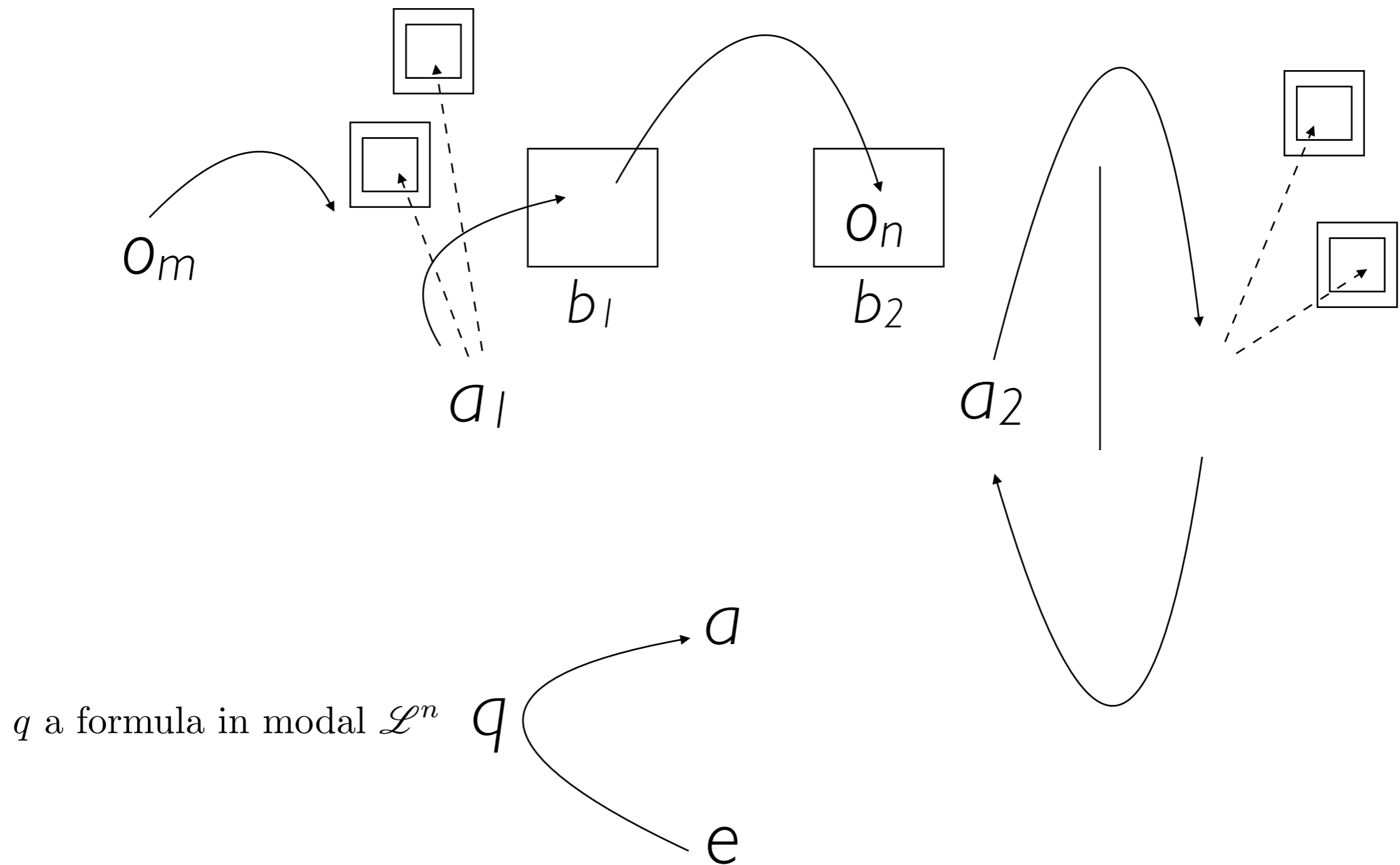
Framework for FBT¹₅

(ten timepoints)



Framework for FBT^1_5

(ten timepoints)



Humans Can Succeed

Neurobiologically normal, nurtured, educated, and sufficiently motivated humans can correctly answer any relevant query q for the infinite progression, and prove that their answer is correct. For the obvious subclass of queries (the form of which appear in the box below), they can prove and exploit the following lemma.

Lemma: Suppose $\text{FBT}_k, k \in \mathbb{Z}^+$, holds; (i.e. that level k of FBT holds). Then, if k is even, $\mathbf{B}_2\mathbf{B}_1 \dots \mathbf{B}_2 \iota$, where there are $k + 1$ iterated \mathbf{B}_i operators; otherwise $\mathbf{B}_1\mathbf{B}_2 \dots \mathbf{B}_1\mathbf{B}_2 \iota$, where there again there are $k + 1$ iterated \mathbf{B}_i operators.

Passing to Probing Mastery of the Specific Subclass

Experimenter to a : “At level k ,
from which box will a_2 attempt to
retrieve the objects o_n ? Prove it!”

Theoretical Machine Success on Infinite FBT!

Theorem: $\forall q \in \mathcal{CC}, \mathfrak{M}$ can correctly answer and justify q .
I.e., \mathfrak{M} can pass FBT_ω .

Ok, so this logic machine exists in the *mathematical* universe; but does there exist an *implemented* machine with this power?

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Ok, so this logic machine exists in the *mathematical* universe; but does there exist an *implemented* machine with this power?

Simulation Courtesy of ...

ShadowProver!



Level 1

```
:name "Level 1: False Belief Task "  
  
:description "Agent a1 puts an object o into b1 in plain view of a2.  
Agent a2 then leaves, and in the absence of a2, a1 moves o  
from b1 into b2 ; this movement isn't perceived by a2 . Agent  
a2 now returns, and a is asked by the experimenter e: "If a2  
desires to retrieve o, which box will a2 look in?" If younger  
than four or five, a will reply "In b " (which of course fails 2  
the task); after this age subjects respond with the correct "In b1."  
  
Level1 Belief: a1 believes a2 believes o is in b1."  
  
:date "Monday July 22, 2019"  
  
:assumptions {  
  :P1 (Perceives! a1 t1 (Perceives! a2 t1 (holds (In o b1) t1)))  
  
  :P2 (Believes! a1 t2 (Believes! a2 t2 (not (exists [?e] (terminates ?e (In o b1))))))  
  
  :P3 (holds (In o b1) t1)  
  
  :C1 (Common! t0 (forall [?f ?t2 ?t2]  
    (if (and (not (exists [?e] (terminates ?e ?f))) (holds ?f ?t1) (< ?t1 ?t2))  
      (holds ?f ?t2))))  
  
  :C2 (Common! t0 (and (< t1 t2) (< t2 t3) (< t1 t3)))  
}  
  
:goal (Believes! a1 t3 (Believes! a2 t3 (holds (In o b1) t3)))}
```

Level 2

```
{:name      "Level 2: False Belief Task "  
  
:description "Agent a1 puts an object o into b1 in plain view of a2.  
Agent a2 then leaves, and in the absence of a2, a1 moves o  
from b1 into b2 ; this movement isn't perceived by a2 . Agent  
a2 now returns, and a is asked by the experimenter e: "If a2  
desires to retrieve o, which box will a2 look in?" If younger  
than four or five, a will reply "In b " (which of course fails 2  
the task); after this age subjects respond with the correct "In b1."  
  
Level2 Belief: a2 believes a1 believes a2 believes o is in b1.  
"  
  
:date      "Monday July 22, 2019"  
  
:assumptions {  
  
  :P1 (Perceives! a2 t1 (Perceives! a1 t1 (Perceives! a2 t1 (holds (In o b1) t1))))  
  
  :P2 (Believes! a2 t2 (Believes! a1 t2 (Believes! a2 t2 (not (exists [?e] (terminates ?e (In o b1)))))))  
  
  :P3 (holds (In o b1) t1)  
  
  :C1 (Common! t0  
      (forall [?f ?t2 ?t2]  
            (if (and (not (exists [?e] (terminates ?e ?f))) (holds ?f ?t1) (< ?t1 ?t2))  
                (holds ?f ?t2))))  
  
  :C2 (Common! t0 (and (< t1 t2) (< t2 t3) (< t1 t3)))}  
  
:goal      (Believes! a2 t3 (Believes! a1 t3 (Believes! a2 t3 (holds (In o b1) t3))))}
```

Level 3

```
{:name "Level 3: False Belief Task "  
  
:description "Agent a1 puts an object o into b1 in plain view of a2.  
Agent a2 then leaves, and in the absence of a2, a1 moves o  
from b1 into b2 ; this movement isn't perceived by a2 . Agent  
a2 now returns, and a is asked by the experimenter e: "If a2  
desires to retrieve o, which box will a2 look in?" If younger  
than four or five, a will reply "In b " (which of course fails 2  
the task); after this age subjects respond with the correct "In b1."  
  
Level3 Belief: a2 believes a1 believes a2 believes o is in b1.  
"  
  
:date "Monday July 22, 2019"  
  
:assumptions {  
  
:P1 (Perceives! a1 t1 (Perceives! a2 t1 (Perceives! a1 t1 (Perceives! a2 t1 (holds (In o b1) t1))))  
:P2 (Believes! a1 t2 (Believes! a2 t2 (Believes! a1 t2 (Believes! a2 t2 (not (exists [?e] (terminates ?e (In o b1)))))))  
  
:P3 (holds (In o b1) t1)  
  
:C1 (Common! t0  
| (forall [?f ?t2 ?t2]  
| | (if (and (not (exists [?e] (terminates ?e ?f))) (holds ?f ?t1) (< ?t1 ?t2))  
| | (holds ?f ?t2))))  
  
:C2 (Common! t0 (and (< t1 t2) (< t2 t3) (< t1 t3)))}  
  
:goal (Believes! a1 t3 (Believes! a2 t3 (Believes! a1 t3 (Believes! a2 t3 (holds (In o b1) t3))))))}
```


Level 4

```
{:name      "Level 4: False Belief Task "
:description "Agent a1 puts an object o into b1 in plain view of a2.
Agent a2 then leaves, and in the absence of a2, a1 moves o
from b1 into b2 ; this movement isn't perceived by a2 . Agent
a2 now returns, and a is asked by the experimenter e: "If a2
desires to retrieve o, which box will a2 look in?" If younger
than four or five, a will reply "In b " (which of course fails 2
the task); after this age subjects respond with the correct "In b1."

Level4 Belief: a2 believes a1 believes a2 believes a1 believes a2 believes o is in b1.
"

:date      "Monday July 22, 2019"

:assumptions {

  :P1 (Perceives! a2 t1 (Perceives! a1 t1 (Perceives! a2 t1 (Perceives! a1 t1 (Perceives! a2 t1 (holds (In o b1) t1))))))
  :P2 (Believes! a2 t2 (Believes! a1 t2 (Believes! a2 t2 (Believes! a1 t2 (Believes! a2 t2 (not (exists [?e] (terminates ?e (In o b1))))))))))
  :P3 (holds (In o b1) t1)

  :C1 (Common! t0
      (forall [?f ?t2 ?t2]
        (if (and (not (exists [?e] (terminates ?e ?f))) (holds ?f ?t1) (< ?t1 ?t2))
            (holds ?f ?t2))))
  :C2 (Common! t0 (and (< t1 t2) (< t2 t3) (< t1 t3)))}

:goal      (Believes! a2 t3 (Believes! a1 t3 (Believes! a2 t3 (Believes! a1 t3 (Believes! a2 t3 (holds (In o b1) t3))))))})
```

Level 5

```
{:name "Level 5: False Belief Task "

:description "Agent a1 puts an object o into b1 in plain view of a2.
Agent a2 then leaves, and in the absence of a2, a1 moves o
from b1 into b2 ; this movement isn't perceived by a2 . Agent
a2 now returns, and a is asked by the experimenter e: "If a2
desires to retrieve o, which box will a2 look in?" If younger
than four or five, a will reply "In b " (which of course fails 2
the task); after this age subjects respond with the correct "In b1."

Level5 Belief: a1 believes a2 believes a1 believes a2 believes a1 believes a2 believes o is in b1.
"

:date "Monday July 22, 2019"

:assumptions {

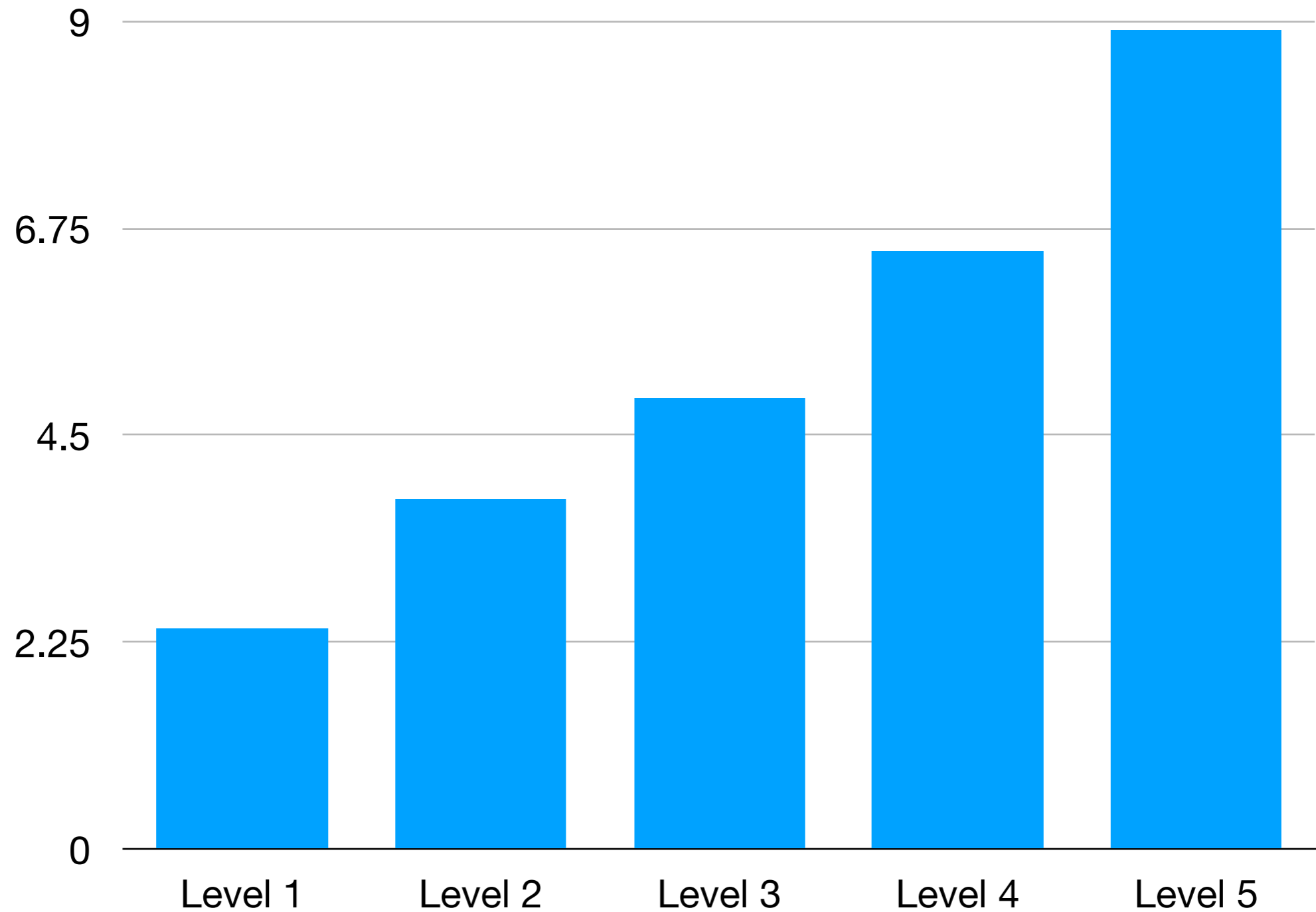
:P1 (Perceives! a1 t1 (Perceives! a2 t1 (Perceives! a1 t1 (Perceives! a2 t1 (Perceives! a1 t1 (Perceives! a2 t1 (holds (In o b1) t1))))))
:P2 (Believes! a1 t2 (Believes! a2 t2 (Believes! a1 t2 (Believes! a2 t2 (Believes! a1 t2 (Believes! a2 t2 (not (exists [?e] (terminates ?e (In o b1))))))))))
:P3 (holds (In o b1) t1)

:C1 (Common! t0
      (forall [?f ?t2 ?t2]
              (if (and (not (exists [?e] (terminates ?e ?f))) (holds ?f ?t1) (< ?t1 ?t2))
                  (holds ?f ?t2))))

:C2 (Common! t0 (and (< t1 t2) (< t2 t3) (< t1 t3)))}

:goal (Believes! a1 t3 (Believes! a2 t3 (Believes! a1 t3 (Believes! a2 t3 (Believes! a1 t3 (Believes! a2 t3 (holds (In o b1) t3))))))})
```


Time (in seconds) to Prove



Simulation of Level 5 in Real Time

```
/Library/Java/JavaVirtualMachines/jdk1.8.0_131.jdk/Contents/Home/bin/java ...
```

```
objc[16653]: Class JavaLaunchHelper is implemented in both /Library/Java/JavaVirtualMachines/jdk1.8.0_131.jdk/Contents/Home/bin/java (0x102a2d4c0) and /Library/Java/JavaVirtualMachines/jdk1.8.0_131.jdk/Contents/Home/jre/lib/libinstrument.dylib (0x102ab94e0)
```

```
----- Level 5 -----
```

Simulation of Level 5 in Real Time

```
/Library/Java/JavaVirtualMachines/jdk1.8.0_131.jdk/Contents/Home/bin/java ...
```

```
objc[16653]: Class JavaLaunchHelper is implemented in both /Library/Java/JavaVirtualMachines/jdk1.8.0_131.jdk/Contents/Home/bin/java (0x102a2d4c0) and /Library/Java/JavaVirtualMachines/jdk1.8.0_131.jdk/Contents/Home/jre/lib/libinstrument.dylib (0x102ab94e0)
```

```
----- Level 5 -----
```

Encapsulation

Slate - K.slt

K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ K \vdash ✓ ∞ \Box	T. $\Box\varphi \rightarrow \varphi$ K \vdash ✗ ∞ \Box	4. $\Box\varphi \rightarrow \Box\Box\varphi$ K \vdash ✗ ∞ \Box	5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ K \vdash ✗ ∞ \Box
--------------------------------------------------------------------------------------------------------------------	----------------------------------------------------------------------	------------------------------------------------------------------------------	--------------------------------------------------------------------------------------

Encapsulation

The image shows two screenshots of a Slate editor window, illustrating the concept of encapsulation in modal logic. The top window is titled "Slate - K.slt" and the bottom window is titled "Slate - T.slt".

Slate - K.slt

- K.** $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$
 $K \vdash \checkmark \infty \Box$
- T.** $\Box\varphi \rightarrow \varphi$
 $K \vdash \times \infty \Box$
- 4.** $\Box\varphi \rightarrow \Box\Box\varphi$
 $K \vdash \times \infty \Box$
- 5.** $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$
 $K \vdash \times \infty \Box$

Slate - T.slt

- K.** $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$
 $M \vdash \checkmark \infty \Box$
- T.** $\Box\varphi \rightarrow \varphi$
 $M \vdash \checkmark \infty \Box$
- 4.** $\Box\varphi \rightarrow \Box\Box\varphi$
 $M \vdash \times \infty \Box$
- 5.** $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$
 $M \vdash \times \infty \Box$

Encapsulation

The image displays three overlapping windows from the Slate application, each showing a set of modal logic formulas and their derivability in various systems. The windows are titled "Slate - K.slt", "Slate - T.slt", and "Slate - D.slt".

Slate - K.slt

- K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$
K $\vdash \checkmark \infty \Box$
- T. $\Box\varphi \rightarrow \varphi$
K $\vdash \times \infty \Box$
- 4. $\Box\varphi \rightarrow \Box\Box\varphi$
K $\vdash \times \infty \Box$
- 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$
K $\vdash \times \infty \Box$

Slate - T.slt

- K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$
M $\vdash \checkmark \infty \Box$
- T. $\Box\varphi \rightarrow \varphi$
M $\vdash \checkmark \infty \Box$
- 4. $\Box\varphi \rightarrow \Box\Box\varphi$
M $\vdash \times \infty \Box$
- 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$
M $\vdash \times \infty \Box$

Slate - D.slt

- K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$
D $\vdash \checkmark \infty \Box$
- T. $\Box\varphi \rightarrow \varphi$
D $\vdash \times \infty \Box$
- D. $\Box\varphi \rightarrow \Diamond\varphi$
D $\vdash \checkmark \infty \Box$
- 4. $\Box\varphi \rightarrow \Box\Box\varphi$
D $\vdash \times \infty \Box$
- 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$
D $\vdash \times \infty \Box$
- INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$
D $\vdash \checkmark \infty \Box$

Encapsulation

The image displays four overlapping Slate windows, each showing a set of modal logic formulas and their derivability status in a specific system. The windows are titled 'Slate - K.slt', 'Slate - T.slt', 'Slate - D.slt', and 'Slate - S4.slt'.

Slate - K.slt

- K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $K \vdash \checkmark \infty \Box$
- T. $\Box\varphi \rightarrow \varphi$ $K \vdash \times \infty \Box$
- 4. $\Box\varphi \rightarrow \Box\Box\varphi$ $K \vdash \times \infty \Box$
- 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $K \vdash \times \infty \Box$

Slate - T.slt

- K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $M \vdash \checkmark \infty \Box$
- T. $\Box\varphi \rightarrow \varphi$ $M \vdash \checkmark \infty \Box$
- 4. $\Box\varphi \rightarrow \Box\Box\varphi$ $M \vdash \times \infty \Box$
- 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $M \vdash \times \infty \Box$

Slate - D.slt

- K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $D \vdash \checkmark \infty \Box$
- T. $\Box\varphi \rightarrow \varphi$ $D \vdash \times \infty \Box$
- D. $\Box\varphi \rightarrow \Diamond\varphi$ $D \vdash \checkmark \infty \Box$
- 4. $\Box\varphi \rightarrow \Box\Box\varphi$ $D \vdash \times \infty \Box$
- 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $D \vdash \times \infty \Box$
- INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$ $D \vdash \checkmark \infty \Box$

Slate - S4.slt

- K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $S4 \vdash \checkmark \infty \Box$
- T. $\Box\varphi \rightarrow \varphi$ $S4 \vdash \checkmark \infty \Box$
- D. $\Box\varphi \rightarrow \Diamond\varphi$ $S4 \vdash \checkmark \infty \Box$
- 4. $\Box\varphi \rightarrow \Box\Box\varphi$ $S4 \vdash \checkmark \infty \Box$
- 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $S4 \vdash \times \infty \Box$
- INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$ $\{INTER\} \text{ Assume } \checkmark$

Encapsulation

K

T

D

4 = S4

5 = S5

The image shows five Slate windows, each representing a different modal logic system. Each window contains several boxes with a formula and its status in that system (derivable or not, with a checkmark or an 'X').

- Slate - K.slt:**
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ (K $\vdash \checkmark \infty \Box$)
 - T. $\Box\varphi \rightarrow \varphi$ (K $\vdash \times \infty \Box$)
 - 4. $\Box\varphi \rightarrow \Box\Box\varphi$ (K $\vdash \times \infty \Box$)
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ (K $\vdash \times \infty \Box$)
- Slate - T.slt:**
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ (M $\vdash \checkmark \infty \Box$)
 - T. $\Box\varphi \rightarrow \varphi$ (M $\vdash \checkmark \infty \Box$)
 - 4. $\Box\varphi \rightarrow \Box\Box\varphi$ (M $\vdash \times \infty \Box$)
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ (M $\vdash \times \infty \Box$)
- Slate - D.slt:**
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ (D $\vdash \checkmark \infty \Box$)
 - T. $\Box\varphi \rightarrow \varphi$ (D $\vdash \times \infty \Box$)
 - D. $\Box\varphi \rightarrow \Diamond\varphi$ (D $\vdash \checkmark \infty \Box$)
 - 4. $\Box\varphi \rightarrow \Box\Box\varphi$ (D $\vdash \times \infty \Box$)
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ (D $\vdash \times \infty \Box$)
 - INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$ (D $\vdash \checkmark \infty \Box$)
- Slate - S4.slt:**
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ (S4 $\vdash \checkmark \infty \Box$)
 - T. $\Box\varphi \rightarrow \varphi$ (S4 $\vdash \checkmark \infty \Box$)
 - D. $\Box\varphi \rightarrow \Diamond\varphi$ (S4 $\vdash \checkmark \infty \Box$)
 - 4. $\Box\varphi \rightarrow \Box\Box\varphi$ (S4 $\vdash \checkmark \infty \Box$)
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ (S4 $\vdash \times \infty \Box$)
 - INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$ ({INTER} Assume \checkmark)
- Slate - S5.slt:**
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ (S5 $\vdash \checkmark \infty \Box$)
 - T. $\Box\varphi \rightarrow \varphi$ (S5 $\vdash \checkmark \infty \Box$)
 - D. $\Box\varphi \rightarrow \Diamond\varphi$ ({D} Assume \checkmark)
 - 4. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ ({4} Assume \checkmark)
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ (S5 $\vdash \checkmark \infty \Box$)
 - INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$ ({INTER} Assume \checkmark)

Encapsulation

K

T

D

4 = S4

5 = S5

The image shows five overlapping Slate windows, each displaying a grid of modal logic formulas and their derivability in a specific system. The windows are titled 'Slate - K.slt', 'Slate - T.slt', 'Slate - D.slt', 'Slate - S4.slt', and 'Slate - S5.slt'. The 'Slate - D.slt' window is highlighted with a red border.

Slate - K.slt

- K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $K \vdash \checkmark \infty \Box$
- T. $\Box\varphi \rightarrow \varphi$ $K \vdash \times \infty \Box$
- 4. $\Box\varphi \rightarrow \Box\Box\varphi$ $K \vdash \times \infty \Box$
- 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $K \vdash \times \infty \Box$

Slate - T.slt

- K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $M \vdash \checkmark \infty \Box$
- T. $\Box\varphi \rightarrow \varphi$ $M \vdash \checkmark \infty \Box$
- 4. $\Box\varphi \rightarrow \Box\Box\varphi$ $M \vdash \times \infty \Box$
- 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $M \vdash \times \infty \Box$

Slate - D.slt (highlighted)

- K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $D \vdash \checkmark \infty \Box$
- T. $\Box\varphi \rightarrow \varphi$ $D \vdash \times \infty \Box$
- D. $\Box\varphi \rightarrow \Diamond\varphi$ $D \vdash \checkmark \infty \Box$
- 4. $\Box\varphi \rightarrow \Box\Box\varphi$ $D \vdash \times \infty \Box$
- 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $D \vdash \times \infty \Box$
- INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$ $D \vdash \checkmark \infty \Box$

Slate - S4.slt

- K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $S4 \vdash \checkmark \infty \Box$
- T. $\Box\varphi \rightarrow \varphi$ $S4 \vdash \checkmark \infty \Box$
- D. $\Box\varphi \rightarrow \Diamond\varphi$ $S4 \vdash \checkmark \infty \Box$
- 4. $\Box\varphi \rightarrow \Box\Box\varphi$ $S4 \vdash \checkmark \infty \Box$
- 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $S4 \vdash \times \infty \Box$
- INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$ $\{INTER\} \text{ Assume } \checkmark$

Slate - S5.slt

- K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $S5 \vdash \checkmark \infty \Box$
- T. $\Box\varphi \rightarrow \varphi$ $S5 \vdash \checkmark \infty \Box$
- D. $\Box\varphi \rightarrow \Diamond\varphi$ $\{D\} \text{ Assume } \checkmark$
- 4. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $\{4\} \text{ Assume } \checkmark$
- 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $S5 \vdash \checkmark \infty \Box$
- INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$ $\{INTER\} \text{ Assume } \checkmark$

Encapsulation

K

T

D

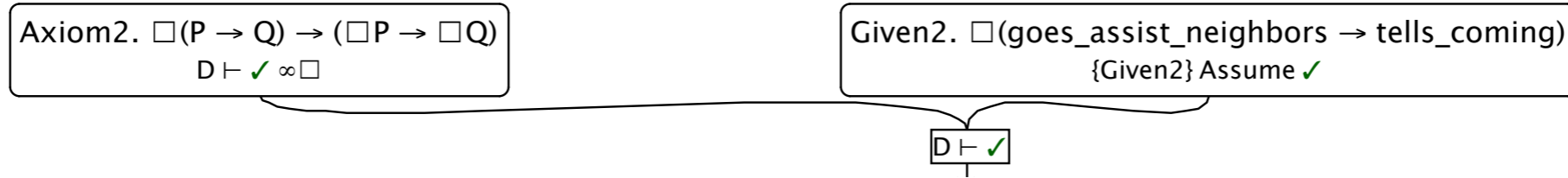
The screenshot displays five windows of the HyperSlate interface, each showing logical formulas and their status in different modal logics. The windows are titled as follows:

- Slate - K.slt**: Shows formulas K, T, 4, and 5. K is derivable (K ⊢ ✓ ∞ □), while T, 4, and 5 are not (K ⊢ ✗ ∞ □).
- Slate - T.slt**: Shows formulas K, T, 4, and 5. K and T are derivable (M ⊢ ✓ ∞ □), while 4 and 5 are not (M ⊢ ✗ ∞ □).
- Slate - D.slt** (highlighted with a red border): Shows formulas K, T, D, 4, 5, and INTER. K, T, and D are derivable (D ⊢ ✓ ∞ □), while 4 and 5 are not (D ⊢ ✗ ∞ □). The INTER formula is derivable (D ⊢ ✓ ∞ □).
- Slate - S4.slt**: Shows formulas K, T, D, 4, 5, and INTER. All formulas are derivable (S4 ⊢ ✓ ∞ □). The INTER formula is marked with "{INTER} Assume ✓".
- Slate - S5.slt**: Shows formulas K, T, D, 4, 5, and INTER. All formulas are derivable (S5 ⊢ ✓ ∞ □). The D and 4 formulas are marked with "{D} Assume ✓" and "{4} Assume ✓" respectively. The INTER formula is marked with "{INTER} Assume ✓".

4 = S4

5 = S5

Chisholm's Paradox

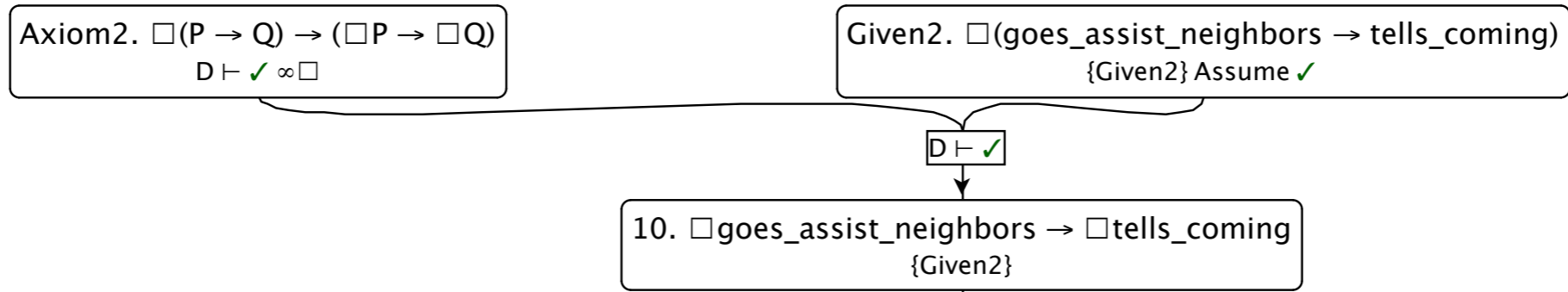


Axiom4. "Modus ponens for provability."
{Axiom4} Assume \checkmark

Axiom5. "Theorems are obligatory."
{Axiom5} Assume \checkmark

Axiom1. "All theorems of the propositional calculus."
{Axiom1} Assume \checkmark

Chisholm's Paradox

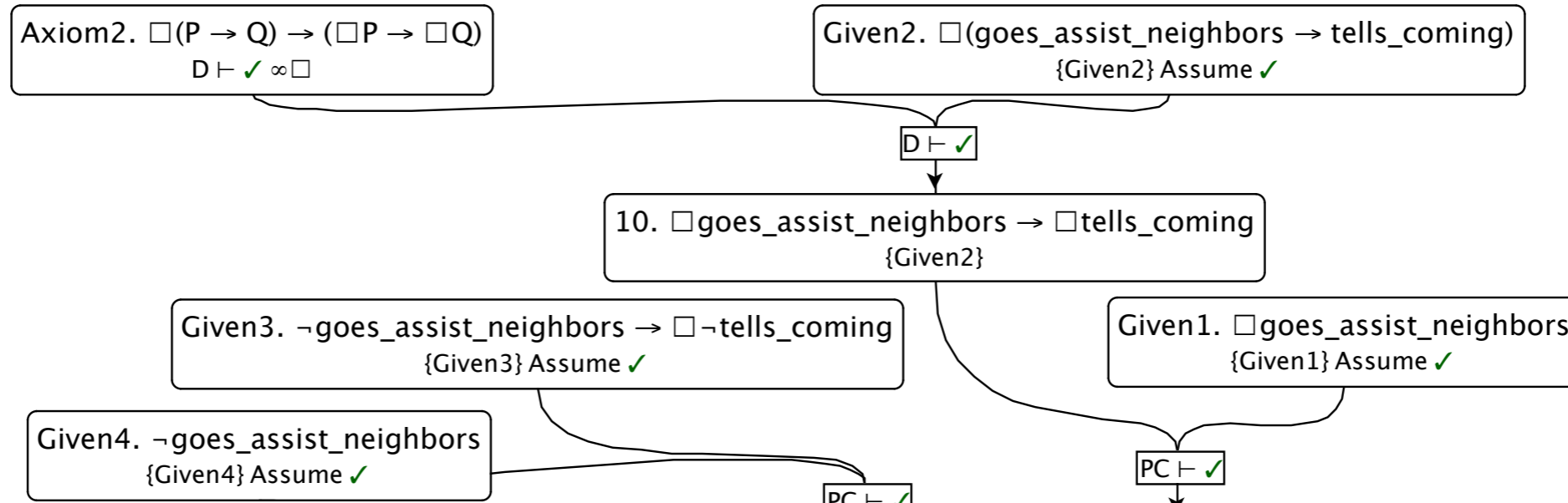


Axiom4. "Modus ponens for provability."
{Axiom4} Assume \checkmark

Axiom5. "Theorems are obligatory."
{Axiom5} Assume \checkmark

Axiom1. "All theorems of the propositional calculus."
{Axiom1} Assume \checkmark

Chisholm's Paradox

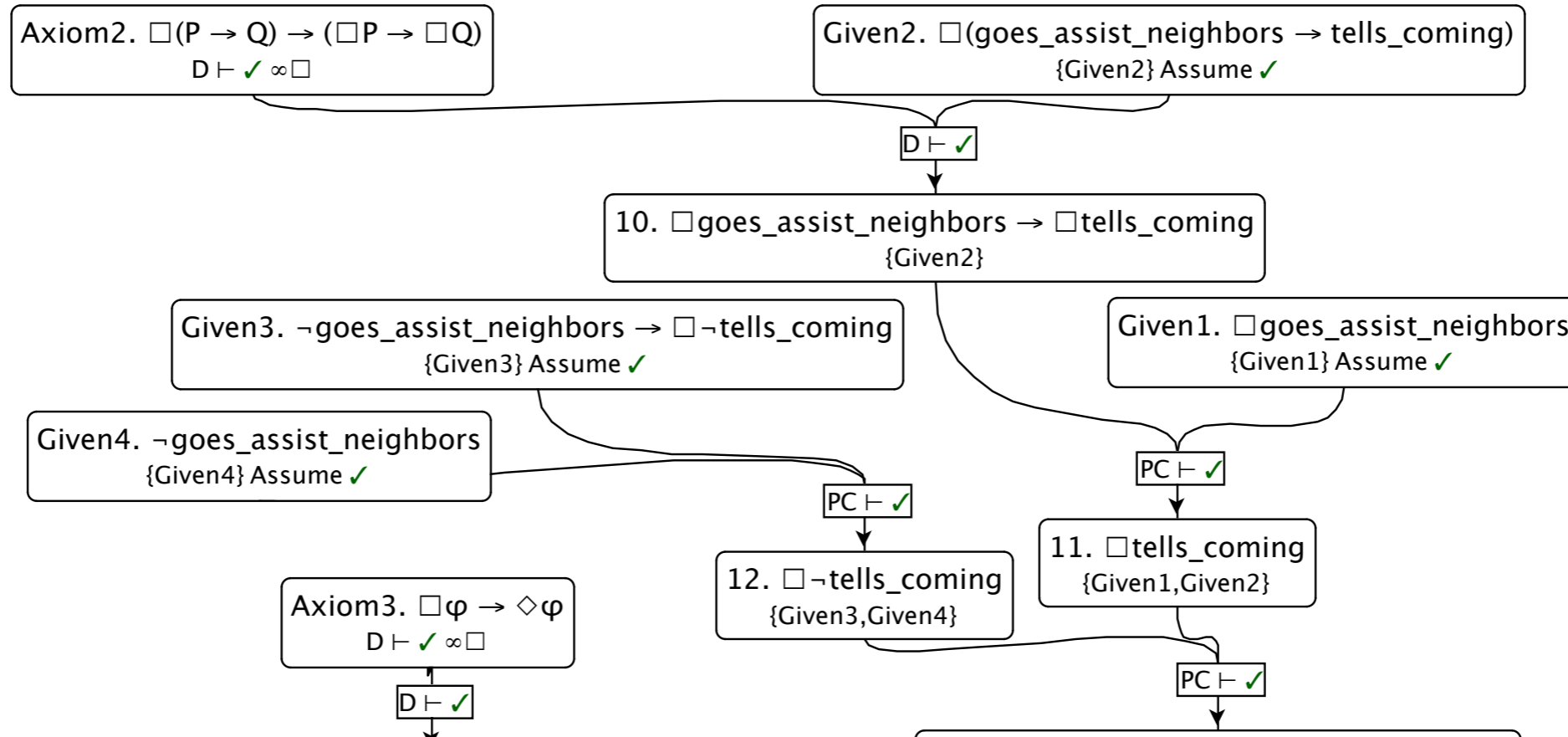


Axiom4. "Modus ponens for provability."
 {Axiom4} Assume ✓

Axiom5. "Theorems are obligatory."
 {Axiom5} Assume ✓

Axiom1. "All theorems of the propositional calculus."
 {Axiom1} Assume ✓

Chisholm's Paradox

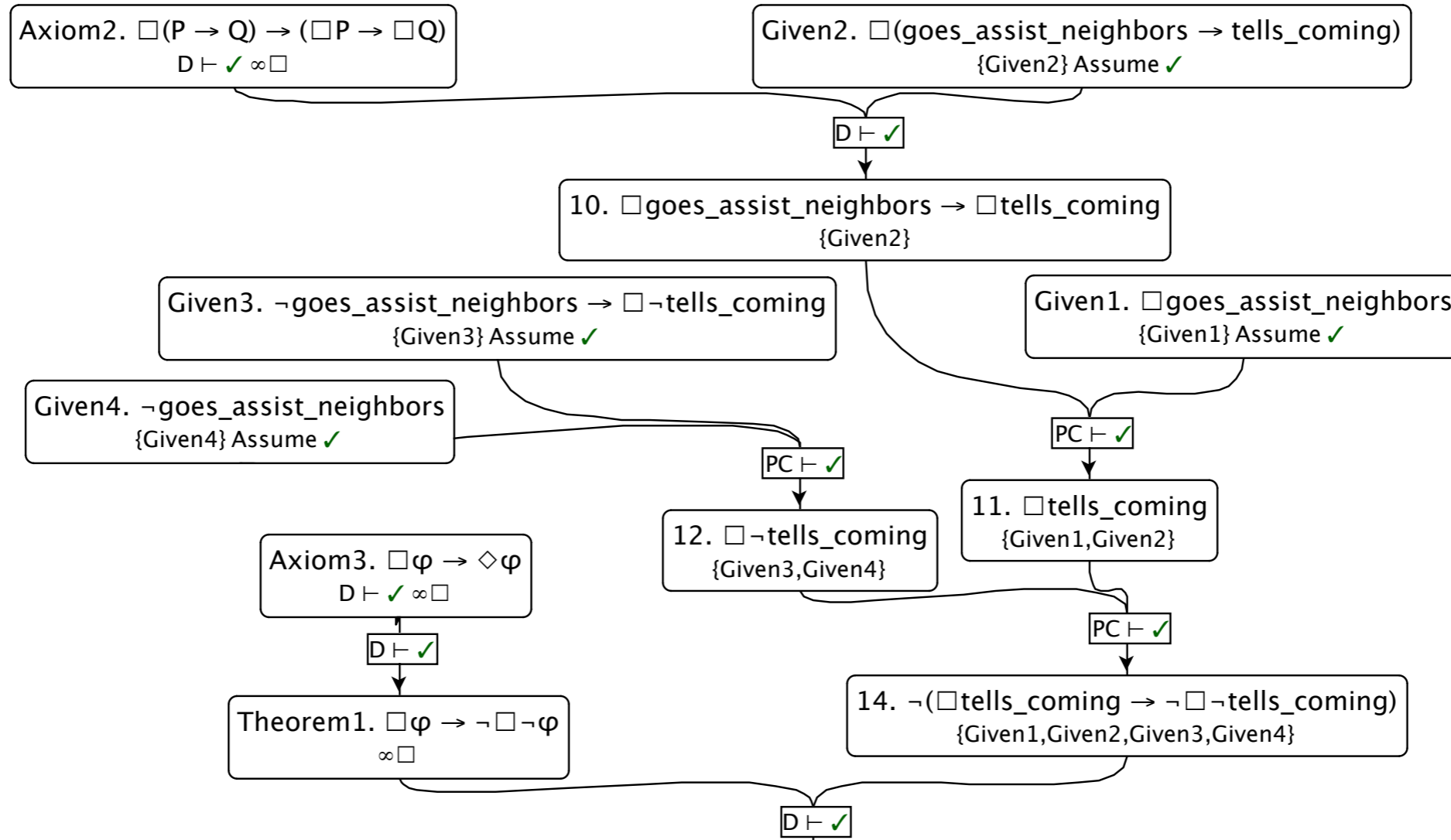


Axiom4. "Modus ponens for provability."
 $\{\text{Axiom4}\} \text{Assume } \checkmark$

Axiom5. "Theorems are obligatory."
 $\{\text{Axiom5}\} \text{Assume } \checkmark$

Axiom1. "All theorems of the propositional calculus."
 $\{\text{Axiom1}\} \text{Assume } \checkmark$

Chisholm's Paradox

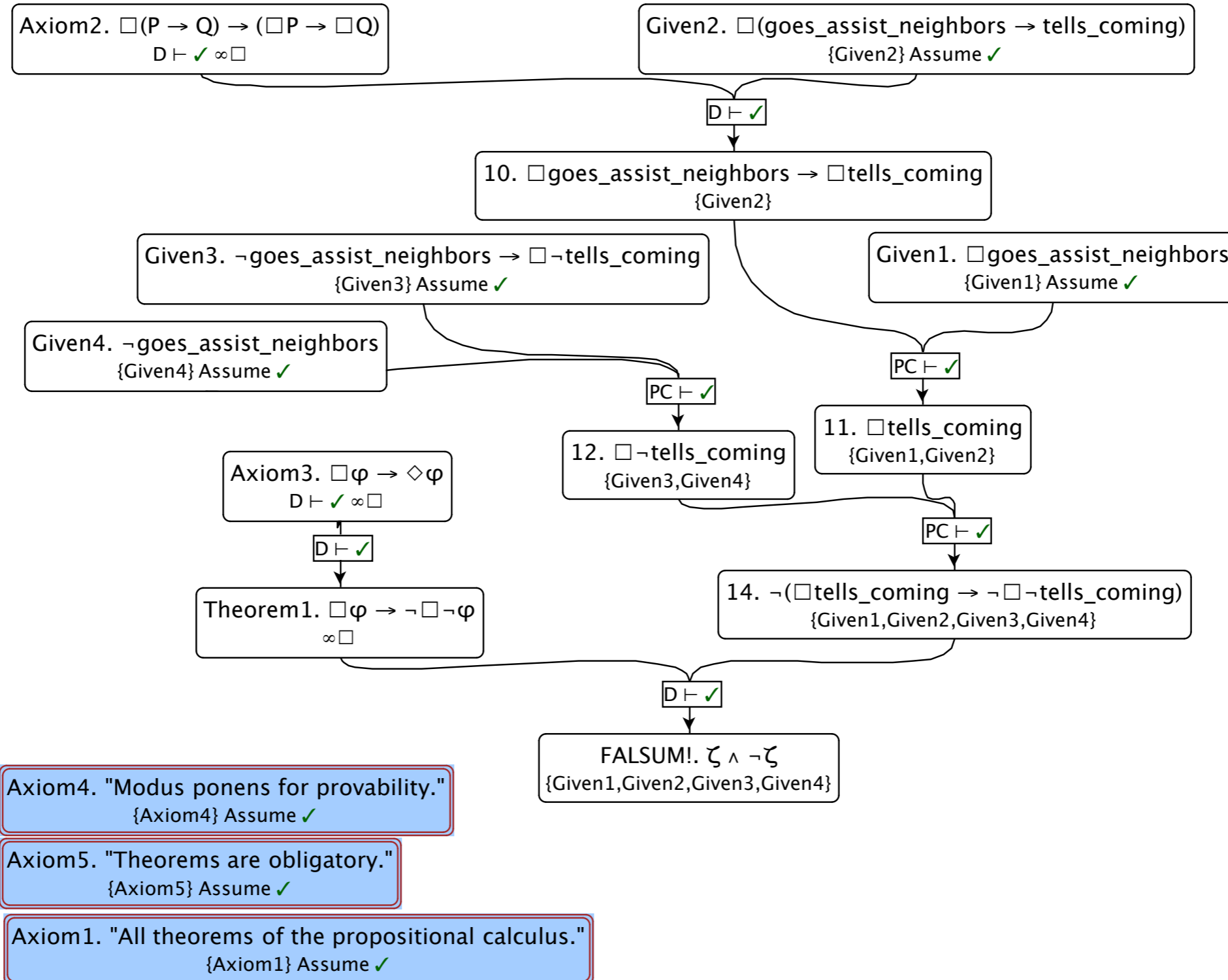


Axiom4. "Modus ponens for provability."
 $\{\text{Axiom4}\} \text{ Assume } \checkmark$

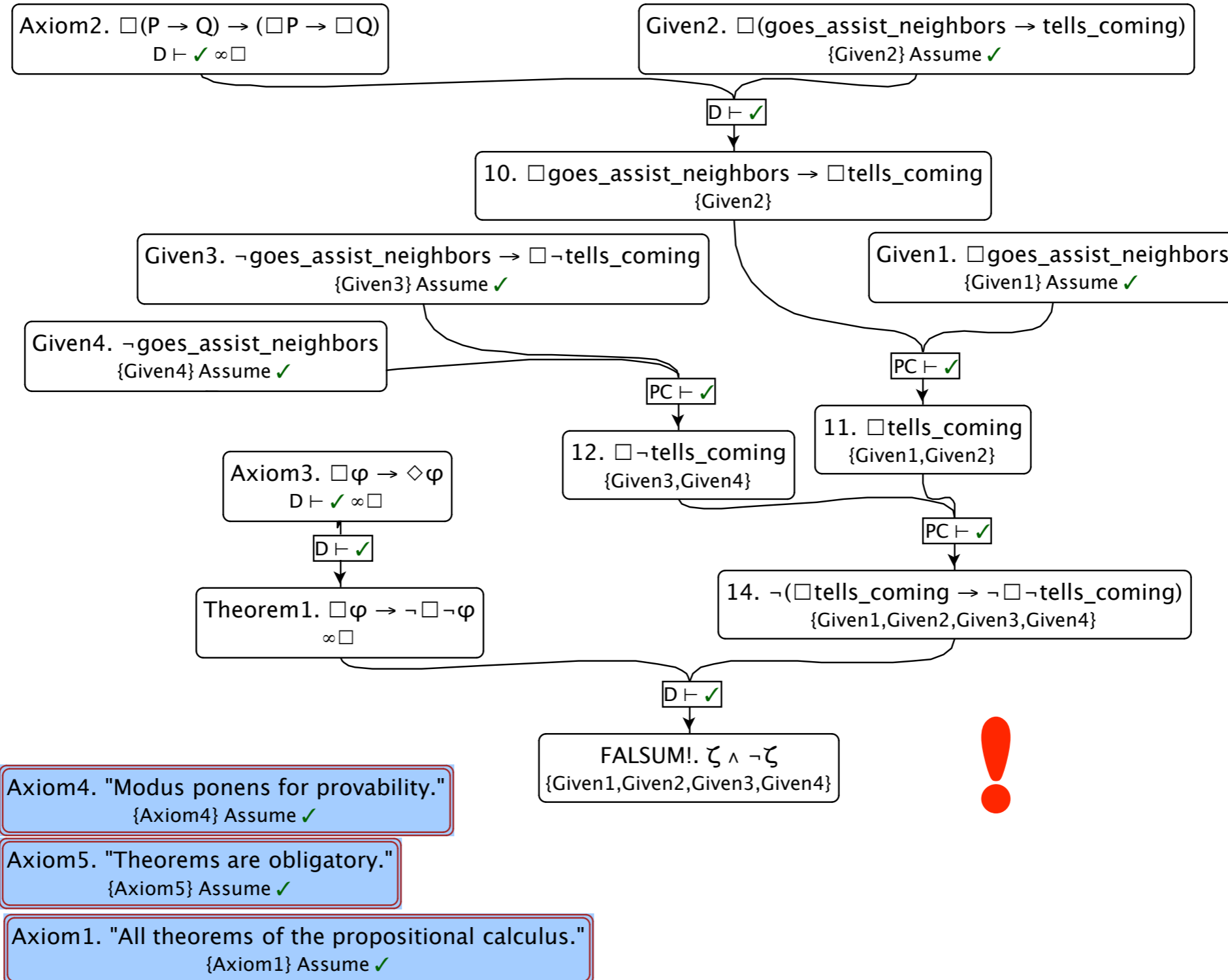
Axiom5. "Theorems are obligatory."
 $\{\text{Axiom5}\} \text{ Assume } \checkmark$

Axiom1. "All theorems of the propositional calculus."
 $\{\text{Axiom1}\} \text{ Assume } \checkmark$

Chisholm's Paradox



Chisholm's Paradox



Review: Encapsulation

K

T

D

4 = S4

5 = S5

The screenshot displays five windows of the HyperSlate interface, each showing a set of modal logic formulas and their derivability in different systems. The systems are K, T, D, 4, 5, and INTER. Checkmarks (✓) indicate derivability, while red X's (✗) indicate non-derivability.

- Slate - K.slt:**
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ (K \vdash ✓ $\infty\Box$)
 - T. $\Box\varphi \rightarrow \varphi$ (K \vdash ✗ $\infty\Box$)
 - 4. $\Box\varphi \rightarrow \Box\Box\varphi$ (K \vdash ✗ $\infty\Box$)
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ (K \vdash ✗ $\infty\Box$)
- Slate - T.slt:**
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ (M \vdash ✓ $\infty\Box$)
 - T. $\Box\varphi \rightarrow \varphi$ (M \vdash ✓ $\infty\Box$)
 - 4. $\Box\varphi \rightarrow \Box\Box\varphi$ (M \vdash ✗ $\infty\Box$)
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ (M \vdash ✗ $\infty\Box$)
- Slate - D.slt:**
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ (D \vdash ✓ $\infty\Box$)
 - T. $\Box\varphi \rightarrow \varphi$ (D \vdash ✗ $\infty\Box$)
 - D. $\Box\varphi \rightarrow \Diamond\varphi$ (D \vdash ✓ $\infty\Box$)
 - 4. $\Box\varphi \rightarrow \Box\Box\varphi$ (D \vdash ✗ $\infty\Box$)
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ (D \vdash ✗ $\infty\Box$)
 - INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$ (D \vdash ✓ $\infty\Box$)
- Slate - S4.slt:**
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ (S4 \vdash ✓ $\infty\Box$)
 - T. $\Box\varphi \rightarrow \varphi$ (S4 \vdash ✓ $\infty\Box$)
 - D. $\Box\varphi \rightarrow \Diamond\varphi$ (S4 \vdash ✓ $\infty\Box$)
 - 4. $\Box\varphi \rightarrow \Box\Box\varphi$ (S4 \vdash ✓ $\infty\Box$)
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ (S4 \vdash ✗ $\infty\Box$)
 - INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$ ({INTER} Assume ✓)
- Slate - S5.slt:**
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ (S5 \vdash ✓ $\infty\Box$)
 - T. $\Box\varphi \rightarrow \varphi$ (S5 \vdash ✓ $\infty\Box$)
 - D. $\Box\varphi \rightarrow \Diamond\varphi$ ({D} Assume ✓)
 - 4. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ ({4} Assume ✓)
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ (S5 \vdash ✓ $\infty\Box$)
 - INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$ ({INTER} Assume ✓)

Review: Encapsulation

K

T

D

The screenshot displays five windows of the HyperSlate interface, each showing logical formulas and their provability status in different modal logics. The windows are titled "Slate - K.slt", "Slate - T.slt", "Slate - D.slt", "Slate - S4.slt", and "Slate - S5.slt".

Slate - K.slt

- K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $K \vdash \checkmark \infty \Box$
- T. $\Box\varphi \rightarrow \varphi$ $K \vdash \times \infty \Box$
- 4. $\Box\varphi \rightarrow \Box\Box\varphi$ $K \vdash \times \infty \Box$
- 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $K \vdash \times \infty \Box$

Slate - T.slt

- K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $M \vdash \checkmark \infty \Box$
- T. $\Box\varphi \rightarrow \varphi$ $M \vdash \checkmark \infty \Box$
- 4. $\Box\varphi \rightarrow \Box\Box\varphi$ $M \vdash \times \infty \Box$
- 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $M \vdash \times \infty \Box$

Slate - D.slt

- K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $D \vdash \checkmark \infty \Box$
- T. $\Box\varphi \rightarrow \varphi$ $D \vdash \times \infty \Box$
- D. $\Box\varphi \rightarrow \Diamond\varphi$ $D \vdash \checkmark \infty \Box$
- 4. $\Box\varphi \rightarrow \Box\Box\varphi$ $D \vdash \times \infty \Box$
- 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $D \vdash \times \infty \Box$
- INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$ $D \vdash \checkmark \infty \Box$

Slate - S4.slt

- K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $S4 \vdash \checkmark \infty \Box$
- T. $\Box\varphi \rightarrow \varphi$ $S4 \vdash \checkmark \infty \Box$
- D. $\Box\varphi \rightarrow \Diamond\varphi$ $S4 \vdash \checkmark \infty \Box$
- 4. $\Box\varphi \rightarrow \Box\Box\varphi$ $S4 \vdash \checkmark \infty \Box$
- 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $S4 \vdash \times \infty \Box$
- INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$ $\{INTER\} \text{ Assume } \checkmark$

Slate - S5.slt

- K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $S5 \vdash \checkmark \infty \Box$
- T. $\Box\varphi \rightarrow \varphi$ $S5 \vdash \checkmark \infty \Box$
- D. $\Box\varphi \rightarrow \Diamond\varphi$ $\{D\} \text{ Assume } \checkmark$
- 4. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $\{4\} \text{ Assume } \checkmark$
- 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $S5 \vdash \checkmark \infty \Box$
- INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$ $\{INTER\} \text{ Assume } \checkmark$

4 = S4

5 = S5

Review: Encapsulation

K

T

D

4 = S4

5 = S5

The screenshot displays the HyperSlate interface with several windows showing logic calculi configurations and their corresponding modal logic formulas. A green box highlights the configuration menu, and a red box highlights the S4 and S5 calculi windows.

Configuration Menu (Green Box):

- Propositional Calculus
- L_0 = Pure Predicate Calculus
- L_1 = First-order Logic
- L_2 = Second-order Logic
- K
- T
- D
- S4
- S5
- DCEC (fragment)
- Hyperlog

Slate - K.slt

- K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $K \vdash \checkmark \infty \Box$
- T. $\Box\varphi \rightarrow \varphi$ $K \vdash \times \infty \Box$
- 4. $\Box\varphi \rightarrow \Box\Box\varphi$ $K \vdash \times \infty \Box$
- 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $K \vdash \times \infty \Box$

Slate - T.slt

- K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $M \vdash \checkmark \infty \Box$
- T. $\Box\varphi \rightarrow \varphi$ $M \vdash \checkmark \infty \Box$
- 4. $\Box\varphi \rightarrow \Box\Box\varphi$ $M \vdash \times \infty \Box$
- 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $M \vdash \times \infty \Box$

Slate - S4.slt

- K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $S4 \vdash \checkmark \infty \Box$
- T. $\Box\varphi \rightarrow \varphi$ $S4 \vdash \checkmark \infty \Box$
- D. $\Box\varphi \rightarrow \Diamond\varphi$ $S4 \vdash \checkmark \infty \Box$
- 4. $\Box\varphi \rightarrow \Box\Box\varphi$ $S4 \vdash \checkmark \infty \Box$
- 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $S4 \vdash \times \infty \Box$
- INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$ $\{INTER\} \text{ Assume } \checkmark$

Slate - S5.slt

- K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $S5 \vdash \checkmark \infty \Box$
- T. $\Box\varphi \rightarrow \varphi$ $S5 \vdash \checkmark \infty \Box$
- D. $\Box\varphi \rightarrow \Diamond\varphi$ $\{D\} \text{ Assume } \checkmark$
- 4. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $\{4\} \text{ Assume } \checkmark$
- 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $S5 \vdash \checkmark \infty \Box$
- INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$ $\{INTER\} \text{ Assume } \checkmark$

Review: Encapsulation

K

T

D

The screenshot displays the HyperSlate interface with several windows showing logic calculi configurations and theorem boxes. A green arrow points to the 'Create file' dialog box, which is highlighted with a green border. The dialog box contains the following options:

- Propositional Calculus
- L_0 = Pure Predicate Calculus
- L_1 = First-order Logic
- L_2 = Second-order Logic
- K
- T
- D
- S4
- S5
- DCEC (fragment)
- Hyperlog

The windows show the following configurations and theorem boxes:

- Slate - K.slt**: K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ (K $\vdash \checkmark \infty \Box$), T. $\Box\varphi \rightarrow \varphi$ (K $\vdash \times \infty \Box$), 4. $\Box\varphi \rightarrow \Box\Box\varphi$ (K $\vdash \times \infty \Box$), 5. $\Box\varphi \rightarrow \Box\neg\Box\varphi$ (K $\vdash \times \infty \Box$)
- Slate - T.slt**: K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ (M $\vdash \checkmark \infty \Box$), T. $\Box\varphi \rightarrow \varphi$ (M $\vdash \checkmark \infty \Box$), 4. $\Box\varphi \rightarrow \Box\Box\varphi$ (M $\vdash \times \infty \Box$), 5. $\Box\varphi \rightarrow \Box\neg\Box\varphi$ (M $\vdash \times \infty \Box$)
- Slate - S4.slt**: K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ (S4 $\vdash \checkmark \infty \Box$), T. $\Box\varphi \rightarrow \varphi$ (S4 $\vdash \checkmark \infty \Box$), D. $\Box\varphi \rightarrow \Diamond\varphi$ (S4 $\vdash \checkmark \infty \Box$), 4. $\Box\varphi \rightarrow \Box\Box\varphi$ (S4 $\vdash \checkmark \infty \Box$), 5. $\Box\varphi \rightarrow \Box\neg\Box\varphi$ (S4 $\vdash \times \infty \Box$), INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$ ({INTER} Assume \checkmark)
- Slate - S5.slt**: K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ (S5 $\vdash \checkmark \infty \Box$), T. $\Box\varphi \rightarrow \varphi$ (S5 $\vdash \checkmark \infty \Box$), D. $\Box\varphi \rightarrow \Diamond\varphi$ ({D} Assume \checkmark), 4. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ ({4} Assume \checkmark), 5. $\Box\varphi \rightarrow \Box\neg\Box\varphi$ (S5 $\vdash \checkmark \infty \Box$), INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$ ({INTER} Assume \checkmark)

4 = S4

5 = S5

*Det er en logikk for
hvert problem!*