Exhortation; Truth Trees; FOL IV: Layered Quantification and Measuring Intelligence Using This Phenomenon

Selmer Bringsjord

Rensselaer AI & Reasoning (RAIR) Lab
Department of Cognitive Science
Department of Computer Science
Lally School of Management & Technology
Rensselaer Polytechnic Institute (RPI)
Troy, New York 12180 USA

Intro to (Formal) Logic
2/28/2022

R A I R
Rensselaer AI and Reasoning Lab

Exhortation …

Make sure you're up-to-date-ish before Spring Break on HyperGrader's current (Required = Homework) Problems, due

Apr 12 11am NY time.

More FOL problems are forthcoming …

New Required Problem: `FregTHEN2`, with corresponding truth-tree Exercise

…

Truth Trees vs. Truth Tables

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Violent breakage between tabular calculation and proof construction.

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LAMA®'s hypergraphs/HyperLogic® achieves
seamless unification of proofs and trees, and provides
AI oracles for their construction and *certification*.

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First very simple: truth-tree for *modus ponens* …

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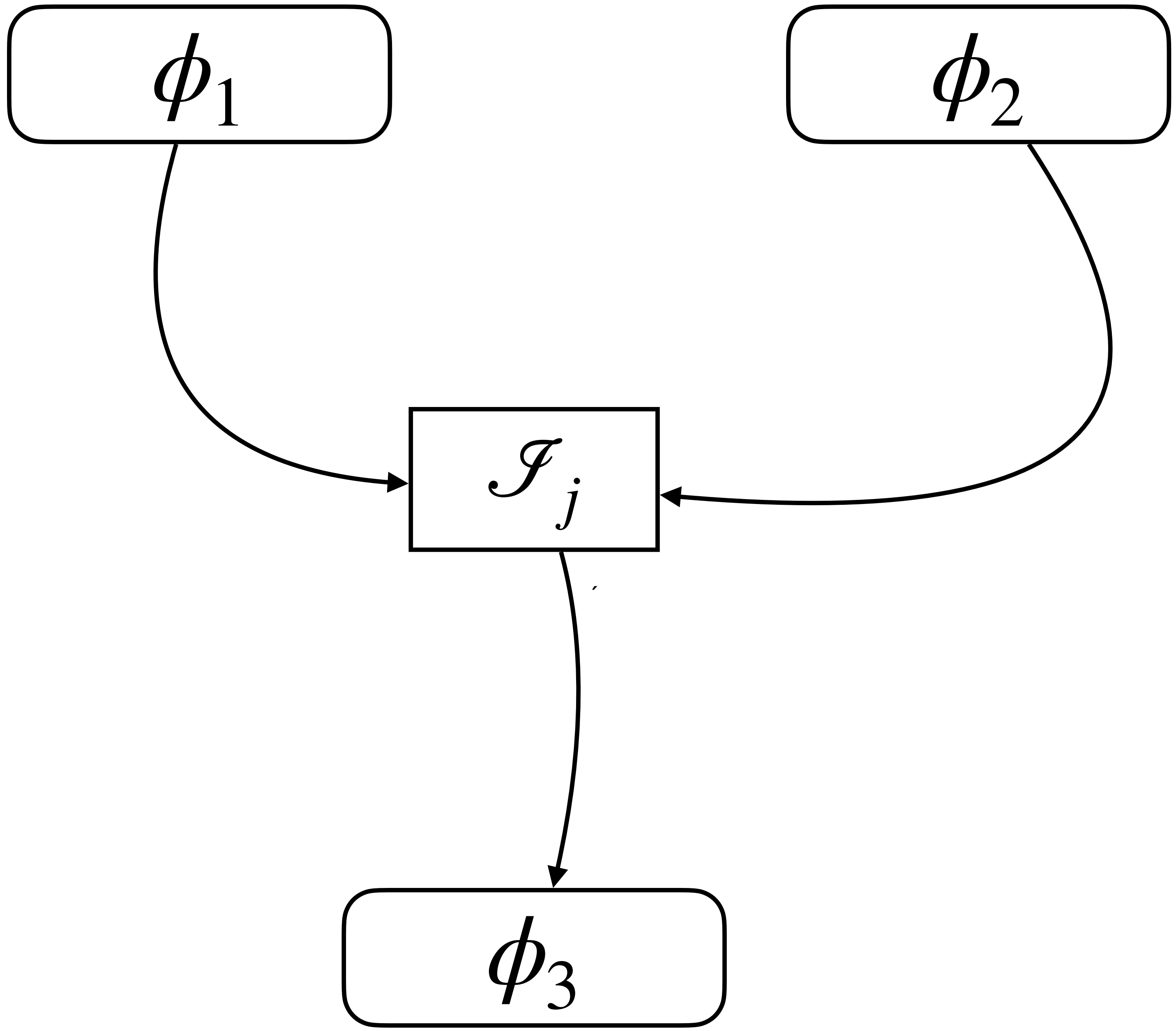
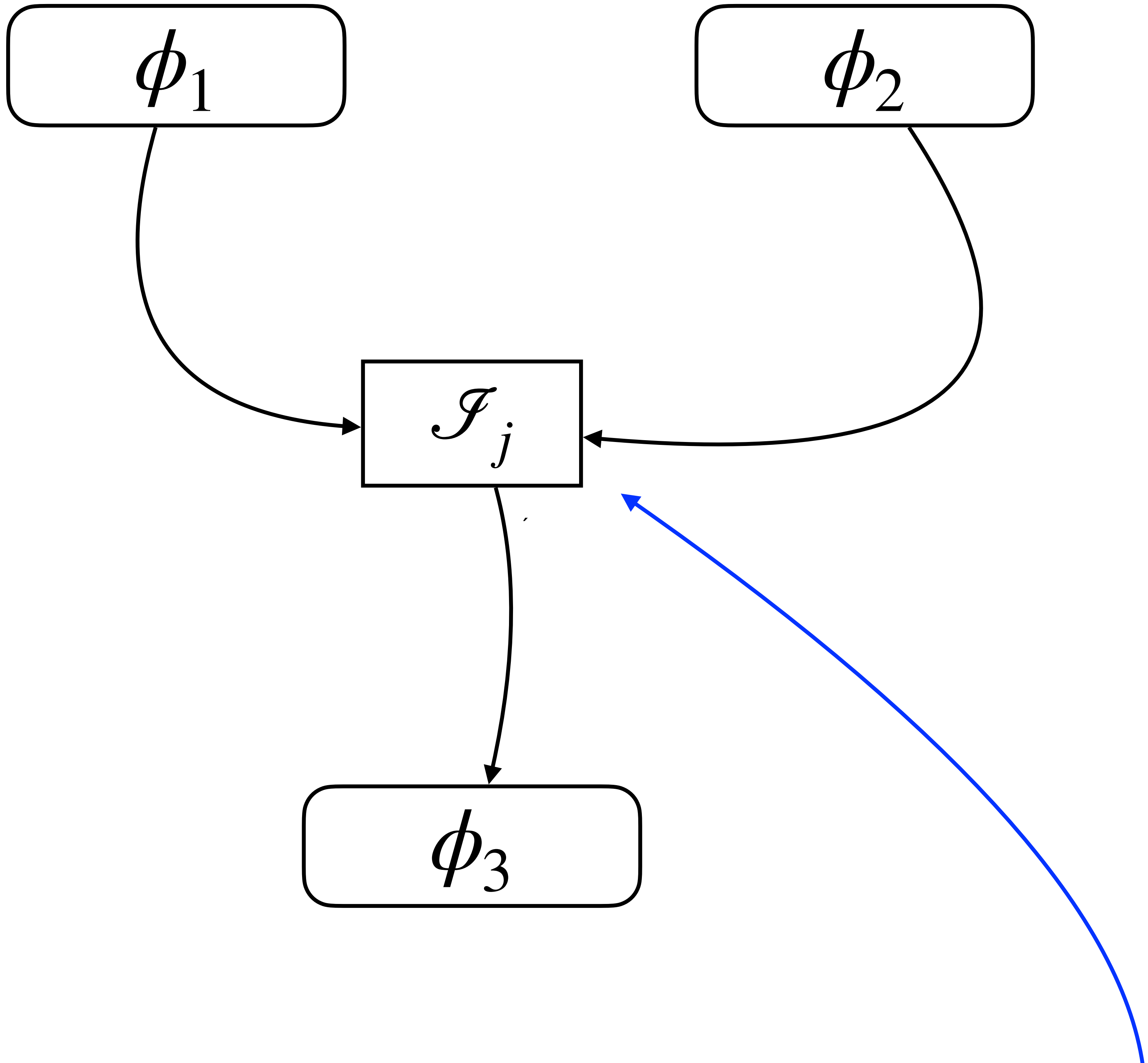


ϕ_1
ϕ_2
$\mathscr{I}_j$
ϕ_3



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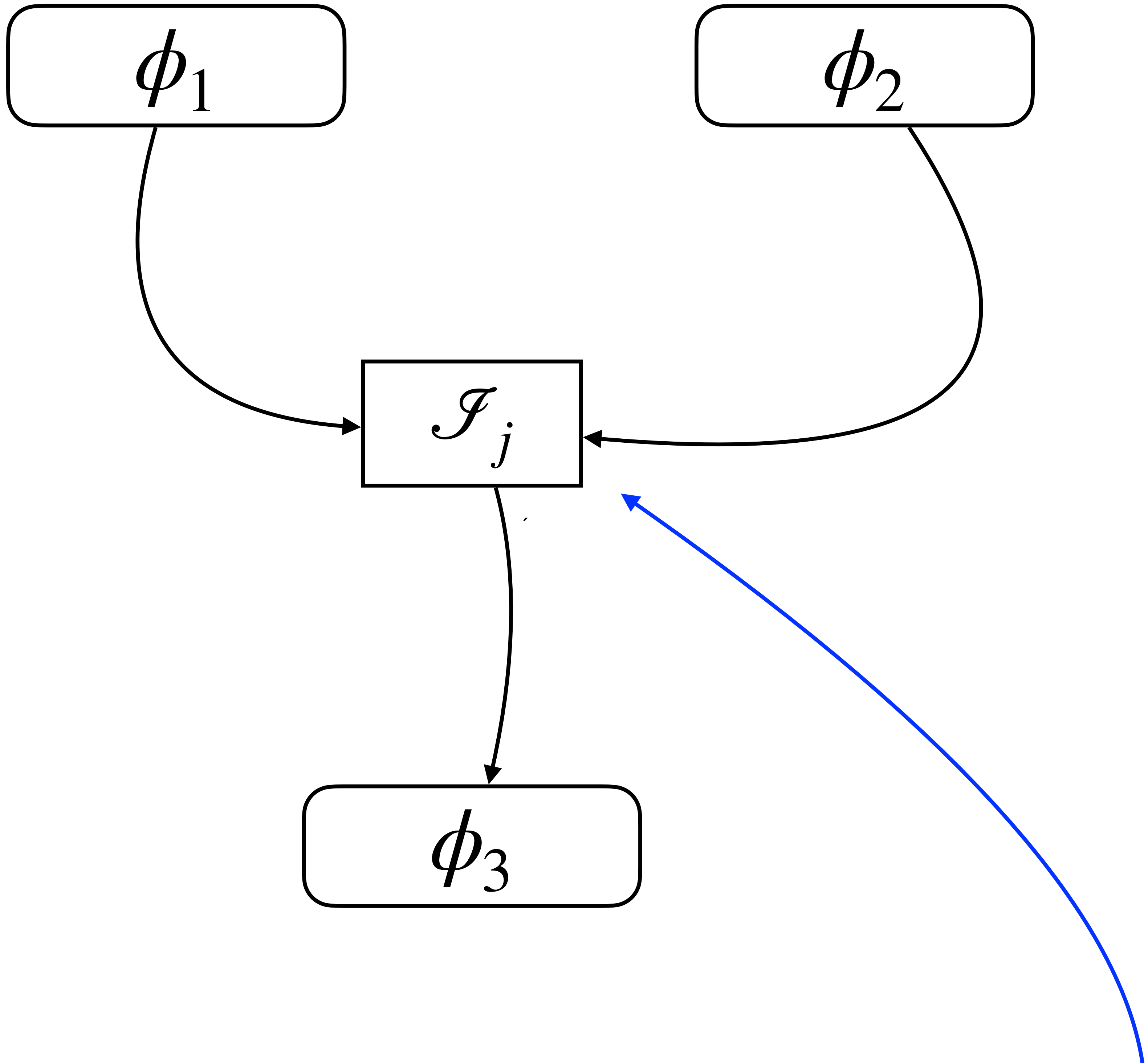
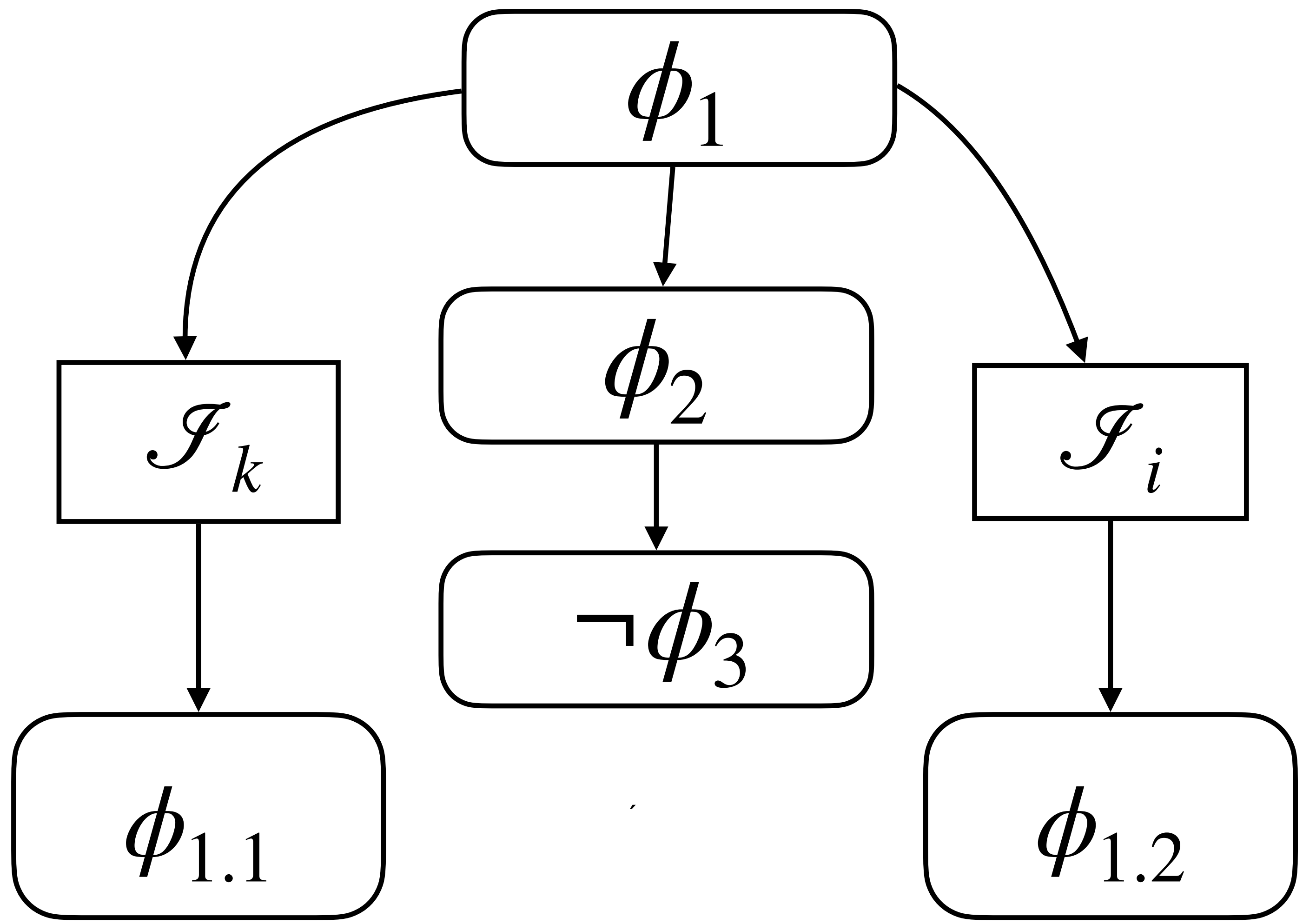


from our collection of natural-
deduction inference rules/schemata



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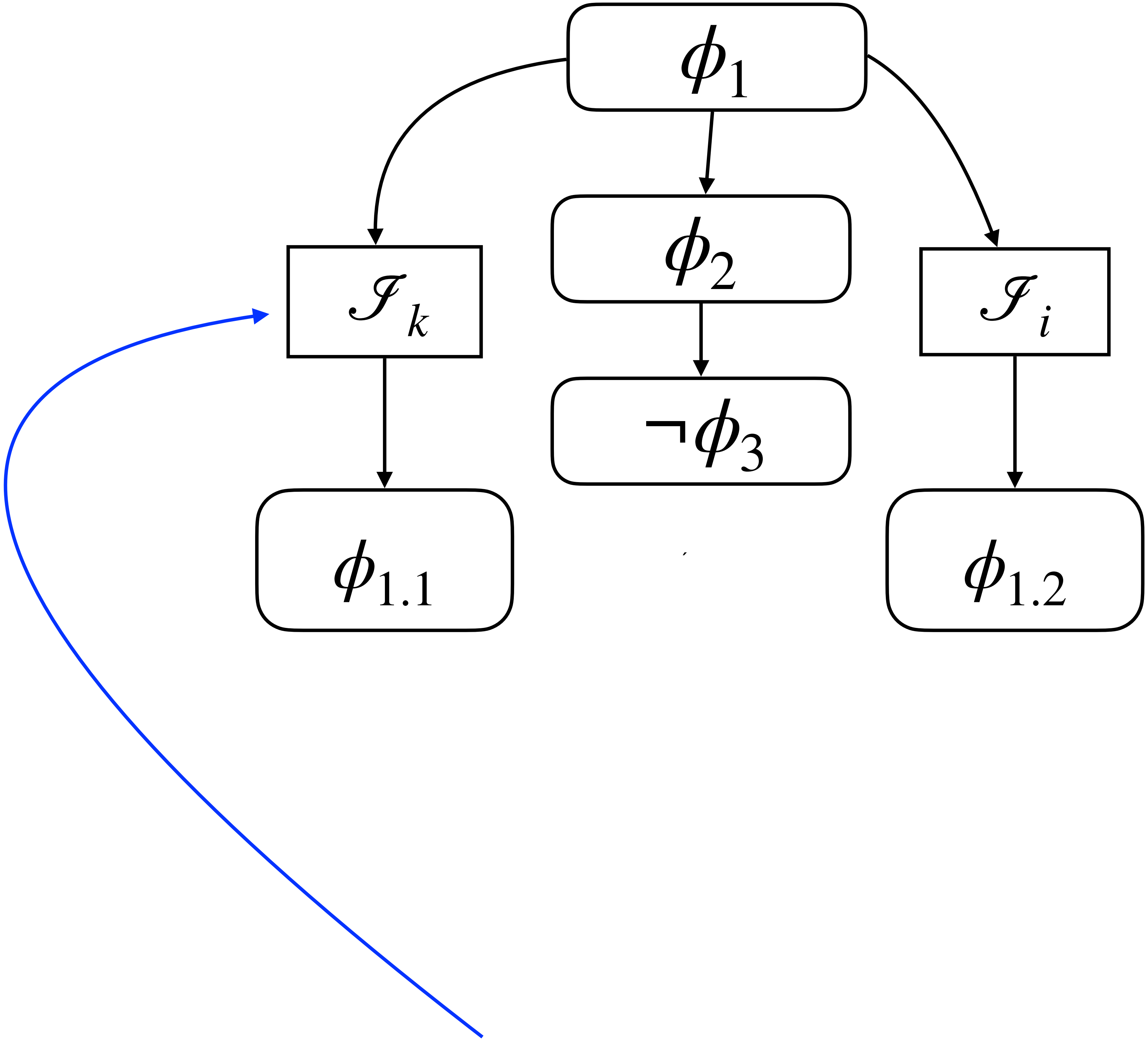
$\phi_{1.1}$
$\phi_{1.2}$



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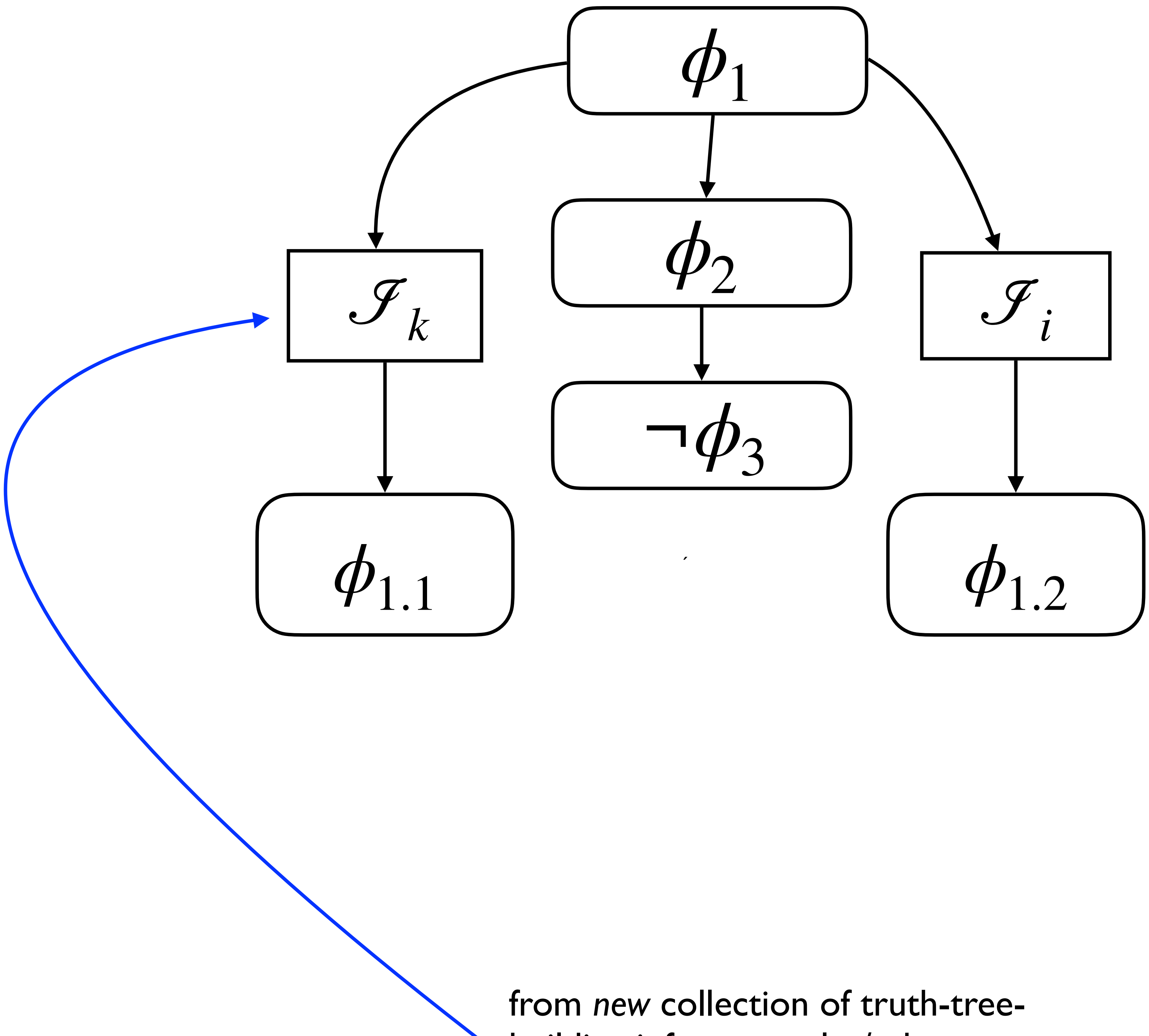

from *new* collection of truth-tree-
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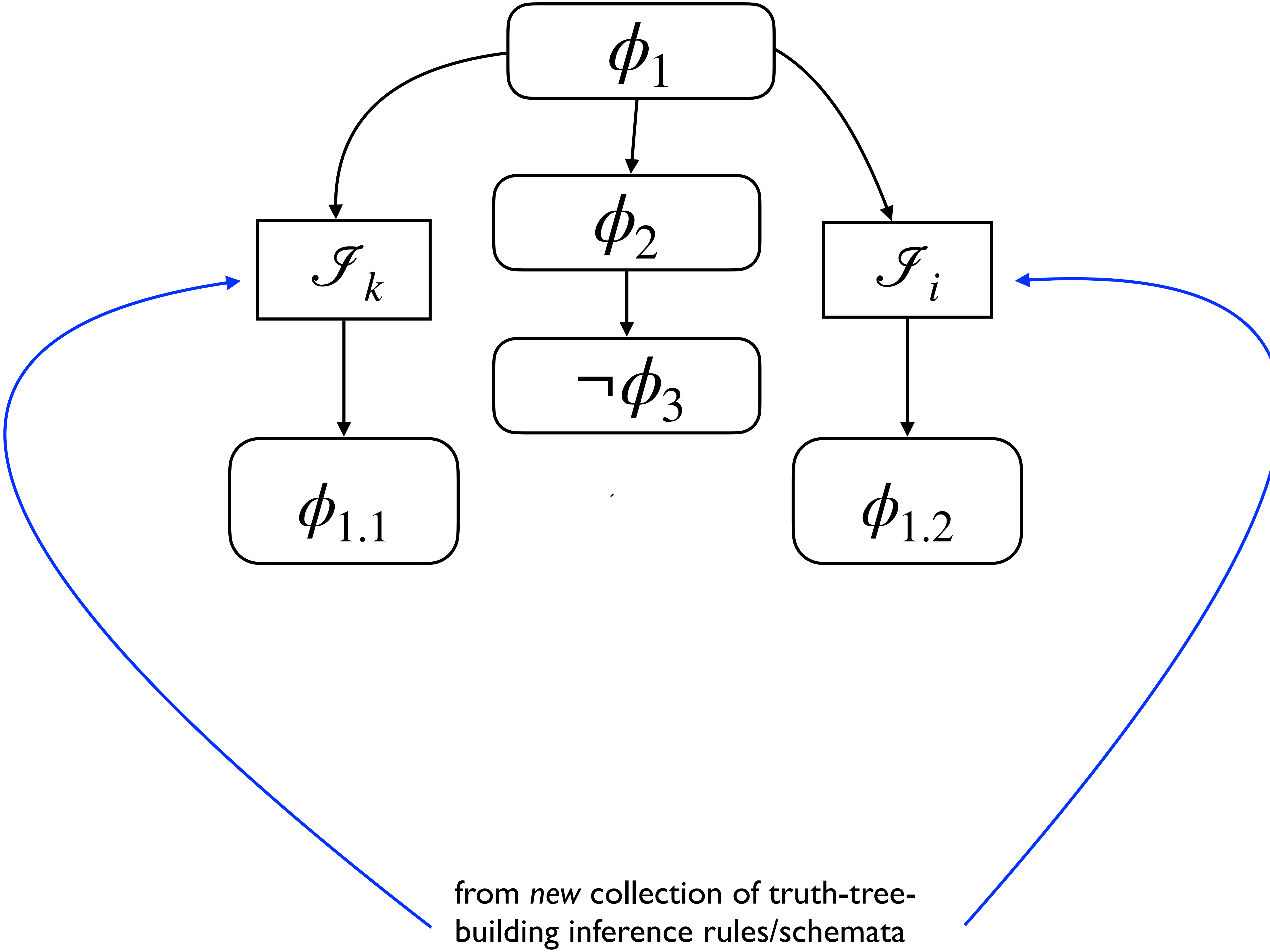
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from *new* collection of truth-tree-building inference rules/schemata

$$\{P \rightarrow Q, P\} \vdash Q$$

GIVEN1. $P \rightarrow Q$

PC $\vdash$ ✗

GIVEN2. P

PC $\vdash$ ✗

NEGATED GOAL. $\neg Q$

PC $\vdash$ ✗

$\{P \to Q, P\} \vdash Q$

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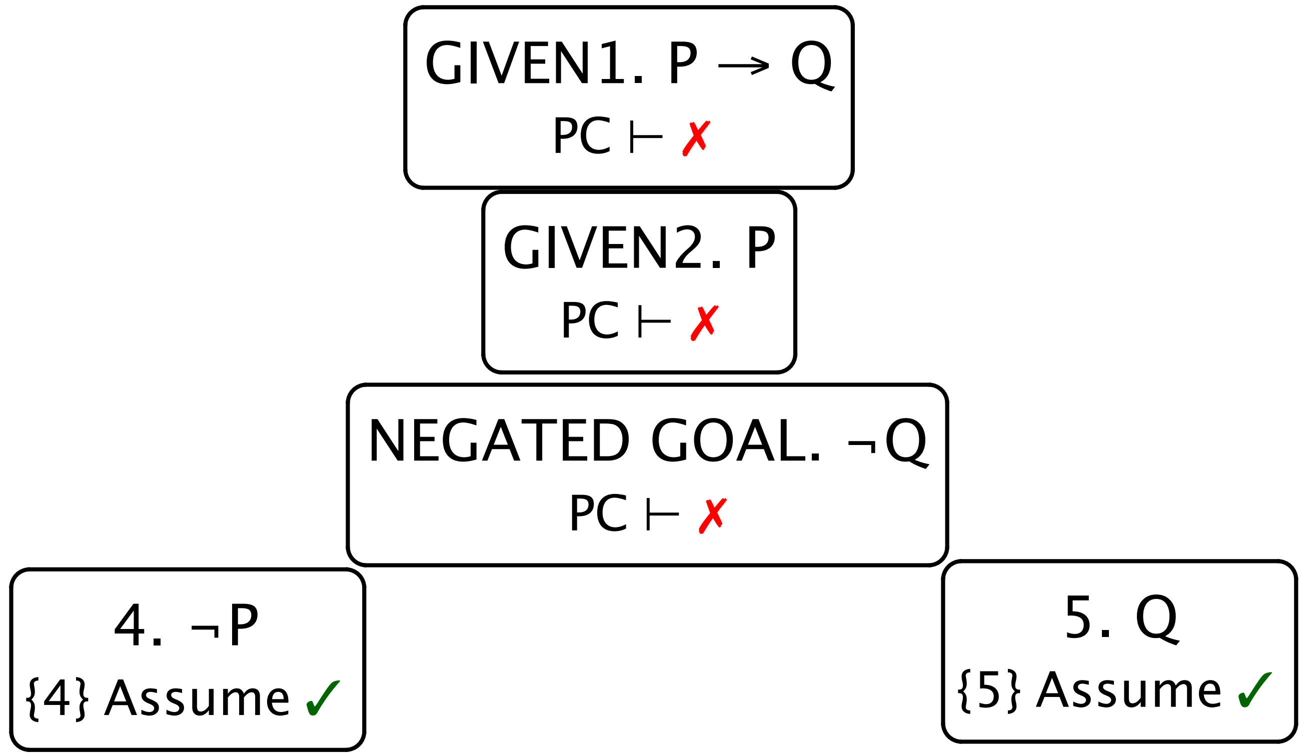
4. $\neg P$

$\{4\}$ Assume ✓

5. Q

$\{5\}$ Assume ✓

$$\{P \to Q, P\} \vdash Q$$



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Either way, a contradiction!

$\{P \rightarrow Q, P\} \vdash Q$

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{4} Assume ✓

$\boxed{\zeta}$

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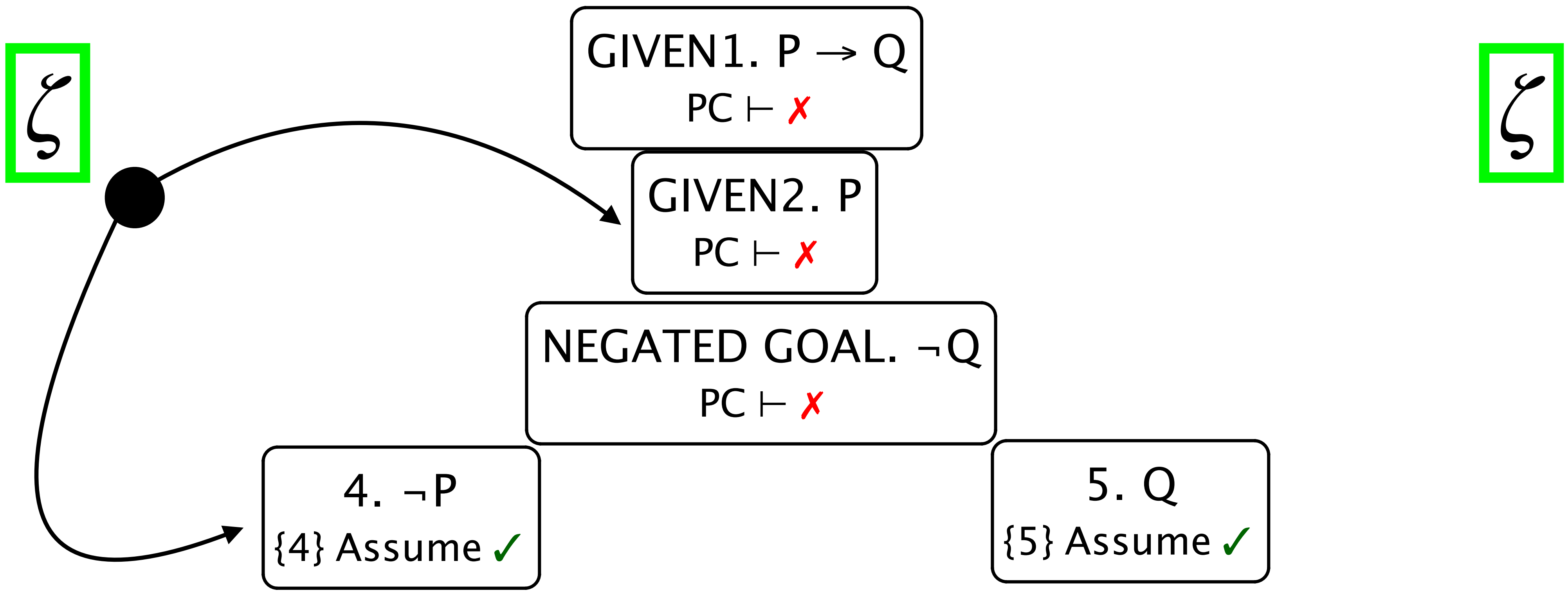

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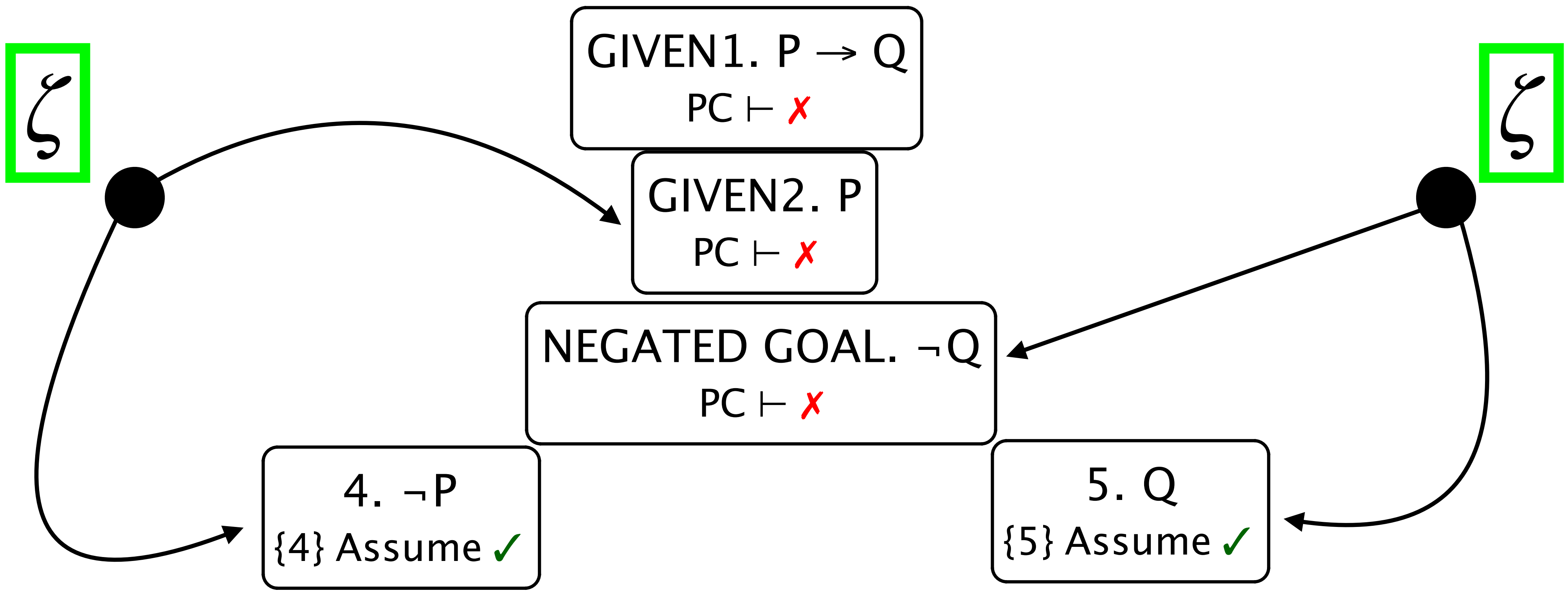

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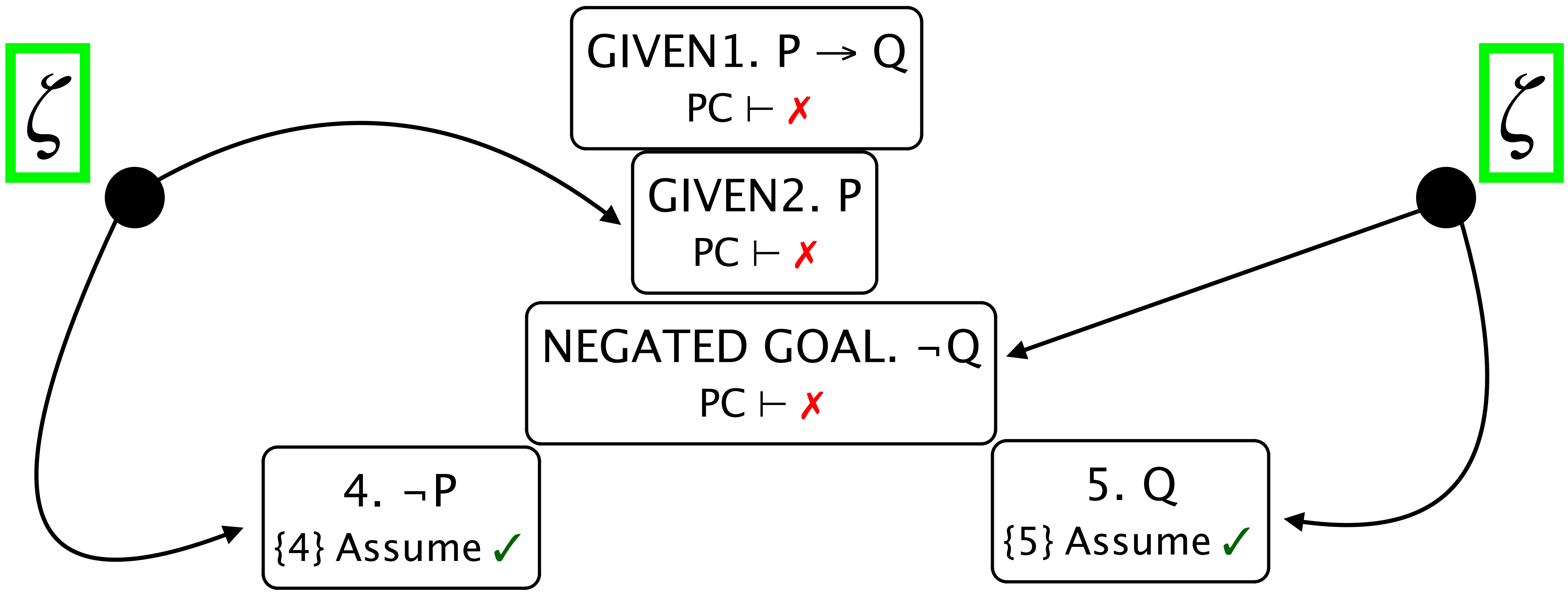

Either way, a contradiction!

Therefore the entailment holds!

Slightly Harder Truth Tree

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$$\vdash (\mathtt{P} \rightarrow (\mathtt{Q} \rightarrow \mathtt{R})) \rightarrow ((\mathtt{P} \rightarrow \mathtt{Q}) \rightarrow (\mathtt{P} \rightarrow \mathtt{R}))$$

Slightly Harder Truth Tree

$$\vdash (\mathtt{P} \to (\mathtt{Q} \to \mathtt{R})) \to ((\mathtt{P} \to \mathtt{Q}) \to (\mathtt{P} \to \mathtt{R}))$$

(This is the axiom THEN-2 in Frege's (brutal) axiomatization of the propositional calculus.)

Frege

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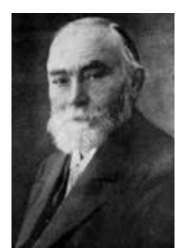

Frege

https://en.wikipedia.org/wiki/Frege%27s_propositional_calculus

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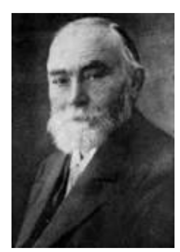

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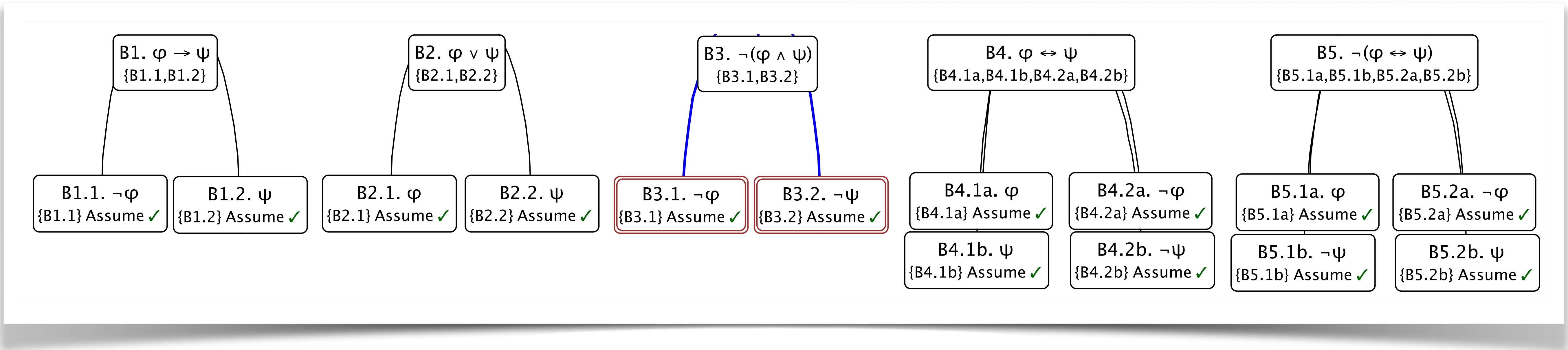

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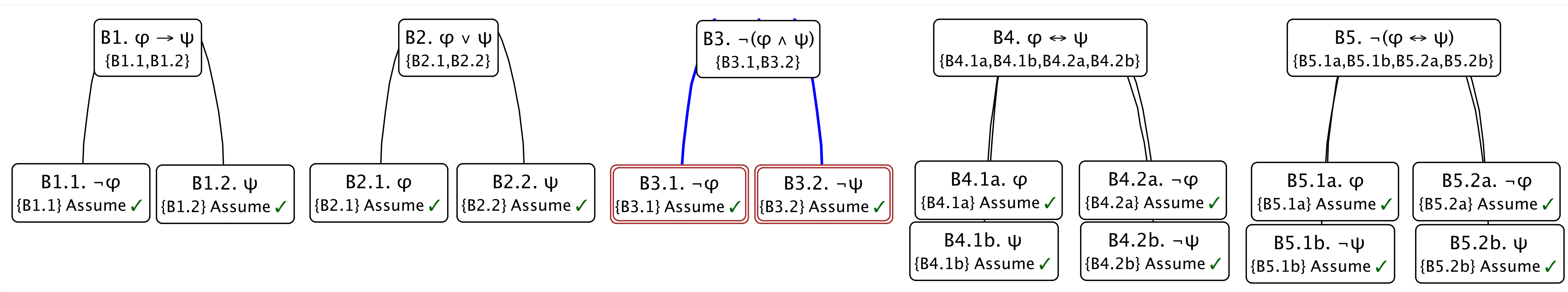

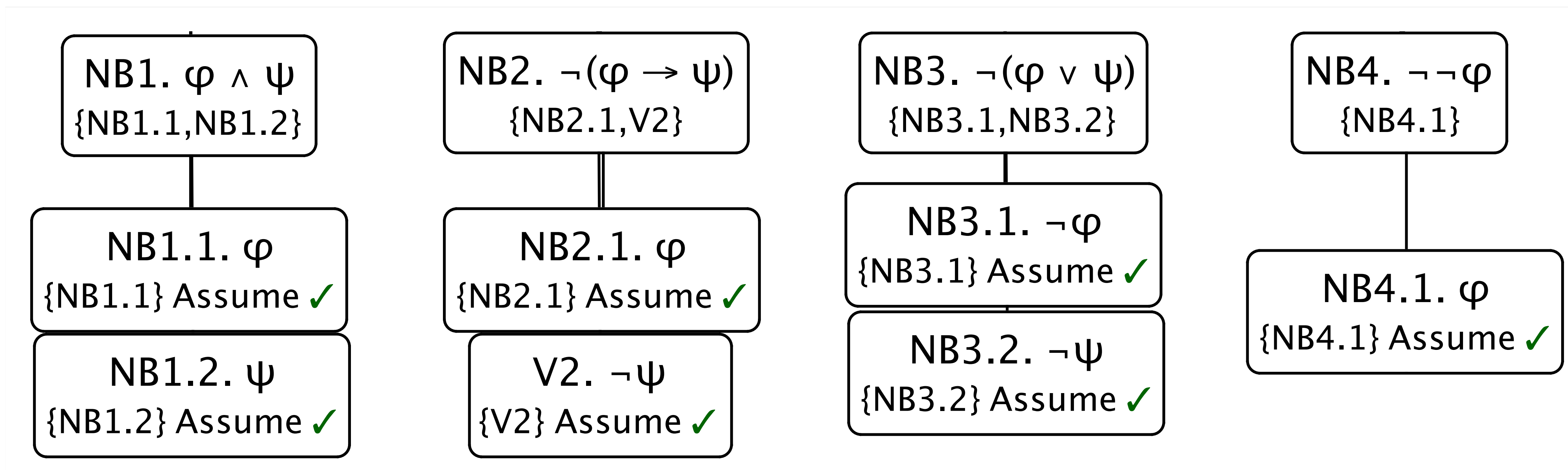

The Rules of the Game!

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Questions?

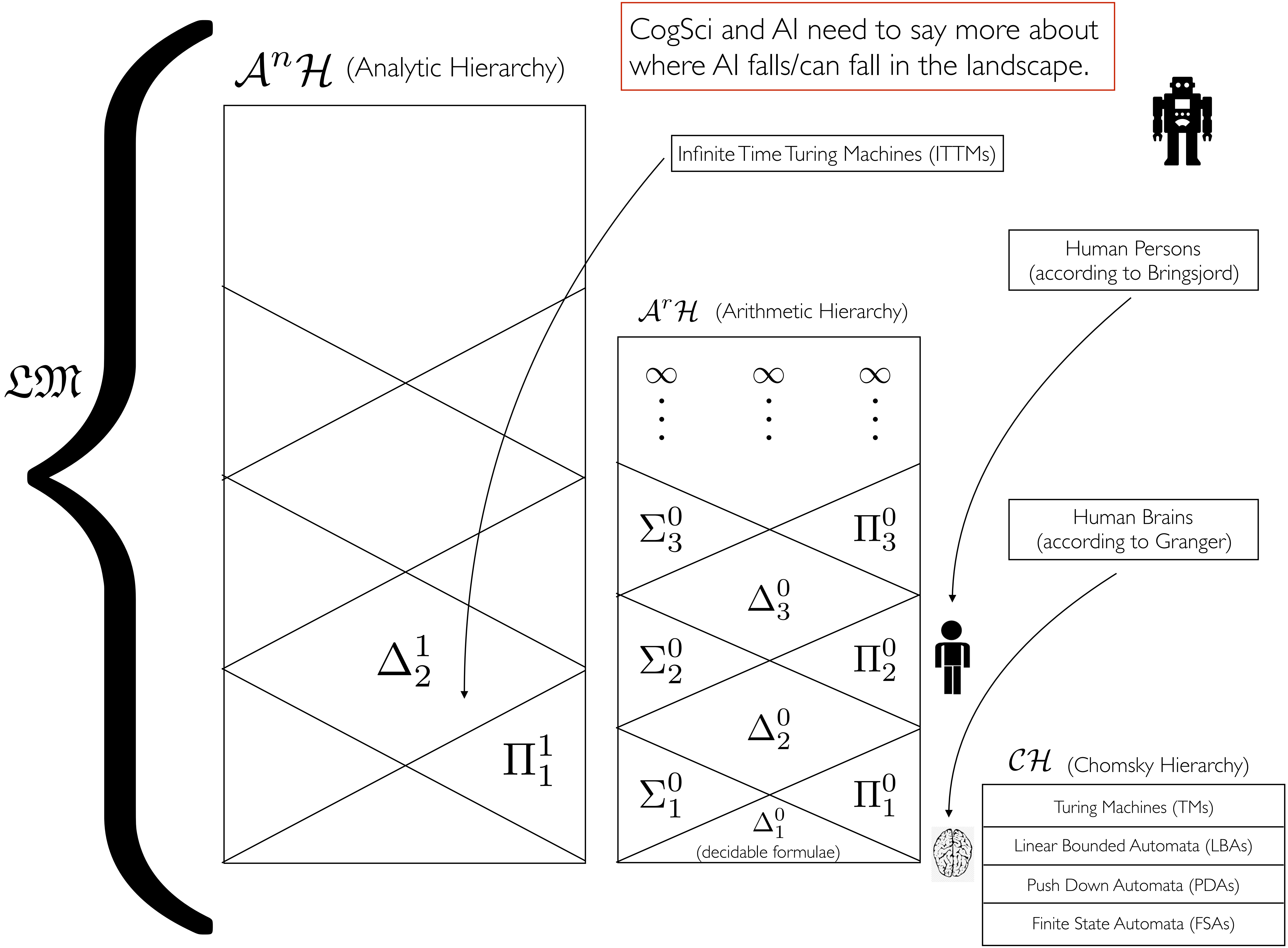
Theorem:

Let ϕ be a theorem in the propositional calculus $= \mathcal{L}_{\mathrm{PC}}$.
Then the truth-tree algorithm will lead to no open branches.

On Measuring The Intelligence of Agents

. . .

using quantification . . .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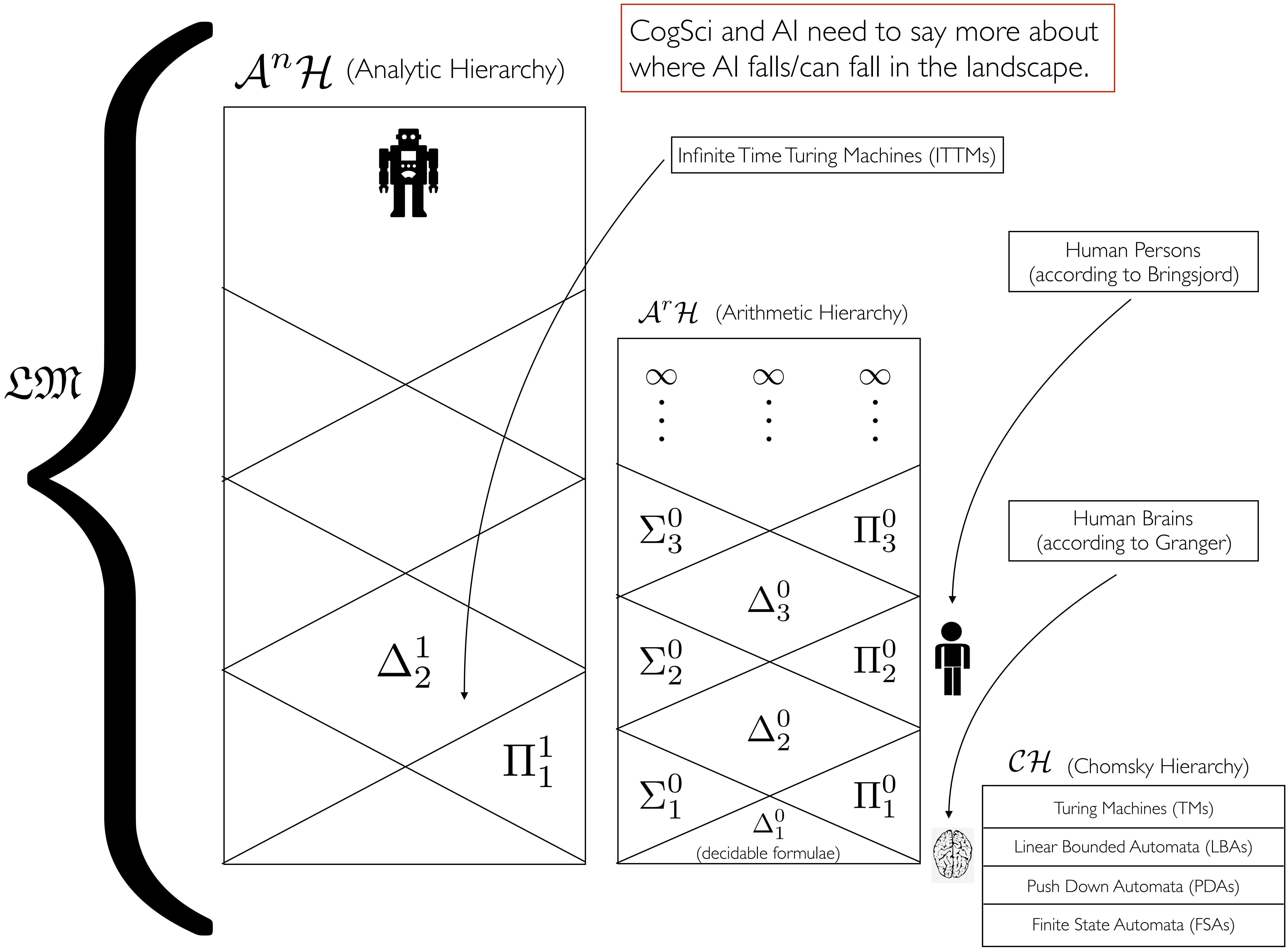
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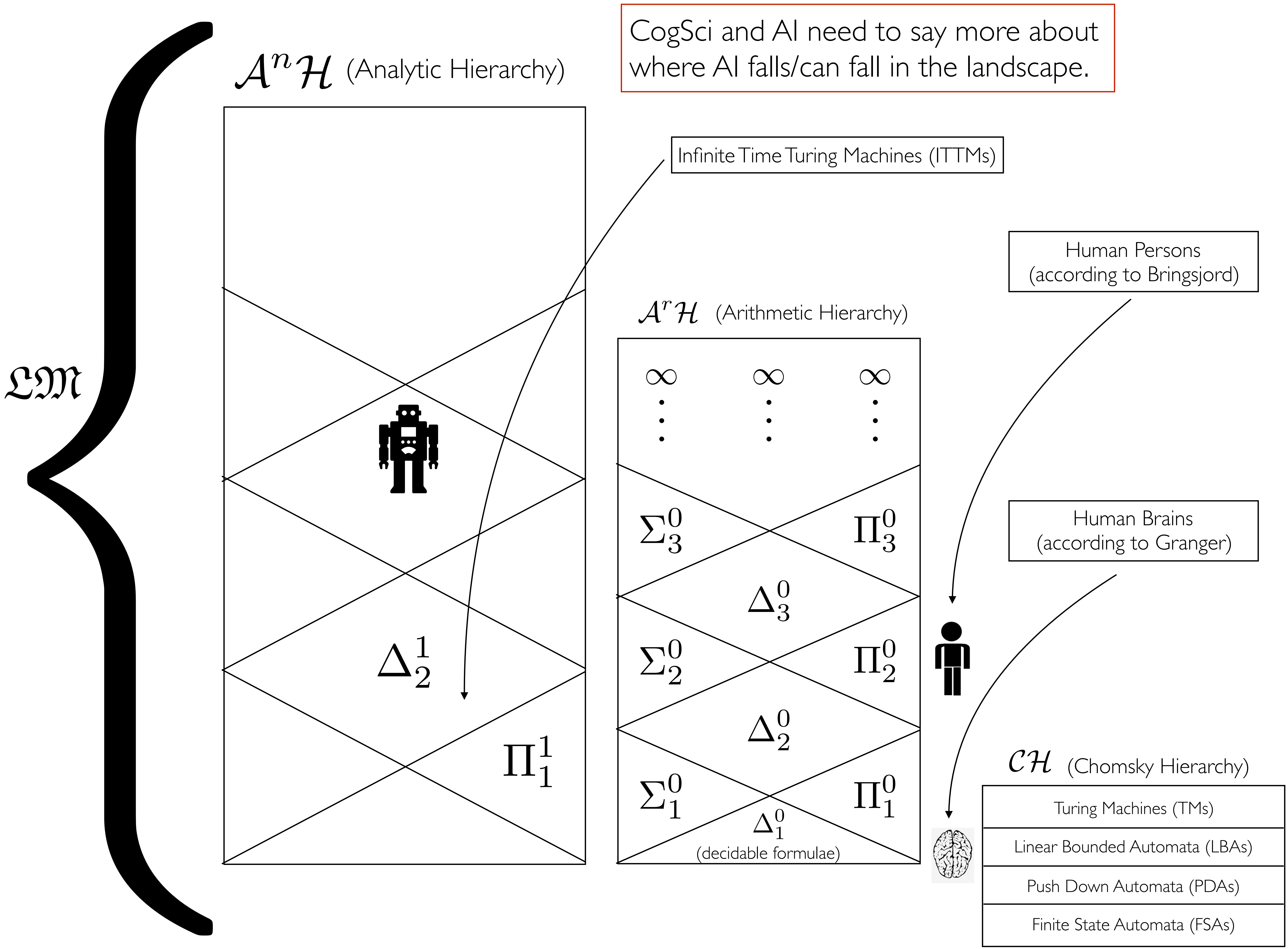
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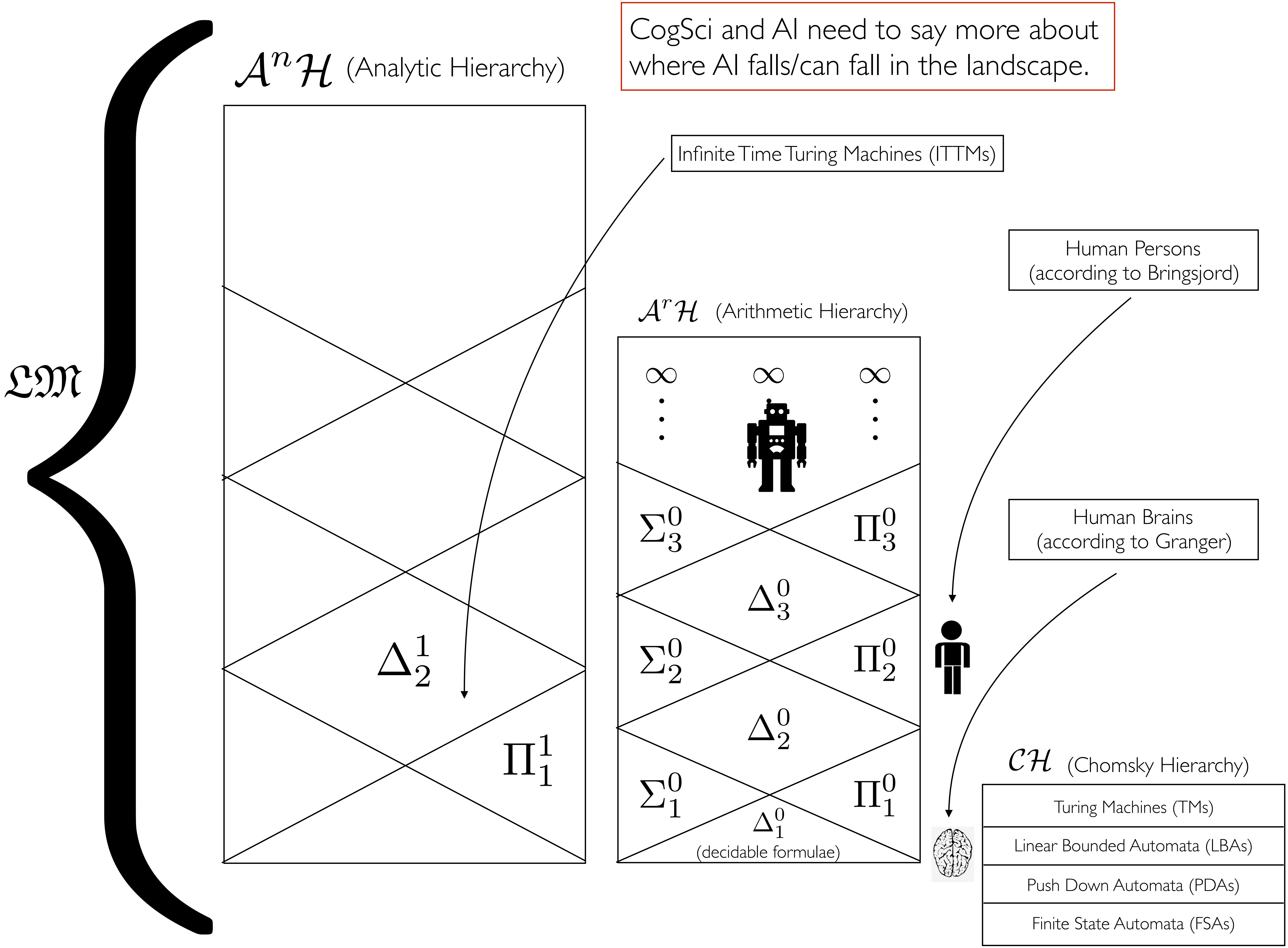
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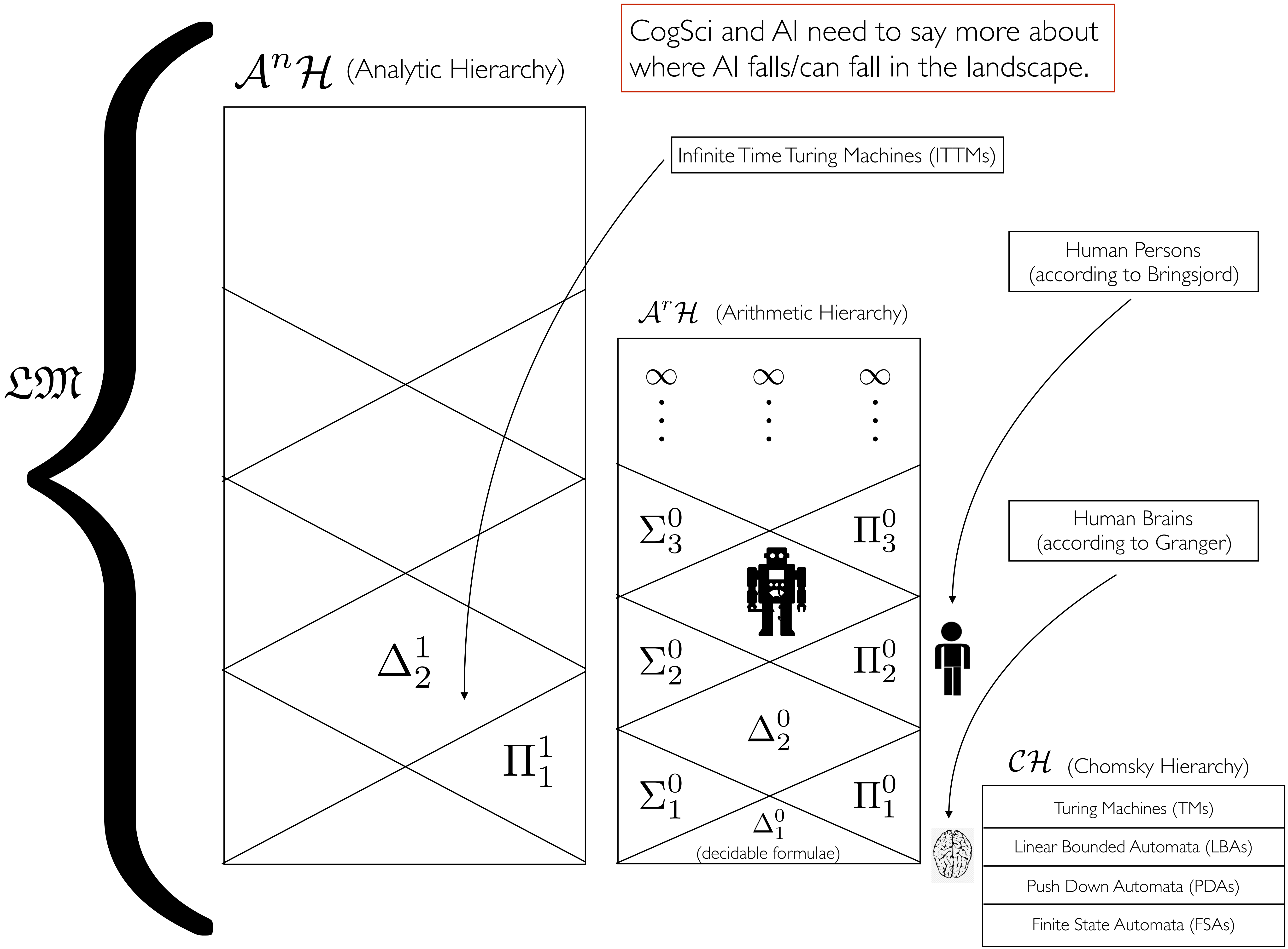
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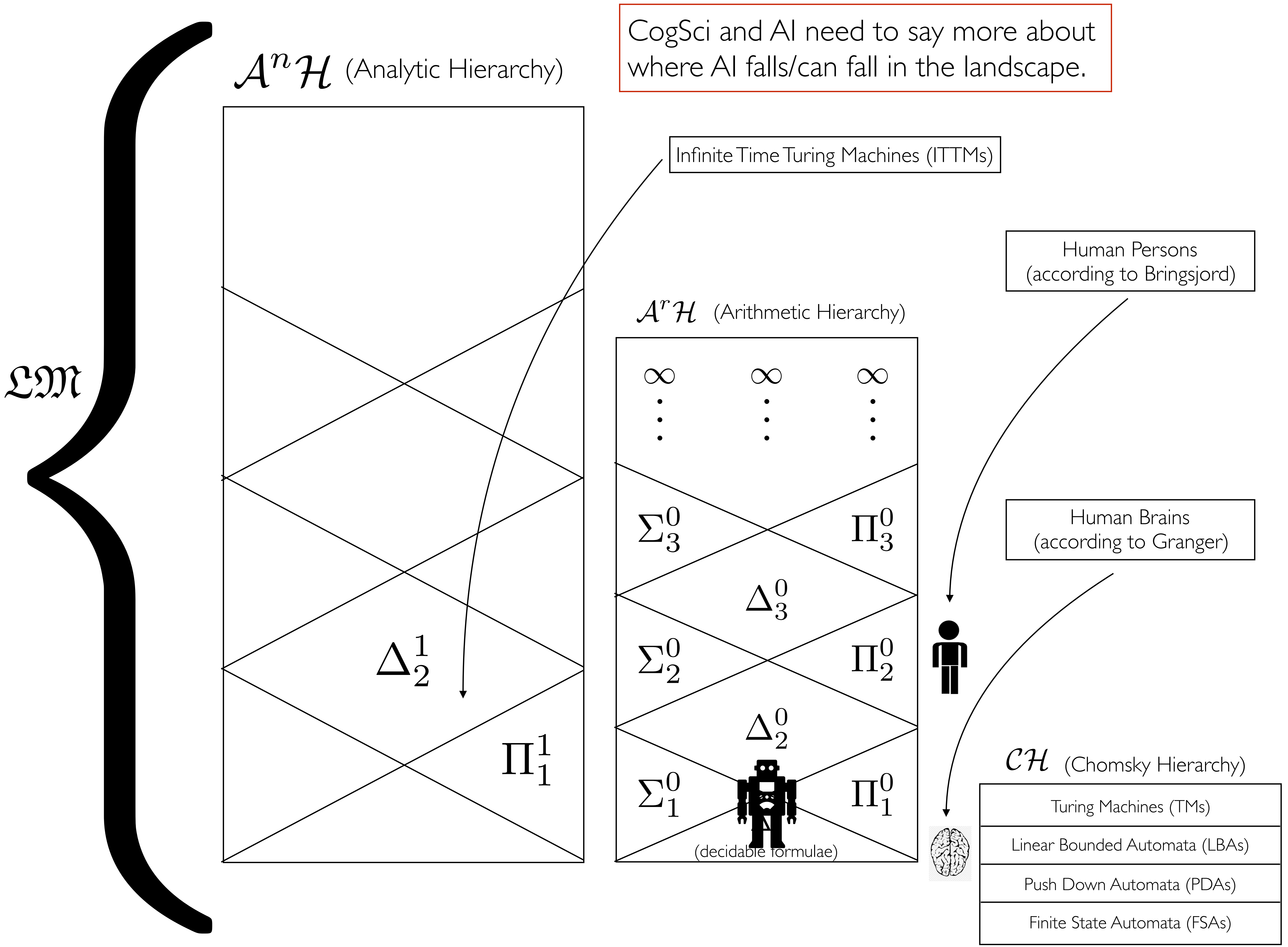

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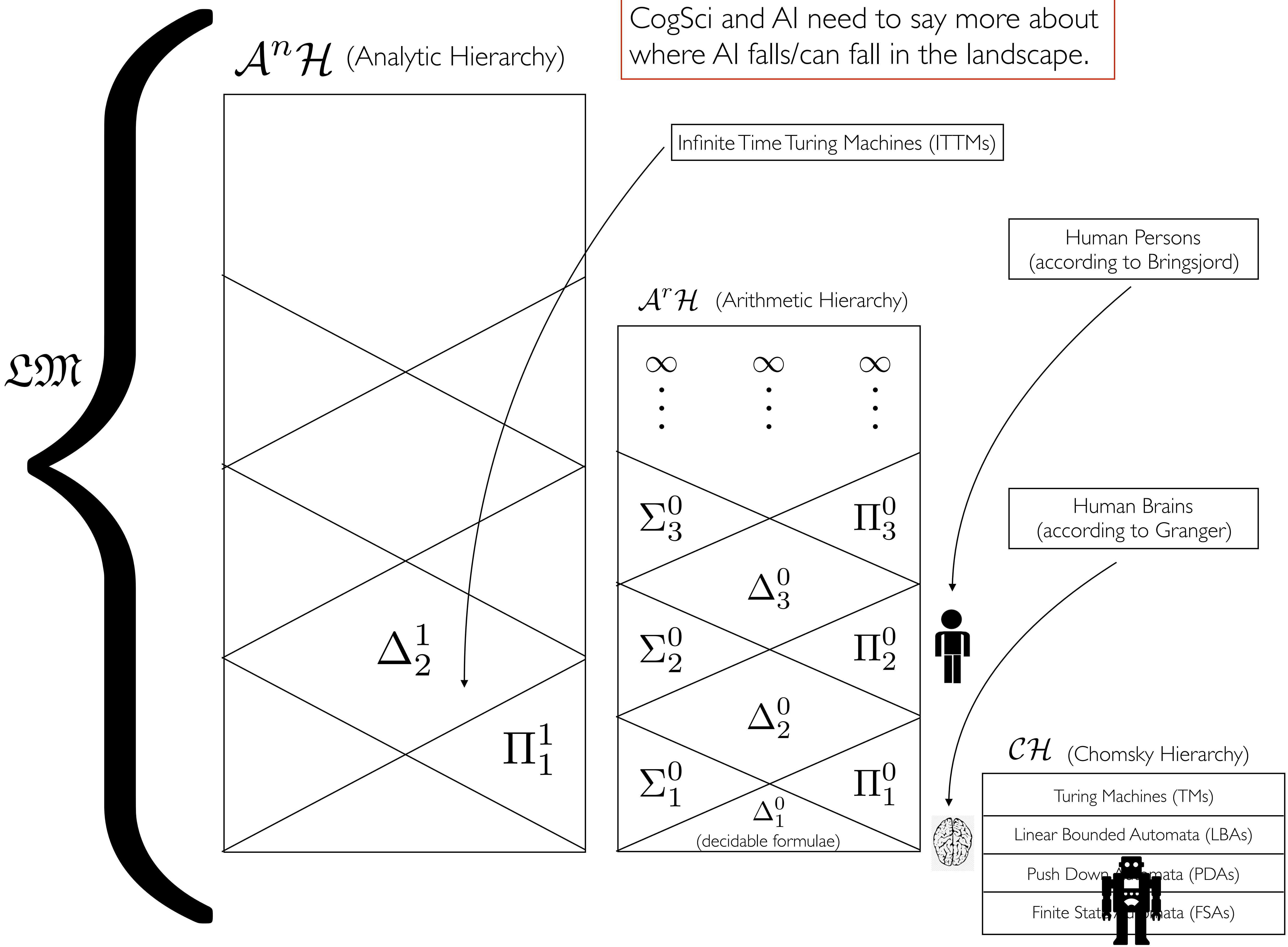

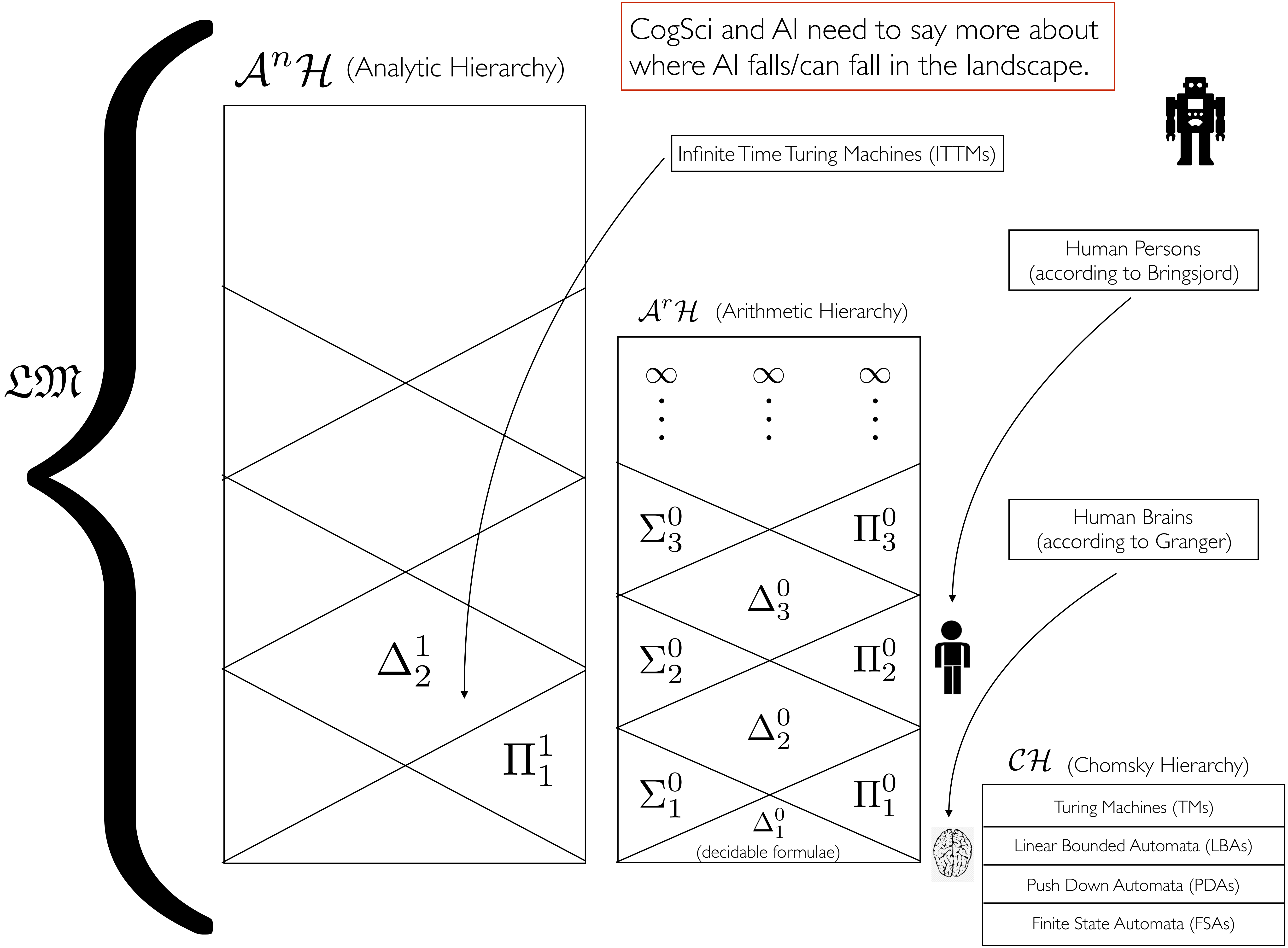
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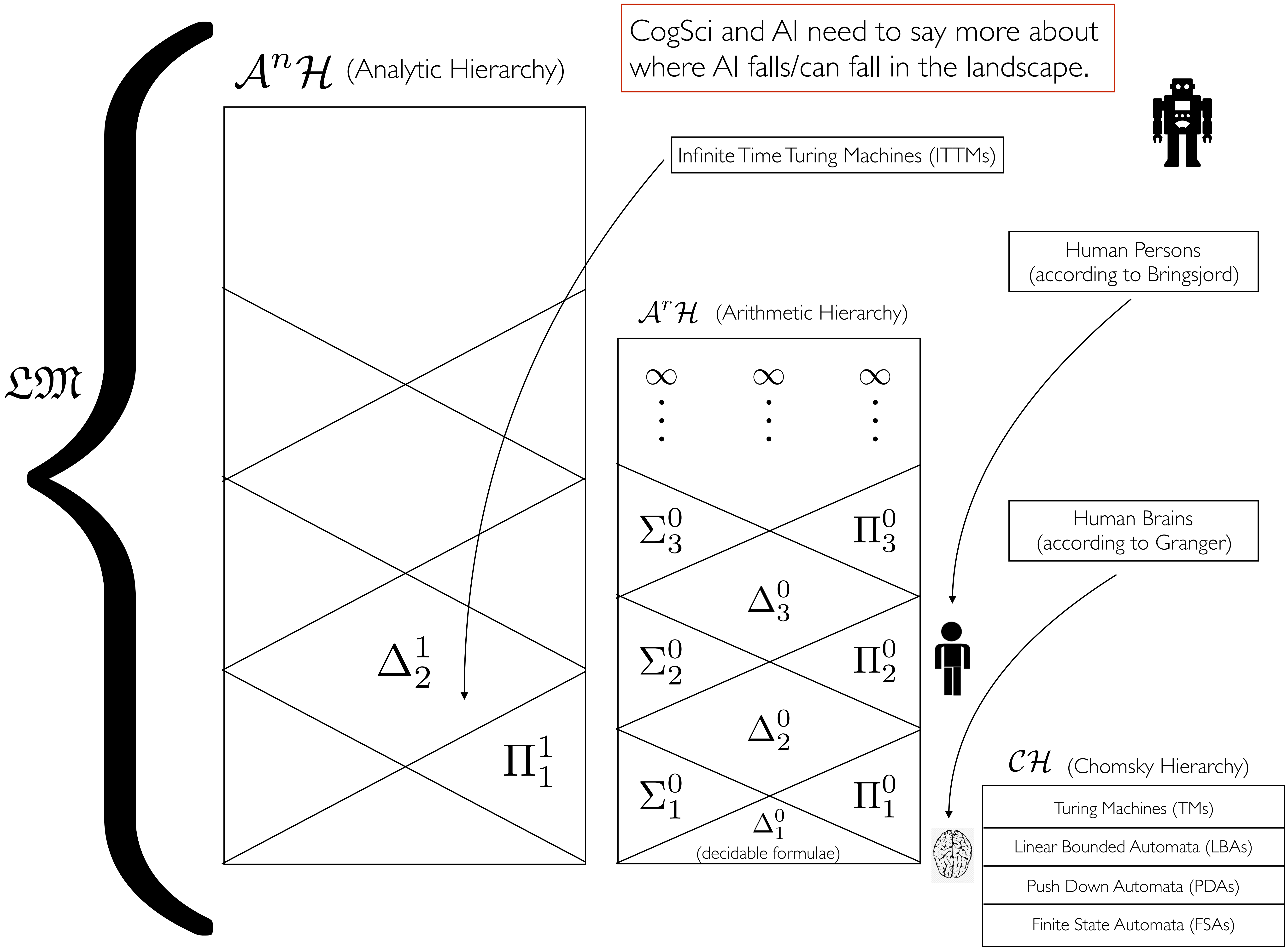

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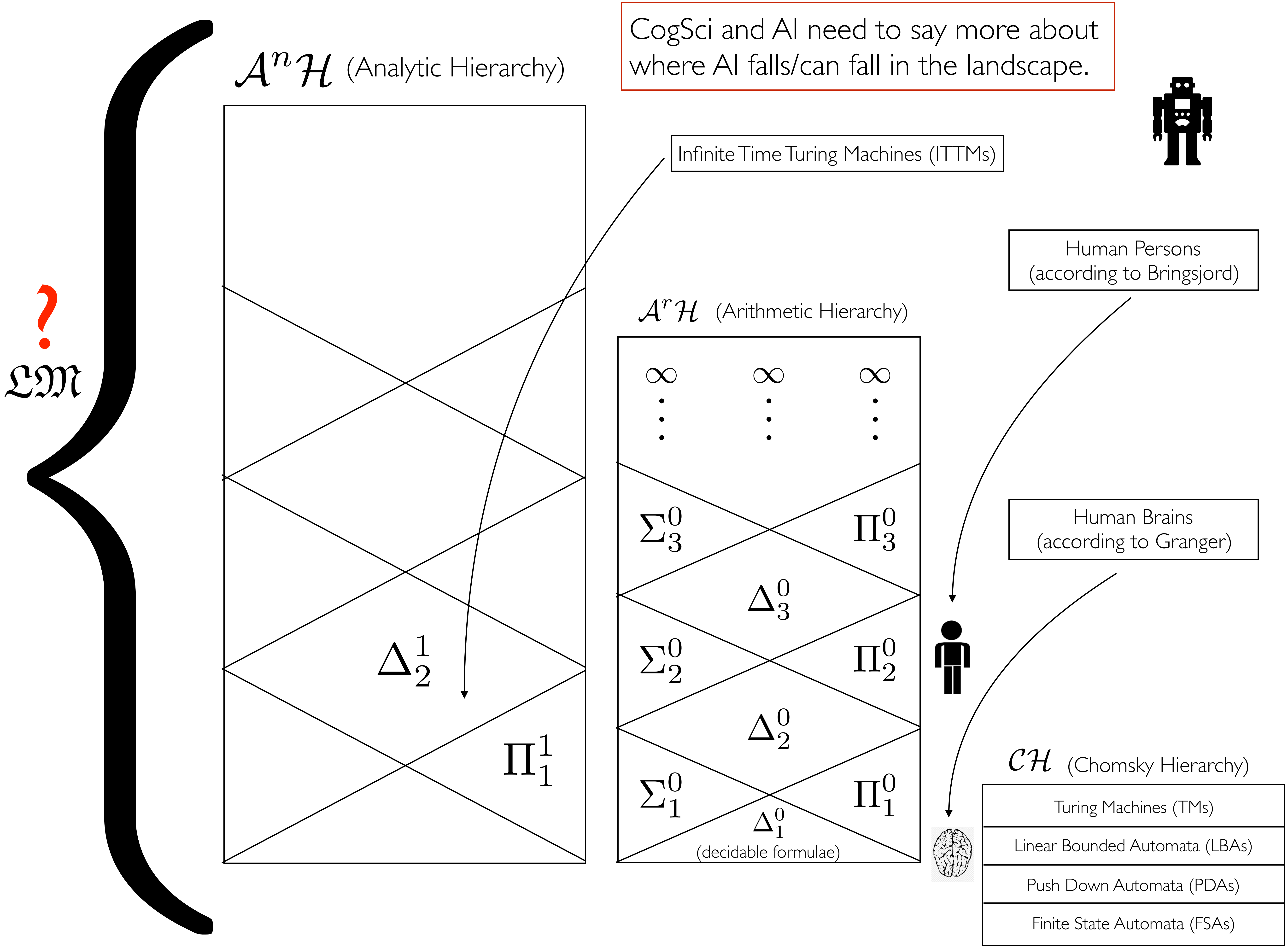
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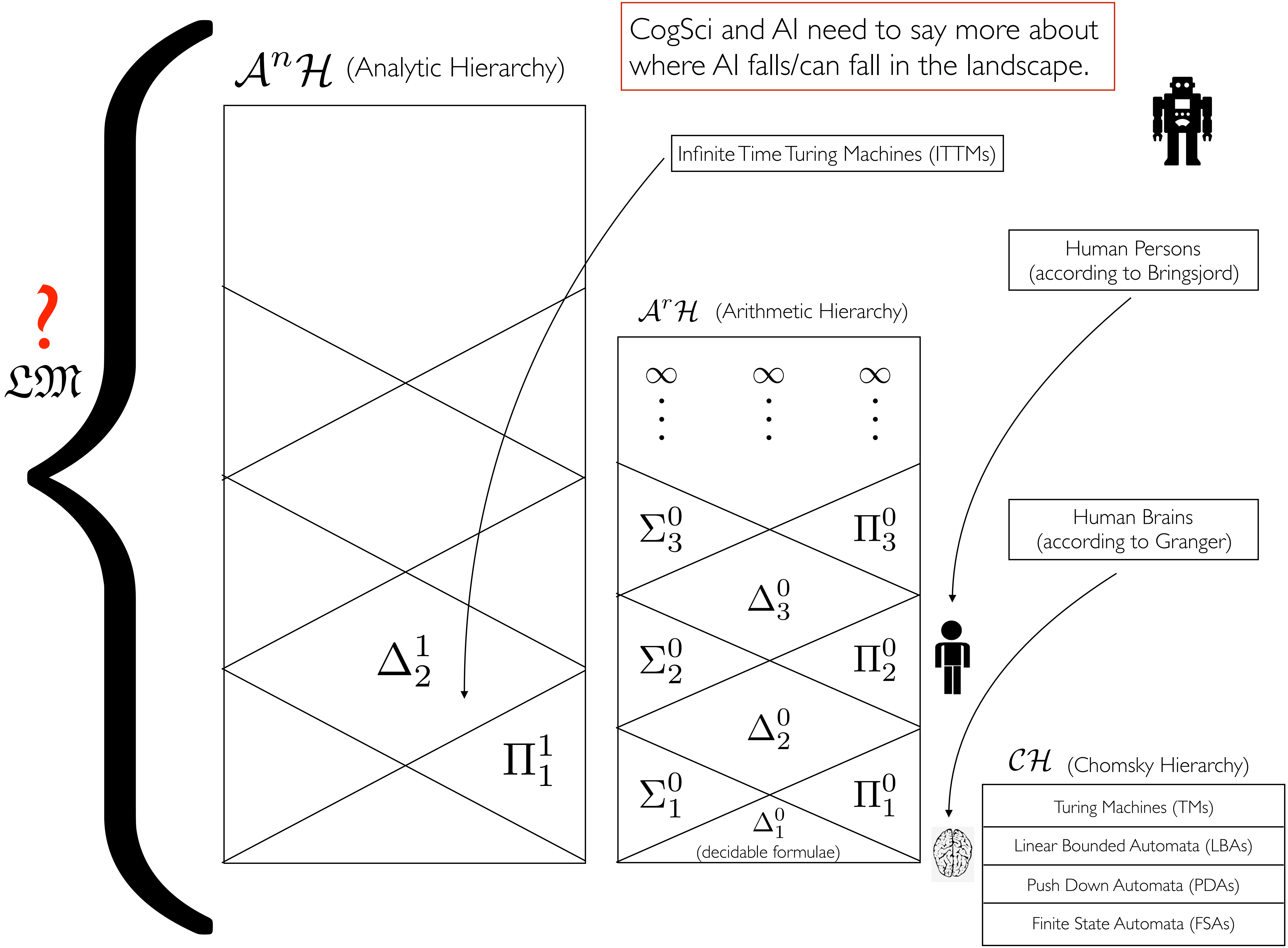
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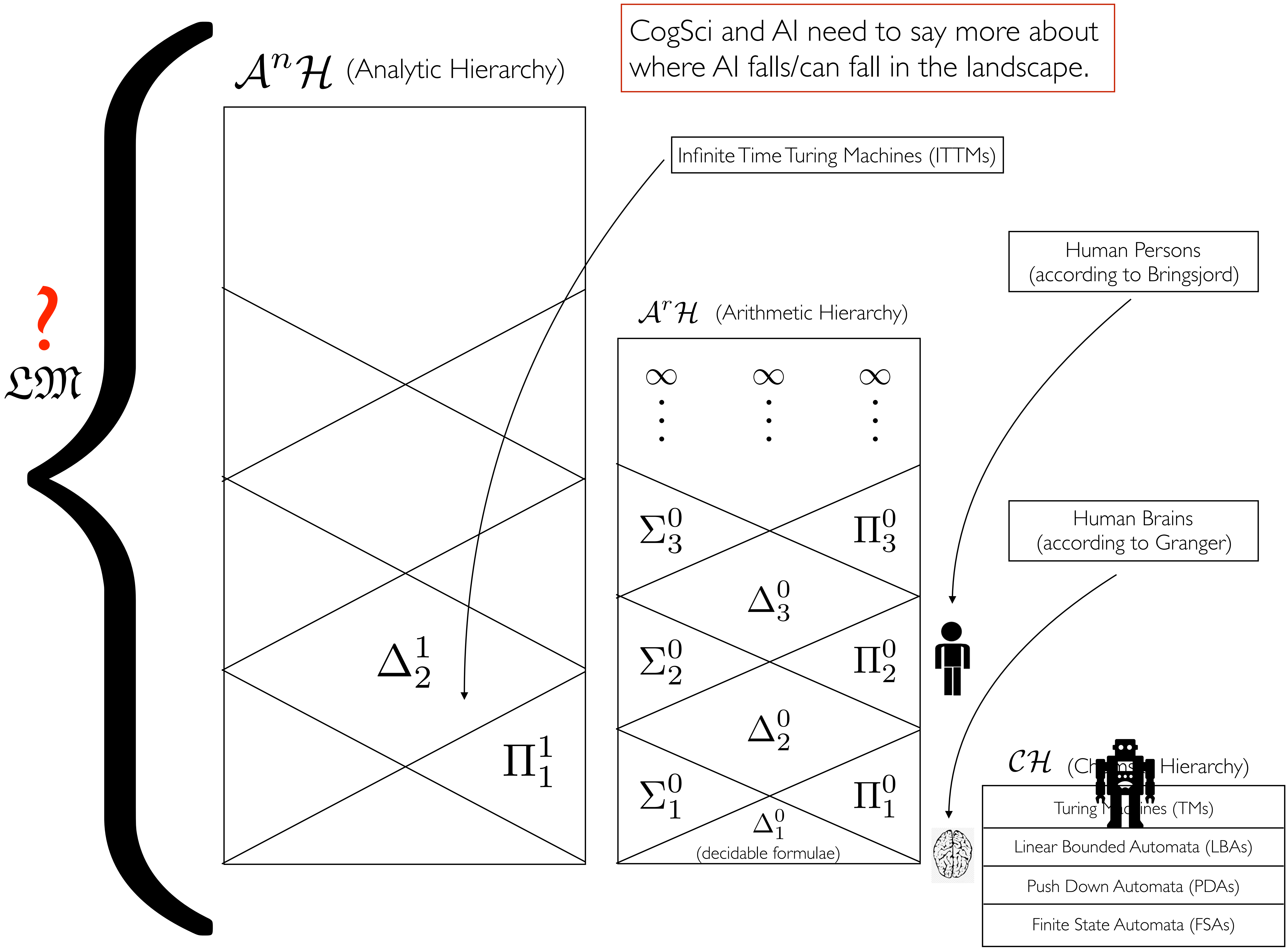

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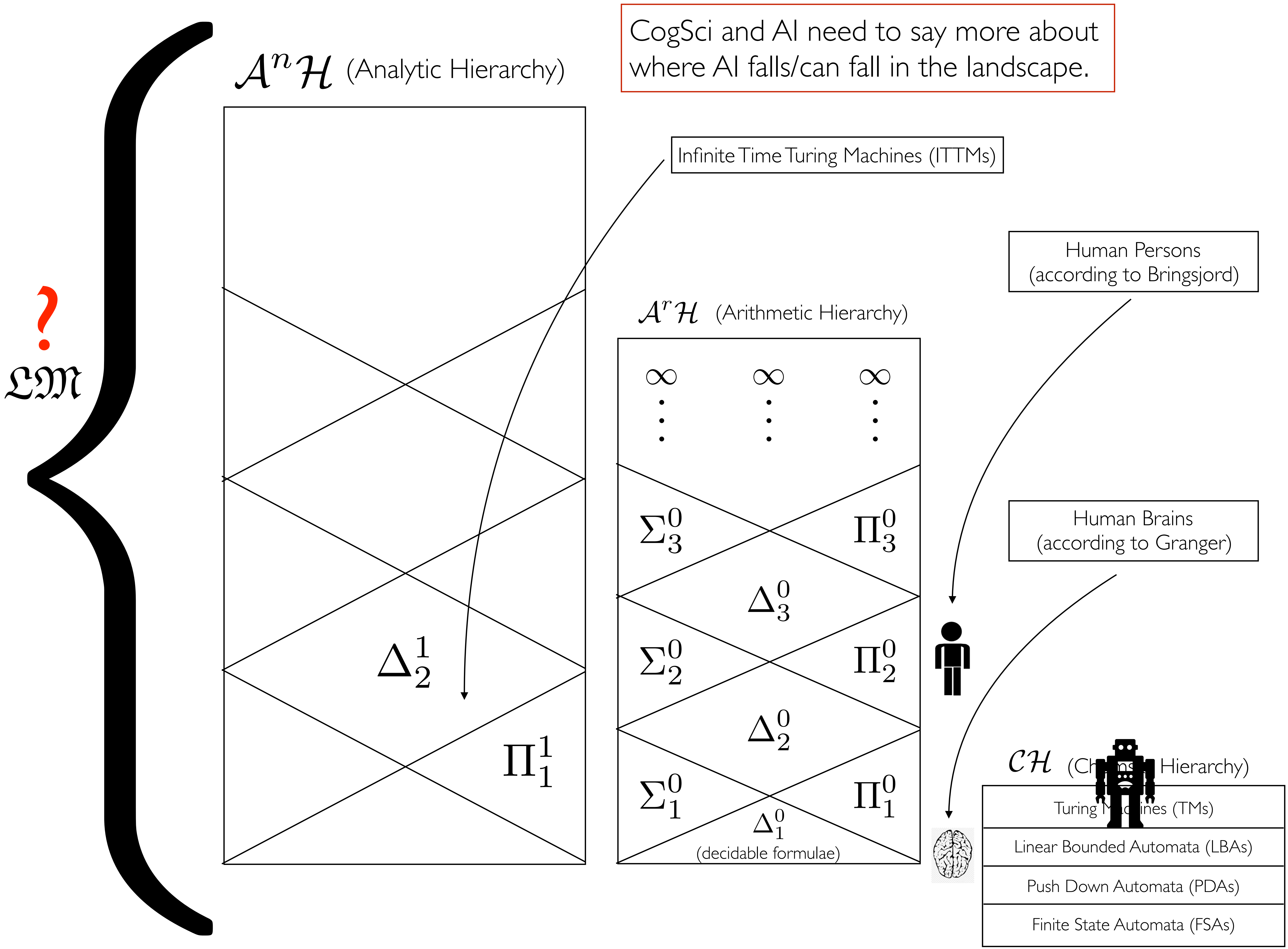
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Measuring Intelligence & AI/The Singularity

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The Singularity (superhuman machine intelligence) is near!!

Measuring Intelligence & AI/The Singularity

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Is that so? And how are you measuring intelligence, pray tell?

Measuring Intelligence & AI/The Singularity

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Polynomial Hierarchy

Measuring Intelligence & AI/The Singularity

Polynomial Hierarchy

$$\mathbf{P} \subseteq \mathbf{NP} \subseteq \mathbf{PSPACE} = \mathbf{NPSPACE} \subseteq \mathbf{EXPTIME} \subseteq \mathbf{NEXPTIME} \subseteq \mathbf{EXPSPACE}$$

Measuring Intelligence & AI/The Singularity

Checkers:Chinook

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Jeopardy!

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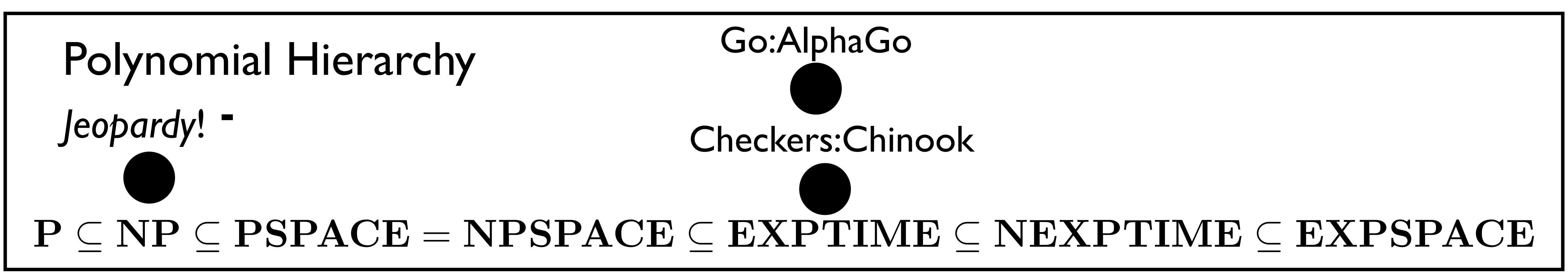

Measuring Intelligence & AI/The Singularity

Polynomial Hierarchy

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Polynomial Hierarchy

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Arithmetical Hierarchy

 

Polynomial Hierarchy

Go:AlphaGo

Jeopardy! -

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Arithmetical Hierarchy

$\vdots$

Π_2

Σ_2

Π_1

Σ_1

Σ_0

Polynomial Hierarchy

Jeopardy! -

Go:AlphaGo

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Measuring Intelligence & AI/The Singularity

Arithmetical Hierarchy

$\vdots$

"Hey, do these two Java programs compute the very same function?" Π_2

Σ_2

Π_1

Σ_1

Σ_0

Polynomial Hierarchy

Go:AlphaGo

Jeopardy!

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Measuring Intelligence & AI/The Singularity

Analytical Hierarchy

Arithmetical Hierarchy

$\vdots$

Π_2

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Σ_2

Π_1

Σ_1

Σ_0

Polynomial Hierarchy

Go:AlphaGo

Jeopardy! -

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Analytical Hierarchy

Arithmetical Hierarchy

This, all of this, is derived from consideration of first-order logic and second-order logic, with an emphasis on *quantification* and proof.

$\vdots$

Π_2

"Hey, do these two Java programs compute the very same function?"

Σ_2

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Polynomial Hierarchy

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An “Advanced” Topic for Measuring Intelligence …

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- FOL formulae that (only) enforce domain size:

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$$\exists x \exists y (x \neq y)$$

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$$\underline{\phi_n}$$

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$\underline{\phi_n}$ domain of at least *n* things

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$\exists x \forall y (y = x)$

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$\exists x \exists y \forall z (z = x \vee z = y)$

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$$\underline{\phi_n}$$ domain of at least *n* things

$$\exists x \forall y (y = x)$$ at most one thing

$$\exists x \exists y \forall z (z = x \lor z = y)$$ at most two things

$$\exists x_1 \exists x_2 \exists x_3 \forall y (y = x_1 \lor y = x_2 \lor y = x_3)$$

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$\vdots$

$\overline{\phi_n}$

For now, let's settle for a focus on quantification. Then …

Measuring AI Intelligence via (in part) Logic:Quantification

Toby Walsh: “The Singularity May Never Be Near”
(http://arxiv.org/pdf/1602.06462v1.pdf)

Measuring AI Intelligence via (in part) Logic:Quantification

Toby Walsh: "The Singularity May Never Be Near" (http://arxiv.org/pdf/1602.06462v1.pdf)

"I will not tackle here head on what we mean by measuring the intelligence of machines (or of humans). I will simply suppose there is such a property as intelligence, that it can be measured and compared, and that the technological singularity is when this measure increases exponentially fast in an appropriate and reasonable scale." (p. 1)

But logico-mathematical definitions of intelligence for animals, humans, machines, aliens, gods … *are* possible; recall our consideration of the *Entscheidungsproblem*. We can specifically challenge today's AI on the basis of simple quantification and simple deduction …

First, need some numerical quantifiers:

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$$\exists x \forall y (y = x \wedge \phi(x)) \text{ will be } \exists^{=1} x \phi(x)$$

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How do we define formulae of this type: $\exists^{=k} x \psi(x)$

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Okay, now AI:

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Okay, now AI:

At least seven kenspeckle blookers are red.

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Okay, now AI:

At least seven kenspeckle blookers are red.

Given this, is it true that there are two red blookers? Why, exactly?

$$
\begin{aligned}
&\forall x \forall y \forall z\ \{[x \neq y \wedge y \neq z \wedge x \neq z \wedge Cx \wedge Cy \wedge Cz\ \wedge \\
&Tz' \wedge \\
&\exists w_1 \exists w_2\ (w_1 \neq w_2 \wedge Aw_1 \wedge Aw_2 \wedge Gz'w_1 \wedge Gz'w_2)\ \wedge \\
&\forall u_1 \forall u_2 \forall u_3\ ((Gz'u_1 \wedge Gz'u_2 \wedge Gz'u_3 \wedge C^b u_1 \wedge C^b u_2 \wedge C^b u_3) \rightarrow \\
&\forall v((Gz'v \wedge C^b v) \rightarrow (v = u_1 \vee v = u_2 \vee v = u_3)))] \\
&\rightarrow \\
&(Gxz' \wedge Gyz' \wedge Gzz')\}
\end{aligned}
$$

Every three cylinders glower at any triangular prism that glowers at at least two arches and at at most three cubes.

$$
\forall x \forall y \forall z \forall z' \left\{ \begin{array}{c}
\left[\begin{array}{c}
x \neq y \land y \neq z \land x \neq z \\
\land \\
Cx \land Cy \land Cz \\
\land \\
Tz' \\
\land \\
\exists w_1 \exists w_2 \ (w_1 \neq w_2 \land Aw_1 \land Aw_2 \land Gz'w_1 \land Gz'w_2) \\
\land \\
\forall u_1 \forall u_2 \forall u_3 \ \left(\begin{array}{c}
\left[Gz'u_1 \land Gz'u_2 \land Gz'u_3 \land C^b u_1 \land C^b u_2 \land C^b u_3 \right] \\
\rightarrow \\
\forall v \left[(Gz'v \land C^b v) \rightarrow (v = u_1 \lor v = u_2 \lor v = u_3) \right]
\end{array} \right)
\end{array} \right] \\
\rightarrow \\
(Gxz' \land Gyz' \land Gzz')
\end{array} \right\}
$$

Intelligent Artificial Multi-Agent

Tentacular AI™ AI
at Work in Problem-Solving in VQ+AJV

RAIR
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The Resurrection of PERI: PERI.2! (MechE?)

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