

Propositional Calculus II:

Two more Rules of Inference/Inference Schemata

(conditional elim = *modus ponens*;

proof by cases = *disjunction elimination*),

Applying Them to Additional Motivating Problems

Selmer Bringsjord

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Lally School of Management & Technology
Rensselaer Polytechnic Institute (RPI)
Troy, New York 12180 USA

Intro to Logic
1/31/2022



Logistics ...

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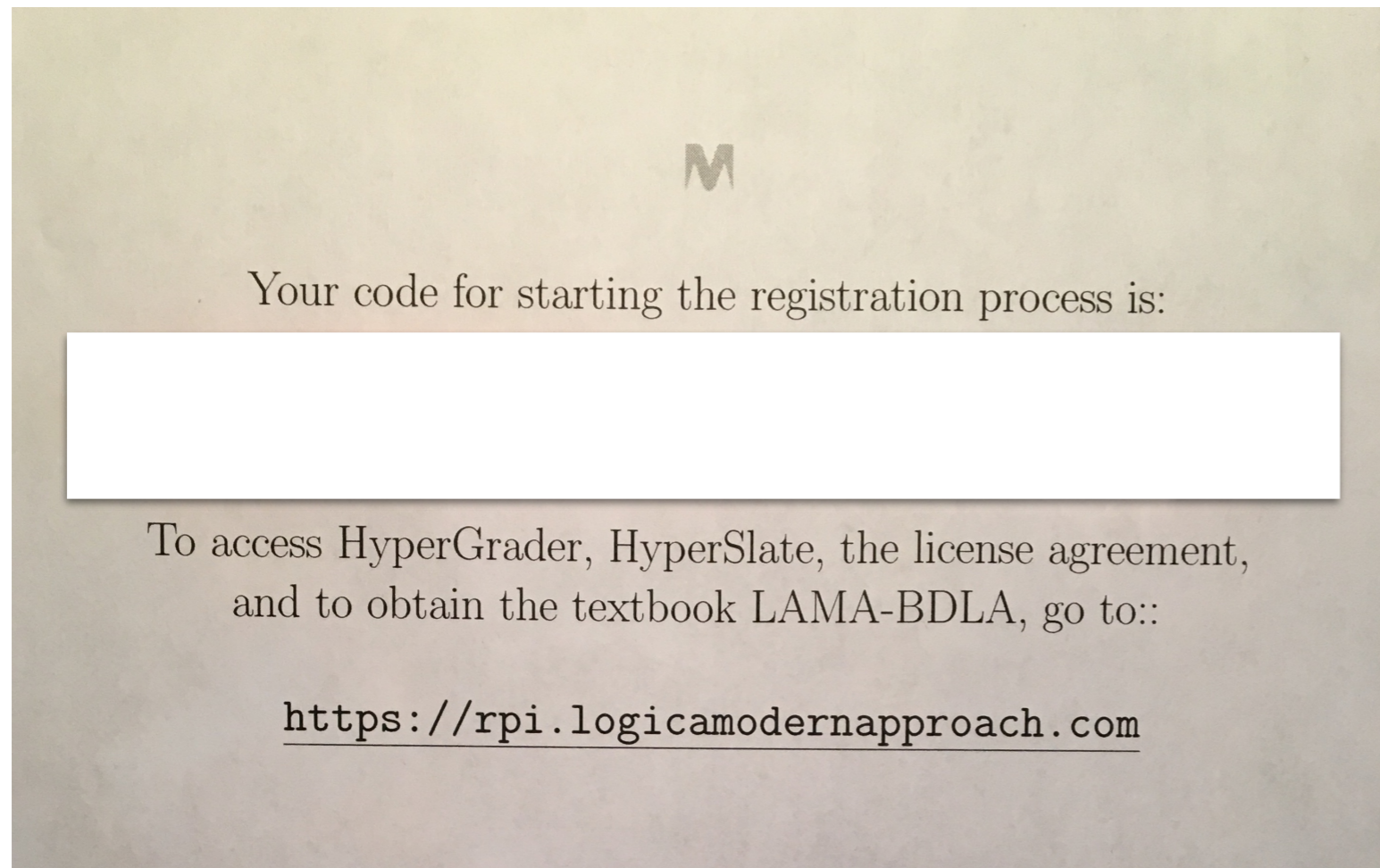
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Your code for starting the registration process is:

To access HyperGrader, HyperSlate, the license agreement,
and to obtain the textbook LAMA-BDLA, go to::

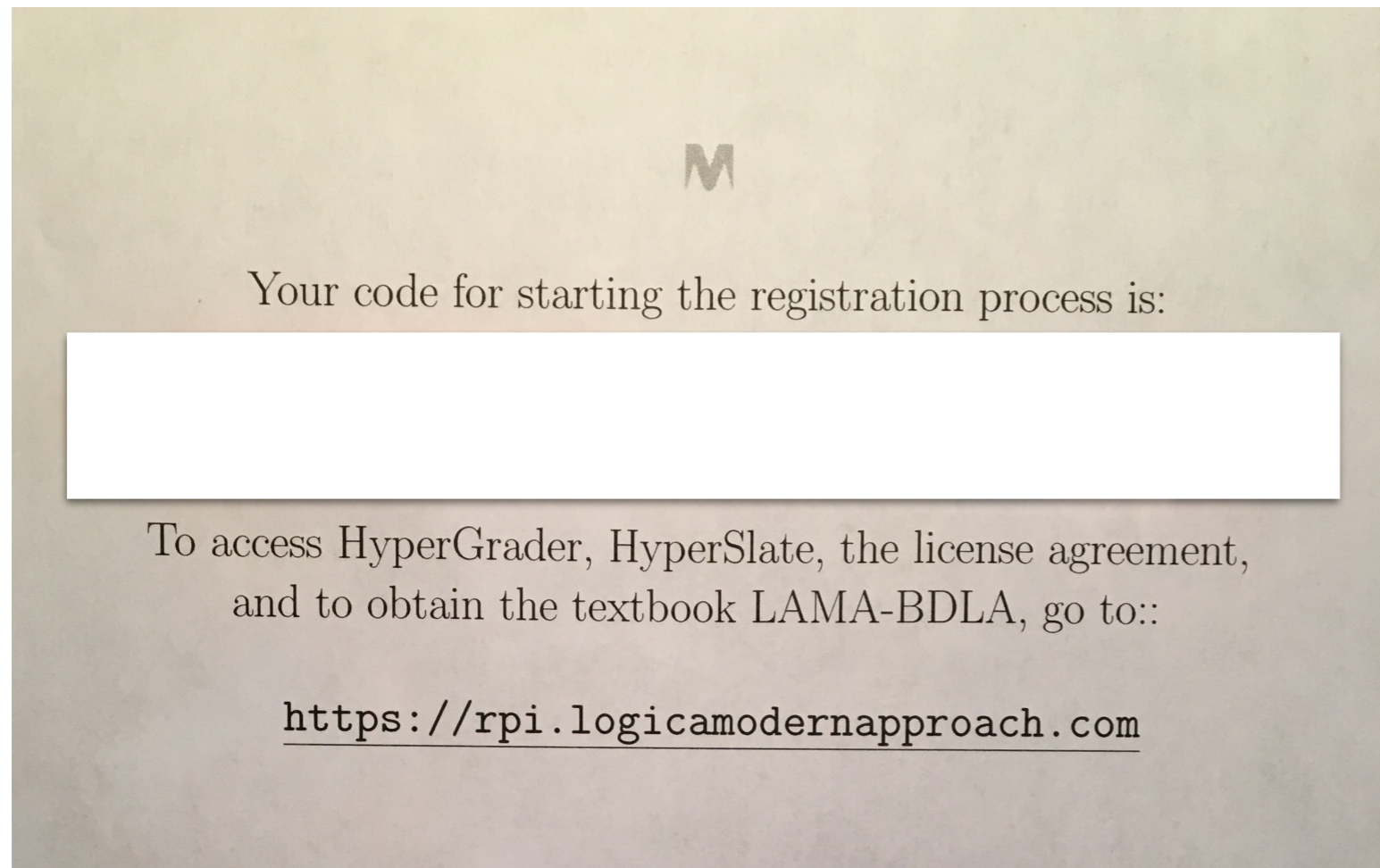
<https://rpi.logicamodernapproach.com>

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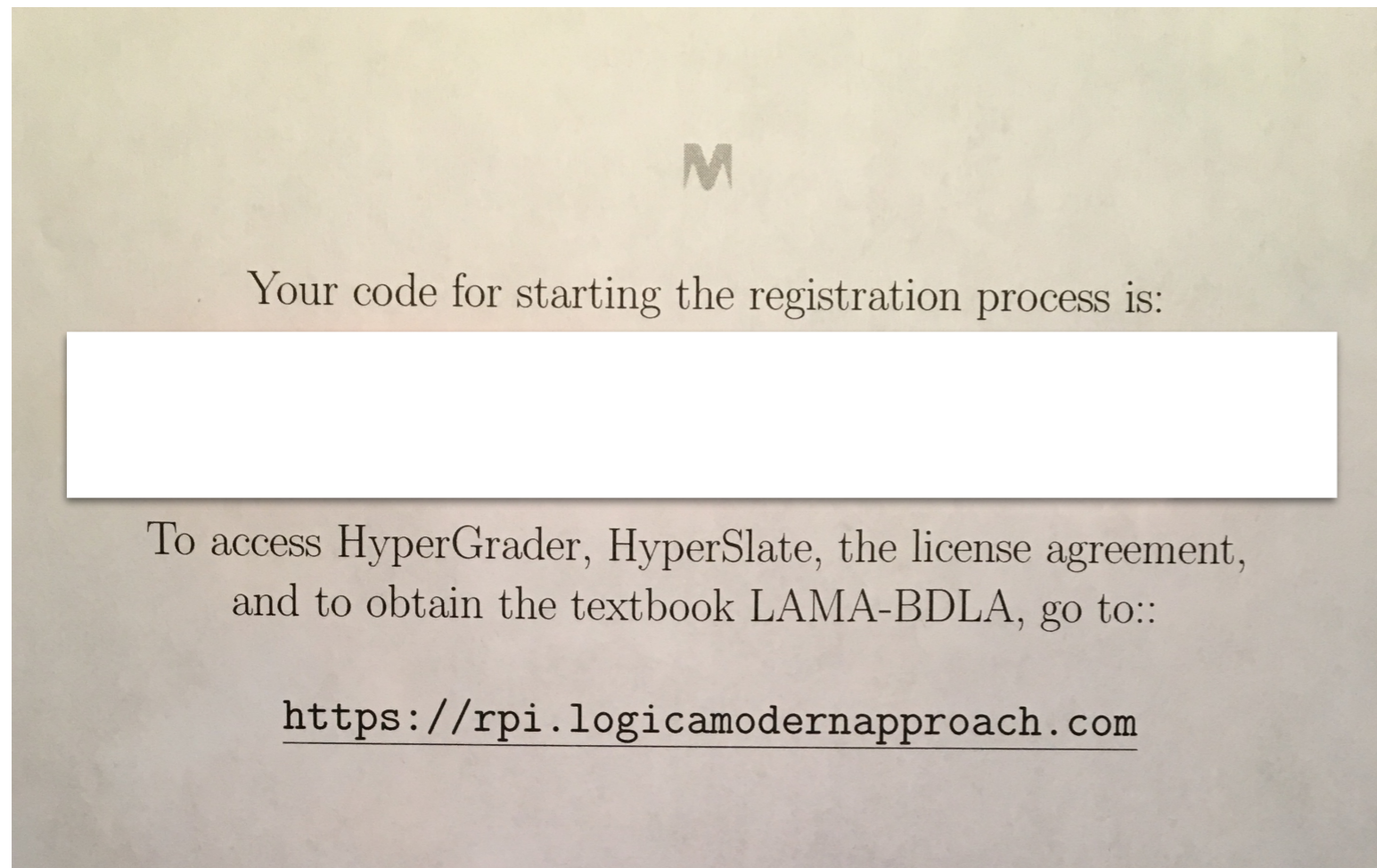
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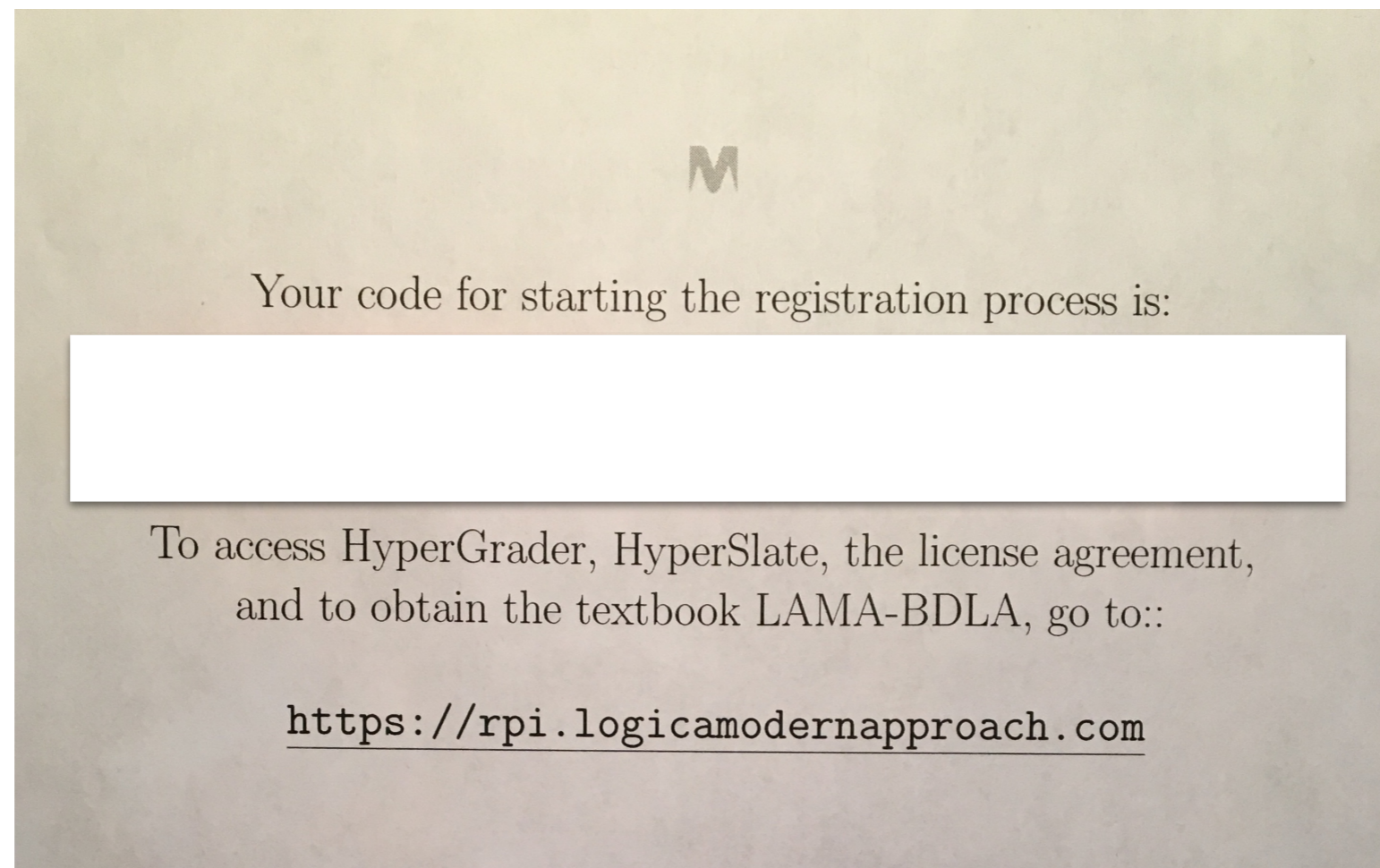
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The email address you enter is case-sensitive!

Your OS and browser must be fully up-to-date; Chrome is the best choice, browser-wise (though I use Safari).

The Starting Code Purchased in Bookstore Should
By Now've Been/Soon Be Used to Register & Subsequently Sign In

First prop. calc. (Exercise) Problems:
switching_conjuncts_fine, switching_disjuncts_fine



E-Housekeeping Pts (again)

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- Must input your RIN. (This is your ID at your university.)

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E-Housekeeping Pts (again)

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- Make sure OS fully up-to-date.
- Make sure browser fully up-to-date.
- Chrome best (but I use Safari).
- Always work in the same browser window with multiple tabs; must do this with email and HyperGrader[®] & HyperSlate[®].

Introduction to (Formal) Logic (and AI)

Spring 2021 edition of IFLAII

[Selmer Bringsjord](#)

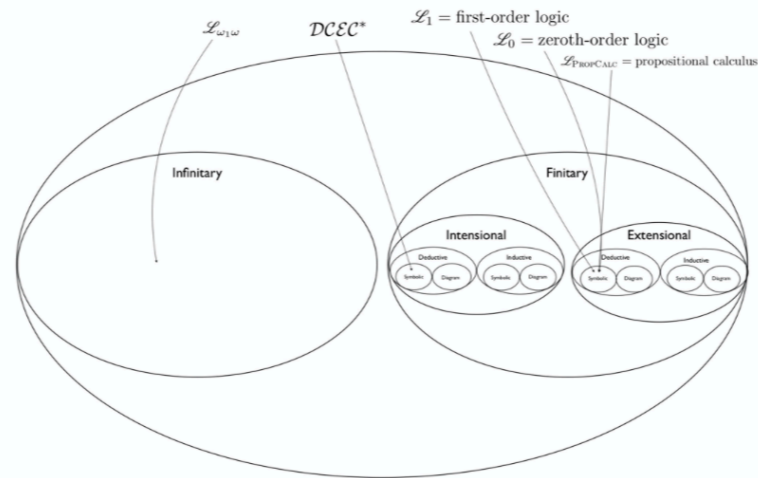
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A fully online course, thanks to singular AI technology.

with [Naveen Sundar G.](#)
^ KB Foush e ^ Joshua Taylor ^ ...

The Universe of Logics



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Intro to Logic
1/31/2022



Last time we introduced and
and lauded the power of
oracles, and questions ...
and now ... picking up
where we left off ...

“NYS 3” Revisited

Given the statements

$$\neg \neg c$$

$$c \rightarrow a$$

$$\neg a \vee b$$

$$b \rightarrow d$$

$$\neg(d \vee e)$$

which one of the following statements must also be true?

$$\neg c$$

e

h

$$\neg a$$

all of the above

“NYS 3” Revisited

Given the statements

$\neg\neg c$

$c \rightarrow a$

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$b \rightarrow d$

$\neg(d \vee e)$

which one of the following statements must also be true?

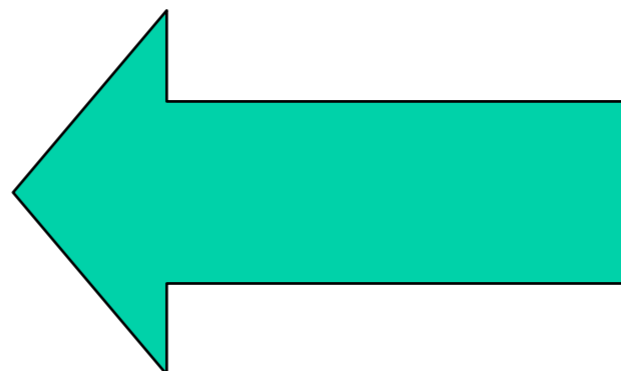
$\neg c$

e

h

$\neg a$

all of the above



“NYS 3” Revisited

Given the statements

$\neg\neg c$

$c \rightarrow a$

$\neg a \vee b$

$b \rightarrow d$

$\neg(d \vee e)$

After last class, should have explored if you are registered ... Show in HyperSlate[®] as I did that each of the first four options can be proved using the PC entailment (= provability) oracle.

which one of the following statements must also be true?

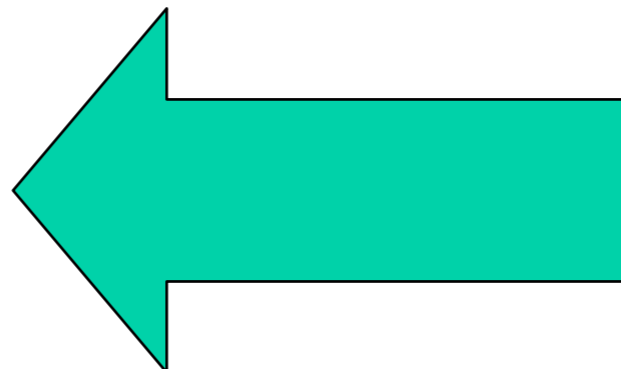
$\neg c$

e

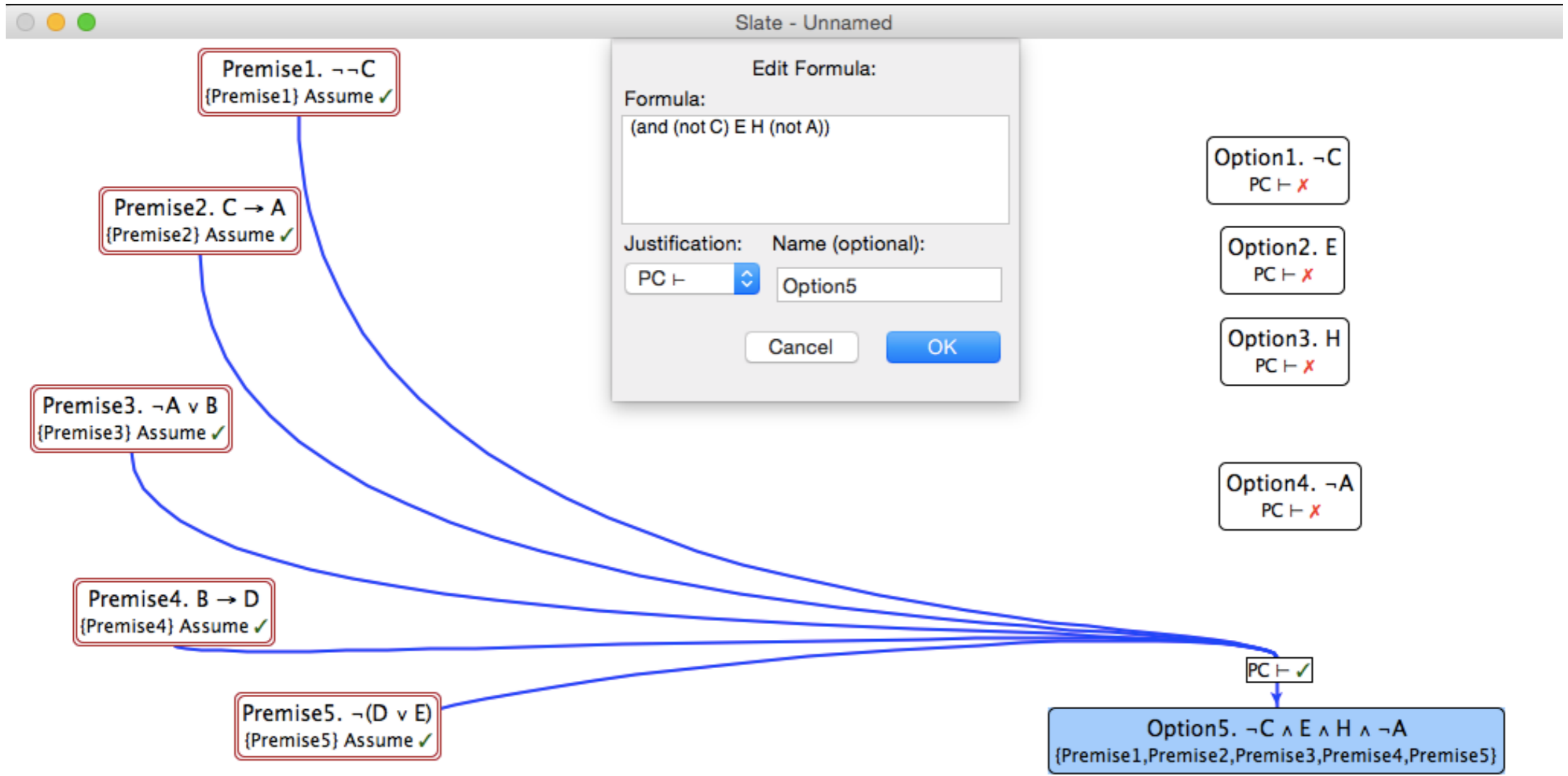
h

$\neg a$

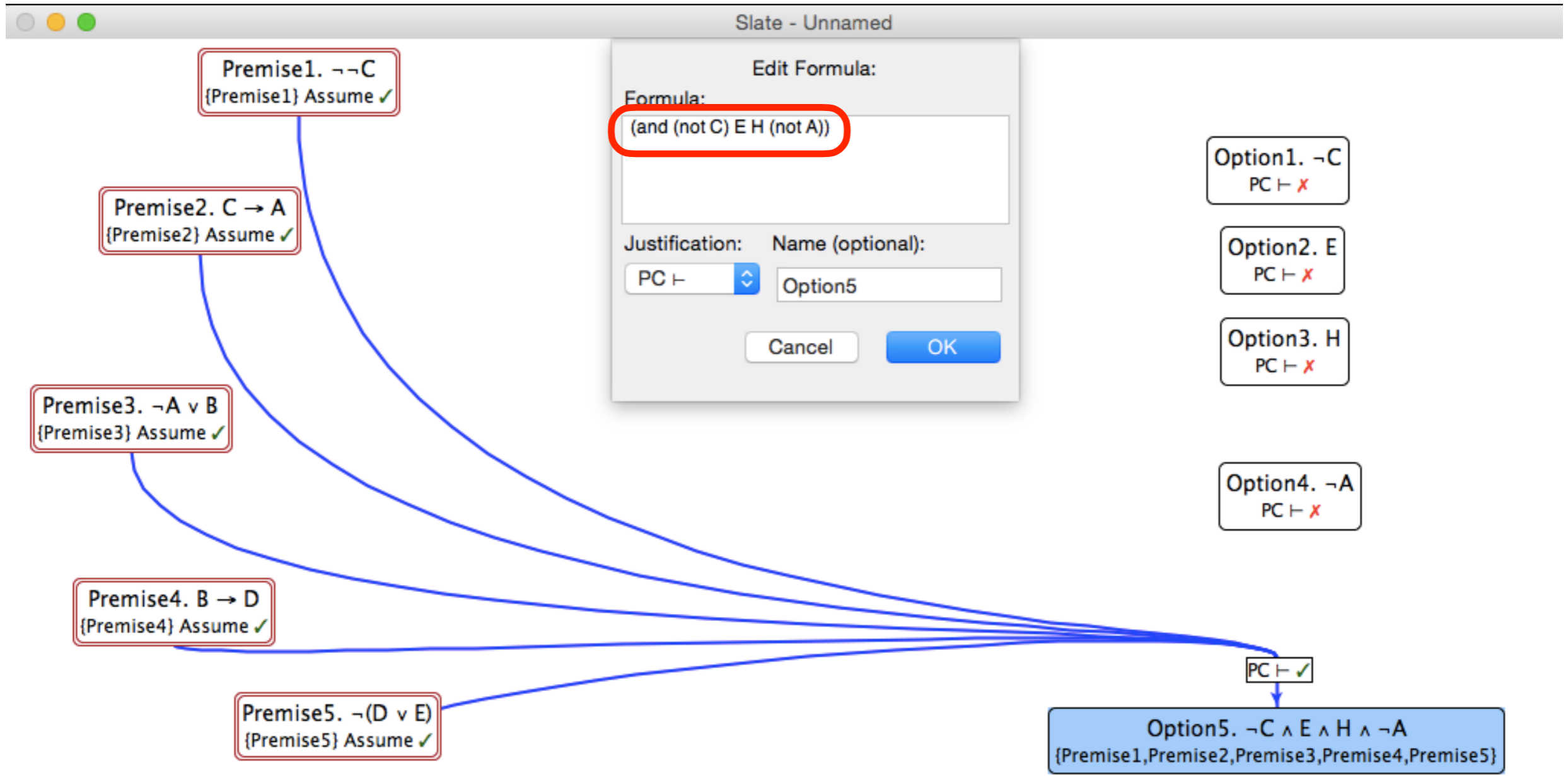
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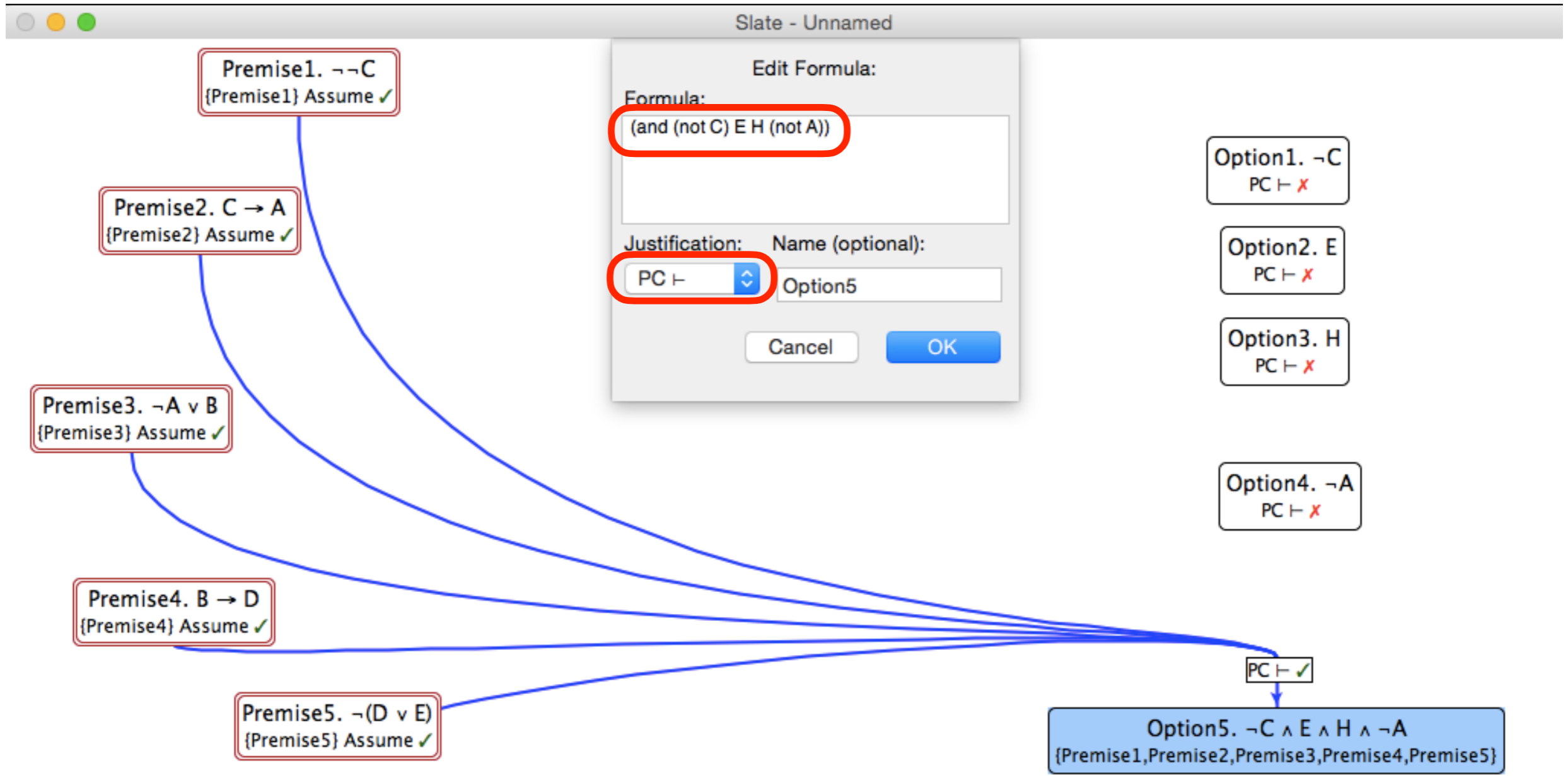
(and (not C) E H (not A))



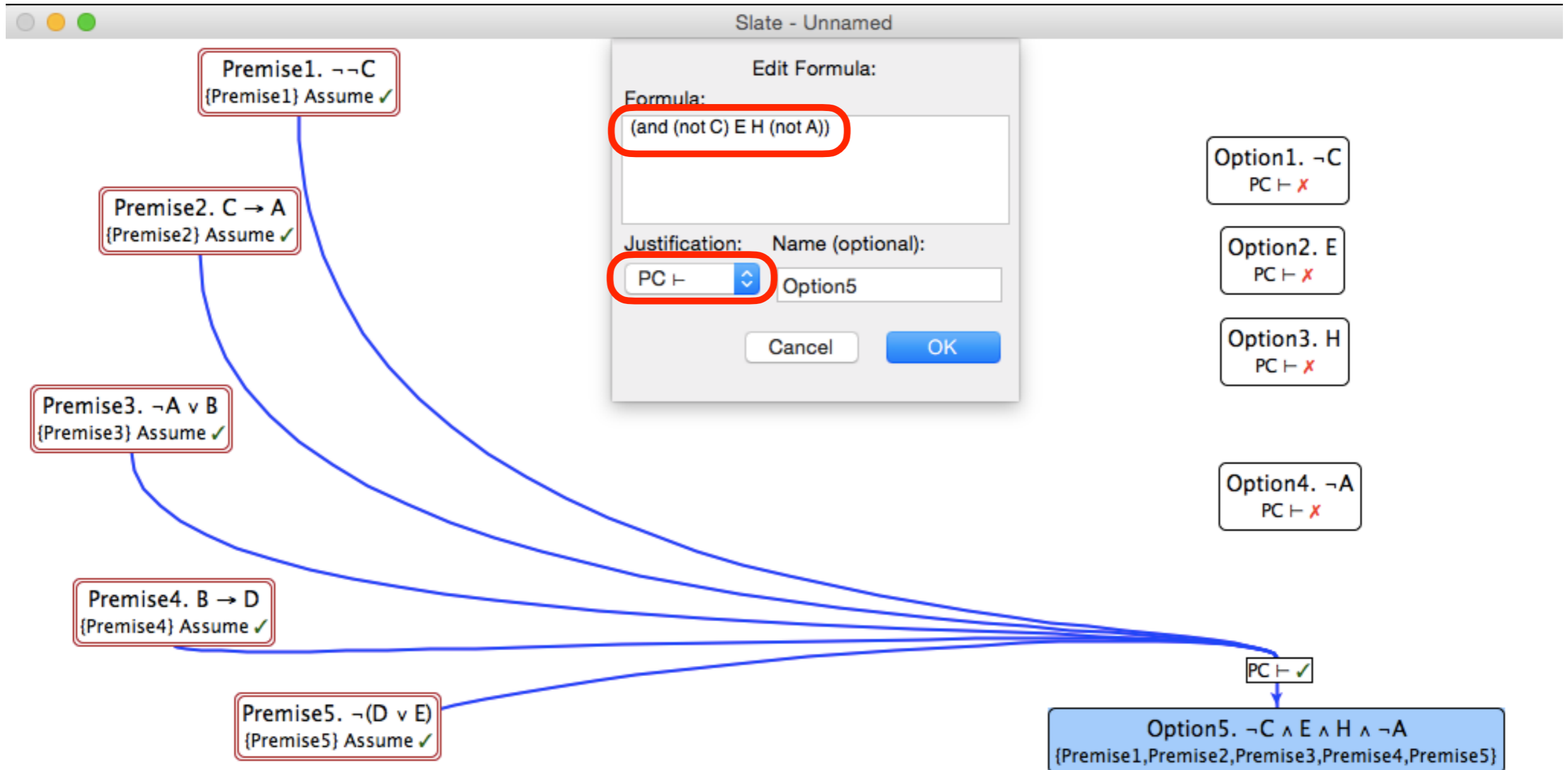
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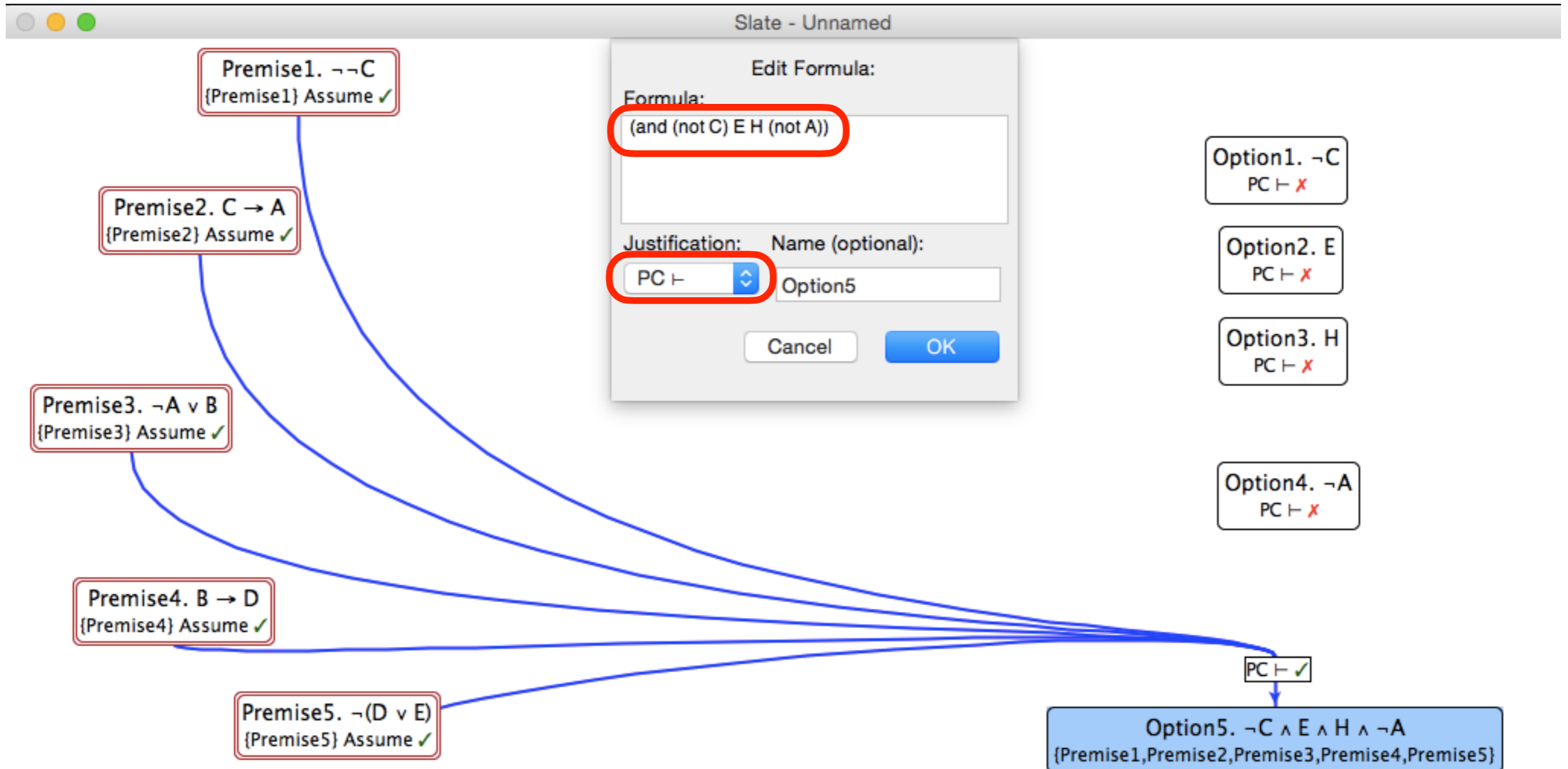
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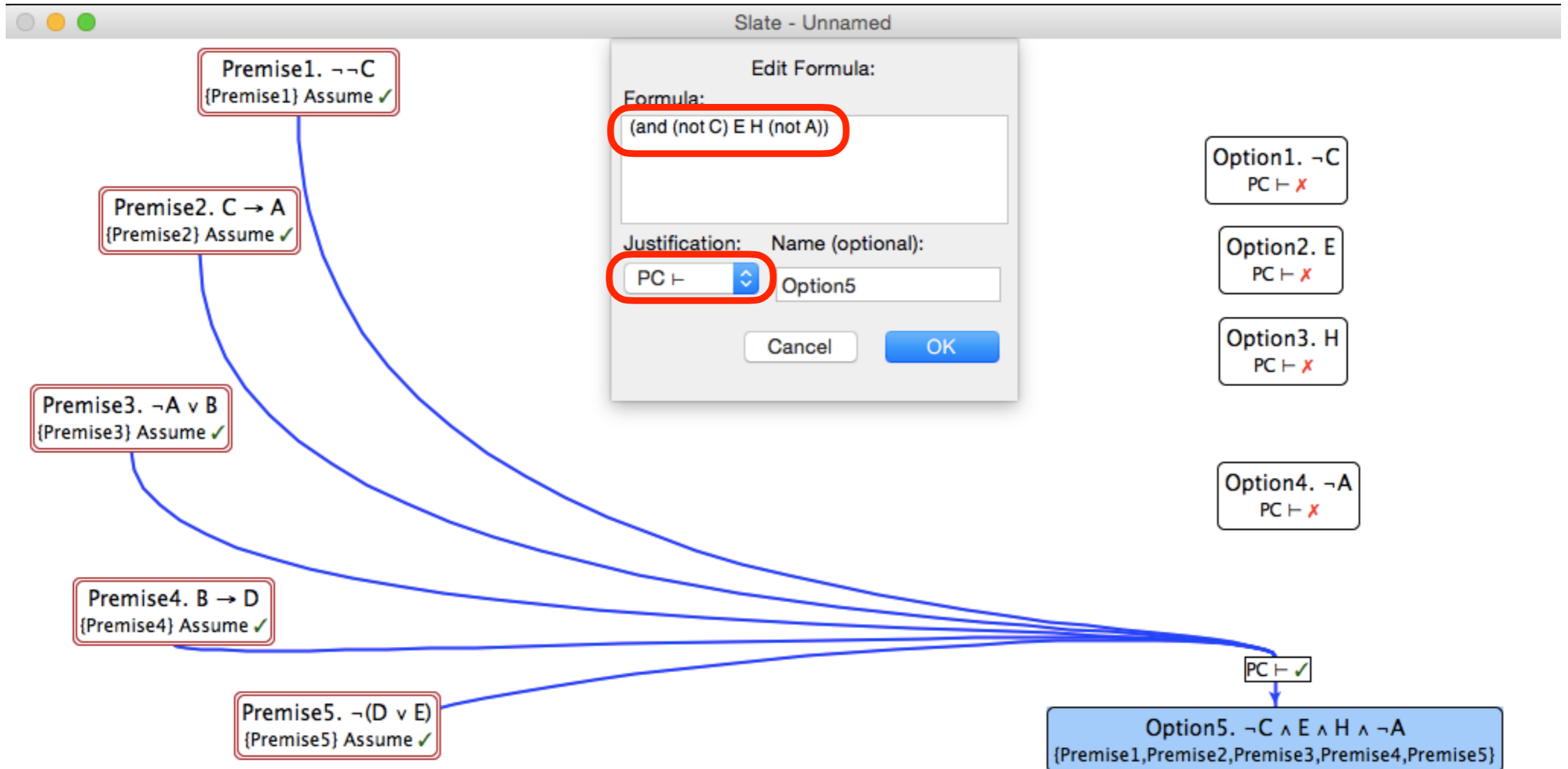
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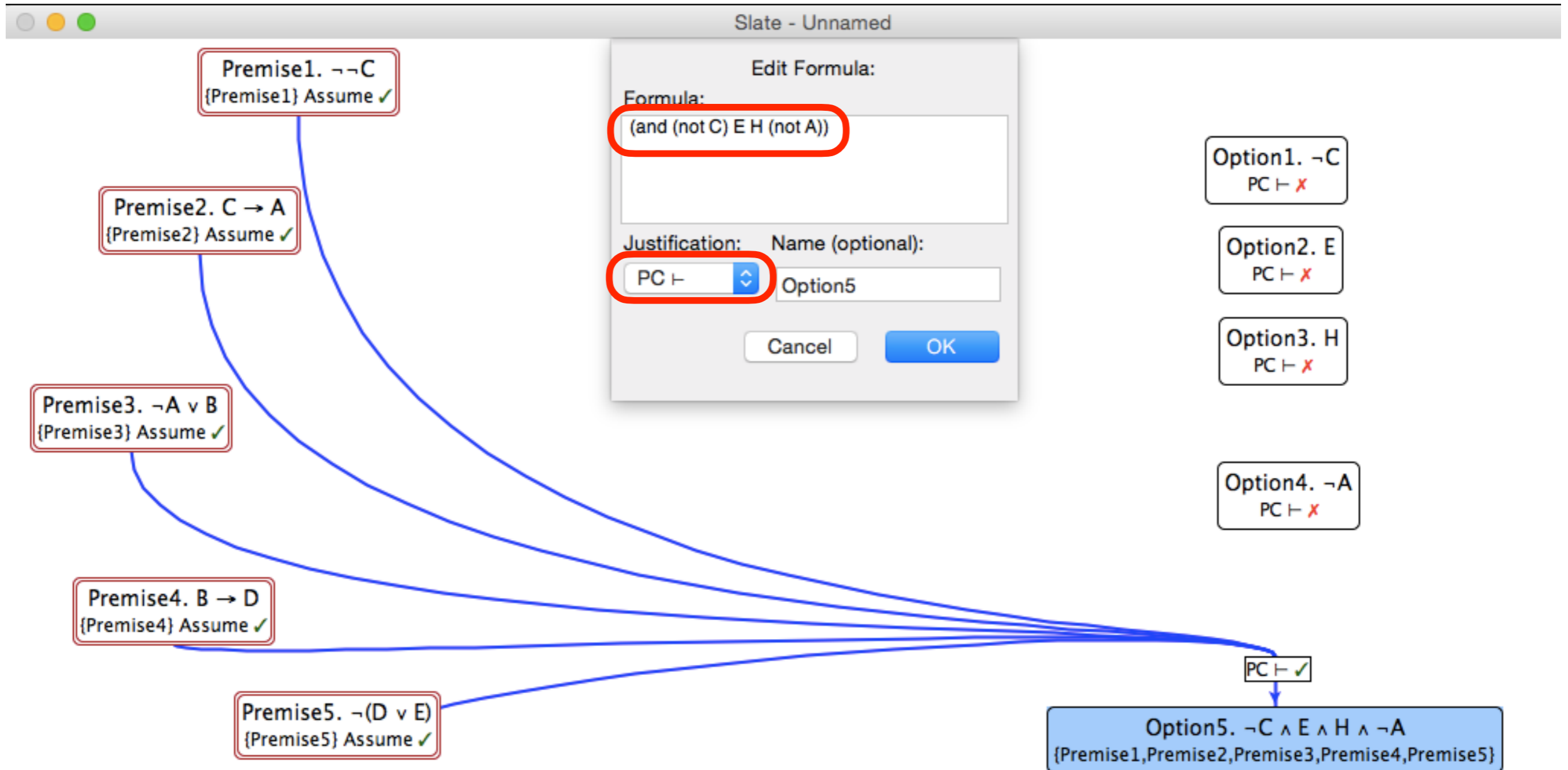
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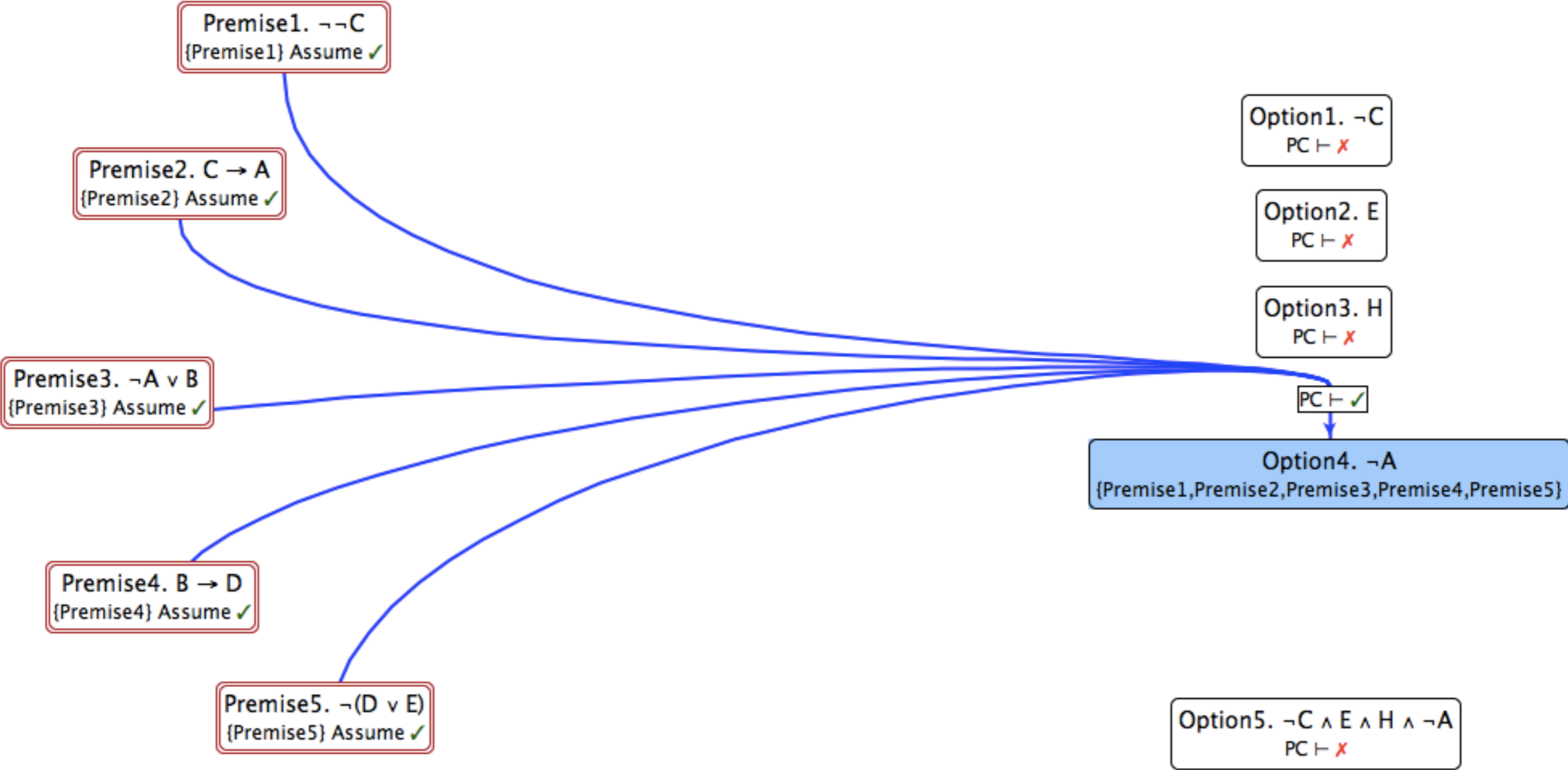


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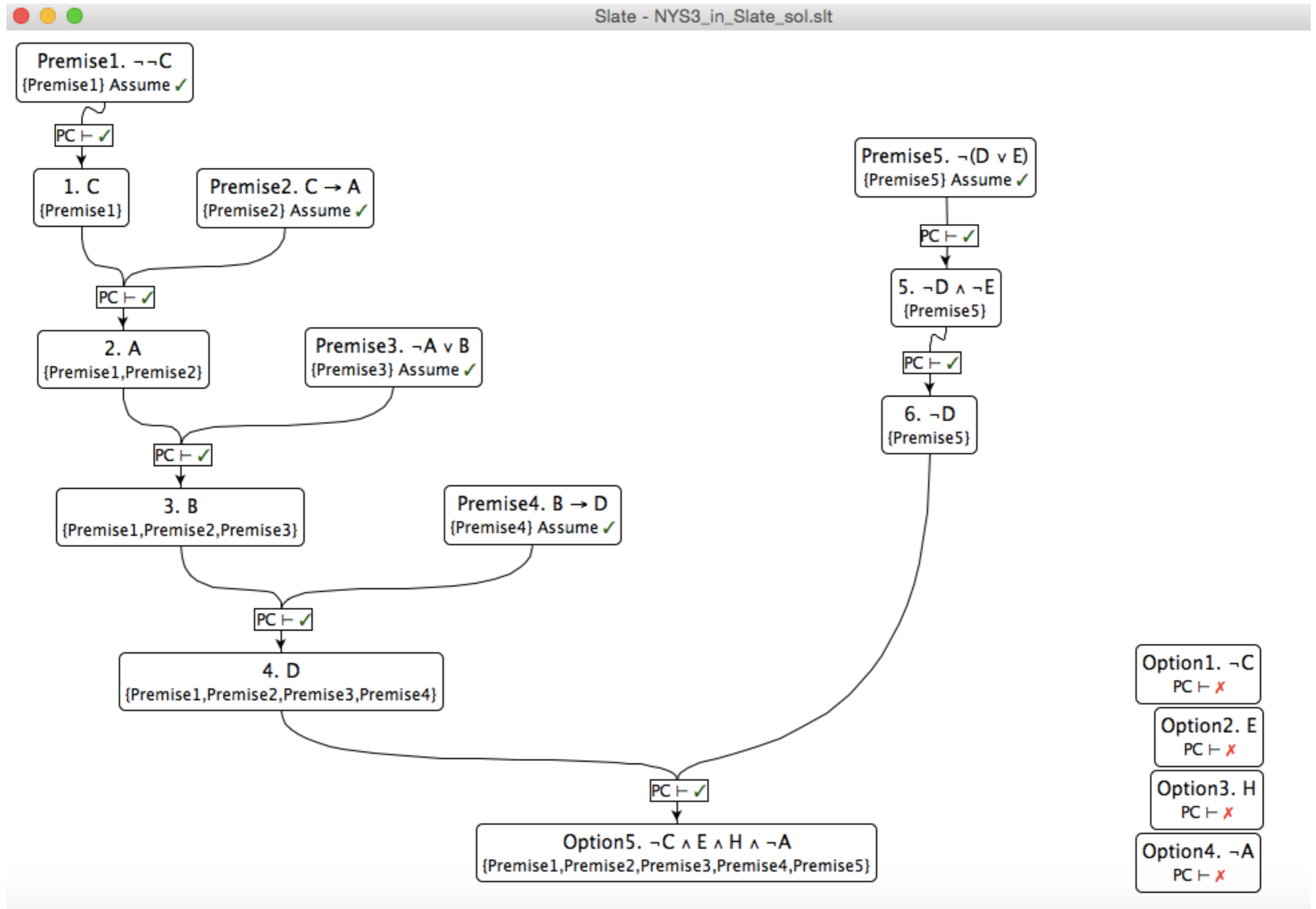


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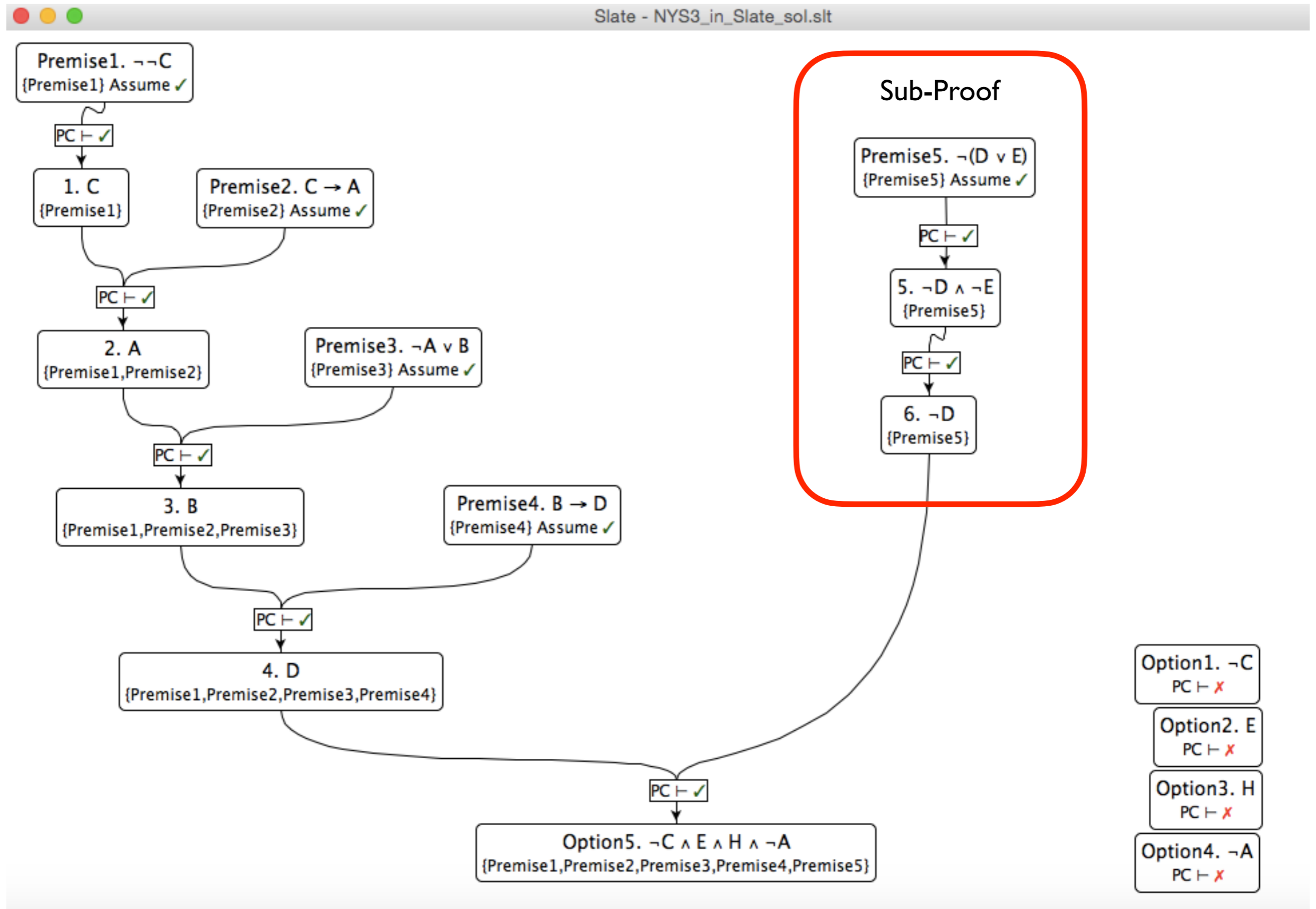
Proof Plan ...

Proof Plan ...

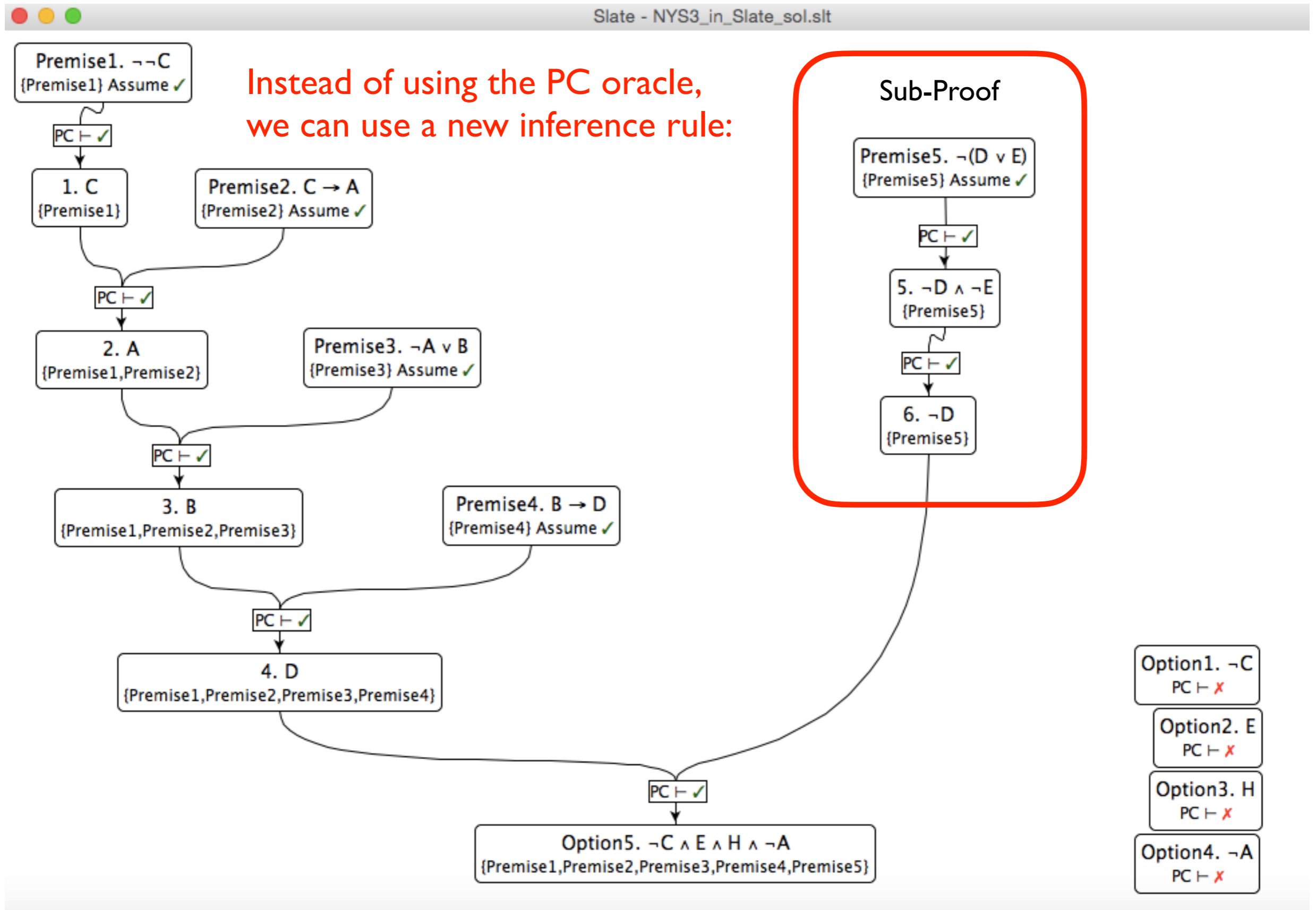
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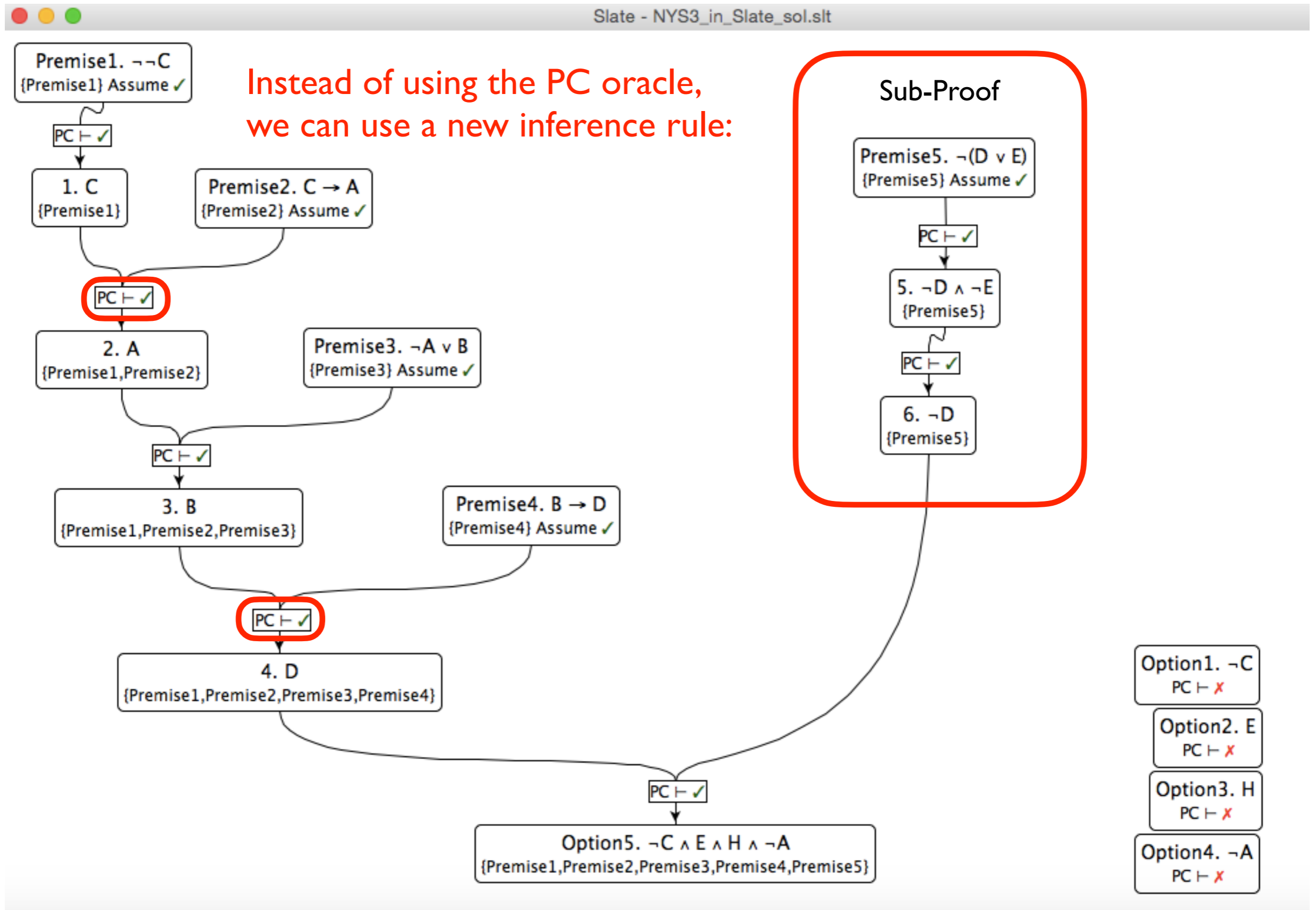
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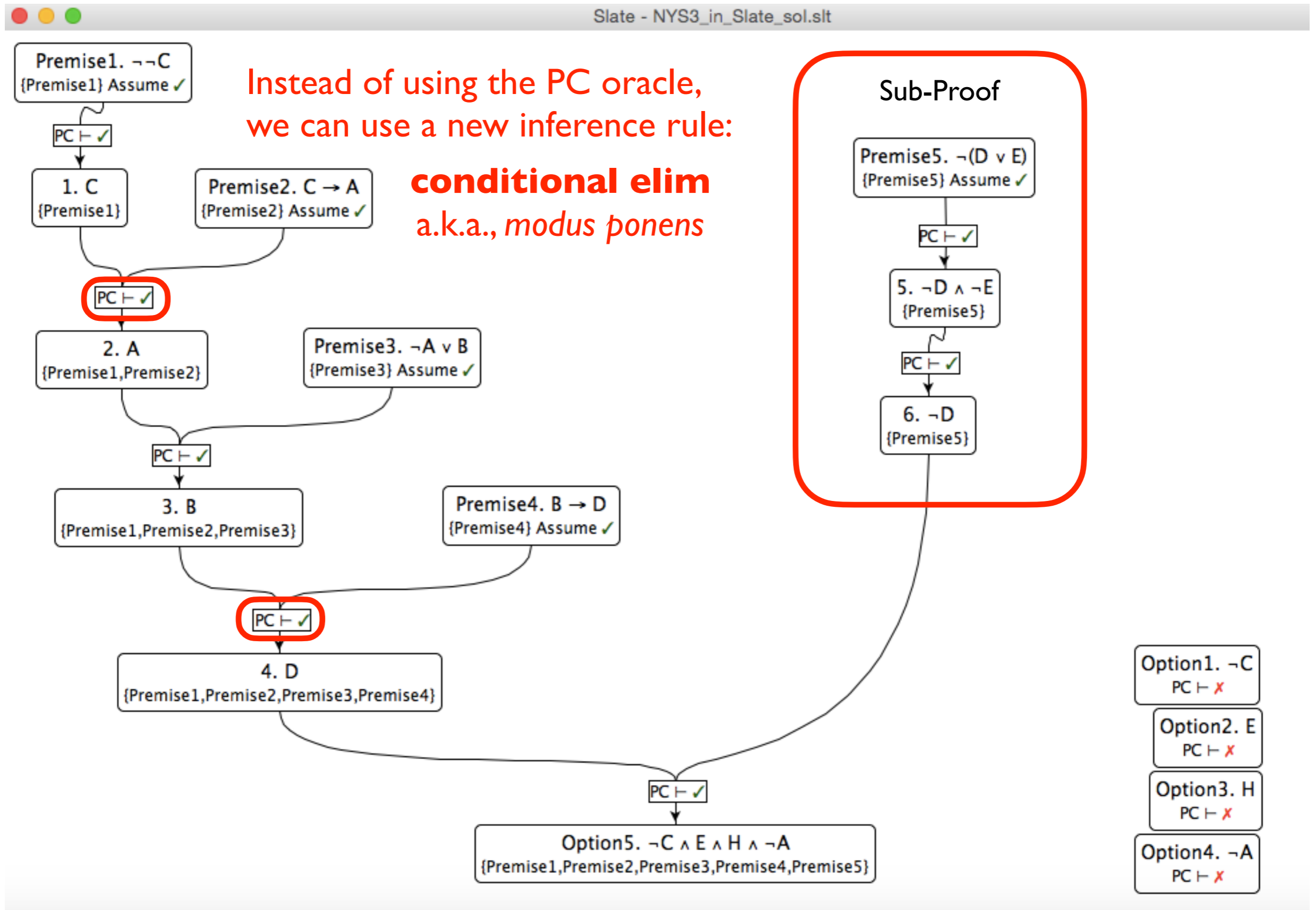
Proof Plan ...



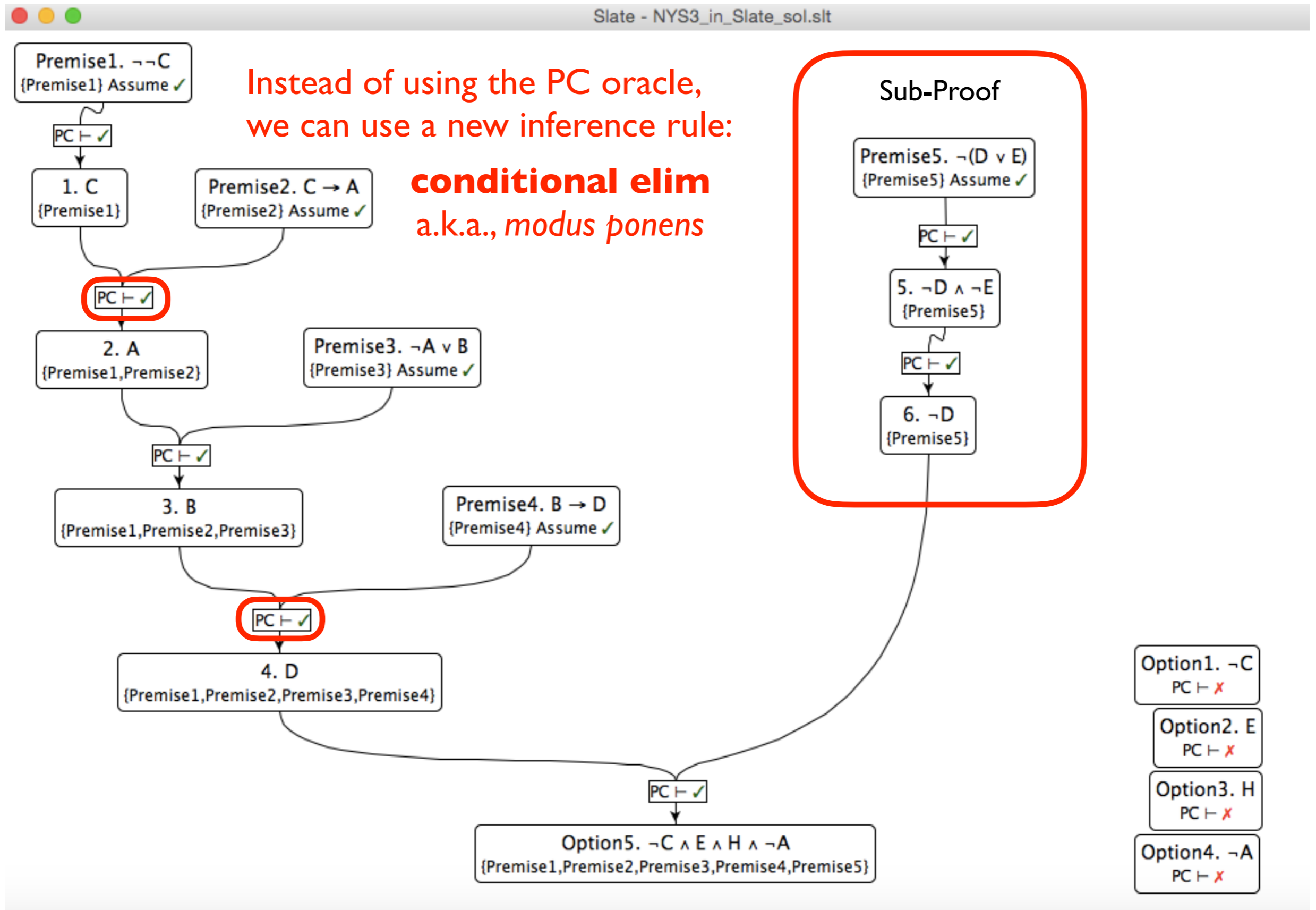
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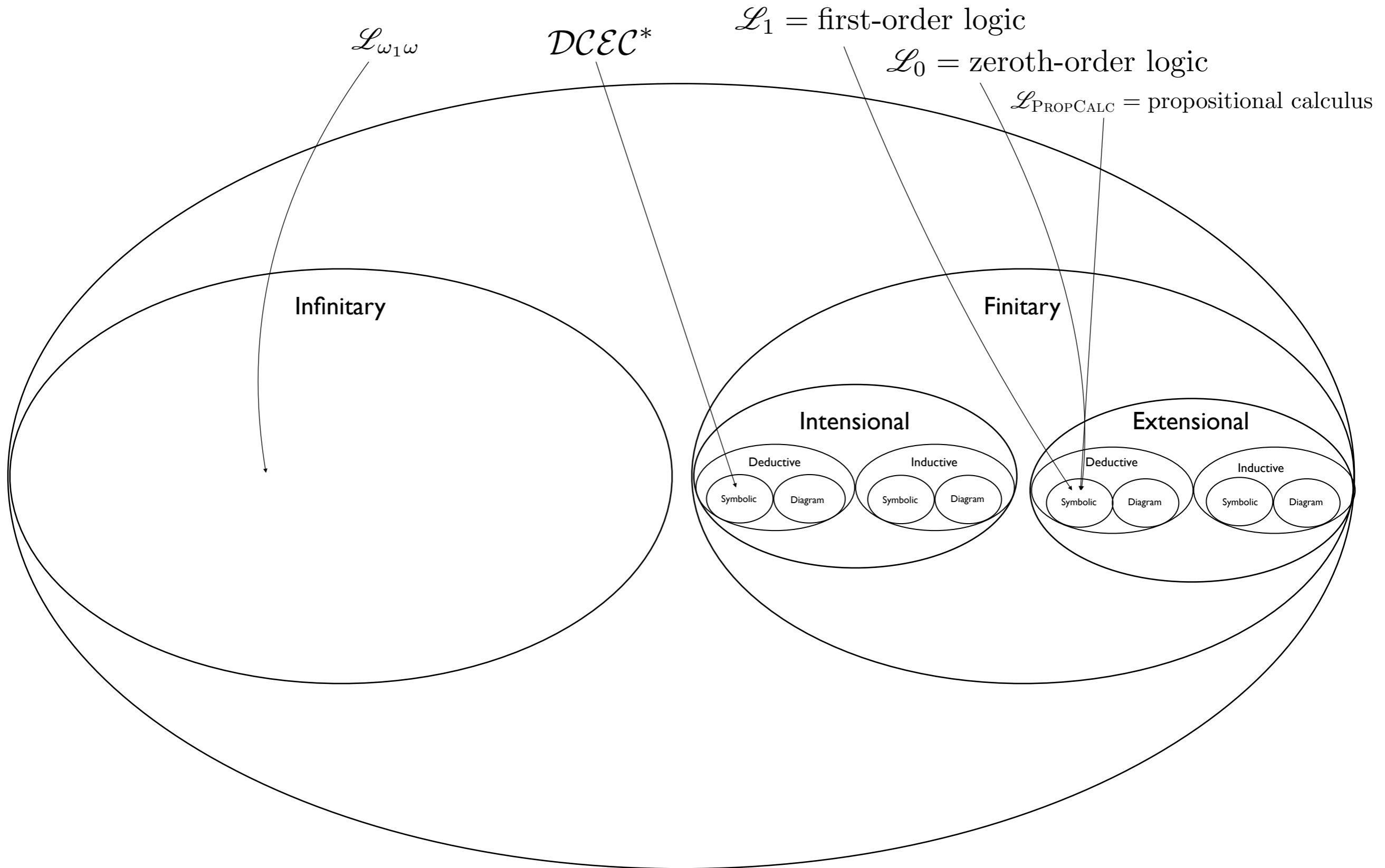
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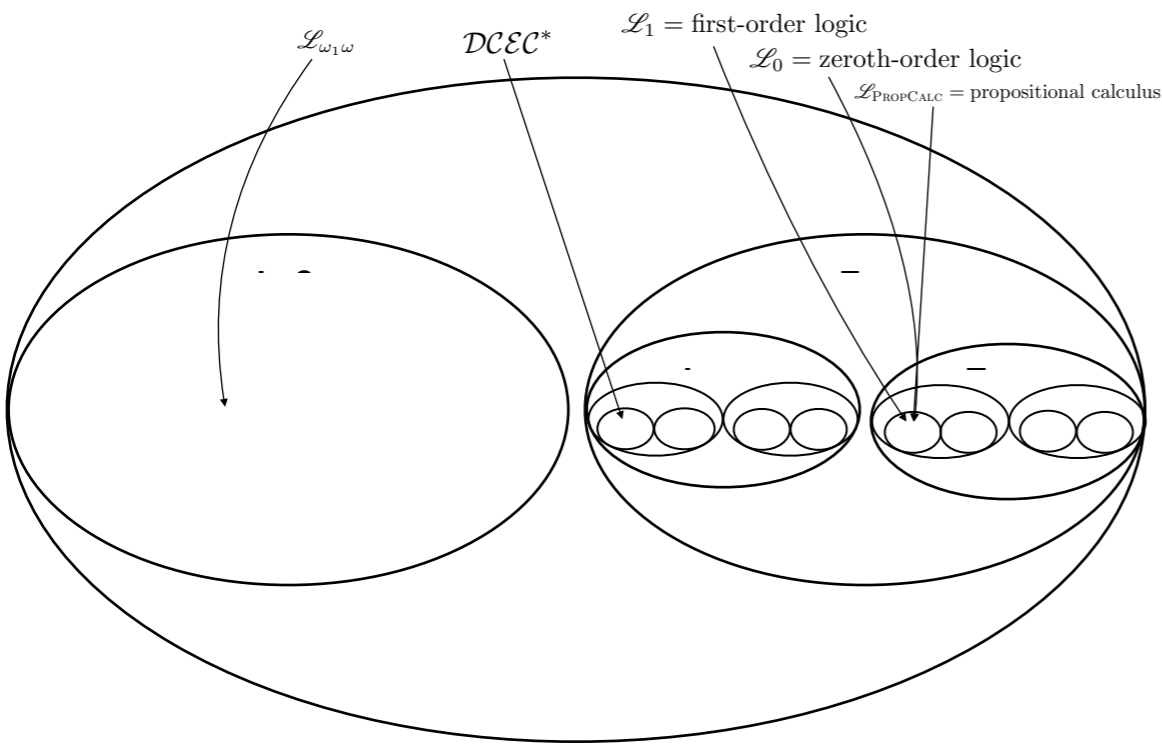
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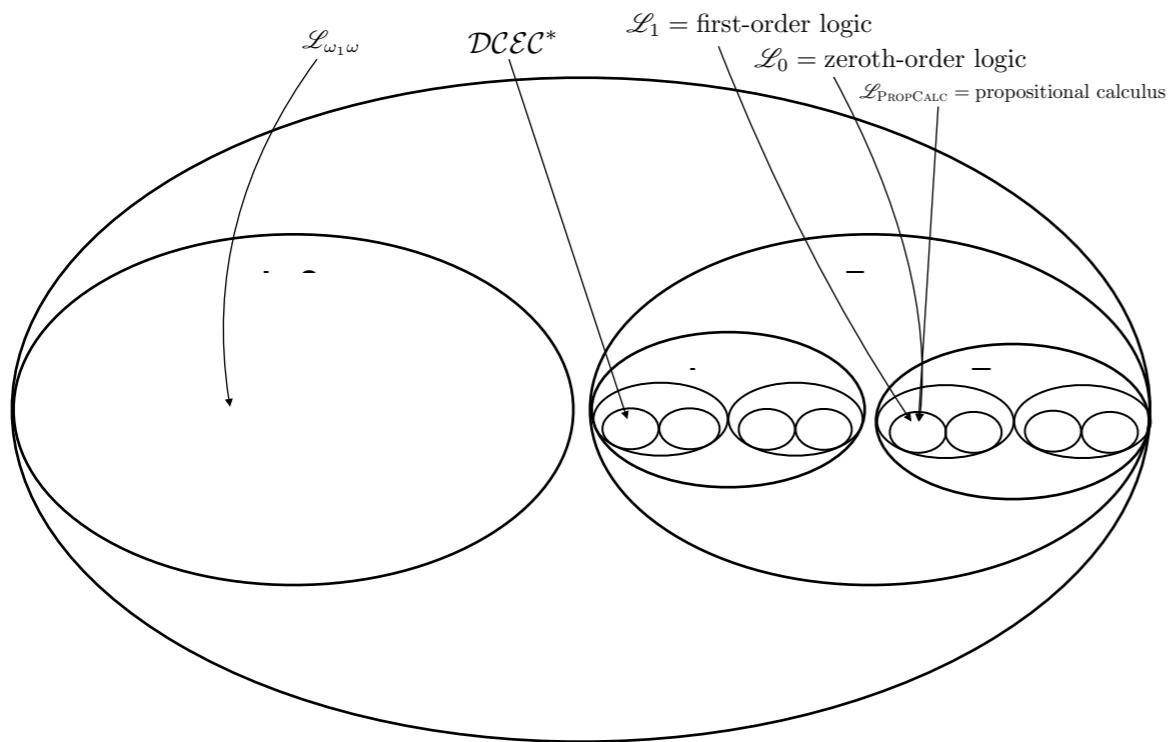
The Universe of Logics



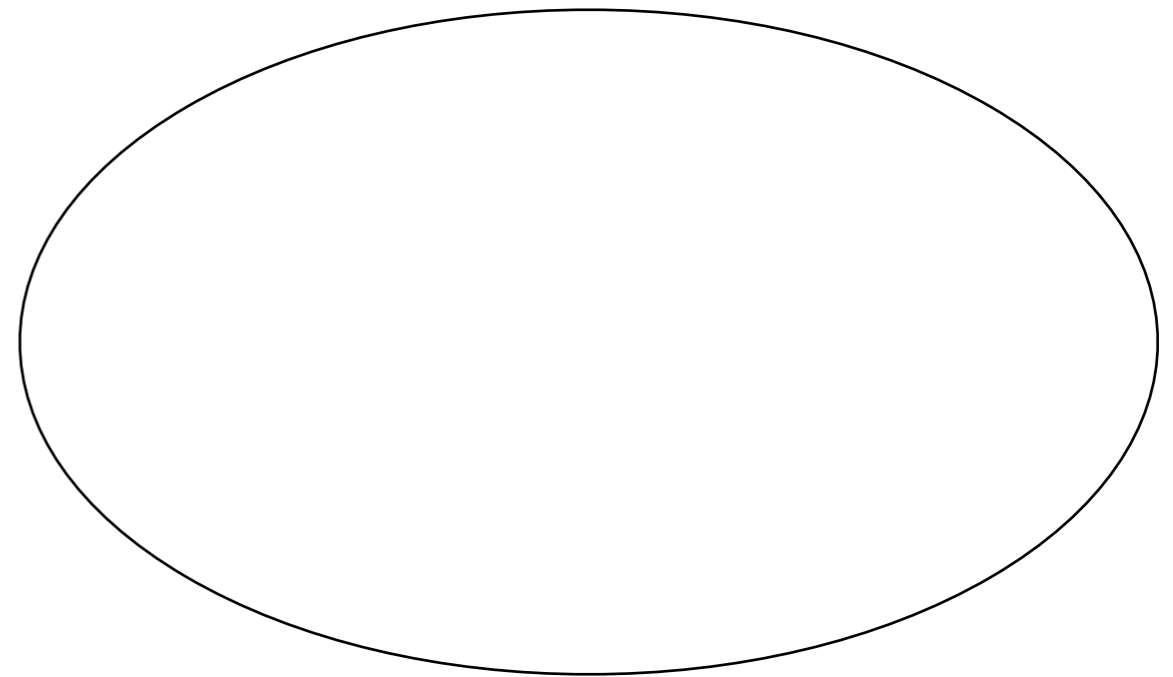
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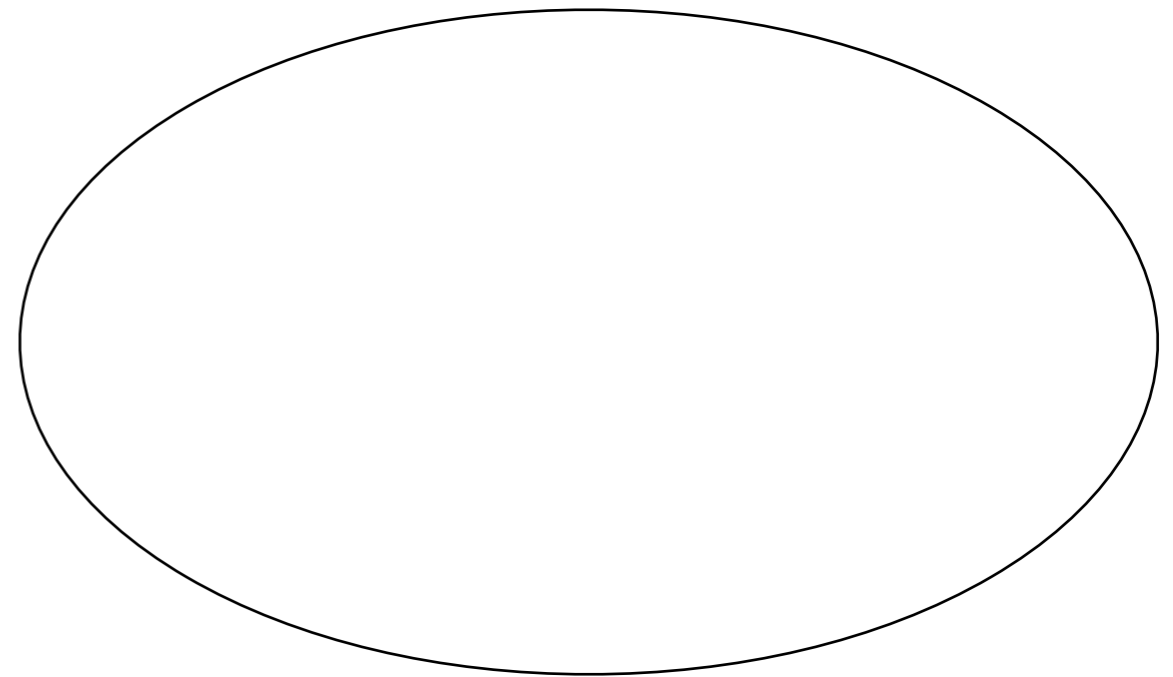
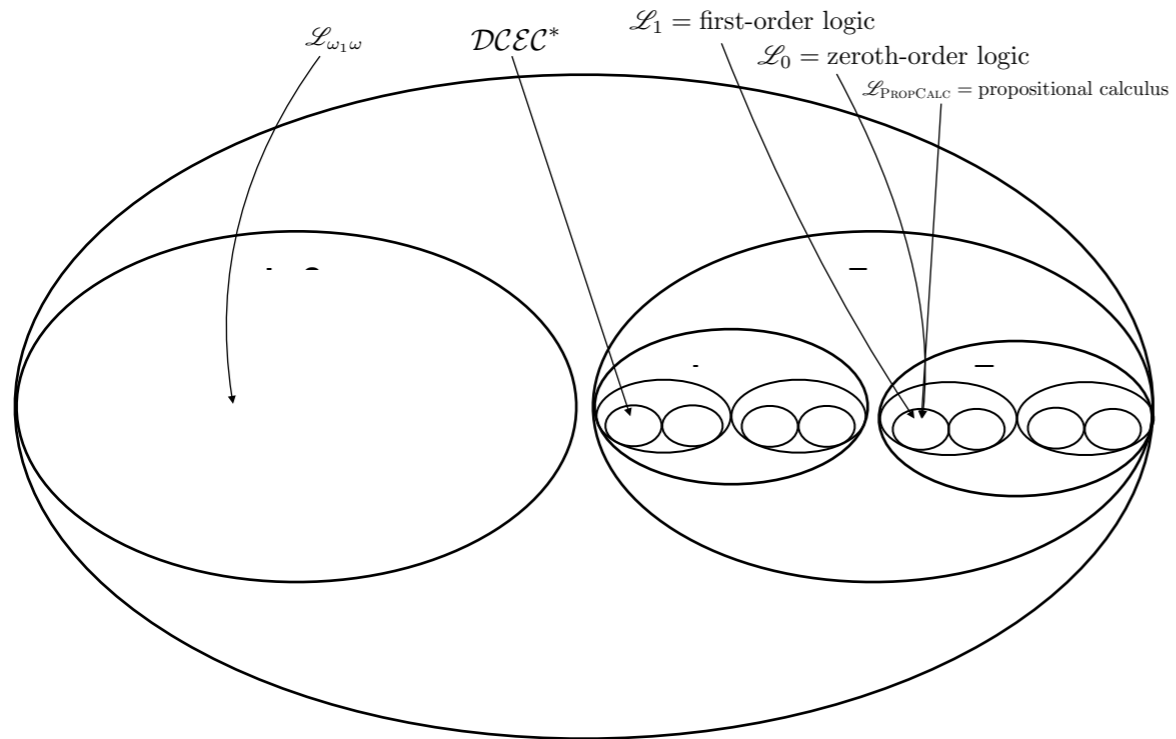


The Physical Universe



The Universe of Logics

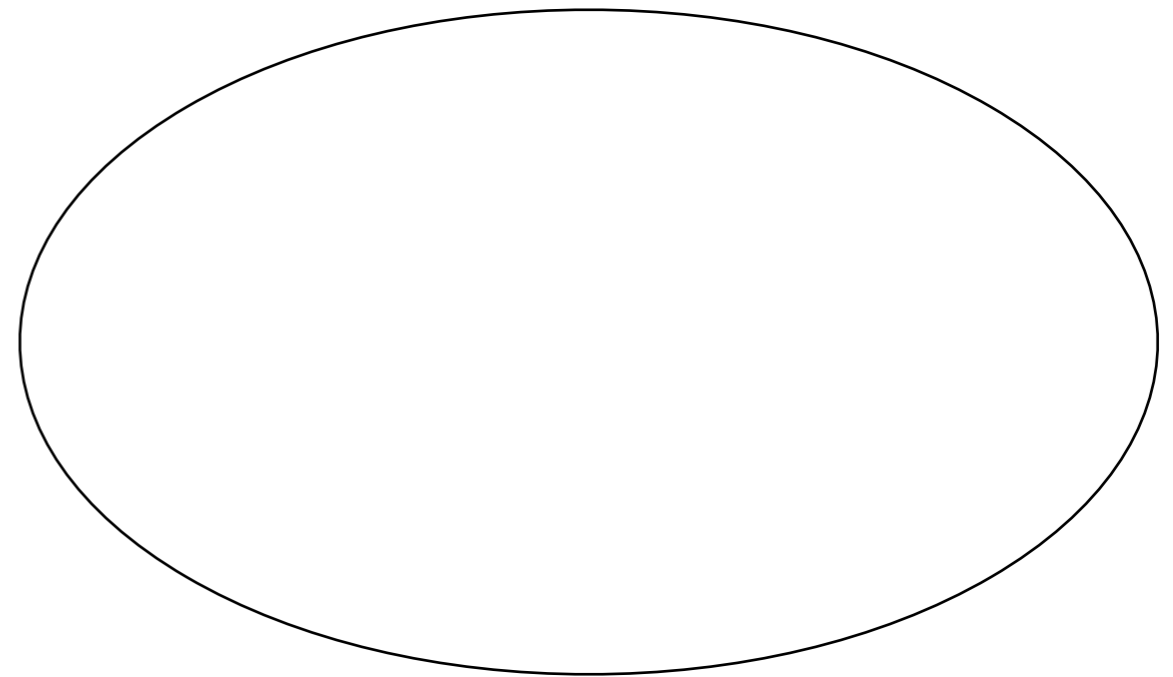
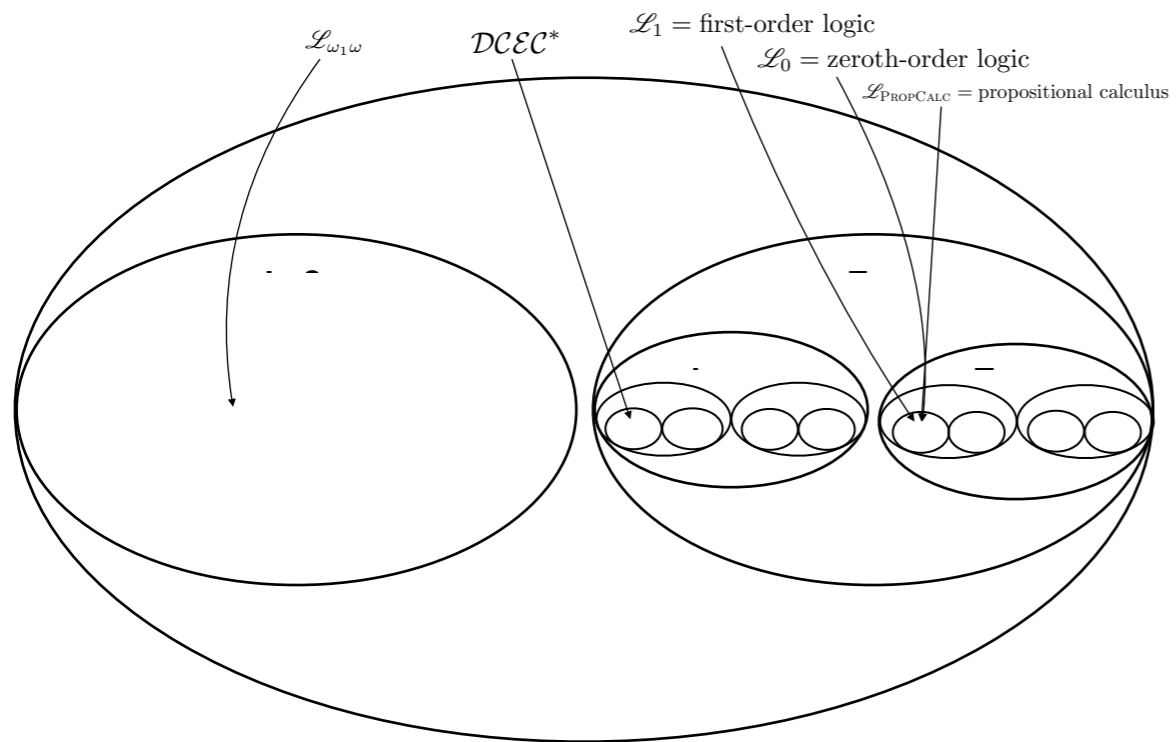
The Physical Universe



Non-Physical

The Universe of Logics

The Physical Universe



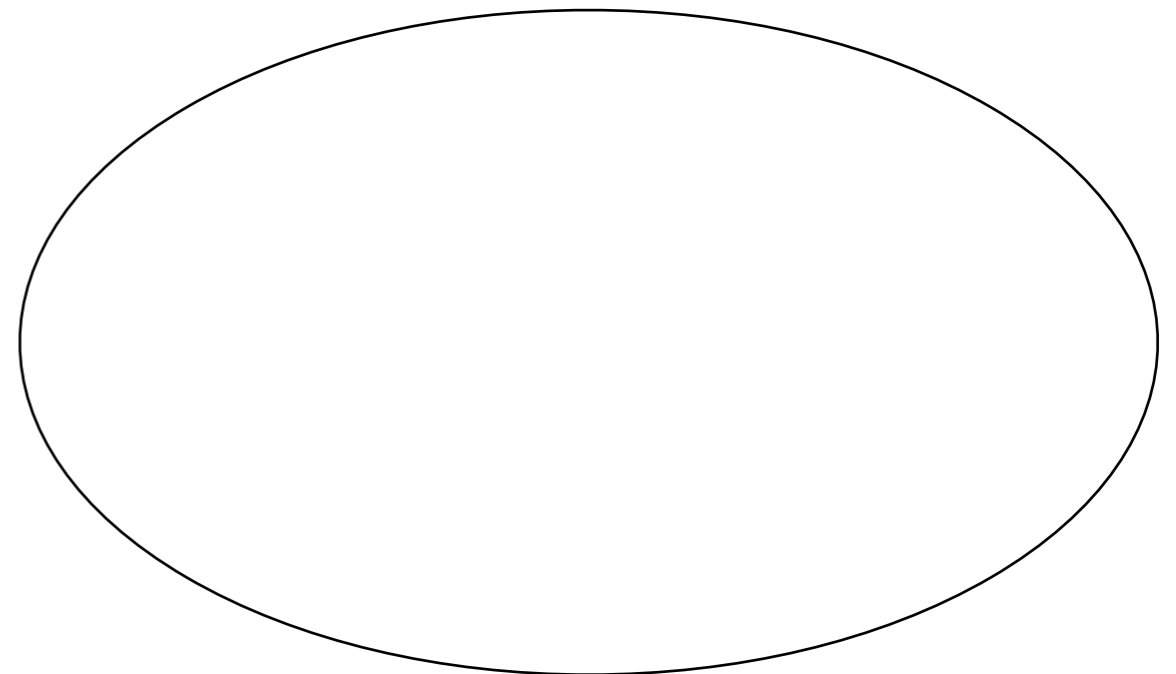
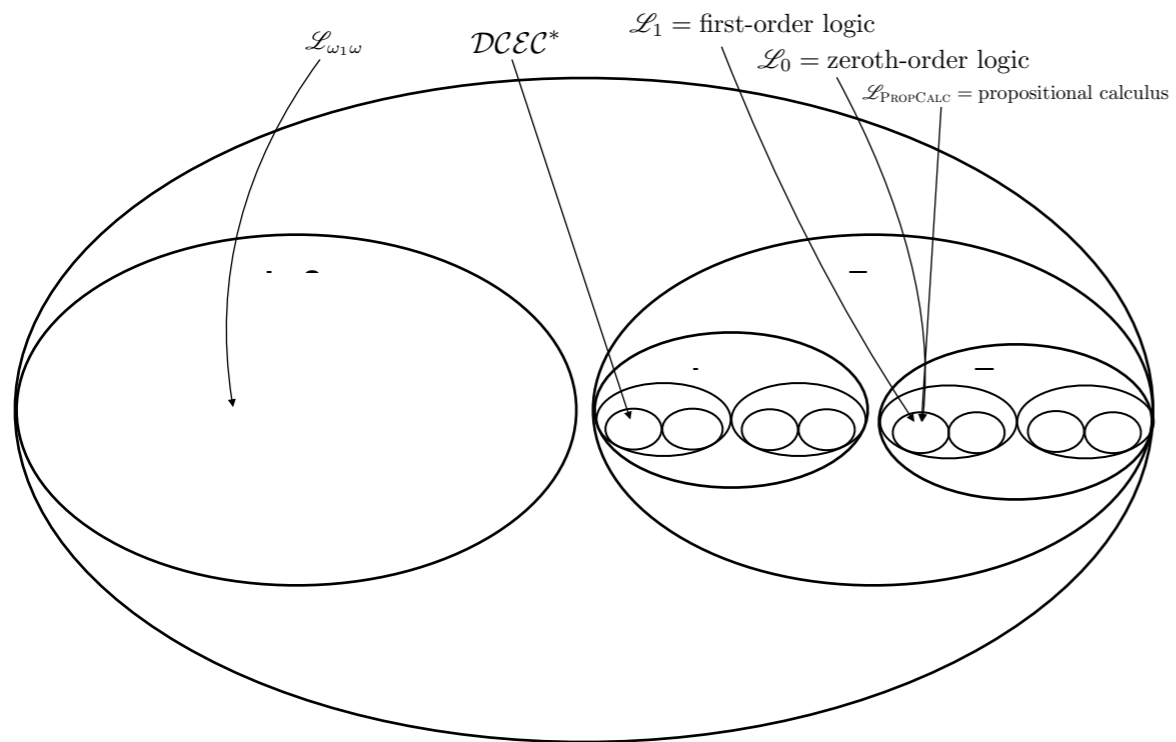
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The Universe of Logics

The Physical Universe



∞

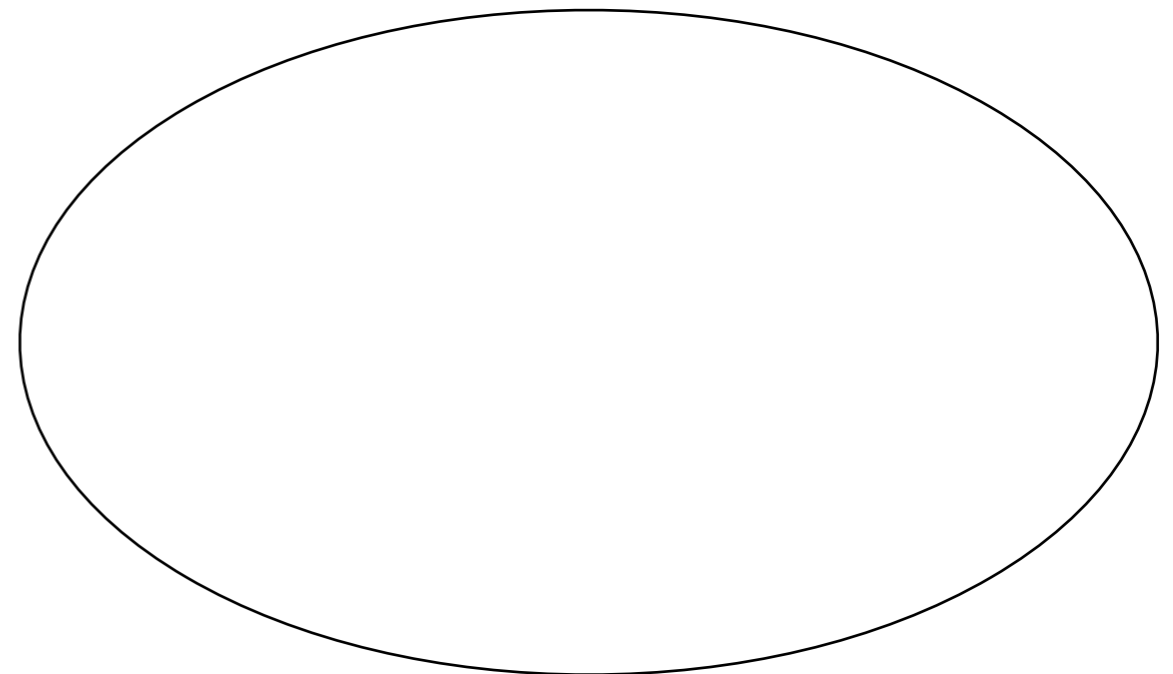
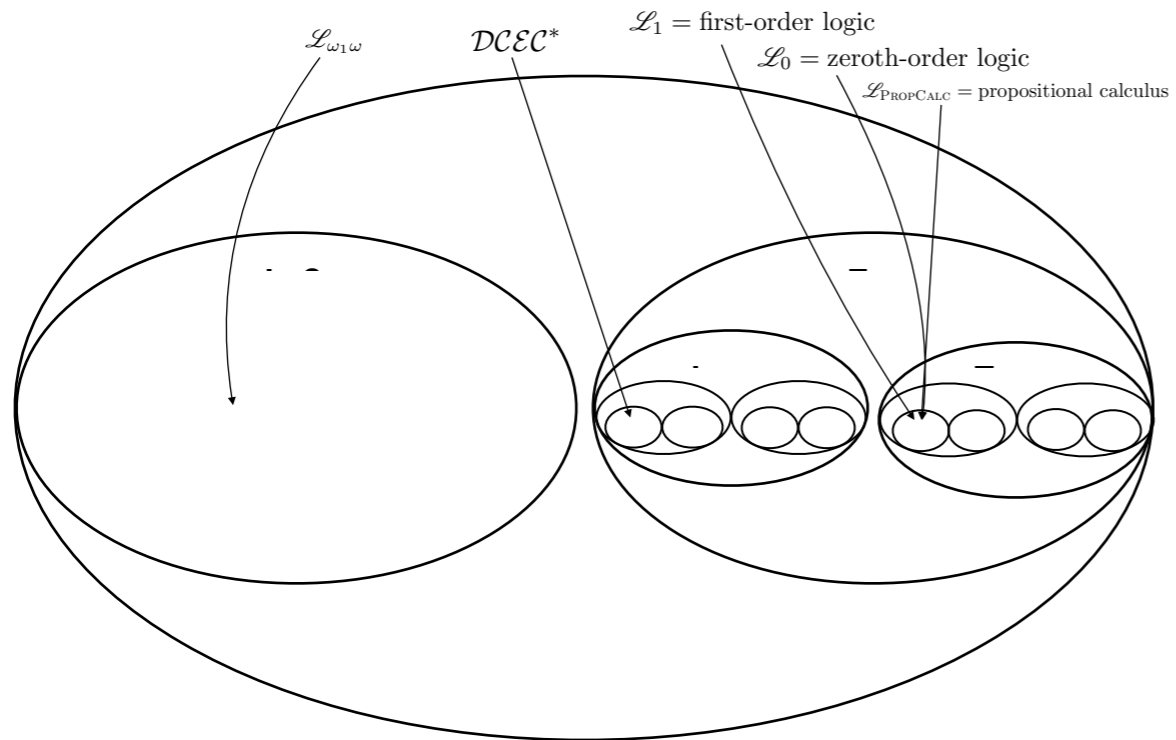
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The Universe of Logics

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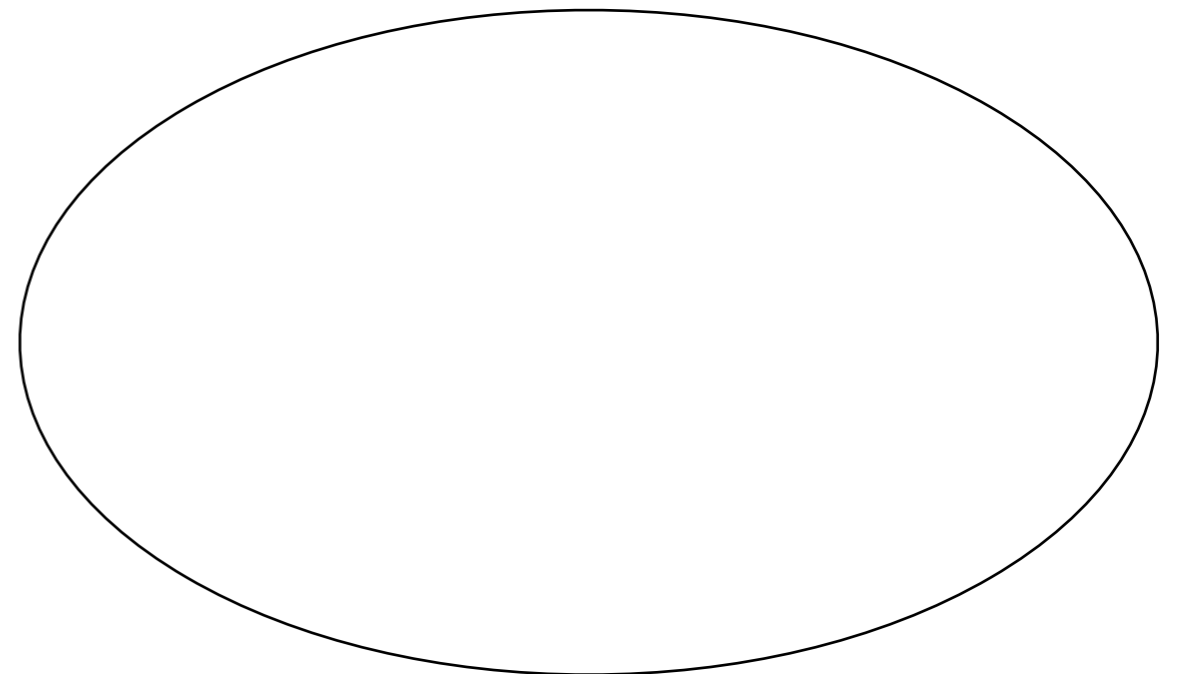
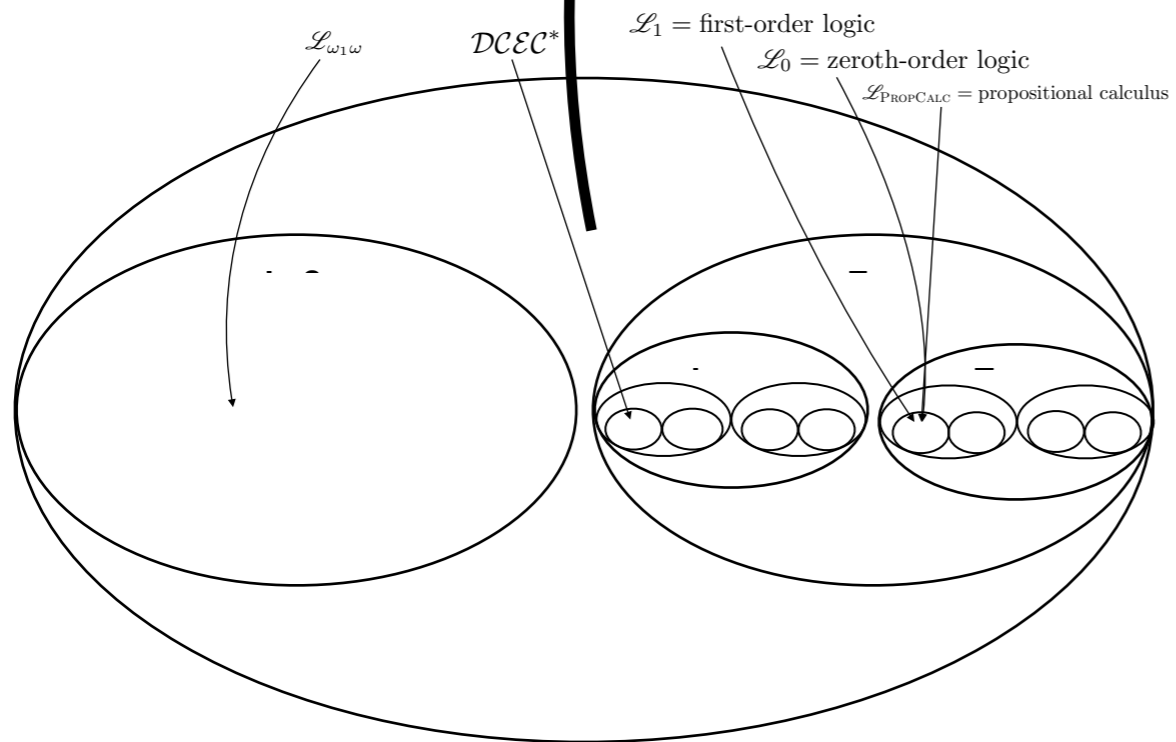
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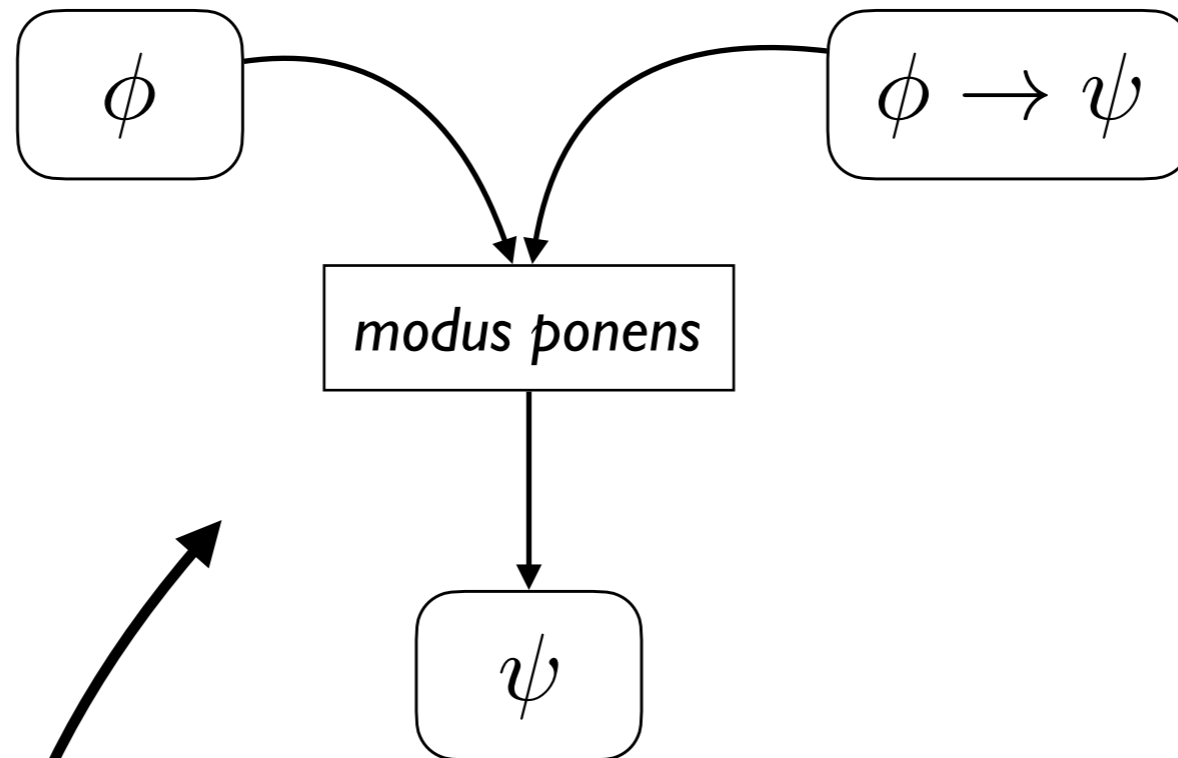
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Non-Physical

The Universe of Logics

The Physical Universe



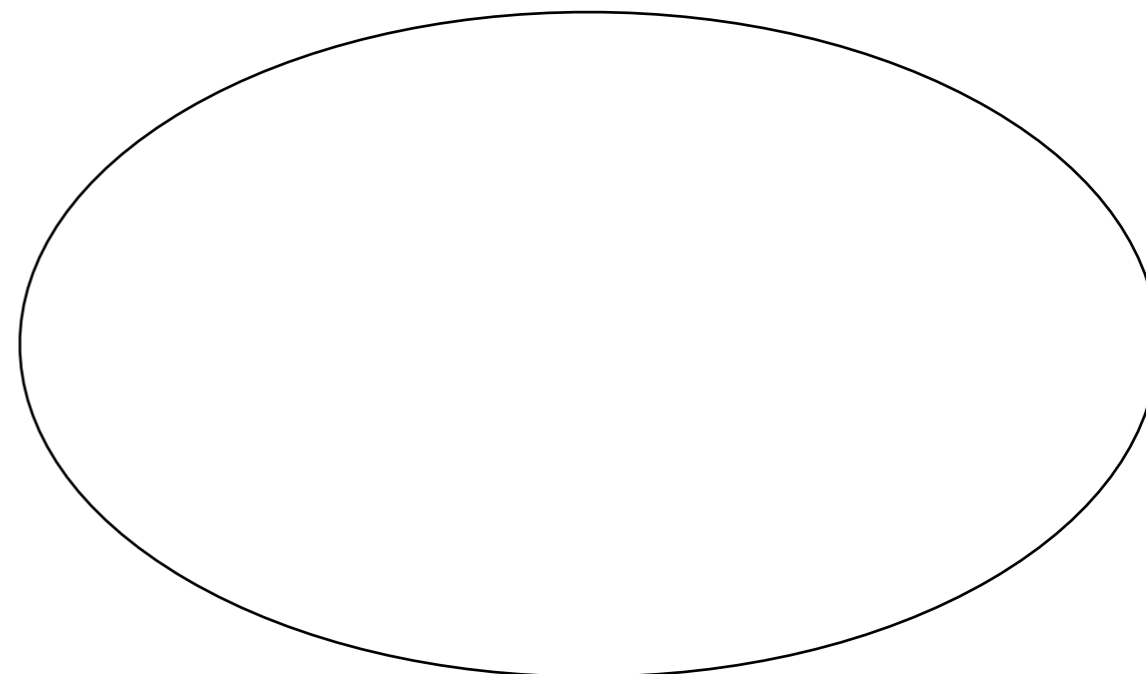
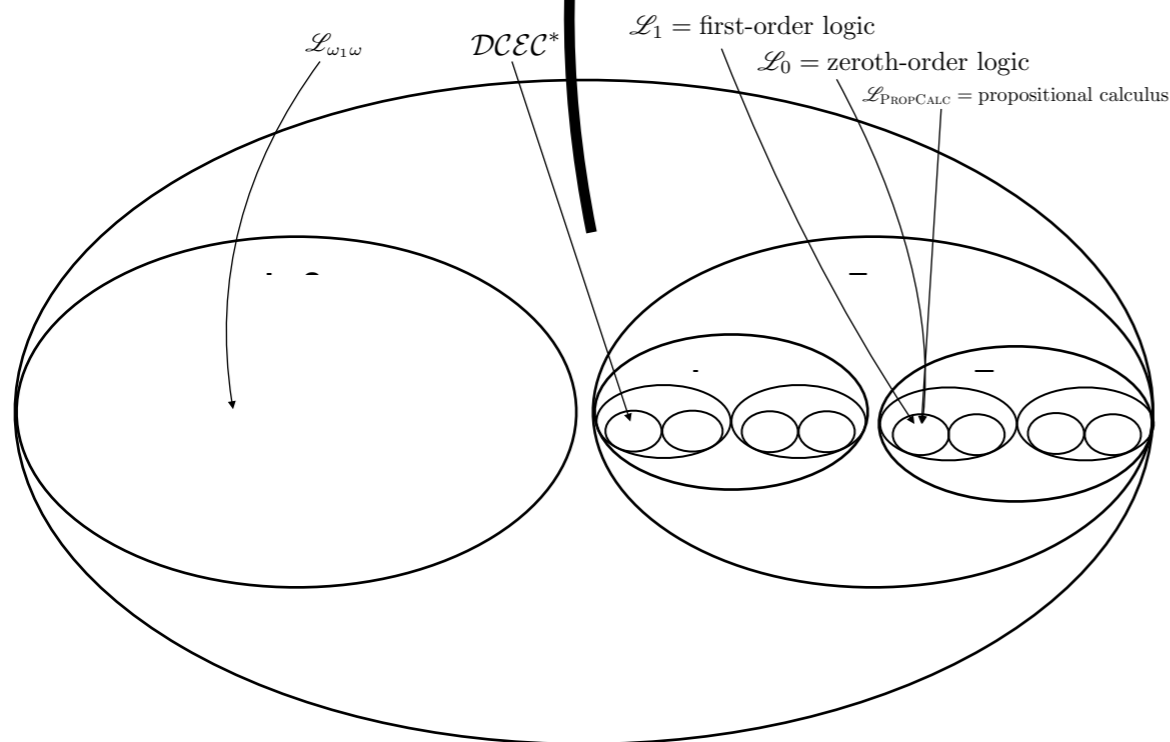


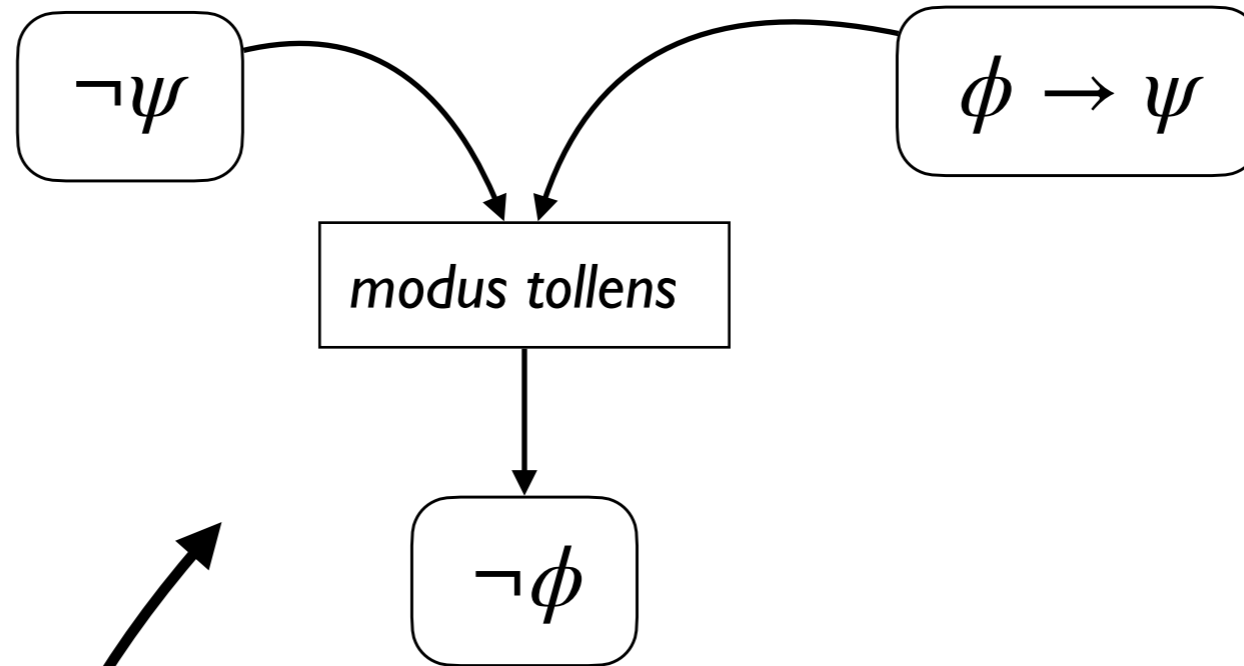
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The Universe of Logics

The Physical Universe

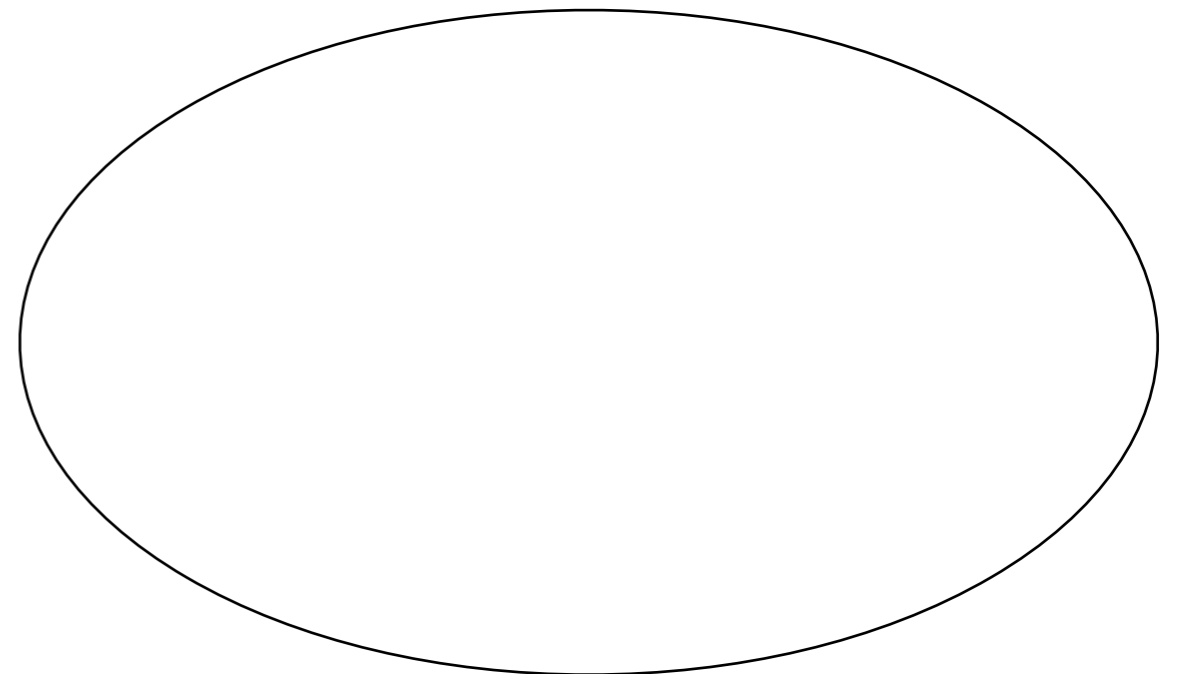
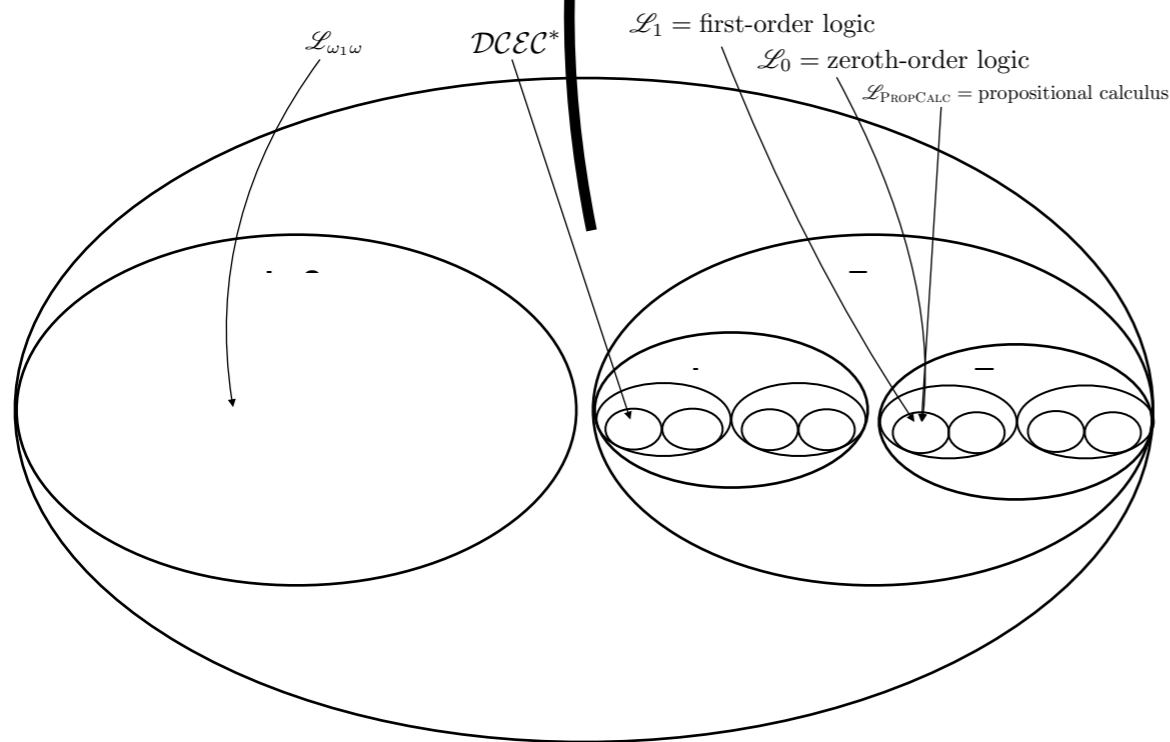




Non-Physical

The Universe of Logics

The Physical Universe



**Mathematical Objects Are Non-Physical,
So We Are Too**

Selmer Bringsjord & Naveen Sundar Govindarajulu

version 0125221730NY

Mathematical Objects Are Non-Physical, So We Are Too

Selmer Bringsjord & Naveen Sundar Govindarajulu

version 0125221730NY

itself is at bottom a simple recursive one. (There are now numerous variants, but we ignore this for efficiency.) The algorithm is to receive an array of ordered objects, for example

$$\langle \boxed{5} \ \boxed{9} \ \boxed{10} \ \boxed{7} \ \boxed{4} \ \boxed{3} \ \boxed{11} \ \boxed{8} \ \boxed{6} \rangle,$$

and to then produce as output the sorted version of this input, which in this case is:

$$\langle \boxed{3} \ \boxed{4} \ \boxed{5} \ \boxed{6} \ \boxed{7} \ \boxed{8} \ \boxed{9} \ \boxed{10} \ \boxed{11} \rangle.$$

So, what's the algorithm? In order to answer this question, we can't avoid resorting to what we can call *embodiments* or *tokens* of the general and abstract *type* Q .⁹ This terminology, and the associated concrete practice, is easy to grasp. For an example, we give one high-level embodiment/token \hat{Q}_1 of Q that views the algorithm as a three-stage one.¹⁰ Before supplying the example in question, we draw the reader's attention to what we just did with a bit of suggestive notation: We used "hat" \hat{O} to indicate that what is being referred to is an embodiment of the thing O (in this case, of course, an algorithm). Hence, the hat in ' \hat{Q}_1 ' says that we have here an embodiment of the algorithm Q itself. Very well, and now to the embodiment in question itself:

- I Pick the rightmost element in the array as the *pivot*.
- II Partition the array so that all elements in the array less than the pivot are before it, and all elements greater than the pivot are placed after it.
- III Recursively apply both I and II to the sub-array now before the pivot, as well as to the sub-array now after the pivot.

This is said to be 'high-level' for obvious reasons. \hat{Q}_1 doesn't tell us how to carry out partitioning, and it relies on an understanding of what recursion means — or at least what it means in this context. But no worries: Stage II can be further specified by saying that we simply move to the left one entry at a time, and decide whether to move an entry to the right of our pivot, or else leave it where it is. And how to decide? Simple: If what we find is greater than our pivot, append it to whatever sub-sequence is to the right of the pivot; otherwise just leave what we find alone. Using a double-box to indicate our pivot, the result of executing Stage I and then Stage II in \hat{Q}_1 on the initial input array will result in this configuration:

$$\langle \boxed{5} \ \boxed{4} \ \boxed{3} \ \boxed{\boxed{6}} \ \boxed{8} \ \boxed{11} \ \boxed{7} \ \boxed{10} \ \boxed{9} \rangle.$$

Now the algorithm calls for Stage III in \hat{Q}_1 , which means that the sub-array to the left of $\boxed{6}$ with $\boxed{3}$ as the pivot of this sub-array is processed; ditto for the sub-array to the right of $\boxed{6}$ with $\boxed{9}$ as the pivot of this sub-array. In the case of the right sub-array, here's the result of running Stage I, which is to be passed to Stage II to be processed (we once again indicate the pivot by a double-box):

$$\langle \boxed{8} \ \boxed{11} \ \boxed{7} \ \boxed{10} \ \boxed{\boxed{9}} \rangle.$$

Stage II applied to the input to it immediately above then results in this:

Mathematical Objects Are Non-Physical, So We Are Too

Selmer Bringsjord & Naveen

version 012522

$$\langle \boxed{8} \ \boxed{7} \ \boxed{9} \ \boxed{10} \ \boxed{11} \rangle.$$

We continue in this way until we reach sub-arrays composed of but one element, which are by definition sorted, and hence processing is guaranteed to terminate.

It should be obvious to the reader that an infinite number of embodiments or tokens of Quicksort are available.¹¹ Many of these embodiments call upon programming languages used today. We shall assume, going forward, that \hat{Q}_2 refers to an embodiment of Quicksort = Q that is expressed in the modern functional programming language known as Clojure.¹²

3.2 Exemplar 2, an Inference Schema: *Modus Tollens*

Next, we use a variant of the famous “Wason Selection Task” (WST) (Wason 1966) to anchor our presentation of *modus tollens* = MT , the gist of which, intuitively, can be thought of as the kernel of a kind of *disconfirmation*, in which if it is claimed that ϕ implies ψ , and one observes that ψ isn’t the case, one can safely infer that ϕ doesn’t hold either. We can be a bit clearer about what *modus tollens* is by way of the following oft-used token of it:

$$\frac{\phi \rightarrow \psi, \neg\psi}{\neg\phi}$$

The token written immediately above, which — following our “hat” technique explained and introduced above — we shall denote by ‘ \widehat{MT}_1 ,’ tells us that if we have two formulae of the form indicated by the two expressions above the horizontal line (the first a conditional and the second the negation of the consequent of that conditional), then the inference schema in question allows us to infer what’s below the horizontal line, namely that the antecedent in the conditional can be negated.

Now here’s our selection-task challenge: Imagine that, operating as a teacher of mathematics trying to transition one of our students to proof (from mere calculation), we have a deck of cards, each member of which has a digit from 1 to 9 inclusive on one side, and a majuscule Roman letter A, B, . . . , K on the other. From this deck, we deal onto a table in front of one of our students the following four cards:

$$\begin{array}{cccc} \boxed{E} & \boxed{T} & \boxed{4} & \boxed{7} \\ c1 & c2 & c3 & c4 \end{array}$$

Now suppose that we inform the student that the following rule R is absolutely guaranteed with respect to the entire deck, and hence specifically also for the four cards c1–c4 now lying in front of the student: “Every card with a vowel on one side has an even positive integer on the other side.” Next, we issue the student the following challenge:

C Does card4 have a vowel on its other side? Supply a proof to justify your answer.

What should the student do in order to succeed? It should be clear that the student should answer in the negative, and provide a proof that makes use of *modus tollens*, such as in the following sequence, which we trust will be readily understood by all our readers, after a bit of inspection:¹³

itself is at bottom a simple recursive one. (There are now numerous variants, but we ignore this for efficiency.) The algorithm is to receive an array of ordered objects, for example

$$\langle \boxed{5} \ \boxed{9} \ \boxed{10} \ \boxed{7} \ \boxed{4} \ \boxed{3} \ \boxed{11} \ \boxed{8} \ \boxed{6} \rangle,$$

and to then produce as output the sorted version of this input, which in this case is:

$$\langle \boxed{3} \ \boxed{4} \ \boxed{5} \ \boxed{6} \ \boxed{7} \ \boxed{8} \ \boxed{9} \ \boxed{10} \ \boxed{11} \rangle.$$

answer this question, we can’t avoid resorting to what I’ll call a general and abstract *type Q*.⁹ This terminology, and the

For an example, we give one high-level embodiment — a three-stage one.¹⁰ Before supplying the example in detail, let us first indicate that what we just did with a bit of suggestive notation: We referred to an embodiment of the thing O (in this case, the thing \hat{Q}_1) says that we have here an embodiment of the thing O (in this case, the thing \hat{Q}_1) in question itself:

say as the *pivot*.

Elements in the array less than the pivot are before it, and elements greater than the pivot are placed after it.

The sub-array now before the pivot, as well as to the

reasons. \hat{Q}_1 doesn’t tell us how to carry out particular recursion means — or at least what it means in detail — but it is further specified by saying that we simply move to the right to move an entry to the right of our pivot, or else to the left: If what we find is greater than our pivot, append it to the right of the pivot; otherwise just leave what we find alone. The result of executing Stage I and then Stage II in \hat{Q}_1 is the following configuration:

$$\langle \boxed{8} \ \boxed{11} \ \boxed{7} \ \boxed{10} \ \boxed{9} \rangle.$$

Stage I, which means that the sub-array to the left of $\boxed{6}$ is processed; ditto for the sub-array to the right of $\boxed{6}$ with the pivot. The result of the right sub-array, here’s the result of running Stage I and then Stage II in \hat{Q}_1 above then results in this:

$$\langle \boxed{7} \ \boxed{10} \ \boxed{9} \rangle.$$

3

Next problem
(King-Ace) ...

King-Ace 2

Suppose that the following premise is true:

If there is a king in the hand, then there is an ace in the hand; or if there isn't a king in the hand, then there is an ace; but not both of these if-then statements are true.

What can you infer from this premise?

King-Ace 2

Suppose that the following premise is true:

If there is a king in the hand, then there is an ace in the hand; or if there isn't a king in the hand, then there is an ace; but not both of these if-then statements are true.

What can you infer from this premise?

There is an ace in the hand.

King-Ace 2

Suppose that the following premise is true:

If there is a king in the hand, then there is an ace in the hand; or if there isn't a king in the hand, then there is an ace; but not both of these if-then statements are true.

What can you infer from this premise?

~~—There is an ace in the hand.—~~

King-Ace 2

Suppose that the following premise is true:

If there is a king in the hand, then there is an ace in the hand; or if there isn't a king in the hand, then there is an ace; but not both of these if-then statements are true.

What can you infer from this premise?

~~NO! — There is an ace in the hand. —~~

King-Ace 2

Suppose that the following premise is true:

If there is a king in the hand, then there is an ace in the hand; or if there isn't a king in the hand, then there is an ace; but not both of these if-then statements are true.

What can you infer from this premise?

~~NO! — There is an ace in the hand. — NO!~~

King-Ace 2

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In fact, what you *can* infer is that there *isn't* an ace in the hand!

King-Ace Solved

Proposition: There is *not* an ace in the hand.

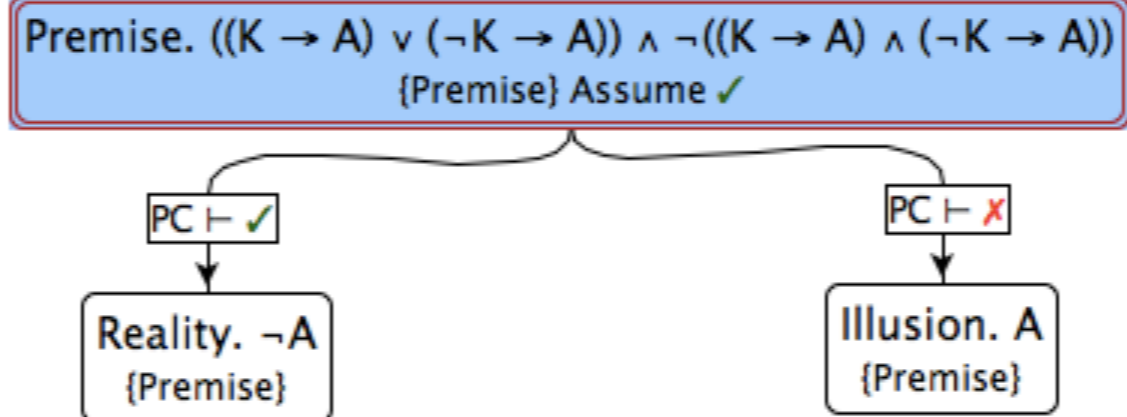
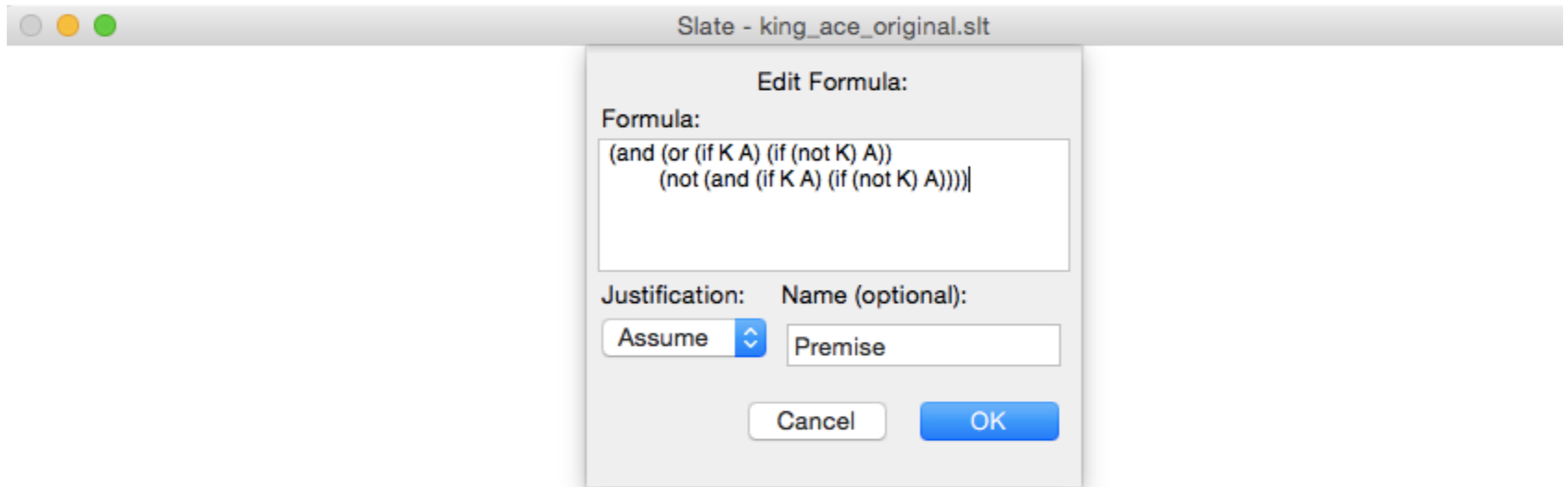
Proof: We know that at least one of the if-thens (i.e., at least one of the **conditionals**) is false. So we have two cases to consider, viz., that $K \Rightarrow A$ is false, and that $\neg K \Rightarrow A$ is false. Take first the first case; accordingly, suppose that $K \Rightarrow A$ is false. Then it follows that K is true (since when a conditional is false, its antecedent holds but its consequent doesn't), and A is false. Now consider the second case, which consists in $\neg K \Rightarrow A$ being false. Here, in a direct parallel, we know $\neg K$ and, once again, $\neg A$. In both of our two cases, which are exhaustive, there is no ace in the hand. The proposition is established. **QED**

King-Ace Solved

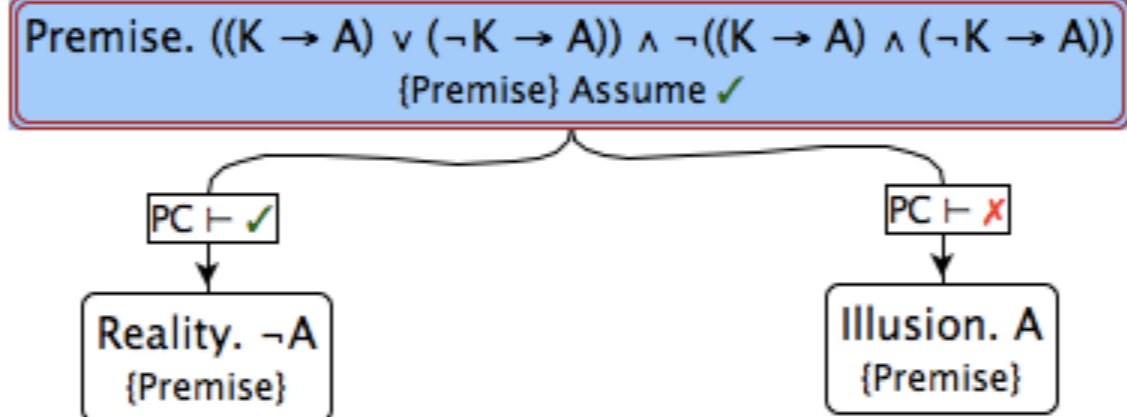
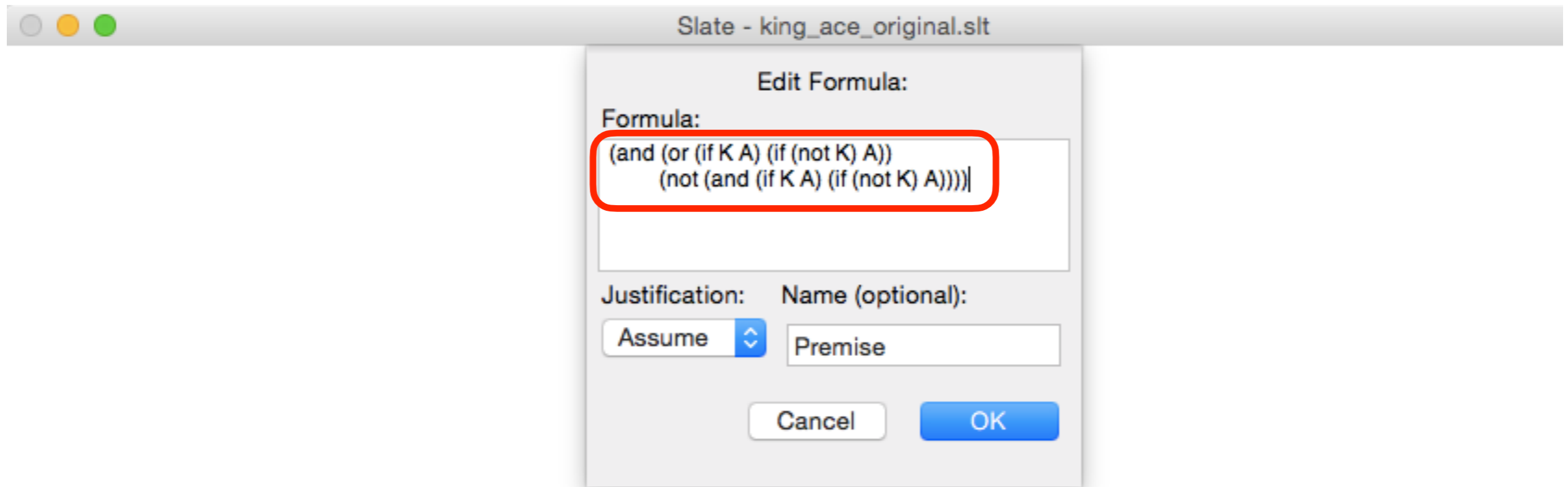
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Study the S-expression



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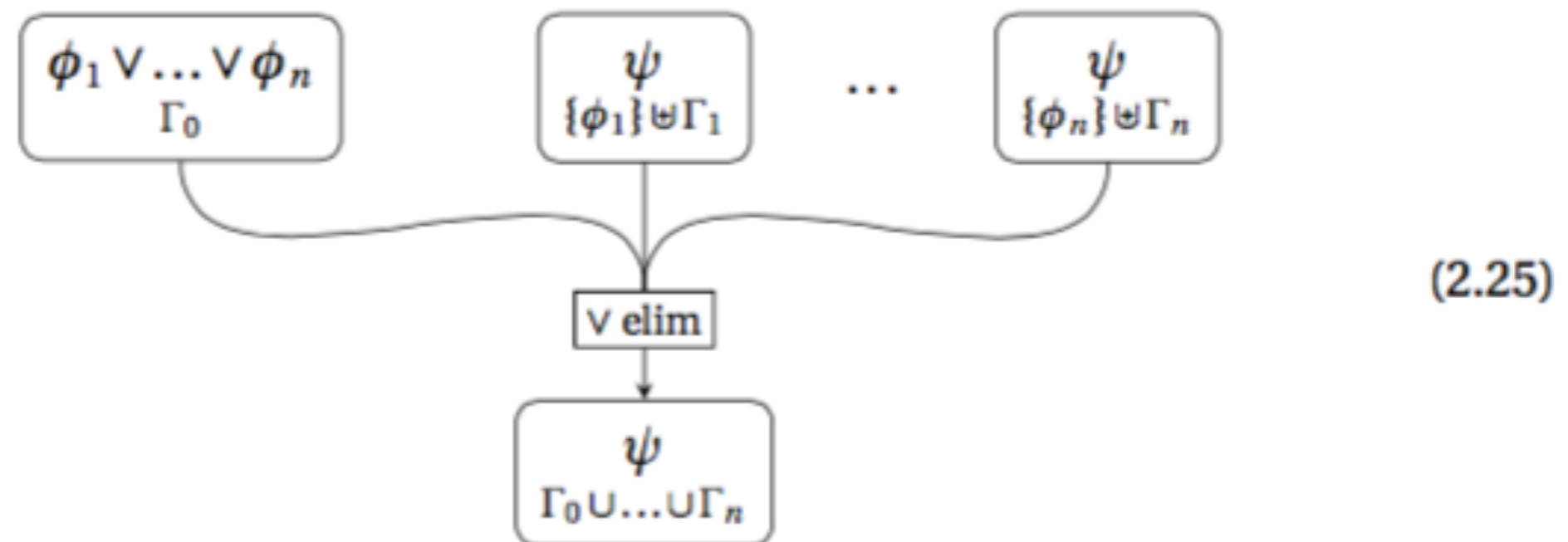
**We need another rule of inference
to crack this problem**

We need another rule of inference
to crack this problem

disjunction elimination

From ~ p. 54 in LAMA-BDLA

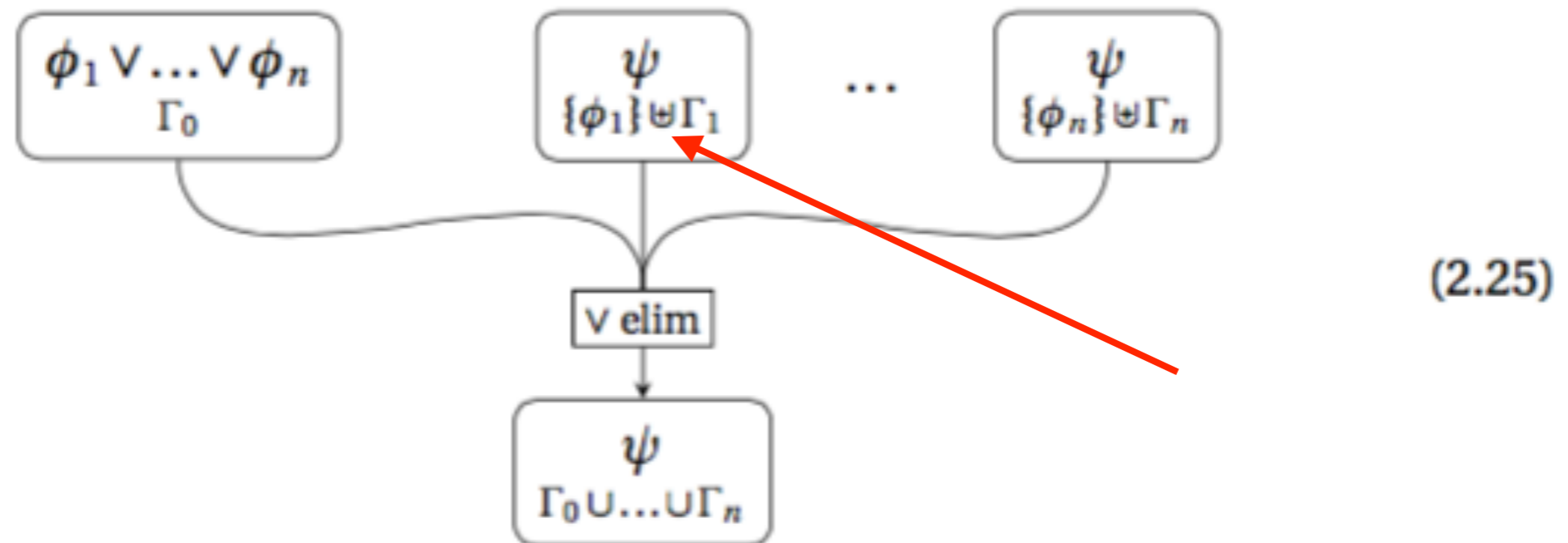
from each ϕ_i , then we may conclude ψ . That is, if we can, for each ϕ_i , assume ϕ_i and show that ψ follows, then we may conclude ψ from the disjunction $\phi_1 \vee \dots \vee \phi_n$ and the derivations of ψ . There is one more subtle point, however. In the days-of-the-week example above, the conclusion that Susan has class on a weekday should not be in the scope of both the assumptions that she has class on Monday and that she has class on Tuesday; these assumptions are *discharged*. Disjunction elimination discharges each assumption ϕ_i from the line of reasoning that corresponds to that case.



The various Γ_i on the premises of disjunction elimination might make this rule seem more complicated than it really is. Their presence makes it clear that the only assumptions discharged from each line of reasoning is the assumption corresponding to that particular case.

From ~ p. 54 in LAMA-BDLA

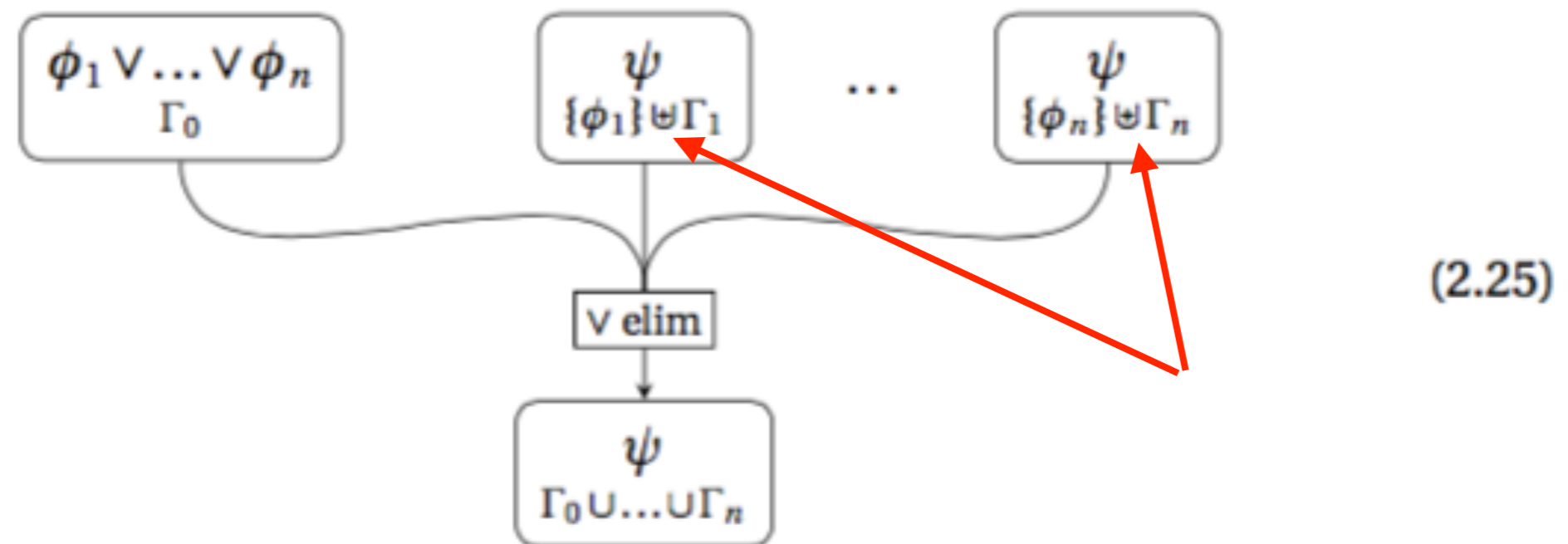
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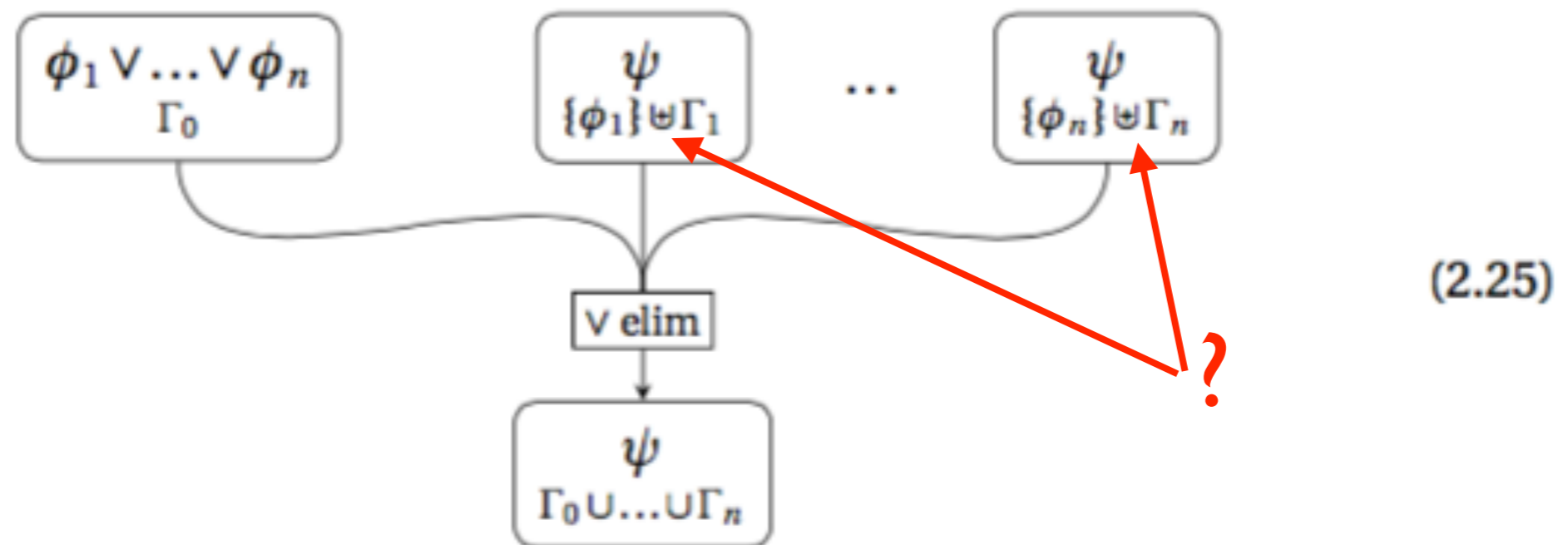
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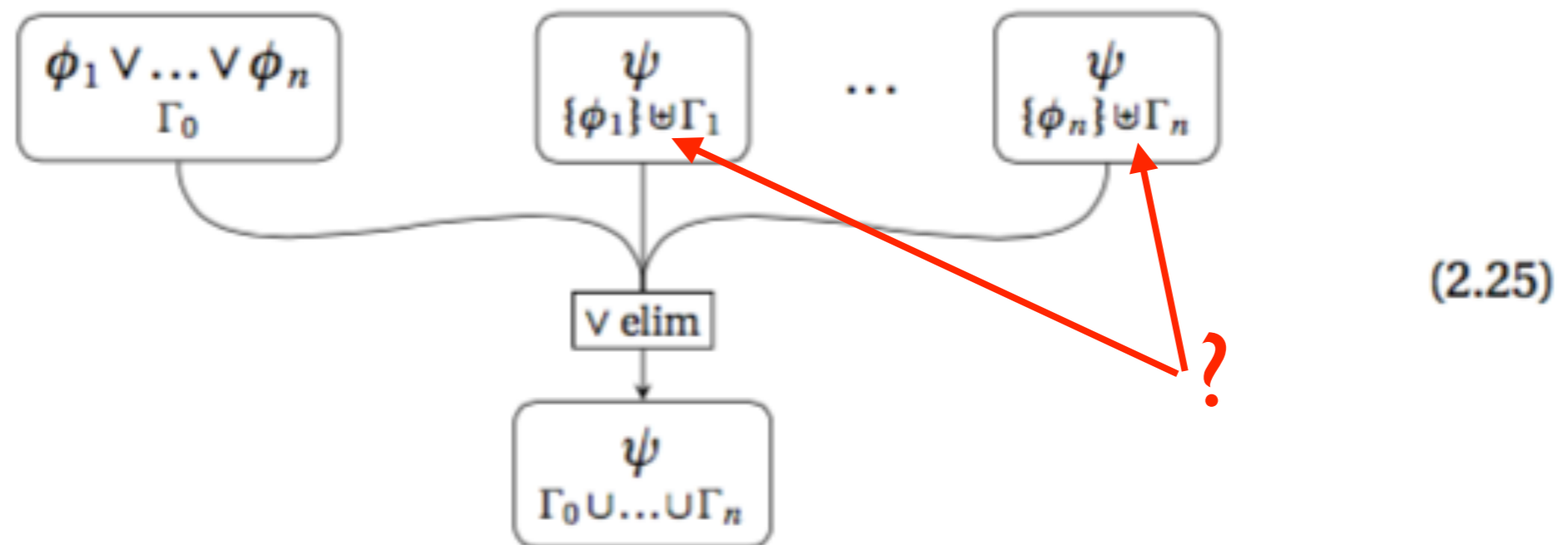
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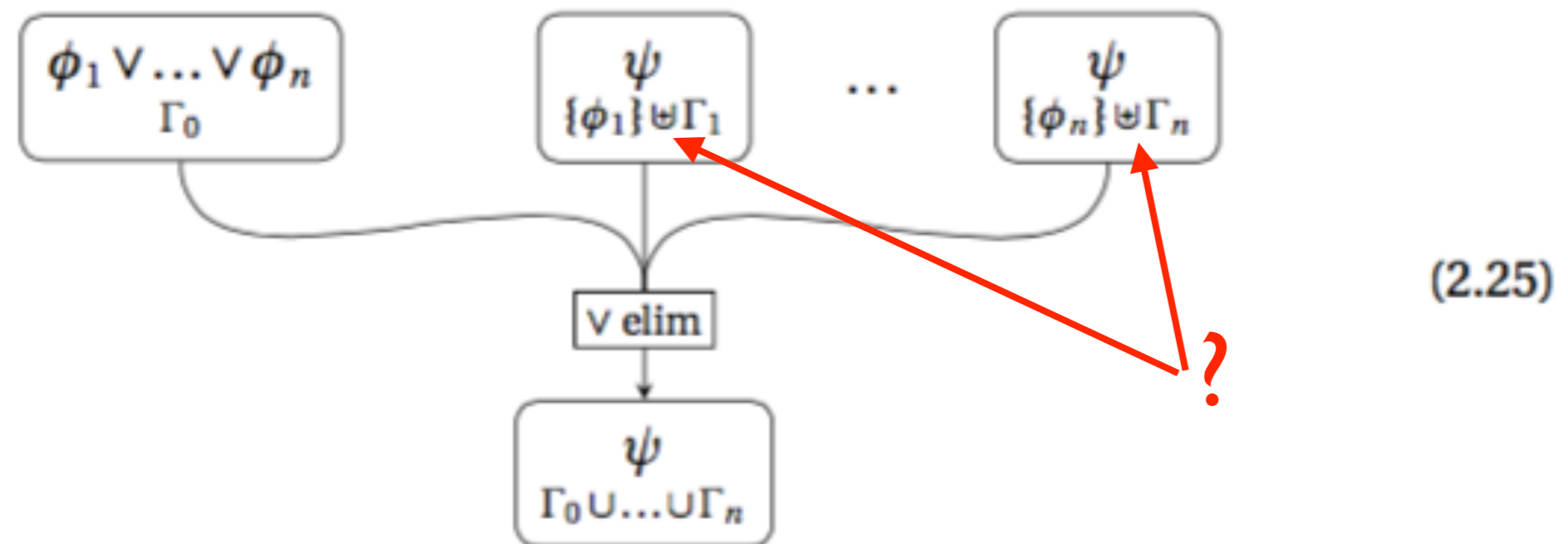
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King-Ace 2

Suppose that the following premise is true:

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Suppose that the following premise is true:

If there is an ace in the hand **Future Required problem (on HyperGrader[®]): You will need to finish the proof in HyperSlate[®] — with no remaining use of an oracle.** *ice in the hand*

there is no ace in the hand **But COVID-19 will be allowed to slow everything down.** *en*

What can you infer?

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Det er en ære å lære formell logikk!