(Part II of the Chapter; Part I is on "The Gödel Game," for IFLAI2)

Selmer Bringsjord
IFLAI2 2022

4/23/23

ver 0423231207NY



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Monographic Context (yet again!)

• • •

- Introduction ("The Wager")
- Brief Preliminaries (e.g. the propositional calculus & FOL)
- The Completeness Theorem
- The First Incompleteness Theorem
- The Second Incompleteness Theorem
- The Speedup Theorem
- The Continuum-Hypothesis Theorem
- The Time-Travel Theorem
- Gödel's "God Theorem"
- Could a Finite Machine Match Gödel's Greatness?



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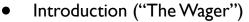


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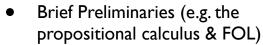
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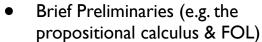
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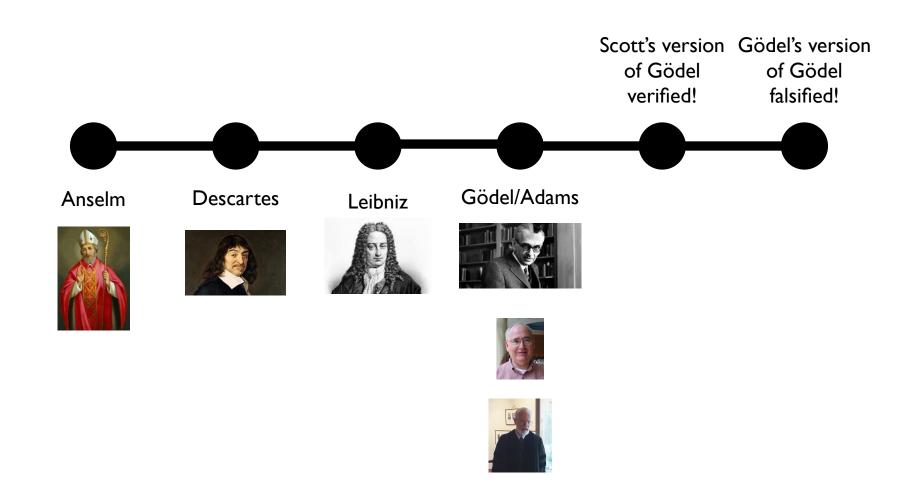




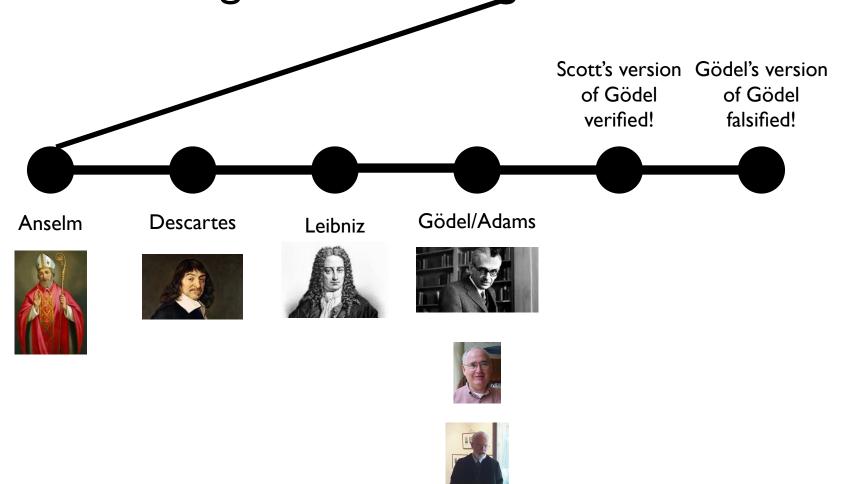
Re Gödel's "God Theorem" ...

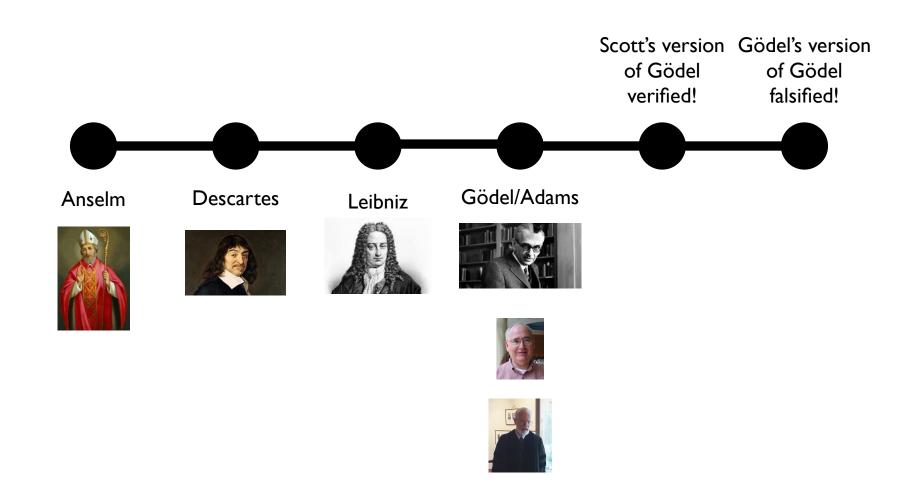
Recommended Podcast:)

https://mindmatters.ai/podcast/ep81









T. Schaub et al. (Eds.)

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Automating Gödel's Ontological Proof of God's Existence with Higher-order Automated Theorem Provers

Christoph Benzmüller¹ and Bruno Woltzenlogel Paleo²

Abstract. Kurt Gödel's ontological argument for God's existence has been formalized and automated on a computer with higher-order automated theorem provers. From Gödel's premises, the computer proved: necessarily, there exists God. On the other hand, the theorem provers have also confirmed prominent criticism on Gödel's ontological argument, and they found some new results about it.

The background theory of the work presented here offers a novel perspective towards a *computational theoretical philosophy*.

1 INTRODUCTION

Kurt Gödel proposed an argumentation formalism to prove the existence of God [23, 30]. Attempts to prove the existence (or non-existence) of God by means of abstract, ontological arguments are an old tradition in western philosophy. Before Gödel, several prominent philosophers, including St. Anselm of Canterbury, Descartes and Leibniz, have presented similar arguments. Moreover, there is an impressive body of recent and ongoing work (cf. [31, 19, 18] and the references therein). Ontological arguments, for or against the existence of God, illustrate well an essential aspect of metaphysics: some (necessary) facts for our existing world are deduced by purely a priori, analytical means from some abstract definitions and axioms.

What motivated Gödel as a logician was the question, whether it

A1 Either a property or its negation is positive, but not both:

 $\forall \phi [P(\neg \phi) \equiv \neg P(\phi)]$

A2 A property necessarily implied by a positive property is positive: $\forall \phi \forall \psi [(P(\phi) \land \Box \forall x [\phi(x) \supset \psi(x)]) \supset P(\psi)]$

T1 Positive properties are possibly exemplified:

 $\forall \phi [P(\phi) \supset \diamondsuit \exists x \phi(x)]$

D1 A God-like being possesses all positive properties:

 $G(x) \equiv \forall \phi [P(\phi) \supset \phi(x)]$

A3 The property of being God-like is positive: P(G)

Possibly, God exists: $\diamondsuit \exists x G(x)$

A4 Positive properties are necessarily positive:

 $\forall \phi [P(\phi) \supset \Box P(\phi)]$

D2 An *essence* of an individual is a property possessed by it and necessarily implying any of its properties:

 $\phi \; ess. \; x \equiv \phi(x) \; \wedge \; \forall \psi(\psi(x) \supset \Box \forall y (\phi(y) \supset \psi(y)))$

T2 Being God-like is an essence of any God-like being:

 $\forall x[G(x) \supset G \ ess. \ x]$

D3 Necessary existence of an individ. is the necessary exemplification of all its essences: $NE(x) \equiv \forall \phi [\phi \ ess. \ x \supset \Box \exists y \phi(y)]$

A5 Necessary existence is a positive property: P(NE)

T3 Necessarily, God exists: $\Box \exists x G(x)$

Figure 1. Scott's version of Gödel's ontological argument [30].

ersion del ed! Proceedings of the Twenty-Fifth International Joint Conference on Artificial Intelligence (IJCAI-16)

The Inconsistency in Gödel's Ontological Argument: A Success Story for AI in Metaphysics

Christoph Benzmüller*

Freie Universität Berlin & Stanford University c.benzmueller@gmail.com

Abstract

This paper discusses the discovery of the inconsistency in Gödel's ontological argument as a success story for artificial intelligence. Despite the popularity of the argument since the appearance of Gödel's manuscript in the early 1970's, the inconsistency of the axioms used in the argument remained unnoticed until 2013, when it was detected automatically by the bigher-order theorem, prover

some (necessary) facts for our existing world are deduced by purely a priori, analytical means from some abstract definitions and axioms.

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Bruno Woltzenlogel Paleo

Australian National University bruno.wp@gmail.com

on the proof [Fuhrmann, 2016].

The in-depth analysis presented here substantially extends previous computer-assisted studies of Gödel's ontological argument. Similarly to the related work [Benzmüller and Woltzenlogel-Paleo, 2013a; 2014] the analysis has been conducted with automated theorem provers for classical higher-order logic (HOL; cf. [Andrews, 2014] and the references therein), even though Gödel's proof is actually formulated in higher-order *modal* logic (HOML; cf. [Muskens, 2006]

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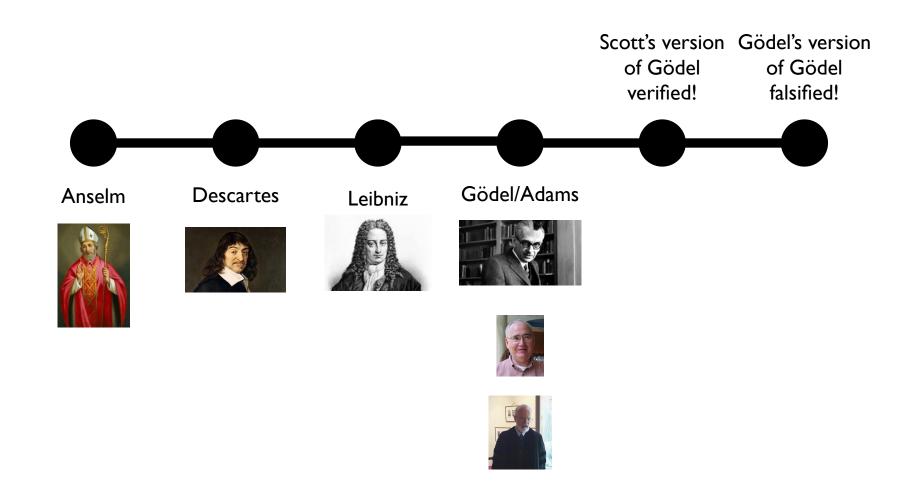
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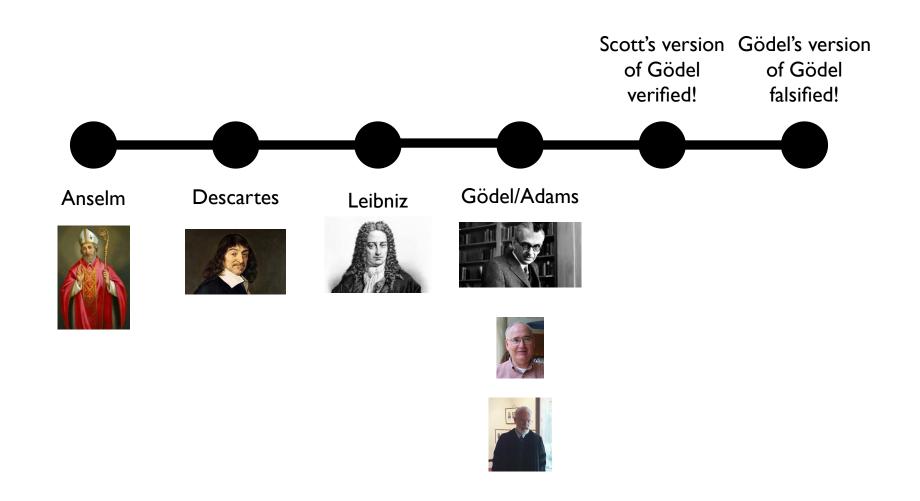
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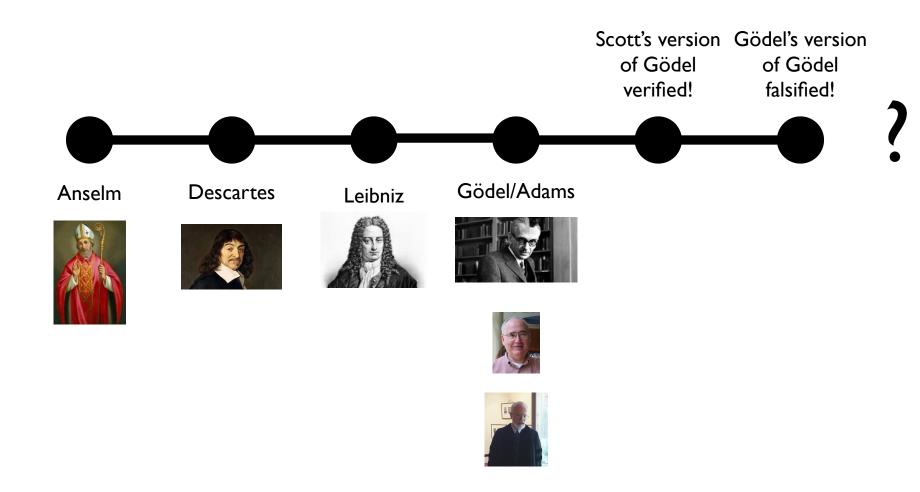
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Figure 1. Scott's version of Gödel's ontological argument [30].

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(**Pos1**)
$$\forall R(Pos(R) \rightarrow \neg Pos(\bar{R}))$$

(**Pos2**)
$$\forall R[(Pos(R) \land \Box \forall x \forall R'(R(x) \rightarrow R'(x))) \rightarrow Pos(R')]$$

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For a wonderfully economical, non-technical overview that includes this observation, see "Chapter 7: Gödel" by Alexander Pruss, in *Ontological Arguments*, G. Oppy, ed. (Cambridge, UK: Cambridge University Press).

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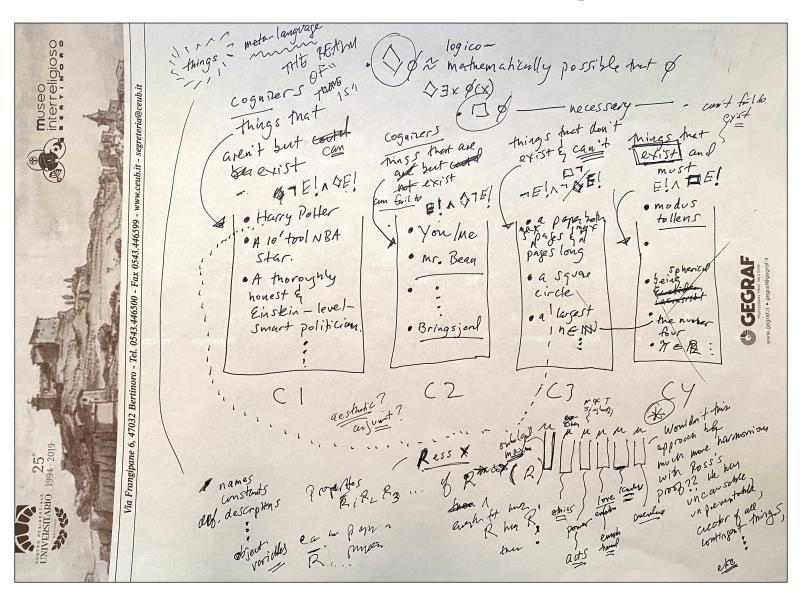
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 $(\mathbf{Pos1}^{\star}) \quad \forall R \,\forall \delta \neq 0 [GPPos(R^{\delta}) \rightarrow \neg GPPos(\bar{R})]$

"The Other Way"



Gödel's Either/Or ...

The Question

Q* Is the human mind more powerful than the class of standard computing machines?

The Question

Q* Is the human mind more powerful than the class of standard computing machines?

(= finite machines)

The Question

Q* Is the human mind more powerful than the class of standard computing machines?

```
(= finite machines)
(= Turing machines)
(= register machines)
(= KU machines)
```

. . .

Gödel's Either/Or

"[E]ither ... the human mind (even within the realm of pure mathematics) infinitely surpasses the power of any finite machine, or else there exist absolutely unsolvable diophantine problems."

— Gödel, 1951, Providence RI

PT as a Diophantine Equation

Equations of this sort were introduced to you in middle-school, when you were asked to find the hypotenuse of a right triangle when you knew its sides; the familiar equation, the famous Pythagorean Theorem that most adults will remember at least echoes of into their old age, is:

(PT)
$$a^2 + b^2 = c^2$$
,

and this is of course equivalent to

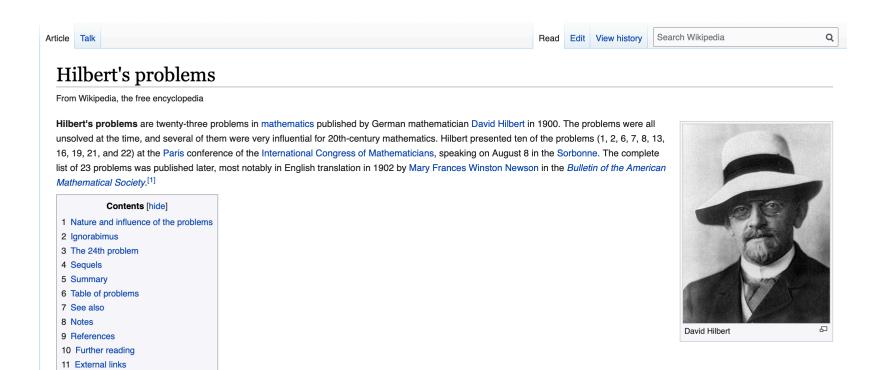
(PT')
$$a^2 + b^2 - c^2 = 0$$
,

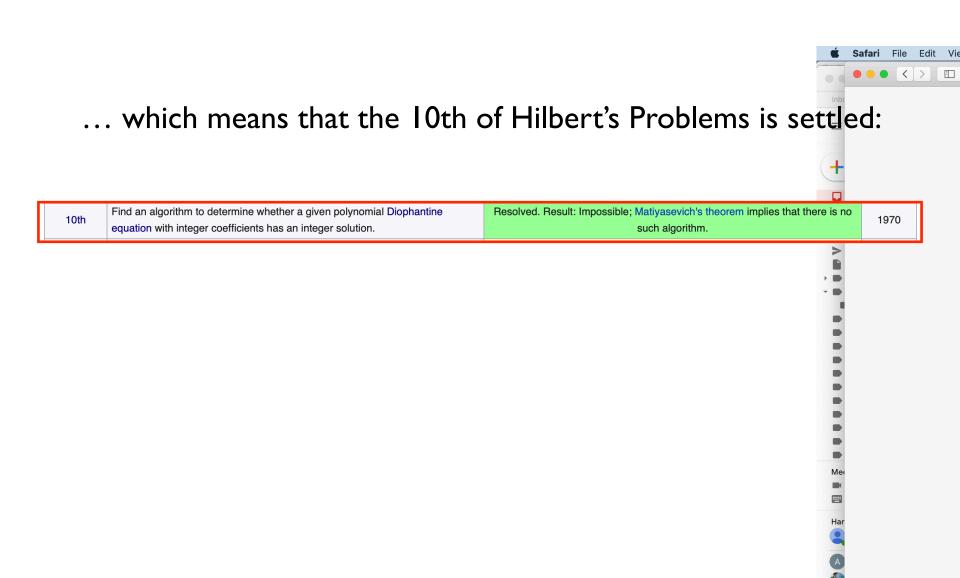
which is a Diophantine equation. Such equations have at least two unknowns (here, we of course have three: a, b, c), and the equation is solved when positive integers for the unknowns are found that render the equation true. Three positive integers that render (PT') true are

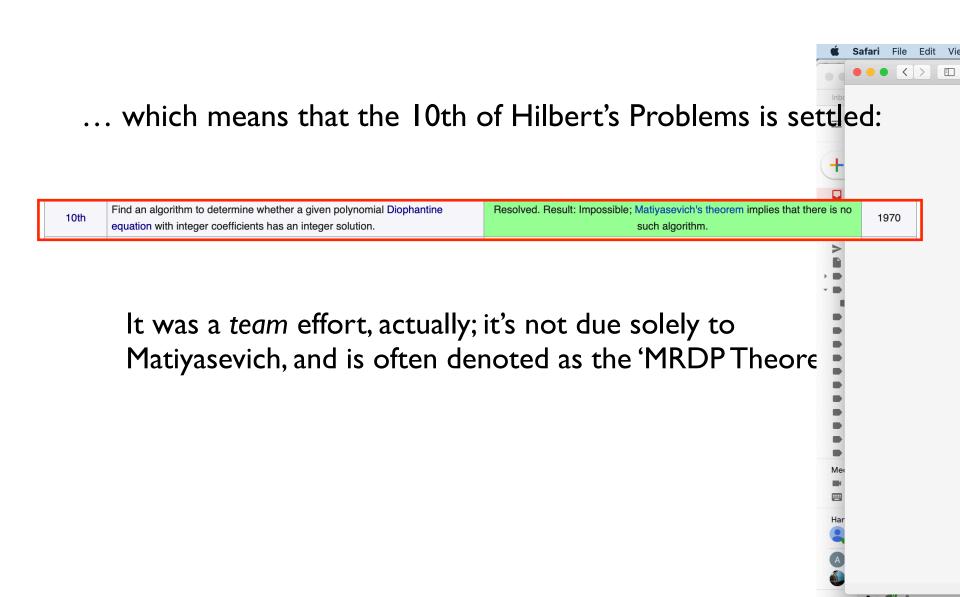
$$a = 4, b = 3, c = 5.$$

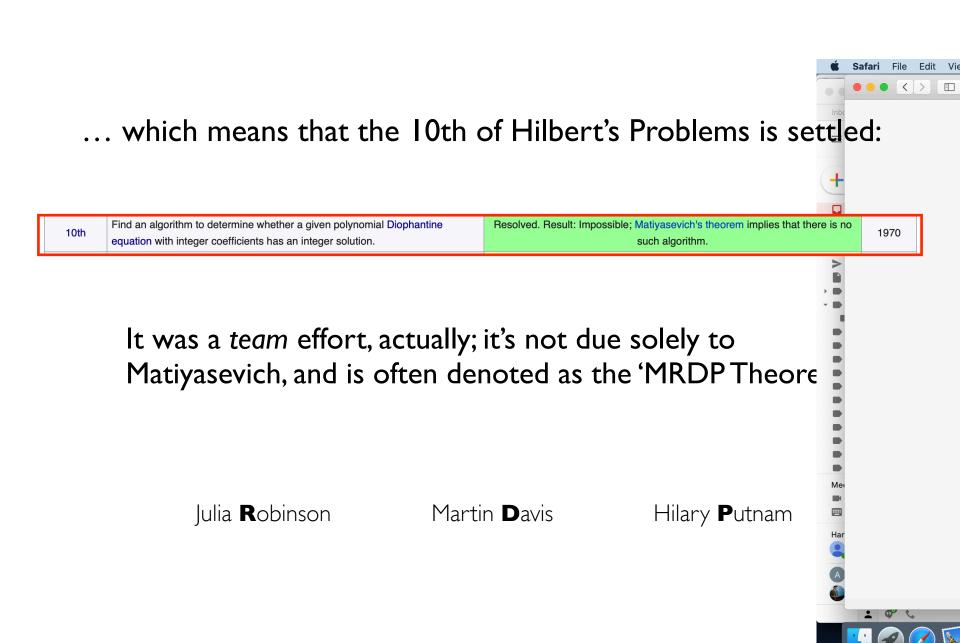
It is mathematically impossible that there is a finite computing machine capable of solving any Diophantine equation given to it as a challenge (!).

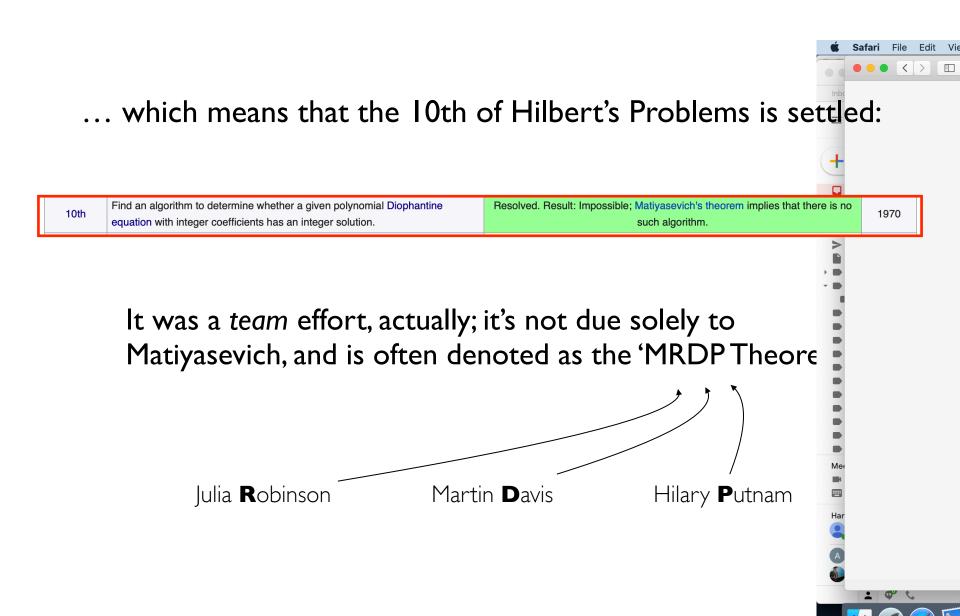
... which means that the 10th of Hilbert's Problems is settled:











Background

problem?⁷ In his lecture, Gödel precisely defines diophantine problems, but we don't need to bother with all of the details here; we only need to appreciate the general structure of such a problem, and that can be achieved quickly as follows, given what was introduced in Chapter 2.

Each diophantine problem has at its core a polynomial \mathcal{P} whose variables are comprised by two lists, x_1, x_2, \ldots, x_n and y_1, y_2, \ldots, y_m ; all variables must be integers, and the same for subscripts n and m. To represent a polynomial in a manner that announces its variables, we can write

$$\mathcal{P}(x_1,x_2,\ldots,x_k,y_1,y_2,\ldots,y_j).$$

But Gödel was specifically interested in whether, for all integers that can be set to the variables x_i , there are integers that can be set to the y_j , such that the polynomial equals 0. To make this clearer, first, here are two particular, simple equations that employ polynomials that are both instances of the needed form:

$$E1 3x - 2y = 0$$

E2
$$2x^2 - y = 0$$

All we need to do now is prefix these equations with quantifiers in the pattern Gödel gave. This pattern is quite simple: universally quantify over each x_i variable (using the now-familiar \forall), after which we existentially quantify over each y_i variable (using the also-now-familiar \exists). Thus, here are the two diophantine problems that correspond to the pair E1 and E2 from just above:

P1 Is it true that $\forall x \exists y (3x - 2y = 0)$?

P2 Is it true that $\forall x \exists y 2x^2 - y = 0$?



Hilbert's Tenth Problem is Unsolvable

Author(s): Martin Davis

Source: The American Mathematical Monthly, Vol. 80, No. 3 (Mar., 1973), pp. 233-269

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1. Diophantine Sets. In this article the usual problem of Diophantine equations will be inverted. Instead of being given an equation and seeking its solutions, one will begin with the set of "solutions" and seek a corresponding Diophantine equation. More precisely:

DEFINITION. A set S of ordered n-tuples of positive integers is called **Diophantine** if there is a polynomial $P(x_1, \dots, x_n, y_1, \dots, y_m)$, where $m \ge 0$, with integer coefficients such that a given n-tuple $\langle x_1, \dots, x_n \rangle$ belongs to S if and only if there exist positive integers y_1, \dots, y_m for which

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HILBERT'S TENTH PROBLEM IS UNSOLVABLE

235

 $P(x_1, \dots, x_n, y_1, \dots, y_m) = 0.$

as:

1973]

Borrowing from logic the symbols "∃" for "there exists" and "⇔" for "if and only if", the relation between the set S and the polynomial P can be written succinctly

$$\langle x_1, \dots, x_n \rangle \in S \Leftrightarrow (\exists y_1, \dots, y_m) [P(x_1, \dots, x_n, y_1, \dots, y_m) = 0],$$

or equivalently:

$$S = \{\langle x_1, \dots, x_n \rangle \mid (\exists y_1, \dots, y_m) \ [P(x_1, \dots, x_n, y_1, \dots, y_m) = 0]\}.$$

Note that P may (and in non-trivial cases always will) have negative coefficients. The word "polynomial" should always be so construed in the article except where the contrary is explicitly stated. Also all numbers in this article are positive integers unless the contrary is stated.



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Notice that this is a perfect fit with how we used formal logic to present and understand the Polynomial Hierarchy and the Arithmetic Hierarchy.

Unsolvable

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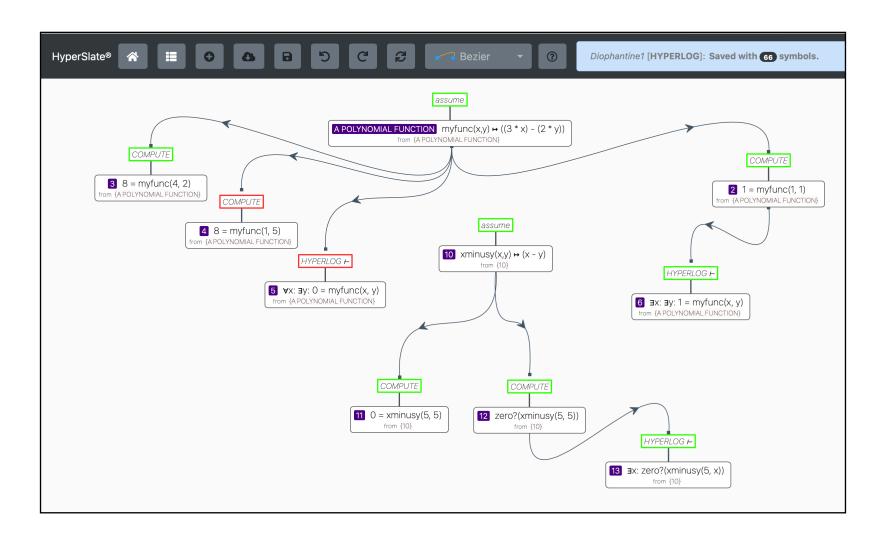
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Diophantine "Threat" in the New Programming Language Hyperlog®

(Another IFLAI2 Topic/Technology)

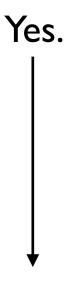


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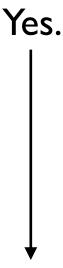
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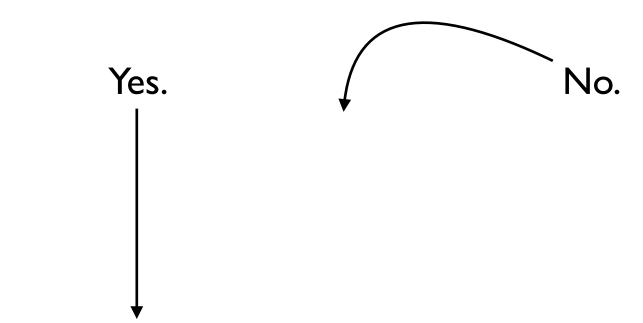
The human mind is *not* infinitely more powerful than any standard computing machine.

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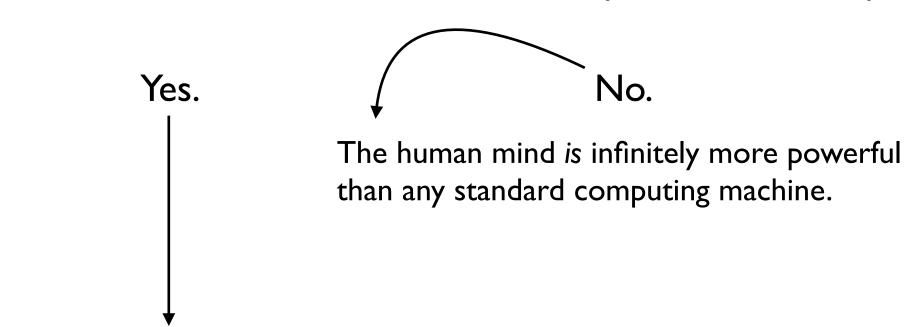
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The Crux

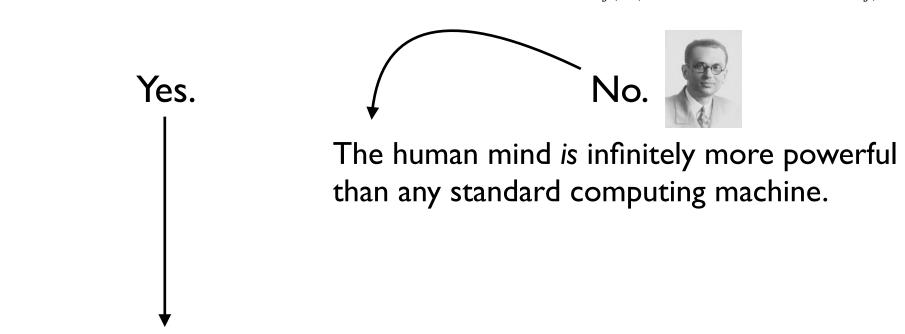
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The human mind is *not* infinitely more powerful than any standard computing machine.

Earlier Gödelian Argument for the "No."



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Outline

Abstract

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- 2. Clarifying computationalism, the view to be overthro...
- 3. The essence of hypercomputation: harnessing the in...
- 4. Gödel on minds exceeding (Turing) machines by "co...
- 5. Setting the context: the busy beaver problem
- 6. The new Gödelian argument
- 7. Objections
- 8. Conclusion

References

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Figures (1)



Tables (1)



Applied Mathematics and Computation

Volume 176, Issue 2, 15 May 2006, Pages 516-530



A new Gödelian argument for hypercomputing minds based on the busy beaver problem ★

Selmer Bringsjord A ☎ ⊕, Owen Kellett, Andrew Shilliday, Joshua Taylor, Bram van Heuveln, Yingrui Yang, Jeffrey Baumes, Kyle Ross

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https://doi.org/10.1016/j.amc.2005.09.071

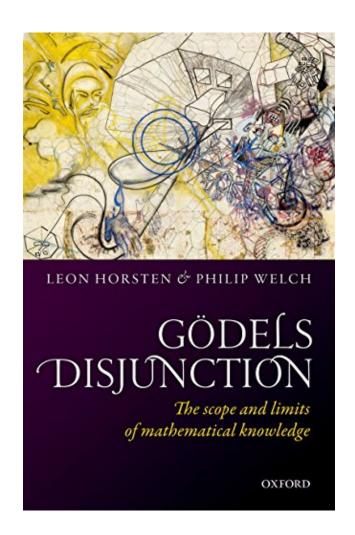
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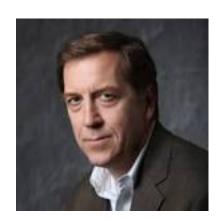
Abstract

Do human persons hypercompute? Or, as the doctrine of *computationalism* holds, are they information processors at or below the Turing Limit? If the former, given the essence of hypercomputation, persons must in some real way be capable of infinitary information processing. Using as a springboard Gödel's little-known assertion that the human mind has a power "converging to infinity", and as an anchoring problem Rado's [T. Rado, On non-computable functions, Bell System Technical Journal 41 (1963) 877–884] Turing-uncomputable "busy beaver" (or Σ) function, we present in this short paper a new argument that, in fact, human persons can hypercompute. The argument is intended to be formidable, not conclusive: it brings Gödel's intuition to a greater level of precision, and places it within a sensible case against computationalism.

And the book! ...

And the book! ...









Yes.

Will AI Match (Or Even Exceed) Human Intellligence?



No. Yes.



No. Yes.



No. Yes.

I: "Negative" enumerative induction for $\neg \exists year_k(Al = Hl@year_k)$ from $Al \neq Hl@year_{1958} \land ... \land Al \neq Hl@year_{2021}$. Plus the proposition that Al is in fact not improving — relative to the intellectual stuff that matters most.



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3: Amundsen and The Explorer Argument.

4: And finally, the sledgehammer is used: phenomenal consciousness.

Og på det glade merknaden for Selmer (men ikke for Bill), er forelesningene våre nå fullført ... men ...

Finally, finally, ...

The Particular Work	Nutshell Diagnosis	Beyond AI?
	·	

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Time-Travel Thm. (Ch. 8)	Unqualified to even guess.	Unknown.
"God Theorem" (Ch. 9)	An ancient trajectory from Anselm.	Yes
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*Philosophical Reasoning	Undeniably beyond foreseeable AI.	Yes

And now let's wrap up with extensions etc.

• • •

Med nok penger, kan logikk løse alle problemer.