

On to *Intensional* Logics

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IFLAI
3/24/2023

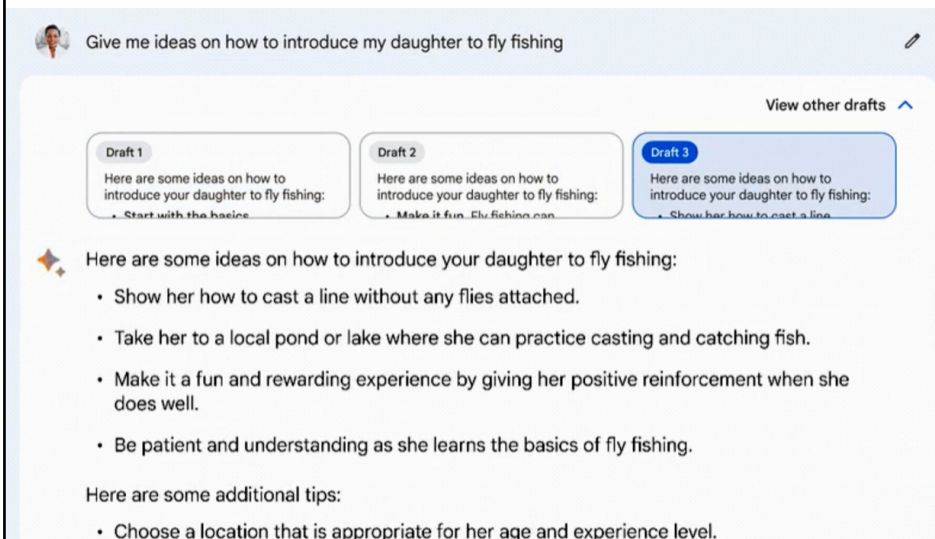


In The Logic-and-AI News

...

What Google Bard Can Do (and What It Can't)

Google has released a new chatbot to a limited number of people in the U.S. and Britain. How does it compare with what is already out there?



The screenshot shows the Google Bard chat interface. At the top, a user's question is entered: "Give me ideas on how to introduce my daughter to fly fishing". Below the question, there are three draft responses labeled "Draft 1", "Draft 2", and "Draft 3". Each draft contains a list of ideas. Draft 1 suggests starting with the basics. Draft 2 suggests making it fun. Draft 3 suggests showing her how to cast a line. Below the drafts, the main response is displayed, starting with "Here are some ideas on how to introduce your daughter to fly fishing:" followed by a list of four ideas: showing her how to cast a line without flies attached, taking her to a local pond or lake, making it a fun and rewarding experience, and being patient and understanding. Below this, there are additional tips, including choosing a location appropriate for her age and experience level.

Give me ideas on how to introduce my daughter to fly fishing

View other drafts

Draft 1
Here are some ideas on how to introduce your daughter to fly fishing:
• Start with the basics

Draft 2
Here are some ideas on how to introduce your daughter to fly fishing:
• Make it fun. Fly fishing can

Draft 3
Here are some ideas on how to introduce your daughter to fly fishing:
• Show her how to cast a line

Here are some ideas on how to introduce your daughter to fly fishing:

- Show her how to cast a line without any flies attached.
- Take her to a local pond or lake where she can practice casting and catching fish.
- Make it a fun and rewarding experience by giving her positive reinforcement when she does well.
- Be patient and understanding as she learns the basics of fly fishing.

Here are some additional tips:

- Choose a location that is appropriate for her age and experience level.

Bard provides different potential answers to its users' questions. via Google



By Cade Metz

Cade Metz has written about artificial intelligence for more than a decade.

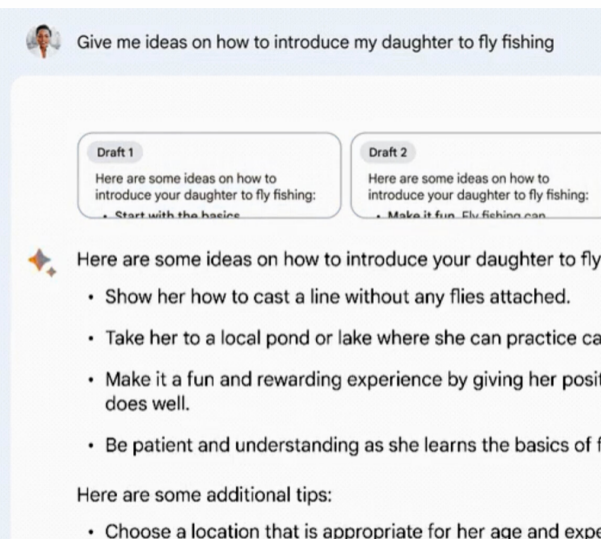
March 21, 2023

3 MIN READ

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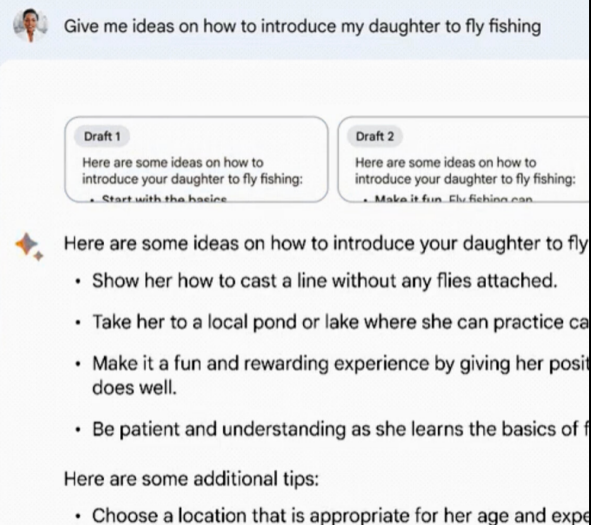
When asked why it had cited that particular source, the bot insisted that it had cited Wikipedia.

It is more cautious than ChatGPT.

[When using the latest version of OpenAI's ChatGPT](#) this month, Oren Etzioni, an A.I. researcher and professor, asked the bot: "What is the relationship between Oren Etzioni and Eli Etzioni?" It responded correctly that Oren and Eli are father and son.

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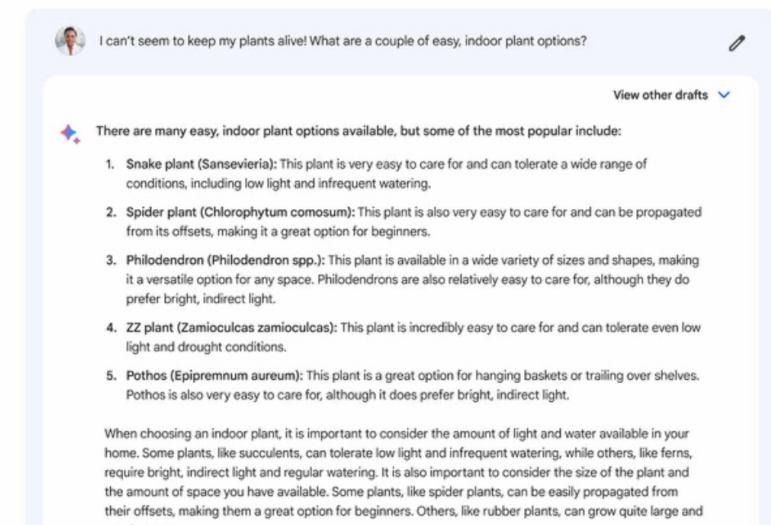
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ChatGPT will respond to similar prompts (and, yes, it will make up websites). Mr. Collins said Google Bard tended to avoid providing medical, legal or financial advice because it could lead to incorrect information.



Google Releases Bard, Its Competitor in the Race to Create A.I. Chatbots

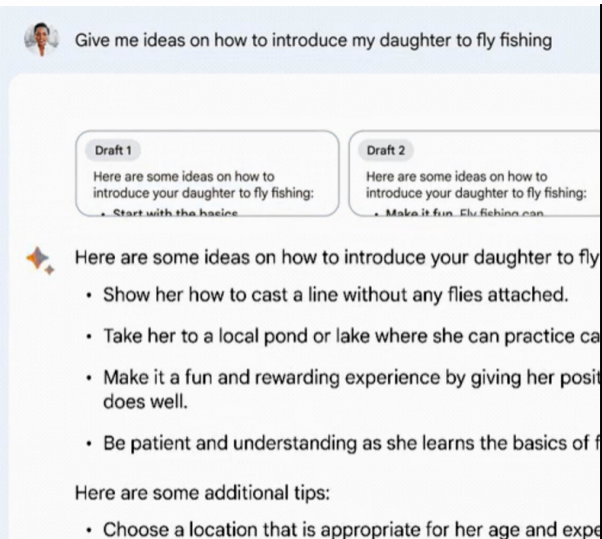
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Does Bard *actually* know, believe, desire, ...??

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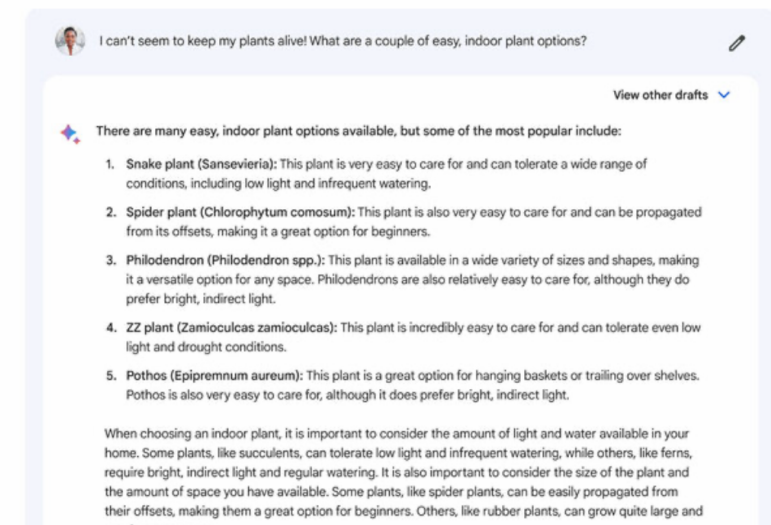
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On the extensional-
logic ladder ...
questions?

Climbing the k -order Ladder

Climbing the k -order Ladder

a is a llama, as is b , a likes b , and the father of a is a llama as well.

Climbing the k -order Ladder

$Llama(a) \wedge Llama(b) \wedge Likes(a, b) \wedge Llama(fatherOf(a))$

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Climbing the k -order Ladder

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There's some thing which is a llama and likes b (which is also a llama), and whose father is a llama too.

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Climbing the k -order Ladder

Things x and y , along with the father of x , share a certain property; and, $x R^2s y$, where R^2 is a positive property.

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Climbing the k -order Ladder

⋮

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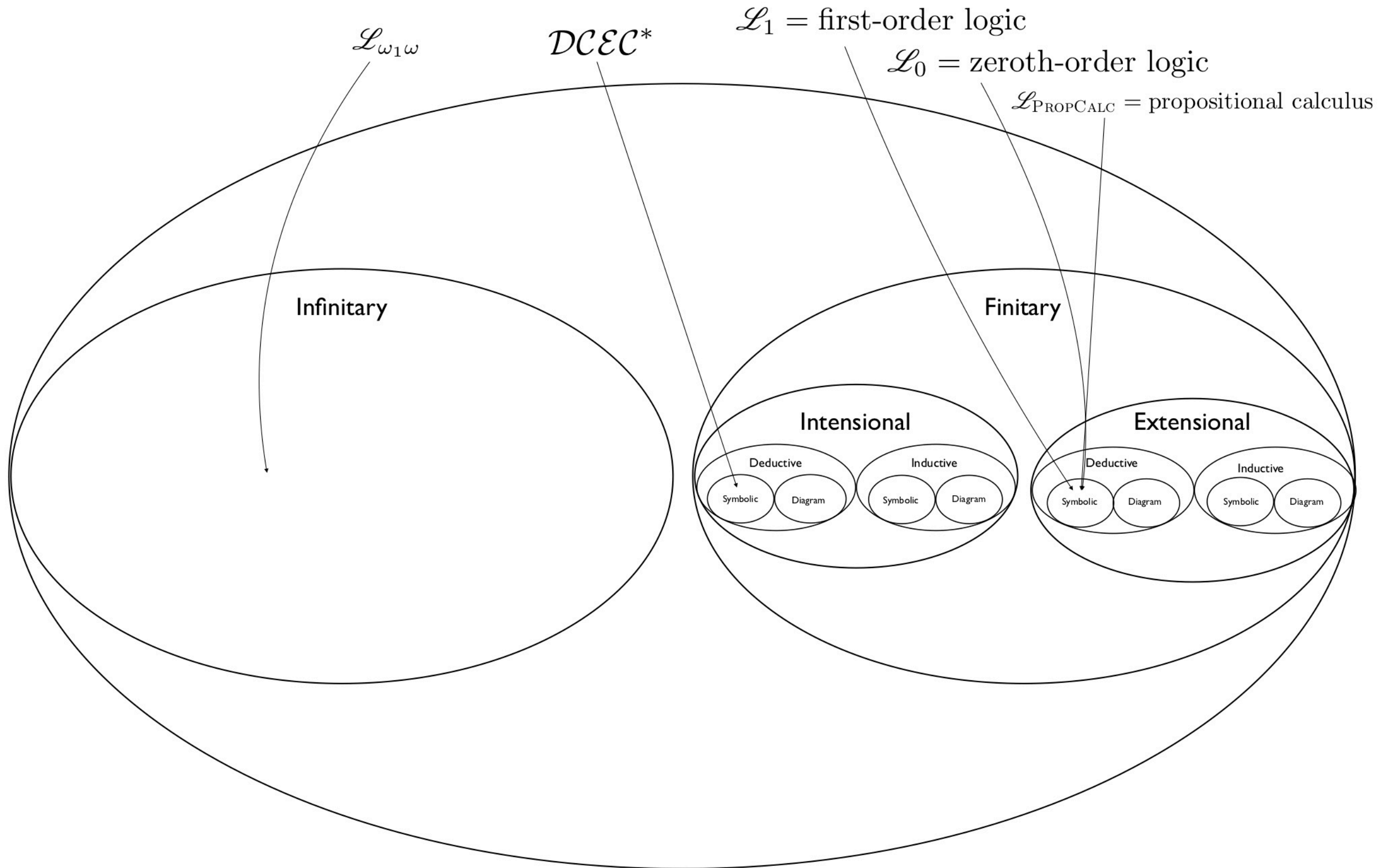
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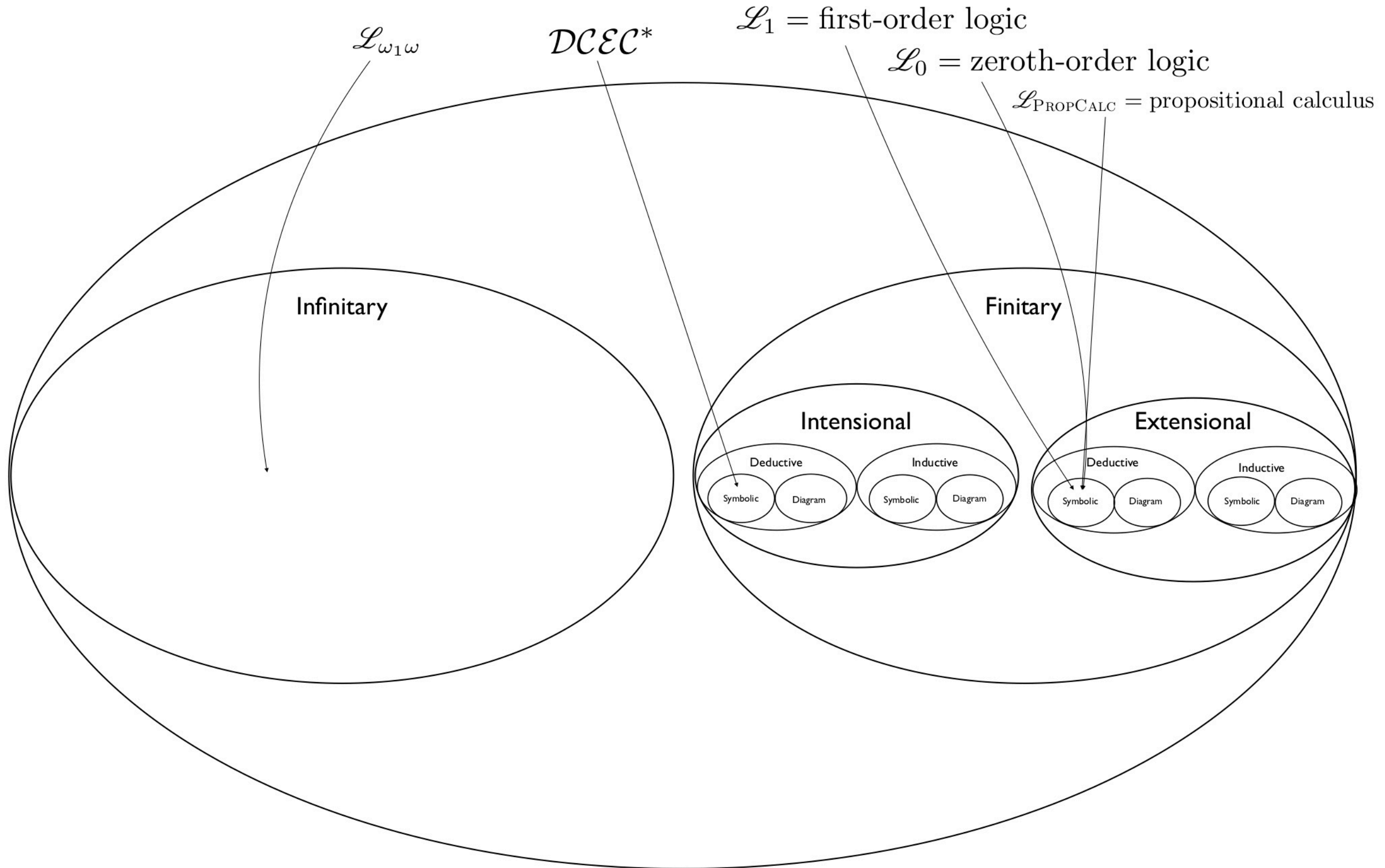
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The Universe of Logics

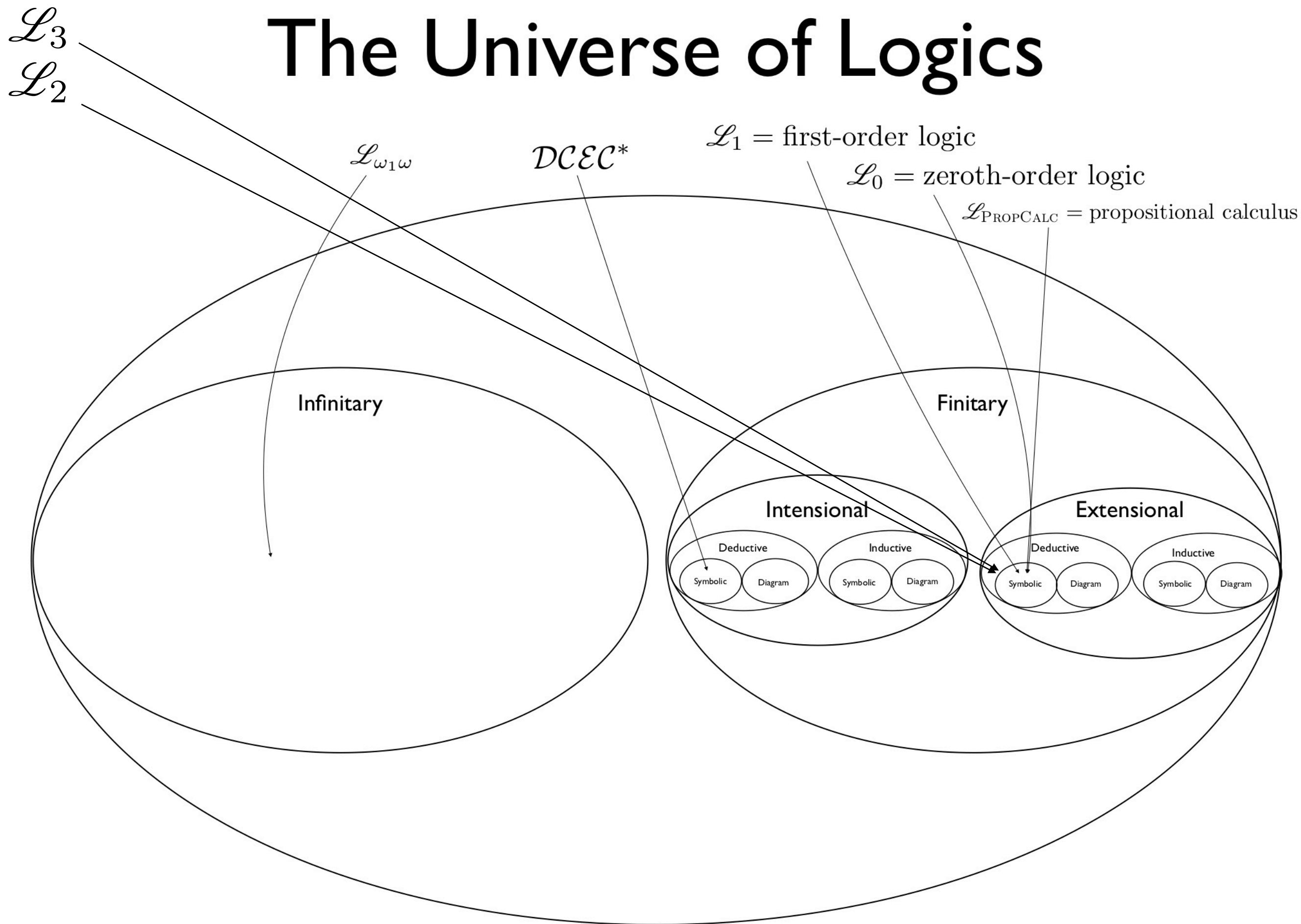


\mathcal{L}_3
 \mathcal{L}_2

The Universe of Logics



The Universe of Logics



Climbing the k -order Ladder

⋮

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Climbing the k -order Ladder

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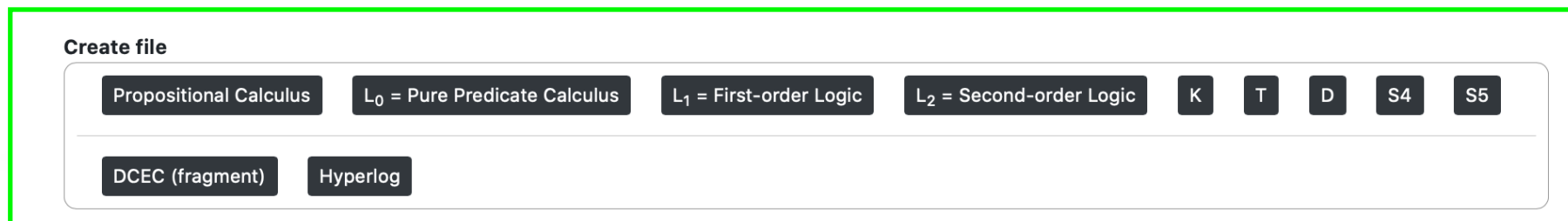
TOL

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SOL



\mathcal{L}_2

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Climbing the k -order Ladder

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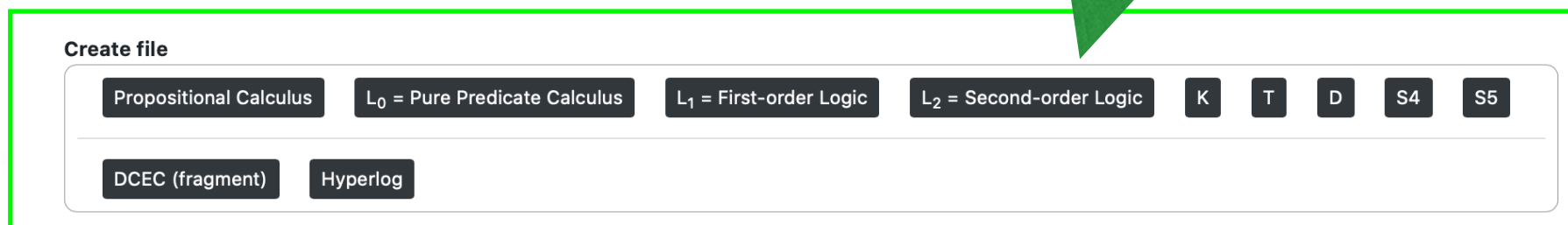
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FOL

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ZOL

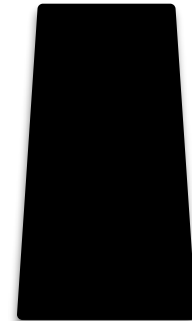
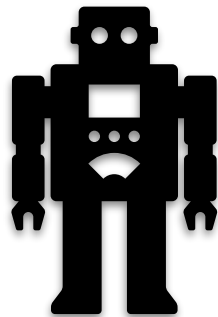
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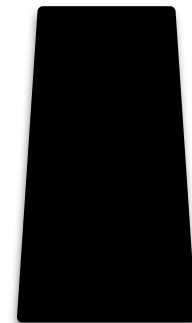
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**Blinky as portal to
intensional logics ...**

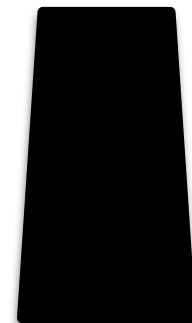
Blinky



1

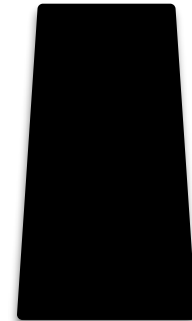
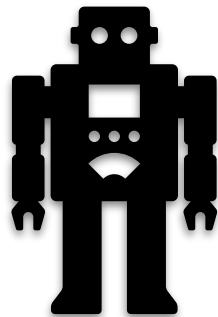


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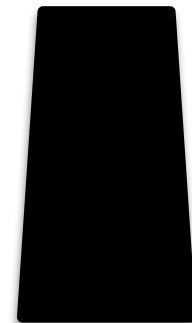


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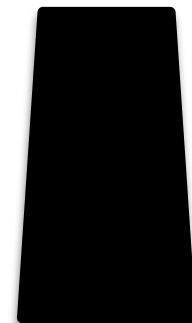
Blinky



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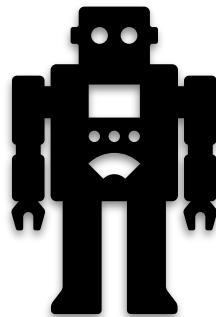
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3



Blinky



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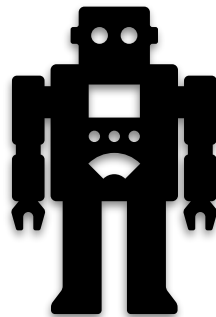


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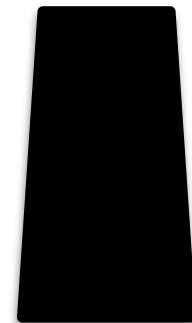
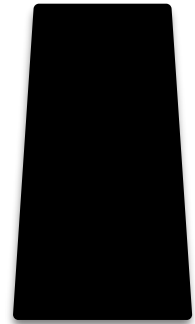
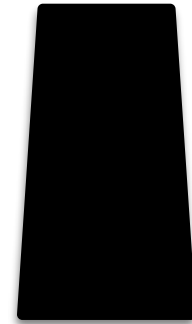
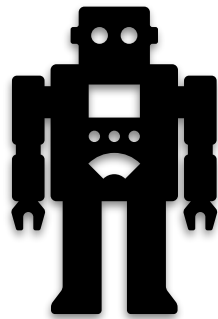


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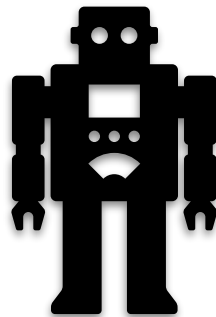
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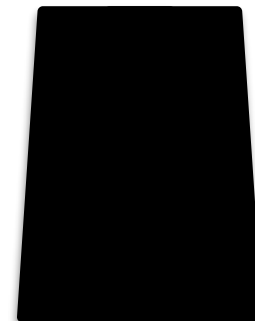
2

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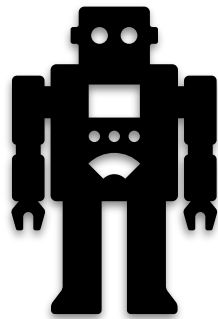
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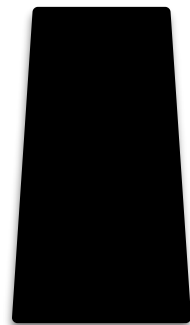
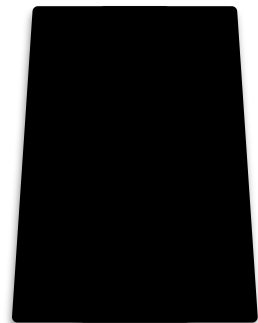
Blinky



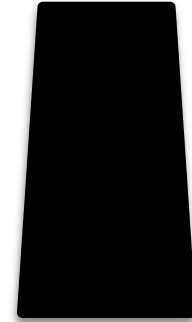
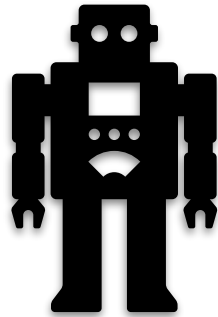
1

2

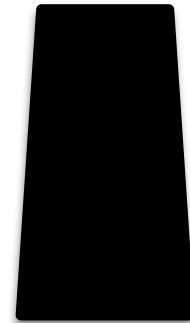
3



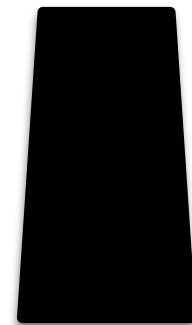
Blinky



1

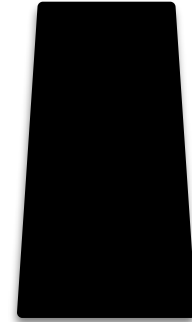
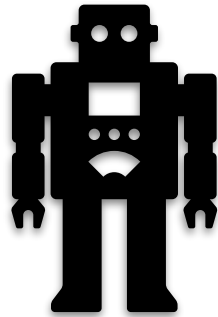


2

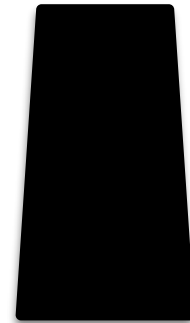


3

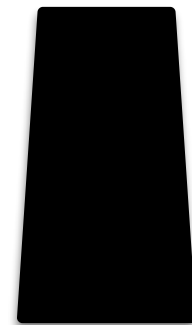
Blinky



1

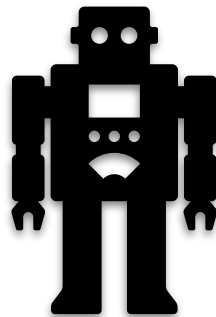


2



3

Blinky



1



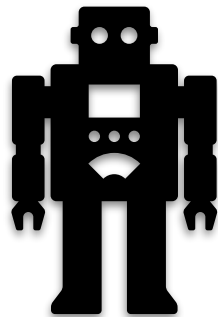
2



3

The ball is in the cup at location #1.

Blinky



1



2

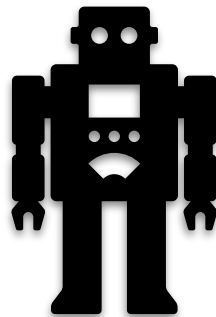


3

The ball is in the cup at location #1.

Loc(ball,1)

Blinky



1



2



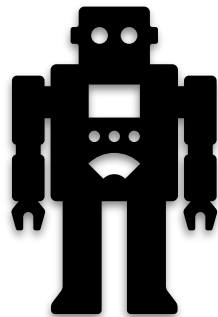
3

The ball is in the cup at location #1.

Loc(ball,1)

(Loc ball 1)

Blinky



1



2



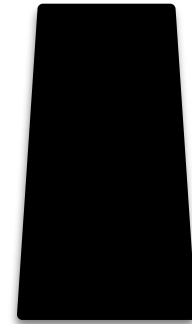
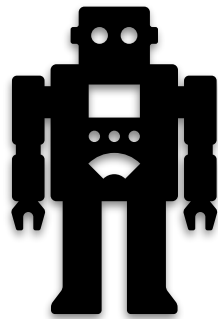
3

The ball is in the cup at location #1.

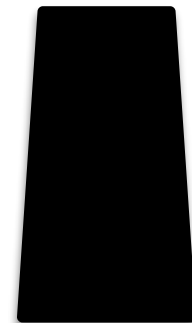
FALSE Loc(ball,1)

(Loc ball 1)

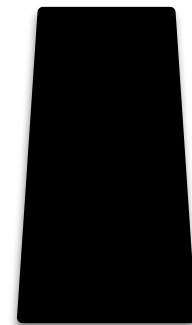
Blinky



1



2

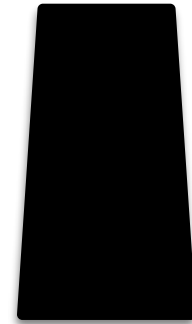
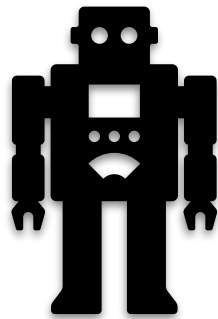


3

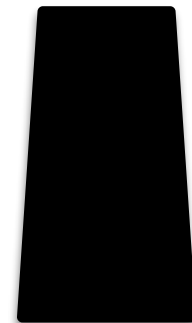
FALSE Loc(ball,1)

(Loc ball 1)

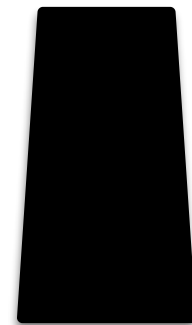
Blinky



1



2

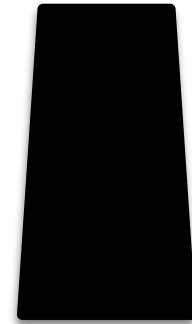
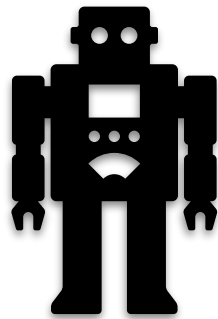


3

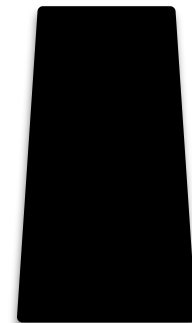
FALSE

(Loc ball 1)

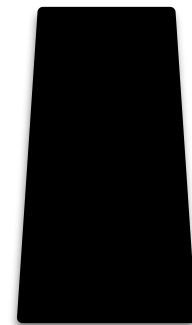
Blinky



1

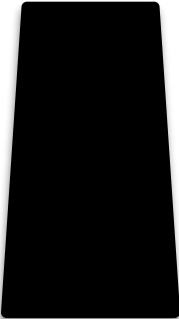


2

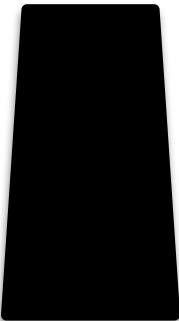


3

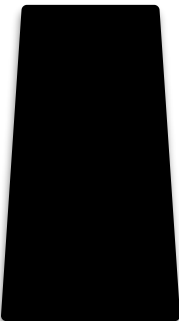
(Loc ball 1)



1

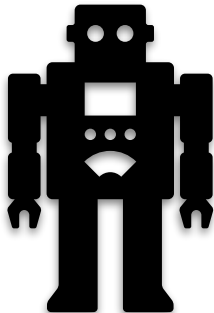


2

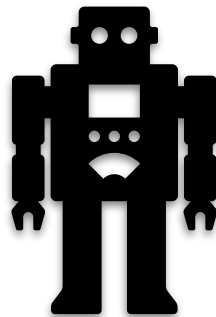


3

Blinky



Blinky



1



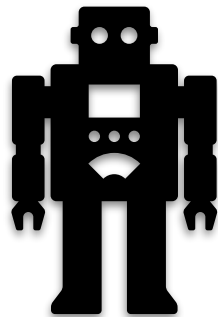
2



3

The ball is in the cup at location #1 or the ball is at location #3.

Blinky



1



2

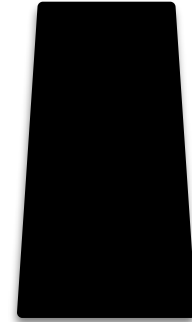
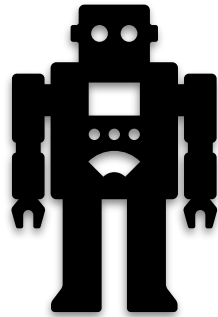


3

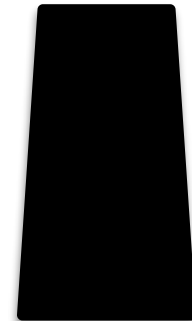
The ball is in the cup at location #1 or the ball is at location #3.

$\text{Loc}(\text{ball}, 1) \vee \text{Loc}(\text{ball}, 3)$

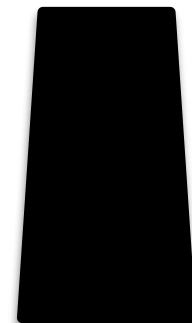
Blinky



1



2



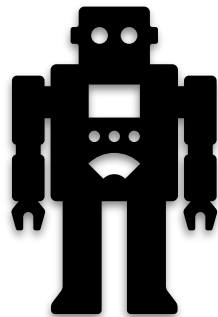
3

The ball is in the cup at location #1 or the ball is at location #3.

$\text{Loc}(\text{ball}, 1) \vee \text{Loc}(\text{ball}, 3)$

(or (Loc ball 1) (Loc ball 3))

Blinky



1



2



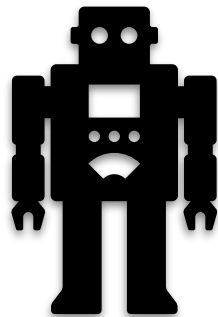
3

The ball is in the cup at location #1 or the ball is at location #3.

FALSE $\text{Loc}(\text{ball}, 1) \vee \text{Loc}(\text{ball}, 3)$

(or (Loc ball 1) (Loc ball 3))

Blinky



1



2

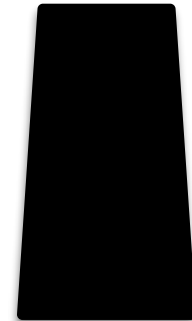
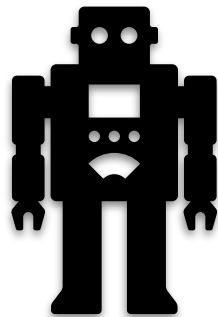


3

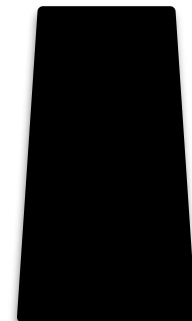
FALSE $\text{Loc}(\text{ball}, 1) \vee \text{Loc}(\text{ball}, 3)$

(or (Loc ball 1) (Loc ball 3))

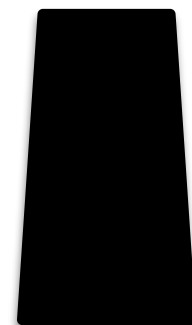
Blinky



1



2

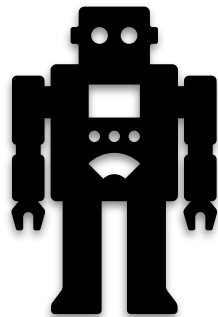


3

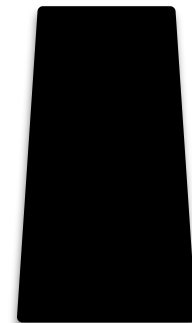
FALSE

(or (Loc ball 1) (Loc ball 3))

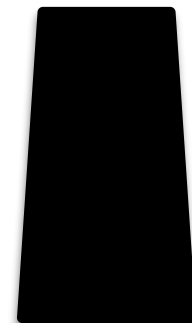
Blinky



1



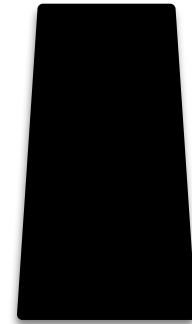
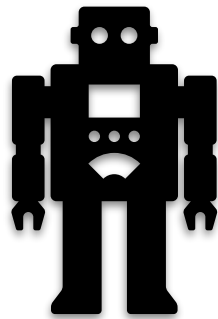
2



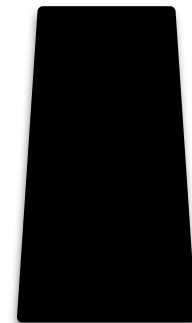
3

FALSE

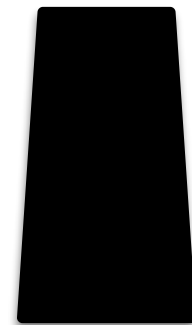
Blinky



1

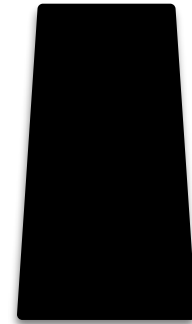
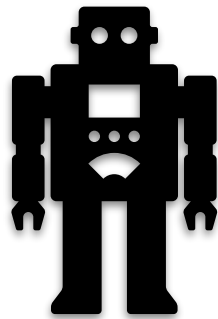


2

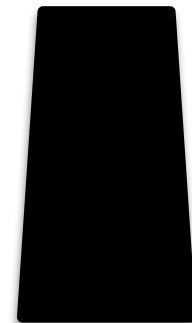


3

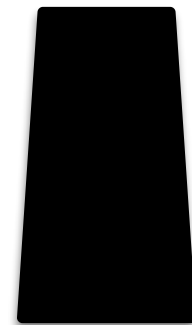
Blinky



1



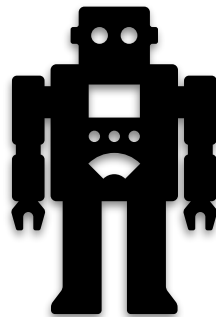
2



3

Blinky believes that the ball is in the cup at location #1.

Blinky



1



2

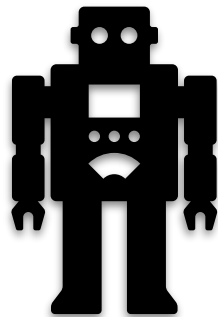


3

Blinky believes that the ball is in the cup at location #1.

$B(\text{blinky}, \text{Loc}(\text{ball}, 1))$

Blinky



1



2



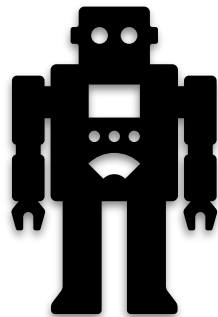
3

Blinky believes that the ball is in the cup at location #1.

$B(\text{blinky}, \text{Loc}(\text{ball}, 1))$

(Believes! blinky (Loc ball 1))

Blinky



1



2



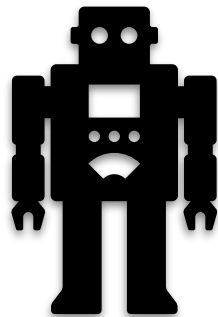
3

Blinky believes that the ball is in the cup at location #1.

??? B(blinky, Loc(ball,1))

(Believes! blinky (Loc ball 1))

Blinky



1



2



3

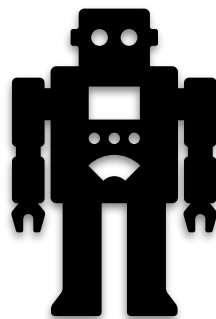
Blinky believes that the ball is in the cup at location #1.

???

B(blinky, Loc(ball,1))

(Believes! blinky (Loc ball 1))

Blinky



1



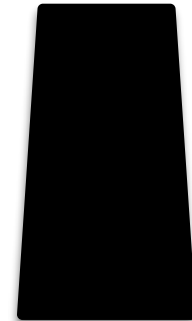
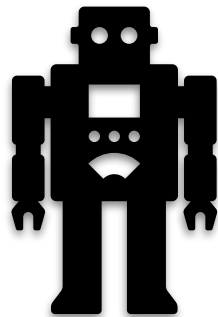
2



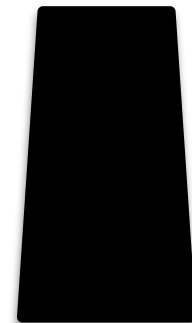
3

In extensional logics, what is denoted is conflated with meaning (the latter being naïvely compositional), but intensional attitudes like *believes*, *knows*, *hopes*, *fears*, etc cannot be represented and reasoned over smoothly (e.g. without fear of inconsistency rising up).

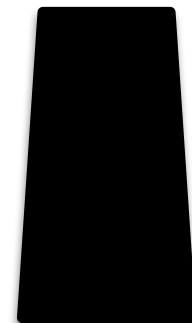
Blinky



1

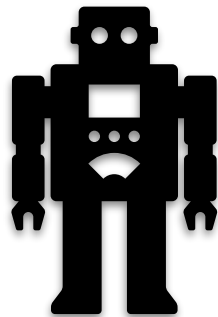


2

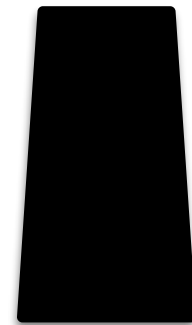


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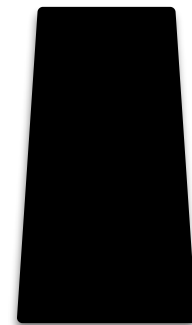
Blinky



1



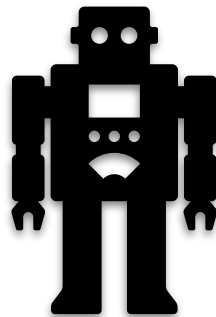
2



3



Blinky



1

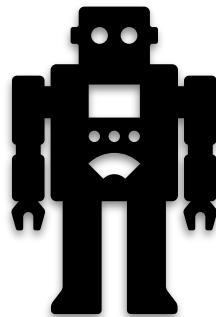


2



3

Blinky



1

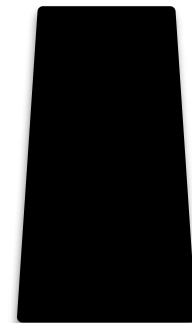
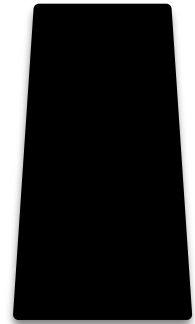
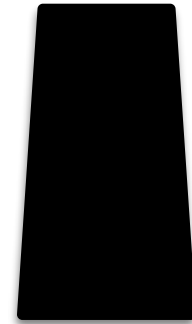
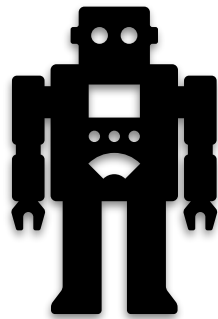


2



3

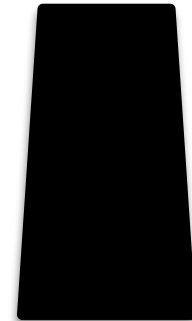
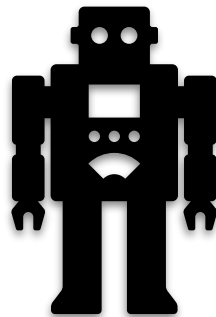
Blinky



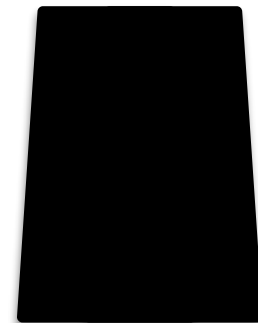
2

3

Blinky



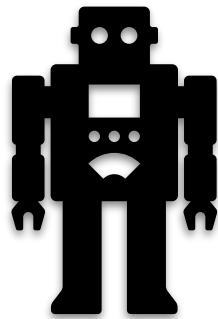
1



2

3

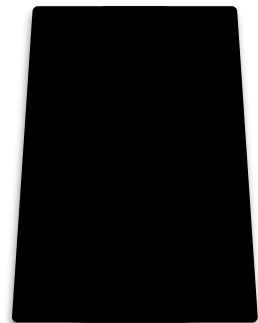
Blinky



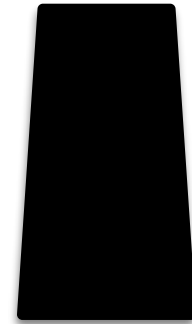
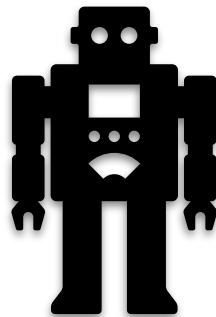
1

2

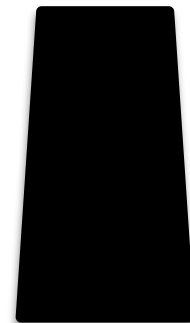
3



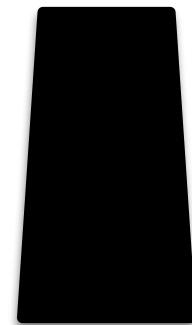
Blinky



1

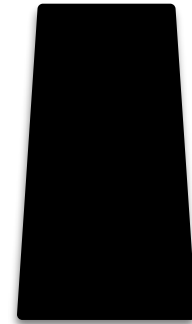
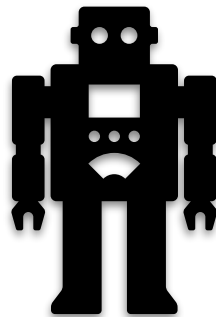


2



3

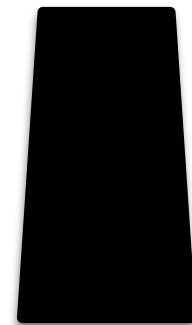
Blinky



1

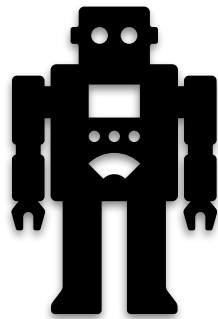


2

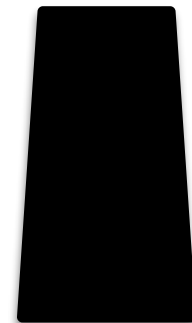


3

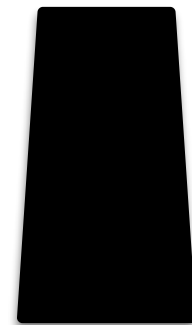
Blinky



1



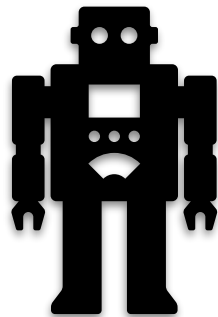
2



3

Blinky believes that the ball is in the cup at location #1.

Blinky



1



2

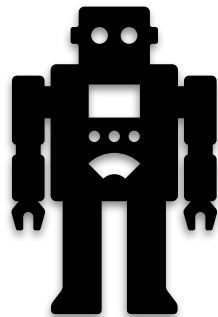


3

Blinky believes that the ball is in the cup at location #1.

$B(\text{blinky}, \text{loc-ball-1})$

Blinky



1



2



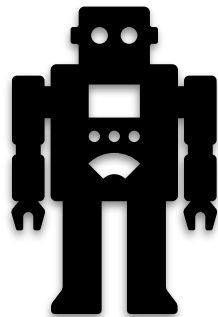
3

Blinky believes that the ball is in the cup at location #1.

$B(\text{blinky}, \text{loc-ball-1})$

(Believes! blinky loc-ball-1)

Blinky



1



2



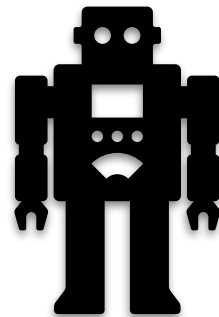
3

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(Believes! blinky loc-ball-1)

Blinky



1



2



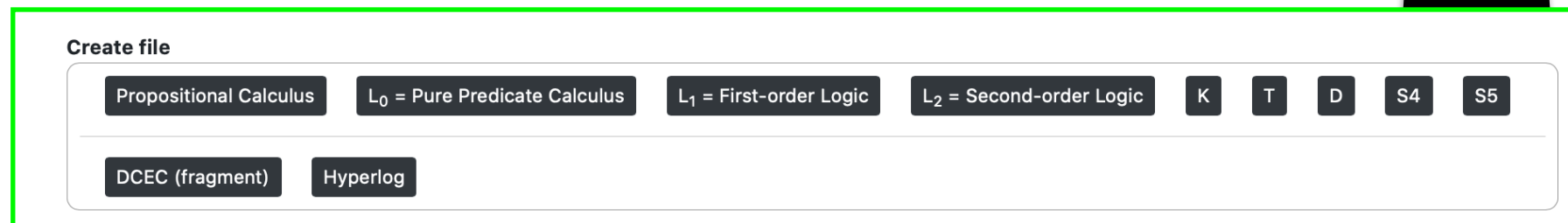
3

In intensional logics, meaning and designation are separated, and compositionality is abandoned.

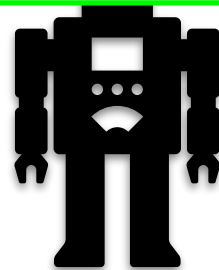
Blinky believes that the ball is in the cup at location #1.

$B(\text{blinky}, \text{loc-ball-1})$

(Believes! blinky loc-ball-1)



Blinky



1

2

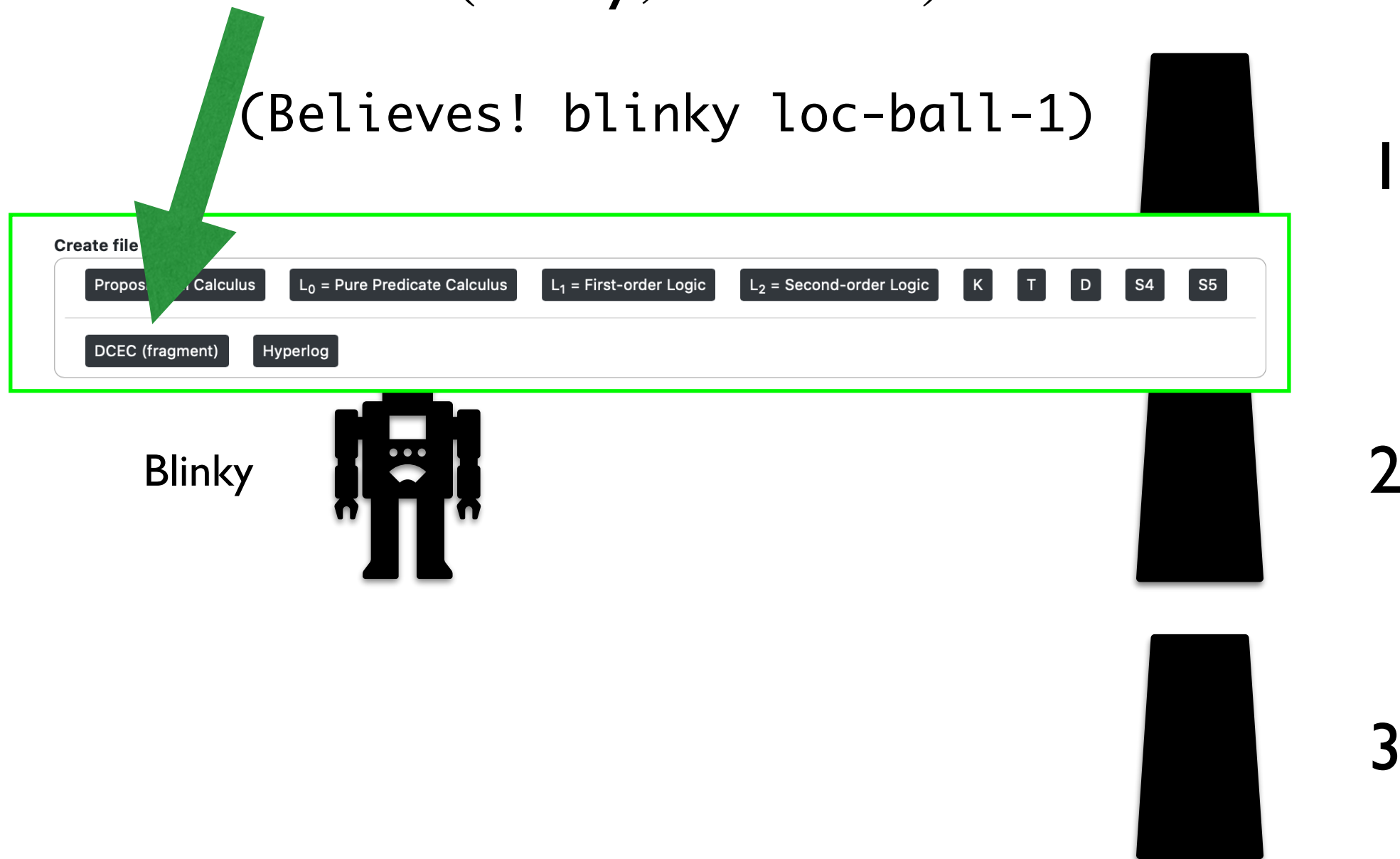
3

In intensional logics, meaning and designation are separated, and compositionality is abandoned.

Blinky believes that the ball is in the cup at location #1.

$B(\text{blinky}, \text{loc-ball-1})$

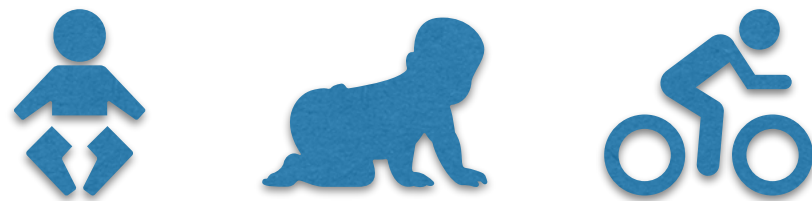
(Believes! blinky loc-ball-1)



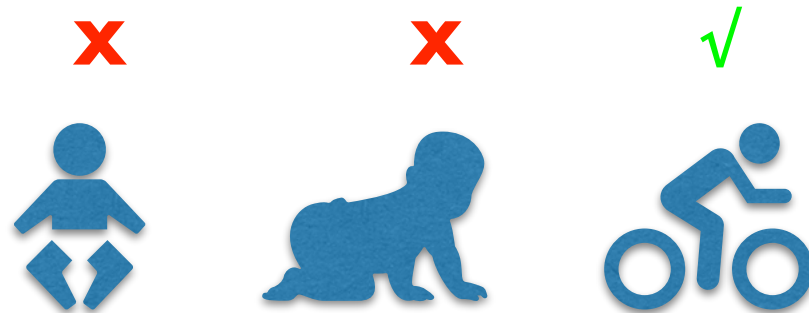
In intensional logics, meaning and designation are separated, and compositionality is abandoned.

False Belief Task Demands Intensional Logic ...

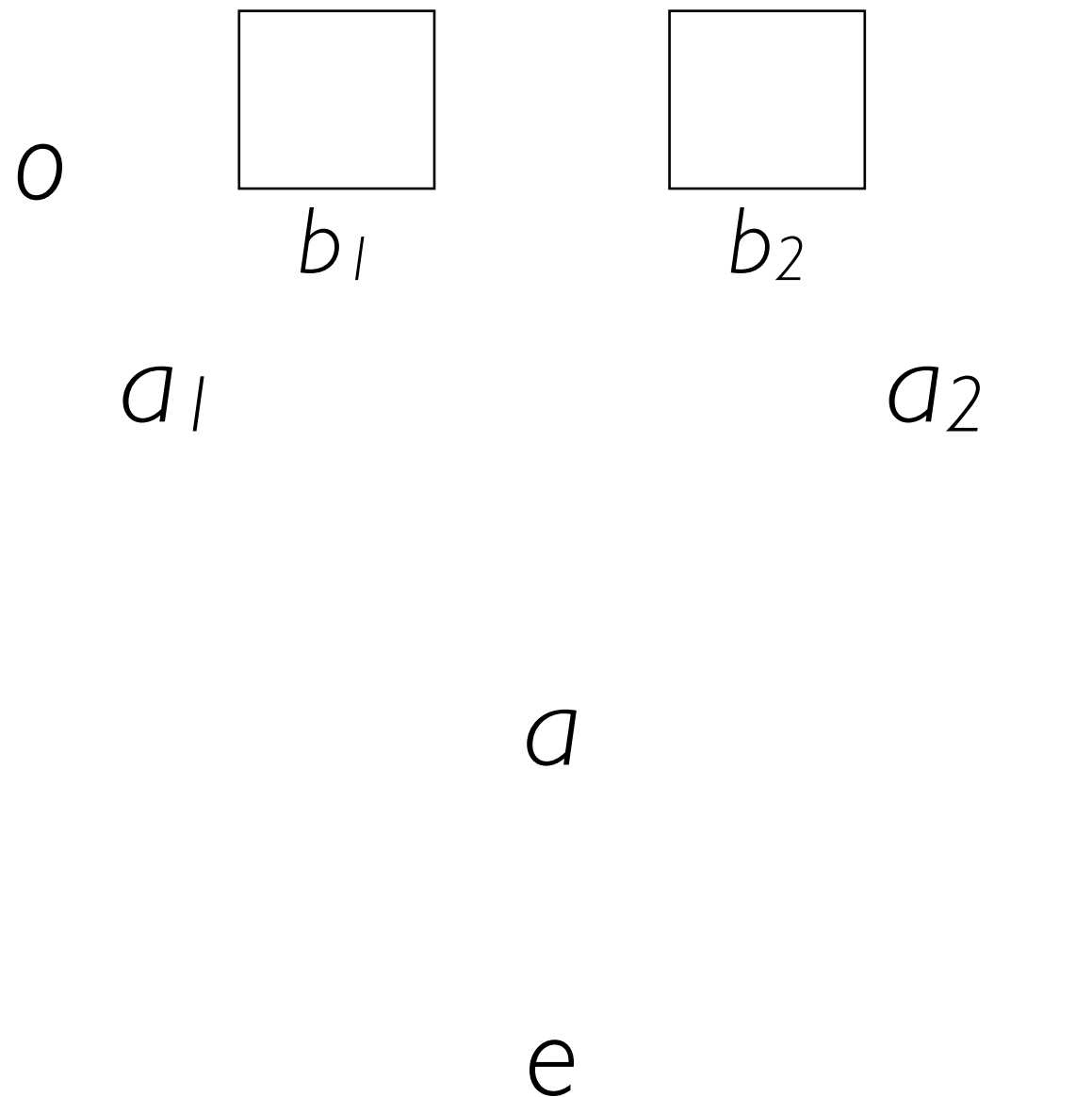
False Belief Task Demands Intensional Logic ...



False Belief Task Demands Intensional Logic ...

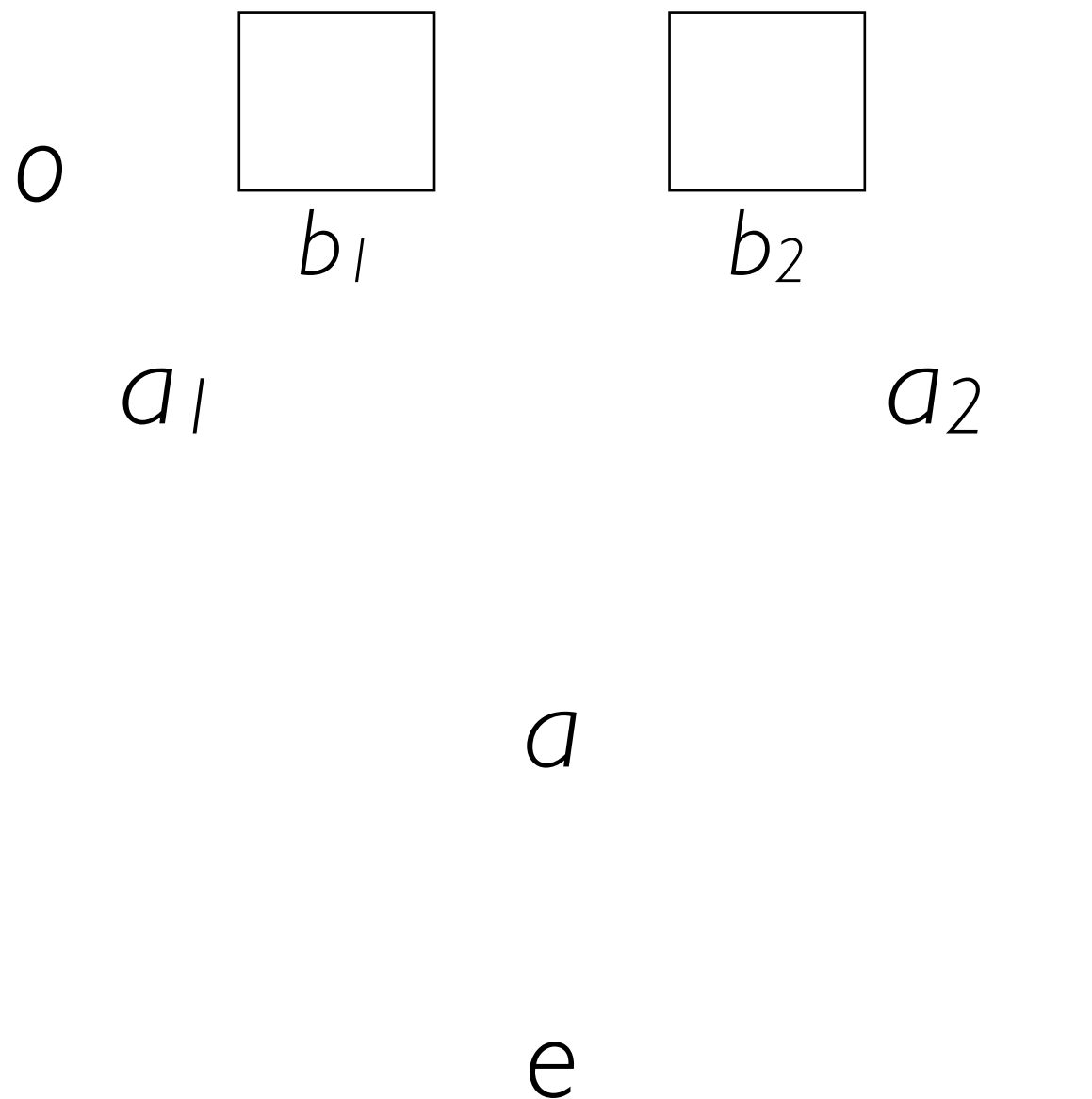


Framework for FBT^0_1



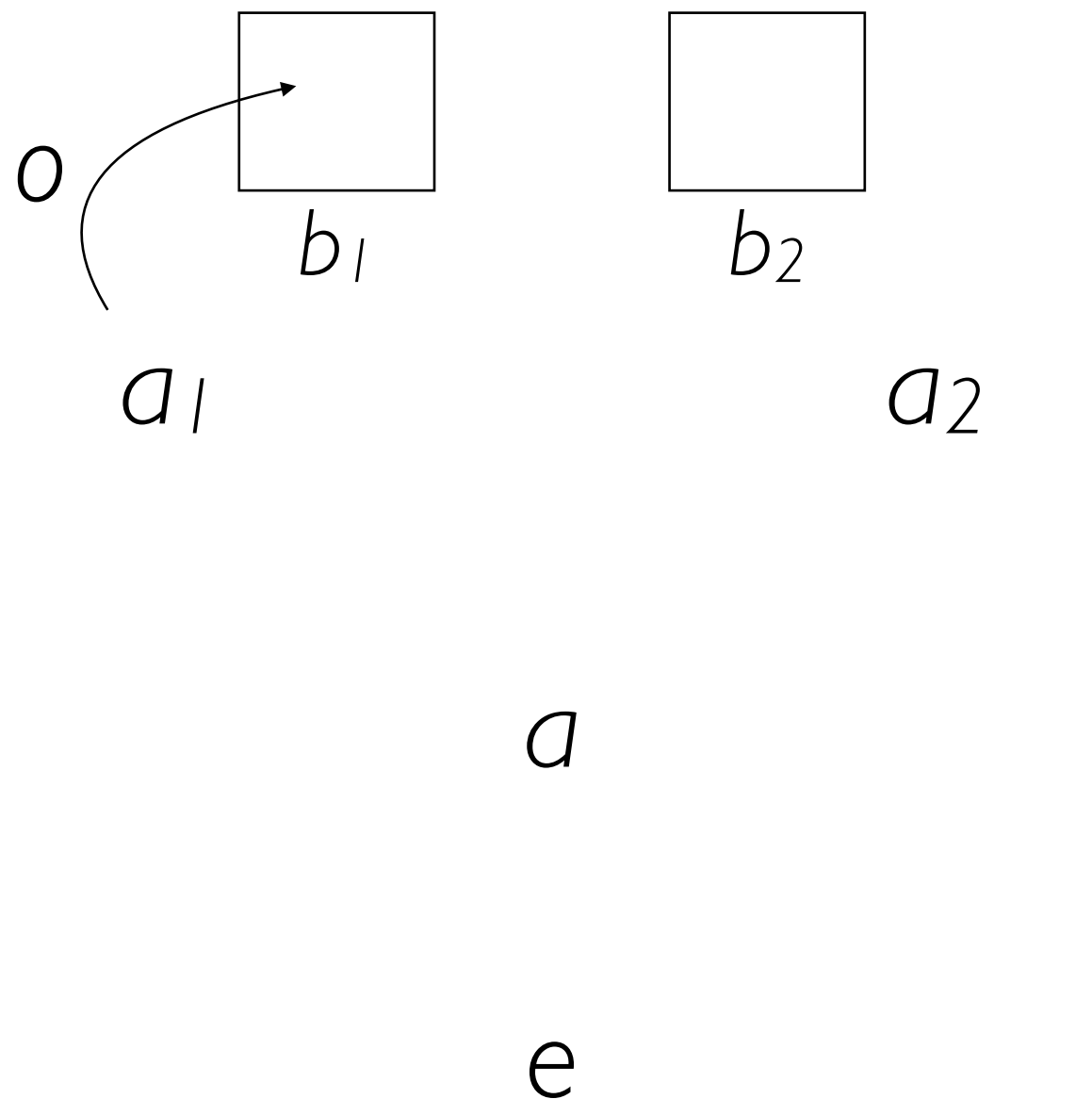
Framework for FBT^0_1

(five timepoints)



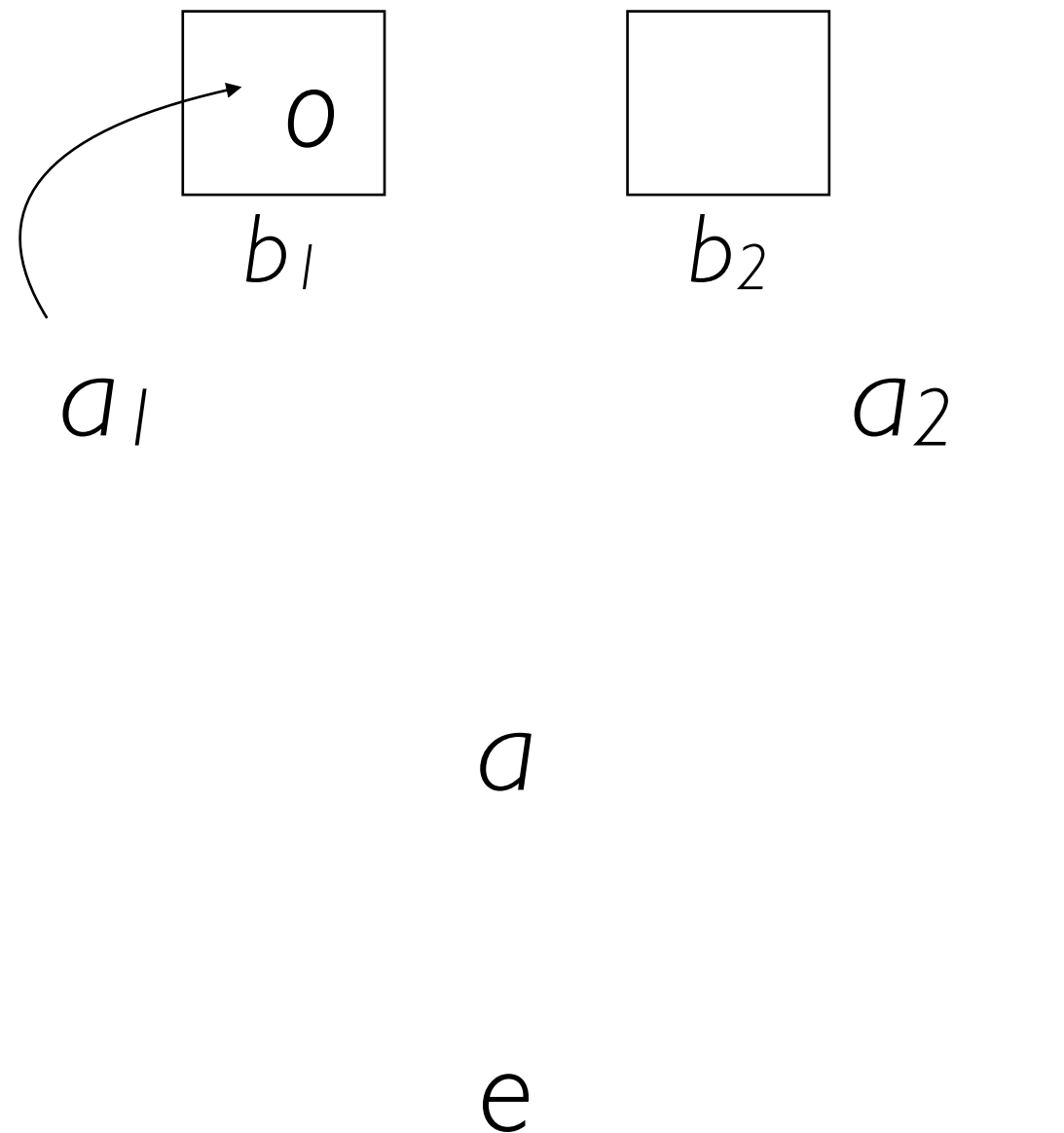
Framework for FBT^0_1

(five timepoints)



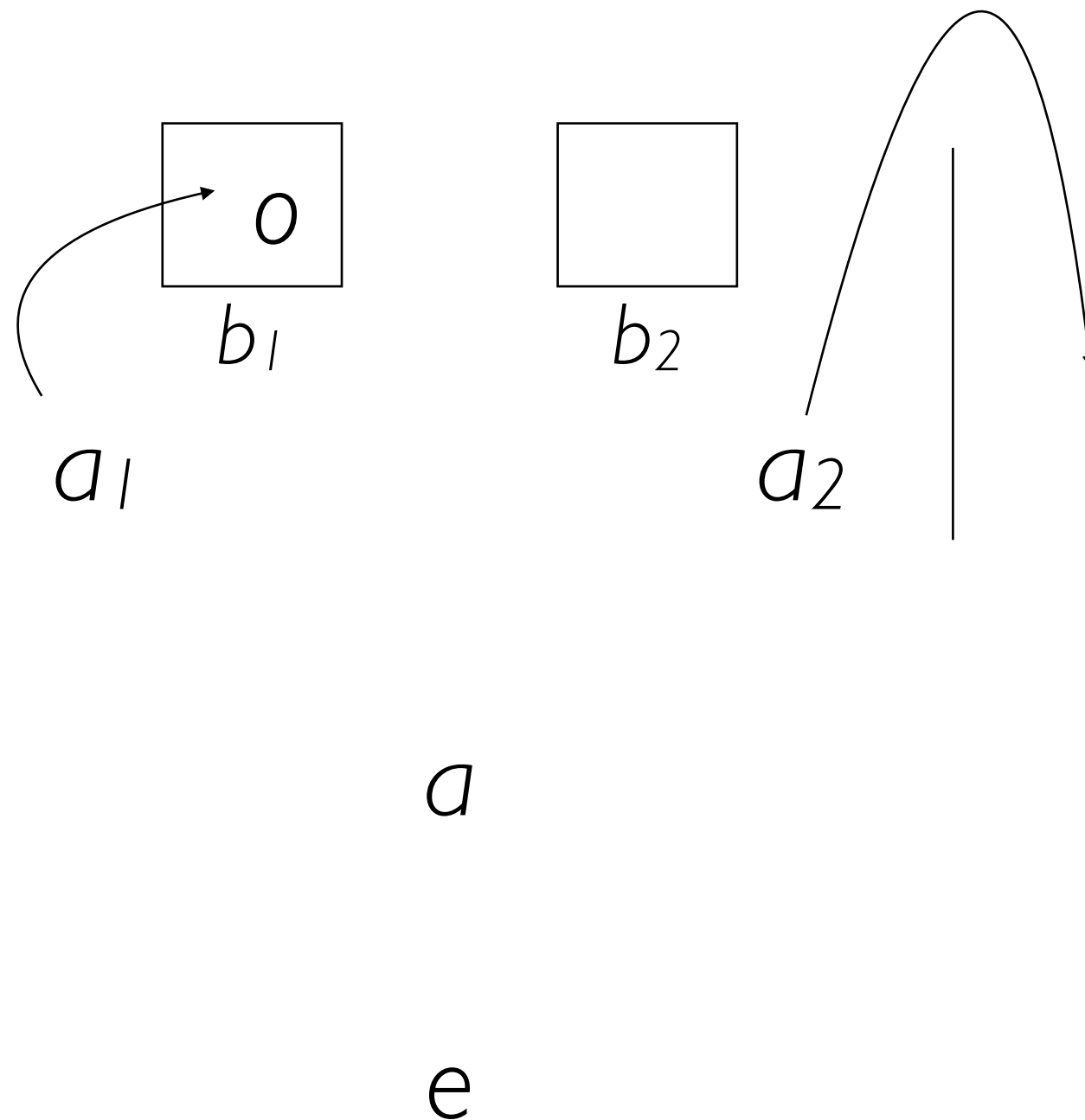
Framework for FBT^0_1

(five timepoints)



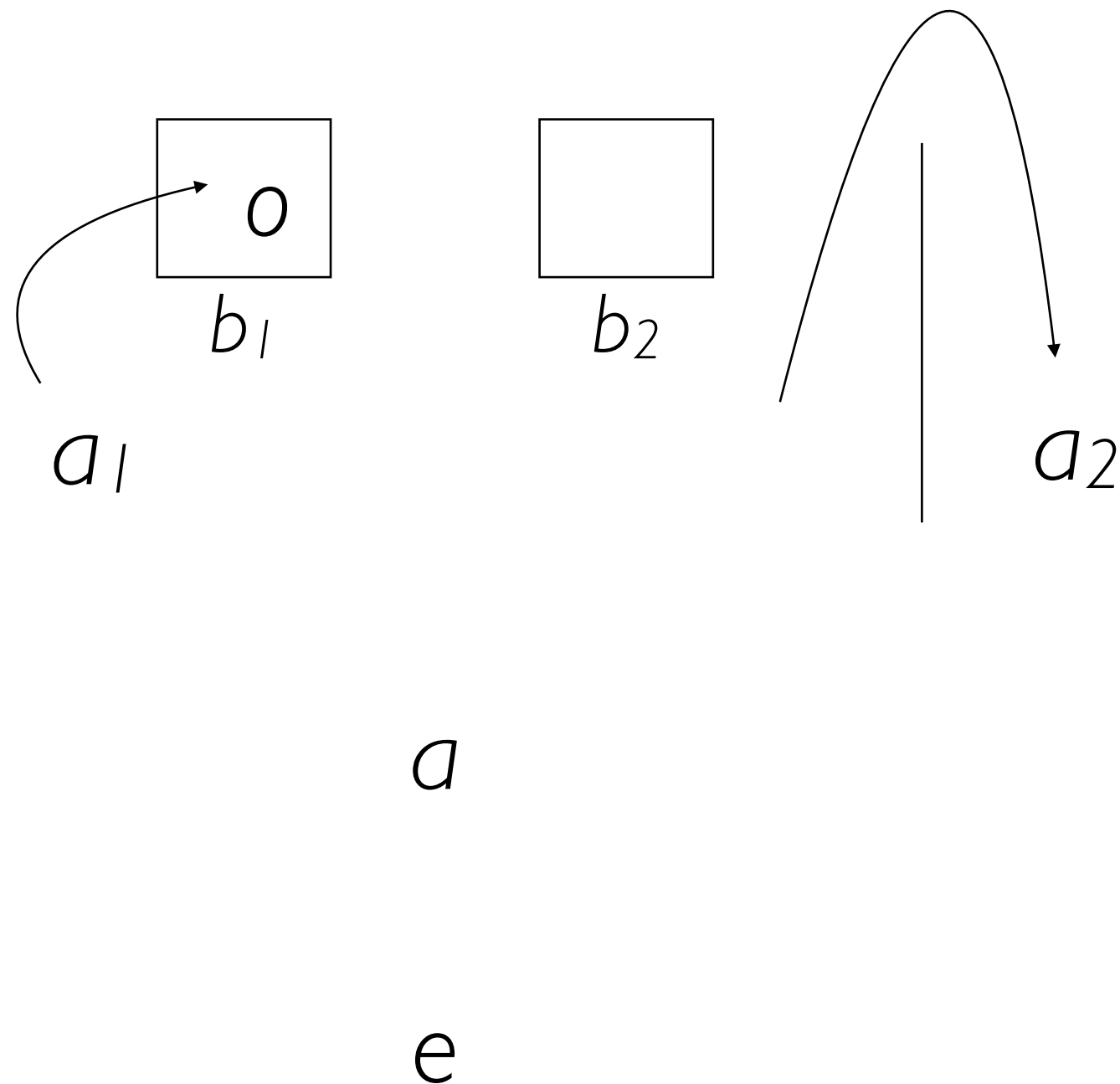
Framework for FBT^0_1

(five timepoints)



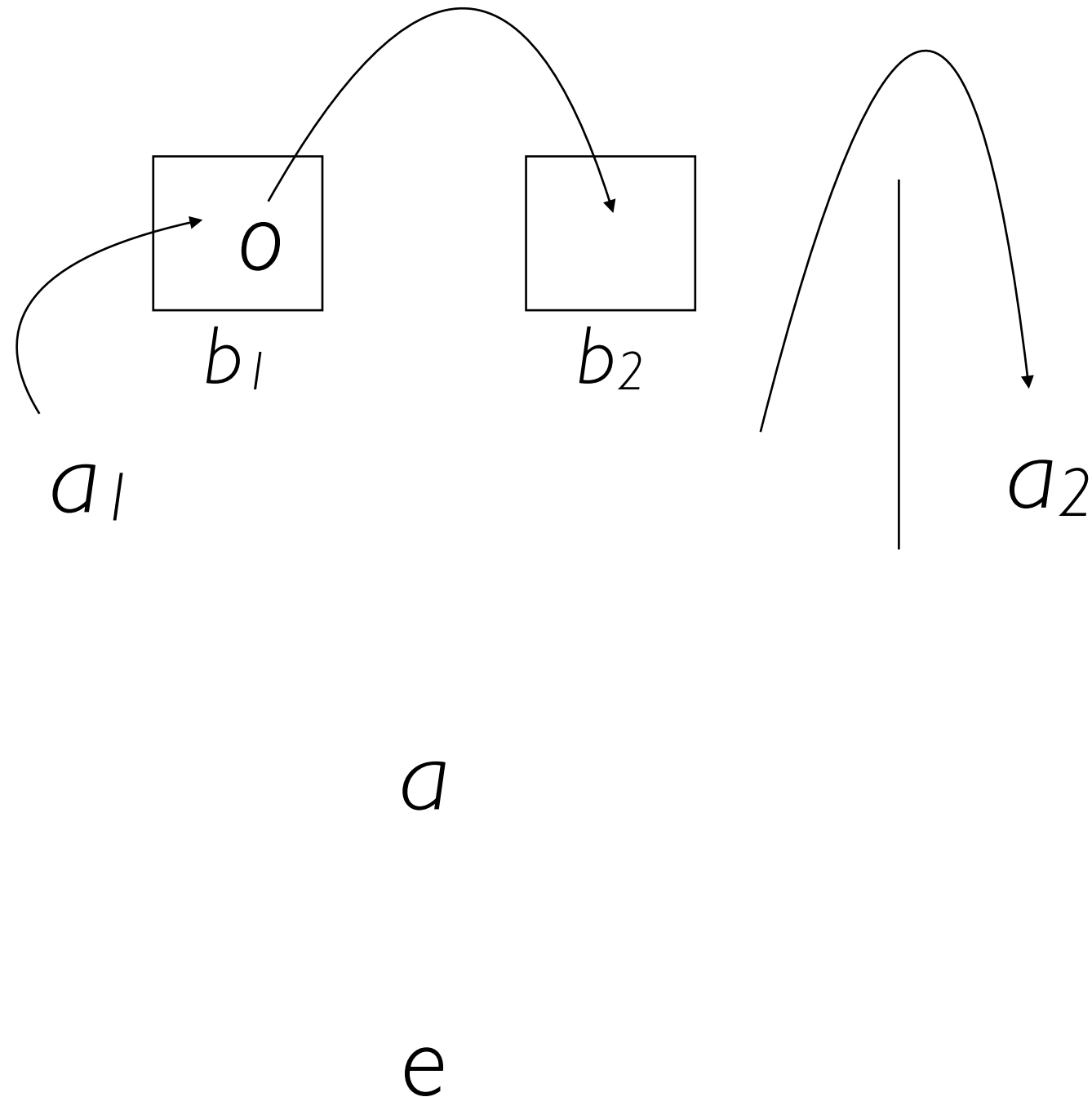
Framework for FBT^0_1

(five timepoints)



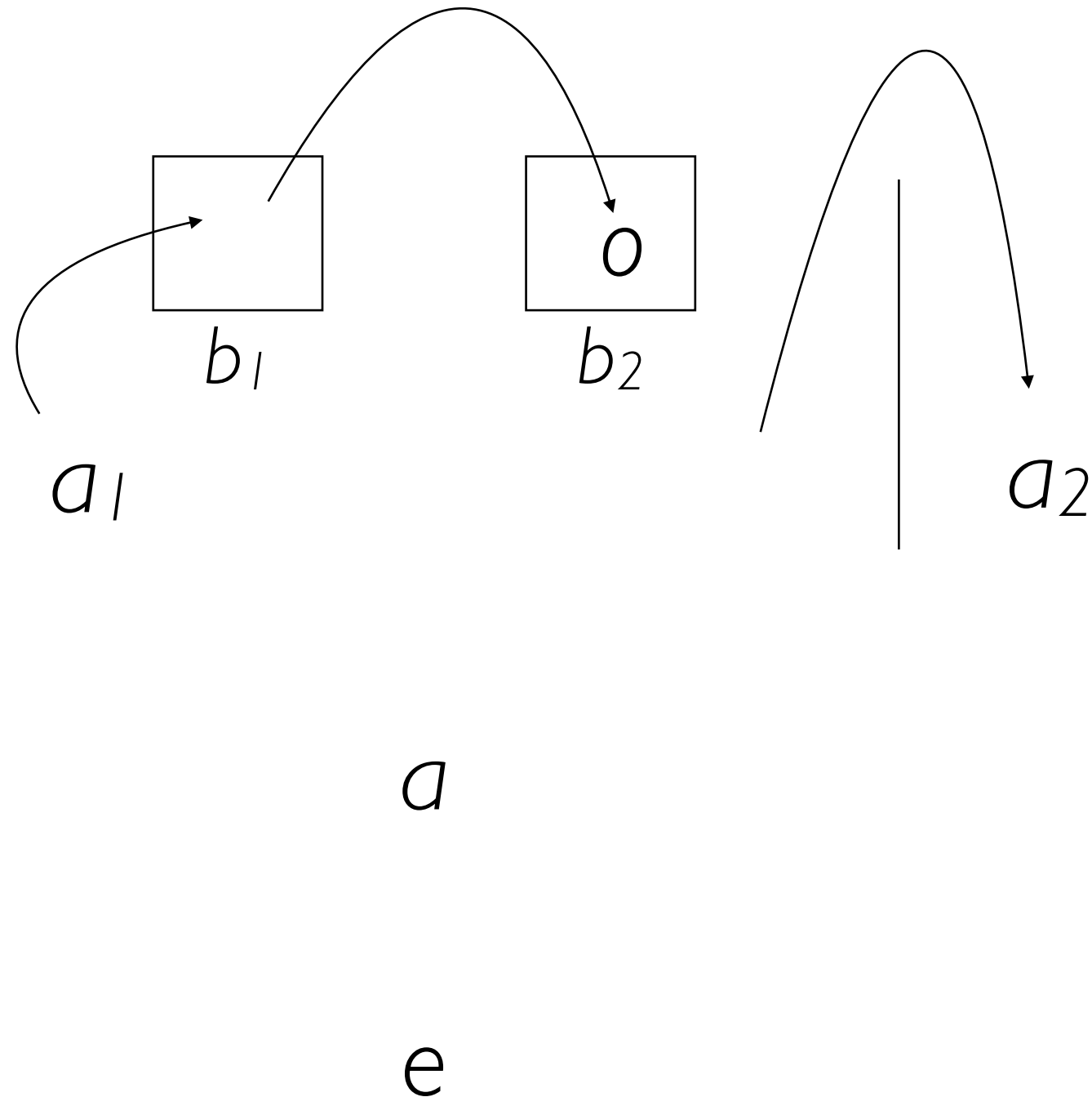
Framework for FBT^0_1

(five timepoints)



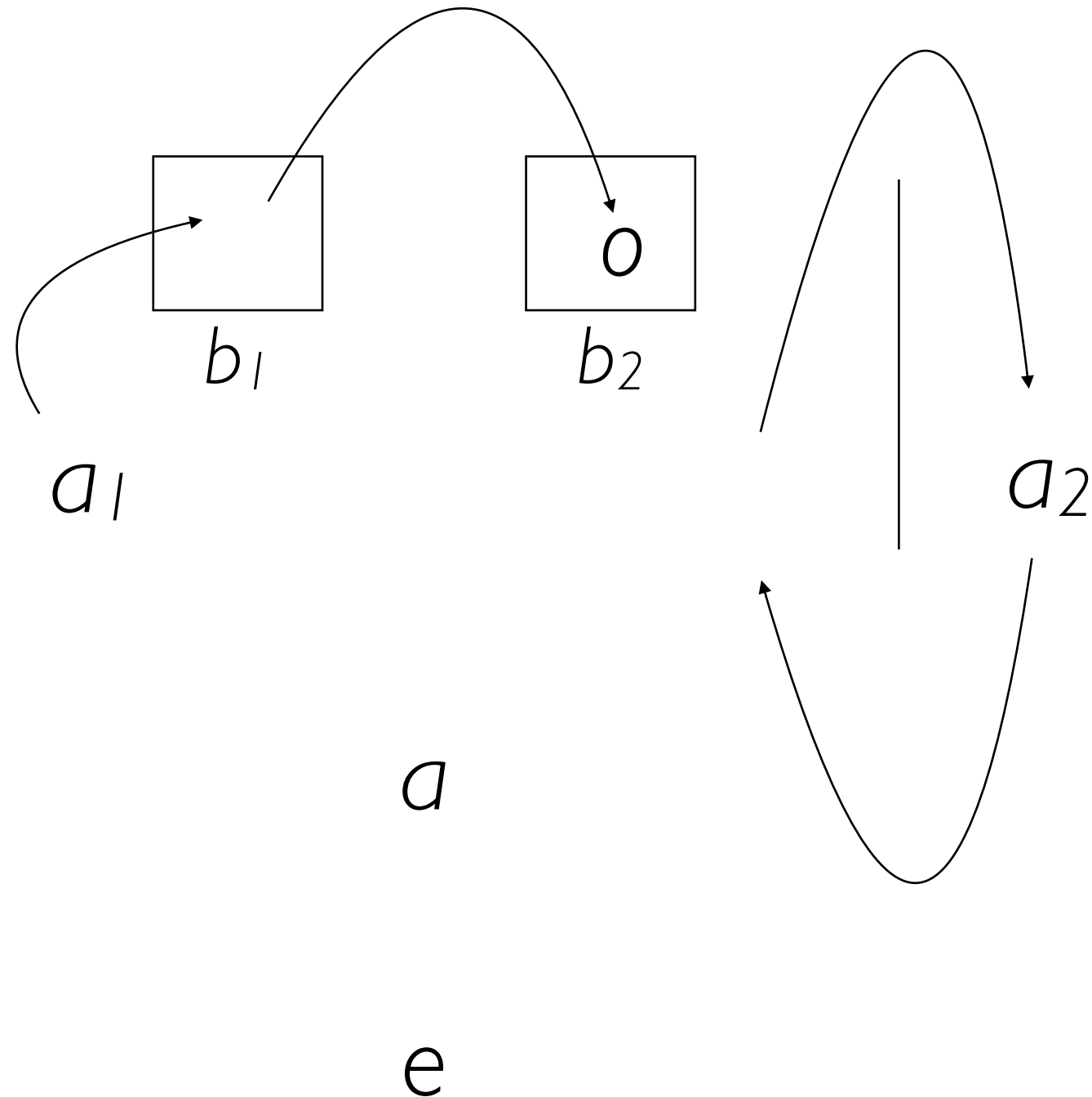
Framework for FBT^0_1

(five timepoints)



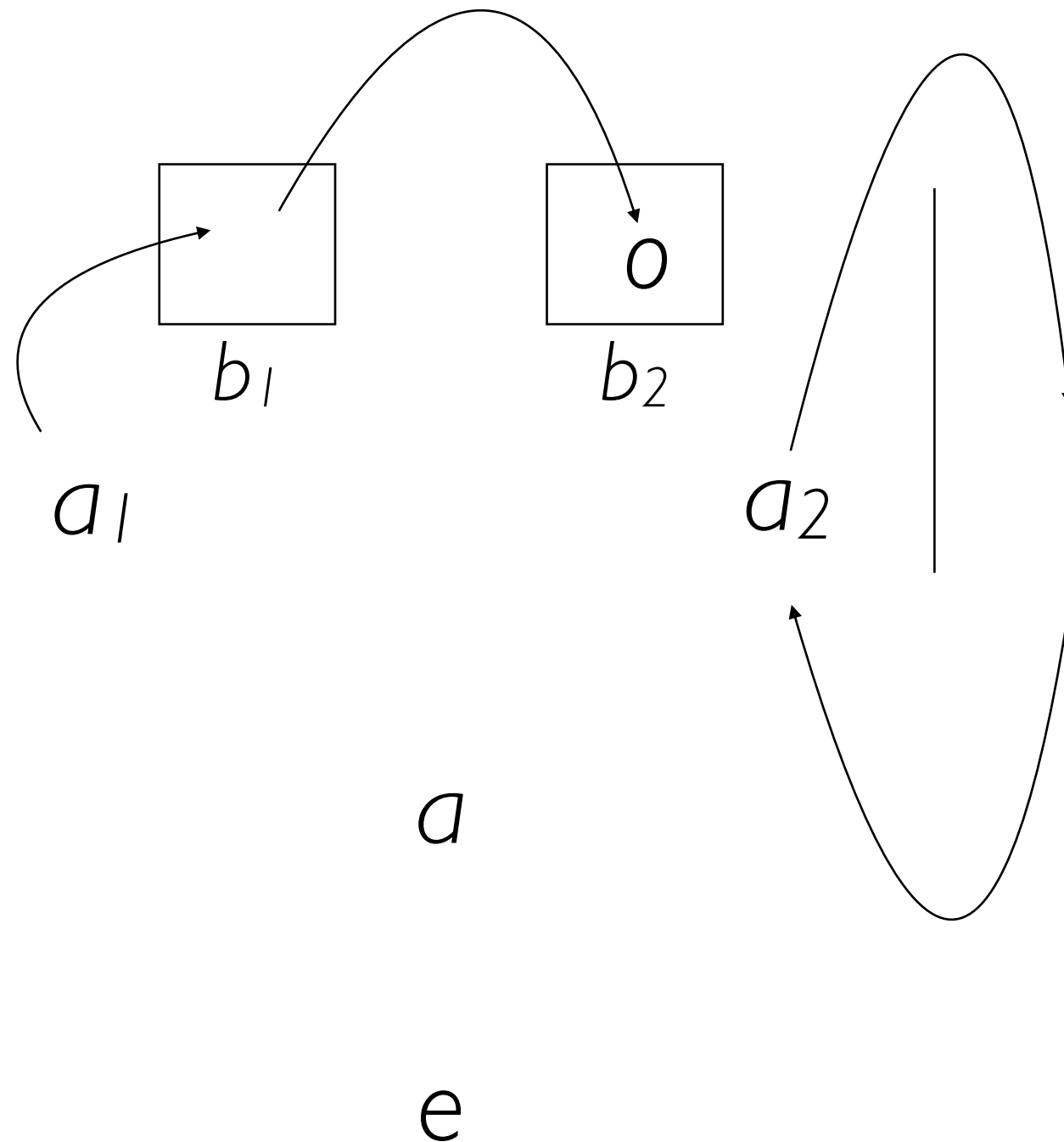
Framework for FBT^0_1

(five timepoints)



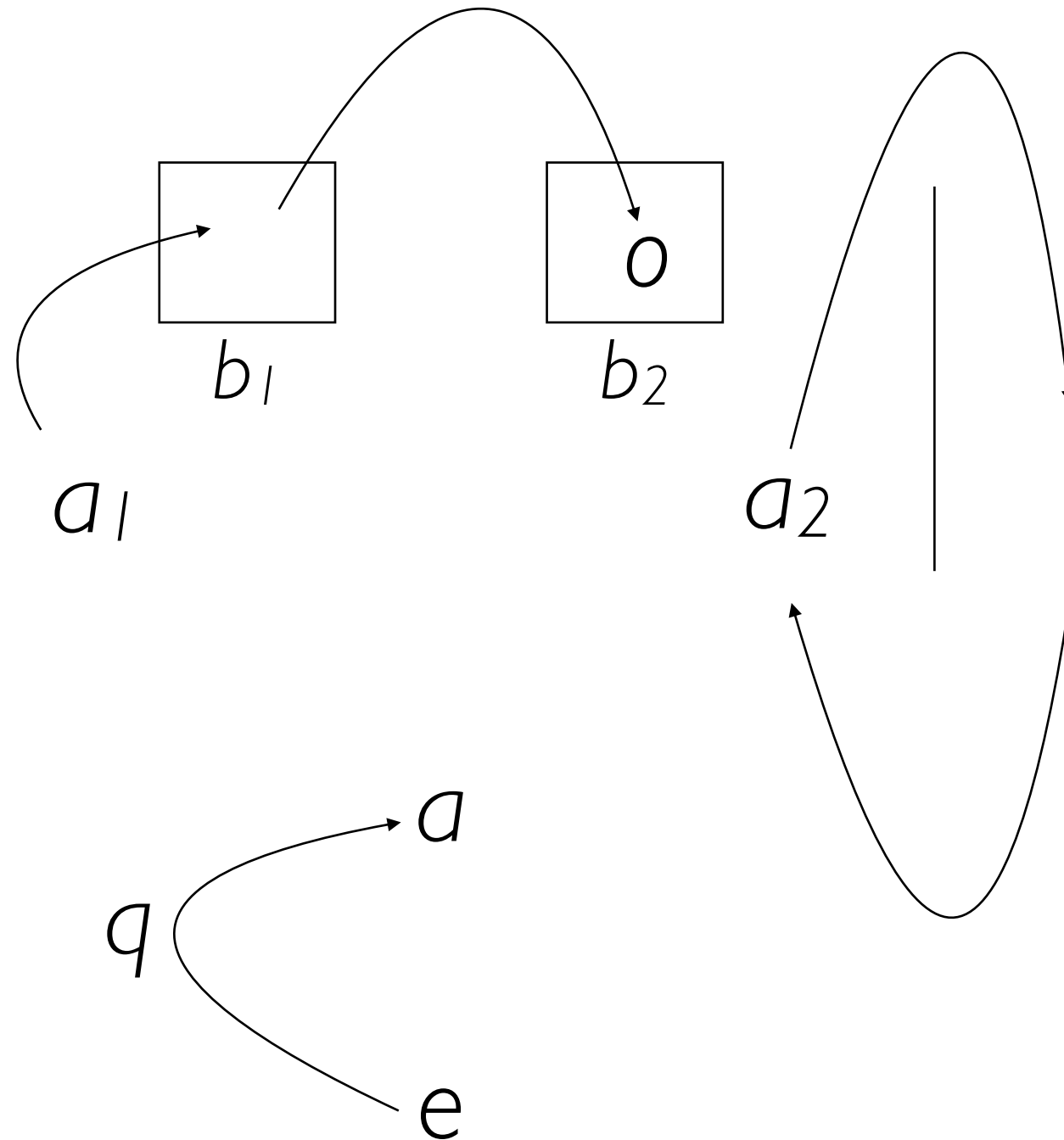
Framework for FBT^0_1

(five timepoints)



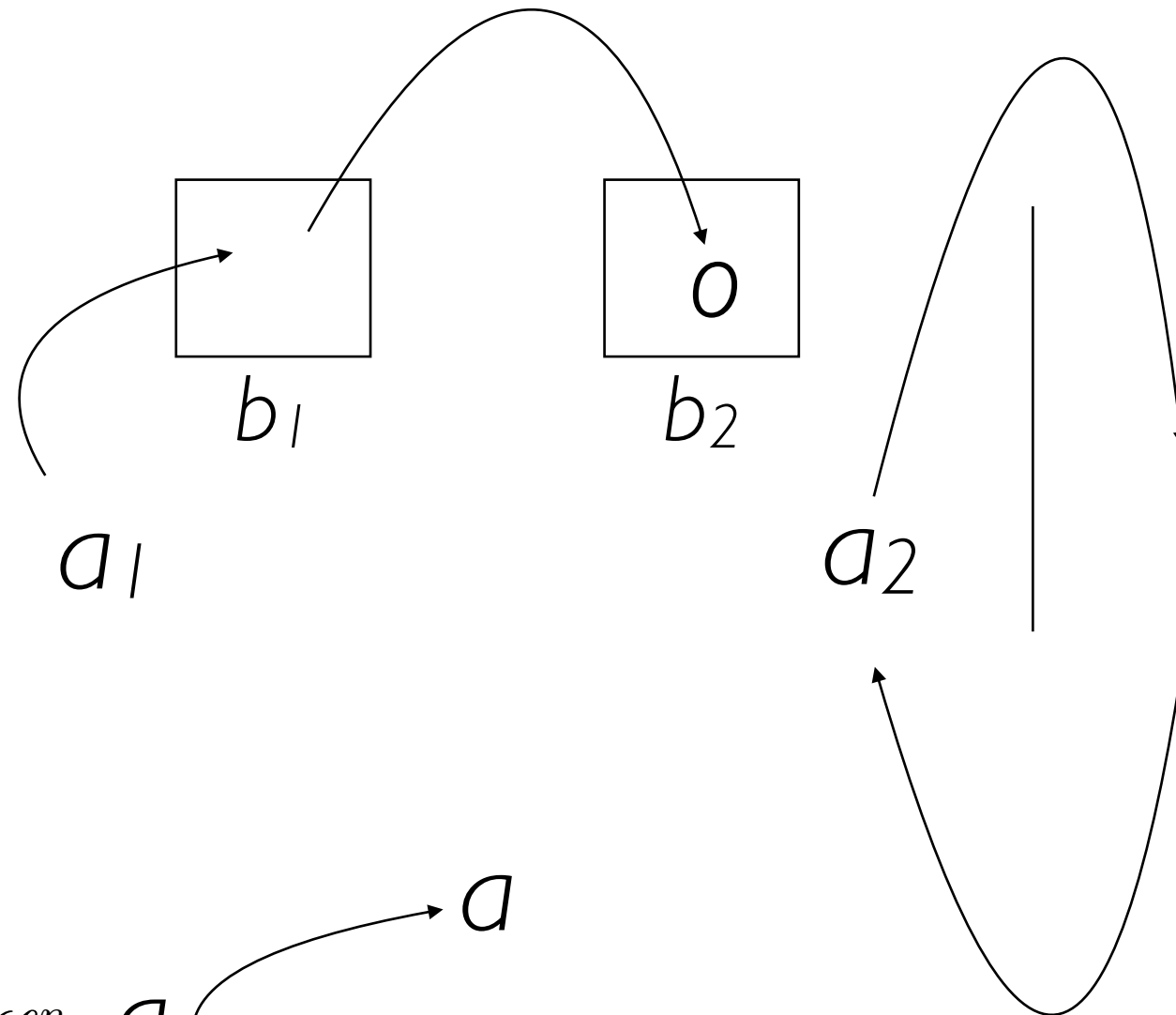
Framework for FBT^0_1

(five timepoints)

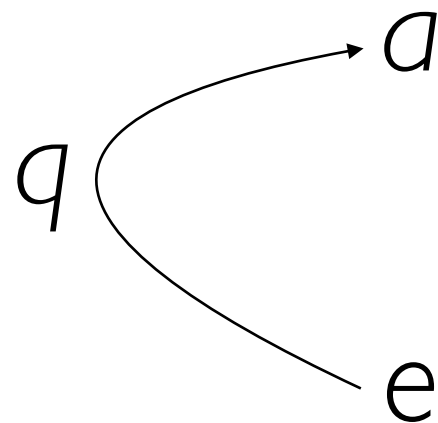


Framework for FBT^0_1

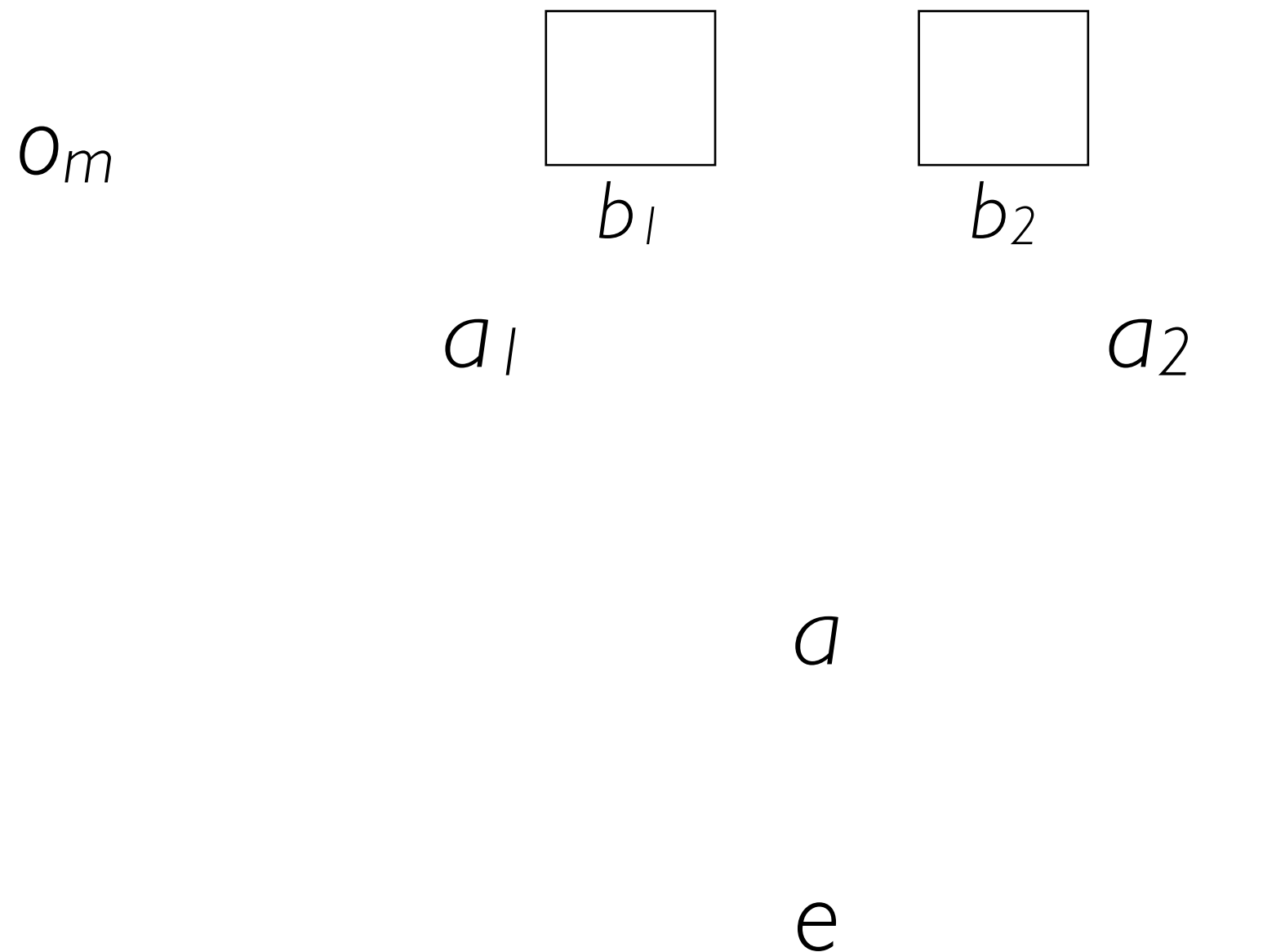
(five timepoints)



q a formula in modal \mathcal{L}^n

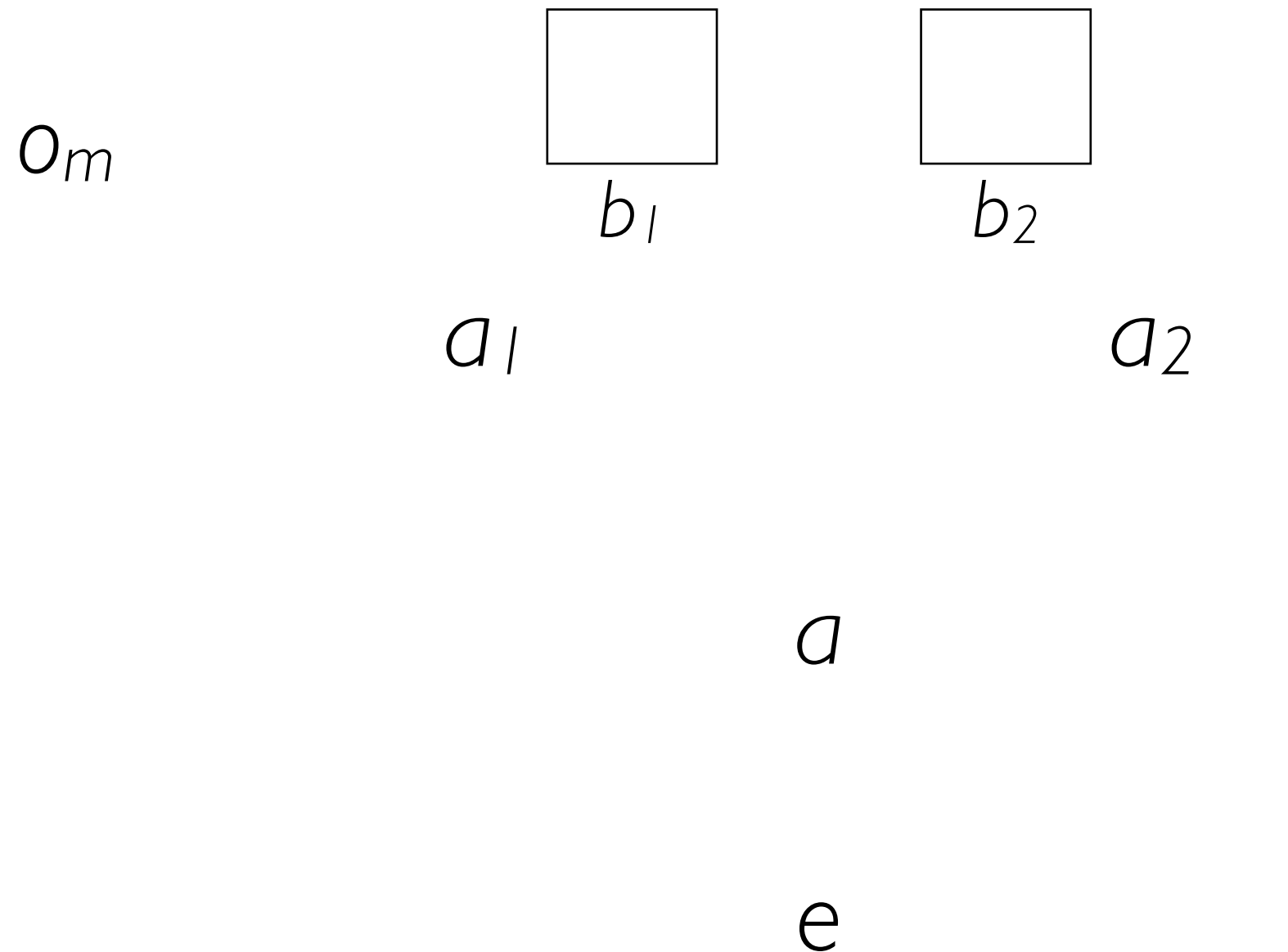


Framework for FBT₁



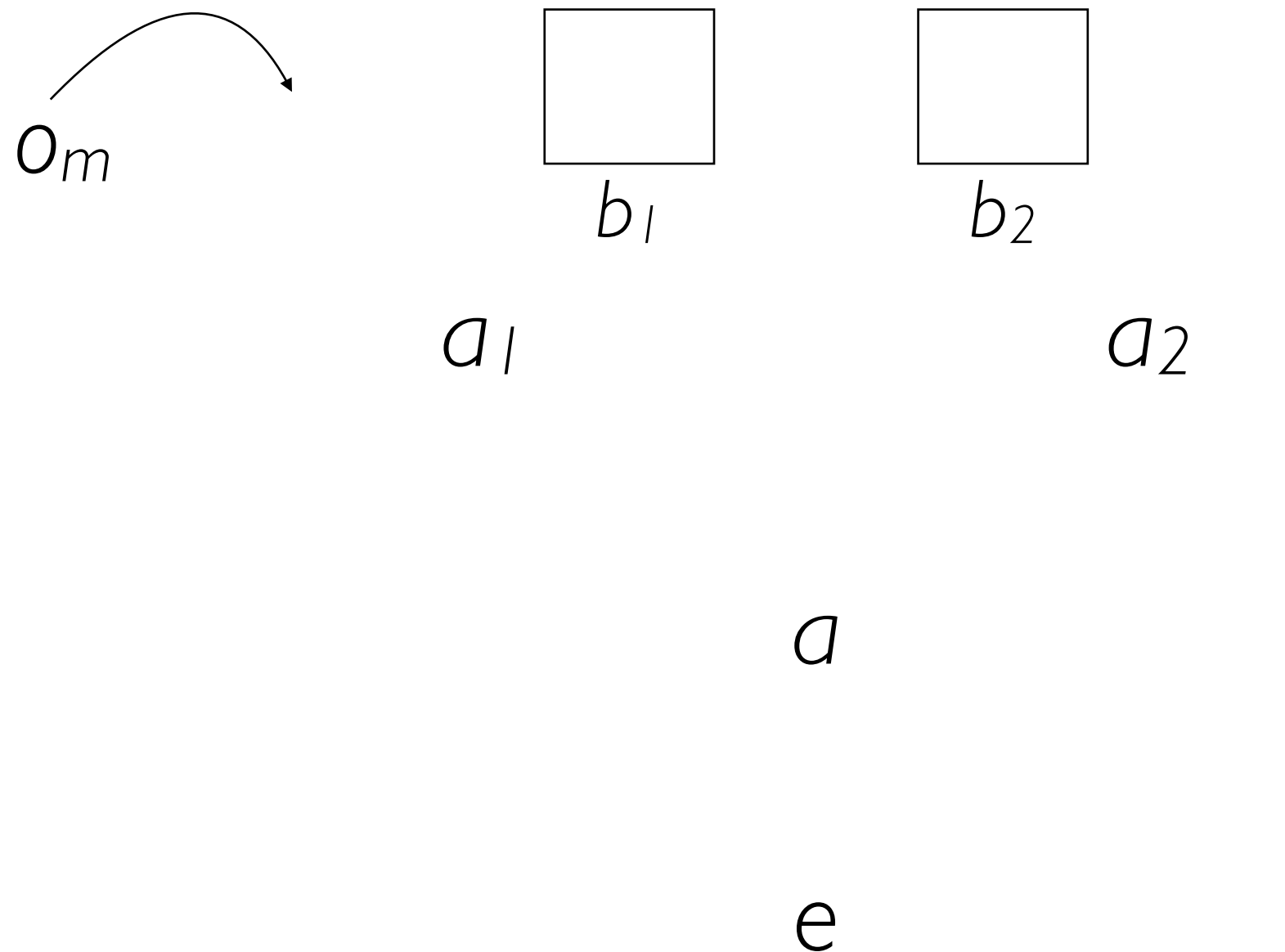
Framework for FBT₁

(six timepoints)



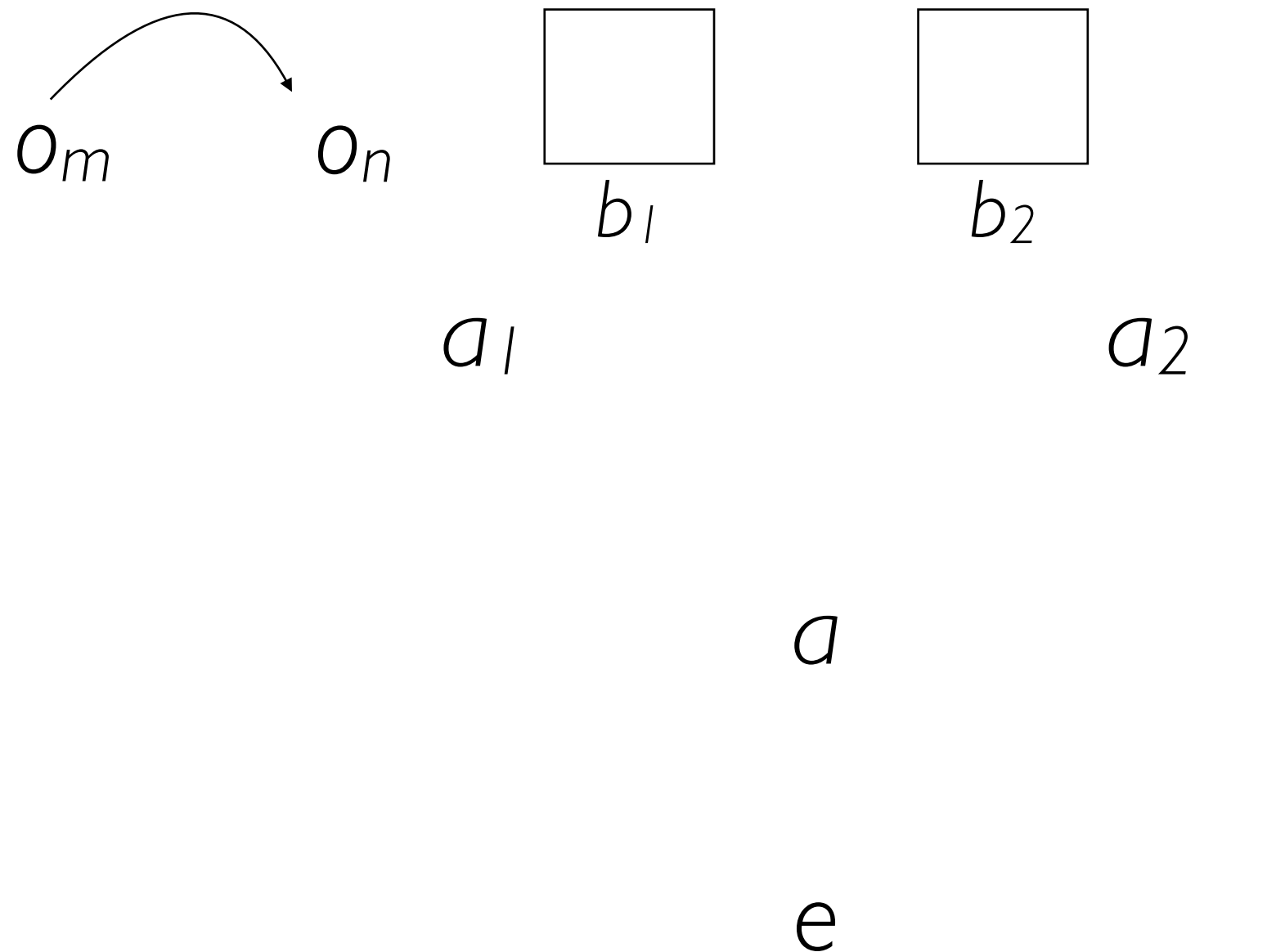
Framework for FBT₁

(six timepoints)



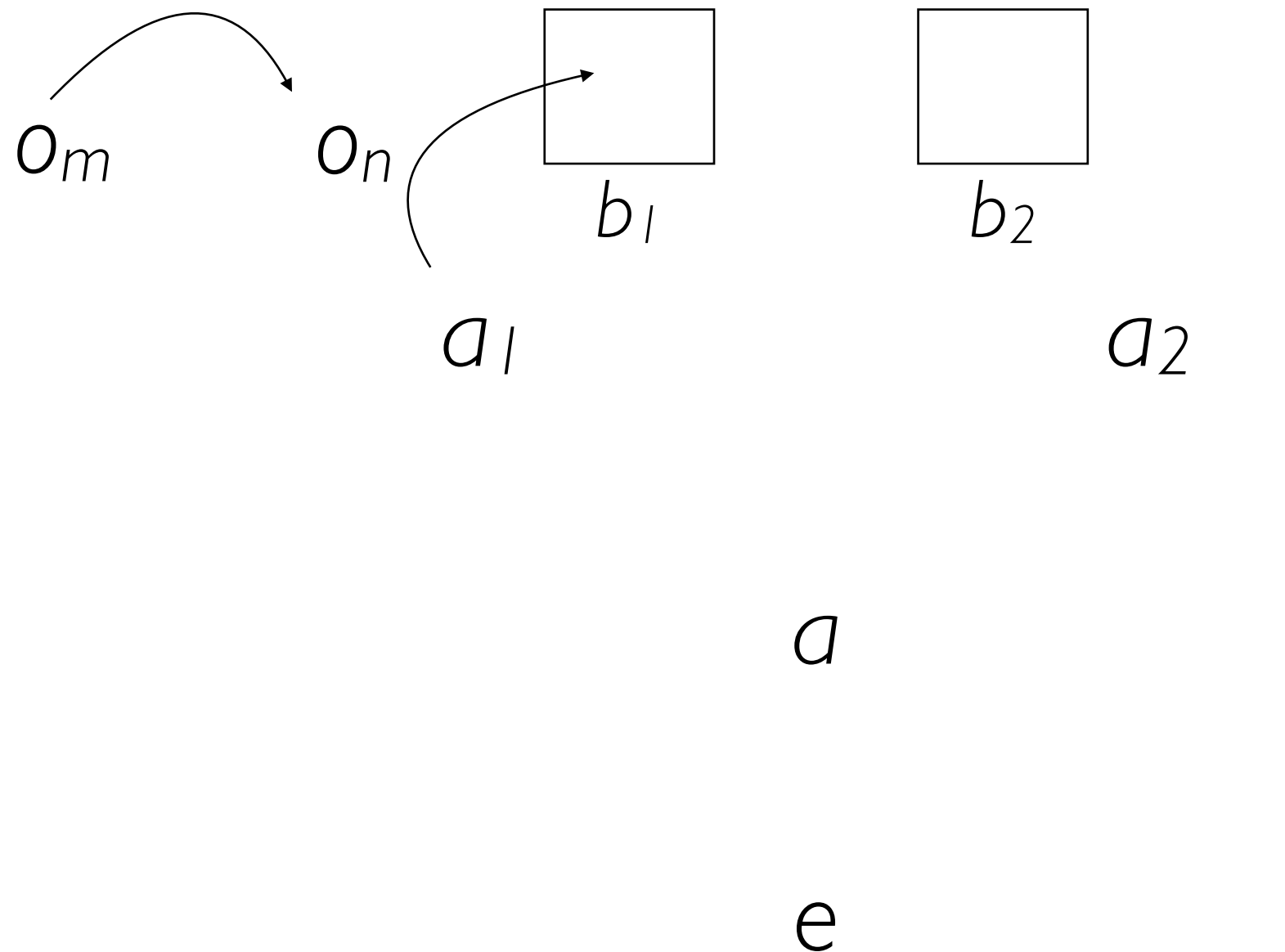
Framework for FBT₁

(six timepoints)



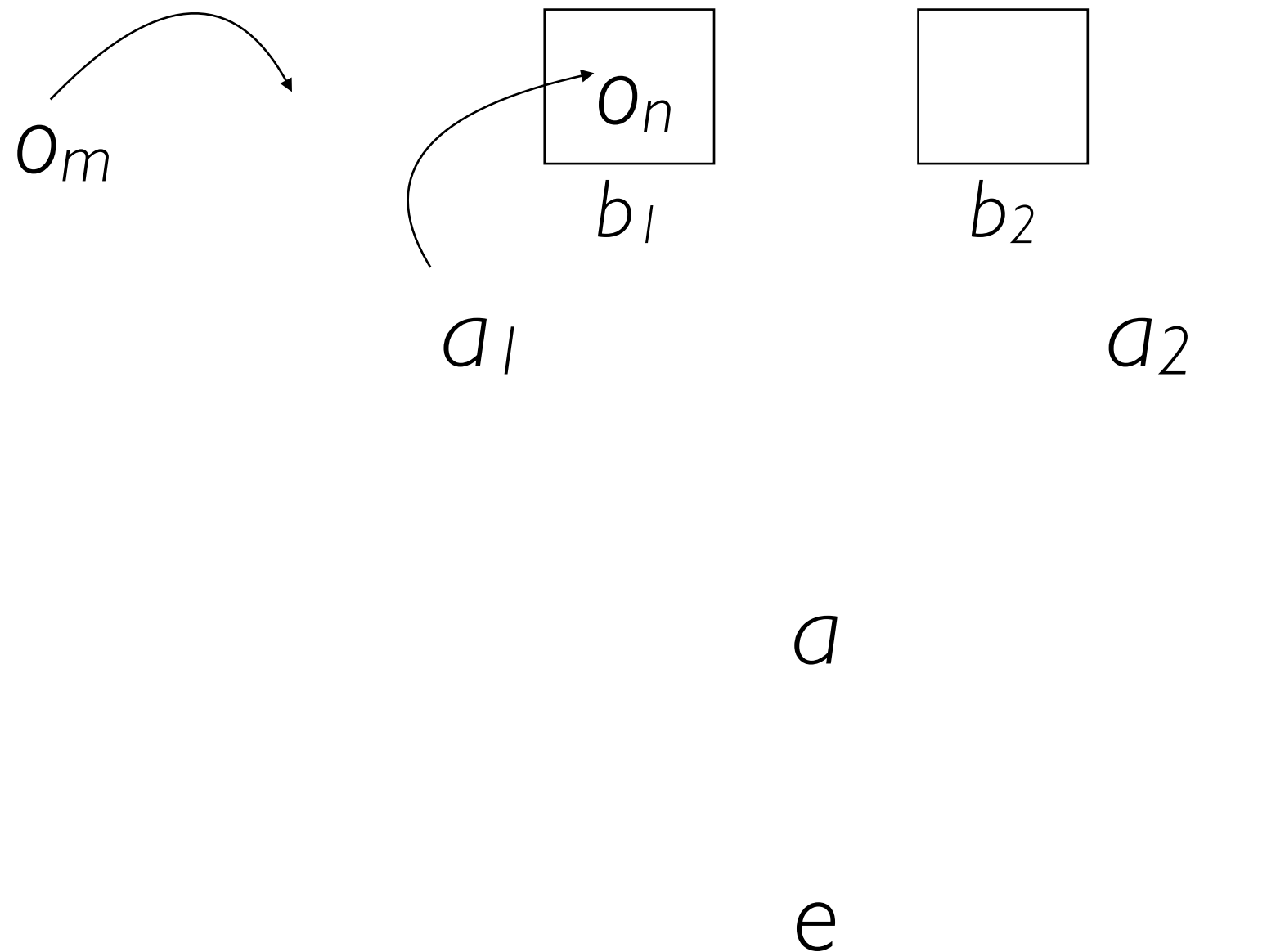
Framework for FBT₁

(six timepoints)



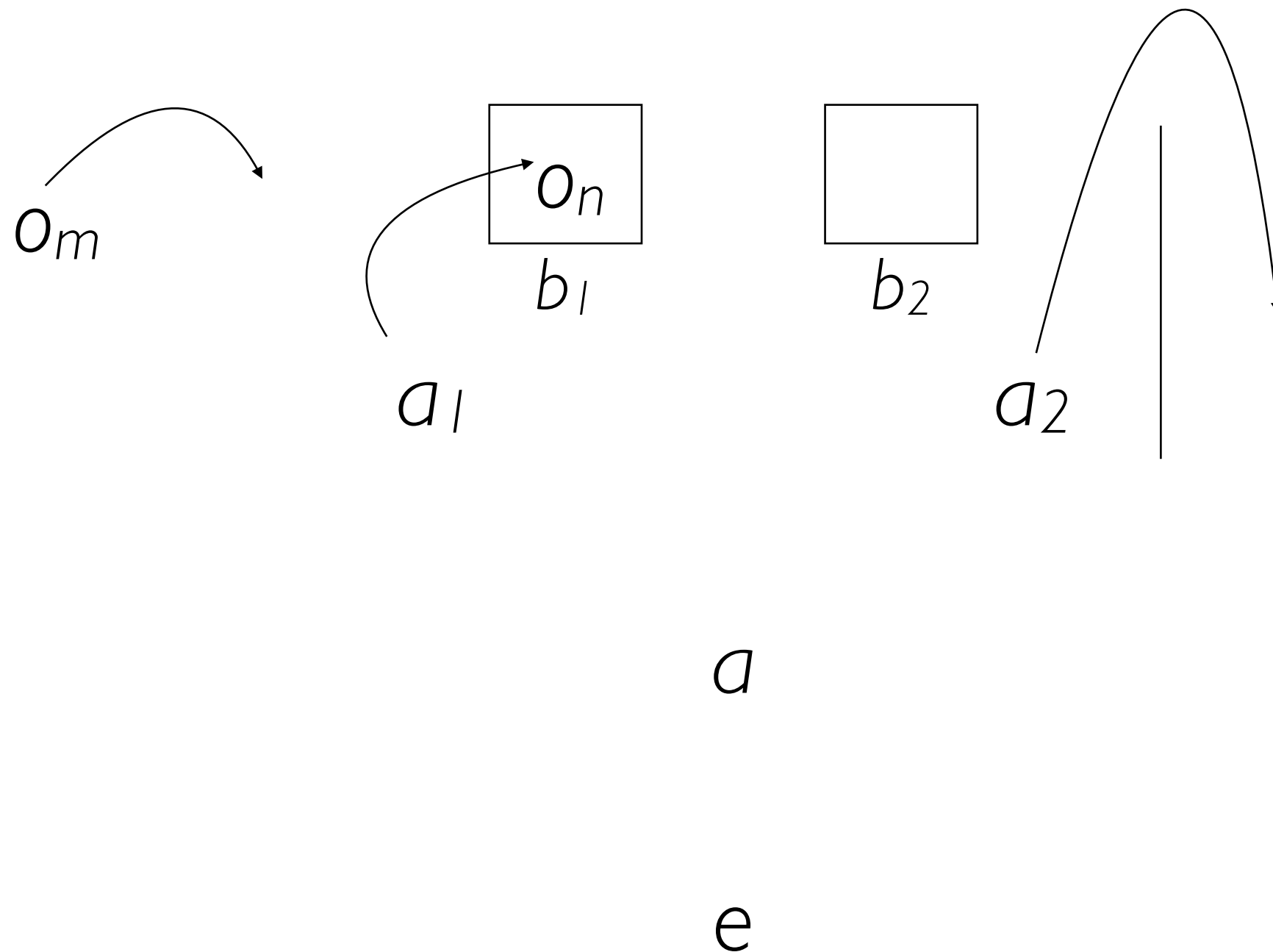
Framework for FBT₁

(six timepoints)



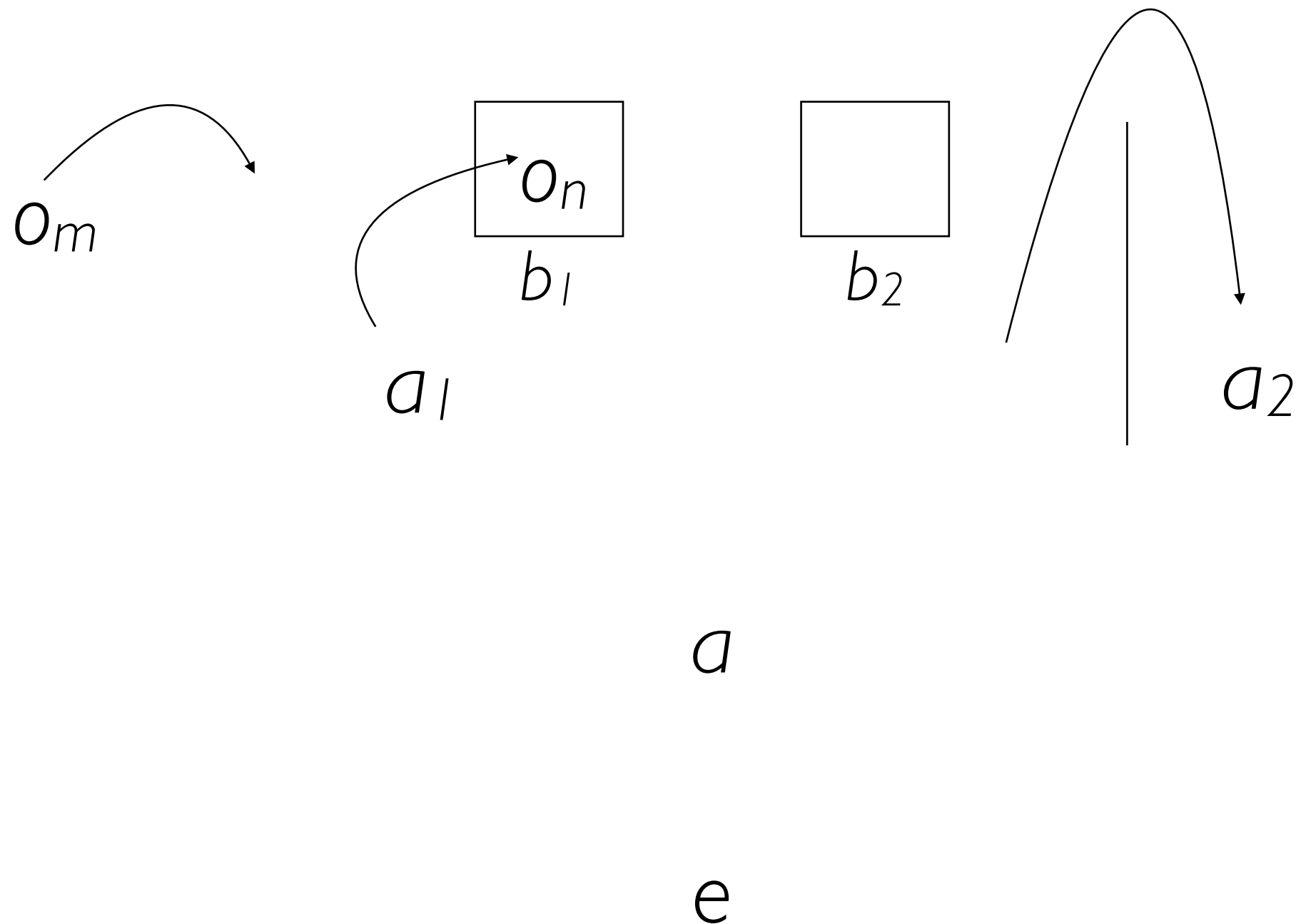
Framework for FBT₁

(six timepoints)



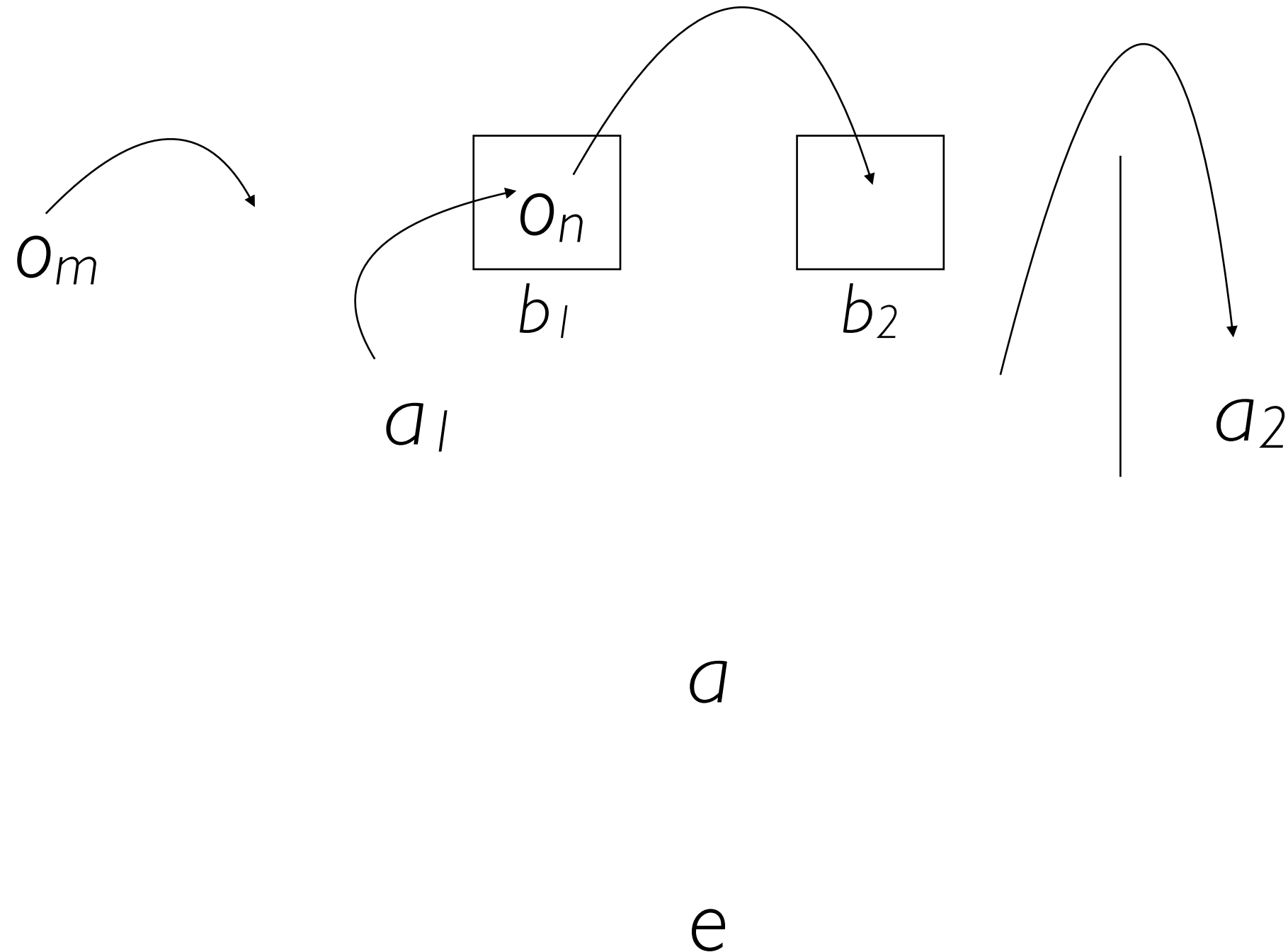
Framework for FBT₁

(six timepoints)



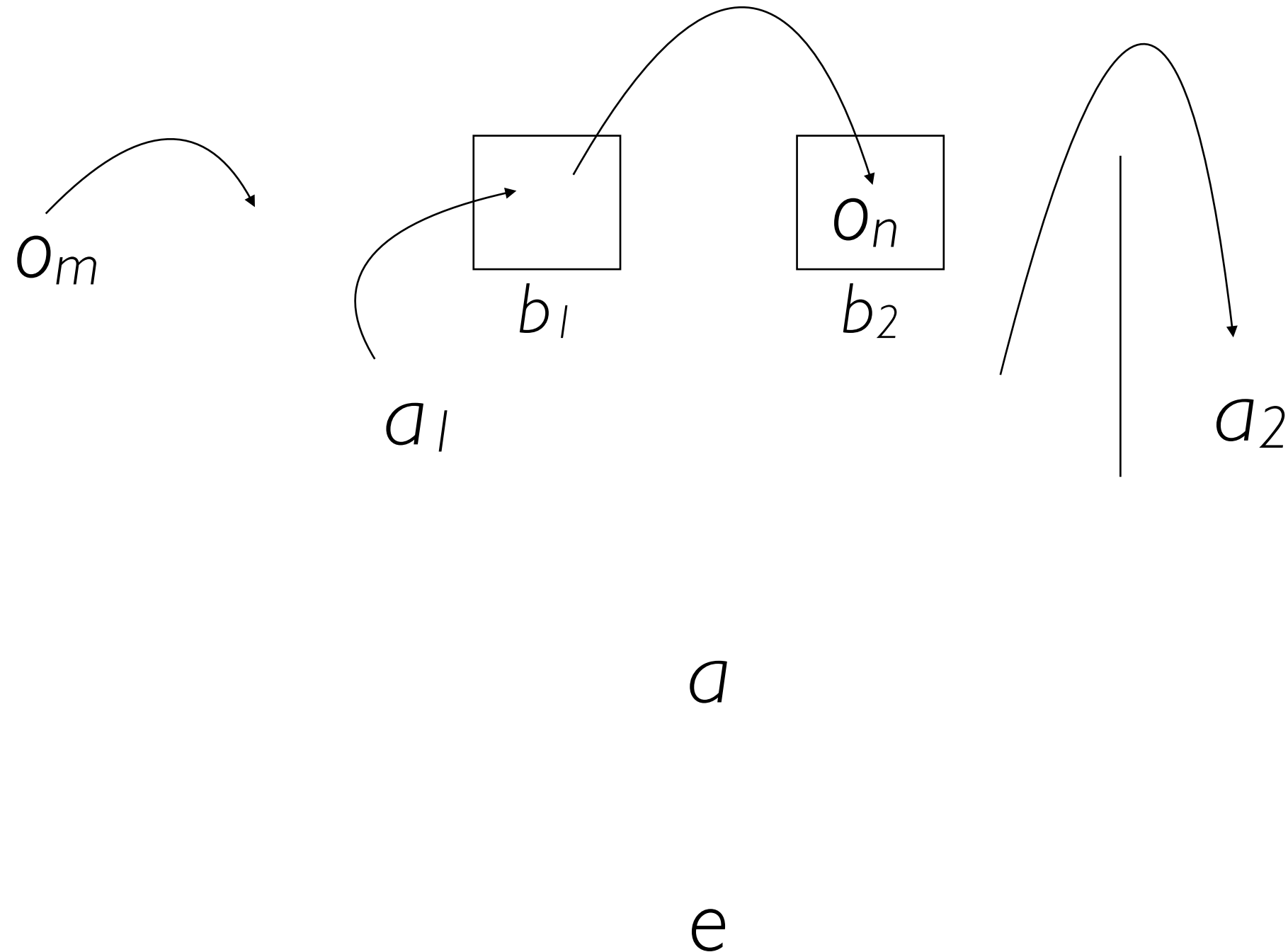
Framework for FBT₁

(six timepoints)



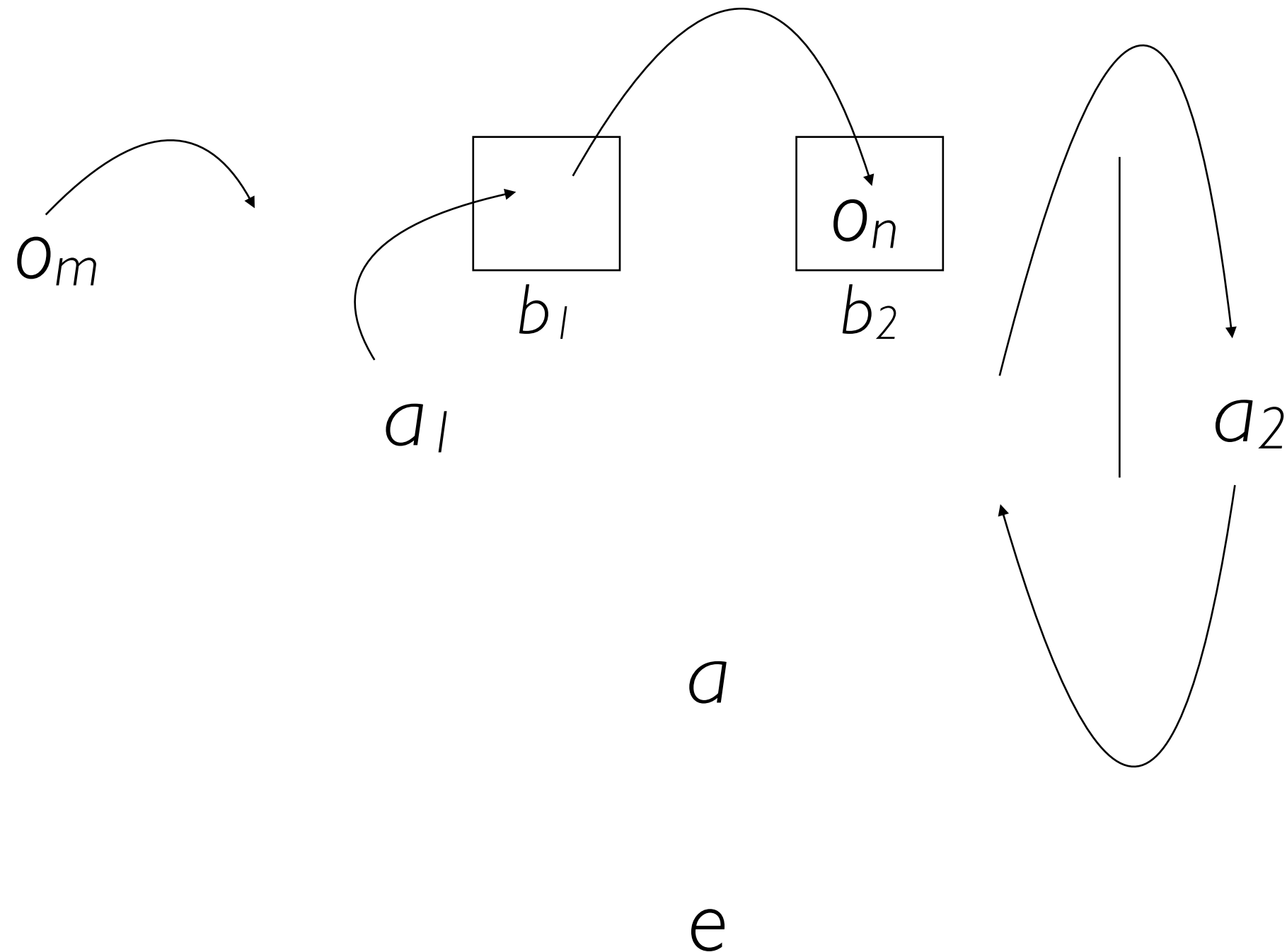
Framework for FBT₁

(six timepoints)



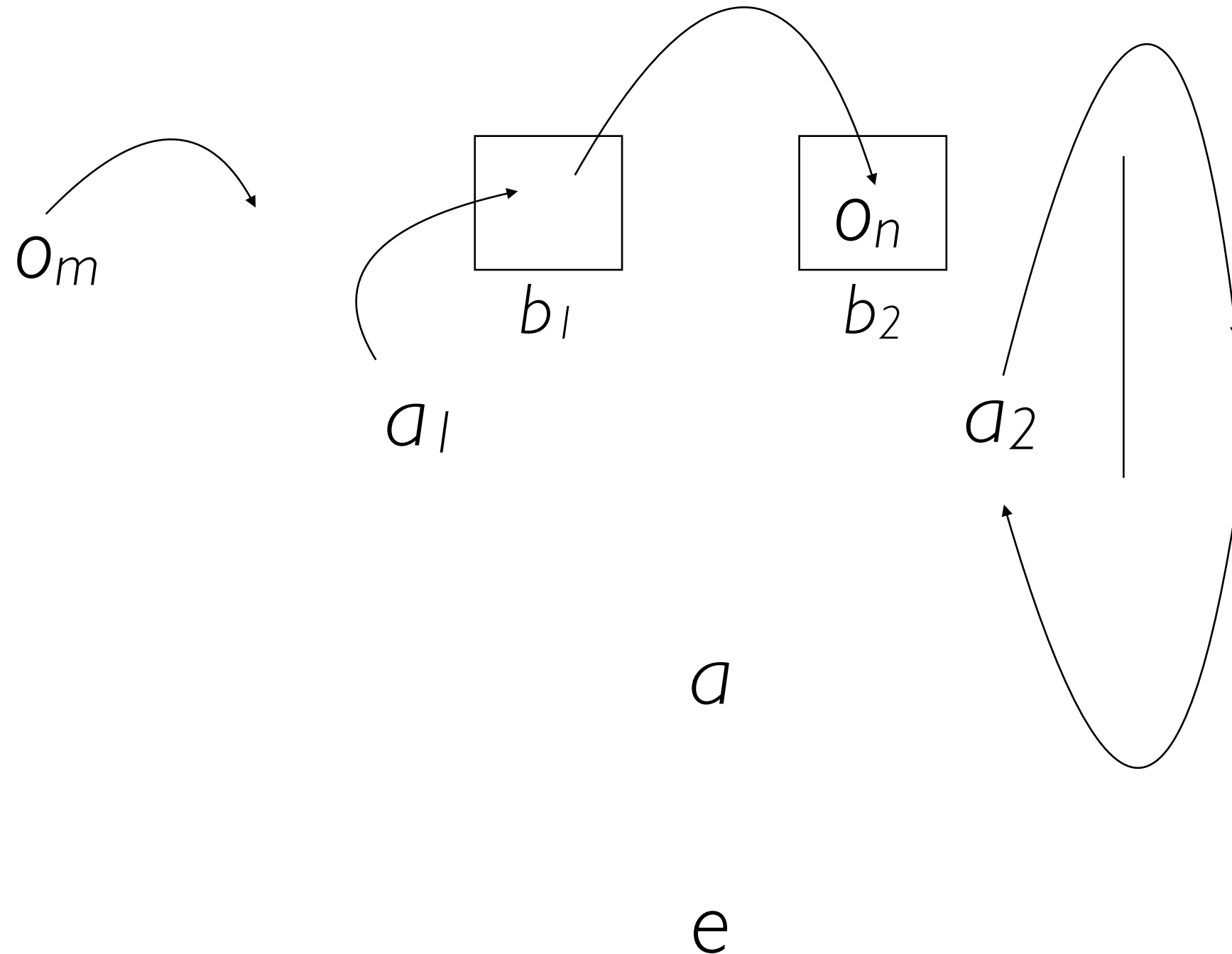
Framework for FBT¹

(six timepoints)



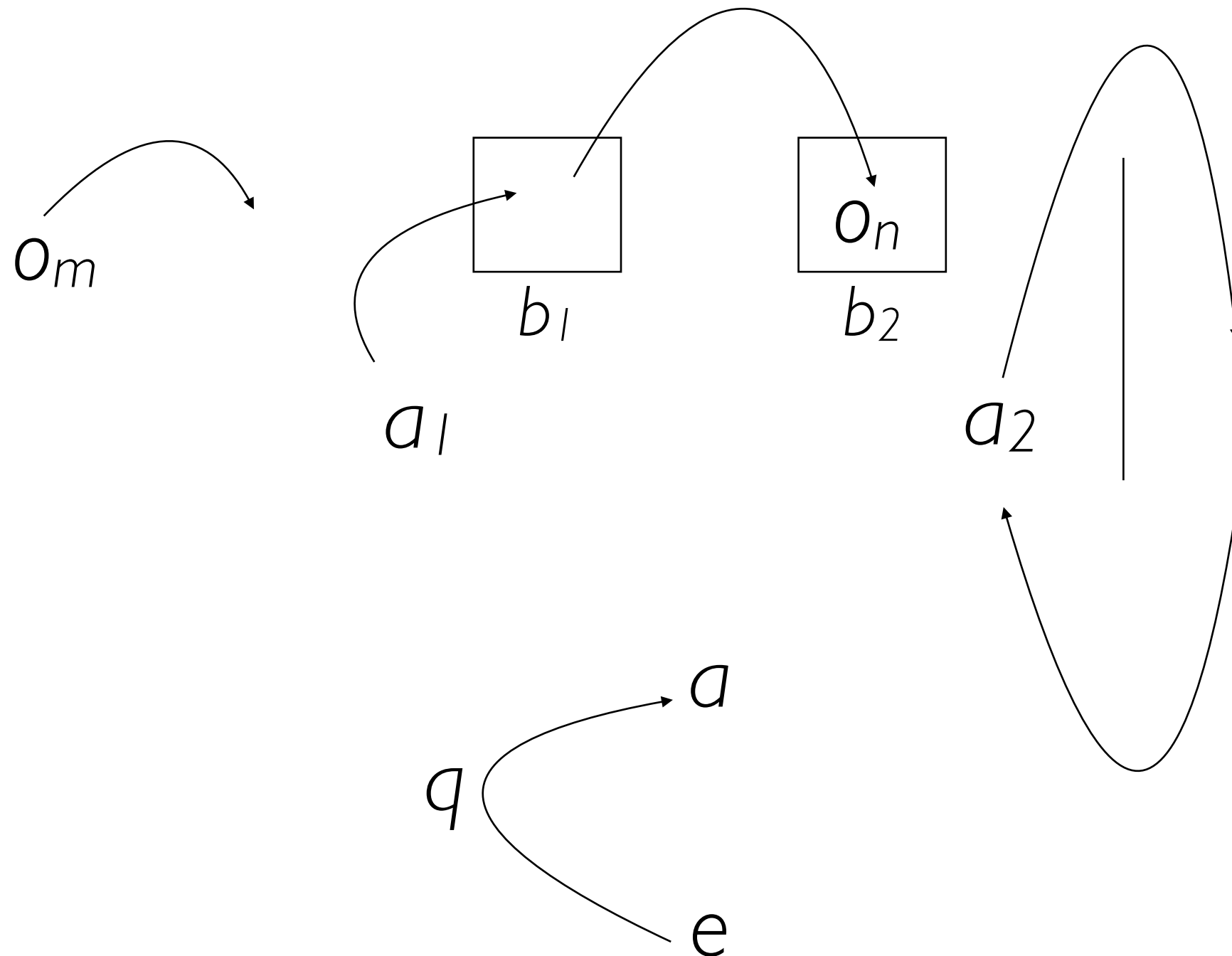
Framework for FBT₁

(six timepoints)



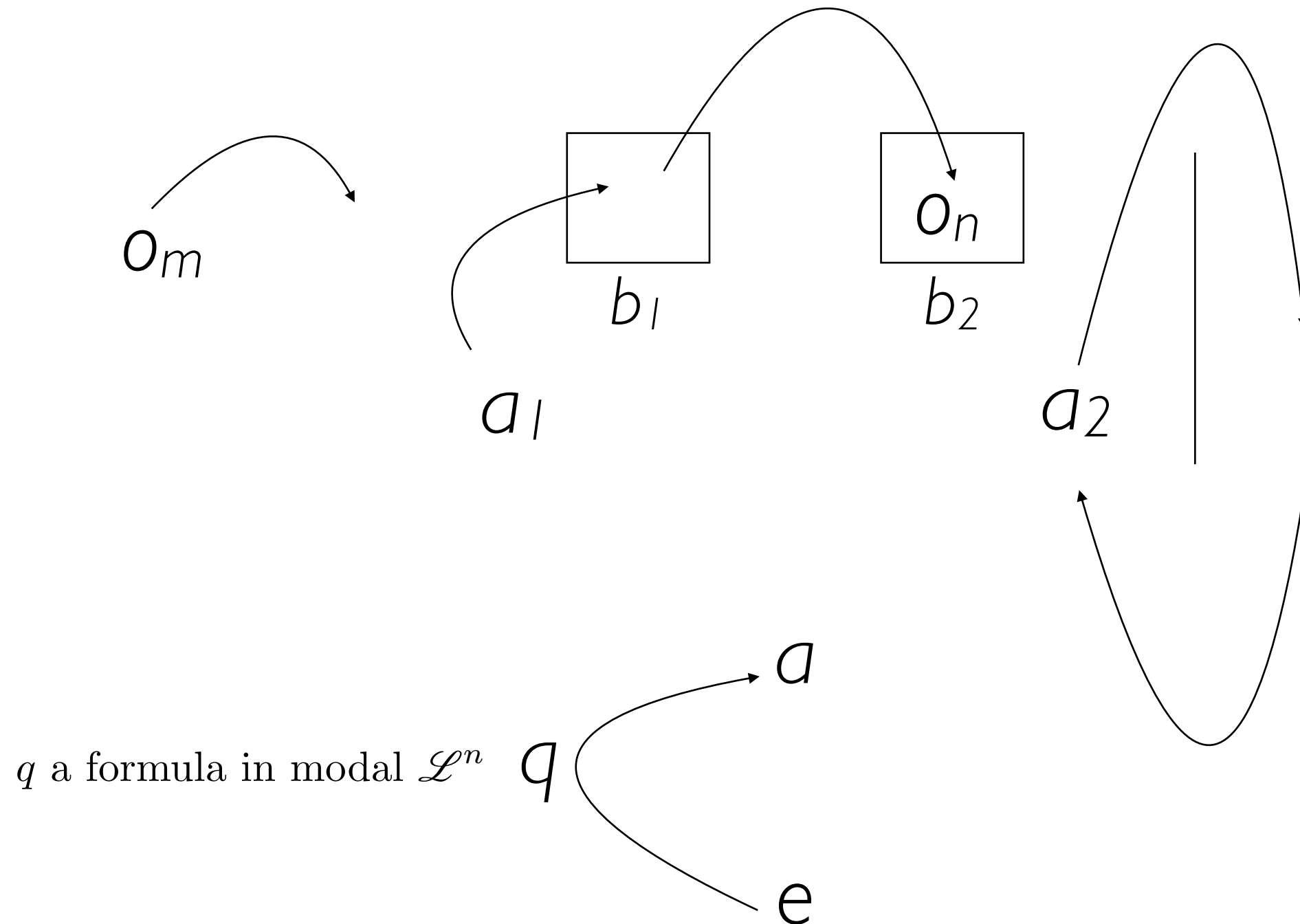
Framework for FBT₁

(six timepoints)



Framework for FBT^1_1

(six timepoints)



Done, a Decade Ago, Formally & Implementation/Simulation

Arkoudas, K. & Bringsjord, S.
(2009) “Propositional
Attitudes and Causation”
*International Journal of Software
and Informatics* **3.1**: 47–65.

http://kryten.mm.rpi.edu/PRICAI_w_sequentcalc_041709.pdf

Propositional attitudes and causation

Konstantine Arkoudas and Selmer Bringsjord

Cognitive Science and Computer Science Departments, RPI
arkouk@rpi.edu, brings@rpi.edu

Abstract. Predicting and explaining the behavior of others in terms of mental states is indispensable for everyday life. It will be equally important for artificial agents. We present an inference system for representing and reasoning about mental states, and use it to provide a formal analysis of the false-belief task. The system allows for the representation of information about events, causation, and perceptual, doxastic, and epistemic states (vision, belief, and knowledge), incorporating ideas from the event calculus and multi-agent epistemic logic. Unlike previous AI formalisms, our focus here is on mechanized proofs and proof programmability, not on metamathematical results. Reasoning is performed via relatively cognitively plausible inference rules, and a degree of automation is achieved by general-purpose inference methods and by a syntactic embedding of the system in first-order logic.

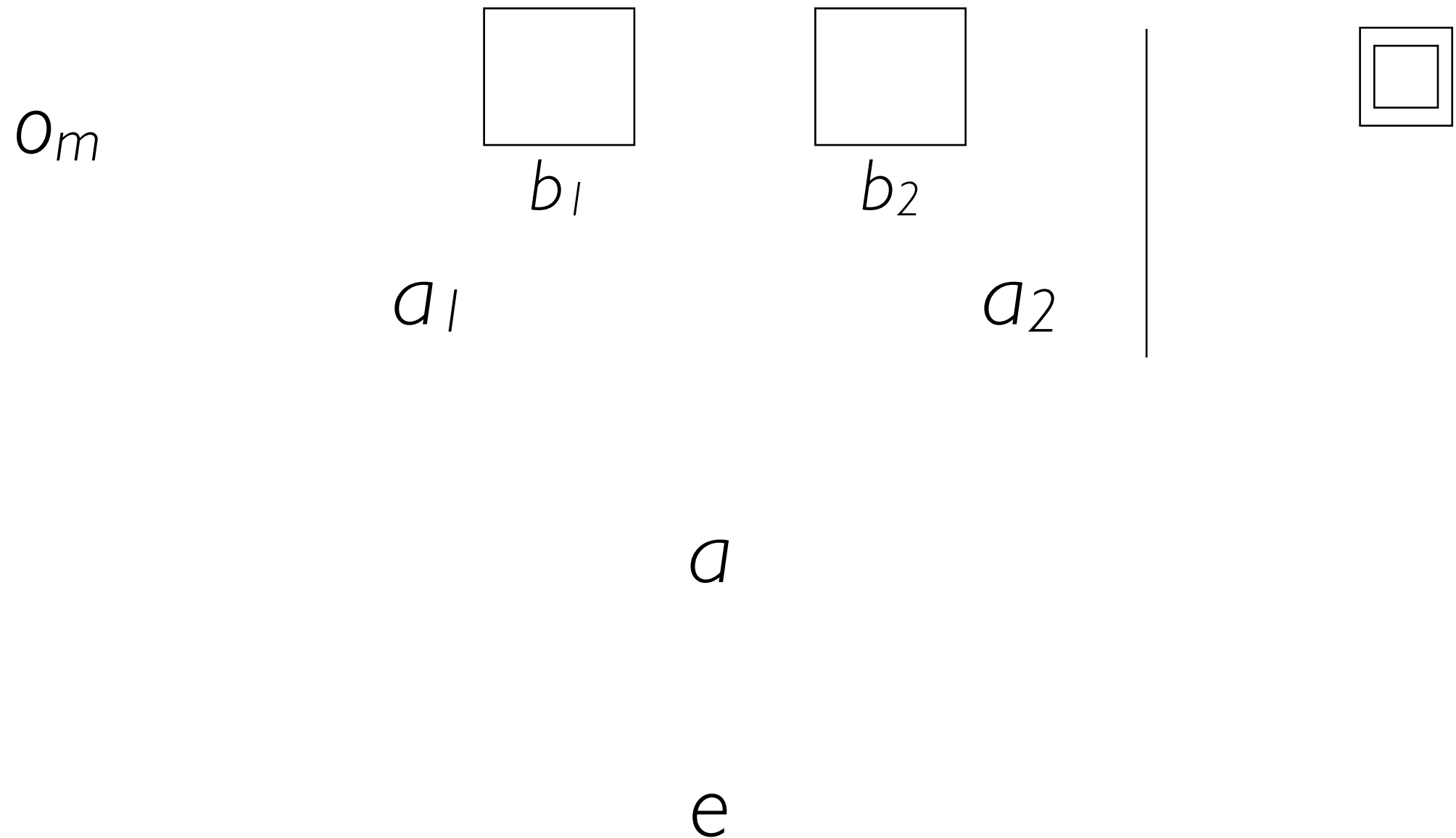
1 Introduction

Interpreting the behavior of other people is indispensable for everyday life. It is something that we do constantly, on a daily basis, and it helps us not only to make sense of human behavior, but also to predict it and—to a certain extent—to control it. How exactly do we manage that? That is not currently known, but many have argued that the ability to ascribe mental states to others and to reason about such mental states is a key component of our capacity to understand human behavior. In particular, all social transactions, from engaging in commerce and negotiating to making jokes and empathizing with other people's pain or joy, appear to require at least a rudimentary grasp of common-sense psychology (CSP), i.e., a large body of truisms such as the following: When an agent a (1) wants to achieve a certain state of affairs p , and (2) believes that some action c can bring about p , and (3) a knows how to carry out c ; then, *ceteris paribus*,¹ a will carry out c ; when a sees that p , a knows that p ; when a fears that p and a discovers that p is the case, a is disappointed; and so on.

Artificial agents without a mastery of CSP would be severely handicapped in their interactions with humans. This could present problems not only for artificial agents trying to interpret human behavior, but also for artificial agents trying to interpret the behavior of one another. When a system exhibits a complex but rational behavior, and detailed knowledge of its internal structure is not

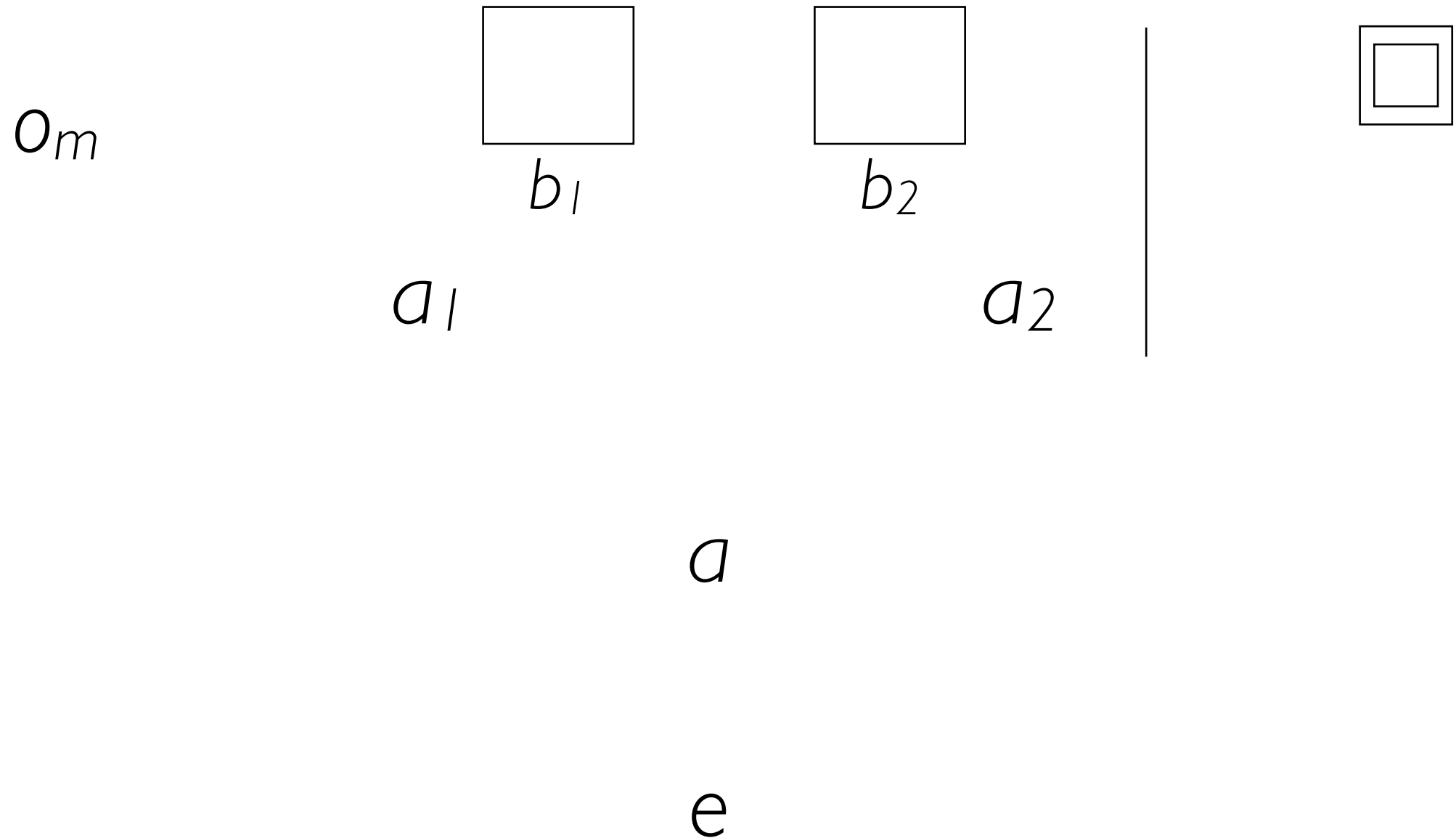
¹ Assuming that a is able to carry out c , that a has no conflicting desires that override his goal that p ; and so on.

Framework for FBT₂

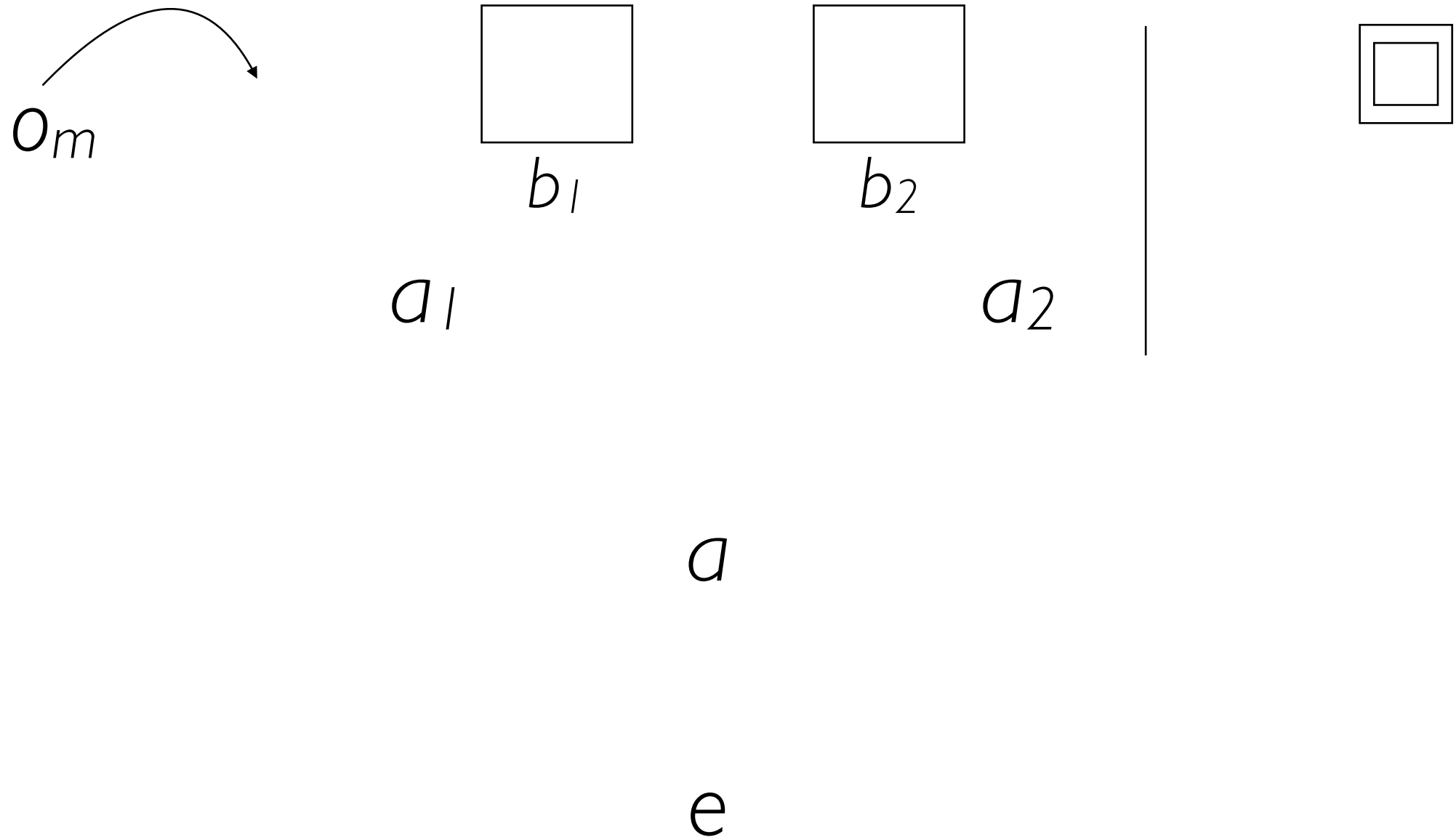


Framework for FBT₂

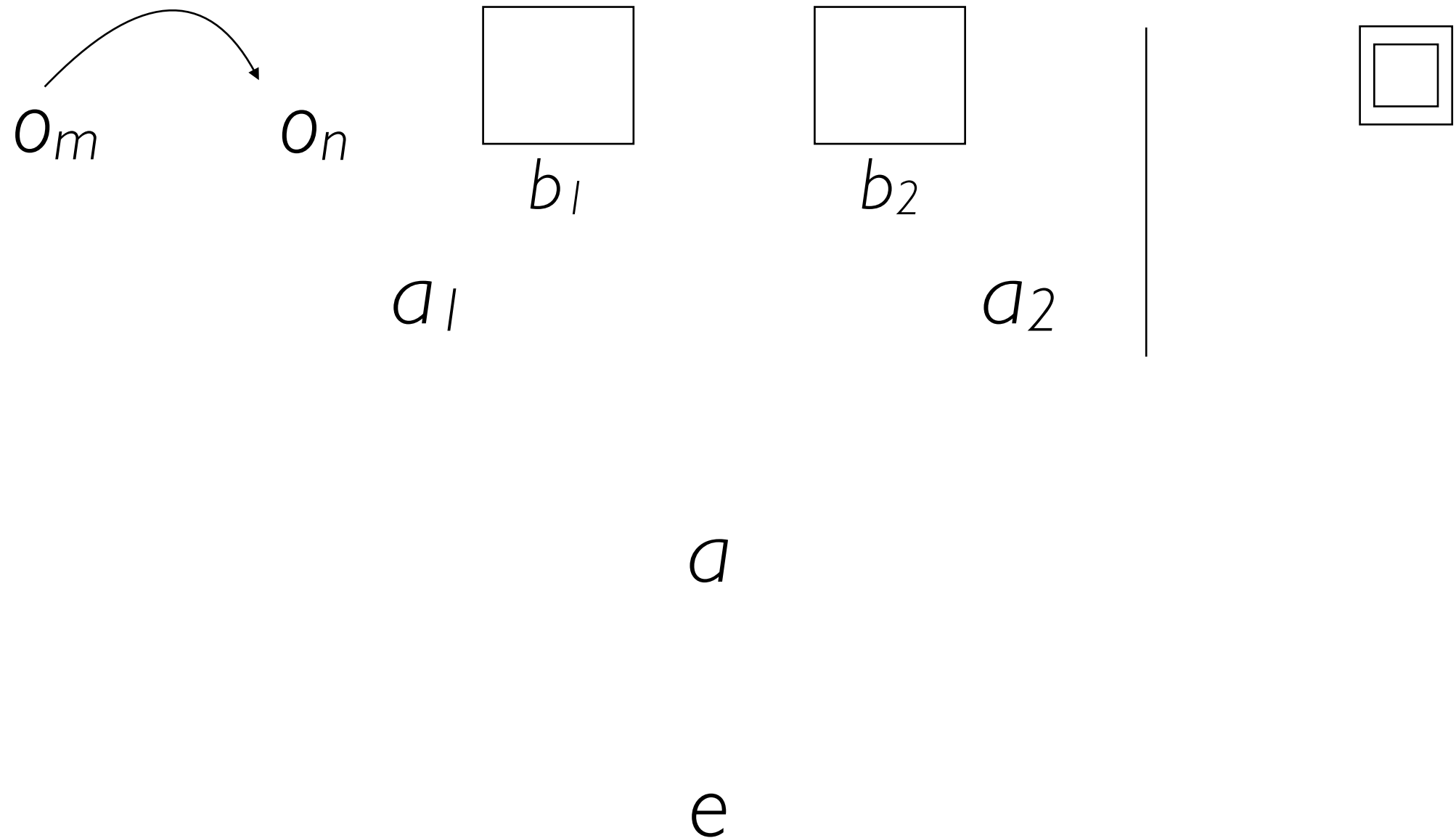
(seven timepoints)



Framework for FBT₂ (seven timepoints)

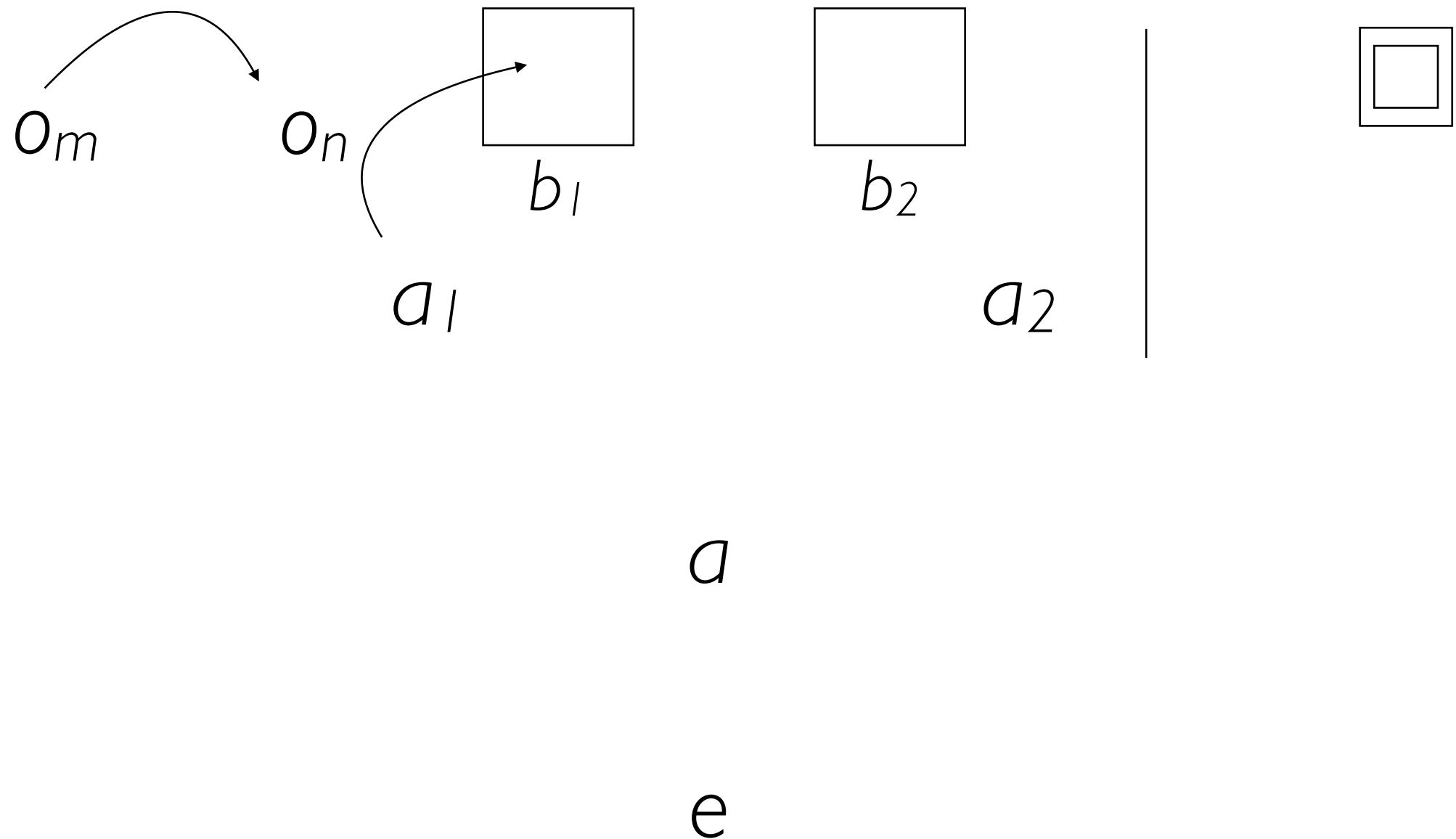


Framework for FBT₂ (seven timepoints)



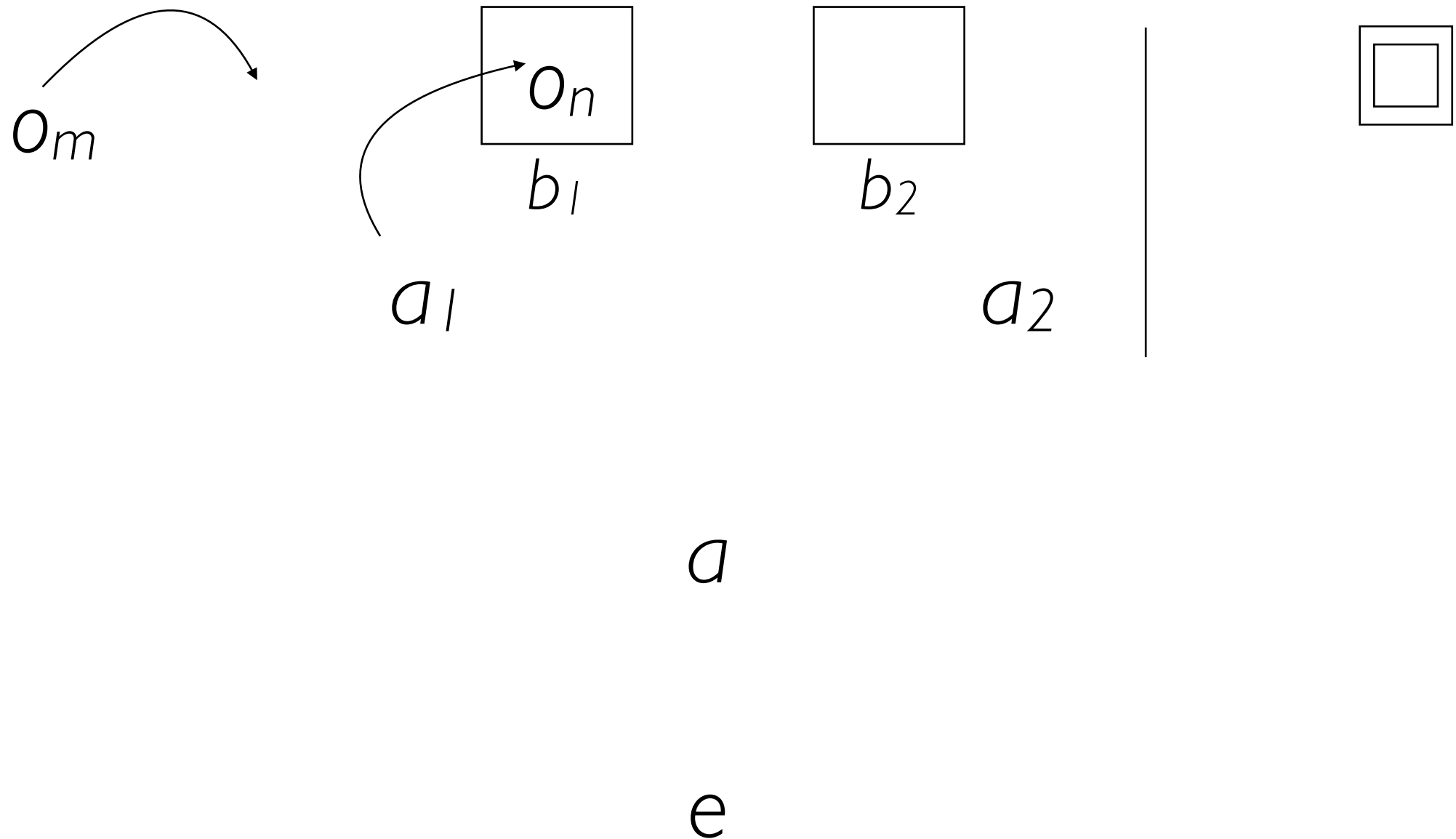
Framework for FBT^I_2

(seven timepoints)



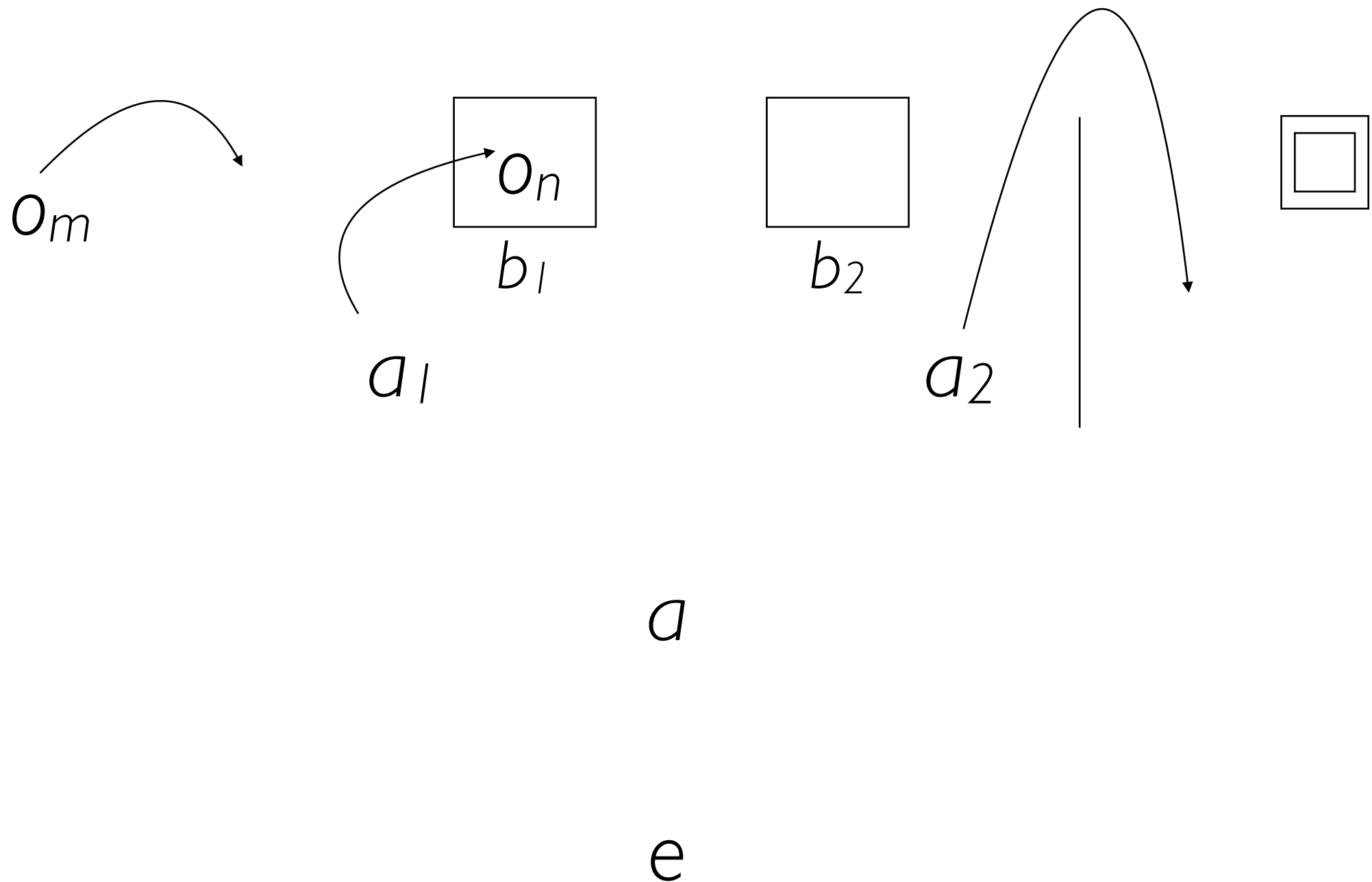
Framework for FBT^1_2

(seven timepoints)



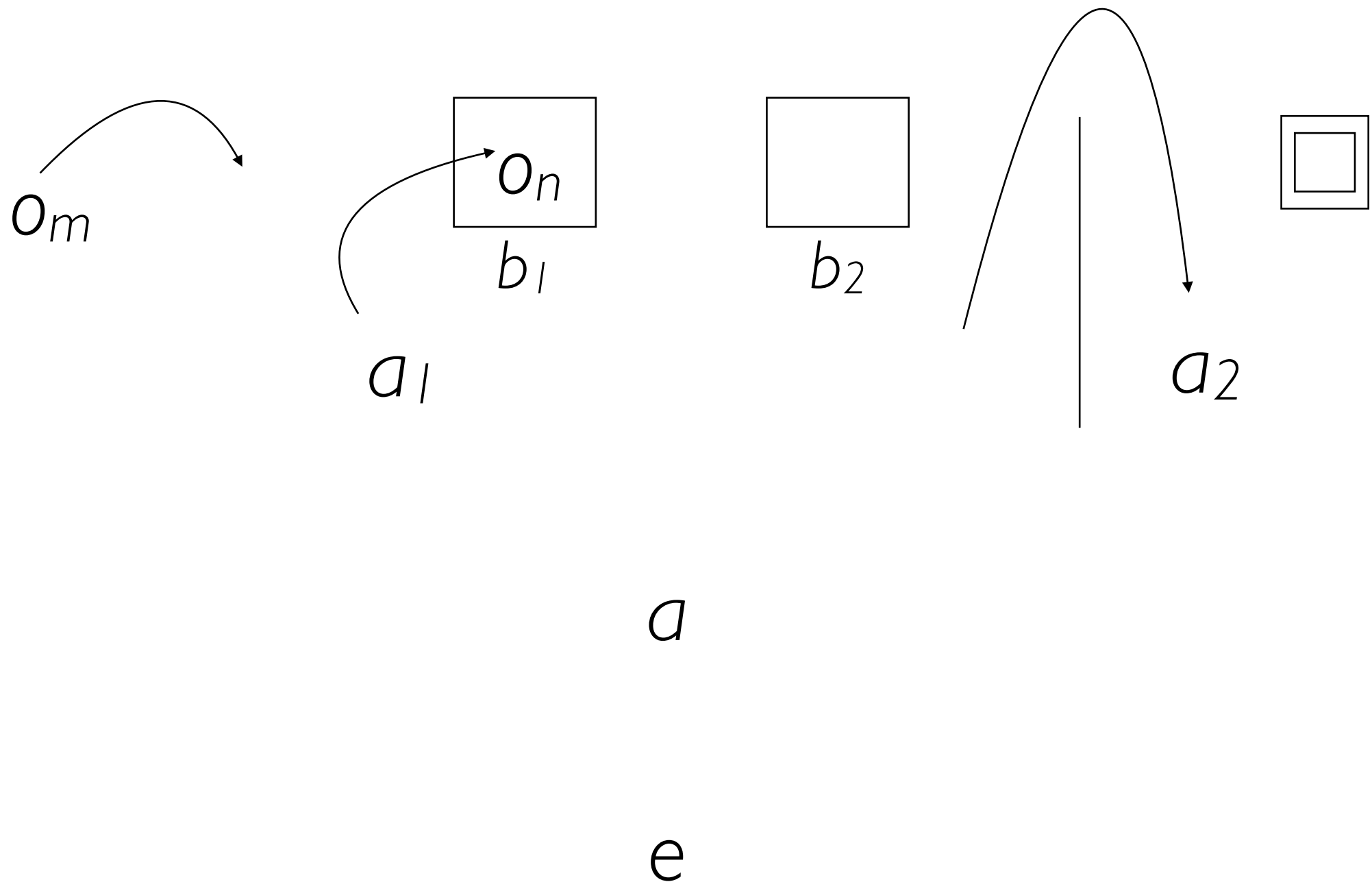
Framework for FBT^I_2

(seven timepoints)



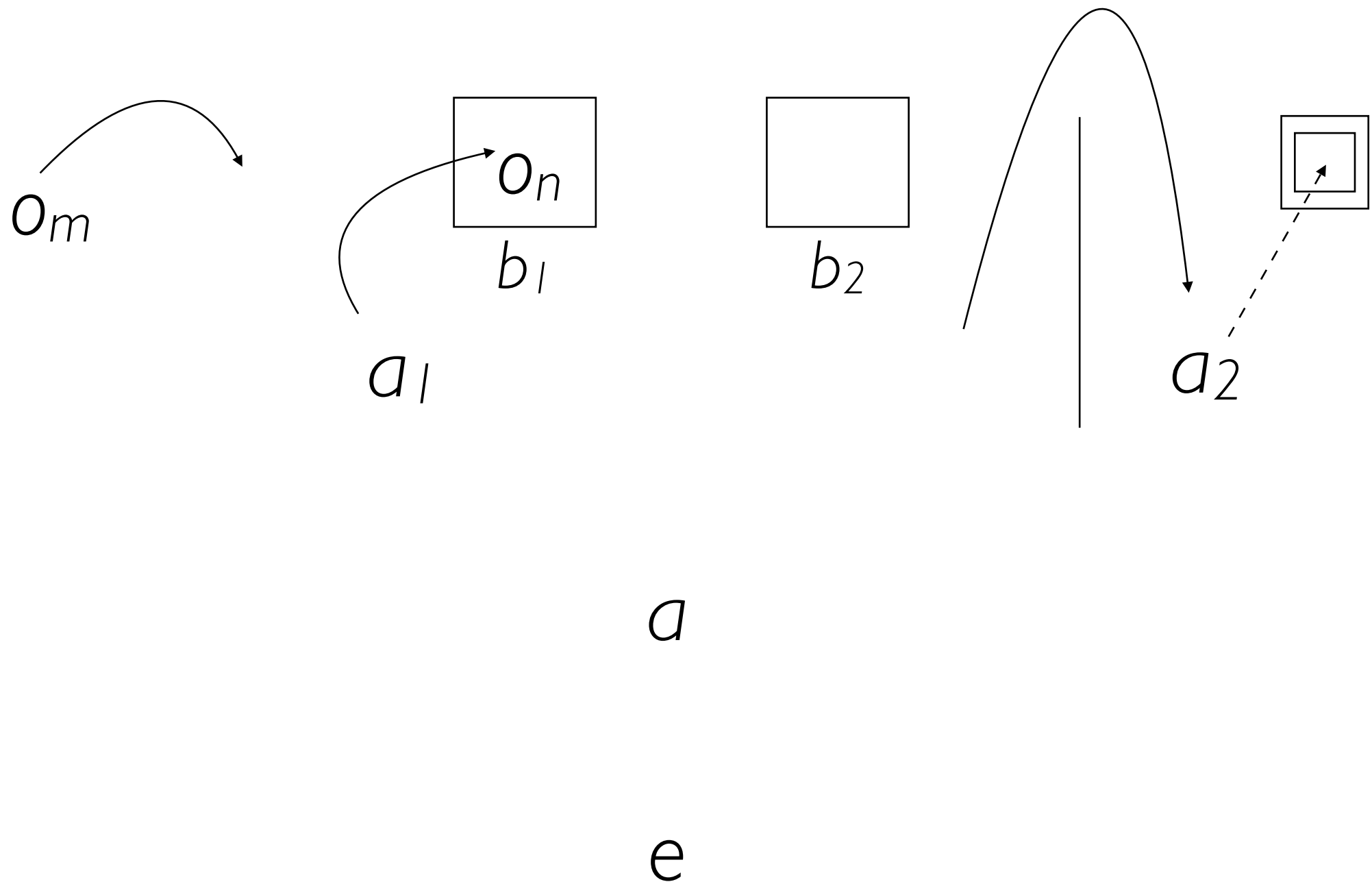
Framework for FBT^I_2

(seven timepoints)



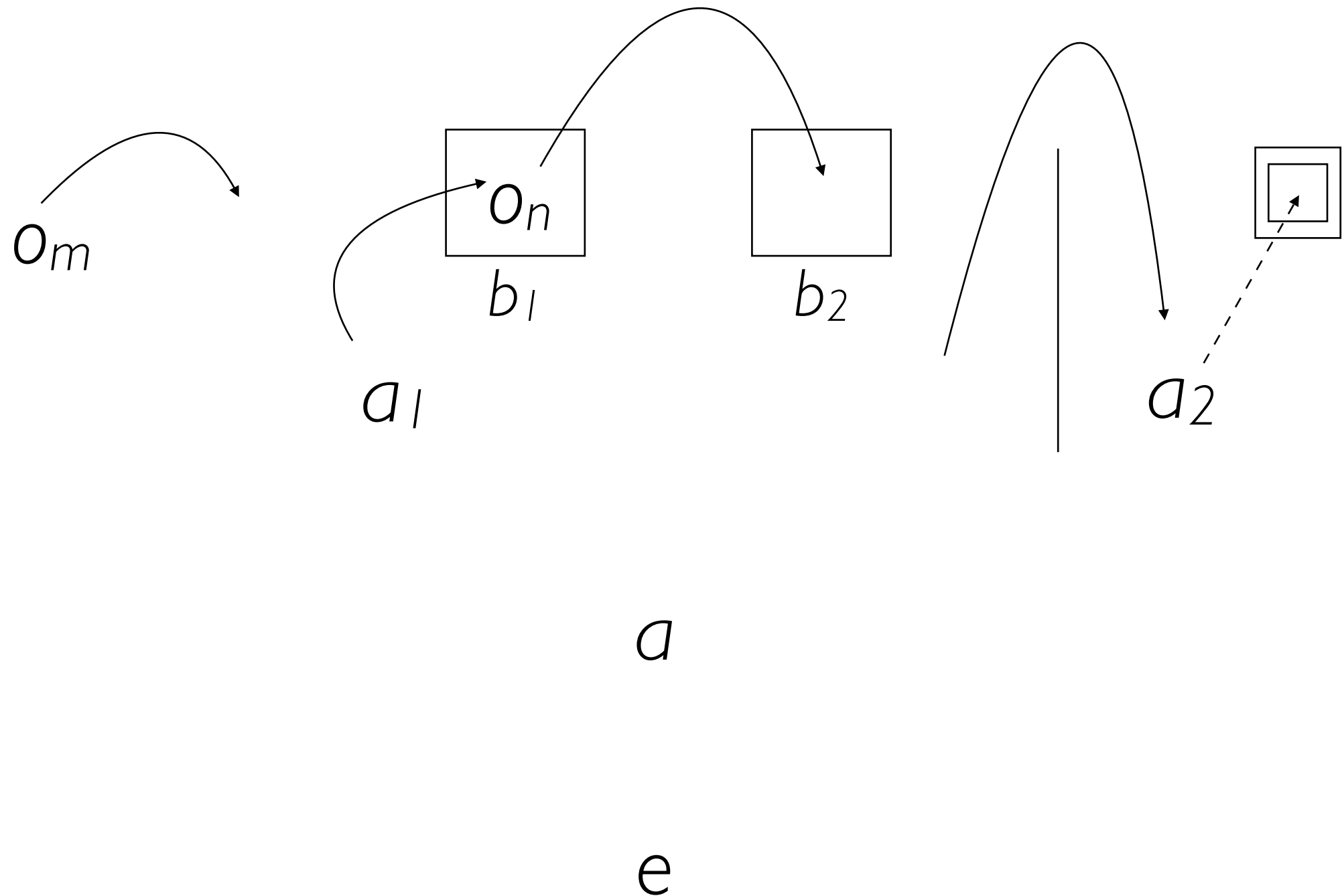
Framework for FBT^1_2

(seven timepoints)



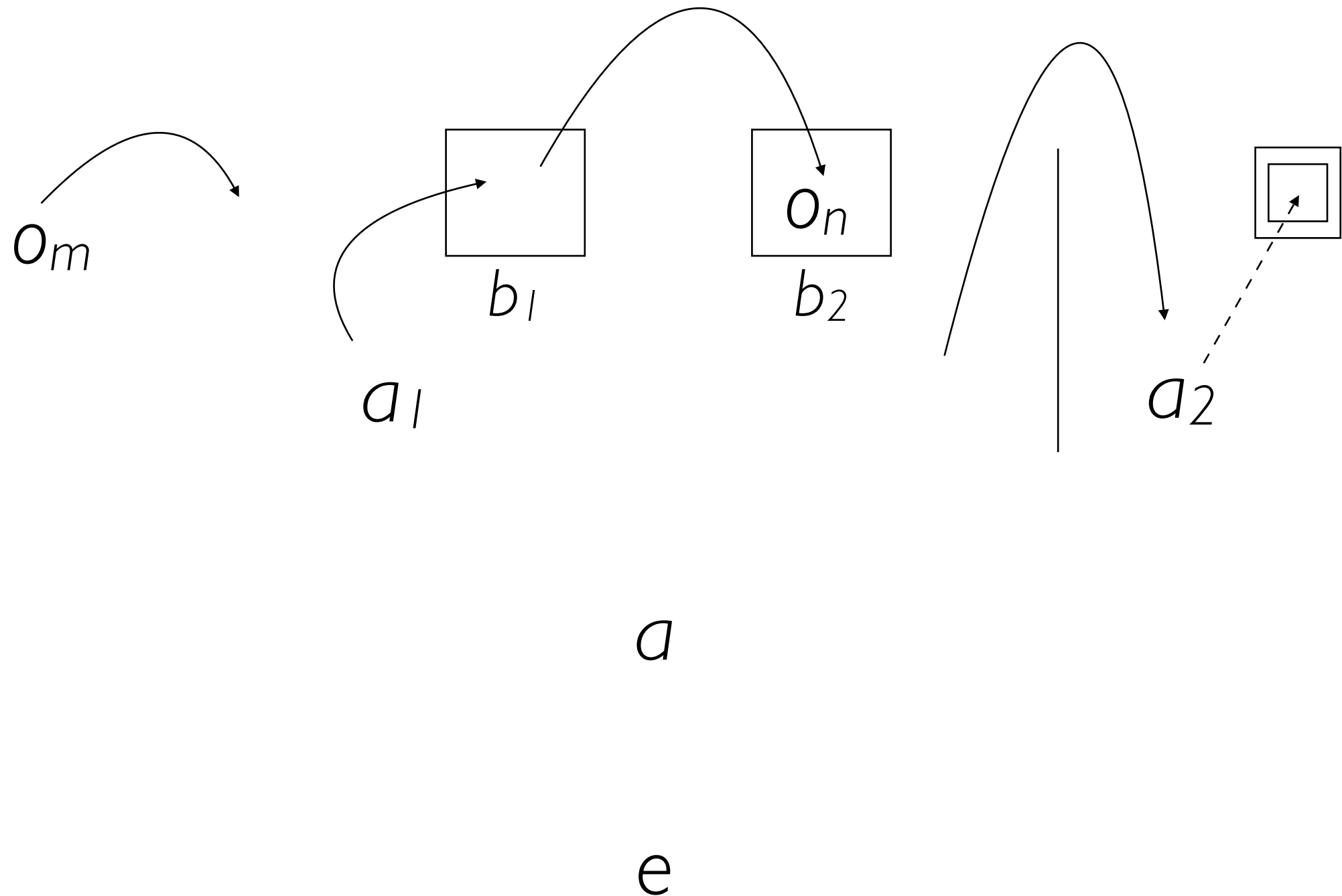
Framework for FBT^1_2

(seven timepoints)



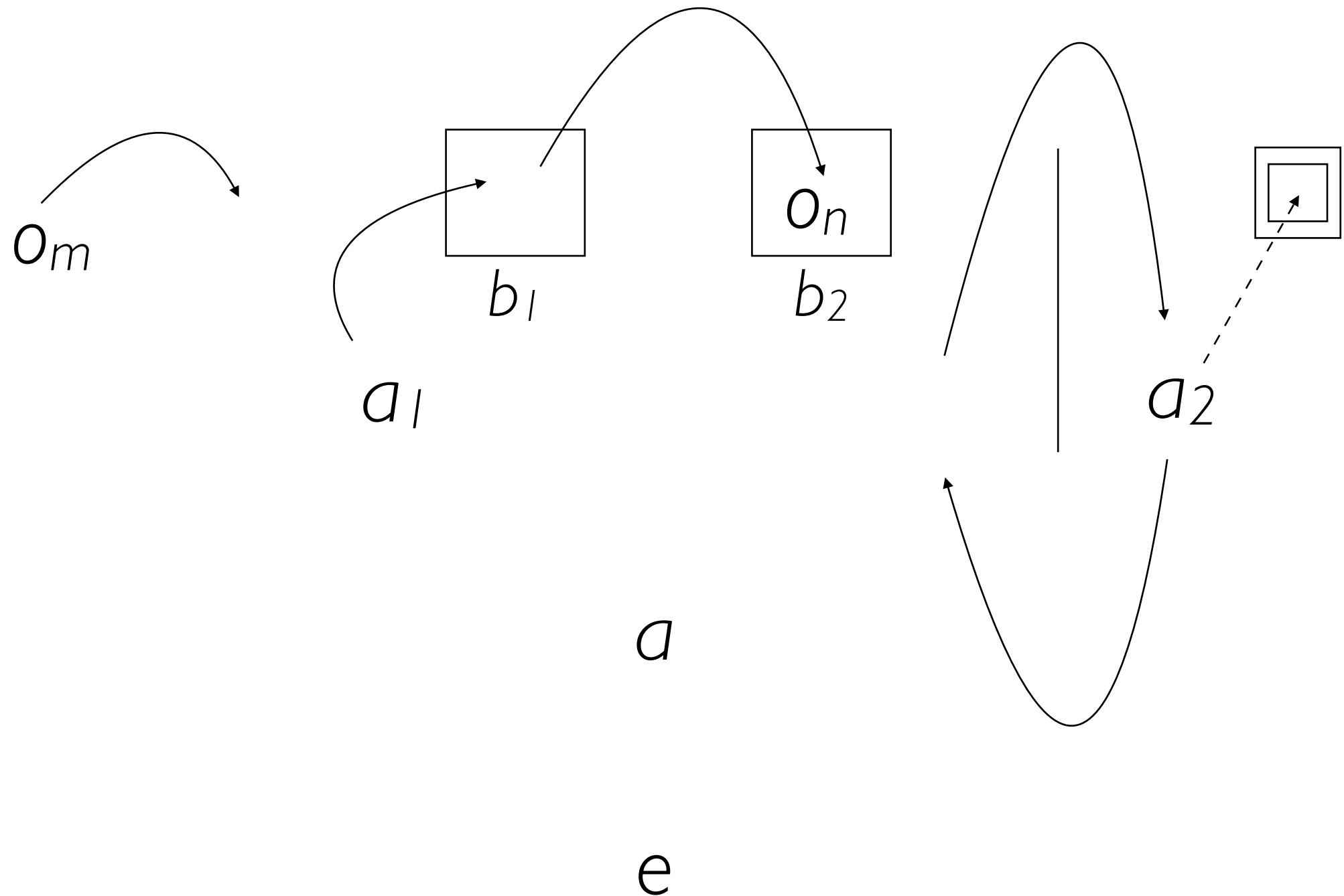
Framework for FBT^1_2

(seven timepoints)



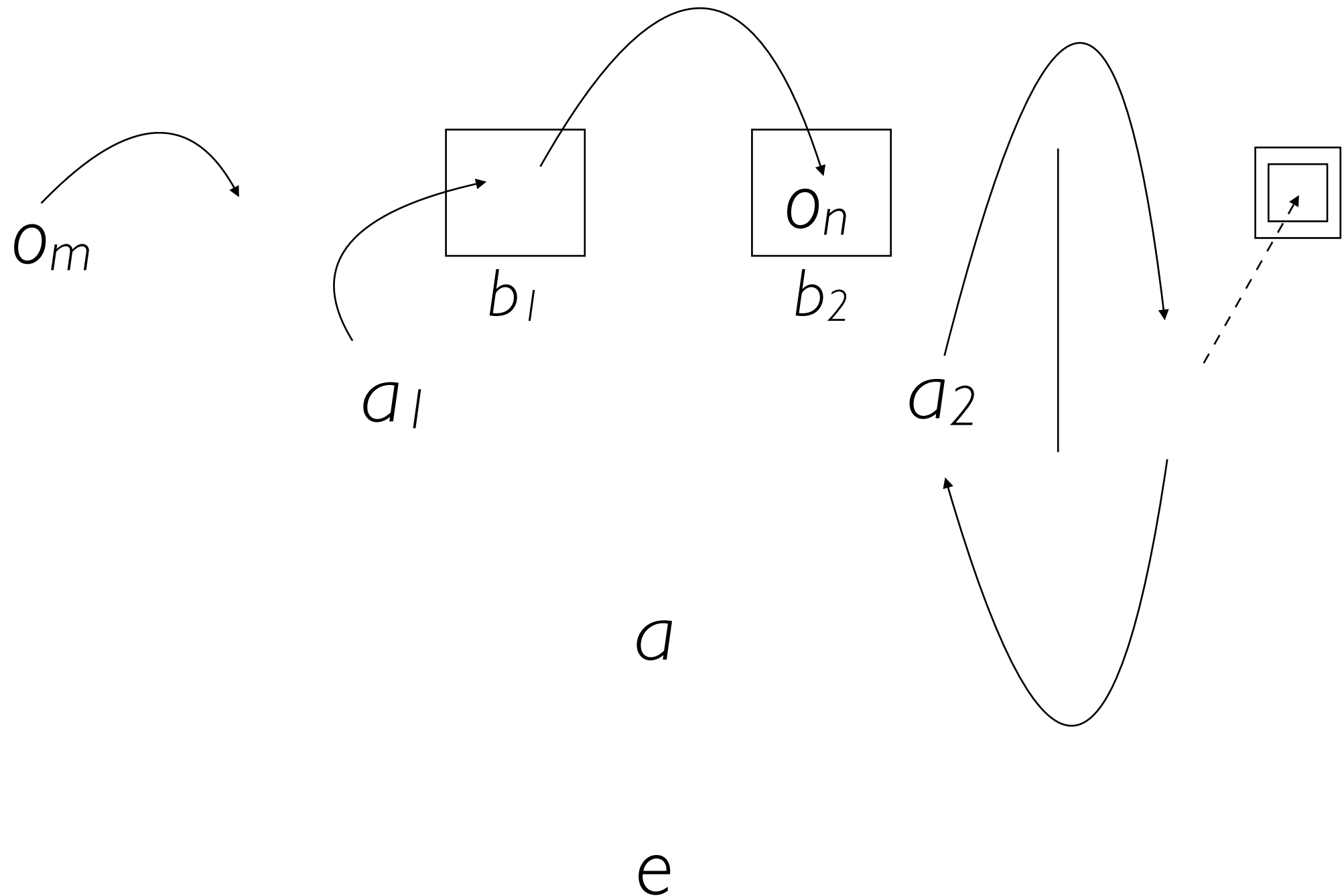
Framework for FBT^1_2

(seven timepoints)



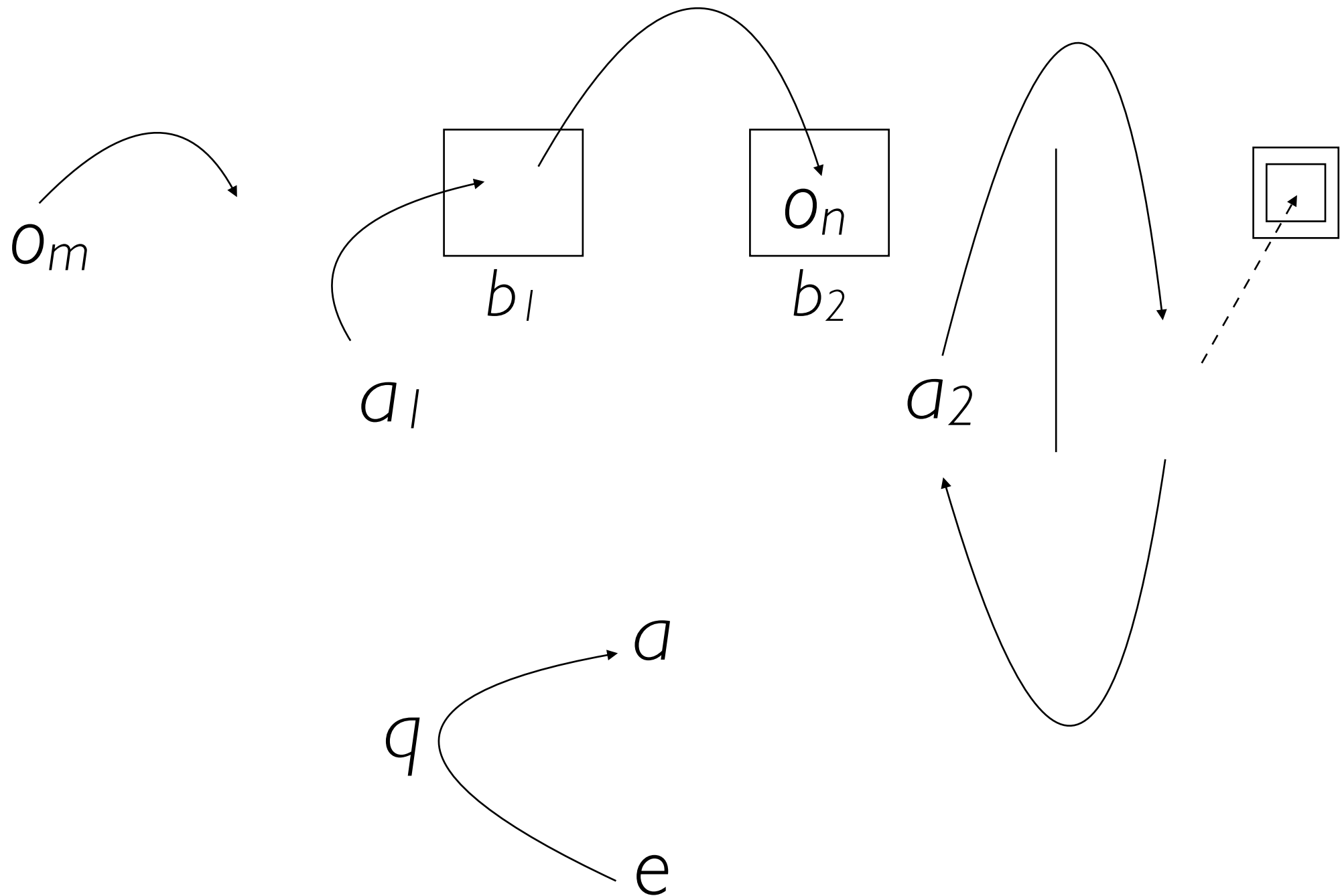
Framework for FBT^1_2

(seven timepoints)



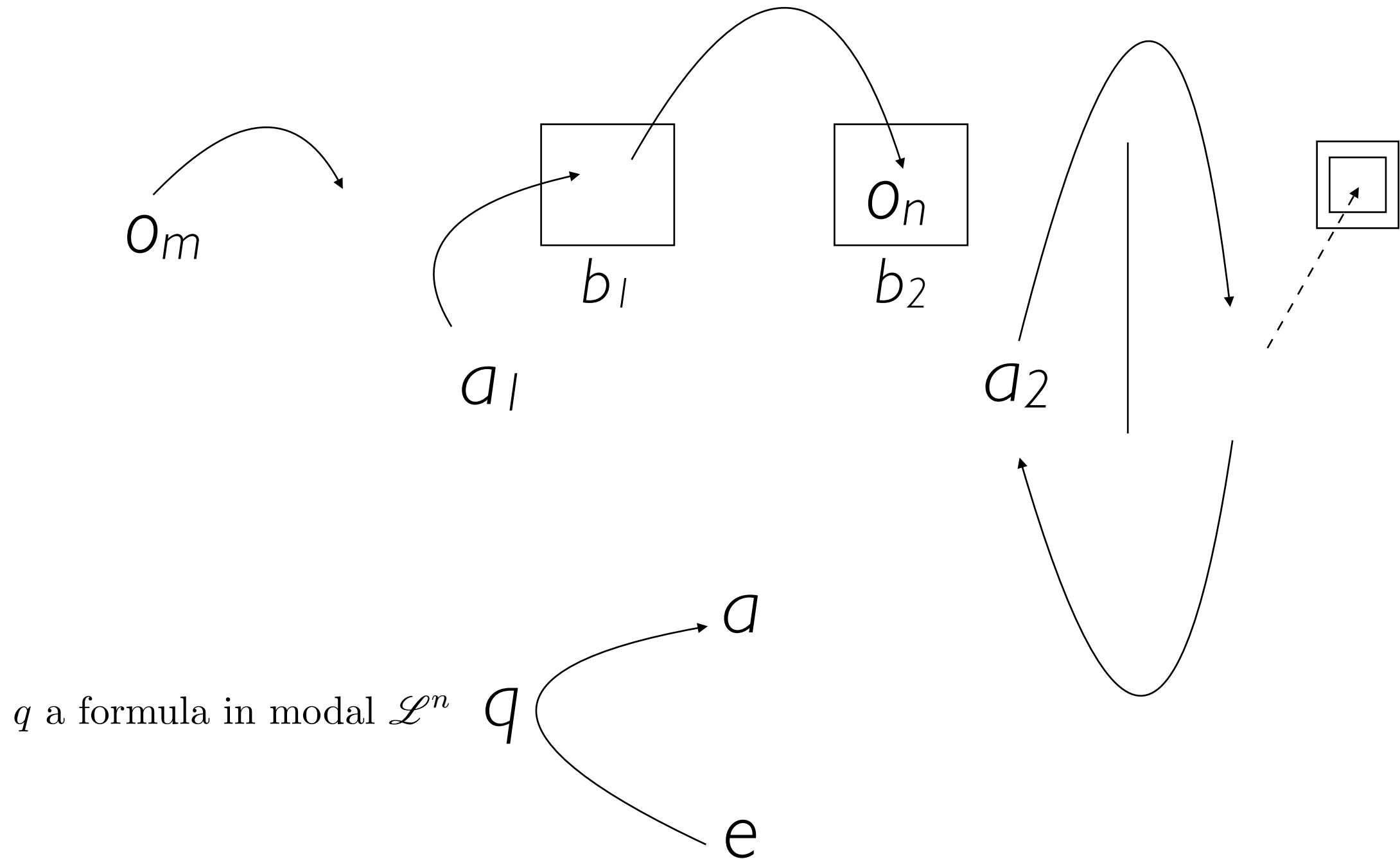
Framework for FBT^1_2

(seven timepoints)

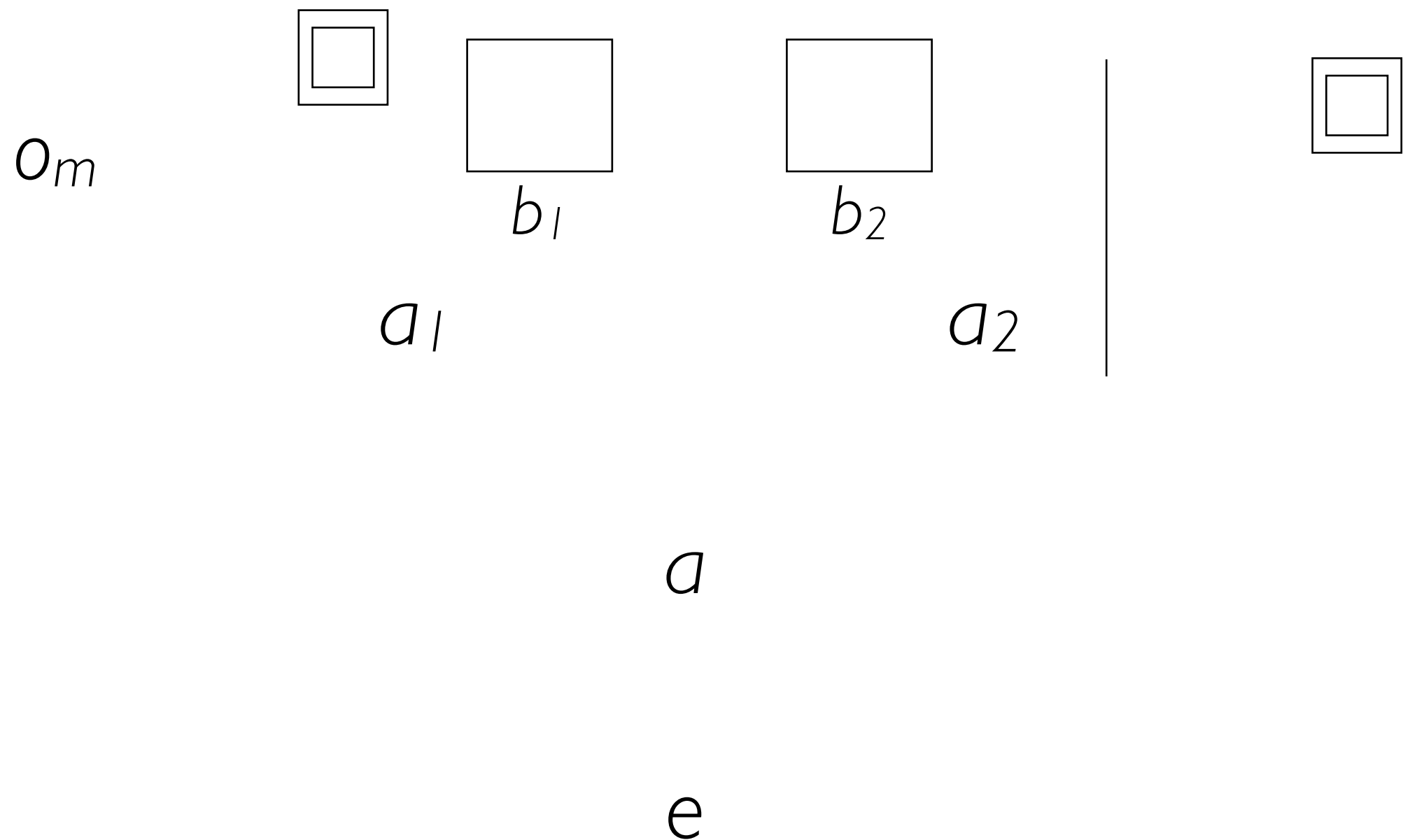


Framework for FBT^1_2

(seven timepoints)

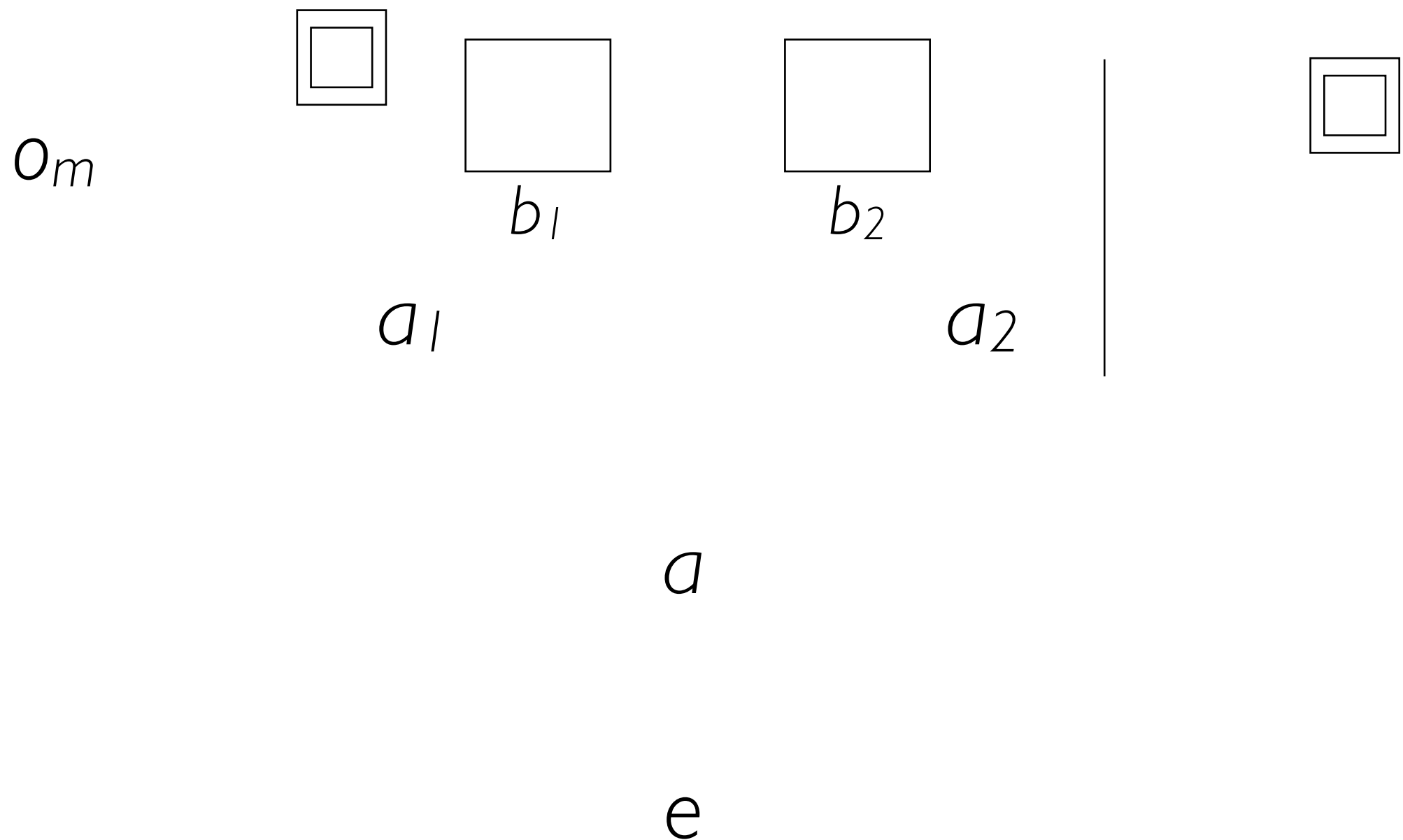


Framework for FBT^I_3



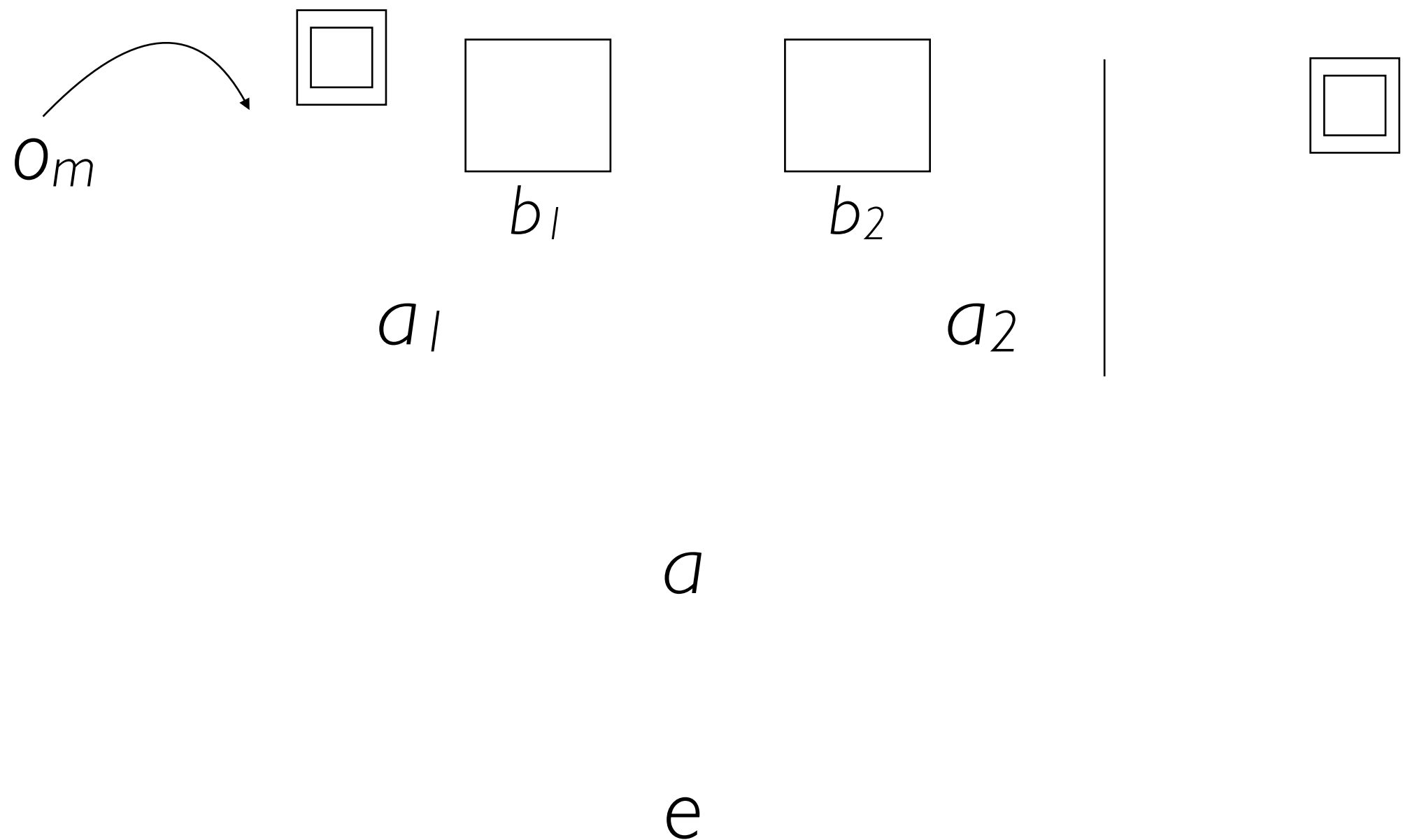
Framework for FBT^I_3

(eight timepoints)



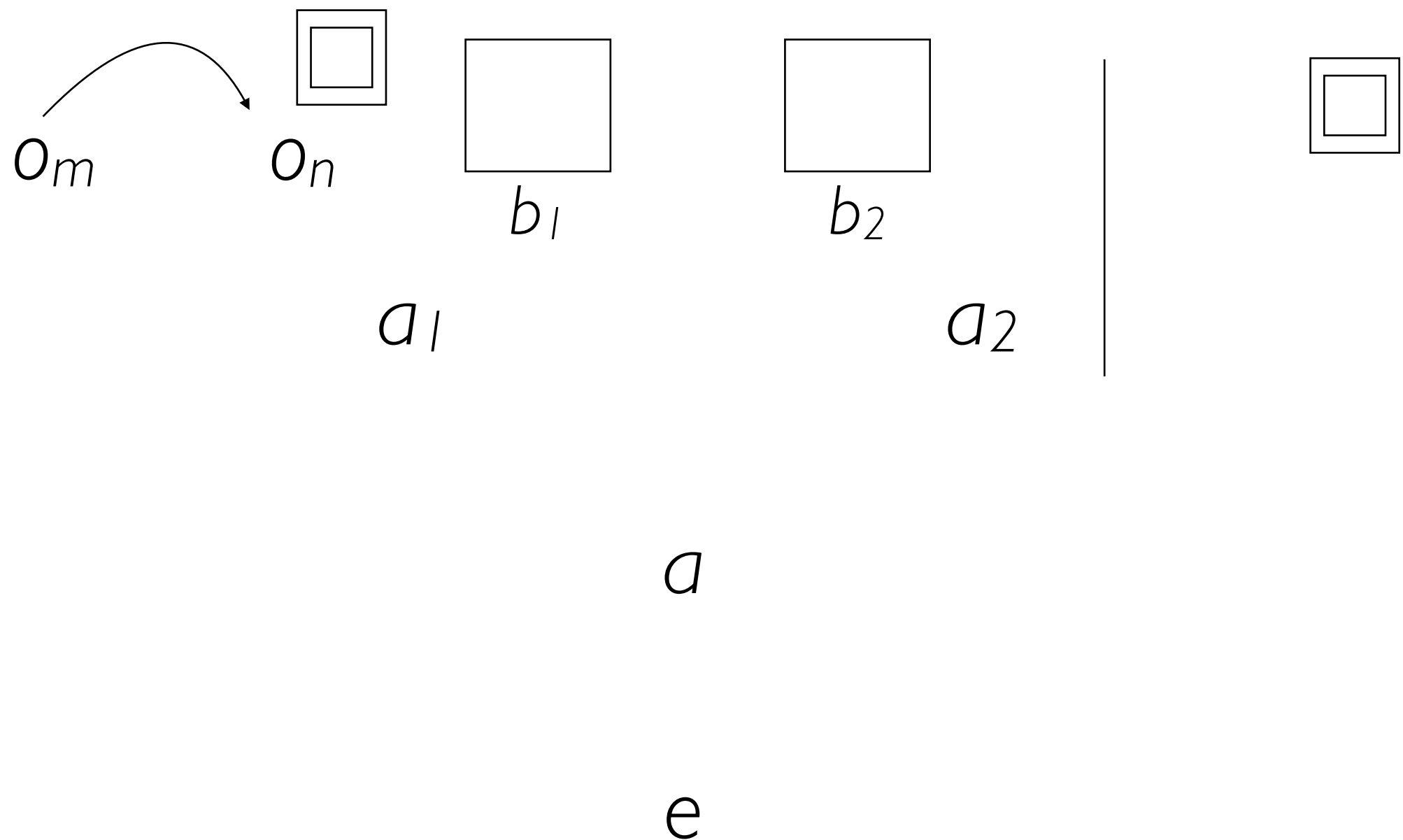
Framework for FBT^I_3

(eight timepoints)



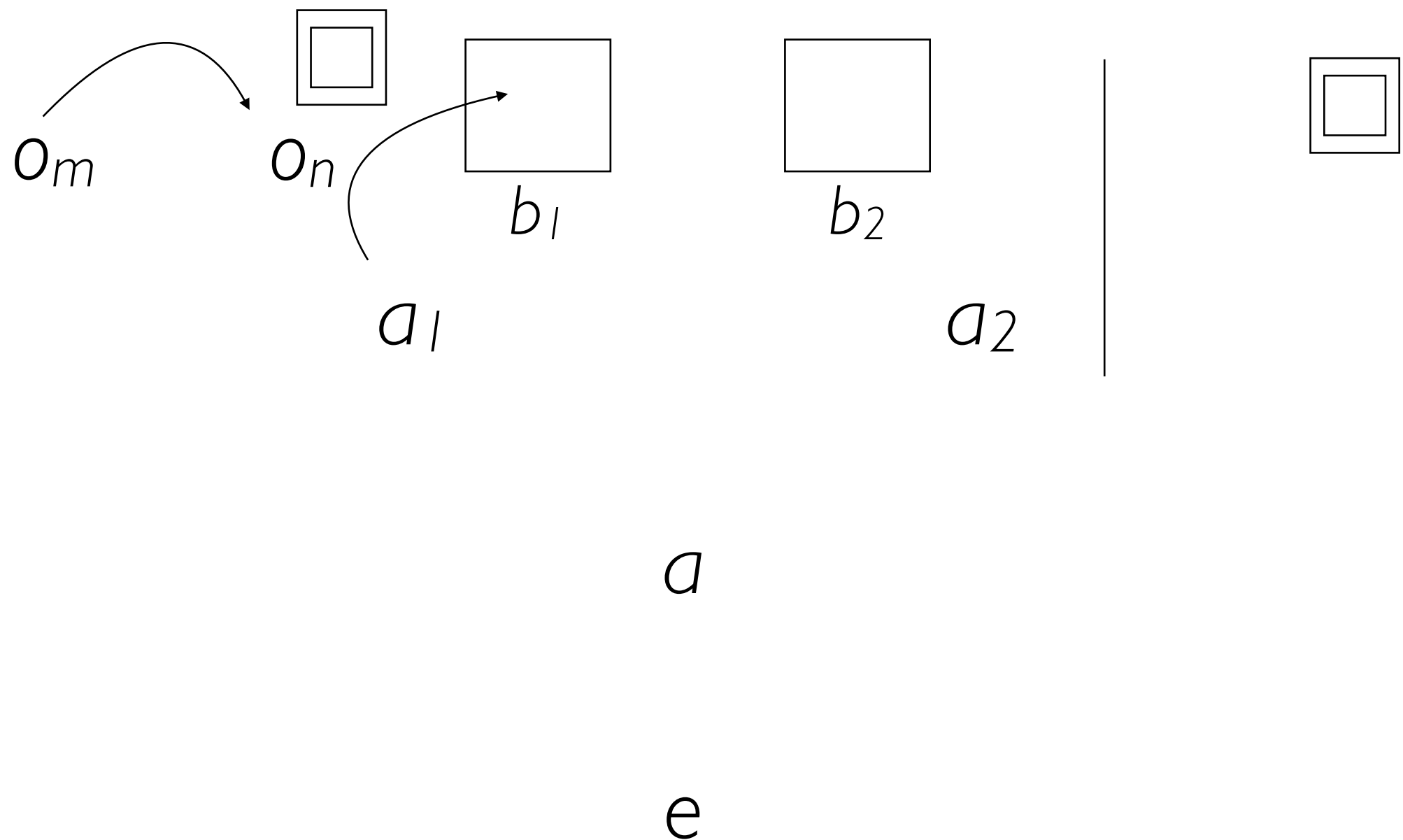
Framework for FBT^I_3

(eight timepoints)



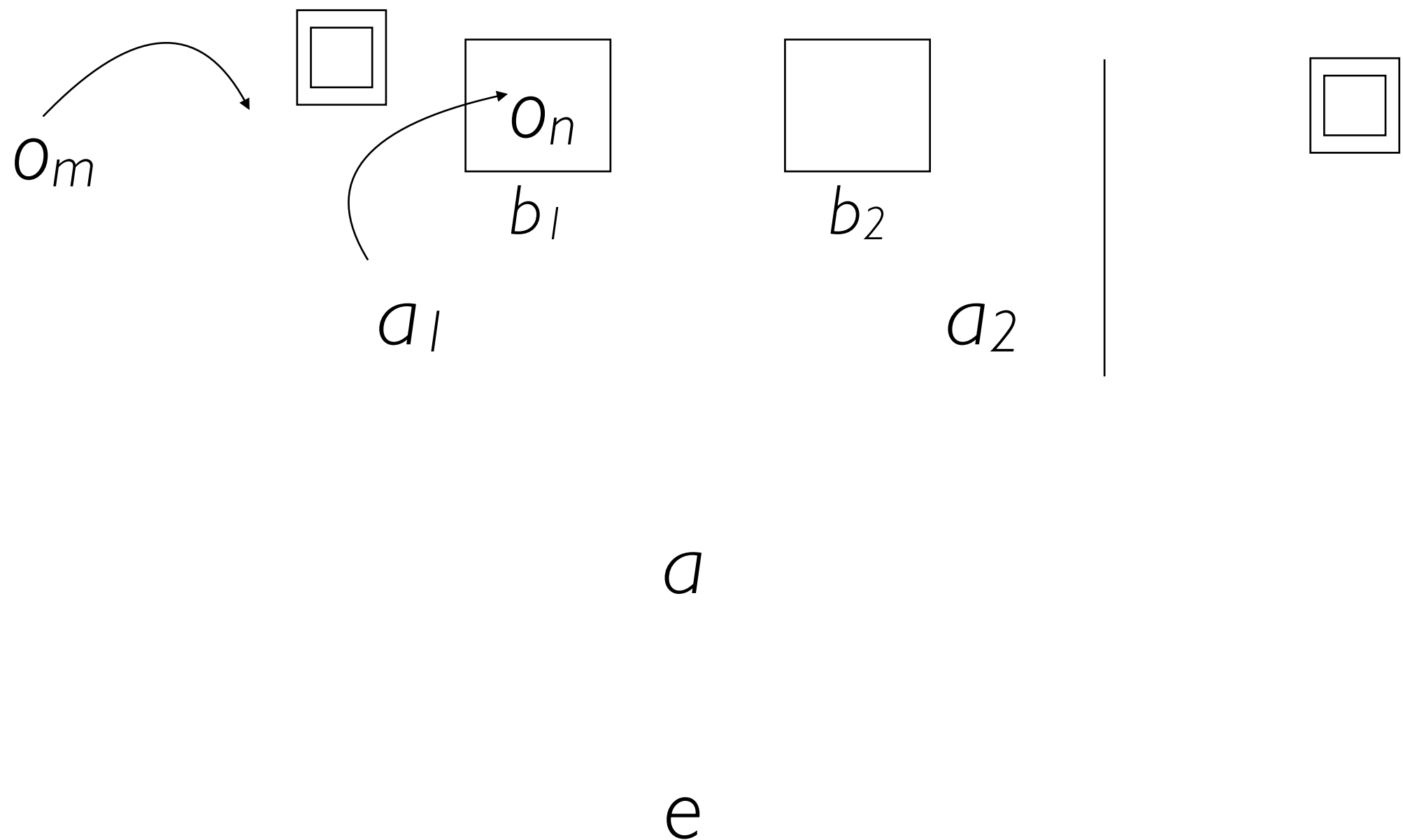
Framework for FBT^I_3

(eight timepoints)



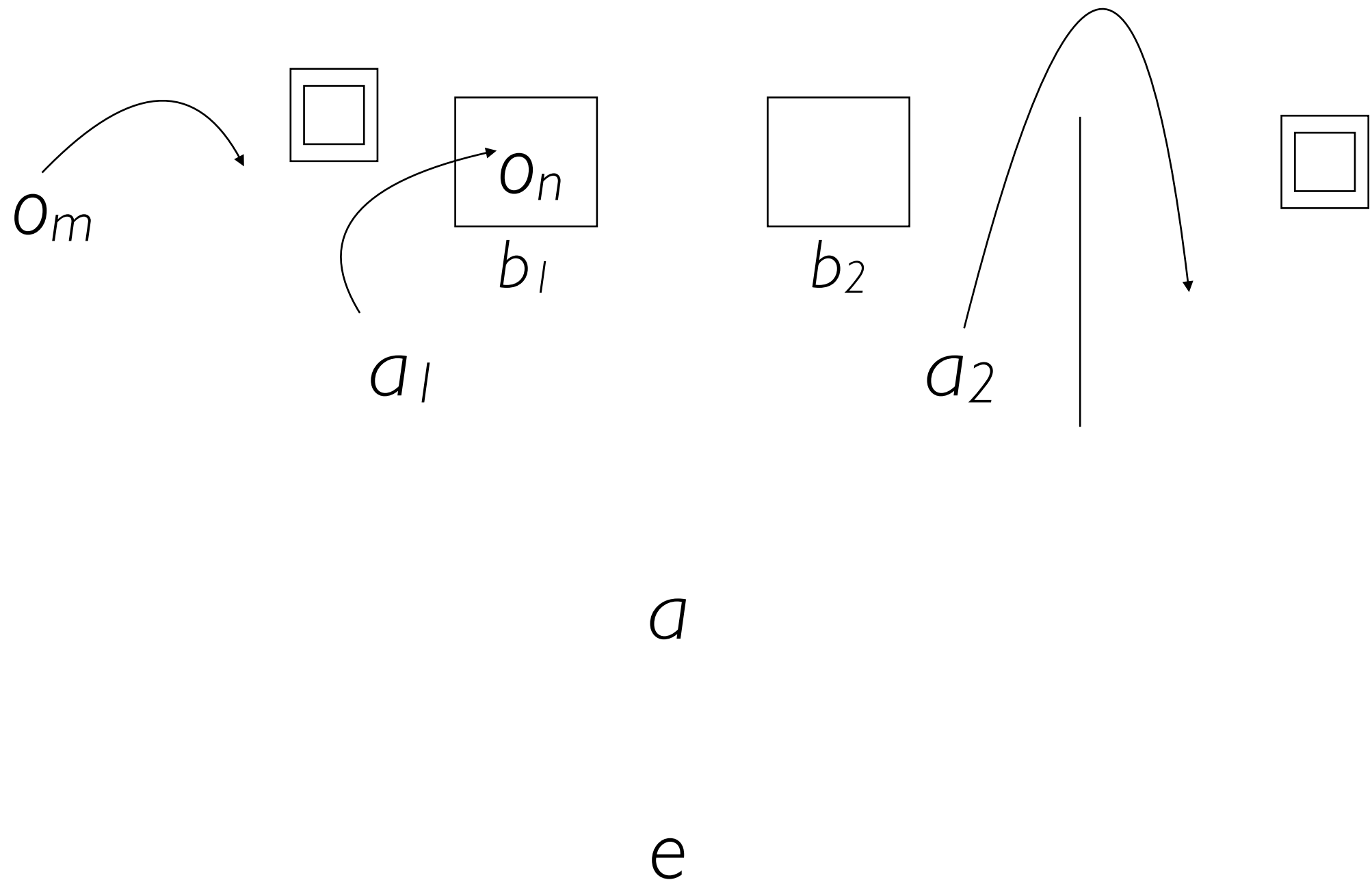
Framework for FBT^I_3

(eight timepoints)



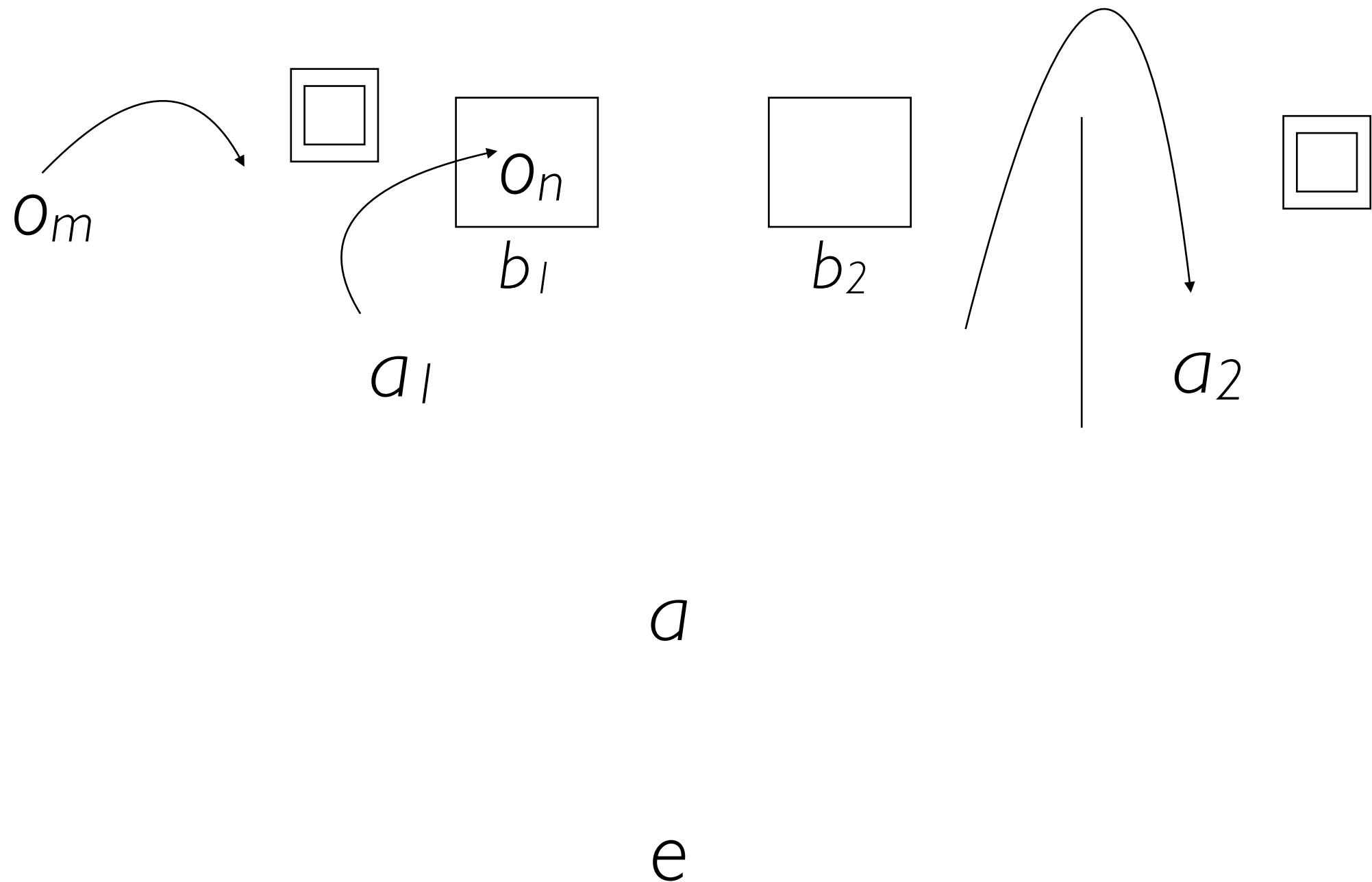
Framework for FBT^I_3

(eight timepoints)



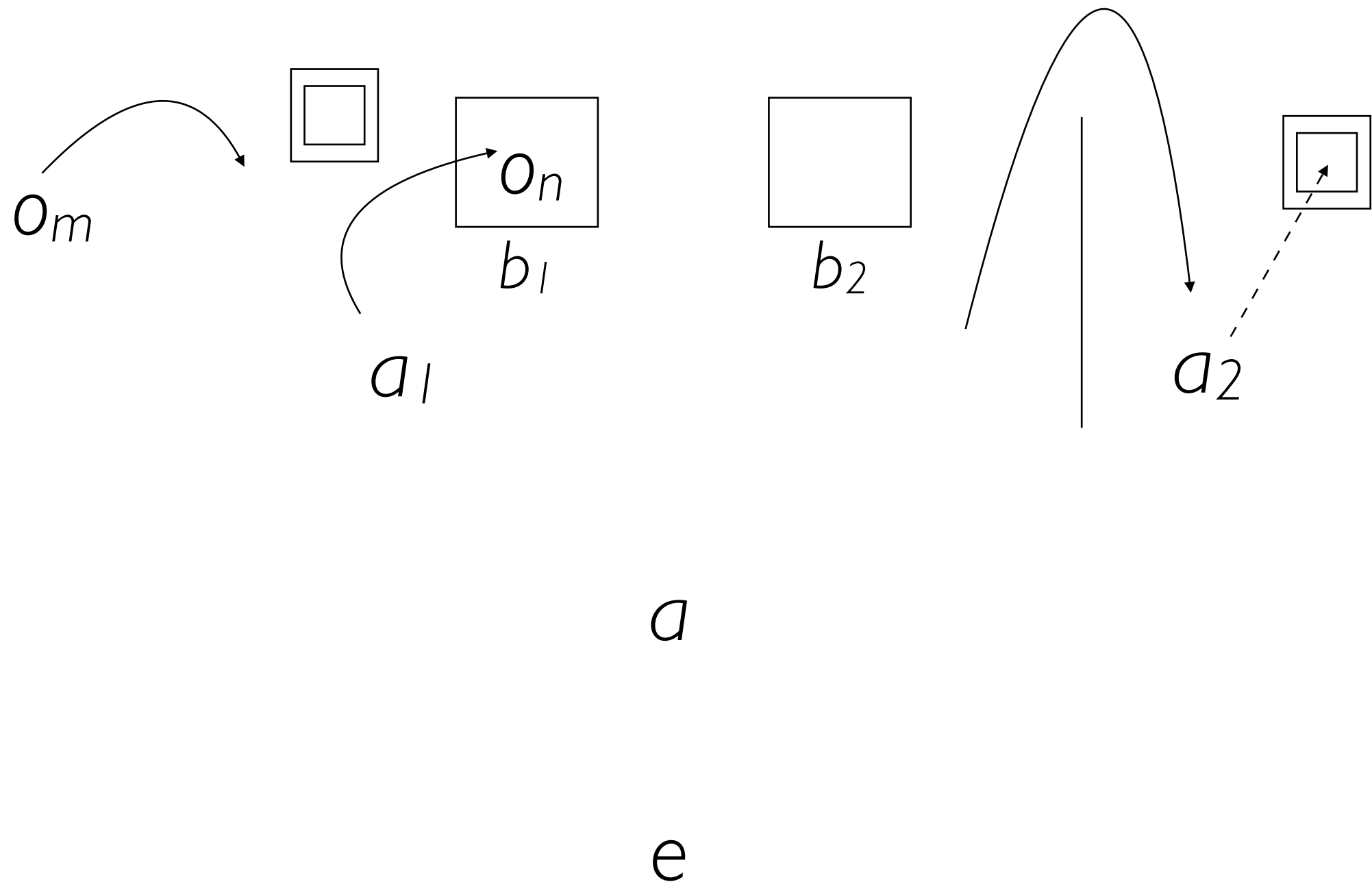
Framework for FBT^I_3

(eight timepoints)



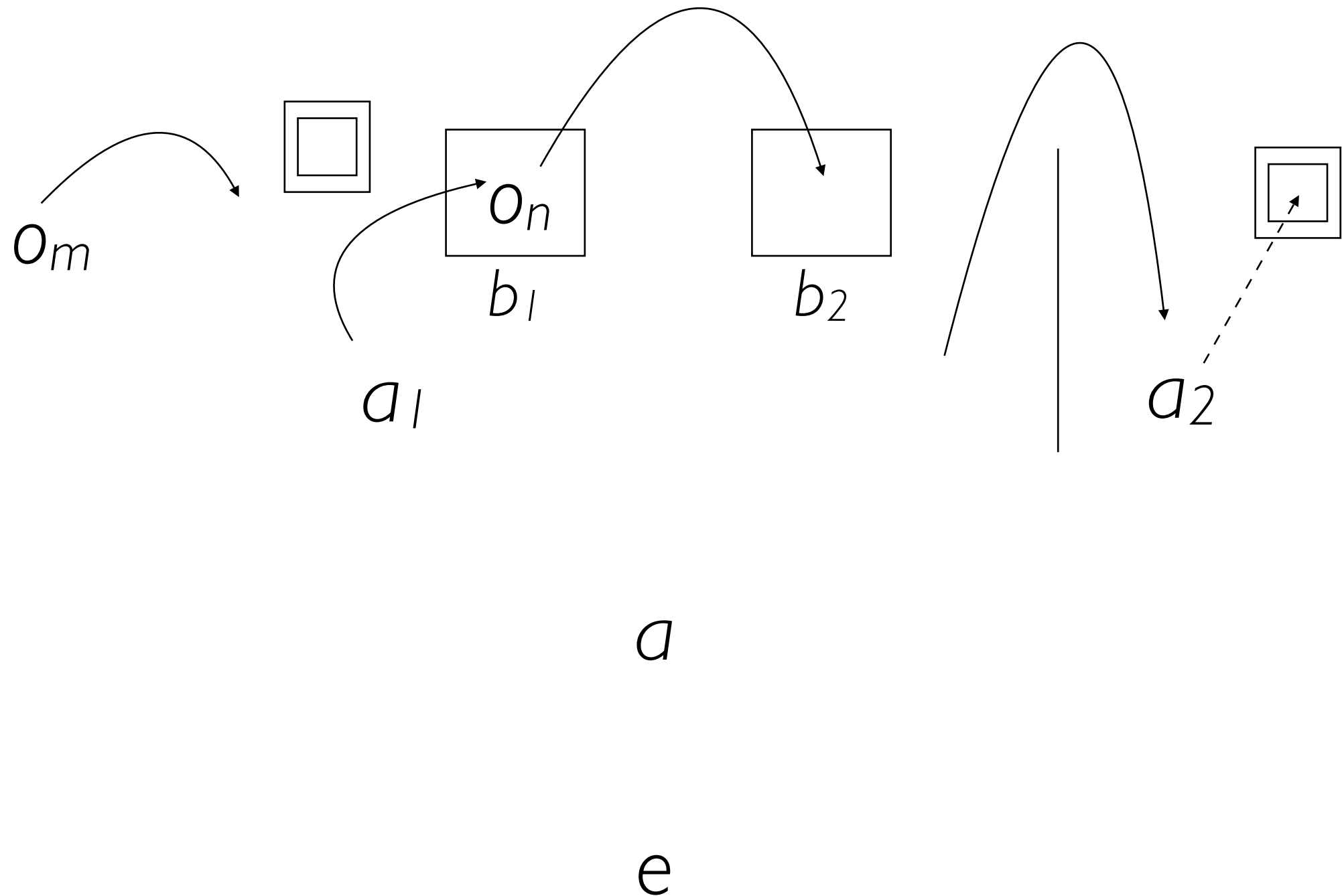
Framework for FBT^I_3

(eight timepoints)



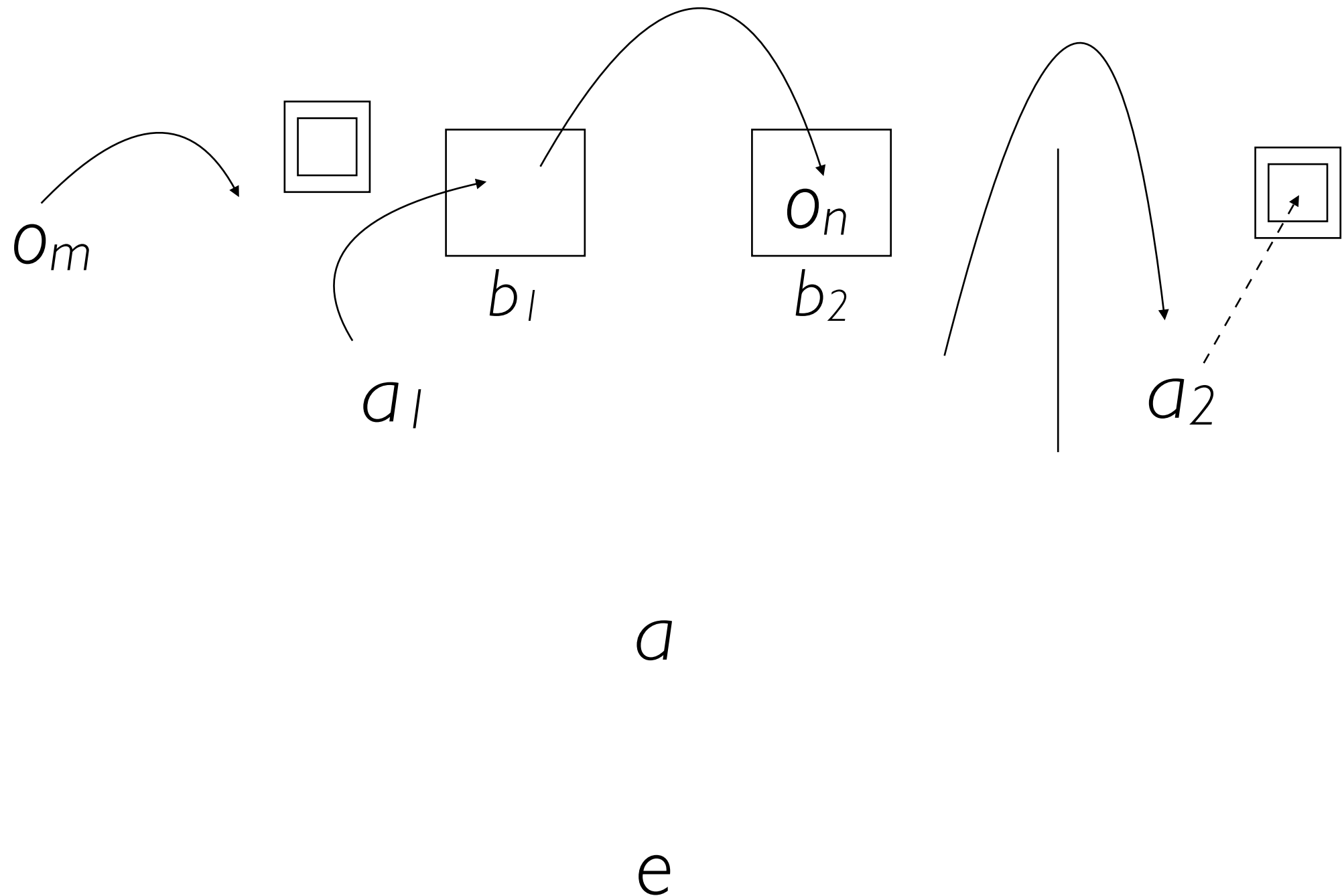
Framework for FBT^I_3

(eight timepoints)



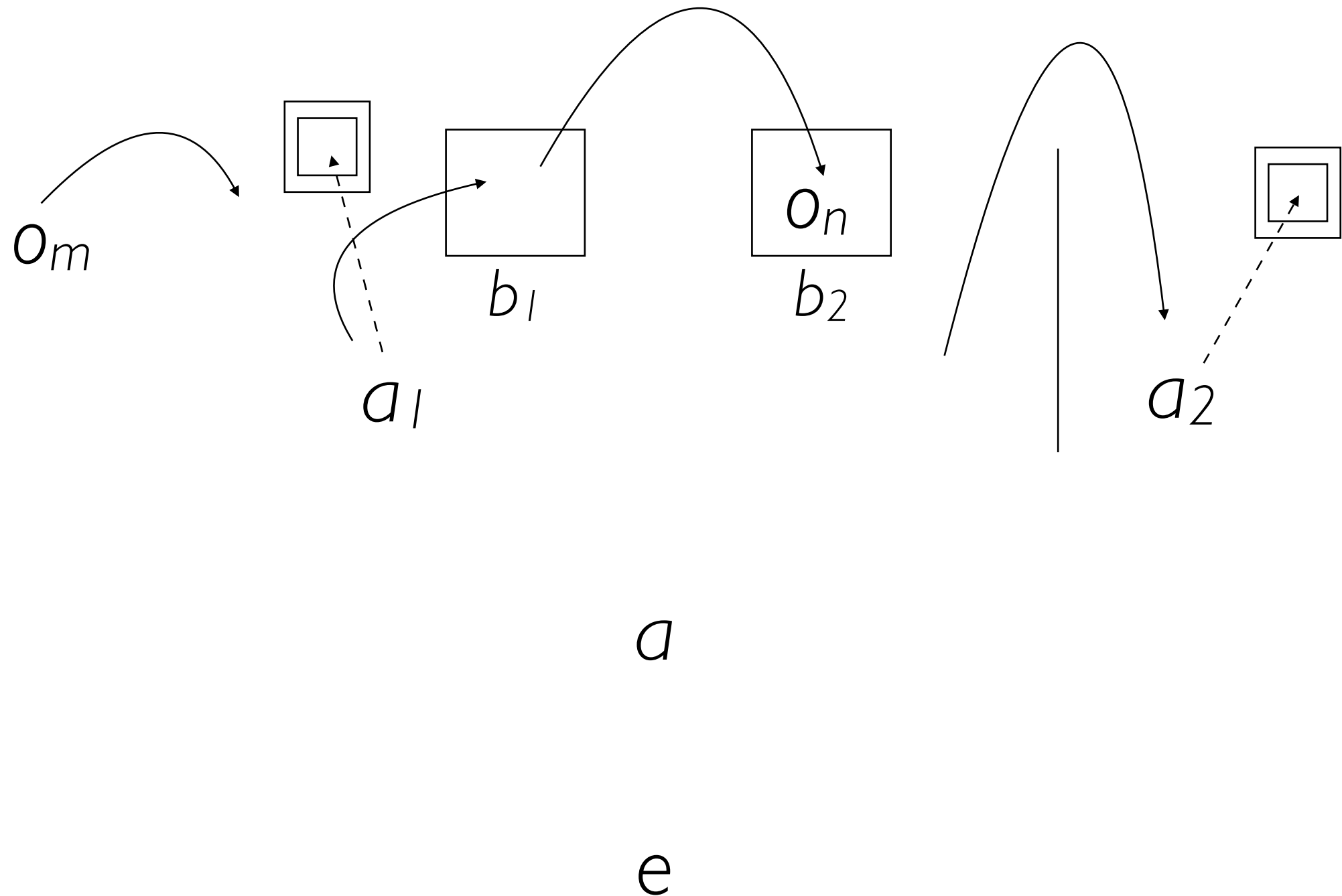
Framework for FBT^I_3

(eight timepoints)



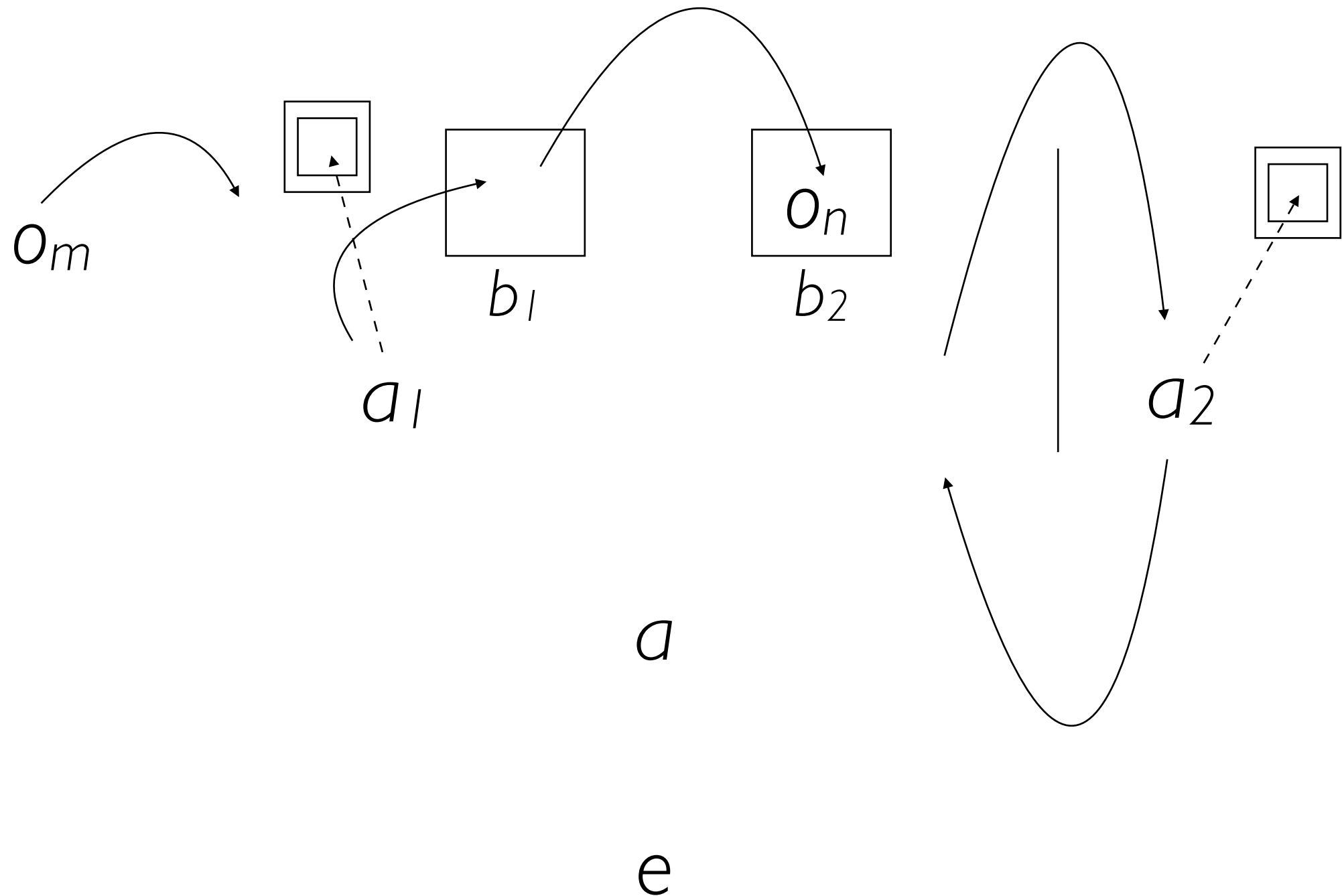
Framework for FBT^I_3

(eight timepoints)



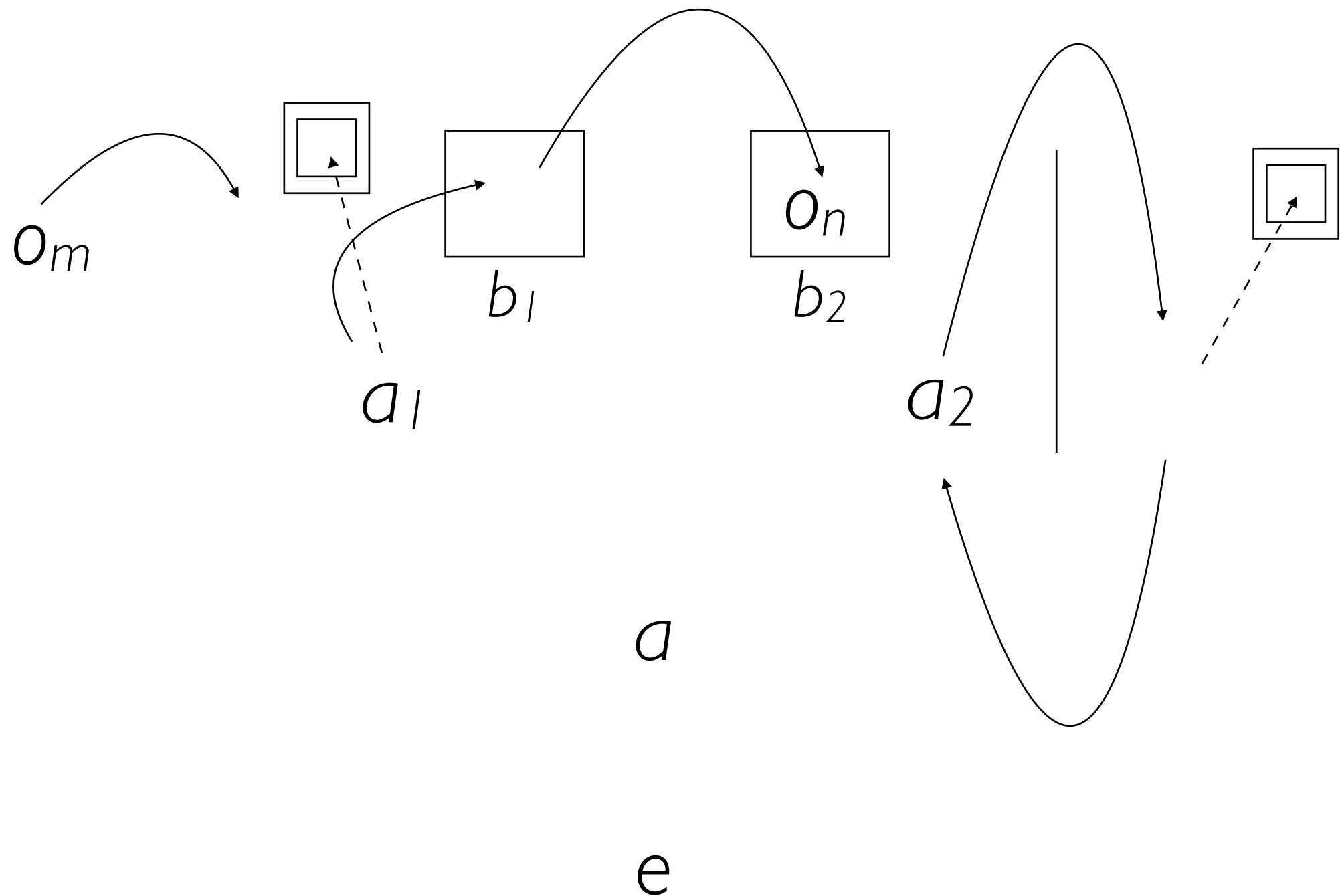
Framework for FBT^I_3

(eight timepoints)



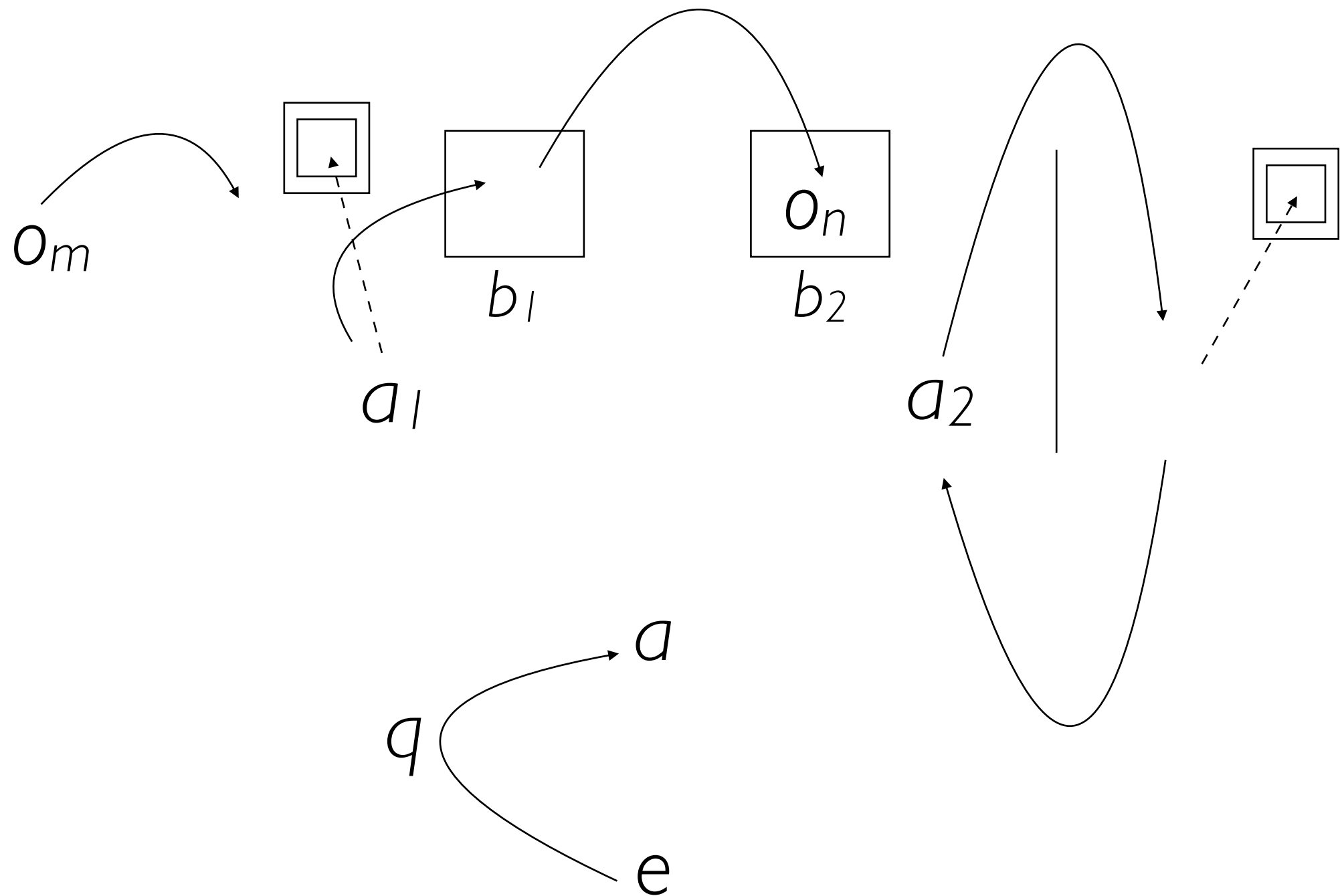
Framework for FBT^I_3

(eight timepoints)



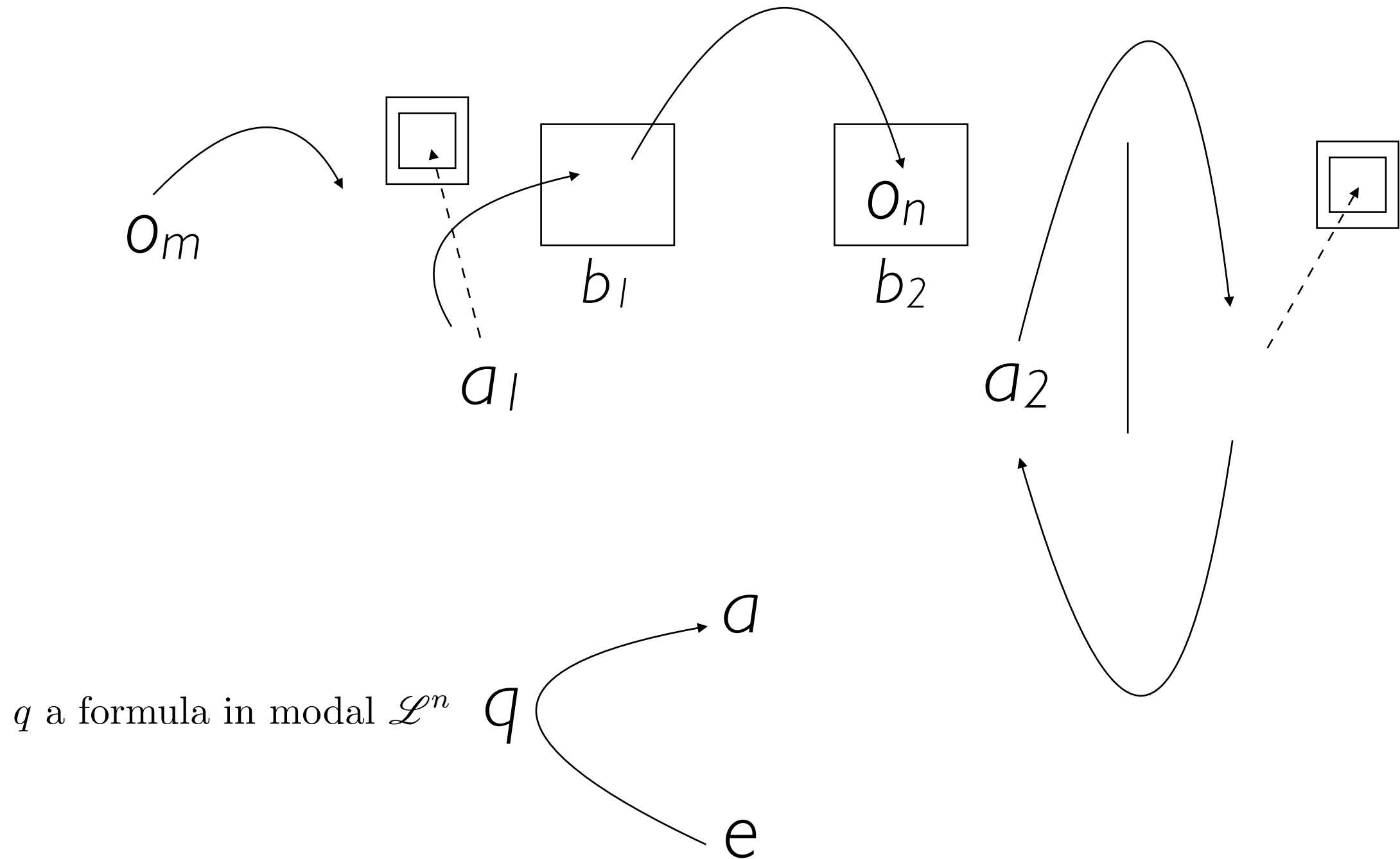
Framework for FBT^I_3

(eight timepoints)

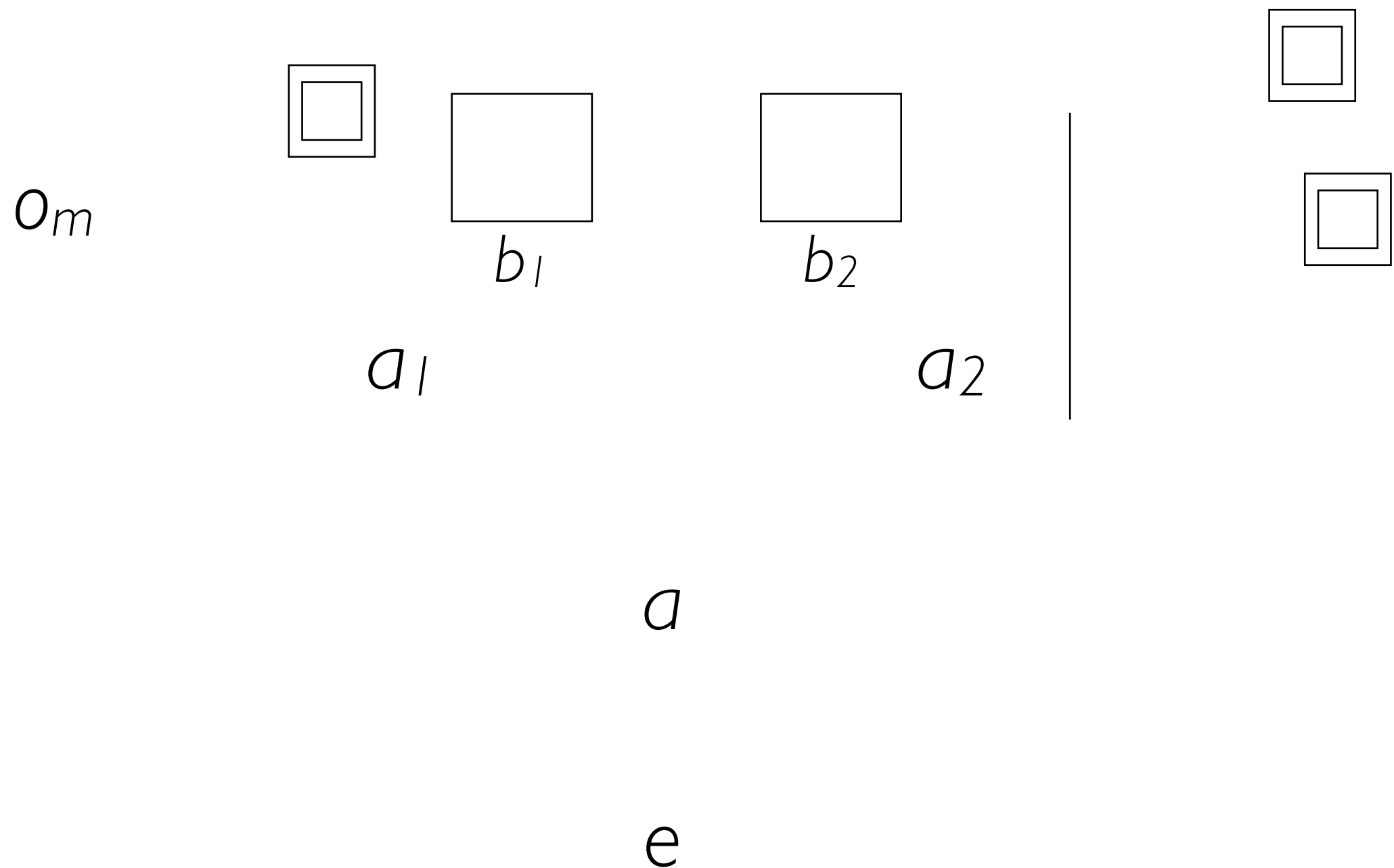


Framework for FBT^I_3

(eight timepoints)

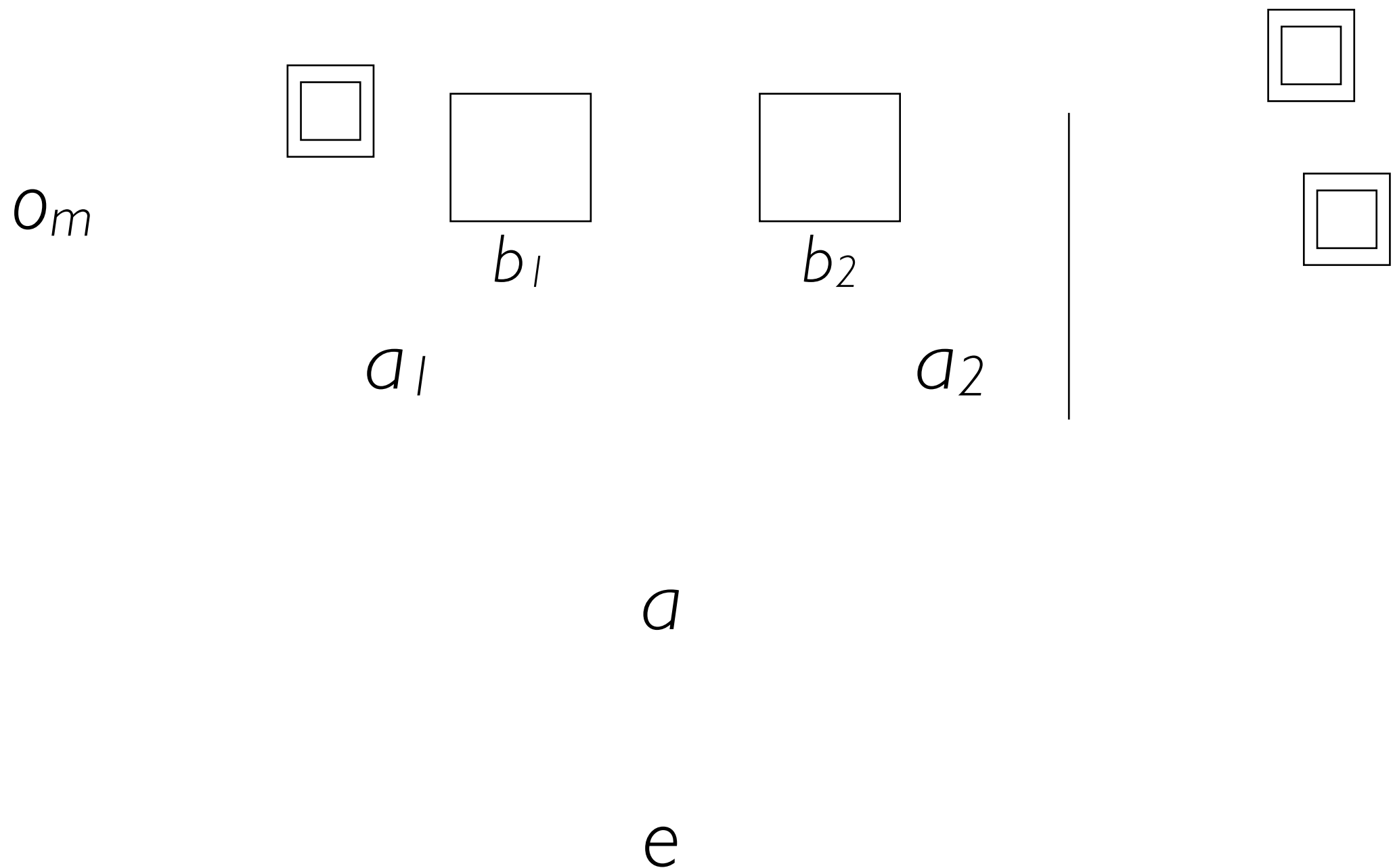


Framework for FBT^I_4



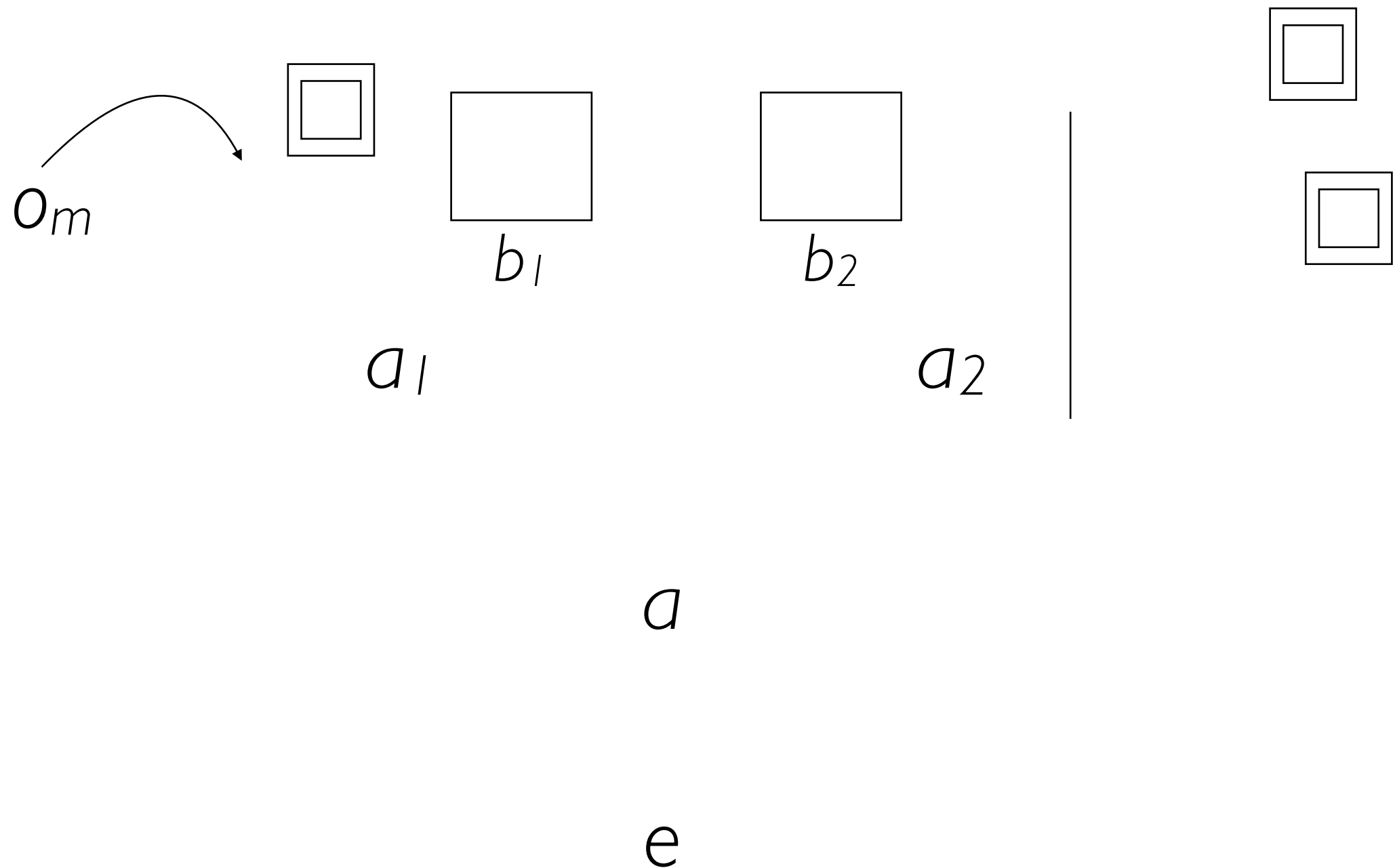
Framework for FBT^I_4

(nine timepoints)



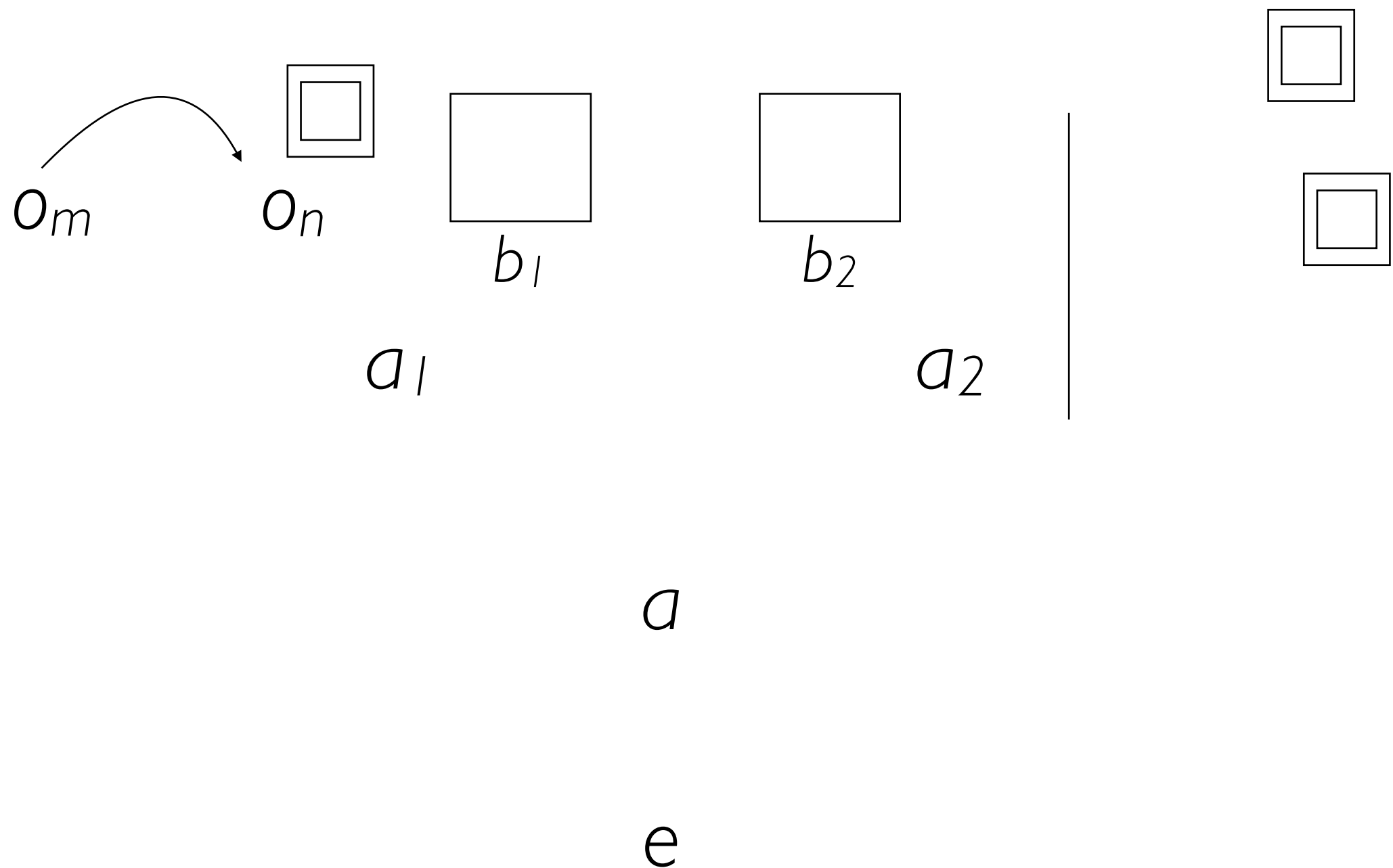
Framework for FBT^I_4

(nine timepoints)



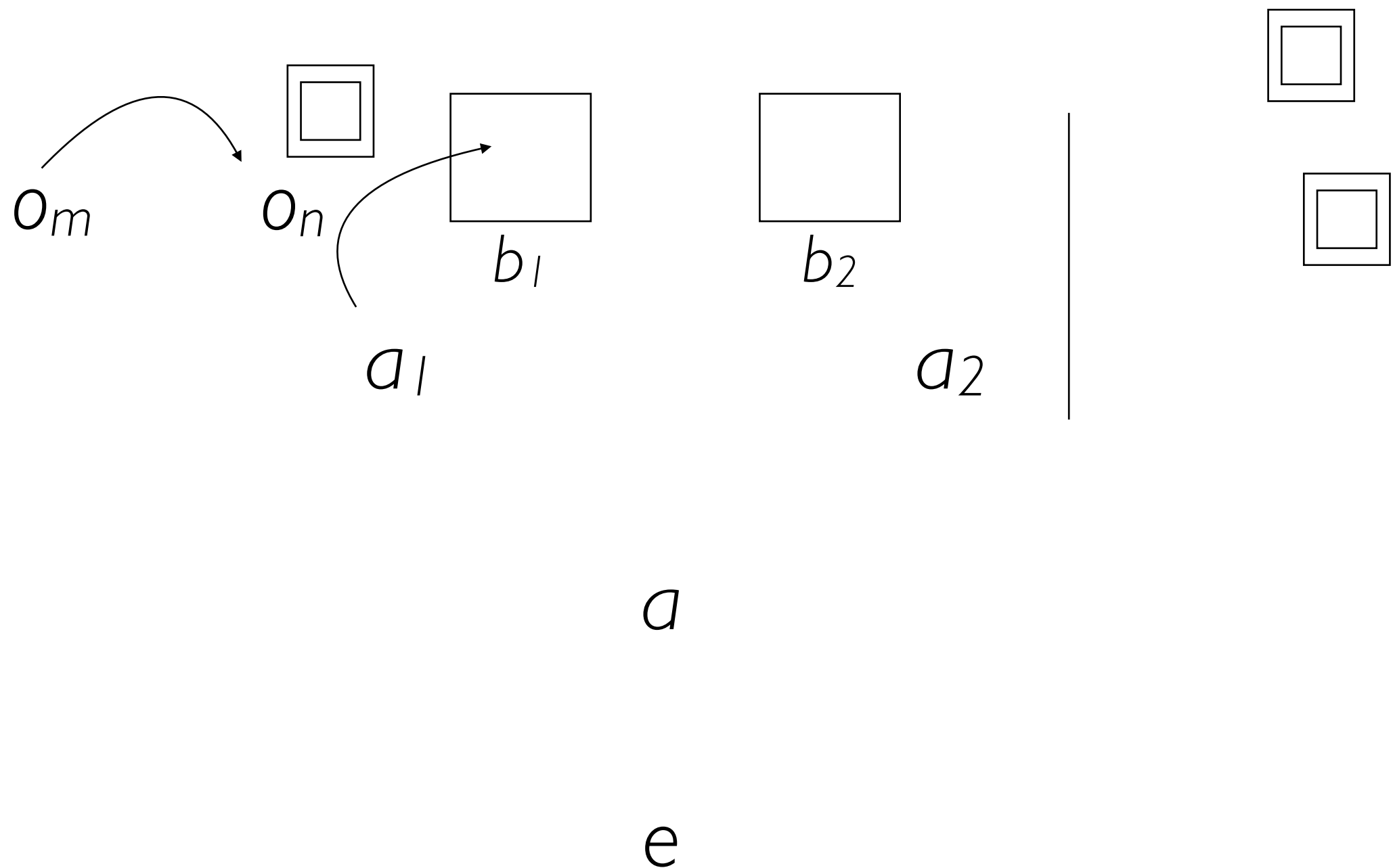
Framework for FBT^I_4

(nine timepoints)



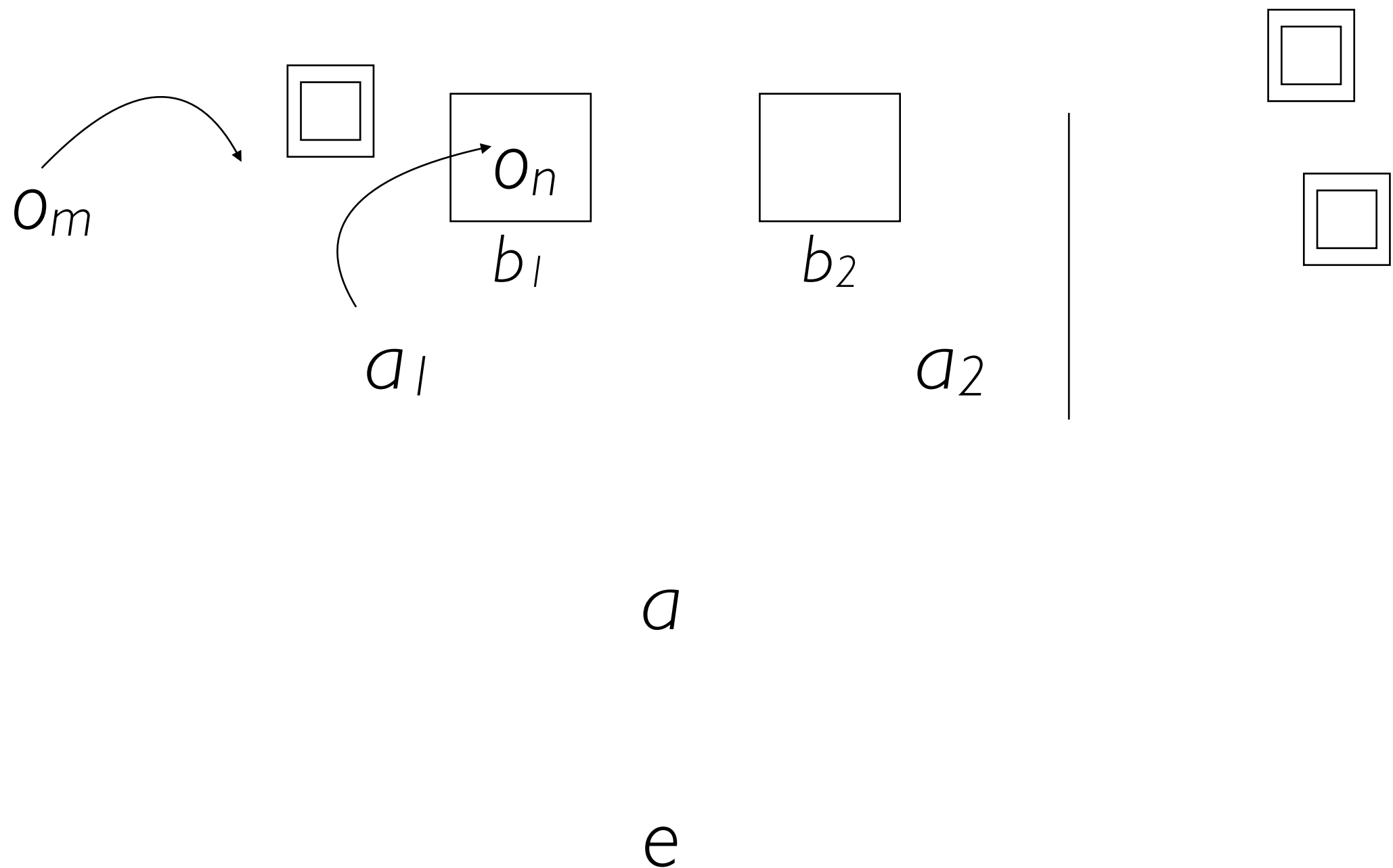
Framework for FBT^I_4

(nine timepoints)



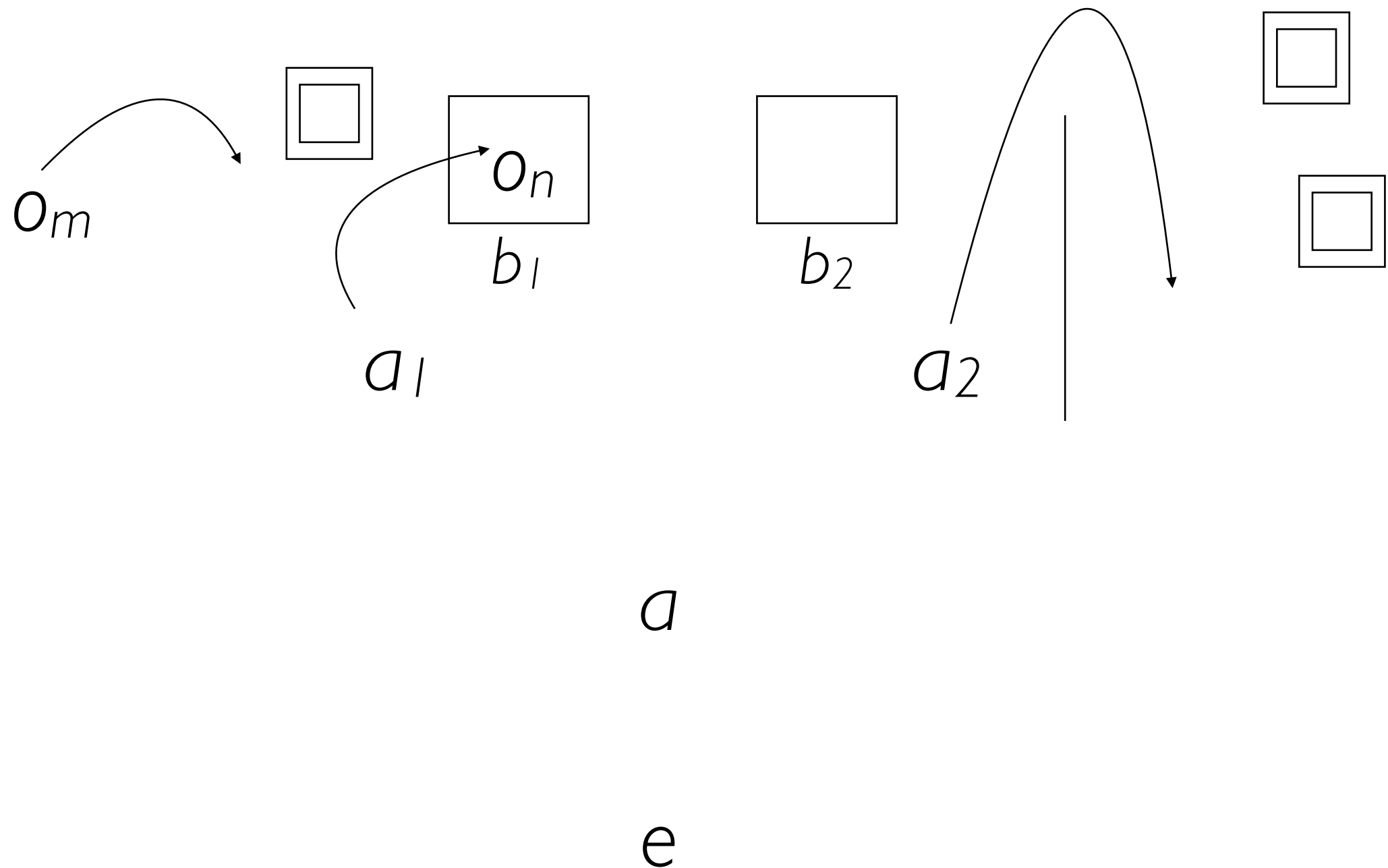
Framework for FBT^I_4

(nine timepoints)



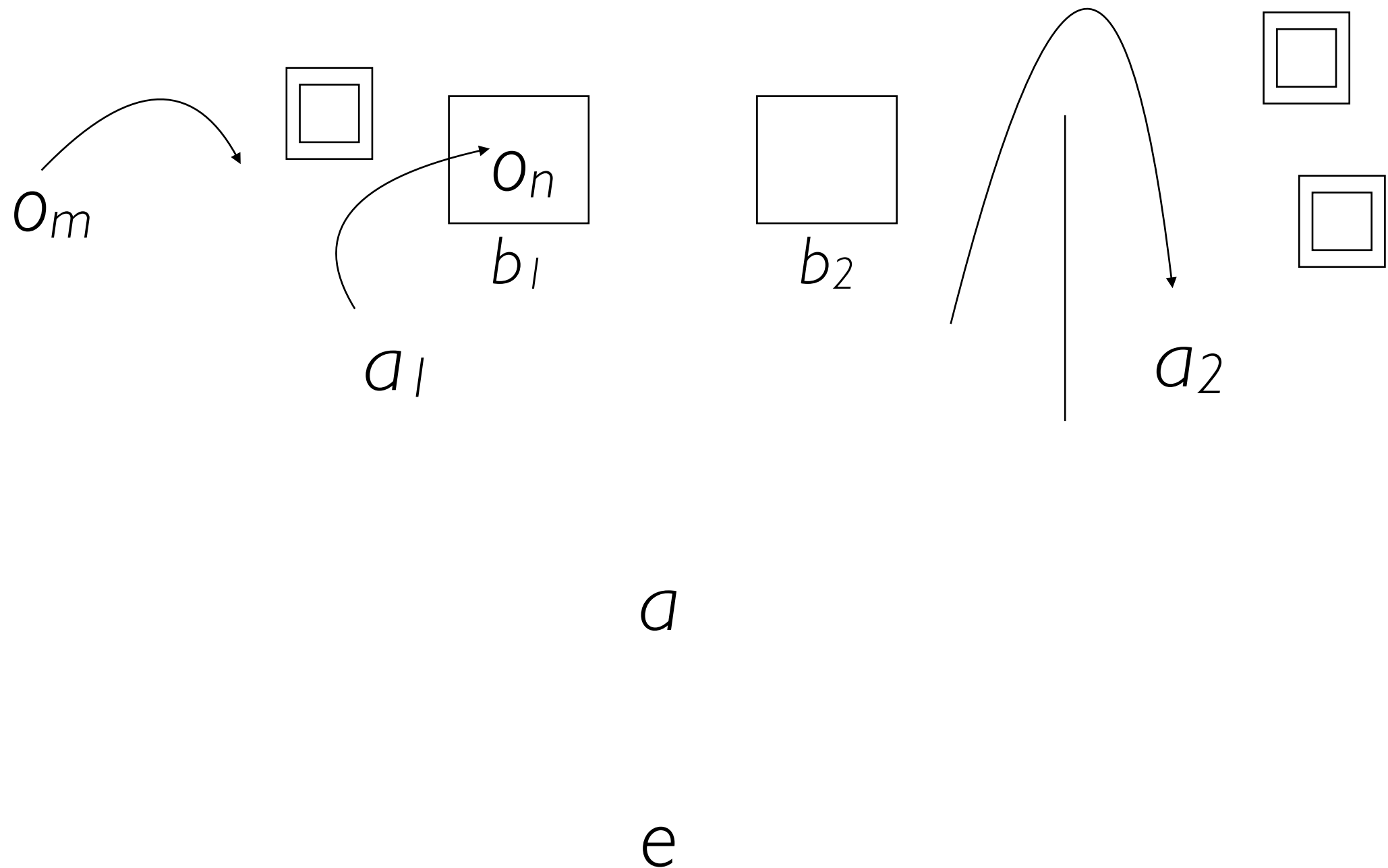
Framework for FBT^I_4

(nine timepoints)



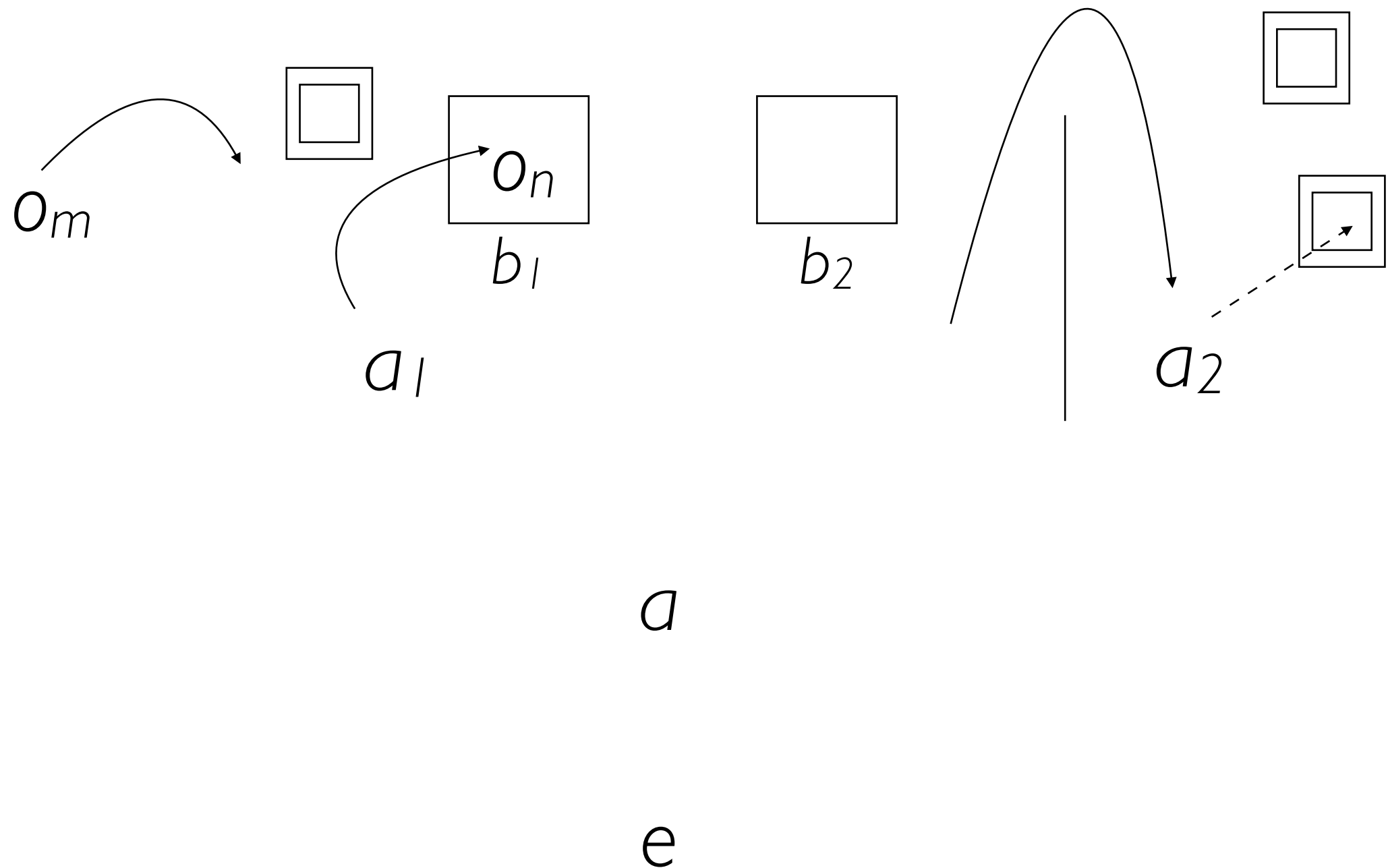
Framework for FBT^I_4

(nine timepoints)



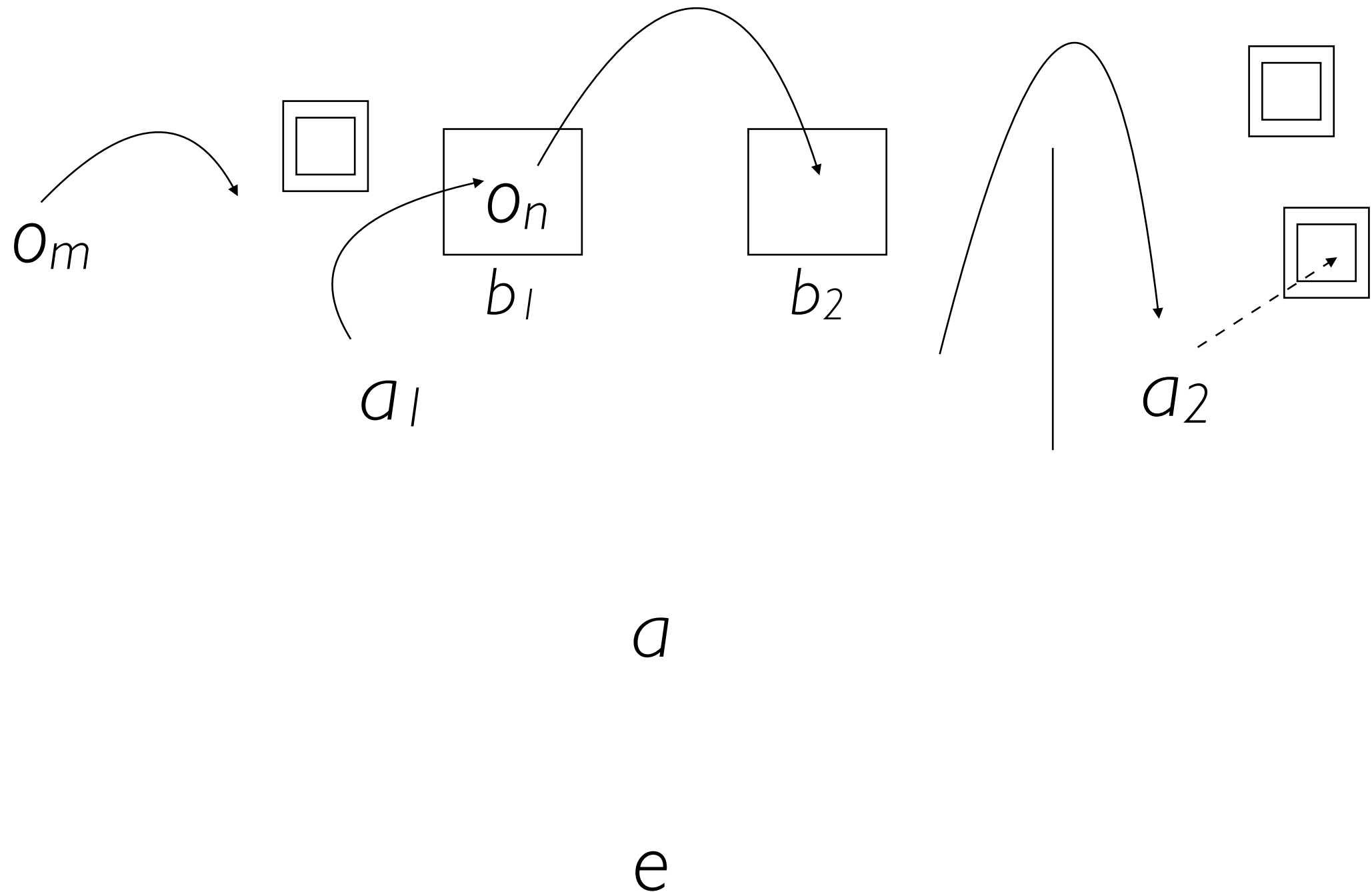
Framework for FBT^I_4

(nine timepoints)



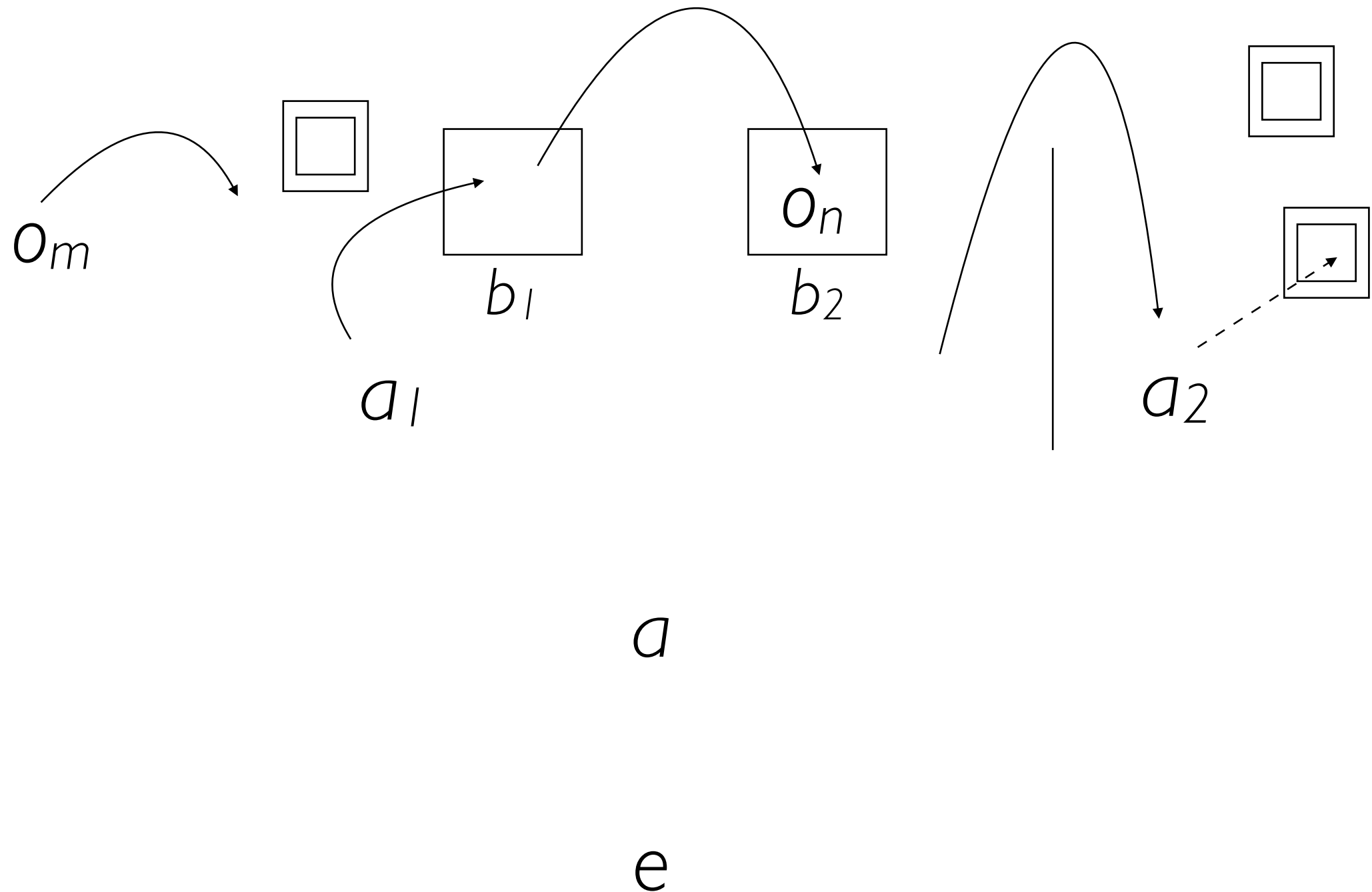
Framework for FBT^I_4

(nine timepoints)



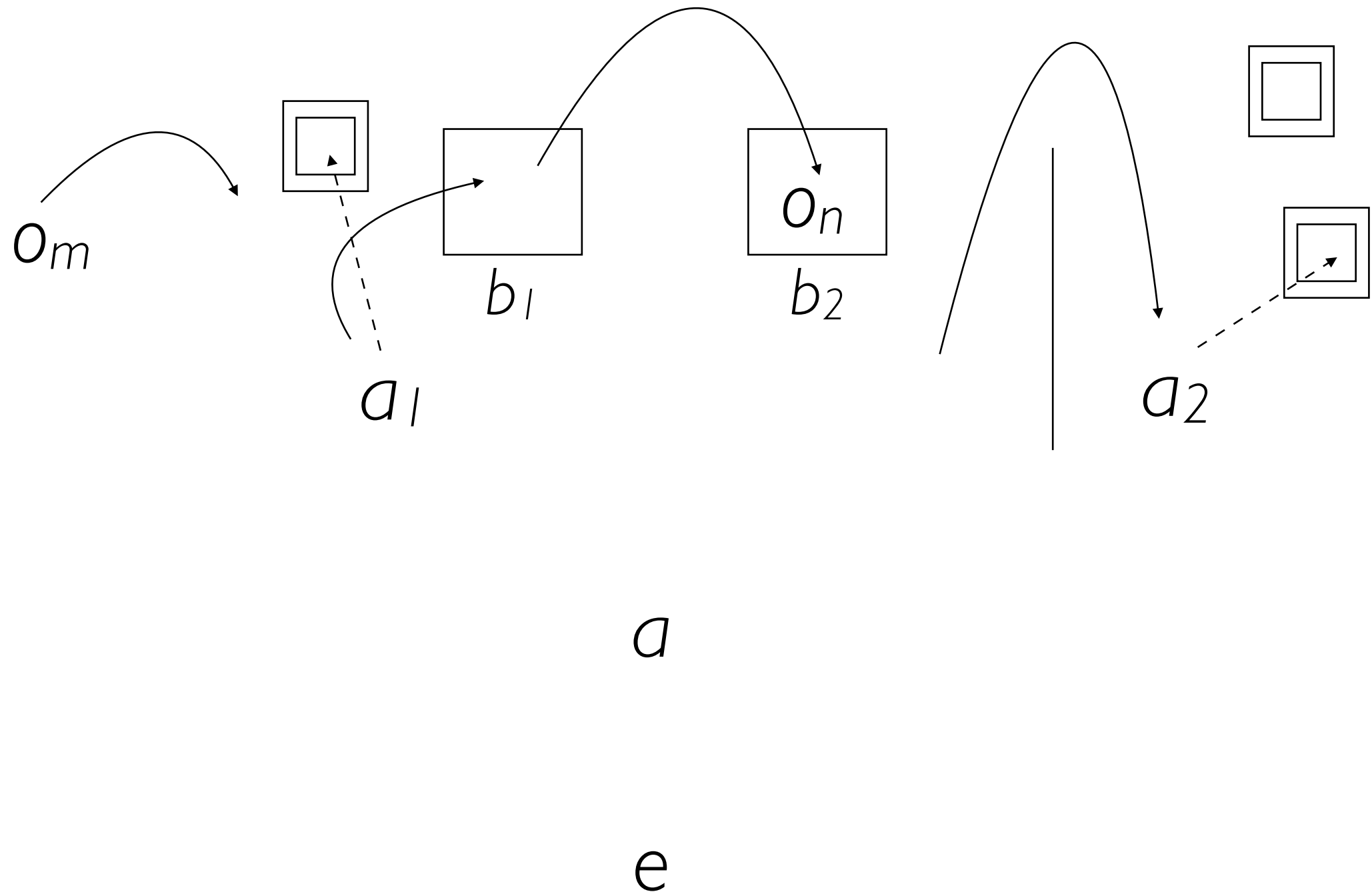
Framework for FBT^I_4

(nine timepoints)



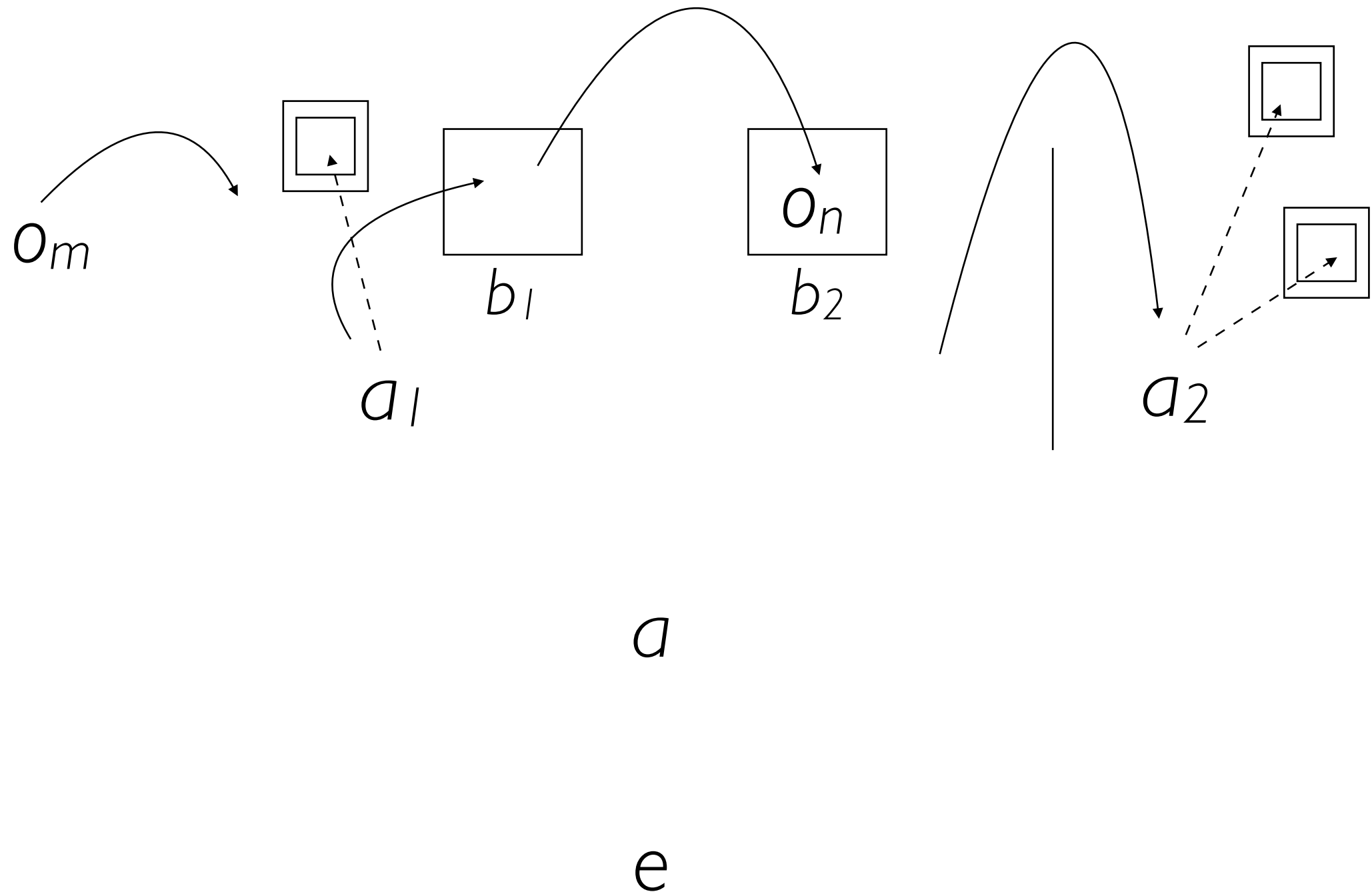
Framework for FBT^I_4

(nine timepoints)



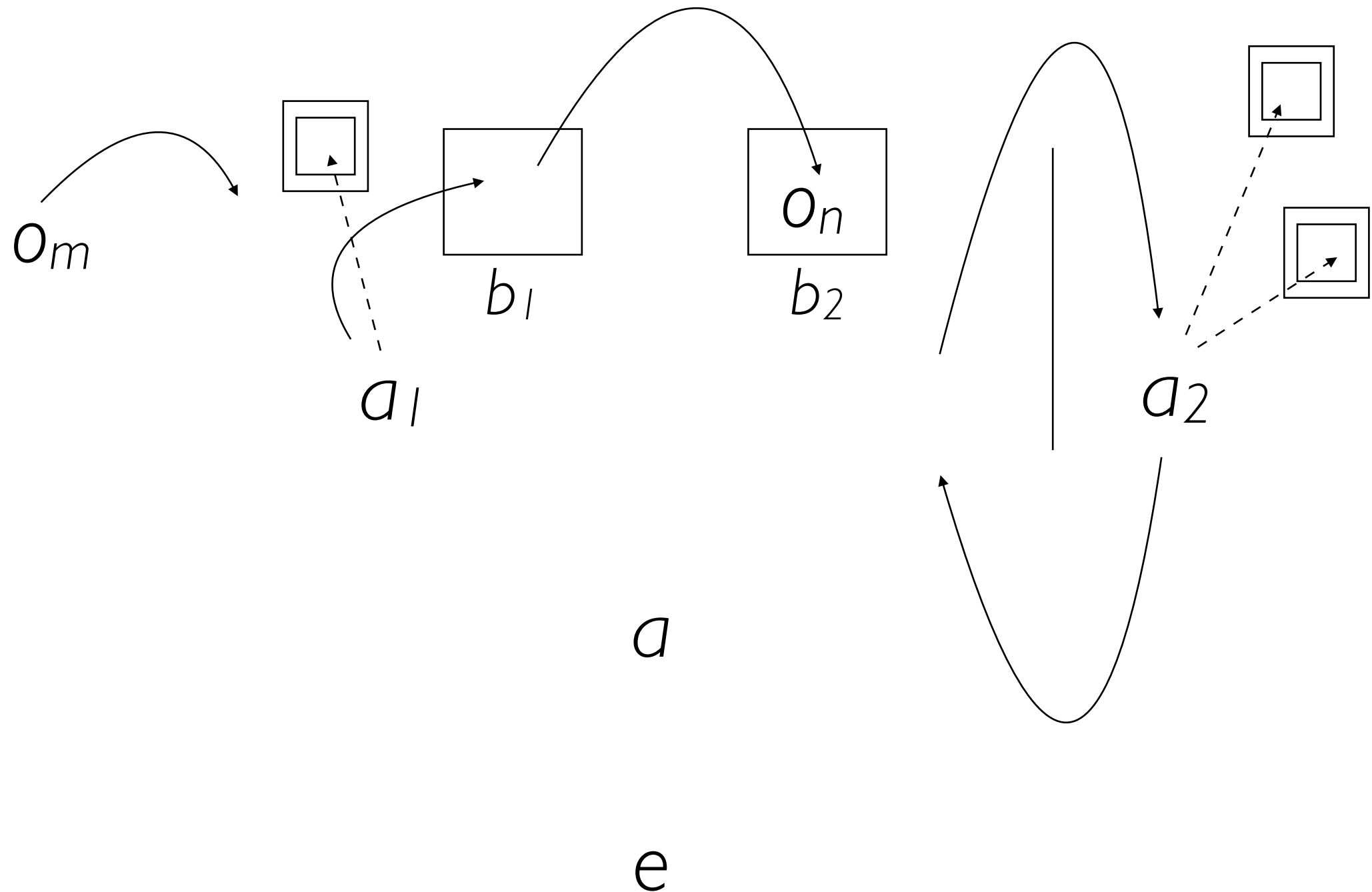
Framework for FBT^I_4

(nine timepoints)



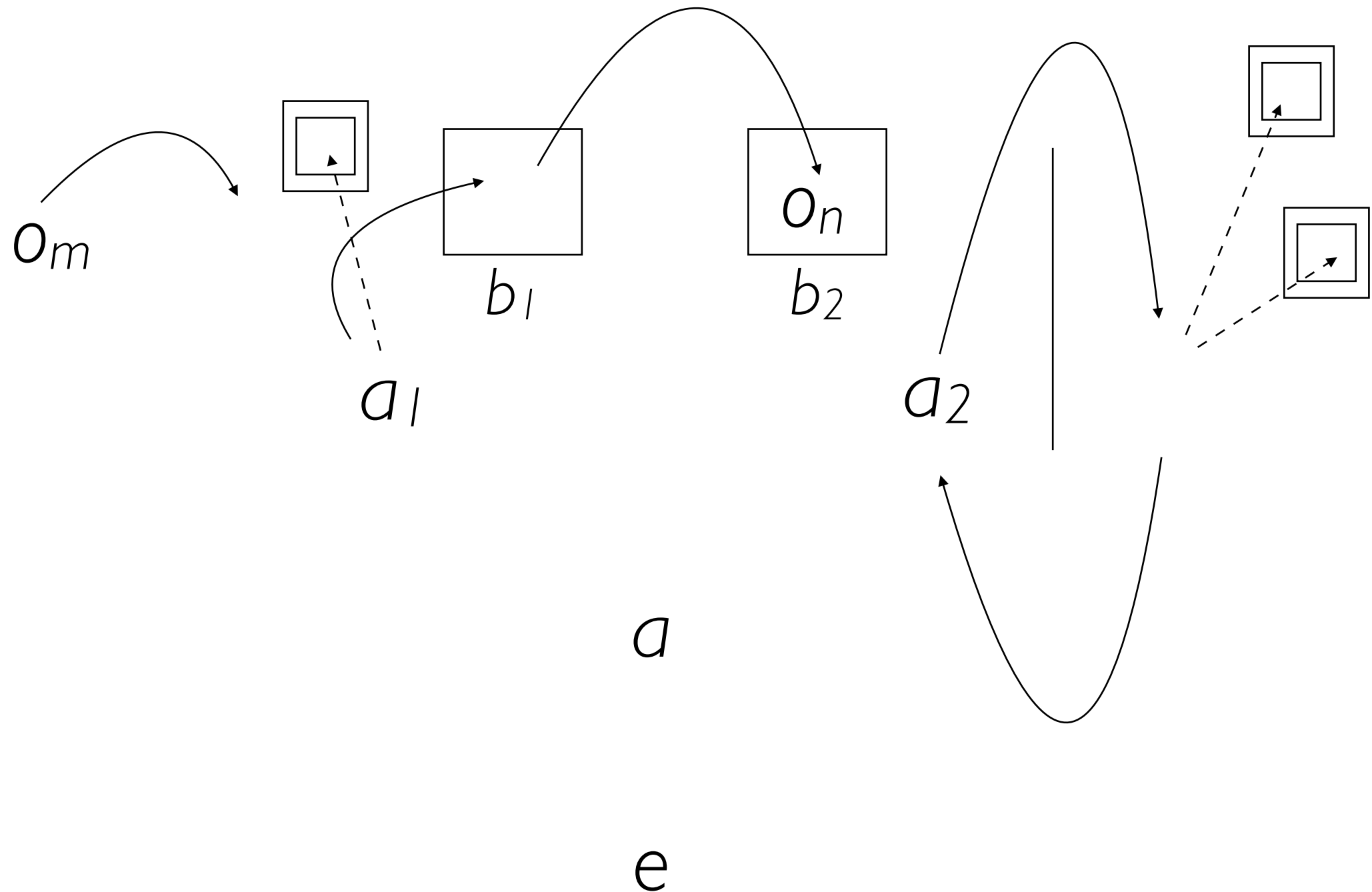
Framework for FBT^I_4

(nine timepoints)



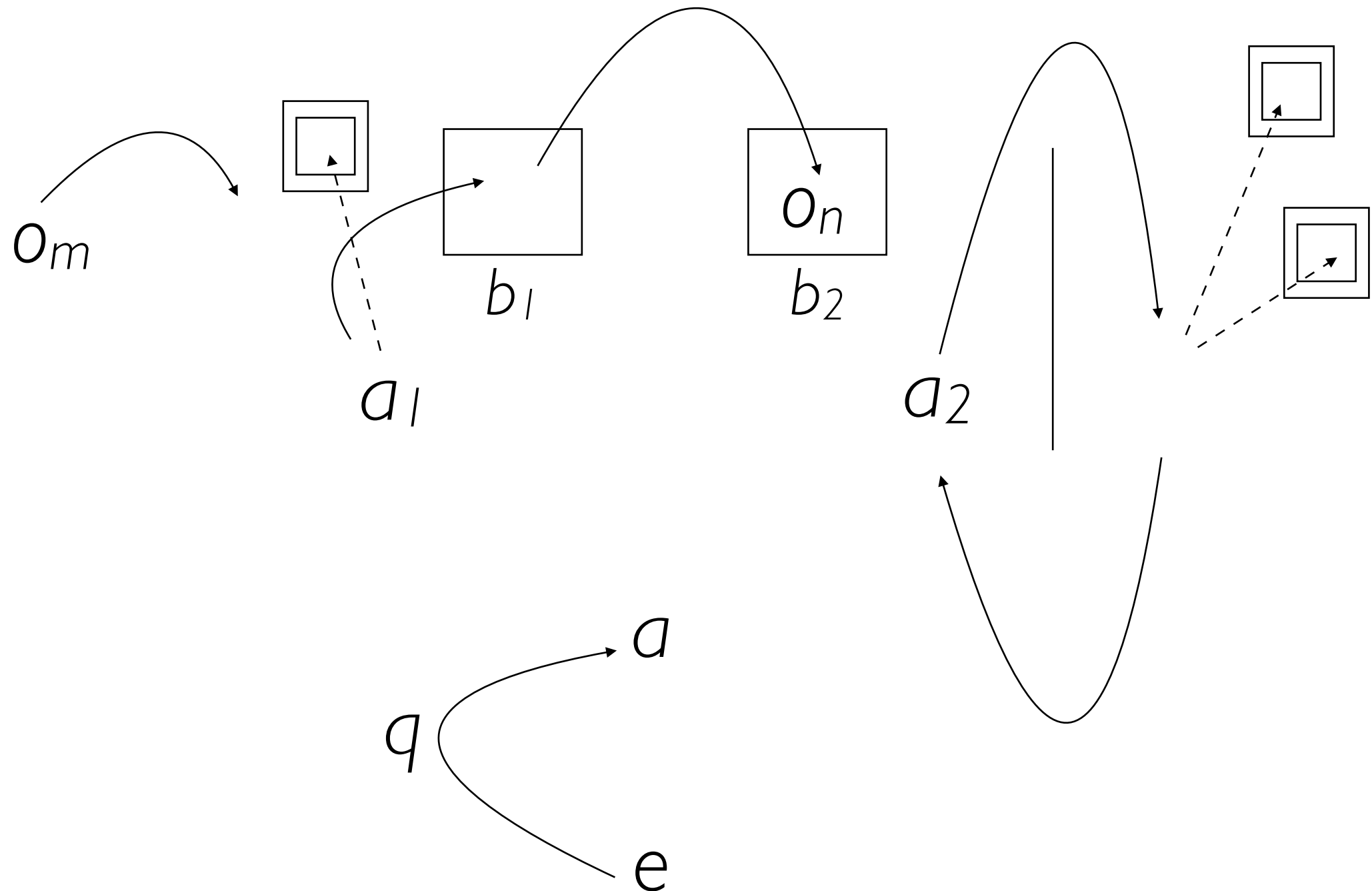
Framework for FBT^I_4

(nine timepoints)



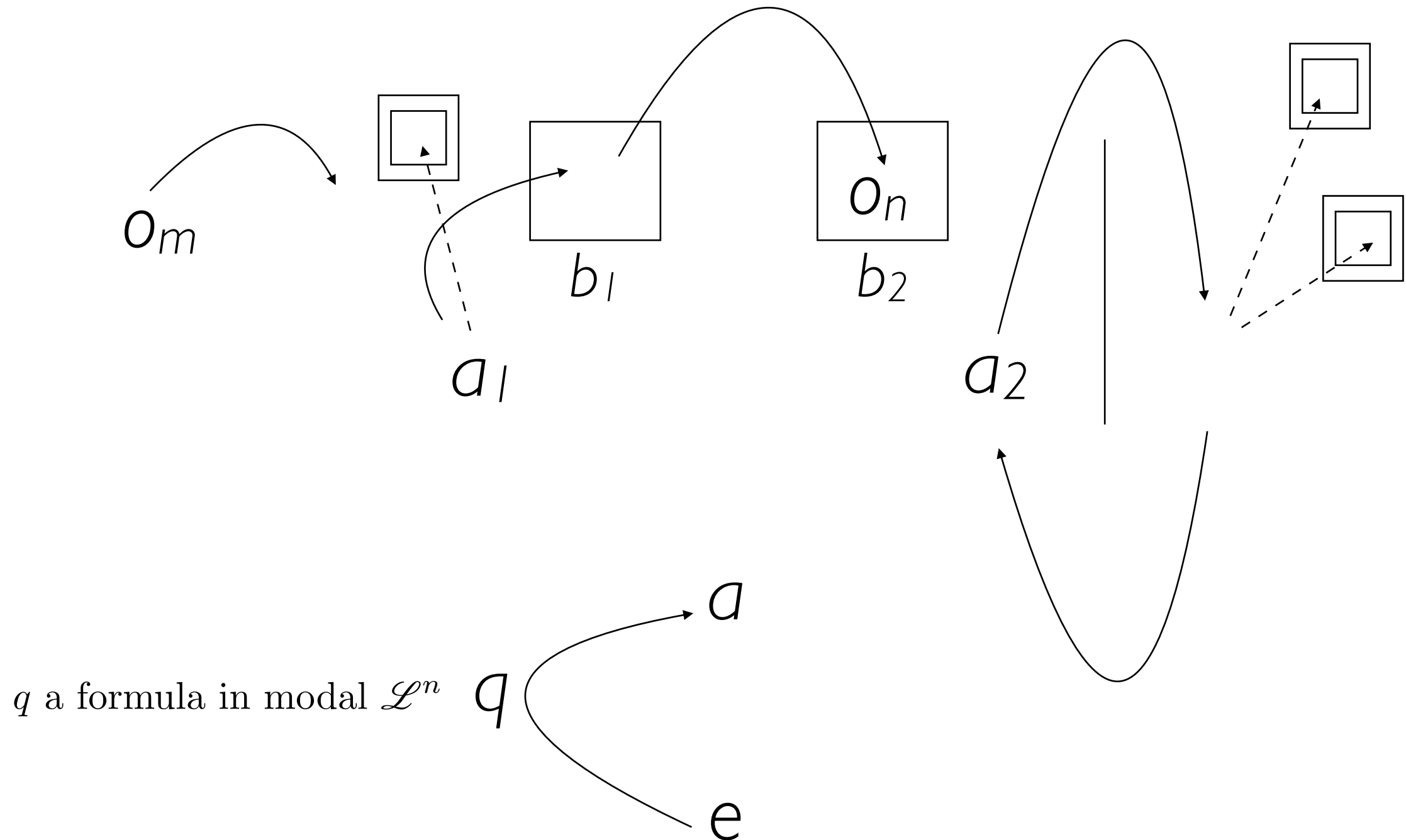
Framework for FBT¹₄

(nine timepoints)

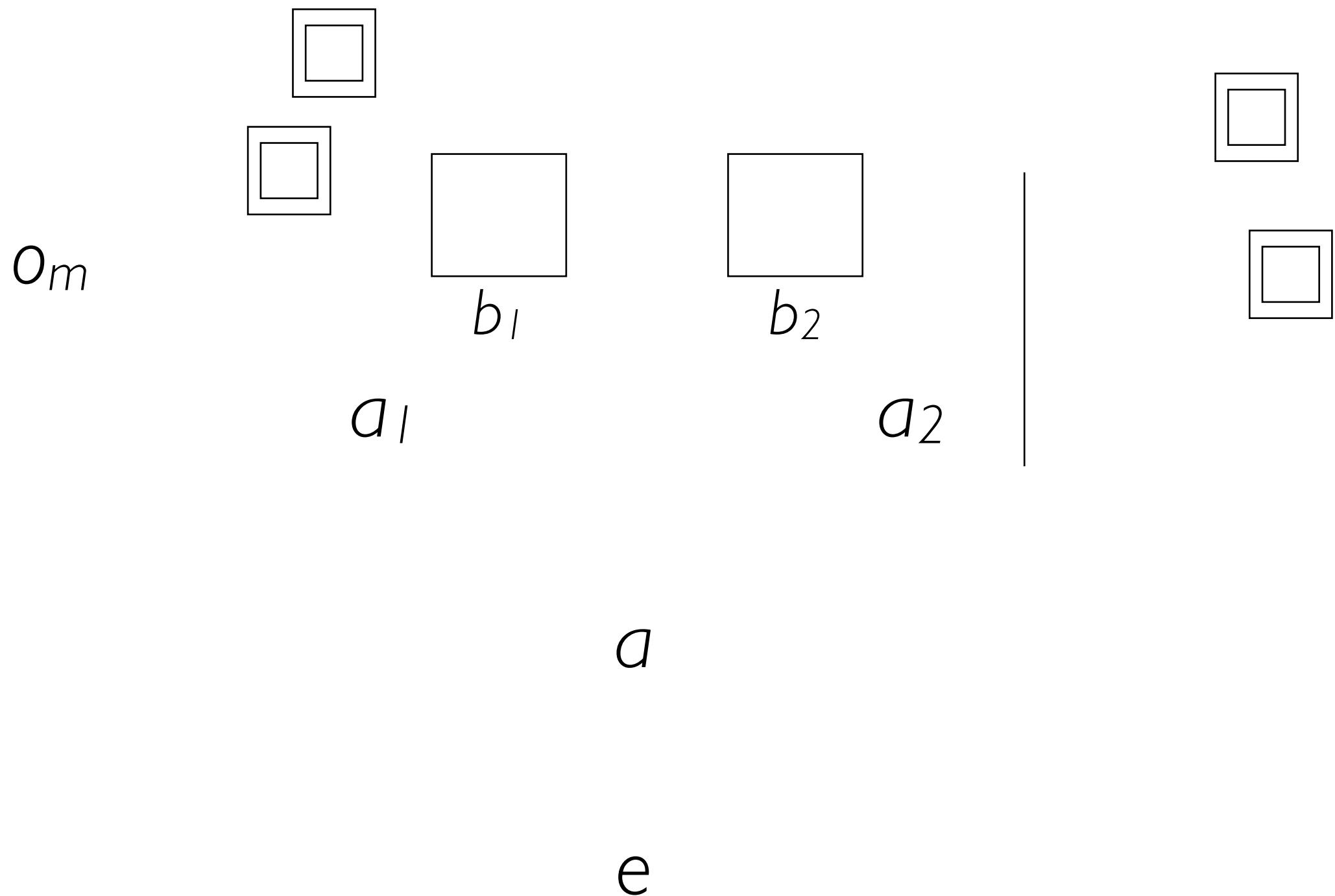


Framework for FBT^I_4

(nine timepoints)

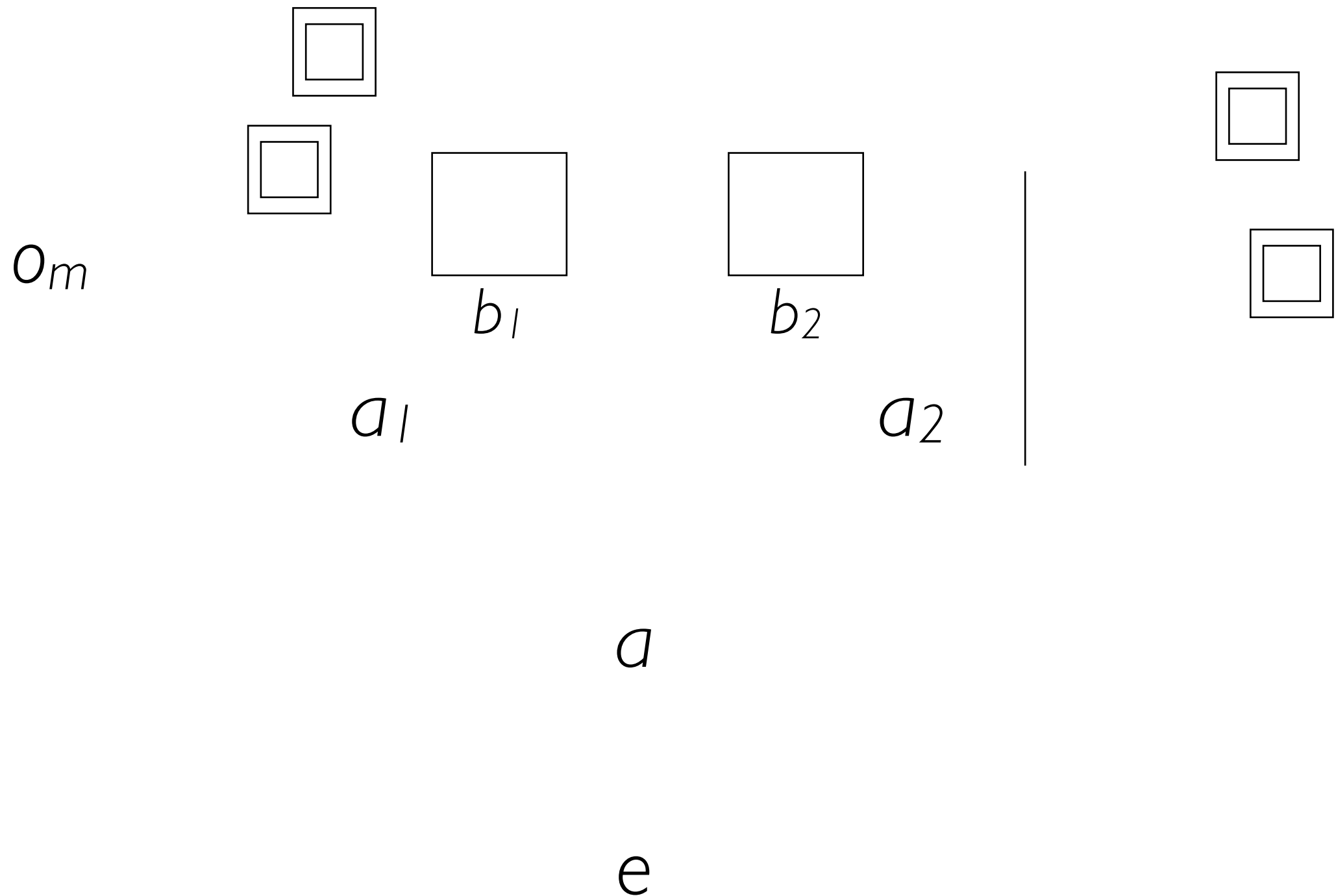


Framework for FBT^I_5



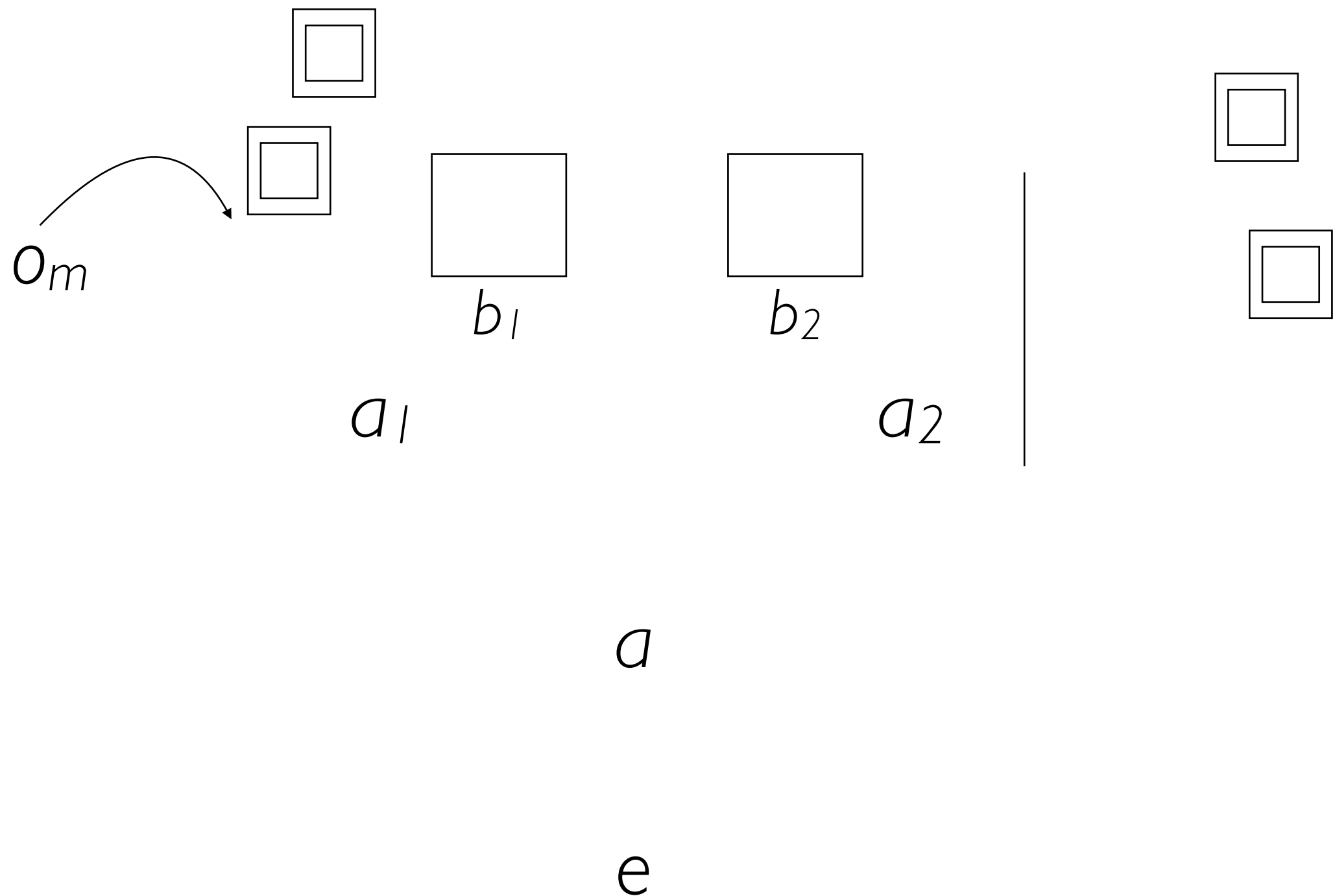
Framework for FBT^I_5

(ten timepoints)



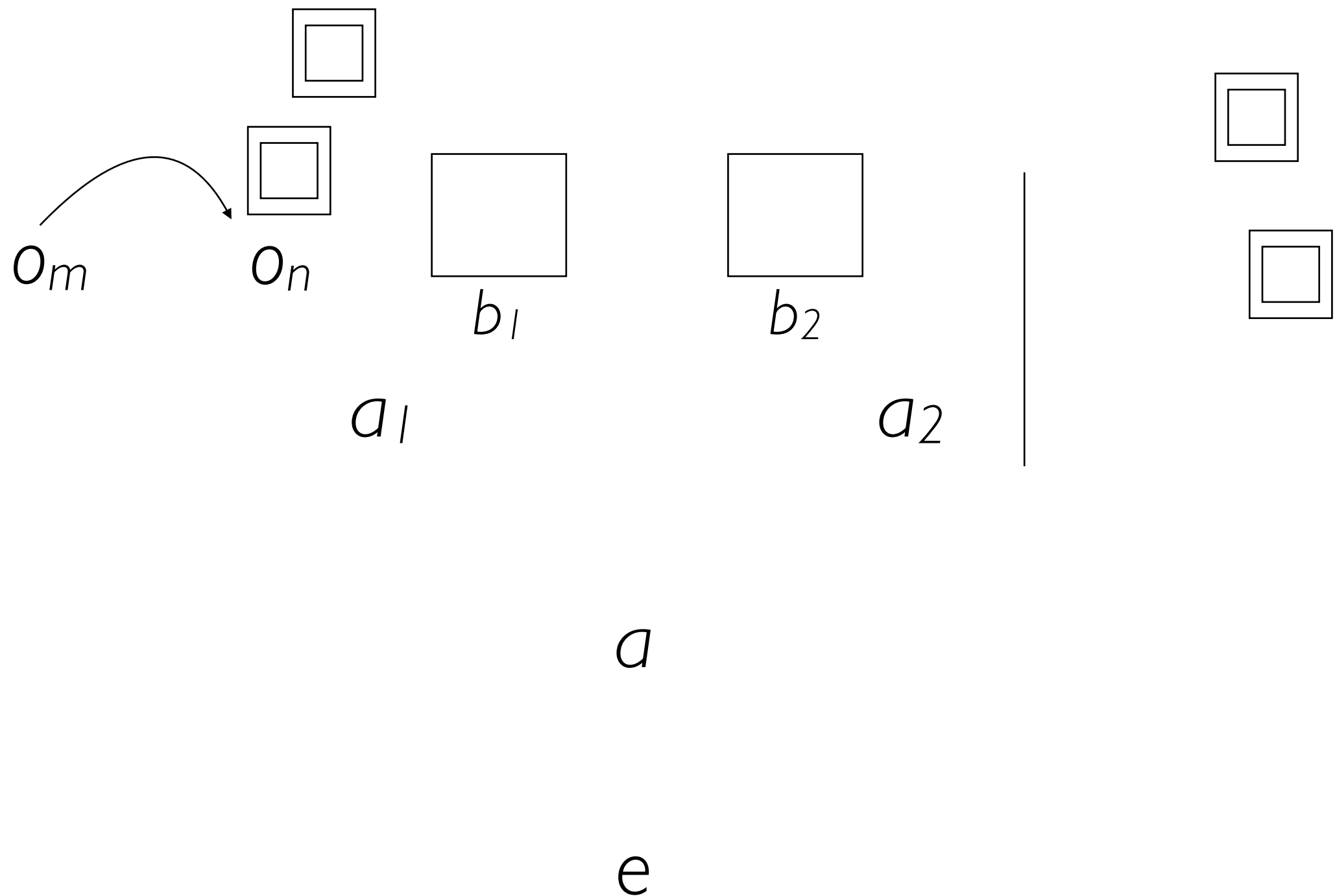
Framework for FBT^I_5

(ten timepoints)



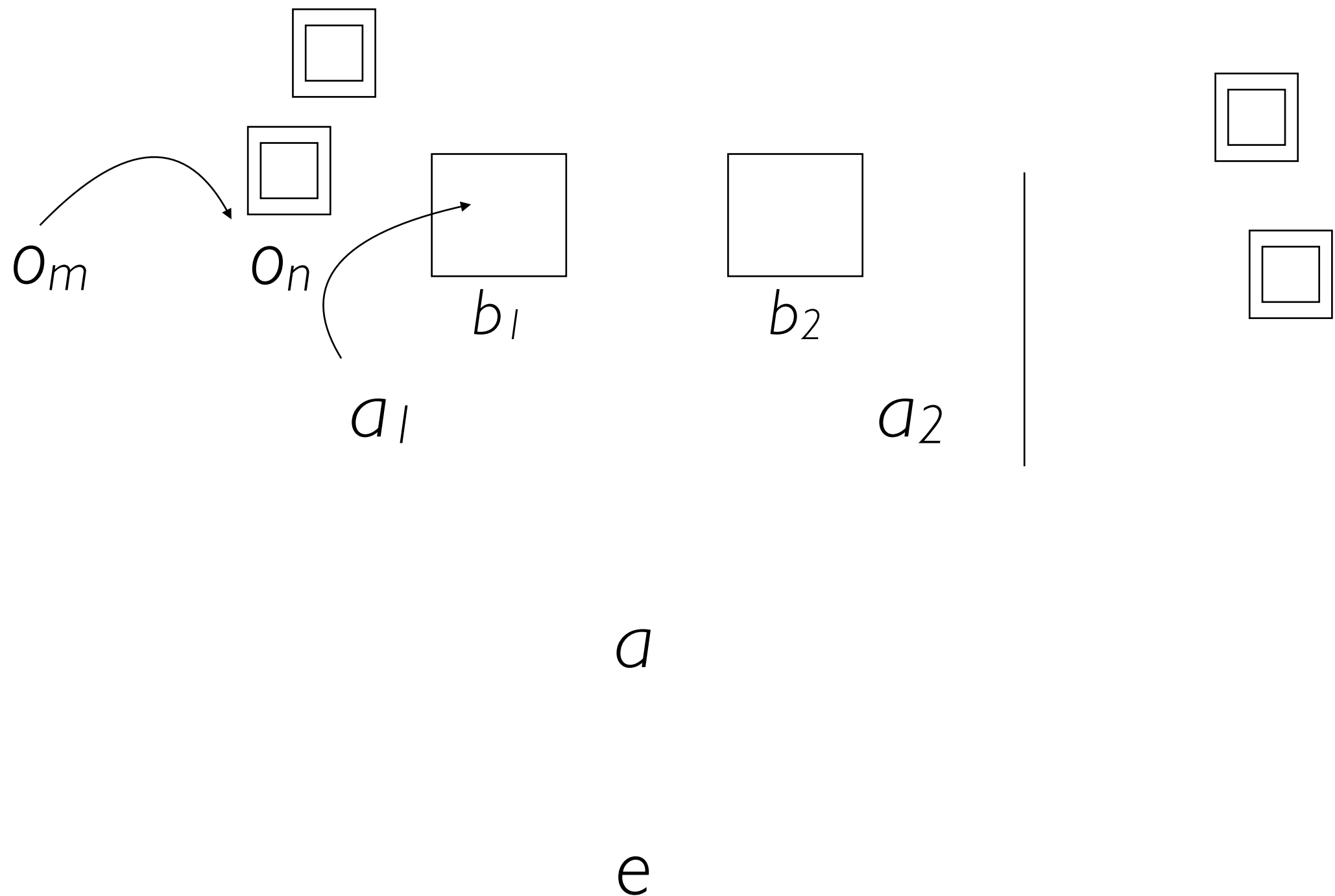
Framework for FBT^I_5

(ten timepoints)



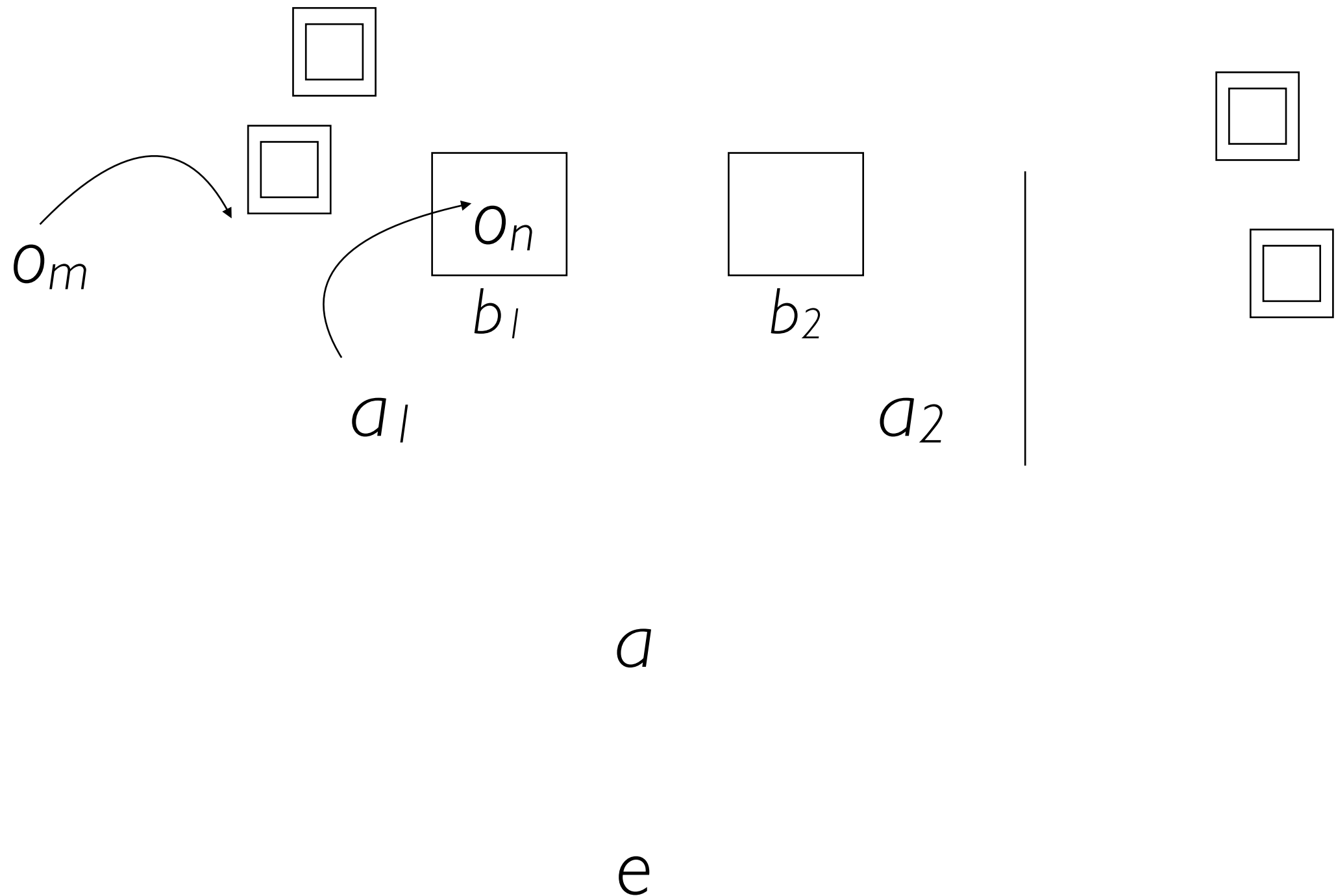
Framework for FBT^I_5

(ten timepoints)



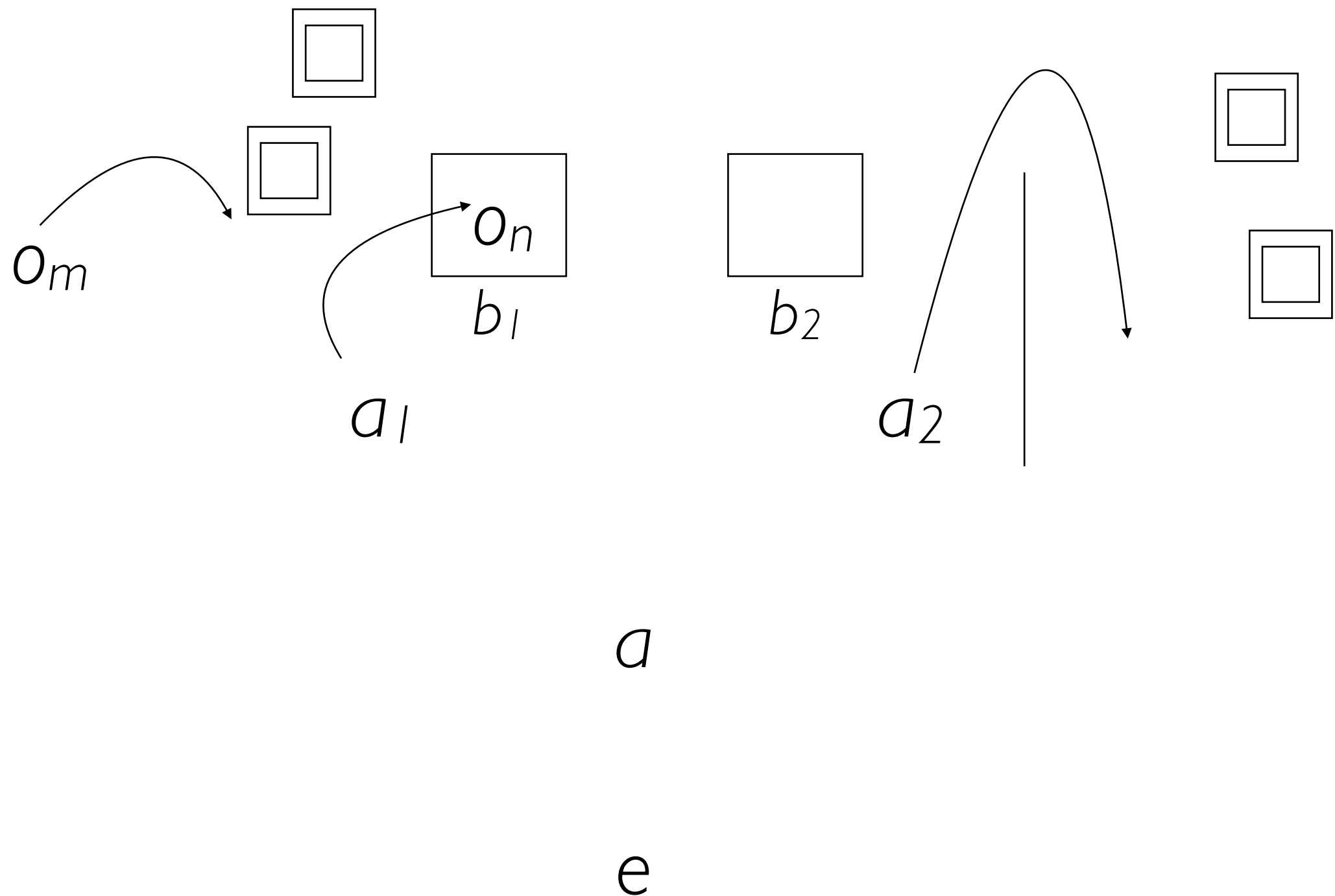
Framework for FBT^I_5

(ten timepoints)



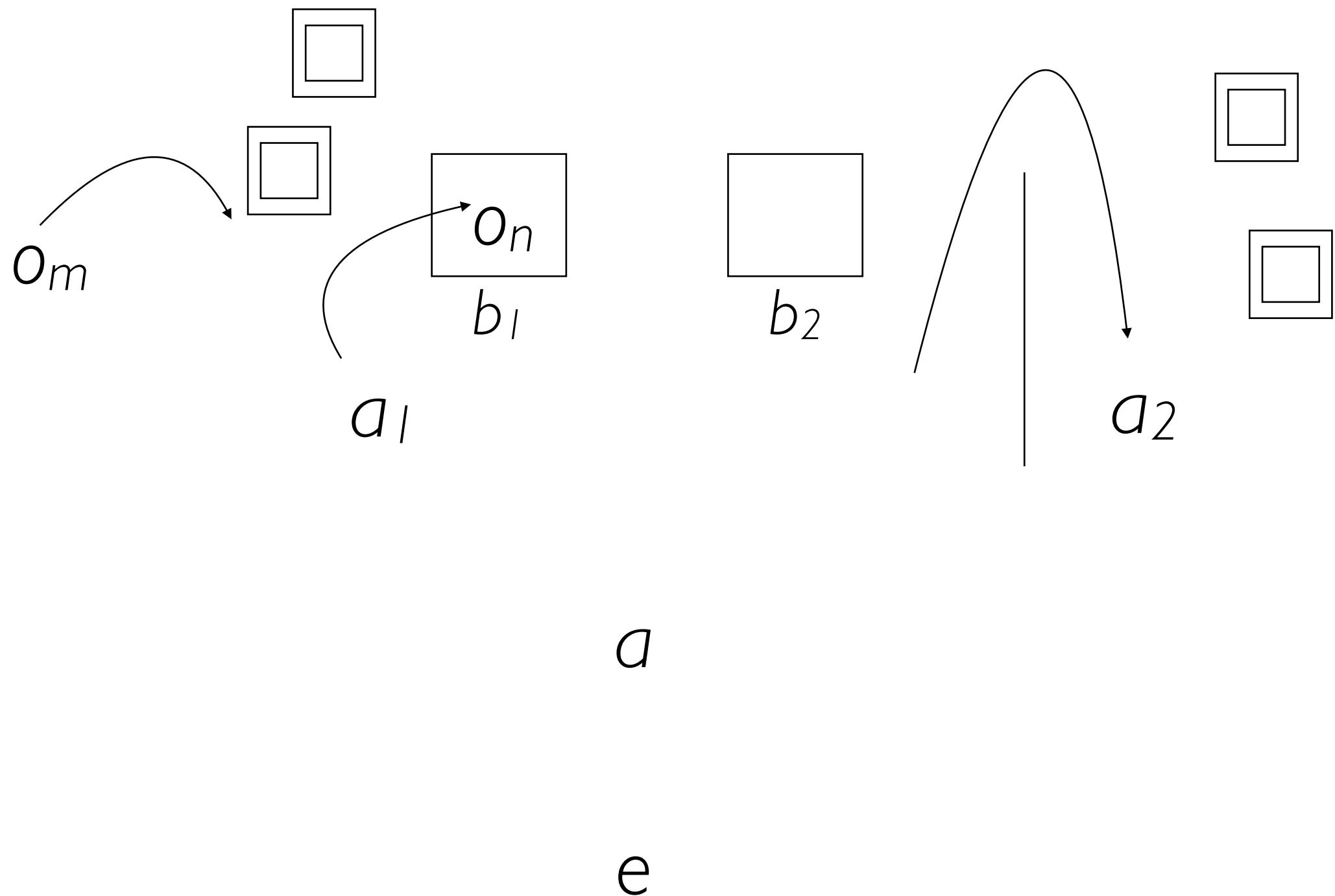
Framework for FBT^I_5

(ten timepoints)



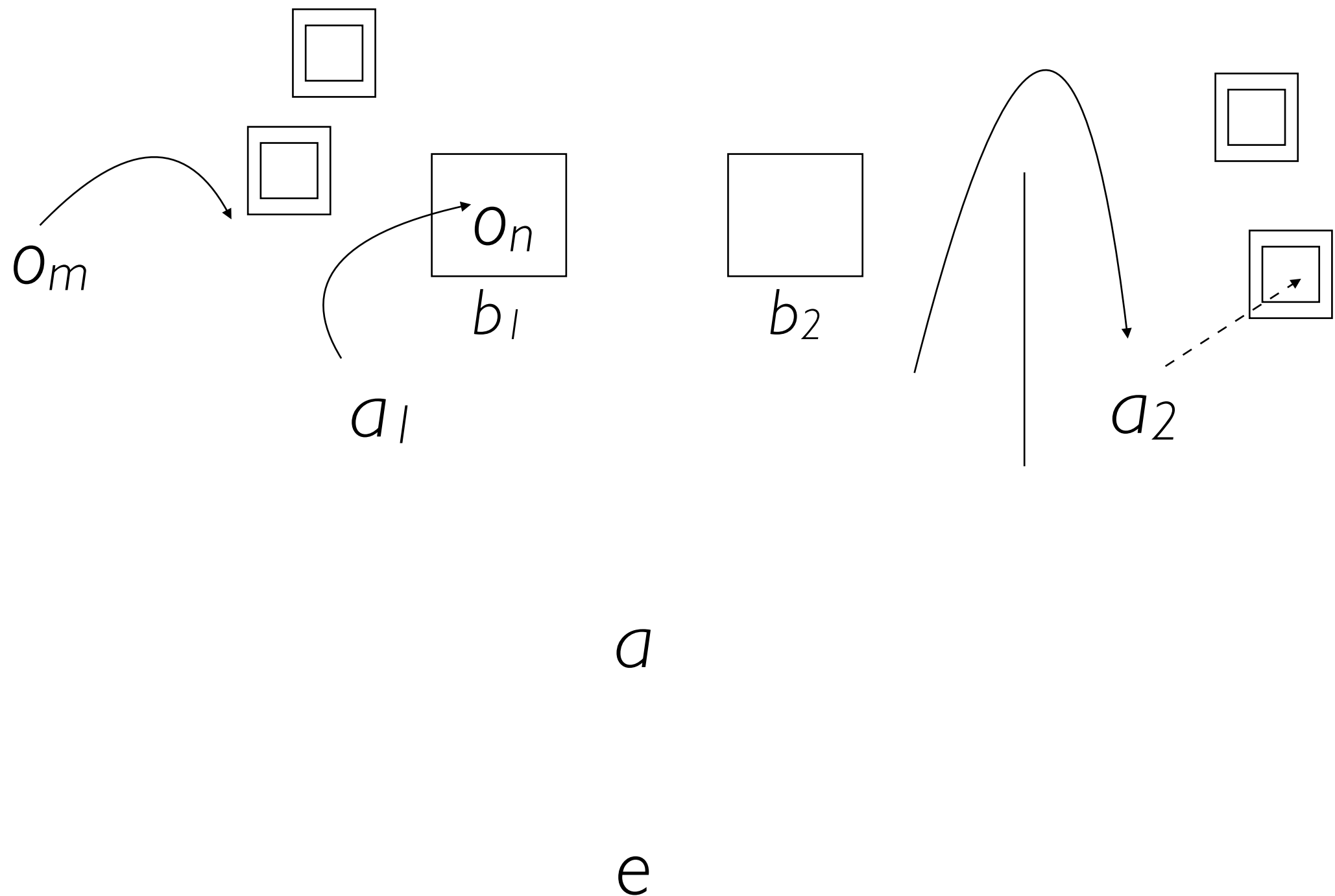
Framework for FBT^I_5

(ten timepoints)



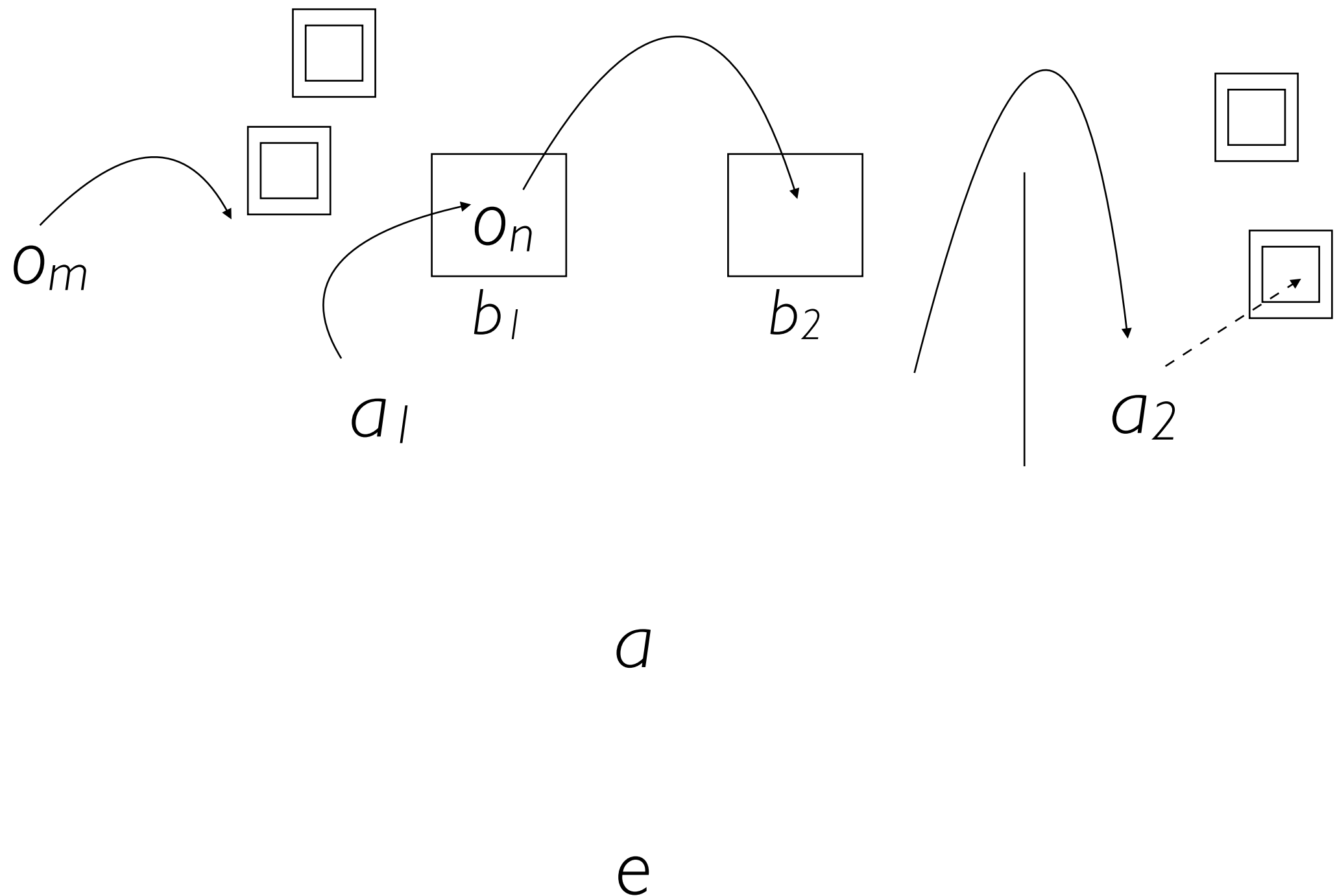
Framework for FBT^I_5

(ten timepoints)



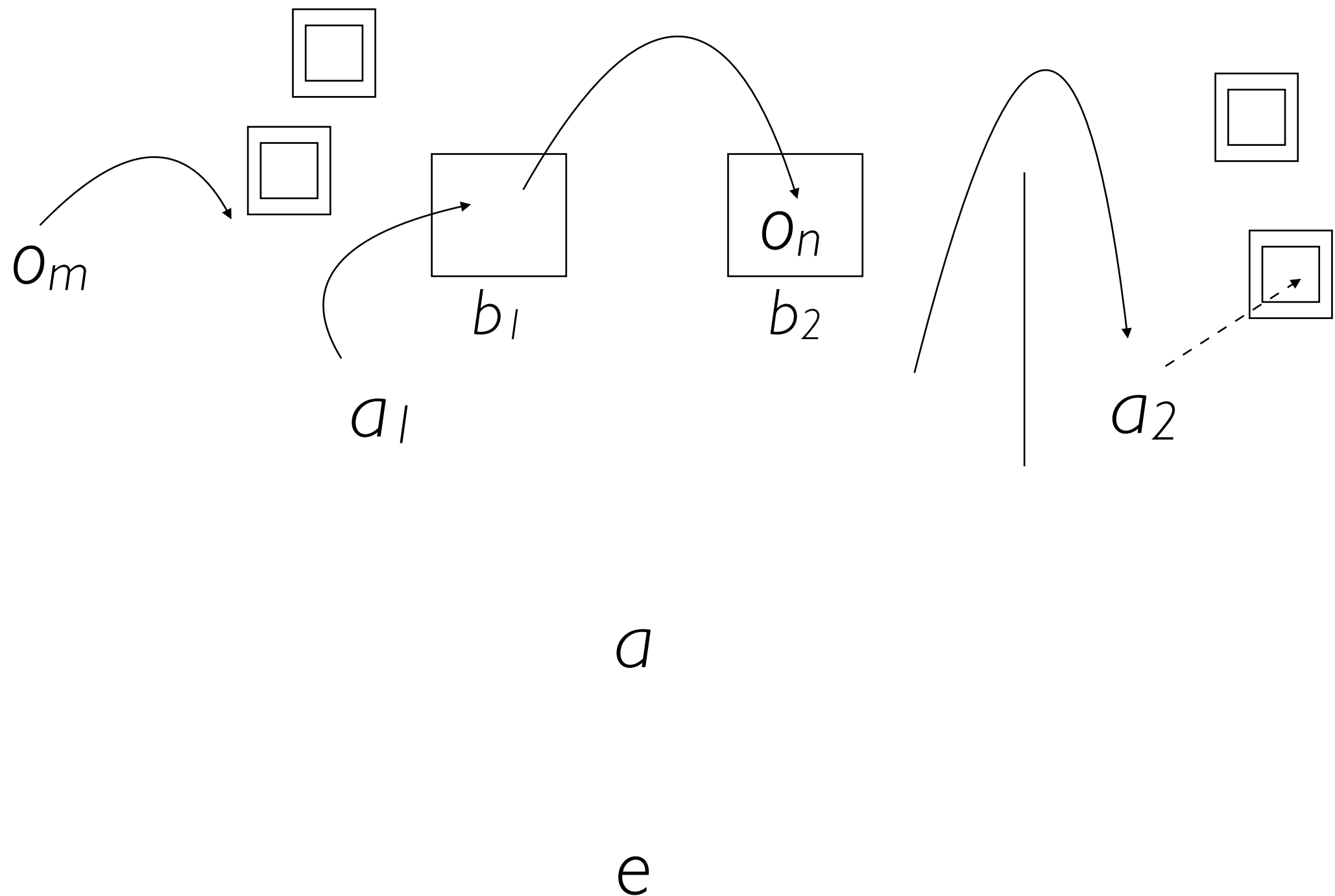
Framework for FBT^I₅

(ten timepoints)



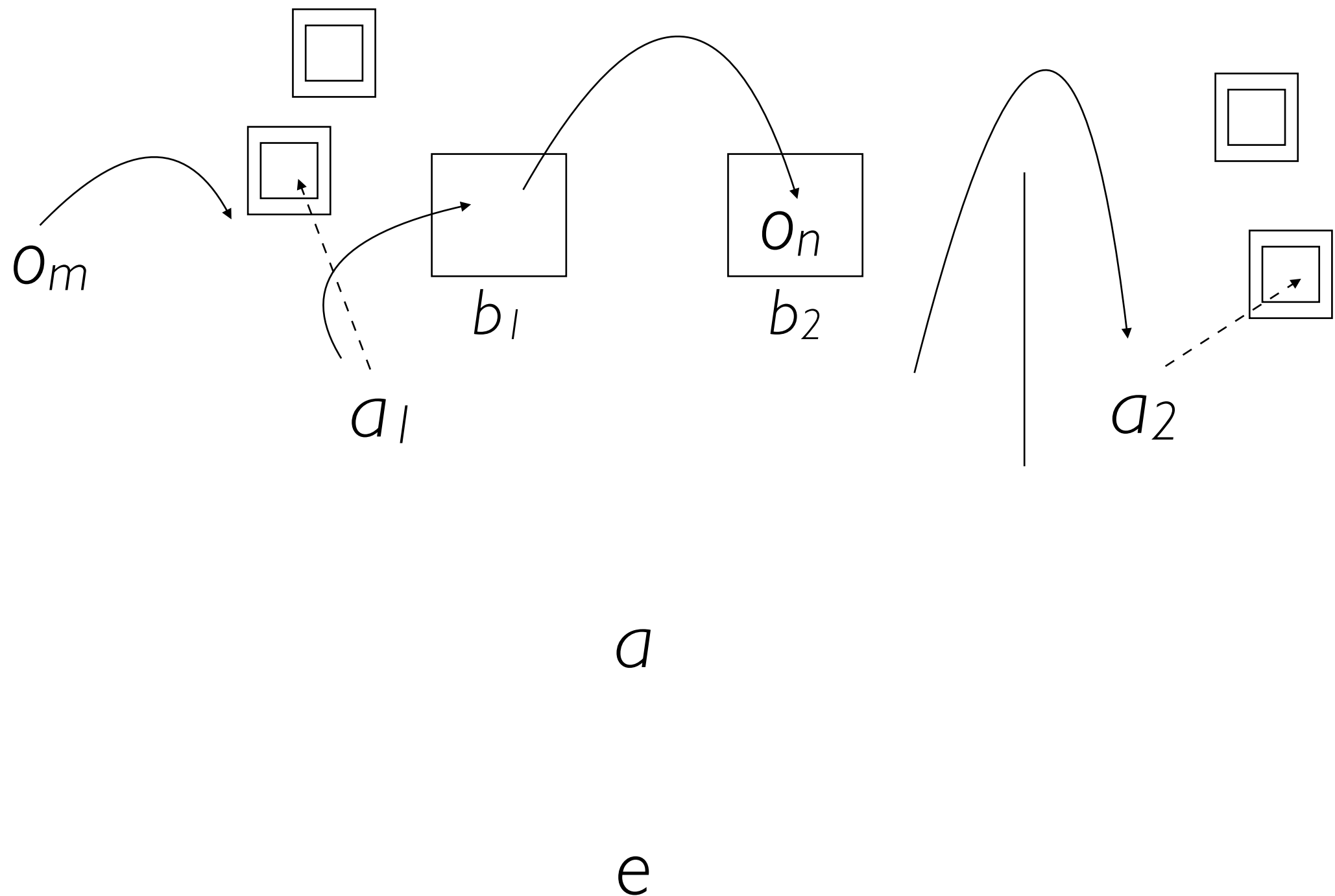
Framework for FBT^I_5

(ten timepoints)



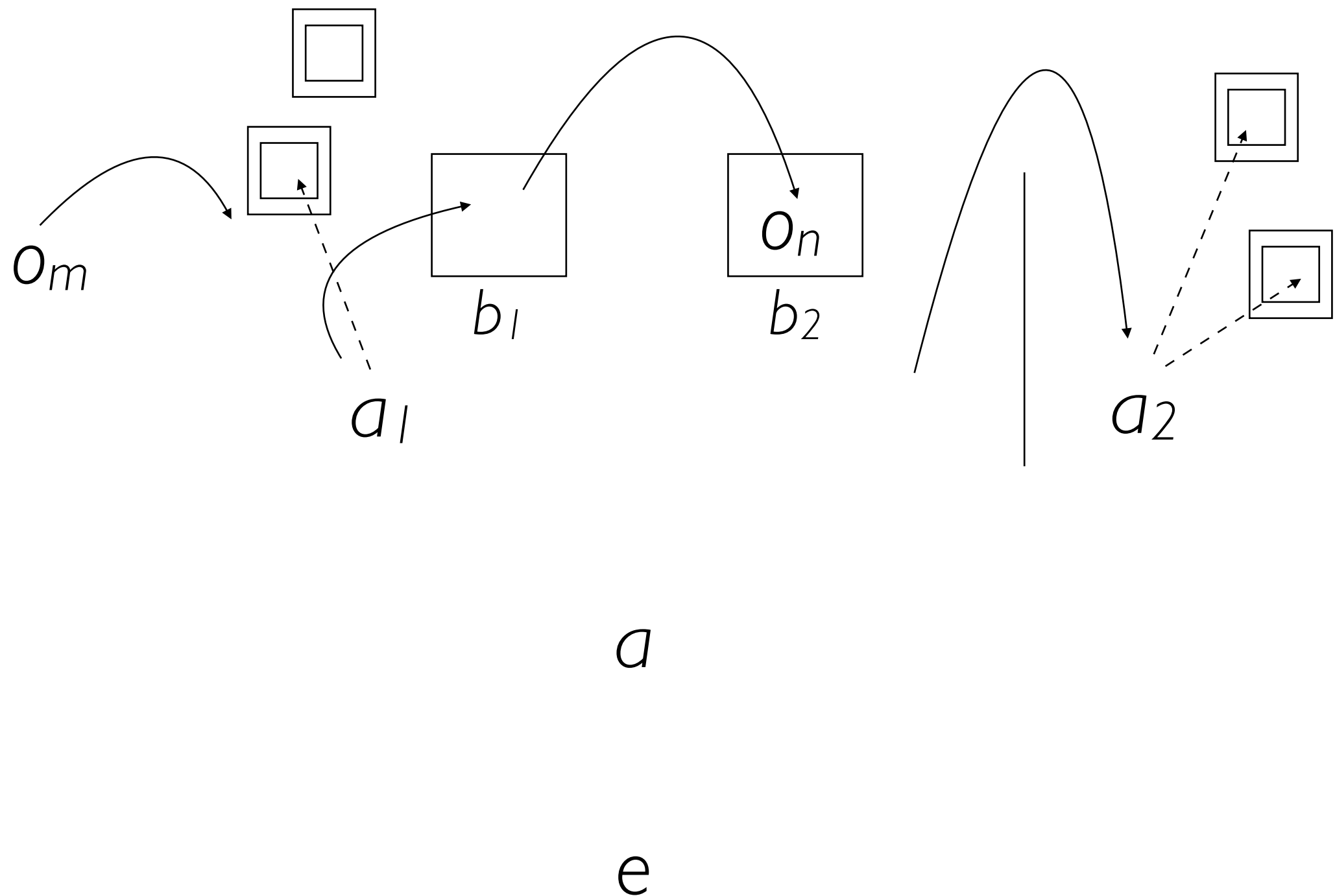
Framework for FBT^I_5

(ten timepoints)



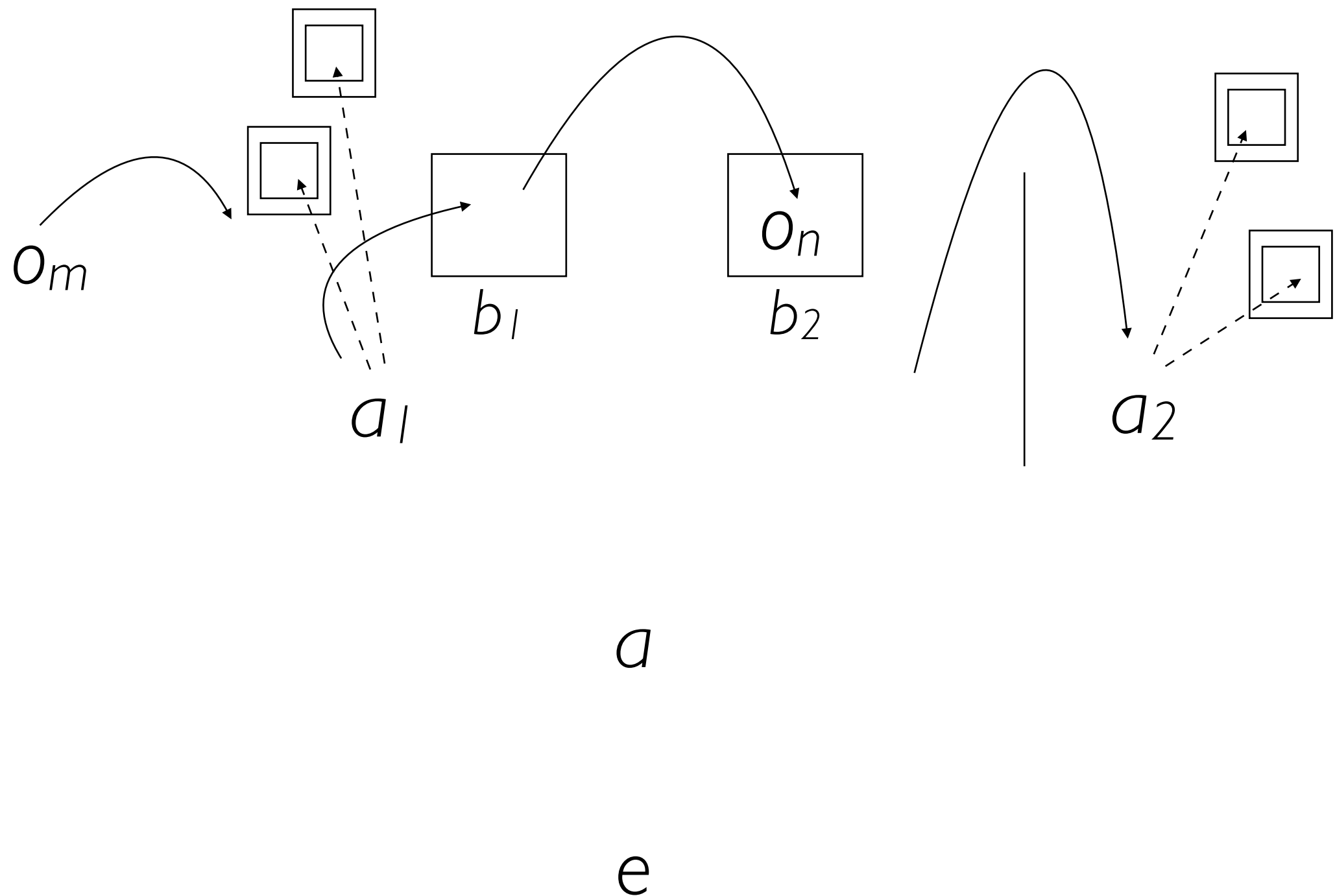
Framework for FBT^I_5

(ten timepoints)



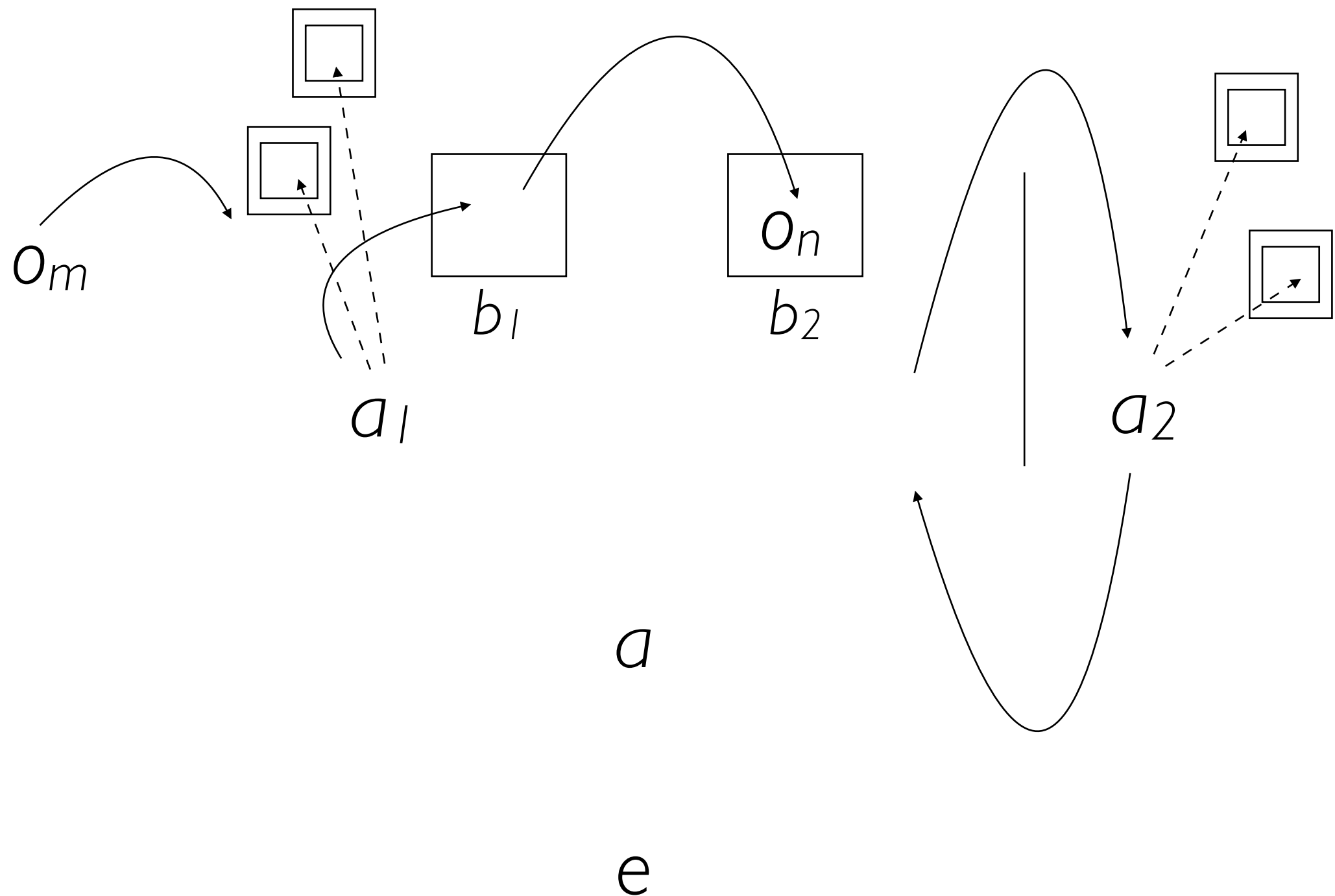
Framework for FBT^I_5

(ten timepoints)



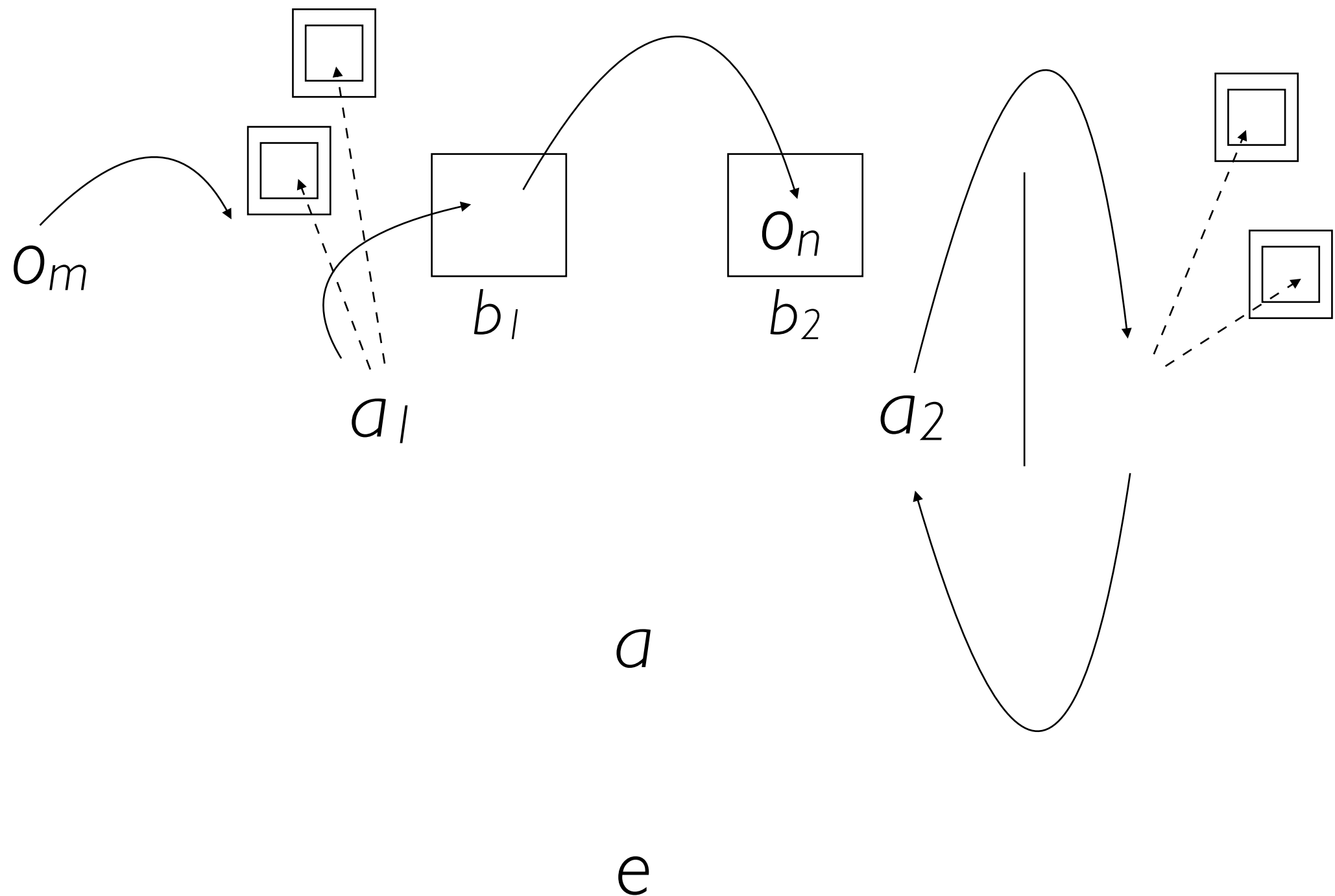
Framework for FBT^I_5

(ten timepoints)



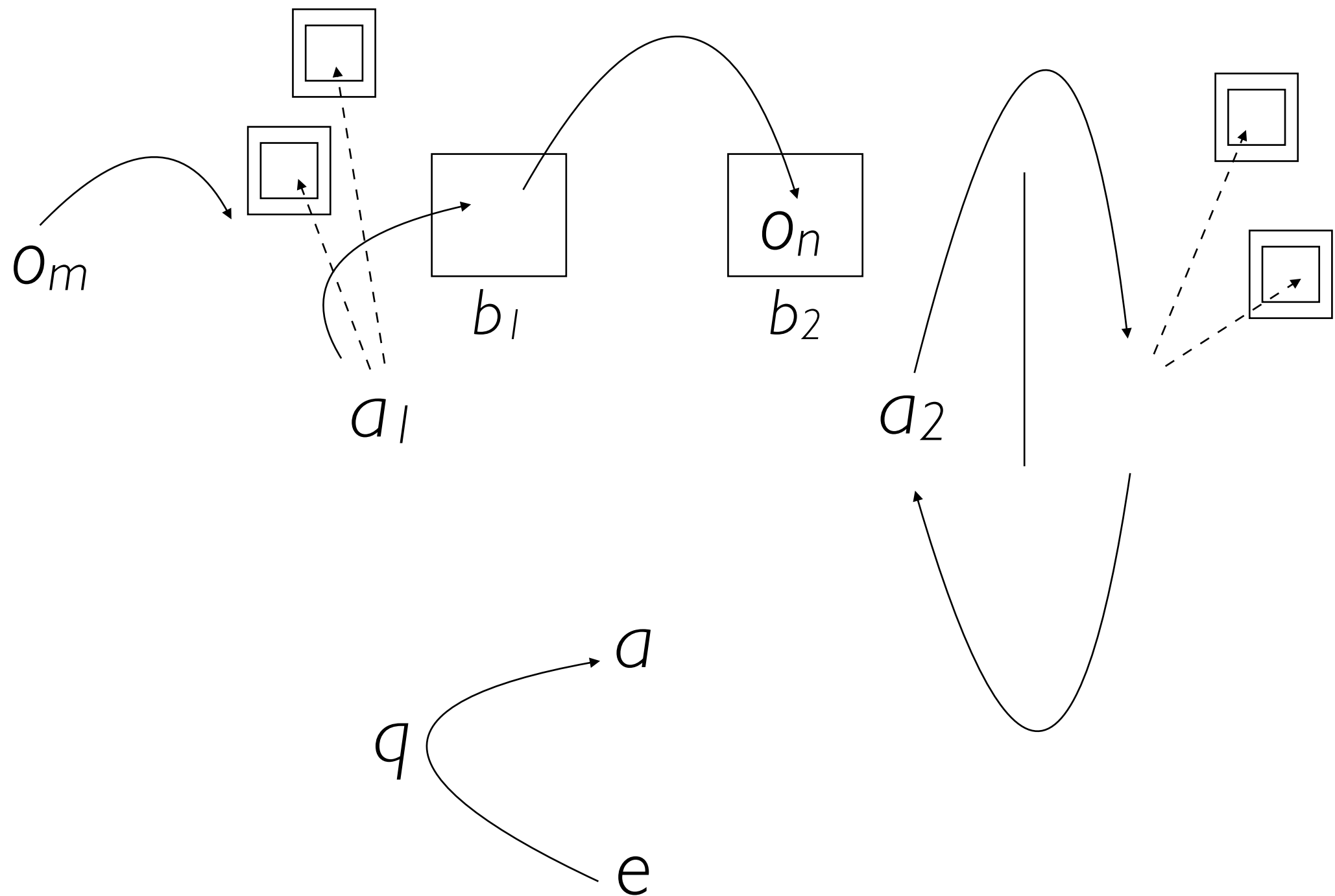
Framework for FBT^I_5

(ten timepoints)



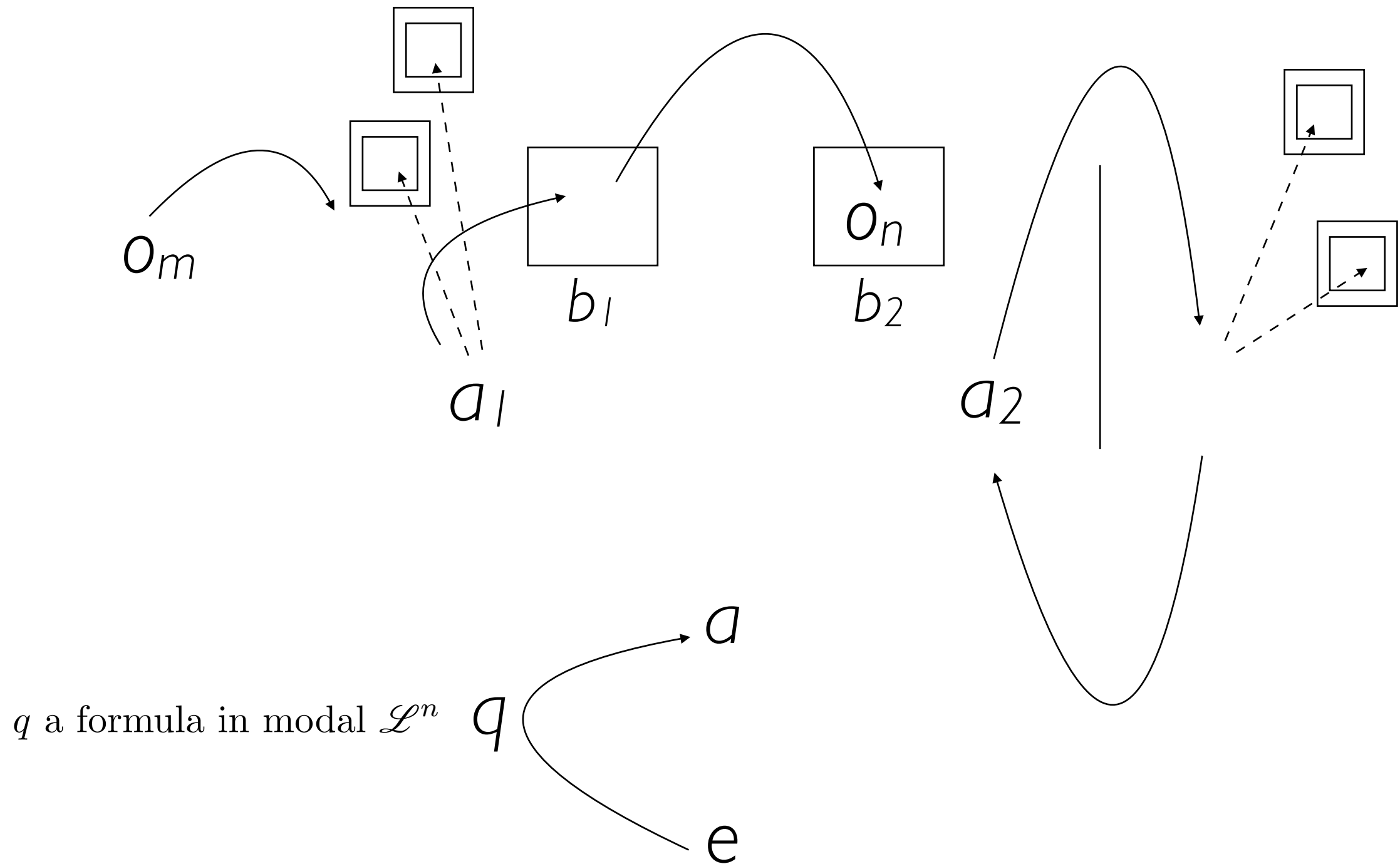
Framework for FBT¹₅

(ten timepoints)



Framework for FBT^I_5

(ten timepoints)



Humans Can Succeed

Neurobiologically normal, nurtured, educated, and sufficiently motivated humans can correctly answer any relevant query q for the infinite progression, and prove that their answer is correct. For the obvious subclass of queries (the form of which appear in the box below), they can prove and exploit the following lemma.

Lemma: Suppose $\text{FBT}_k, k \in \mathbb{Z}^+$, holds; (i.e. that level k of FBT holds). Then, if k is even, $\mathbf{B}_2\mathbf{B}_1 \dots \mathbf{B}_2 \vdash$, where there are $k + 1$ iterated \mathbf{B}_i operators; otherwise $\mathbf{B}_1\mathbf{B}_2 \dots \mathbf{B}_1\mathbf{B}_2 \vdash$, where there again there are $k + 1$ iterated \mathbf{B}_i operators.

Passing to Probing Mastery of the Specific Subclass

Experimenter to a : “At level k ,
from which box will a_2 attempt to
retrieve the objects o_n ? Prove it!”

Theoretical Machine Success on Infinite FBT!

Theorem: $\forall q \in \mathcal{CC}, \mathfrak{M}$ can correctly answer and justify q .
I.e., \mathfrak{M} can pass FBT_ω .

Ok, so this logic machine exists in the *mathematical* universe; but does there exist an *implemented* machine with this power?

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Ok, so this logic machine exists in the *mathematical* universe; but does there exist an *implemented* machine with this power?

Simulation Courtesy of ...

ShadowProver!



Level I

```
:name      "Level 1: False Belief Task "

:description "Agent a1 puts an object o into b1 in plain view of a2.
Agent a2 then leaves, and in the absence of a2, a1 moves o
from b1 into b2 ; this movement isn't perceived by a2 . Agent
a2 now returns, and a is asked by the experimenter e: "If a2
desires to retrieve o, which box will a2 look in?" If younger
than four or five, a will reply "In b " (which of course fails 2
the task); after this age subjects respond with the correct "In b1."

Level1 Belief: a1 believes a2 believes o is in b1.
"

:date      "Monday July 22, 2019"

:assumptions {
    :P1 (Perceives! a1 t1 (Perceives! a2 t1 (holds (In o b1) t1)))

    :P2 (Believes! a1 t2 (Believes! a2 t2 (not (exists [?e] (terminates ?e (In o b1))))))

    :P3 (holds (In o b1) t1)

    :C1 (Common! t0 (forall [?f ?t2 ?t2]
        (if (and (not (exists [?e] (terminates ?e ?f))) (holds ?f ?t1) (< ?t1 ?t2))
            (holds ?f ?t2))))

    :C2 (Common! t0 (and (< t1 t2) (< t2 t3) (< t1 t3)))
}

:goal      (Believes! a1 t3 (Believes! a2 t3 (holds (In o b1) t3))))}
```

Level 2

```
{:name      "Level 2: False Belief Task "

:description "Agent a1 puts an object o into b1 in plain view of a2.
Agent a2 then leaves, and in the absence of a2, a1 moves o
from b1 into b2 ; this movement isn't perceived by a2 . Agent
a2 now returns, and a is asked by the experimenter e: "If a2
desires to retrieve o, which box will a2 look in?" If younger
than four or five, a will reply "In b " (which of course fails 2
the task); after this age subjects respond with the correct "In b1."

Level2 Belief: a2 believes a1 believes a2 believes o is in b1.
"

:date      "Monday July 22, 2019"

:assumptions {

    :P1 (Perceives! a2 t1 (Perceives! a1 t1 (Perceives! a2 t1 (holds (In o b1) t1))))

    :P2 (Believes! a2 t2 (Believes! a1 t2 (Believes! a2 t2 (not (exists [?e] (terminates ?e (In o b1)))))))

    :P3 (holds (In o b1) t1)

    :C1 (Common! t0
        (forall [?f ?t2 ?t2]
            (if (and (not (exists [?e] (terminates ?e ?f))) (holds ?f ?t1) (< ?t1 ?t2))
                (holds ?f ?t2))))

    :C2 (Common! t0 (and (< t1 t2) (< t2 t3) (< t1 t3)))

:goal      (Believes! a2 t3 (Believes! a1 t3 (Believes! a2 t3 (holds (In o b1) t3))))}
```


Level 3

```
{:name "Level 3: False Belief Task "

:description "Agent a1 puts an object o into b1 in plain view of a2.
Agent a2 then leaves, and in the absence of a2, a1 moves o
from b1 into b2 ; this movement isn't perceived by a2 . Agent
a2 now returns, and a is asked by the experimenter e: "If a2
desires to retrieve o, which box will a2 look in?" If younger
than four or five, a will reply "In b " (which of course fails 2
the task); after this age subjects respond with the correct "In b1."

Level3 Belief: a2 believes a1 believes a2 believes o is in b1.
"

:date "Monday July 22, 2019"

:assumptions {

    :P1 (Perceives! a1 t1 (Perceives! a2 t1 (Perceives! a1 t1 (Perceives! a2 t1 (holds (In o b1) t1))))))
    :P2 (Believes! a1 t2 (Believes! a2 t2 (Believes! a1 t2 (Believes! a2 t2 (not (exists [?e] (terminates ?e (In o b1))))))))))
    :P3 (holds (In o b1) t1)

    :C1 (Common! t0
        (forall [?f ?t2 ?t2]
            (if (and (not (exists [?e] (terminates ?e ?f))) (holds ?f ?t1) (< ?t1 ?t2))
                (holds ?f ?t2))))

    :C2 (Common! t0 (and (< t1 t2) (< t2 t3) (< t1 t3)))}

:goal (Believes! a1 t3 (Believes! a2 t3 (Believes! a1 t3 (Believes! a2 t3 (holds (In o b1) t3))))))}
```

Level 4

```
{:name      "Level 4: False Belief Task "

:description "Agent a1 puts an object o into b1 in plain view of a2.
Agent a2 then leaves, and in the absence of a2, a1 moves o
from b1 into b2 ; this movement isn't perceived by a2 . Agent
a2 now returns, and a is asked by the experimenter e: "If a2
desires to retrieve o, which box will a2 look in?" If younger
than four or five, a will reply "In b " (which of course fails 2
the task); after this age subjects respond with the correct "In b1."

Level4 Belief: a2 believes a1 believes a2 believes a1 believes a2 believes o is in b1.
"

:date      "Monday July 22, 2019"

:assumptions {

    :P1 (Perceives! a2 t1 (Perceives! a1 t1 (Perceives! a2 t1 (Perceives! a1 t1 (Perceives! a2 t1 (holds (In o b1) t1))))))
    :P2 (Believes! a2 t2 (Believes! a1 t2 (Believes! a2 t2 (Believes! a1 t2 (Believes! a2 t2 (not (exists [?e] (terminates ?e (In o b1))))))))))
    :P3 (holds (In o b1) t1)

    :C1 (Common! t0
        (forall [?f ?t2 ?t2]
            (if (and (not (exists [?e] (terminates ?e ?f))) (holds ?f ?t1) (< ?t1 ?t2))
                (holds ?f ?t2))))

    :C2 (Common! t0 (and (< t1 t2) (< t2 t3) (< t1 t3)))}

:goal      (Believes! a2 t3 (Believes! a1 t3 (Believes! a2 t3 (Believes! a1 t3 (Believes! a2 t3 (holds (In o b1) t3))))))}
```

Level 5

```
{:name "Level 5: False Belief Task "

:description "Agent a1 puts an object o into b1 in plain view of a2.
Agent a2 then leaves, and in the absence of a2, a1 moves o
from b1 into b2 ; this movement isn't perceived by a2 . Agent
a2 now returns, and a is asked by the experimenter e: "If a2
desires to retrieve o, which box will a2 look in?" If younger
than four or five, a will reply "In b " (which of course fails 2
the task); after this age subjects respond with the correct "In b1."

Level5 Belief: a1 believes a2 believes a1 believes a2 believes a1 believes a2 believes o is in b1.
"

:date "Monday July 22, 2019"

:assumptions {

  :P1 (Perceives! a1 t1 (Perceives! a2 t1 (Perceives! a1 t1 (Perceives! a2 t1 (Perceives! a1 t1 (Perceives! a2 t1 (holds (In o b1) t1)))))))
  :P2 (Believes! a1 t2 (Believes! a2 t2 (Believes! a1 t2 (Believes! a2 t2 (Believes! a1 t2 (Believes! a2 t2 (not (exists [?e] (terminates ?e (In o b1))))))))))
  :P3 (holds (In o b1) t1)

  :C1 (Common! t0
    (forall [?f ?t2 ?t2]
      (if (and (not (exists [?e] (terminates ?e ?f))) (holds ?f ?t1) (< ?t1 ?t2))
        (holds ?f ?t2))))

  :C2 (Common! t0 (and (< t1 t2) (< t2 t3) (< t1 t3)))}

:goal (Believes! a1 t3 (Believes! a2 t3 (Believes! a1 t3 (Believes! a2 t3 (Believes! a1 t3 (Believes! a2 t3 (holds (In o b1) t3))))))})
```

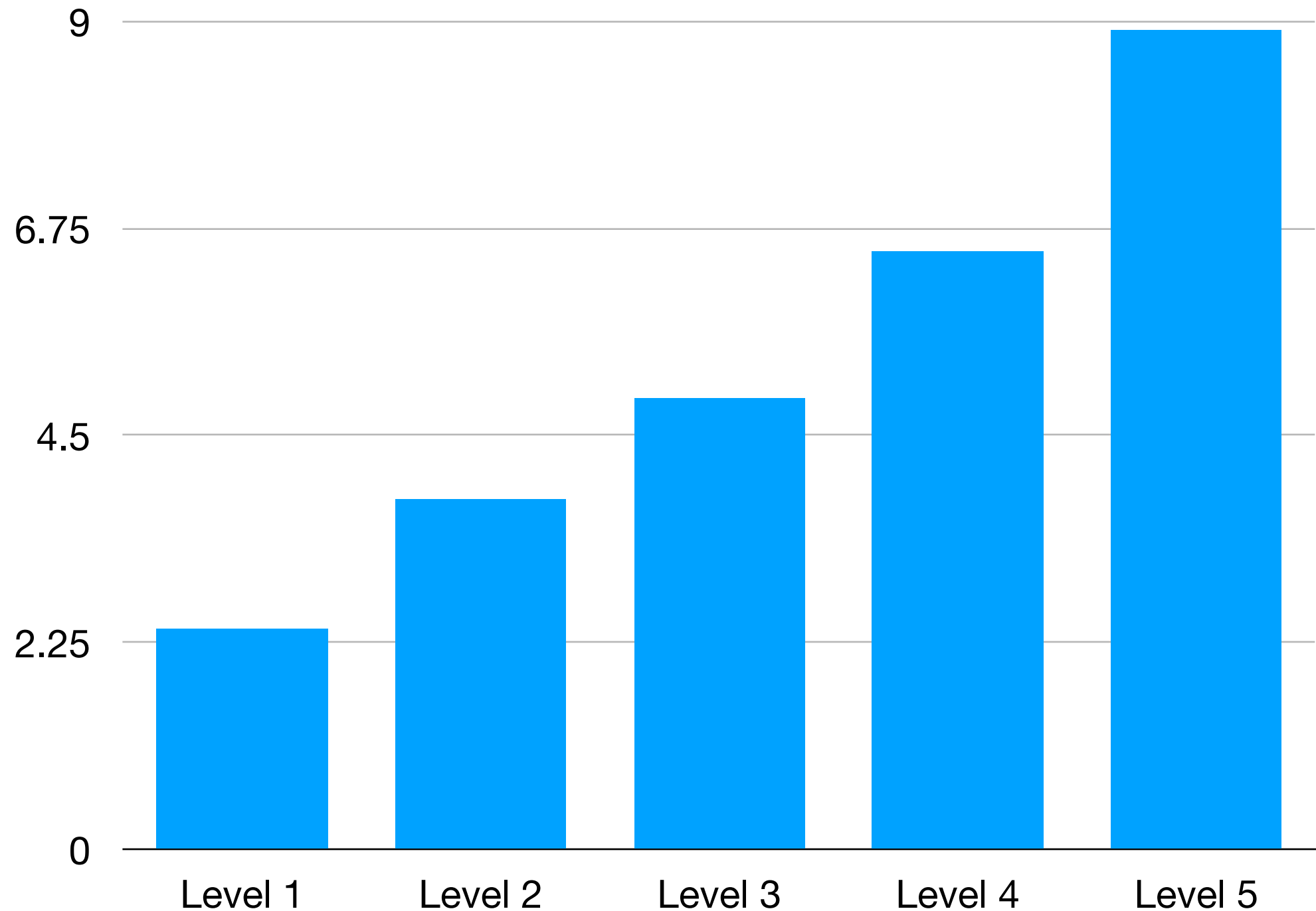
[illegible]

```

    :goal
      (Common! t0
        (forall [?f ?t2 ?t2]
          (if (and (not (exists [?e] (terminates ?e ?f)))
            (holds ?f ?t1) (< ?t1 ?t2))
            (Believes! a1 t3 (Believes! a2 t3 (holds (In o b1) t3))))))}
      )
    )
  )

```

Time (in seconds) to Prove



Simulation of Level 5 in Real Time

```
/Library/Java/JavaVirtualMachines/jdk1.8.0_131.jdk/Contents/Home/bin/java ...  
objc[16653]: Class JavaLaunchHelper is implemented in both /Library/Java/JavaVirtualMachines/jdk1.8.0_131.jdk/Contents/Home/bin/java (0x102a2d4c0) and /Library/Java/JavaVirtualMachines/jdk1.8.0_131.jdk/Contents/Home/jre/lib/libinstrument.dylib (0x102ab94e0)  
----- Level 5 -----
```

Simulation of Level 5 in Real Time

```
/Library/Java/JavaVirtualMachines/jdk1.8.0_131.jdk/Contents/Home/bin/java ...  
objc[16653]: Class JavaLaunchHelper is implemented in both /Library/Java/JavaVirtualMachines/jdk1.8.0_131.jdk/Contents/Home/bin/java (0x102a2d4c0) and /Library/Java/JavaVirtualMachines/jdk1.8.0_131.jdk/Contents/Home/jre/lib/libinstrument.dylib (0x102ab94e0)  
----- Level 5 -----
```


Encapsulation

Slate - K.slt

K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $K \vdash \checkmark \infty \Box$	T. $\Box\varphi \rightarrow \varphi$ $K \vdash \times \infty \Box$	4. $\Box\varphi \rightarrow \Box\Box\varphi$ $K \vdash \times \infty \Box$	5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $K \vdash \times \infty \Box$
---	---	---	---

Encapsulation

The image shows two overlapping Slate editor windows. The top window is titled 'Slate - K.slt' and contains four boxes, each with a modal logic formula and its status in the K system. The bottom window is titled 'Slate - T.slt' and contains the same four boxes, but with their status in the T system.

Formula	K System Status	T System Status
K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$	$K \vdash \checkmark \infty \Box$	$M \vdash \checkmark \infty \Box$
T. $\Box\varphi \rightarrow \varphi$	$K \vdash \times \infty \Box$	$M \vdash \checkmark \infty \Box$
4. $\Box\varphi \rightarrow \Box\Box\varphi$	$K \vdash \times \infty \Box$	$M \vdash \times \infty \Box$
5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$	$K \vdash \times \infty \Box$	$M \vdash \times \infty \Box$

Encapsulation

The image displays three overlapping windows, each representing a different modal logic system. Each window contains four boxes, each with a formula and its status in that system (derivable or not, with a checkmark or cross and a symbol).

Slate - K.slt

- K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$
 $K \vdash \checkmark \infty \Box$
- T. $\Box\varphi \rightarrow \varphi$
 $K \vdash \times \infty \Box$
- 4. $\Box\varphi \rightarrow \Box\Box\varphi$
 $K \vdash \times \infty \Box$
- 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$
 $K \vdash \times \infty \Box$

Slate - T.slt

- K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$
 $M \vdash \checkmark \infty \Box$
- T. $\Box\varphi \rightarrow \varphi$
 $M \vdash \checkmark \infty \Box$
- 4. $\Box\varphi \rightarrow \Box\Box\varphi$
 $M \vdash \times \infty \Box$
- 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$
 $M \vdash \times \infty \Box$

Slate - D.slt

- K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$
 $D \vdash \checkmark \infty \Box$
- T. $\Box\varphi \rightarrow \varphi$
 $D \vdash \times \infty \Box$
- D. $\Box\varphi \rightarrow \Diamond\varphi$
 $D \vdash \checkmark \infty \Box$
- 4. $\Box\varphi \rightarrow \Box\Box\varphi$
 $D \vdash \times \infty \Box$
- 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$
 $D \vdash \times \infty \Box$
- INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$
 $D \vdash \checkmark \infty \Box$

Encapsulation

Slate - K.slt

K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $K \vdash \checkmark \infty \Box$	T. $\Box\varphi \rightarrow \varphi$ $K \vdash \times \infty \Box$	4. $\Box\varphi \rightarrow \Box\Box\varphi$ $K \vdash \times \infty \Box$	5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $K \vdash \times \infty \Box$
---	---	---	---

Slate - T.slt

K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $M \vdash \checkmark \infty \Box$	T. $\Box\varphi \rightarrow \varphi$ $M \vdash \checkmark \infty \Box$	4. $\Box\varphi \rightarrow \Box\Box\varphi$ $M \vdash \times \infty \Box$	5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $M \vdash \times \infty \Box$
---	---	---	---

Slate - D.slt

K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $D \vdash \checkmark \infty \Box$	T. $\Box\varphi \rightarrow \varphi$ $D \vdash \times \infty \Box$	D. $\Box\varphi \rightarrow \Diamond\varphi$ $D \vdash \checkmark \infty \Box$	4. $\Box\varphi \rightarrow \Box\Box\varphi$ $D \vdash \times \infty \Box$
5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $D \vdash \times \infty \Box$		INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$ $D \vdash \checkmark \infty \Box$	

Slate - S4.slt

K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $S4 \vdash \checkmark \infty \Box$	T. $\Box\varphi \rightarrow \varphi$ $S4 \vdash \checkmark \infty \Box$	D. $\Box\varphi \rightarrow \Diamond\varphi$ $S4 \vdash \checkmark \infty \Box$	4. $\Box\varphi \rightarrow \Box\Box\varphi$ $S4 \vdash \checkmark \infty \Box$
5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $S4 \vdash \times \infty \Box$		INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$ {INTER} Assume \checkmark	

Encapsulation

K

T

D

4 = S4

5 = S5

Slate - K.slt

K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $K \vdash \checkmark \infty \Box$	T. $\Box\varphi \rightarrow \varphi$ $K \vdash \times \infty \Box$	4. $\Box\varphi \rightarrow \Box\Box\varphi$ $K \vdash \times \infty \Box$	5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $K \vdash \times \infty \Box$
---	---	---	---

Slate - T.slt

K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $M \vdash \checkmark \infty \Box$	T. $\Box\varphi \rightarrow \varphi$ $M \vdash \checkmark \infty \Box$	4. $\Box\varphi \rightarrow \Box\Box\varphi$ $M \vdash \times \infty \Box$	5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $M \vdash \times \infty \Box$
---	---	---	---

Slate - D.slt

K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $D \vdash \checkmark \infty \Box$	T. $\Box\varphi \rightarrow \varphi$ $D \vdash \times \infty \Box$	D. $\Box\varphi \rightarrow \Diamond\varphi$ $D \vdash \checkmark \infty \Box$	4. $\Box\varphi \rightarrow \Box\Box\varphi$ $D \vdash \times \infty \Box$
5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $D \vdash \times \infty \Box$		INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$ $D \vdash \checkmark \infty \Box$	

Slate - S4.slt

K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $S4 \vdash \checkmark \infty \Box$	T. $\Box\varphi \rightarrow \varphi$ $S4 \vdash \checkmark \infty \Box$	D. $\Box\varphi \rightarrow \Diamond\varphi$ $S4 \vdash \checkmark \infty \Box$	4. $\Box\varphi \rightarrow \Box\Box\varphi$ $S4 \vdash \checkmark \infty \Box$
5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $S4 \vdash \times \infty \Box$		INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$ {INTER} Assume \checkmark	

Slate - S5.slt

K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $S5 \vdash \checkmark \infty \Box$	T. $\Box\varphi \rightarrow \varphi$ $S5 \vdash \checkmark \infty \Box$	D. $\Box\varphi \rightarrow \Diamond\varphi$ {D} Assume \checkmark	4. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ {4} Assume \checkmark
5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $S5 \vdash \checkmark \infty \Box$		INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$ {INTER} Assume \checkmark	

Encapsulation

K

T

D

4 = S4

5 = S5

The image shows a screenshot of the Slate IDE with five windows, each displaying a set of modal logic formulas and their derivability status in a specific system.

- Slate - K.slt**
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $K \vdash \checkmark \infty \Box$
 - T. $\Box\varphi \rightarrow \varphi$ $K \vdash \times \infty \Box$
 - 4. $\Box\varphi \rightarrow \Box\Box\varphi$ $K \vdash \times \infty \Box$
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $K \vdash \times \infty \Box$
- Slate - T.slt**
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $M \vdash \checkmark \infty \Box$
 - T. $\Box\varphi \rightarrow \varphi$ $M \vdash \checkmark \infty \Box$
 - 4. $\Box\varphi \rightarrow \Box\Box\varphi$ $M \vdash \times \infty \Box$
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $M \vdash \times \infty \Box$
- Slate - D.slt** (highlighted with a red border)
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $D \vdash \checkmark \infty \Box$
 - T. $\Box\varphi \rightarrow \varphi$ $D \vdash \times \infty \Box$
 - D. $\Box\varphi \rightarrow \Diamond\varphi$ $D \vdash \checkmark \infty \Box$
 - 4. $\Box\varphi \rightarrow \Box\Box\varphi$ $D \vdash \times \infty \Box$
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $D \vdash \times \infty \Box$
 - INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$ $D \vdash \checkmark \infty \Box$
- Slate - S4.slt**
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $S4 \vdash \checkmark \infty \Box$
 - T. $\Box\varphi \rightarrow \varphi$ $S4 \vdash \checkmark \infty \Box$
 - D. $\Box\varphi \rightarrow \Diamond\varphi$ $S4 \vdash \checkmark \infty \Box$
 - 4. $\Box\varphi \rightarrow \Box\Box\varphi$ $S4 \vdash \checkmark \infty \Box$
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $S4 \vdash \times \infty \Box$
 - INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$ $\{INTER\}$ Assume \checkmark
- Slate - S5.slt**
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $S5 \vdash \checkmark \infty \Box$
 - T. $\Box\varphi \rightarrow \varphi$ $S5 \vdash \checkmark \infty \Box$
 - D. $\Box\varphi \rightarrow \Diamond\varphi$ $\{D\}$ Assume \checkmark
 - 4. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $\{4\}$ Assume \checkmark
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $S5 \vdash \checkmark \infty \Box$
 - INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$ $\{INTER\}$ Assume \checkmark

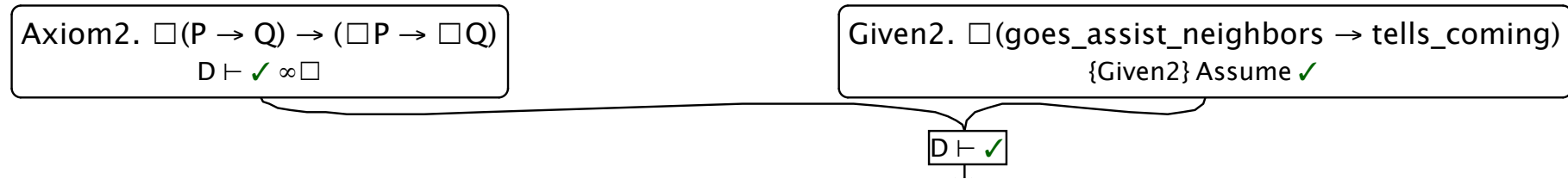
Encapsulation

K
T
D
4 = S4
5 = S5

The screenshot displays five windows of the HyperSlate interface, each representing a different modal logic system. Each window contains a grid of logical formulas and their derivability status (indicated by a green checkmark for true and a red X for false).

- Slate - K.slt**
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $K \vdash \checkmark \infty \Box$
 - T. $\Box\varphi \rightarrow \varphi$ $K \vdash \times \infty \Box$
 - 4. $\Box\varphi \rightarrow \Box\Box\varphi$ $K \vdash \times \infty \Box$
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $K \vdash \times \infty \Box$
- Slate - T.slt**
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $M \vdash \checkmark \infty \Box$
 - T. $\Box\varphi \rightarrow \varphi$ $M \vdash \checkmark \infty \Box$
 - 4. $\Box\varphi \rightarrow \Box\Box\varphi$ $M \vdash \times \infty \Box$
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $M \vdash \times \infty \Box$
- Slate - D.slt** (highlighted with a red border)
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $D \vdash \checkmark \infty \Box$
 - T. $\Box\varphi \rightarrow \varphi$ $D \vdash \times \infty \Box$
 - D. $\Box\varphi \rightarrow \Diamond\varphi$ $D \vdash \checkmark \infty \Box$
 - 4. $\Box\varphi \rightarrow \Box\Box\varphi$ $D \vdash \times \infty \Box$
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $D \vdash \times \infty \Box$
 - INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$ $D \vdash \checkmark \infty \Box$
- Slate - S4.slt**
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $S4 \vdash \checkmark \infty \Box$
 - T. $\Box\varphi \rightarrow \varphi$ $S4 \vdash \checkmark \infty \Box$
 - D. $\Box\varphi \rightarrow \Diamond\varphi$ $S4 \vdash \checkmark \infty \Box$
 - 4. $\Box\varphi \rightarrow \Box\Box\varphi$ $S4 \vdash \checkmark \infty \Box$
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $S4 \vdash \times \infty \Box$
 - INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$ $\{INTER\} \text{ Assume } \checkmark$
- Slate - S5.slt**
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $S5 \vdash \checkmark \infty \Box$
 - T. $\Box\varphi \rightarrow \varphi$ $S5 \vdash \checkmark \infty \Box$
 - D. $\Box\varphi \rightarrow \Diamond\varphi$ $\{D\} \text{ Assume } \checkmark$
 - 4. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $\{4\} \text{ Assume } \checkmark$
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $S5 \vdash \checkmark \infty \Box$
 - INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$ $\{INTER\} \text{ Assume } \checkmark$

Chisholm's Paradox

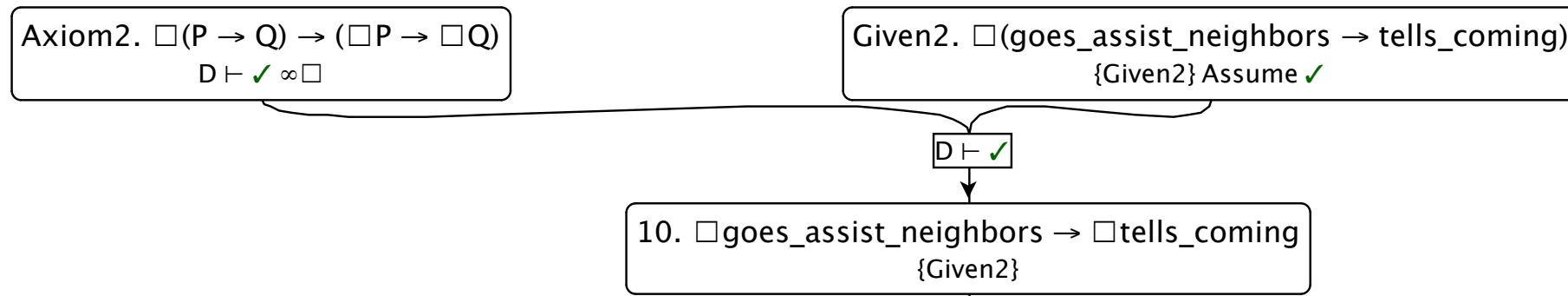


Axiom4. "Modus ponens for provability."
 $\{\text{Axiom4}\} \text{ Assume } \checkmark$

Axiom5. "Theorems are obligatory."
 $\{\text{Axiom5}\} \text{ Assume } \checkmark$

Axiom1. "All theorems of the propositional calculus."
 $\{\text{Axiom1}\} \text{ Assume } \checkmark$

Chisholm's Paradox

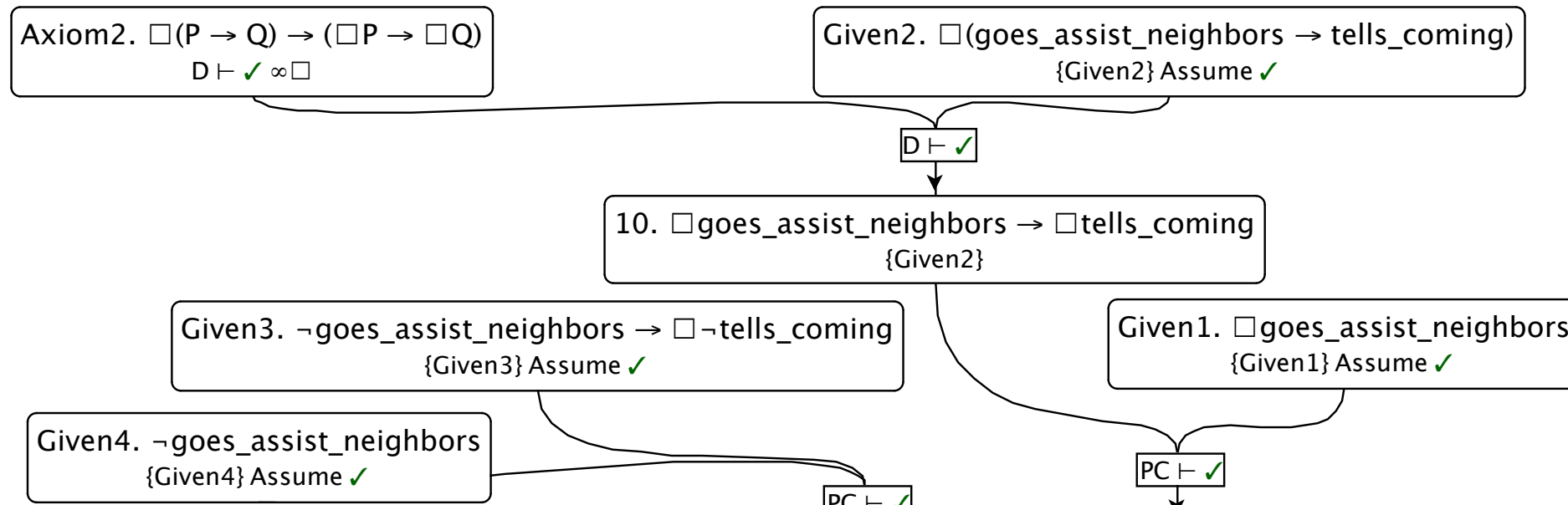


Axiom4. "Modus ponens for provability."
{Axiom4} Assume \checkmark

Axiom5. "Theorems are obligatory."
{Axiom5} Assume \checkmark

Axiom1. "All theorems of the propositional calculus."
{Axiom1} Assume \checkmark

Chisholm's Paradox

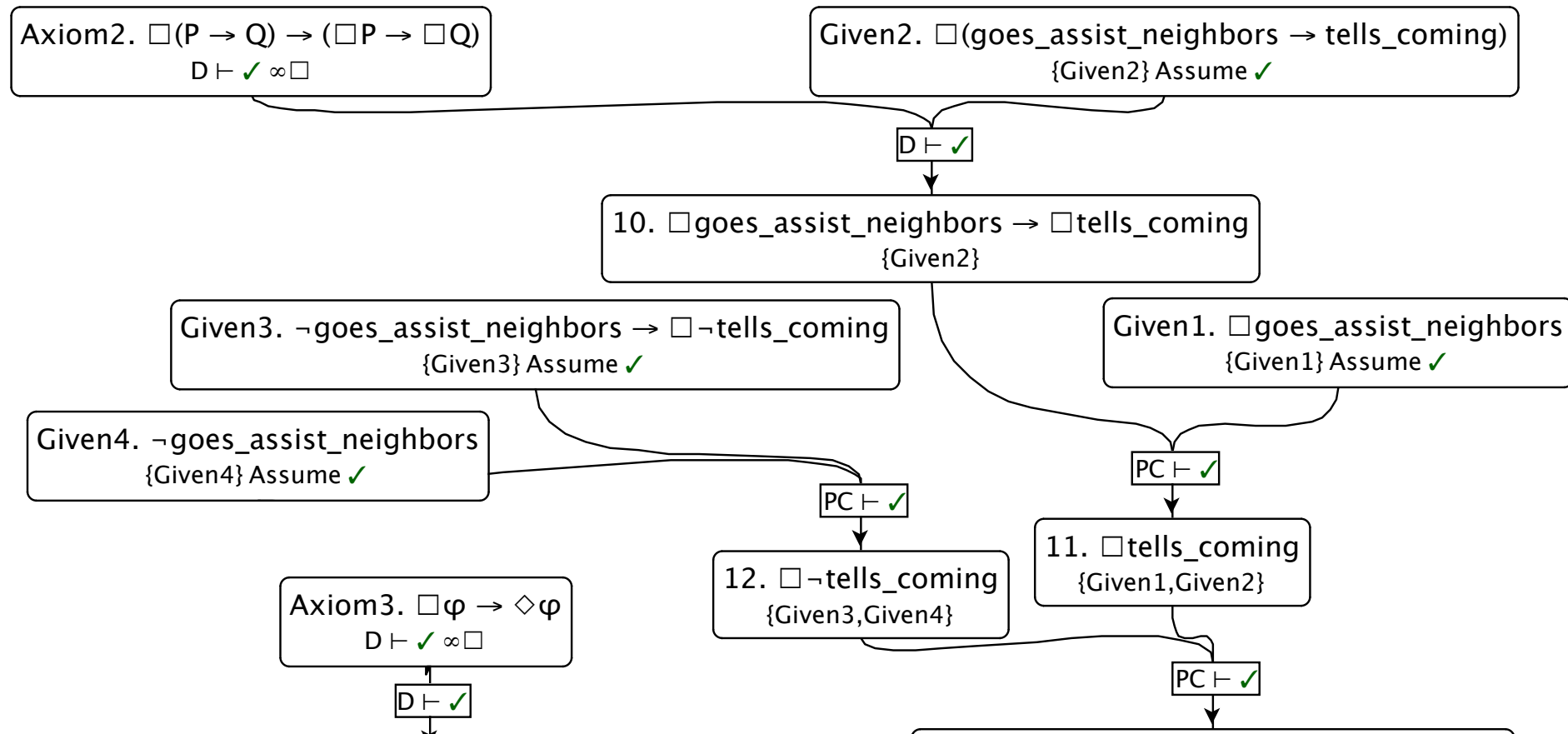


Axiom4. "Modus ponens for provability."
 $\{\text{Axiom4}\} \text{ Assume } \checkmark$

Axiom5. "Theorems are obligatory."
 $\{\text{Axiom5}\} \text{ Assume } \checkmark$

Axiom1. "All theorems of the propositional calculus."
 $\{\text{Axiom1}\} \text{ Assume } \checkmark$

Chisholm's Paradox

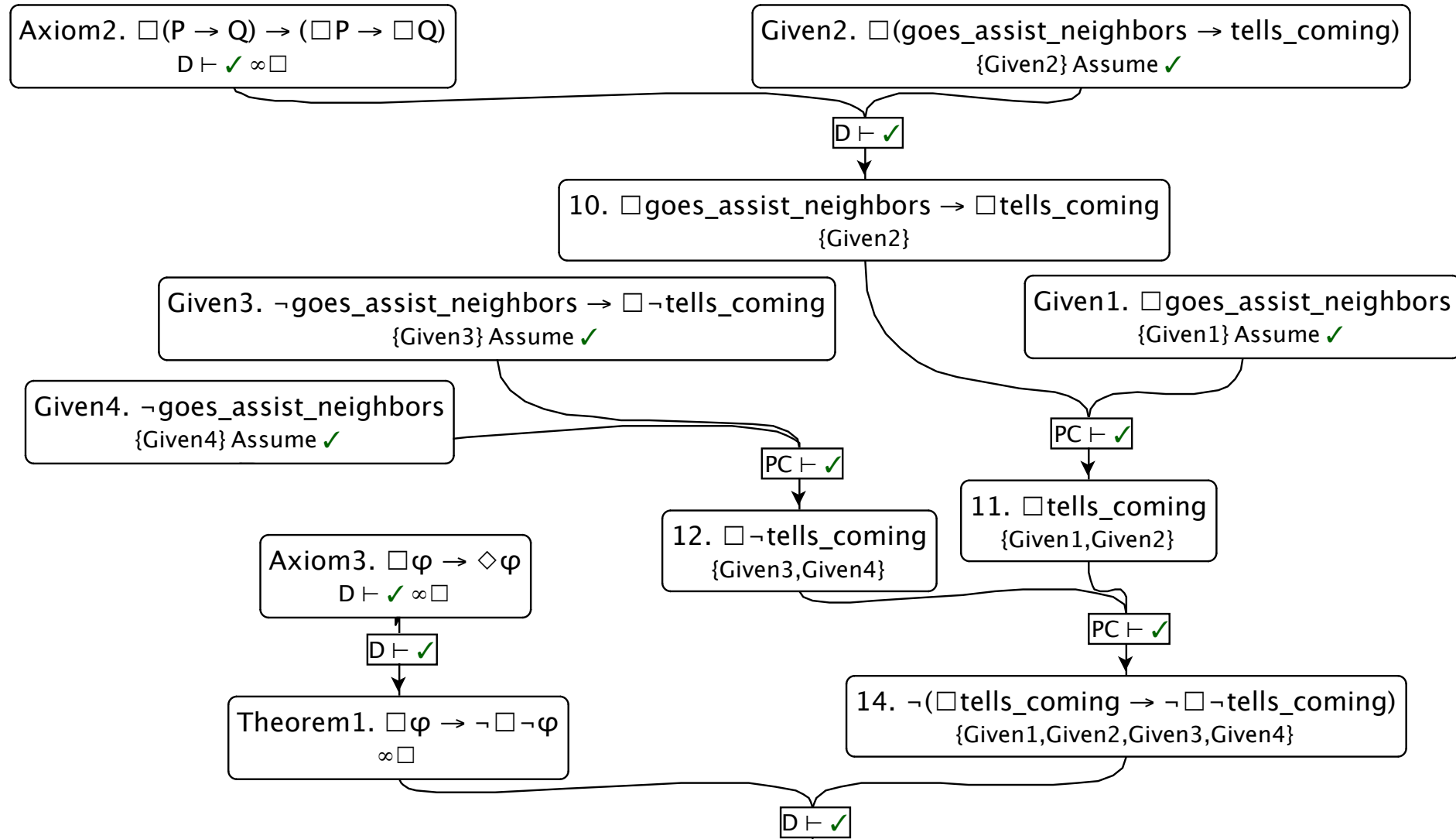


Axiom4. "Modus ponens for provability."
 $\{\text{Axiom4}\} \text{ Assume } \checkmark$

Axiom5. "Theorems are obligatory."
 $\{\text{Axiom5}\} \text{ Assume } \checkmark$

Axiom1. "All theorems of the propositional calculus."
 $\{\text{Axiom1}\} \text{ Assume } \checkmark$

Chisholm's Paradox

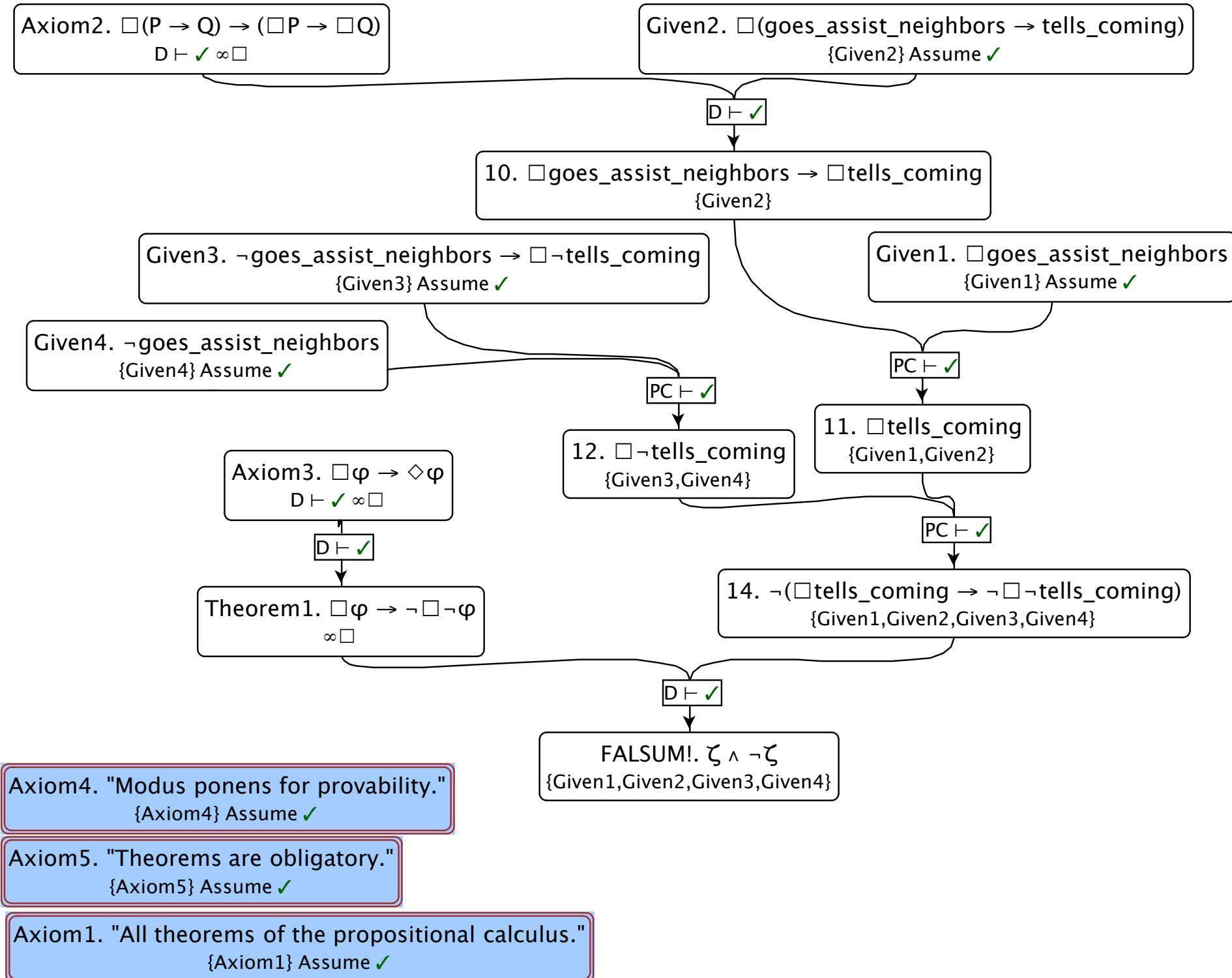


Axiom4. "Modus ponens for provability."
 $\{\text{Axiom4}\} \text{ Assume } \checkmark$

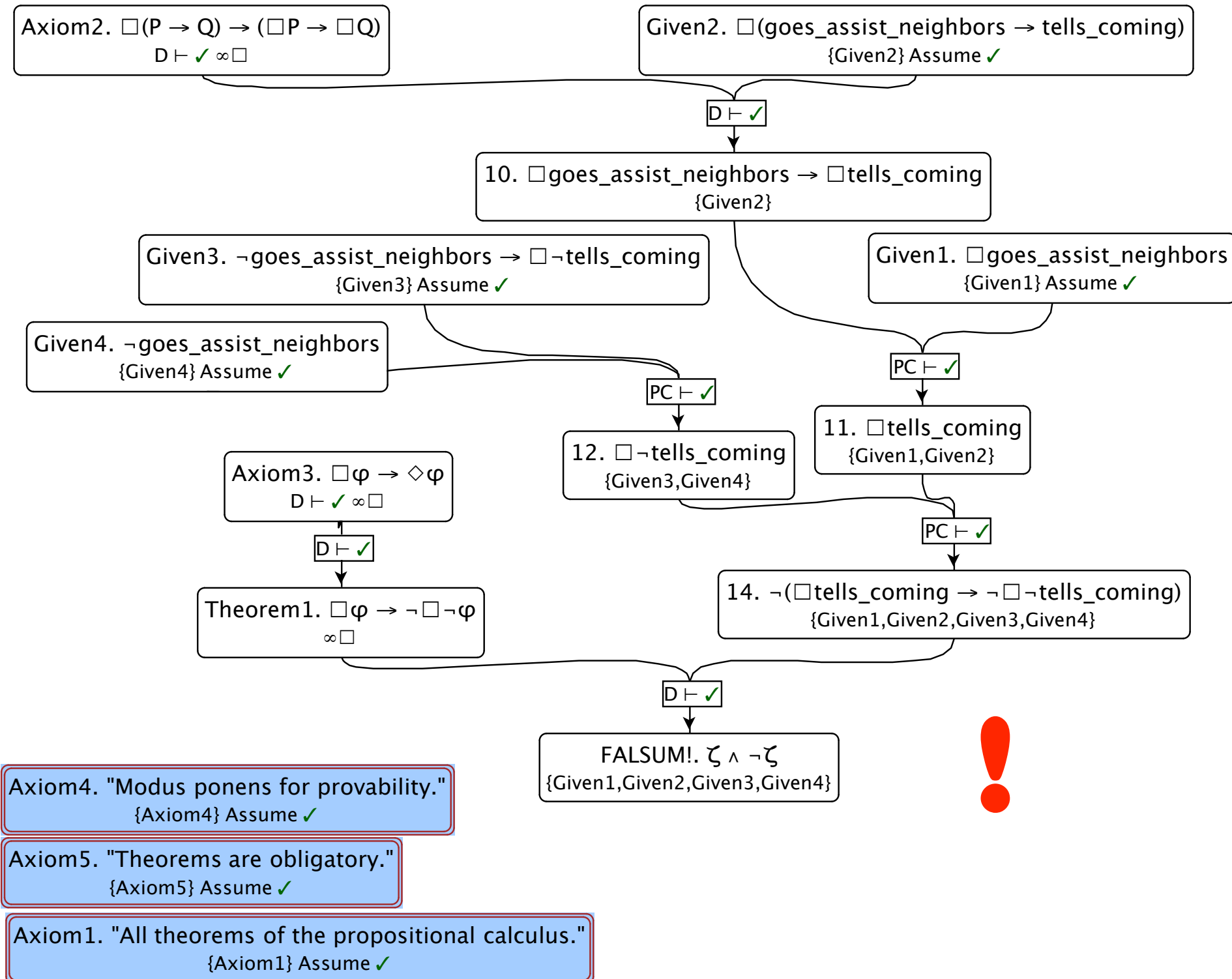
Axiom5. "Theorems are obligatory."
 $\{\text{Axiom5}\} \text{ Assume } \checkmark$

Axiom1. "All theorems of the propositional calculus."
 $\{\text{Axiom1}\} \text{ Assume } \checkmark$

Chisholm's Paradox



Chisholm's Paradox



Review: Encapsulation

K

T

D

4 = S4

5 = S5

The screenshot displays five windows from the HyperSlate application, each showing a set of modal logic formulas and their derivability status in different systems. The windows are titled "Slate - K.slt", "Slate - T.slt", "Slate - D.slt", "Slate - S4.slt", and "Slate - S5.slt".

- Slate - K.slt:**
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $K \vdash \checkmark \infty \Box$
 - T. $\Box\varphi \rightarrow \varphi$ $K \vdash \times \infty \Box$
 - 4. $\Box\varphi \rightarrow \Box\Box\varphi$ $K \vdash \times \infty \Box$
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $K \vdash \times \infty \Box$
- Slate - T.slt:**
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $M \vdash \checkmark \infty \Box$
 - T. $\Box\varphi \rightarrow \varphi$ $M \vdash \checkmark \infty \Box$
 - 4. $\Box\varphi \rightarrow \Box\Box\varphi$ $M \vdash \times \infty \Box$
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $M \vdash \times \infty \Box$
- Slate - D.slt:**
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $D \vdash \checkmark \infty \Box$
 - T. $\Box\varphi \rightarrow \varphi$ $D \vdash \times \infty \Box$
 - D. $\Box\varphi \rightarrow \Diamond\varphi$ $D \vdash \checkmark \infty \Box$
 - 4. $\Box\varphi \rightarrow \Box\Box\varphi$ $D \vdash \times \infty \Box$
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $D \vdash \times \infty \Box$
 - INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$ $D \vdash \checkmark \infty \Box$
- Slate - S4.slt:**
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $S4 \vdash \checkmark \infty \Box$
 - T. $\Box\varphi \rightarrow \varphi$ $S4 \vdash \checkmark \infty \Box$
 - D. $\Box\varphi \rightarrow \Diamond\varphi$ $S4 \vdash \checkmark \infty \Box$
 - 4. $\Box\varphi \rightarrow \Box\Box\varphi$ $S4 \vdash \checkmark \infty \Box$
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $S4 \vdash \times \infty \Box$
 - INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$ $\{INTER\} \text{ Assume } \checkmark$
- Slate - S5.slt:**
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $S5 \vdash \checkmark \infty \Box$
 - T. $\Box\varphi \rightarrow \varphi$ $S5 \vdash \checkmark \infty \Box$
 - D. $\Box\varphi \rightarrow \Diamond\varphi$ $\{D\} \text{ Assume } \checkmark$
 - 4. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $\{4\} \text{ Assume } \checkmark$
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $S5 \vdash \checkmark \infty \Box$
 - INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$ $\{INTER\} \text{ Assume } \checkmark$

Review: Encapsulation

K

T

D

4 = S4

5 = S5

The screenshot displays five windows from the HyperSlate application, each showing a set of modal logic formulas and their derivability status in a specific logic.

- Slate - K.slt**
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $K \vdash \checkmark \infty \Box$
 - T. $\Box\varphi \rightarrow \varphi$ $K \vdash \times \infty \Box$
 - 4. $\Box\varphi \rightarrow \Box\Box\varphi$ $K \vdash \times \infty \Box$
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $K \vdash \times \infty \Box$
- Slate - T.slt**
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $M \vdash \checkmark \infty \Box$
 - T. $\Box\varphi \rightarrow \varphi$ $M \vdash \checkmark \infty \Box$
 - 4. $\Box\varphi \rightarrow \Box\Box\varphi$ $M \vdash \times \infty \Box$
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $M \vdash \times \infty \Box$
- Slate - D.slt**
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $D \vdash \checkmark \infty \Box$
 - T. $\Box\varphi \rightarrow \varphi$ $D \vdash \times \infty \Box$
 - D. $\Box\varphi \rightarrow \Diamond\varphi$ $D \vdash \checkmark \infty \Box$
 - 4. $\Box\varphi \rightarrow \Box\Box\varphi$ $D \vdash \times \infty \Box$
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $D \vdash \times \infty \Box$
 - INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$ $D \vdash \checkmark \infty \Box$
- Slate - S4.slt** (Highlighted with a red border)
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $S4 \vdash \checkmark \infty \Box$
 - T. $\Box\varphi \rightarrow \varphi$ $S4 \vdash \checkmark \infty \Box$
 - D. $\Box\varphi \rightarrow \Diamond\varphi$ $S4 \vdash \checkmark \infty \Box$
 - 4. $\Box\varphi \rightarrow \Box\Box\varphi$ $S4 \vdash \checkmark \infty \Box$
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $S4 \vdash \times \infty \Box$
 - INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$ $\{INTER\} \text{ Assume } \checkmark$
- Slate - S5.slt** (Highlighted with a red border)
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $S5 \vdash \checkmark \infty \Box$
 - T. $\Box\varphi \rightarrow \varphi$ $S5 \vdash \checkmark \infty \Box$
 - D. $\Box\varphi \rightarrow \Diamond\varphi$ $\{D\} \text{ Assume } \checkmark$
 - 4. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ $\{4\} \text{ Assume } \checkmark$
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ $S5 \vdash \checkmark \infty \Box$
 - INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$ $\{INTER\} \text{ Assume } \checkmark$

Review: Encapsulation

K

T

D

4 = S4

5 = S5

The screenshot displays the HyperSlate interface with several windows open, each showing a set of modal logic axioms and their derivability status.

Create file dialog (highlighted with a green box):

- Buttons for logic levels: Propositional Calculus, L_0 = Pure Predicate Calculus, L_1 = First-order Logic, L_2 = Second-order Logic.
- Buttons for modal systems: K, T, D, S4, S5.
- Buttons for fragments: DCEC (fragment), Hyperlog.

Windows and their contents (highlighted with a red box):

- Slate - K.slt:**
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ (K $\vdash \checkmark \infty \Box$)
 - T. $\Box\varphi \rightarrow \varphi$ (K $\vdash \times \infty \Box$)
 - 4. $\Box\varphi \rightarrow \Box\Box\varphi$ (K $\vdash \times \infty \Box$)
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ (K $\vdash \times \infty \Box$)
- Slate - T.slt:**
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ (M $\vdash \checkmark \infty \Box$)
 - T. $\Box\varphi \rightarrow \varphi$ (M $\vdash \checkmark \infty \Box$)
 - 4. $\Box\varphi \rightarrow \Box\Box\varphi$ (M $\vdash \times \infty \Box$)
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ (M $\vdash \times \infty \Box$)
- Slate - S4.slt:**
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ (S4 $\vdash \checkmark \infty \Box$)
 - T. $\Box\varphi \rightarrow \varphi$ (S4 $\vdash \checkmark \infty \Box$)
 - D. $\Box\varphi \rightarrow \Diamond\varphi$ (S4 $\vdash \checkmark \infty \Box$)
 - 4. $\Box\varphi \rightarrow \Box\Box\varphi$ (S4 $\vdash \checkmark \infty \Box$)
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ (S4 $\vdash \times \infty \Box$)
 - INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$ ({INTER} Assume \checkmark)
- Slate - S5.slt:**
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ (S5 $\vdash \checkmark \infty \Box$)
 - T. $\Box\varphi \rightarrow \varphi$ (S5 $\vdash \checkmark \infty \Box$)
 - D. $\Box\varphi \rightarrow \Diamond\varphi$ ({D} Assume \checkmark)
 - 4. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ ({4} Assume \checkmark)
 - 5. $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$ (S5 $\vdash \checkmark \infty \Box$)
 - INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$ ({INTER} Assume \checkmark)

Review: Encapsulation

K

T

D

4 = S4

5 = S5

The screenshot displays the HyperSlate interface with several windows showing logical formulas and their derivability status in different calculi.

Windows and Formulas:

- Slate - K.slt:**
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ (K $\vdash \checkmark \infty \Box$)
 - T. $\Box\varphi \rightarrow \varphi$ (K $\vdash \times \infty \Box$)
 - 4. $\Box\varphi \rightarrow \Box\Box\varphi$ (K $\vdash \times \infty \Box$)
 - 5. $\Box\varphi \rightarrow \Box\neg\Box\varphi$ (K $\vdash \times \infty \Box$)
- Slate - T.slt:**
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ (M $\vdash \checkmark \infty \Box$)
 - T. $\Box\varphi \rightarrow \varphi$ (M $\vdash \checkmark \infty \Box$)
 - 4. $\Box\varphi \rightarrow \Box\Box\varphi$ (M $\vdash \times \infty \Box$)
 - 5. $\Box\varphi \rightarrow \Box\neg\Box\varphi$ (M $\vdash \times \infty \Box$)
- Create file dialog:**
 - Buttons: Propositional Calculus, L_0 = Pure Predicate Calculus, L_1 = First-order Logic, L_2 = Second-order Logic, K, T, D, S4, S5, DCEC (fragment), Hyperlog.
- Slate - S4.slt:**
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ (S4 $\vdash \checkmark \infty \Box$)
 - T. $\Box\varphi \rightarrow \varphi$ (S4 $\vdash \checkmark \infty \Box$)
 - D. $\Box\varphi \rightarrow \Diamond\varphi$ (S4 $\vdash \checkmark \infty \Box$)
 - 4. $\Box\varphi \rightarrow \Box\Box\varphi$ (S4 $\vdash \checkmark \infty \Box$)
 - 5. $\Box\varphi \rightarrow \Box\neg\Box\varphi$ (S4 $\vdash \times \infty \Box$)
 - INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$ ({INTER} Assume \checkmark)
- Slate - S5.slt:**
 - K. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ (S5 $\vdash \checkmark \infty \Box$)
 - T. $\Box\varphi \rightarrow \varphi$ (S5 $\vdash \checkmark \infty \Box$)
 - D. $\Box\varphi \rightarrow \Diamond\varphi$ ({D} Assume \checkmark)
 - 4. $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$ ({4} Assume \checkmark)
 - 5. $\Box\varphi \rightarrow \Box\neg\Box\varphi$ (S5 $\vdash \checkmark \infty \Box$)
 - INTER. $\Box\varphi \leftrightarrow \neg\Diamond\neg\varphi$ ({INTER} Assume \checkmark)

A green arrow points to the 'Create file' dialog, and a red box highlights the S4 and S5 windows.

*Det er en logikk for
hvert problem!*