

Propositional Calculus I: The Formal Language, The Prop. Calc. Oracle (= AI), Application to Some Motivating Problems

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Troy, New York 12180 USA

IFLAI [Intro to (Formal) Logic (and AI)]
1/26/2023



How'd We Arrive Here?

(Selmer's Leibnizian Whirlwind History of Logic,
With Discussion of The Singularity)

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Questions about last time ...?

Intro to Logic

1/23/2023



Logic-and-AI in the news

...



The Atlantic

AA



Michael Brennan; Getty; The Atlantic

IDEAS

HOW CHATGPT WILL DESTABILIZE WHITE- COLLAR WORK

No technology in modern memory has caused
mass job loss among highly educated workers.

Will Oremus | Atlantic Staff



The Atlantic



Michael Brennan; Getty; The Atlantic

IDEAS

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HOW CHATGPT WILL DESTABILIZE WHITE- COLLAR WORK

No technology in modern memory has caused
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Will generative AI be an exception?

By Annie Lowrey

JANUARY 20, 2023

In the next five years, it is likely that AI will begin to reduce employment for college-educated workers. As the technology continues to advance, it will be able to perform tasks that were previously thought to require a high level of education and skill. This could lead to a displacement of workers in certain industries, as companies look to cut costs by automating processes. While it is difficult to predict the exact extent of this trend, it is clear that AI will have a significant impact on the job market for college-educated workers. It will be important for individuals to stay up to date on the latest developments in AI and to consider how their skills and expertise can be leveraged in a world where machines are increasingly able to perform many tasks.



The Atlantic



Michael Brennan; Getty; The Atlantic

IDEAS

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Will generative AI be an exception?

By Annie Hsu

JANUARY 20

In the next five years, it is to reduce employment for college-educated workers. As the technology will be able to perform tasks thought to require a high level of skill. This could lead to a decline in certain industries, as companies begin to automate processes. We predict the exact extent of that AI will have a significant impact on the market for college-educated workers. It is important for individuals to stay up to date on the latest developments in AI and to consider how their skills and expertise can be leveraged in a world where machines are increasingly able to perform many tasks.

There you have it, I guess: ChatGPT is coming for my job and yours, according to ChatGPT itself. The artificially intelligent content creator, whose name is short for “Chat Generative Pre-trained Transformer,” was released two months ago by OpenAI, one of the country’s most influential artificial-intelligence research laboratories. The technology is, put simply, amazing. It generated that first paragraph instantly, working with this prompt: “Write a five-sentence paragraph in the style of *The Atlantic* about whether AI will begin to reduce employment for college-educated workers in the next five years.”

Logistics again ...

The Starting Code to Purchase in Bookstore

M

Your code for starting the registration process is:

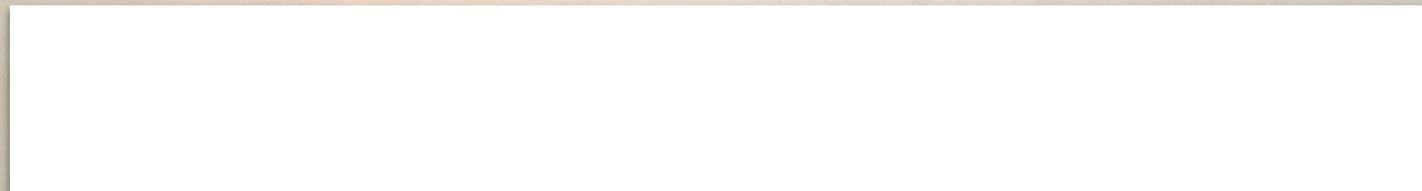
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and to obtain the textbook LAMA-BDLA, go to::

<https://rpi.logicamodernapproach.com>

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Your code for starting the registration process is:

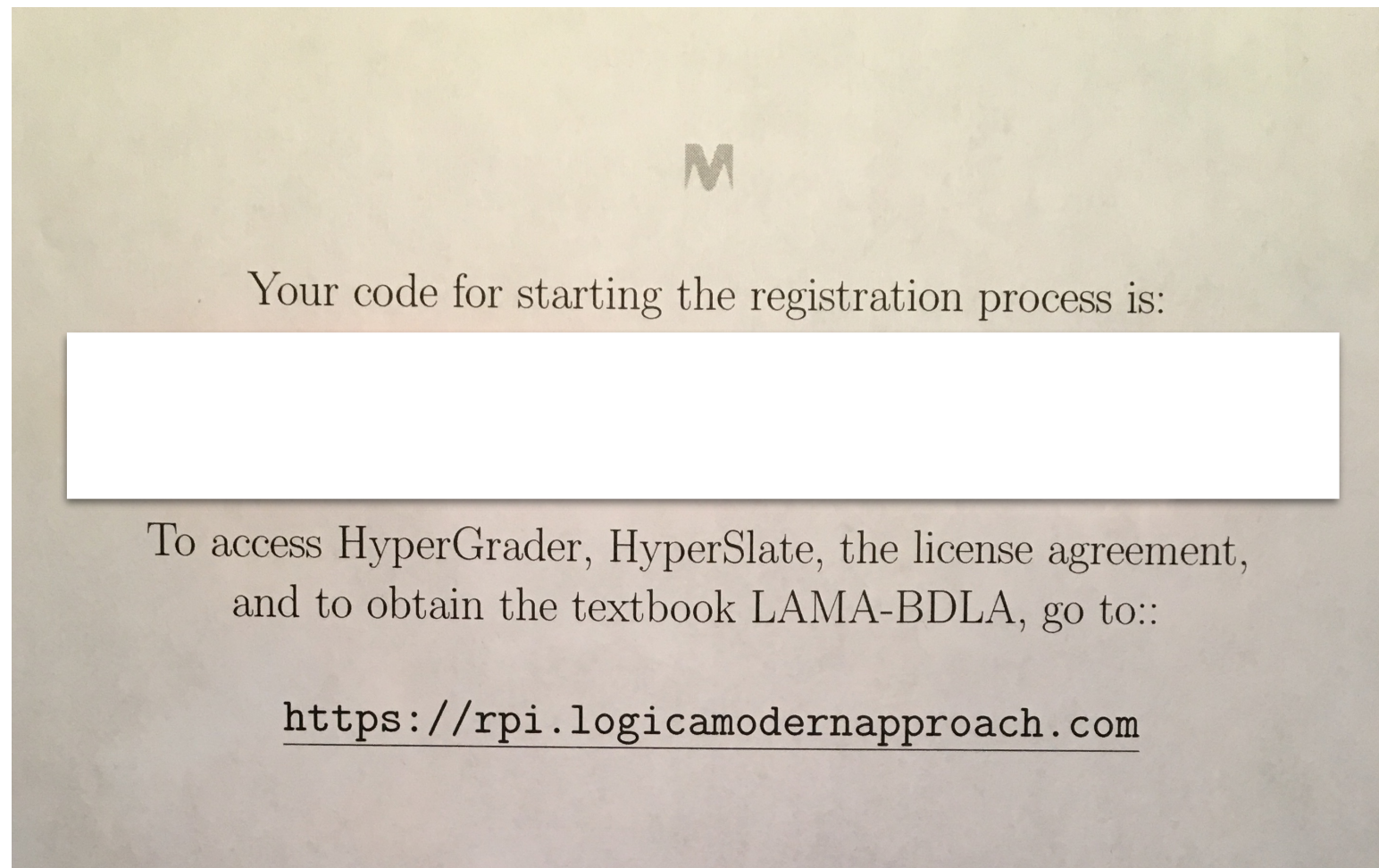


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Once seal broken on envelope, no return. Remember from first class, any reservations, opt for “Stanford” paradigm, with its software instead of LAMA® paradigm!

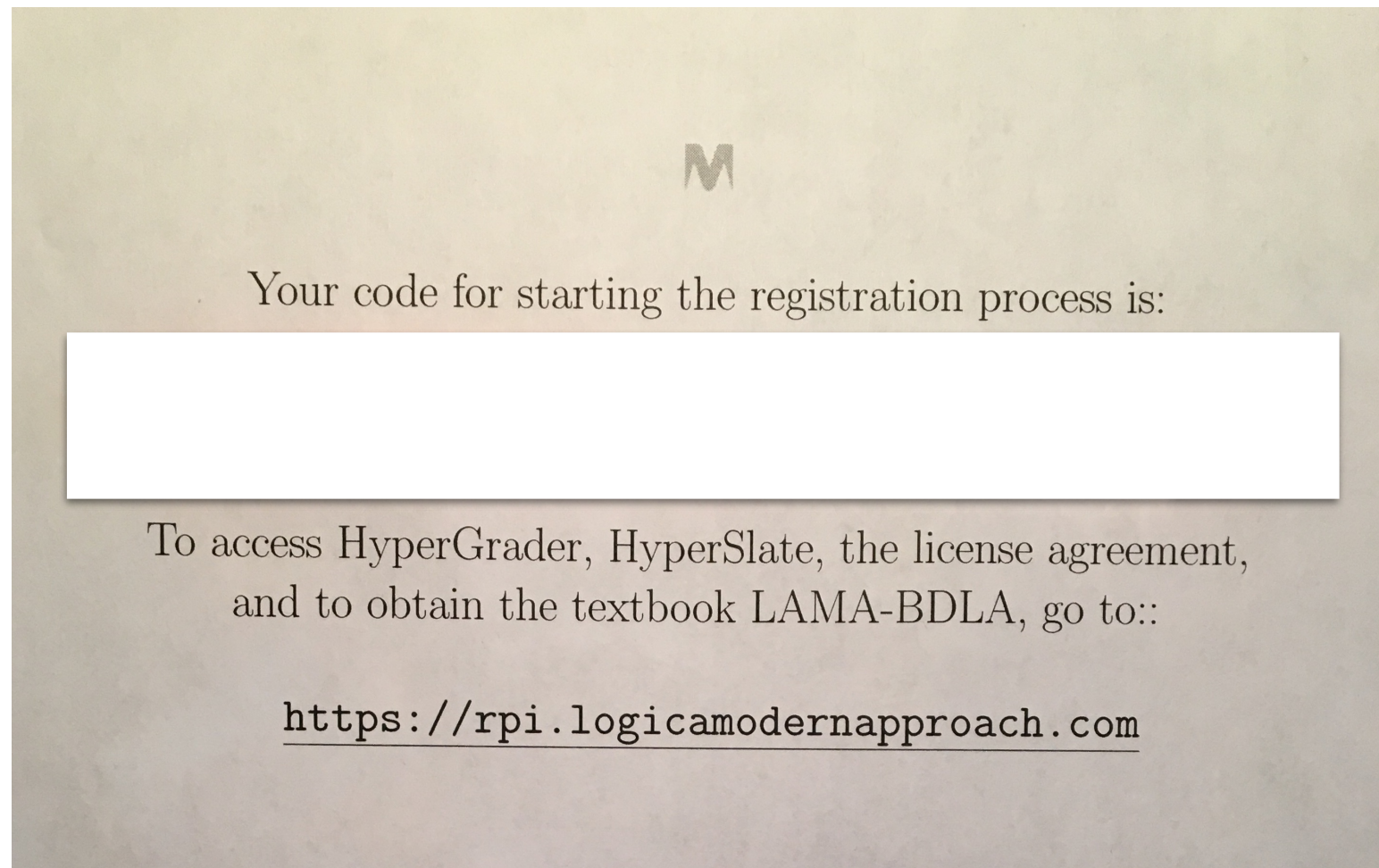
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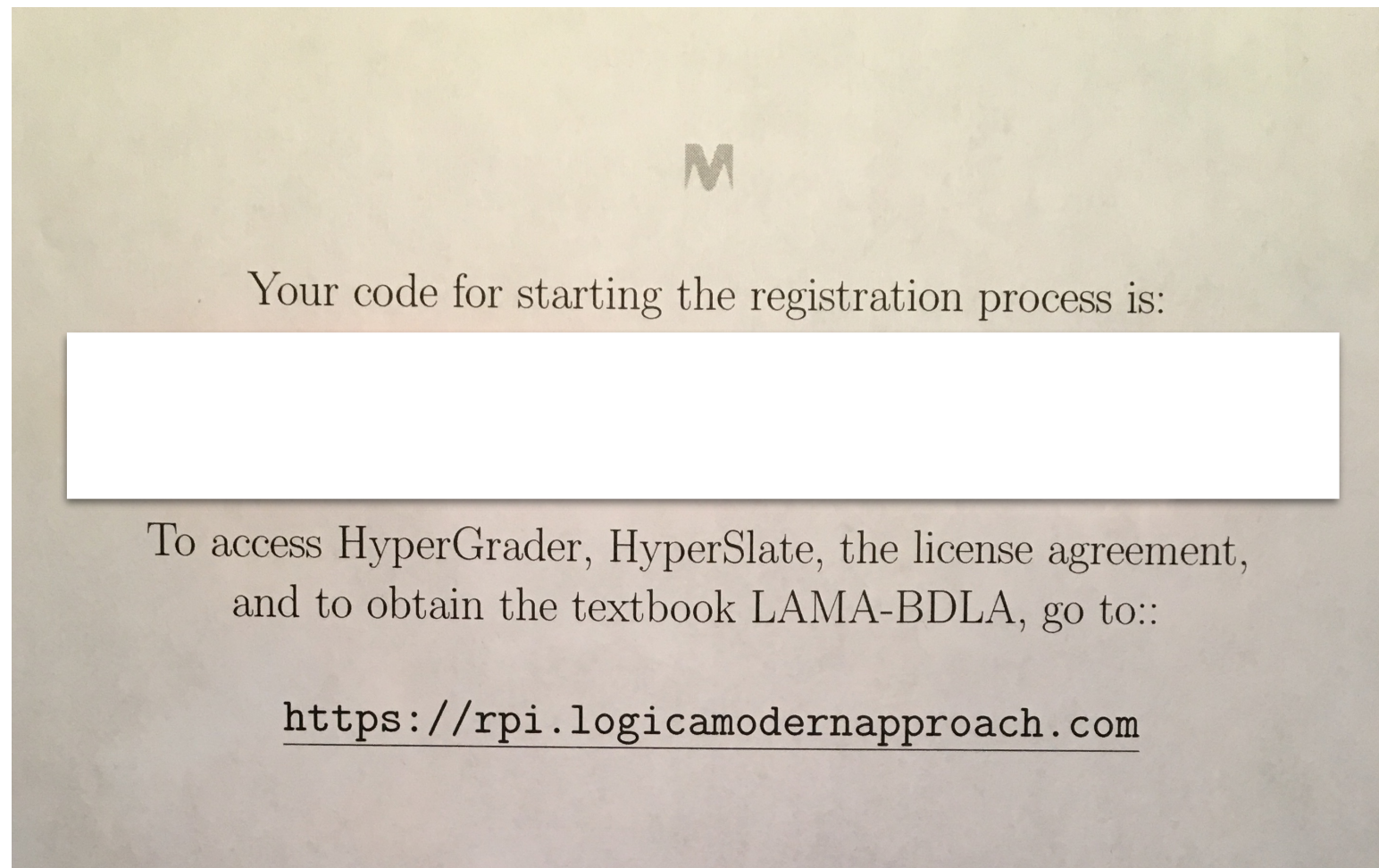


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Watch that the link emailed to you doesn't end up being classified as spam.

Introduction to (Formal) Logic (and **AI**)

Spring 2021 edition of IFLAI1

[Selmer Bringsjord](#)

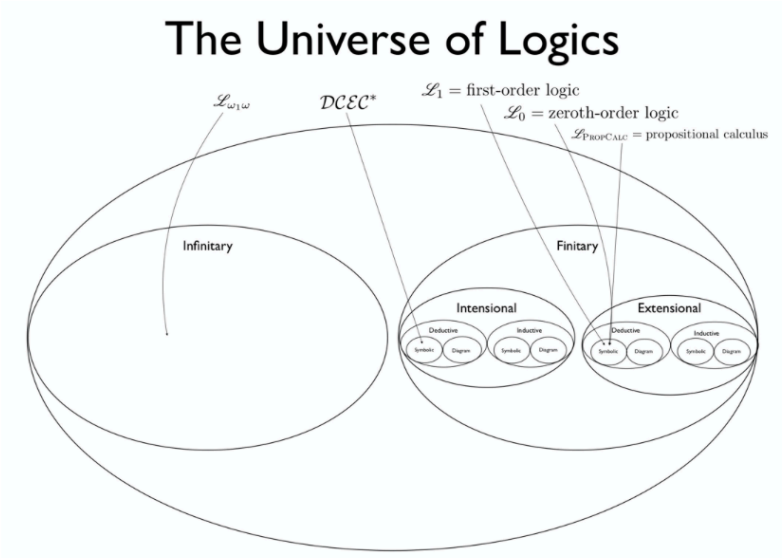
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A fully online course, thanks to singular AI technology.

with [Naveen Sundar G.](#)
^ KB Foush   ^ Joshua Taylor ^ ...



Micro-homily:

Micro-homily:

skipping to ~ p. 34!

Micro-homily:

skipping to ~ p. 34!



Micro-homily:

skipping to ~ p. 34!



M. Chi: Self-testers end up being self-made.

Micro-homily:

skipping to ~ p. 34!



M. Chi: Self-testers end up being self-made.

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M. Chi: Self-testers end up being self-made.

“What category of English sentences does logic focus on?”

The Formal Language

CHAPTER 2. PROPOSITIONAL CALCULUS

Syntax	Formula Type	Sample Representation
$P, P_1, P_2, Q, Q_1, \dots$	Atomic Formulas	"Larry is lucky." as L_l
$\neg\phi$	Negation	"Gary isn't lucky." as $\neg L_g$
$\phi_1 \wedge \dots \wedge \phi_n$	Conjunction	"Both Larry and Carl are lucky." as $L_l \wedge L_c$
$\phi_1 \vee \dots \vee \phi_n$	Disjunction	"Either Billy is lucky or Alvin is." as $L_b \vee L_a$
$\phi \rightarrow \psi$	Conditional (Implication)	"If Ron is lucky, so is Frank." as $L_r \rightarrow L_f$
$\phi \leftrightarrow \psi$	Biconditional (Coimplication)	"Tim is lucky if and only if Kim is." as $L_t \leftrightarrow L_k$

Table 2.1: Syntax of the Propositional Calculus. Note that ϕ , ψ , and ϕ_i stand for arbitrary formulas.

The Formal Language

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Exercise: Is this language Roger-decidable? Prove it!

The Formal Language

(presented as formal grammar)

Formula \Rightarrow *AtomicFormula*

| (*Formula* *Connective* *Formula*)
| \neg *Formula*

AtomicFormula \Rightarrow $P_1 \mid P_2 \mid P_3 \mid \dots$

Connective \Rightarrow $\wedge \mid \vee \mid \rightarrow \mid \leftrightarrow$

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As S-expressions

$$\begin{array}{lcl} \textit{Formula} & \Rightarrow & \textit{AtomicFormula} \\ & & | \\ & & (\textit{Formula} \textit{Connective} \textit{Formula}) \\ & & | \\ & & \neg \textit{Formula} \end{array}$$

$$\textit{AtomicFormula} \Rightarrow \text{P}_1 \mid \text{P}_2 \mid \text{P}_3 \mid \dots$$

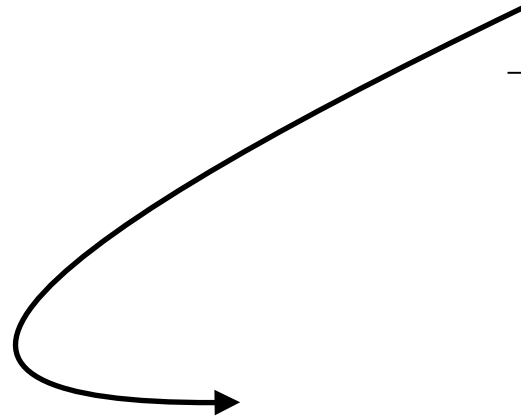
$$\textit{Connective} \Rightarrow \wedge \mid \vee \mid \rightarrow \mid \leftrightarrow$$

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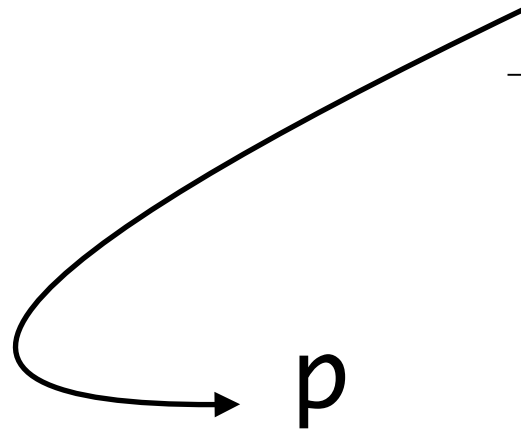
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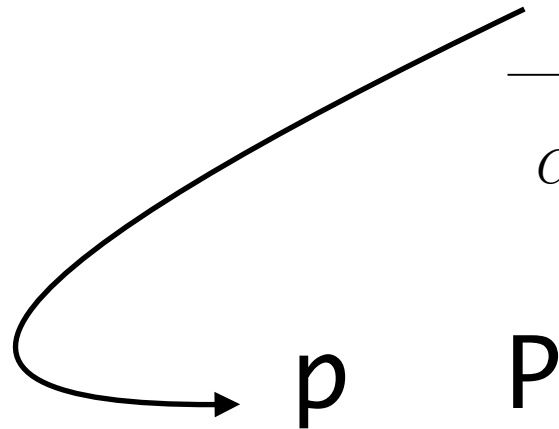
p

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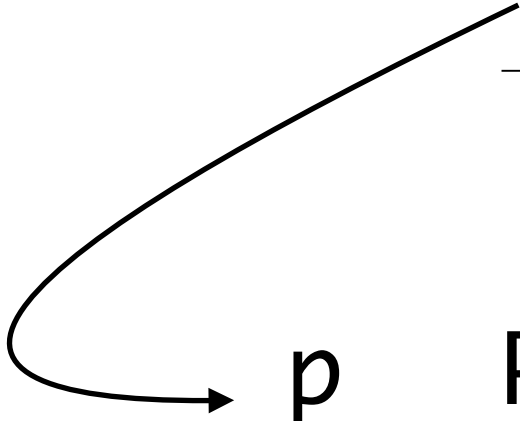
p **P**

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p P bradywillbeback

As S-expressions

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$$\textit{AtomicFormula} \Rightarrow P_1 \mid P_2 \mid P_3 \mid \dots$$

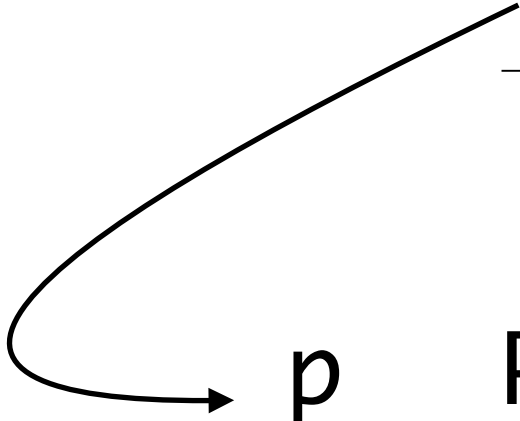
$$\textit{Connective} \Rightarrow \wedge \mid \vee \mid \rightarrow \mid \leftrightarrow$$


p P bradywillbeback P26

As S-expressions

$$\begin{array}{lcl} \textit{Formula} & \Rightarrow & \textit{AtomicFormula} \\ & & | \\ & & (\textit{Formula} \textit{Connective} \textit{Formula}) \\ & & | \\ & & \neg \textit{Formula} \end{array}$$

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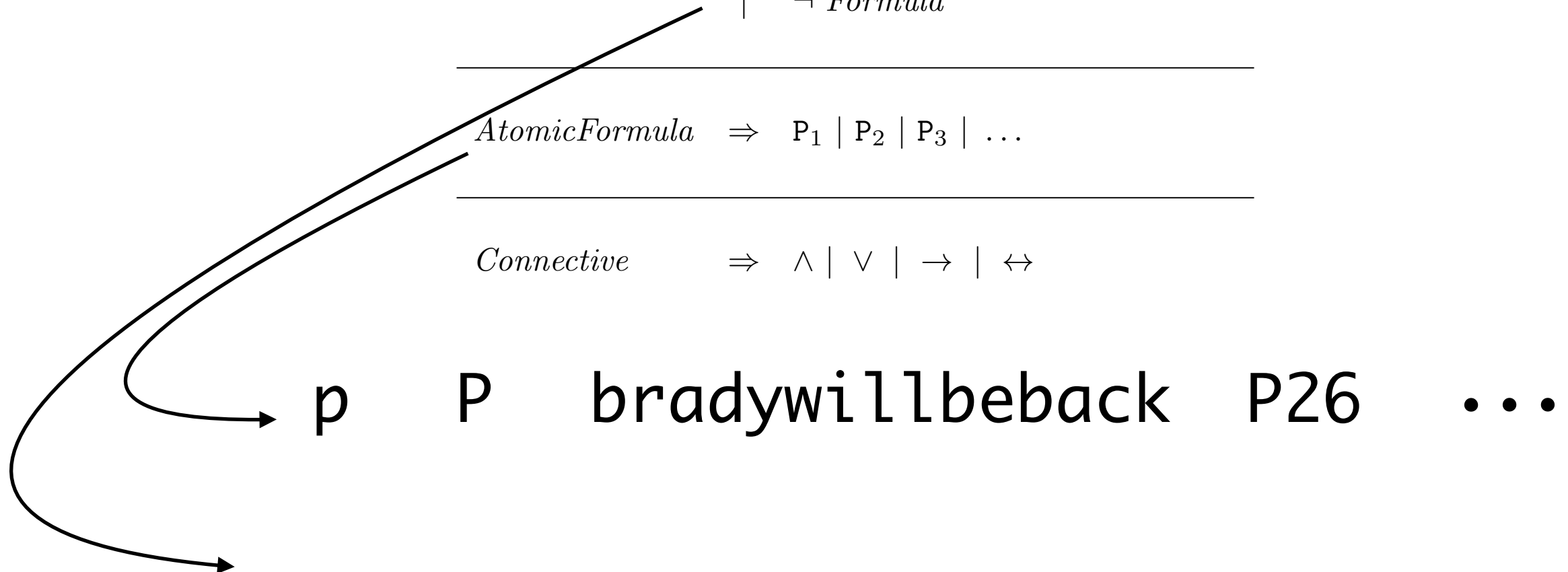
p P bradywillbeback P26 ...

As S-expressions

Formula \Rightarrow *AtomicFormula*
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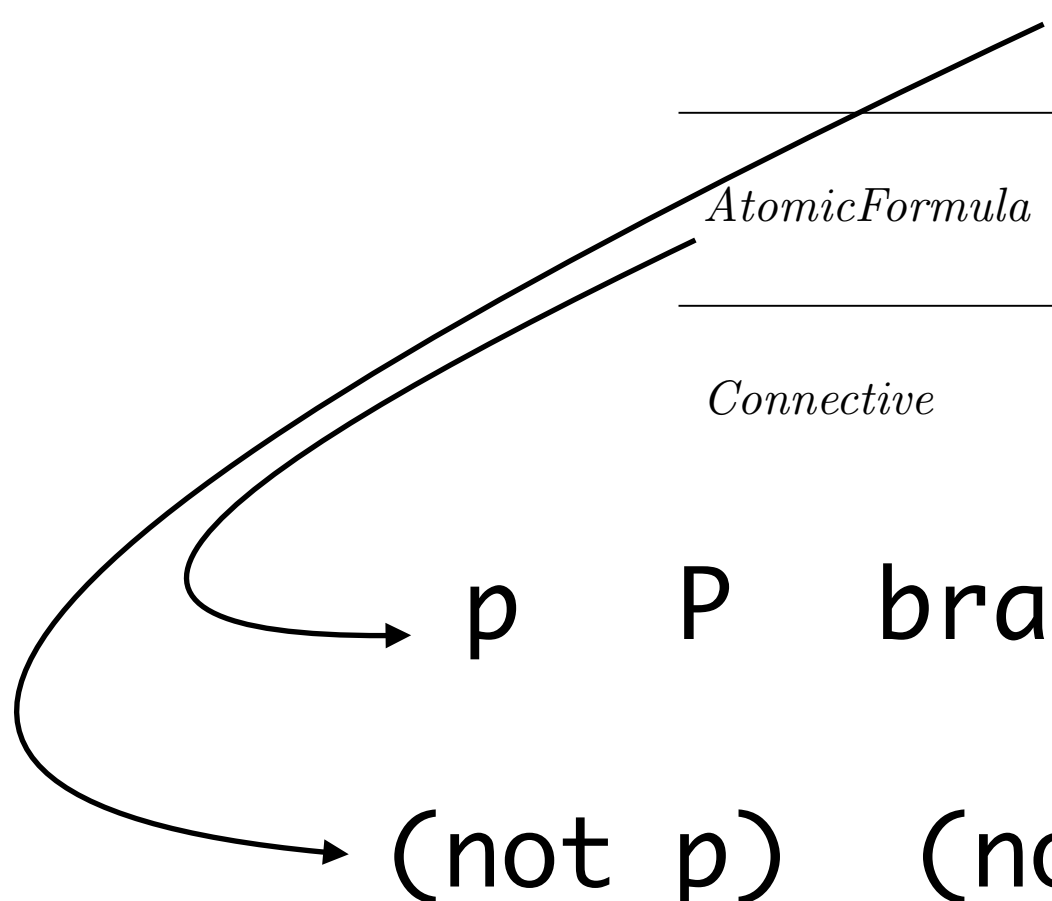
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AtomicFormula \Rightarrow $P_1 \mid P_2 \mid P_3 \mid \dots$

Connective \Rightarrow $\wedge \mid \vee \mid \rightarrow \mid \leftrightarrow$


 p P bradywillbeback P26 ...
 (not p) (not P)

As S-expressions

Formula \Rightarrow *AtomicFormula*

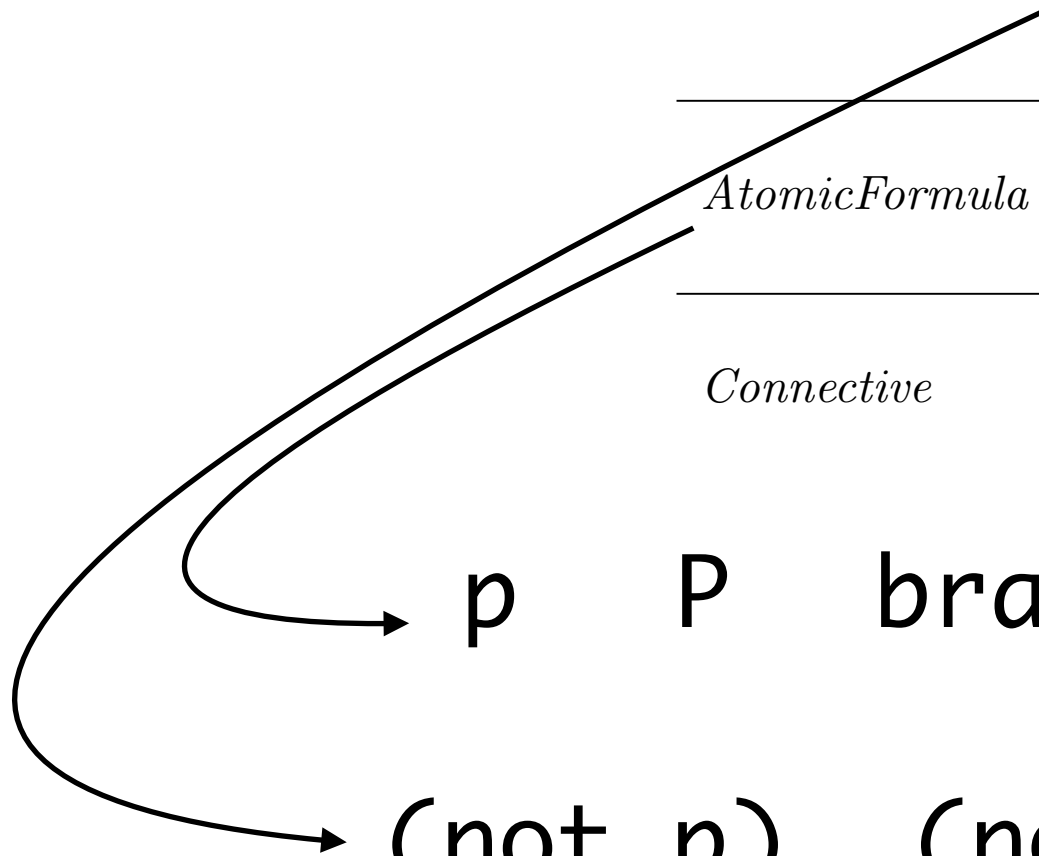
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p P bradywillbeback P26 ...

(not p) (not P) (not P26)



As S-expressions

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p P bradywillbeback P26 ...

(not p) (not P) (not P26) ...

As S-expressions

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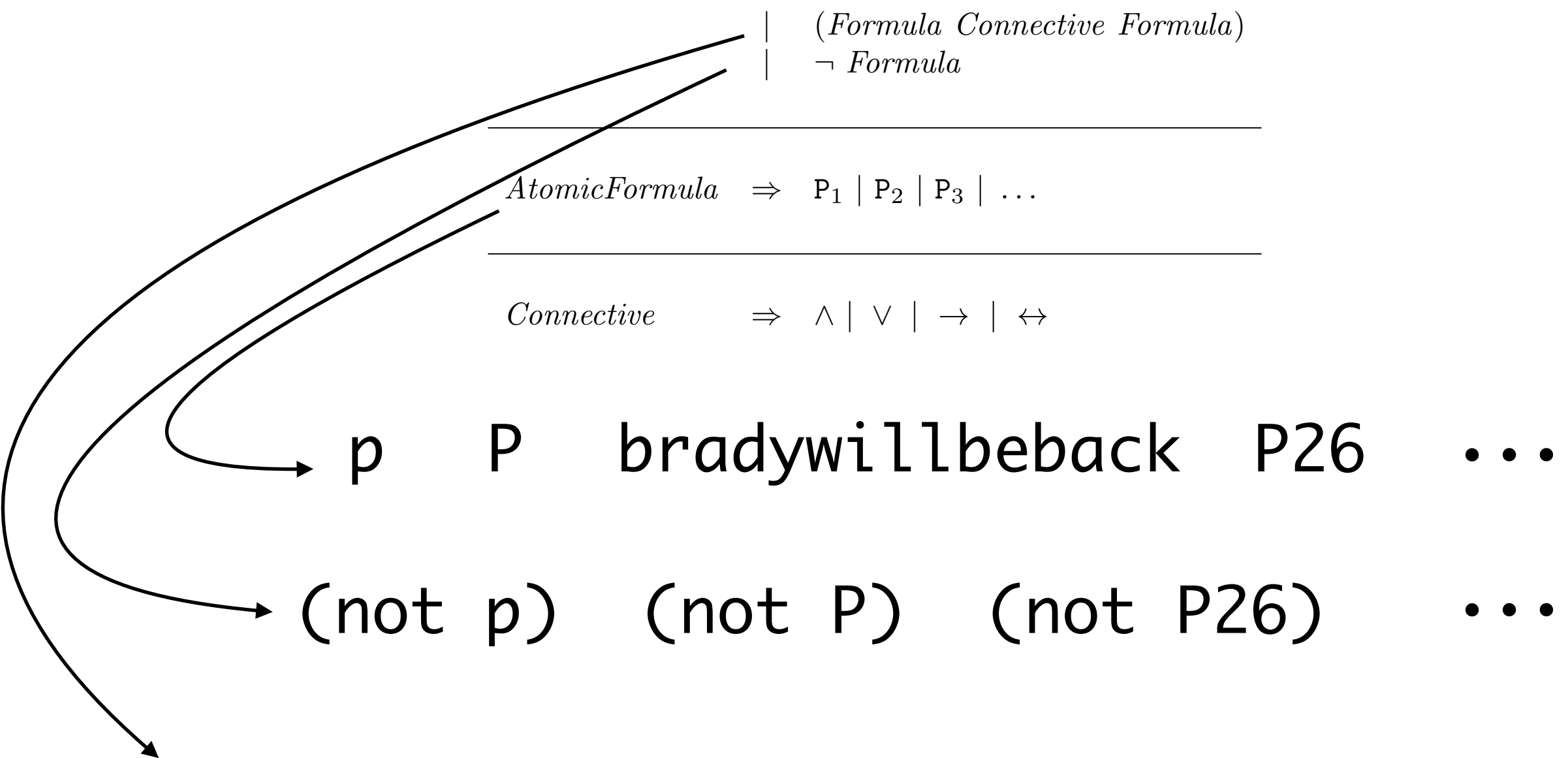
| (*Formula* *Connective* *Formula*)
| \neg *Formula*

AtomicFormula \Rightarrow P₁ | P₂ | P₃ | ...

Connective \Rightarrow \wedge | \vee | \rightarrow | \leftrightarrow

p P bradywillbeback P26 ...

(not p) (not P) (not P26) ...



As S-expressions

Formula \Rightarrow *AtomicFormula*

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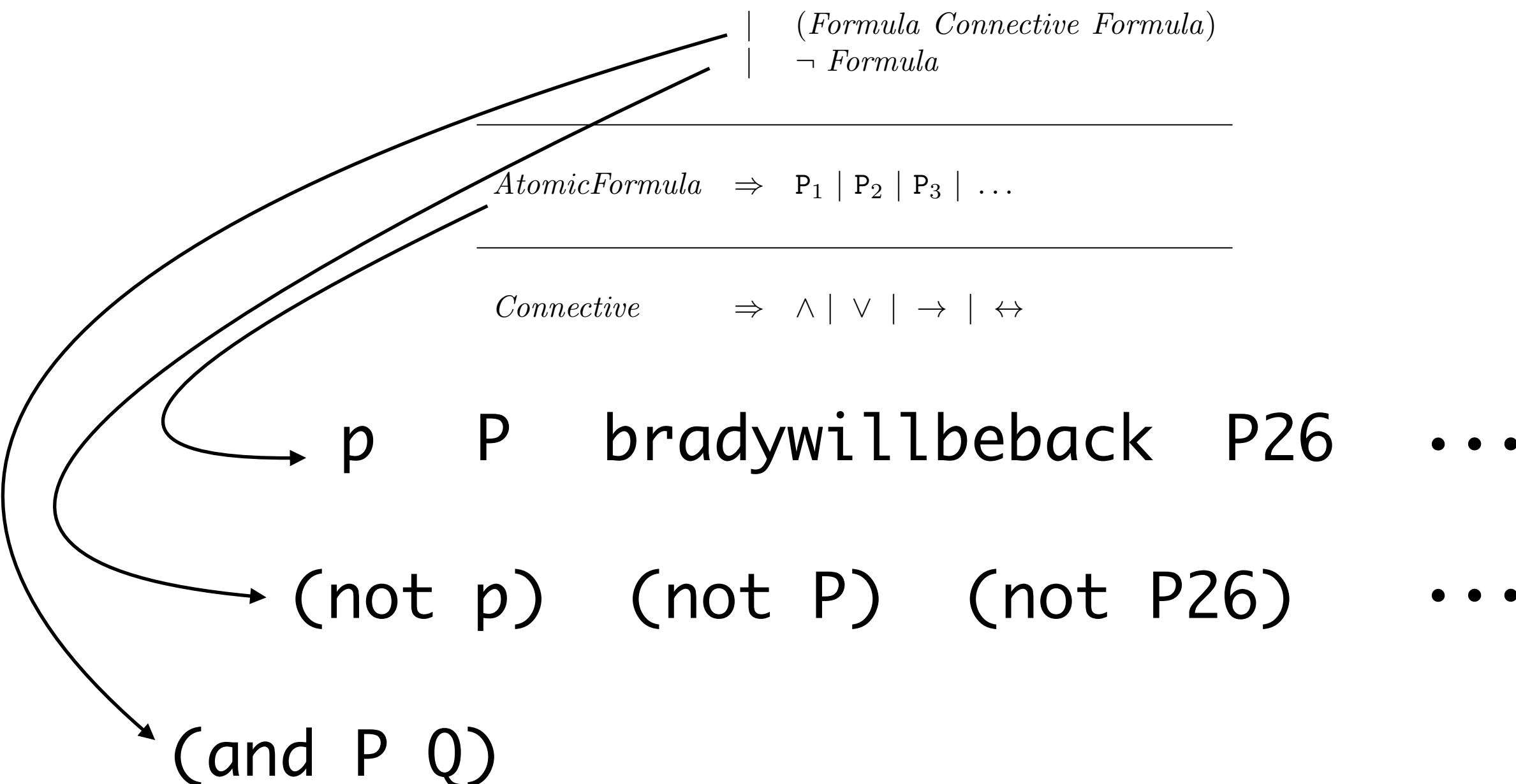
AtomicFormula \Rightarrow P₁ | P₂ | P₃ | ...

Connective \Rightarrow \wedge | \vee | \rightarrow | \leftrightarrow

p P bradywillbeback P26 ...

(not p) (not P) (not P26) ...

(and P Q)



As S-expressions

Formula \Rightarrow *AtomicFormula*

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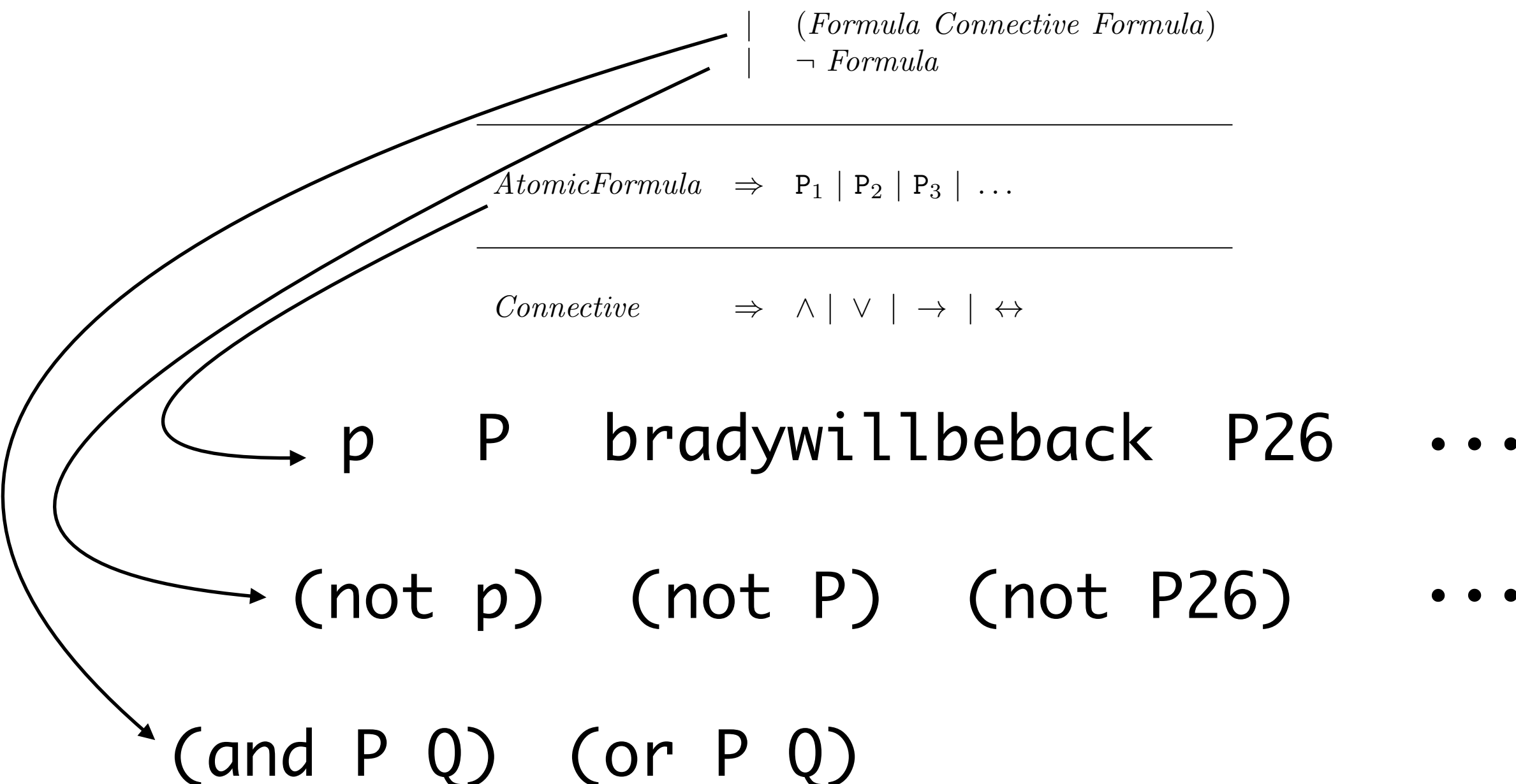
AtomicFormula \Rightarrow P₁ | P₂ | P₃ | ...

Connective \Rightarrow \wedge | \vee | \rightarrow | \leftrightarrow

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(not p) (not P) (not P26) ...

(and P Q) (or P Q)



As S-expressions

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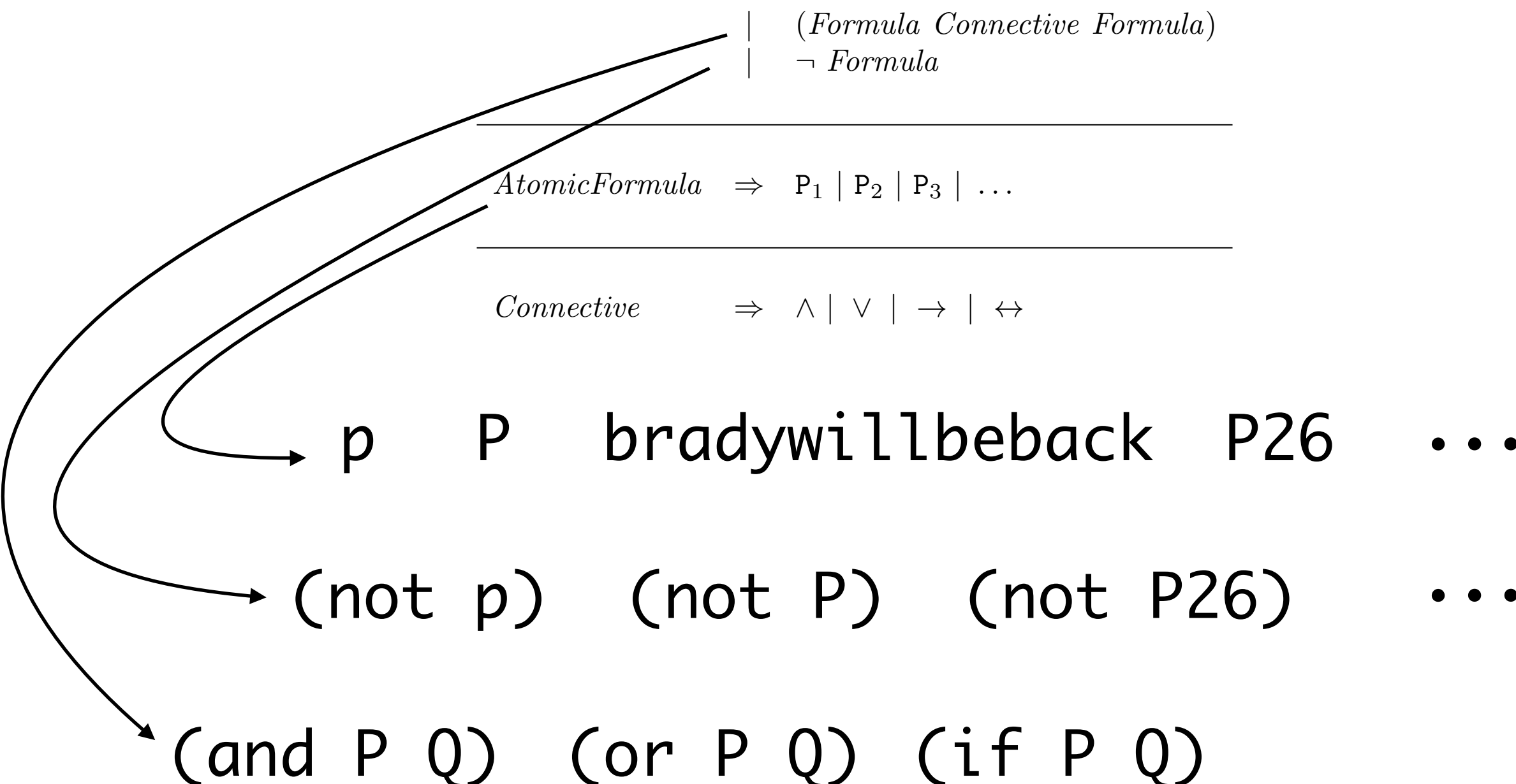
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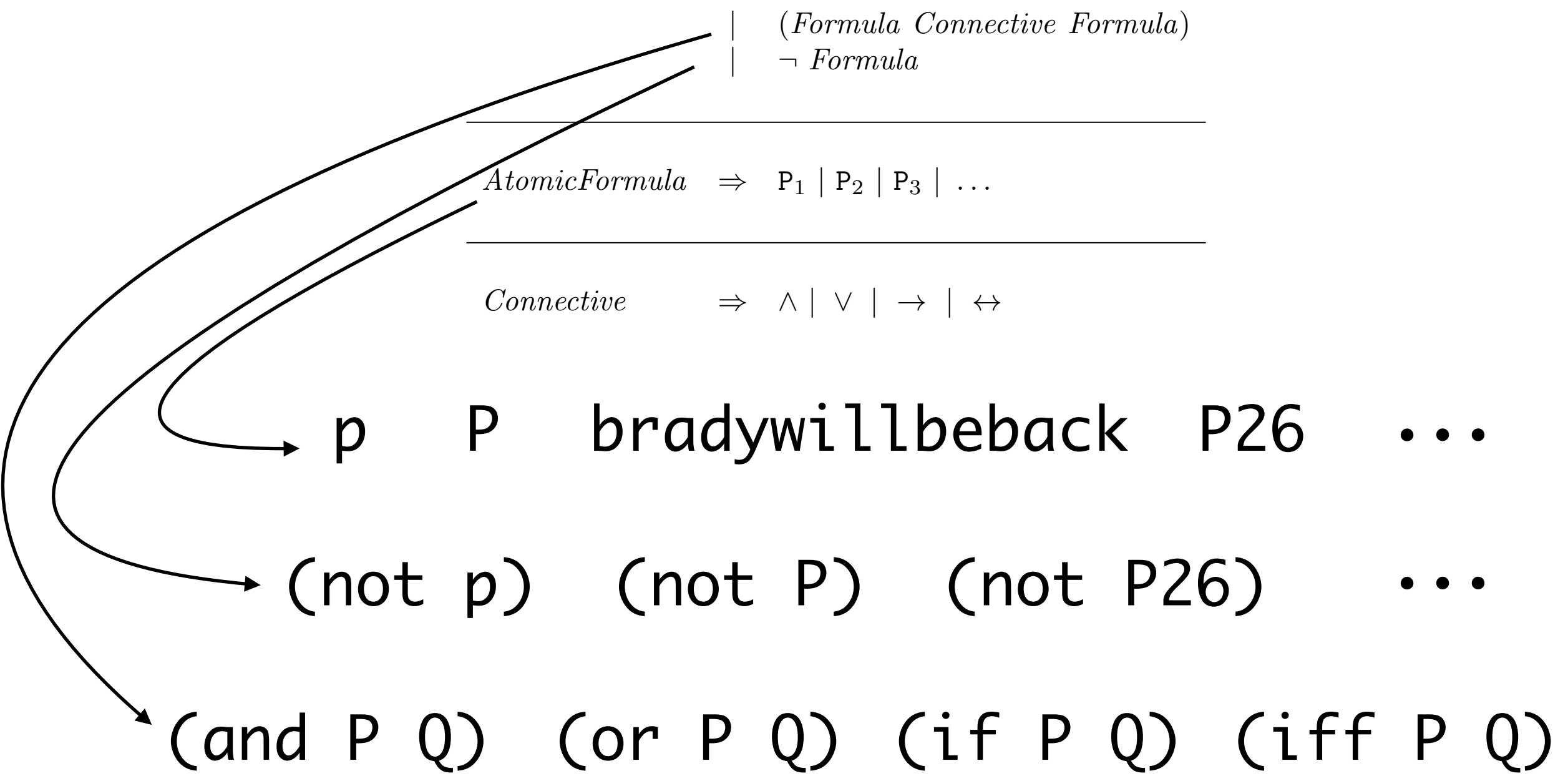
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p P bradywillbeback P26 ...

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(and P Q) (or P Q) (if P Q) (iff P Q)



Better Formal Language: Pure Predicate Calculus (presented via formal grammar)

$$\begin{array}{lcl} \textit{Formula} & \Rightarrow & \textit{AtomicFormula} \\ & | & (\textit{Formula} \textit{Connective} \textit{Formula}) \\ & | & \neg \textit{Formula} \end{array}$$

$$\textit{AtomicFormula} \Rightarrow (\textit{Predicate} \textit{Term}_1 \dots \textit{Term}_k)$$

$$\begin{array}{lcl} \textit{Term} & \Rightarrow & (\textit{Function} \textit{Term}_1 \dots \textit{Term}_k) \\ & | & \textit{Constant} \\ & | & \textit{Variable} \end{array}$$

$$\textit{Connective} \Rightarrow \wedge \mid \vee \mid \rightarrow \mid \leftrightarrow$$

$$\begin{array}{lcl} \textit{Predicate} & \Rightarrow & P_1 \mid P_2 \mid P_3 \dots \\ \textit{Constant} & \Rightarrow & c_1 \mid c_2 \mid c_3 \dots \\ \textit{Variable} & \Rightarrow & v_1 \mid v_2 \mid v_3 \dots \\ \textit{Function} & \Rightarrow & f_1 \mid f_2 \mid f_3 \dots \end{array}$$

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Exercise: Is this language also Roger-decidable? Prove it!

“NYS I” Revisited

Given the statements

$$\neg a \vee \neg b$$

$$b$$

$$c \rightarrow a$$

which one of the following statements is provable?

$$c$$

$$\neg b$$

$$\neg c$$

$$h$$

$$a$$

none of the above

“NYS I” Revisited

Given the statements

$$\neg a \vee \neg b$$

b

$$c \rightarrow a$$

which one of the following statements is provable?

c

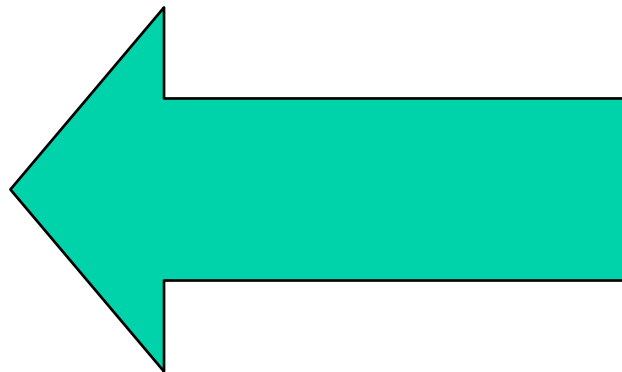
$\neg b$

$\neg c$

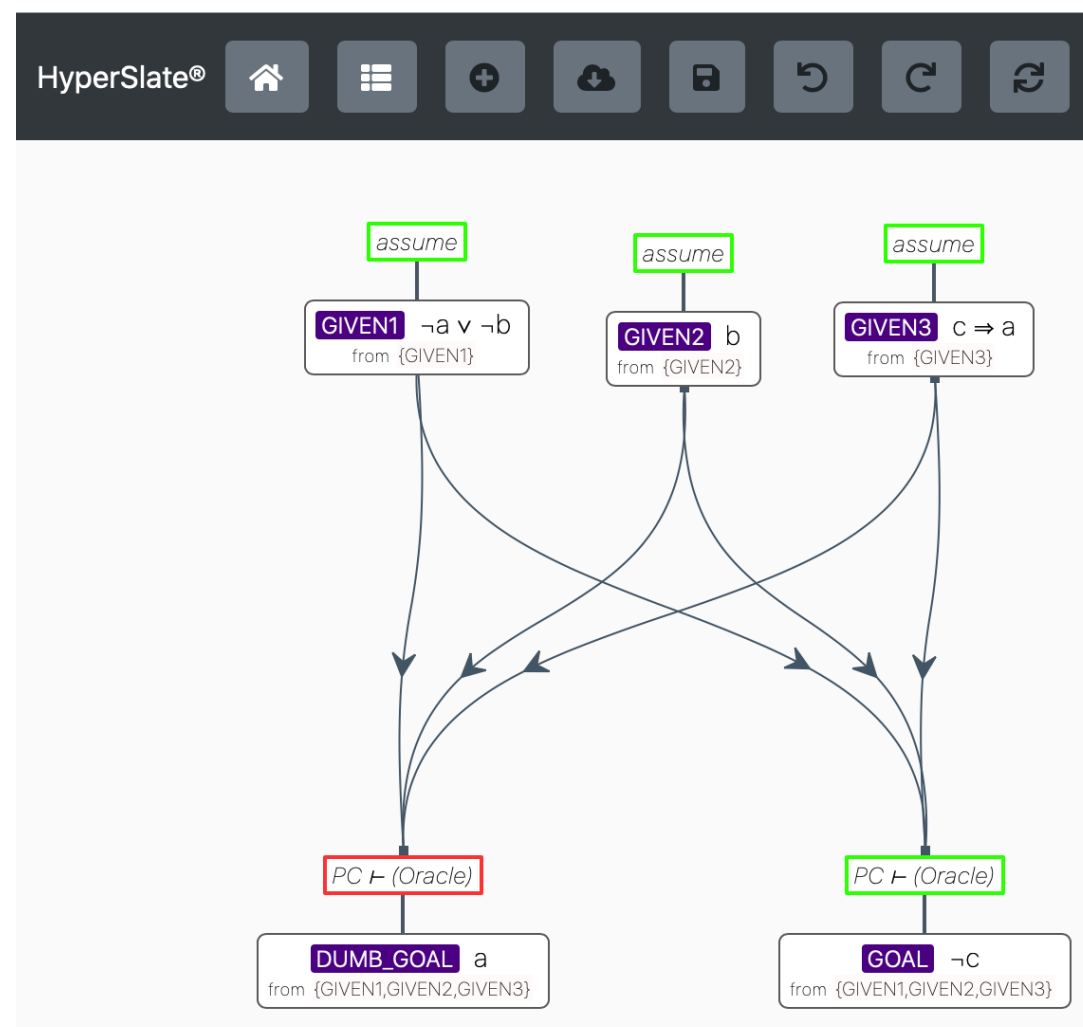
h

a

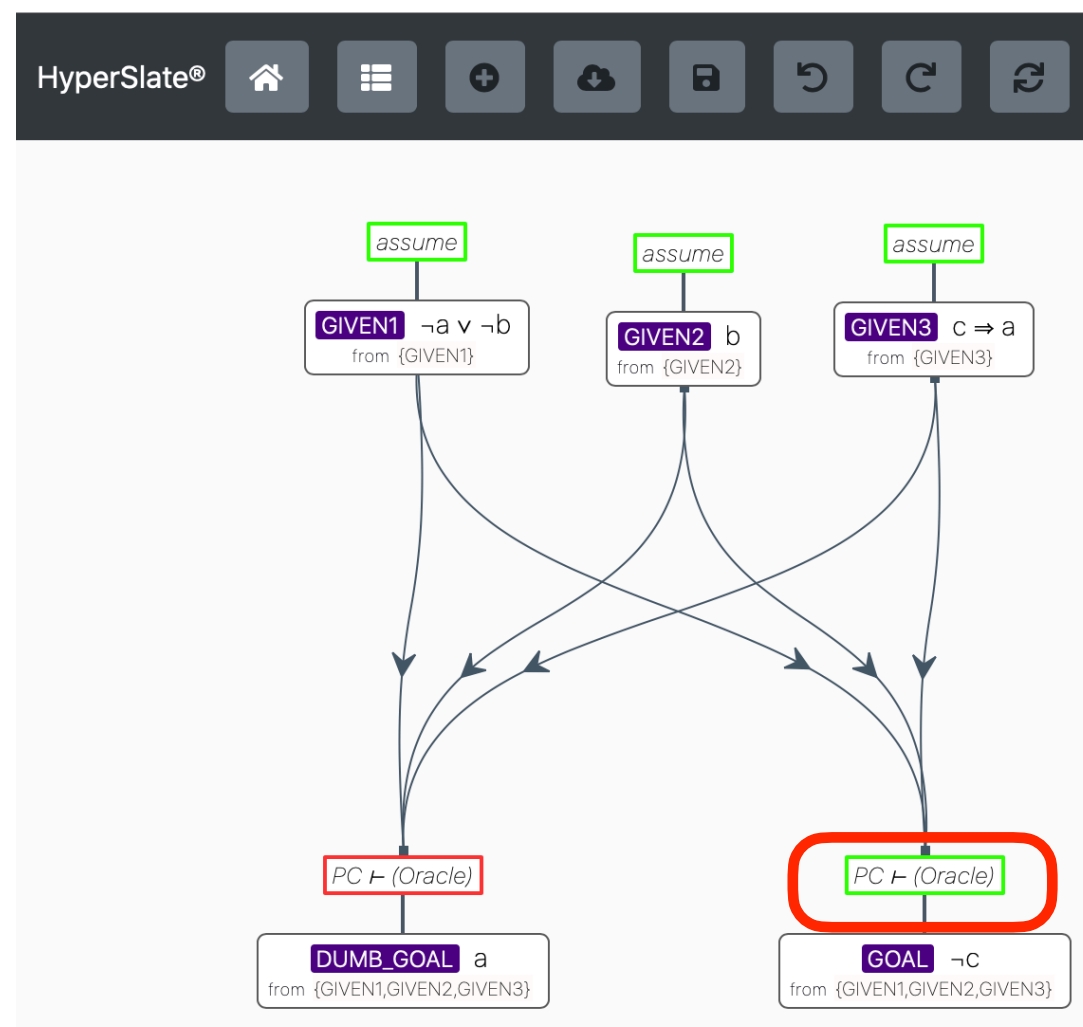
none of the above



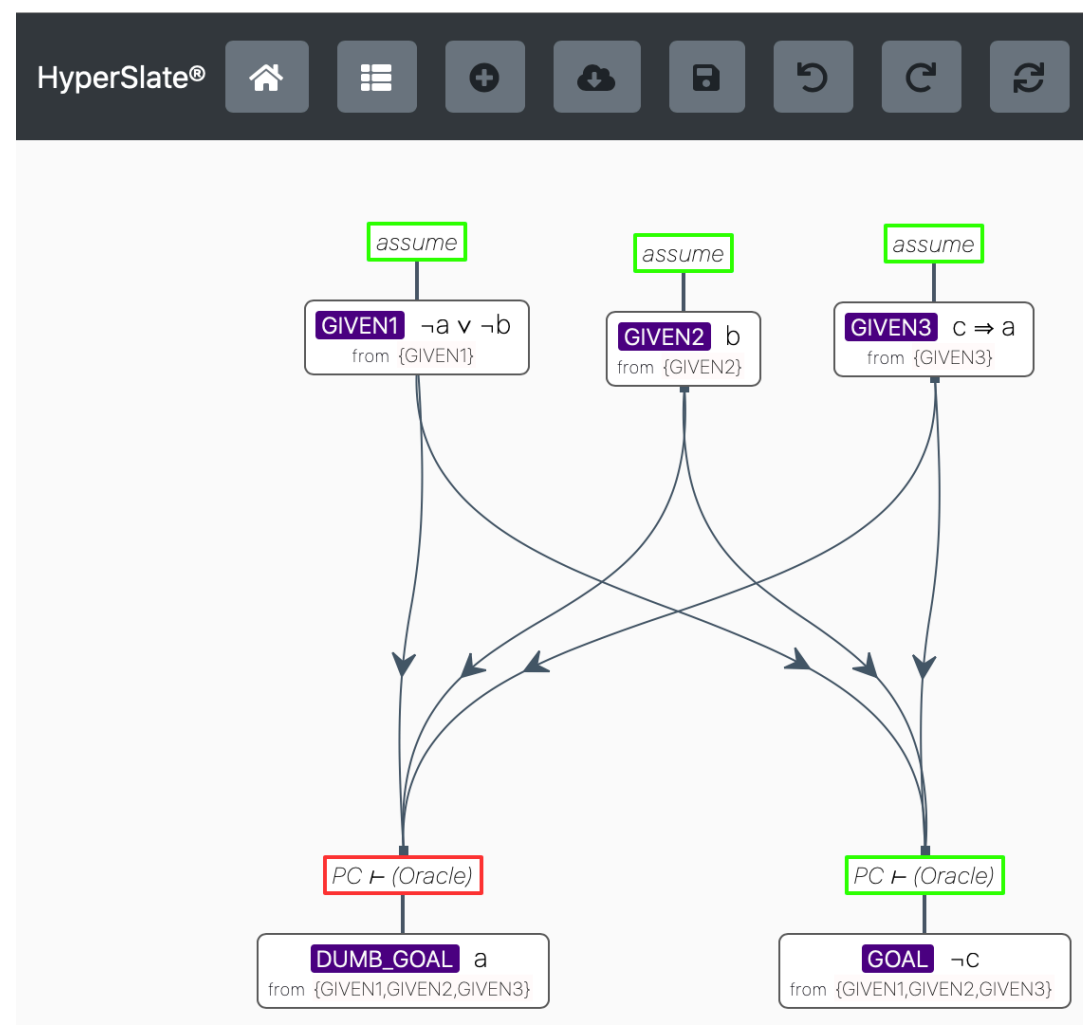
Our First Rule of Inference (= Inference Schema): PC (Entailment) Oracle



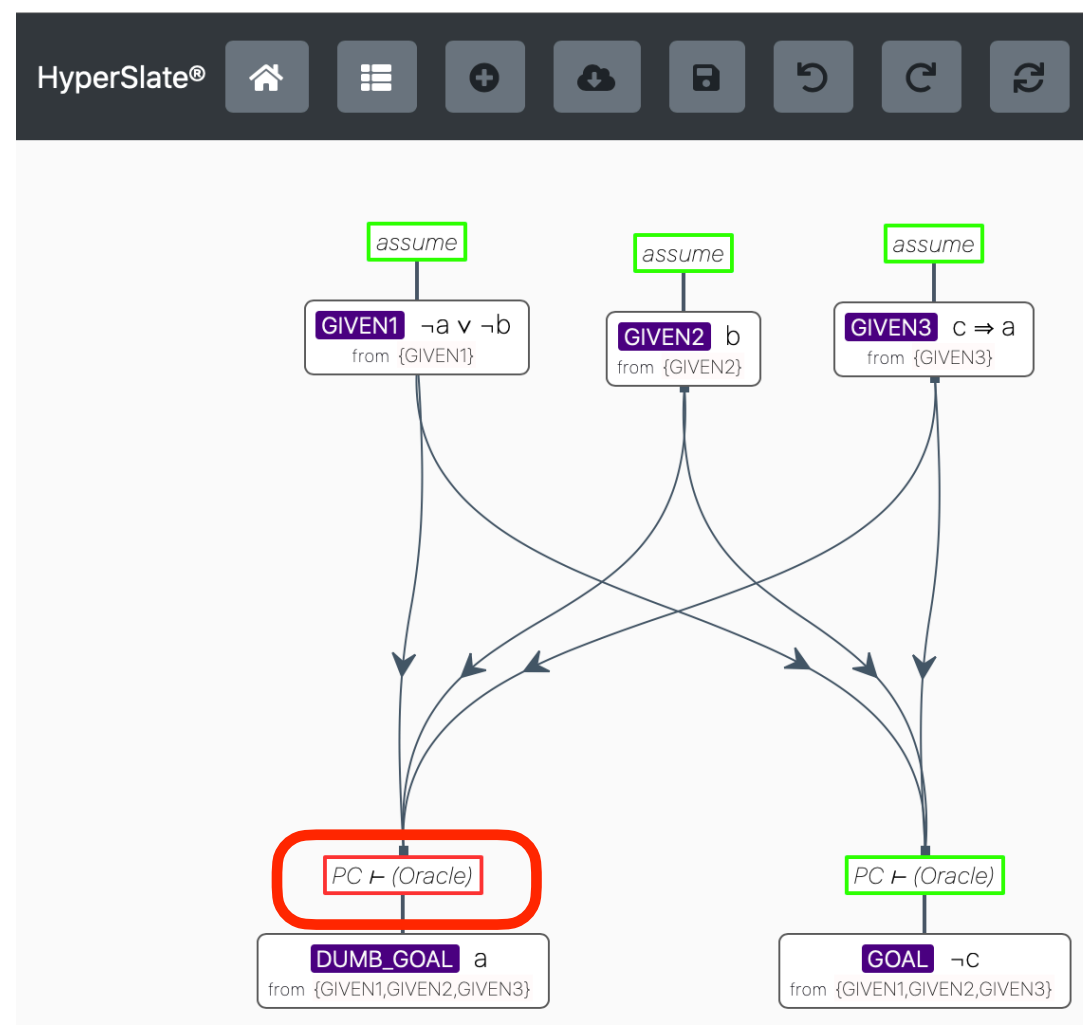
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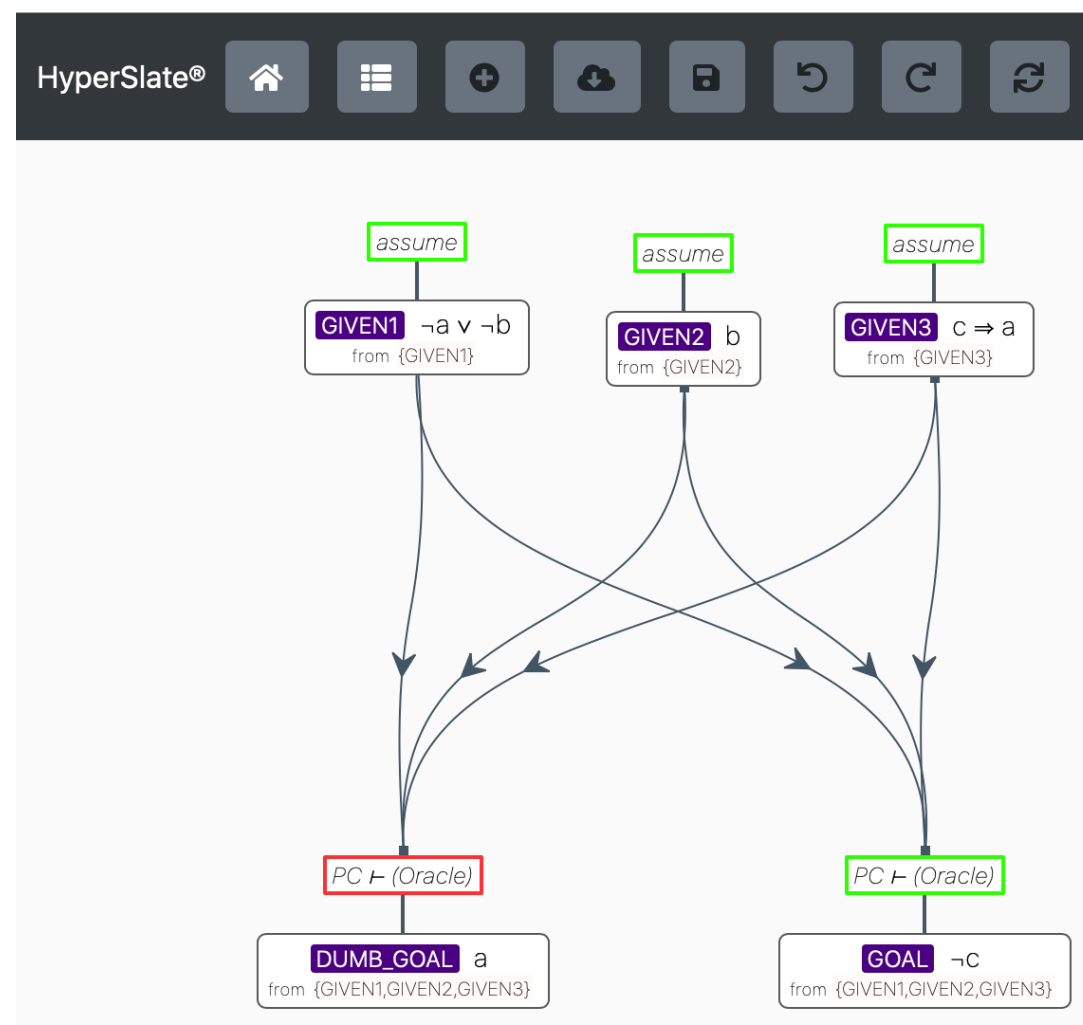
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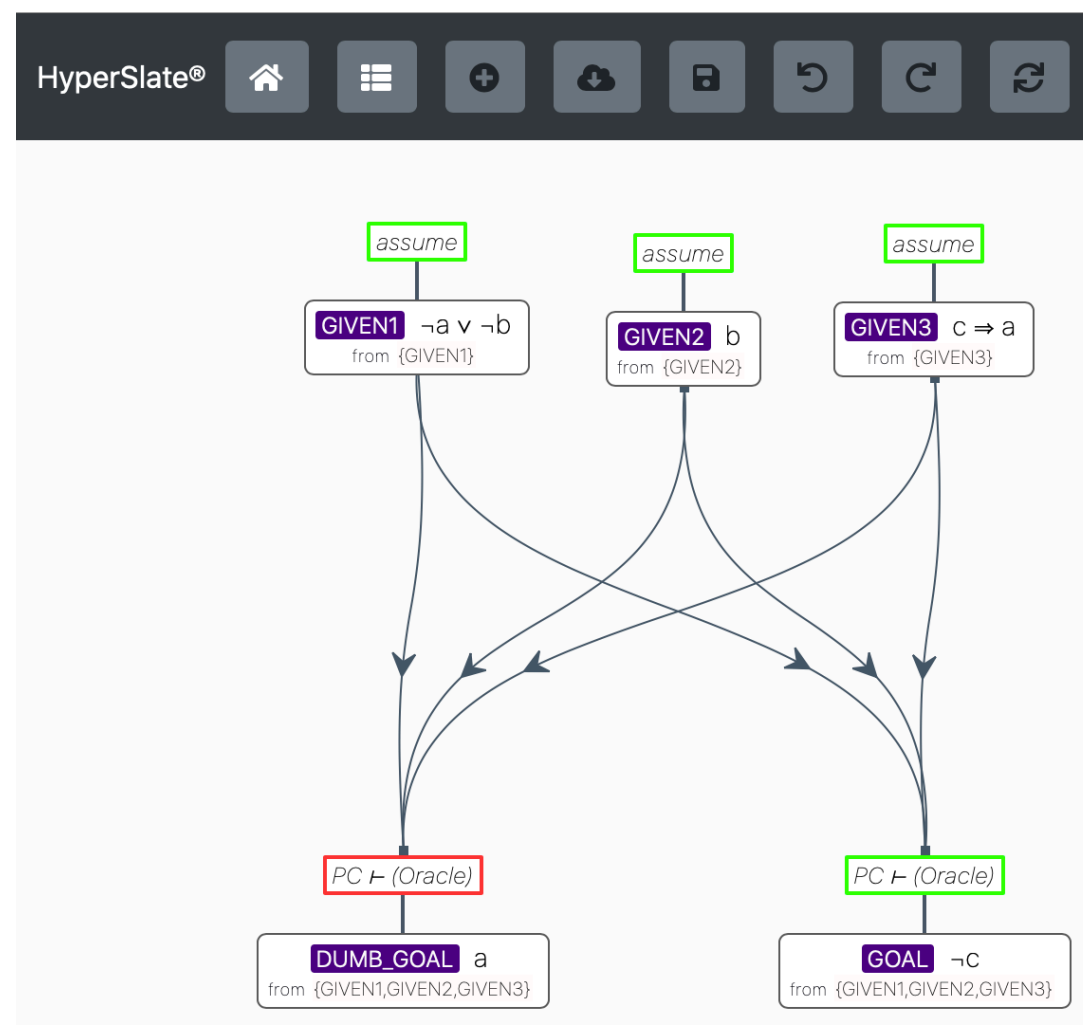
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Our First Rule of Inference (= Inference Schema): PC (Entailment) Oracle



“NYS 3” Revisited

Given the statements

$$\neg\neg c$$

$$c \rightarrow a$$

$$\neg a \vee b$$

$$b \rightarrow d$$

$$\neg(d \vee e)$$

which one of the following statements are provable?

$$\neg c$$

$$e$$

$$h$$

$$\neg a$$

all of the above

“NYS 3” Revisited

Given the statements

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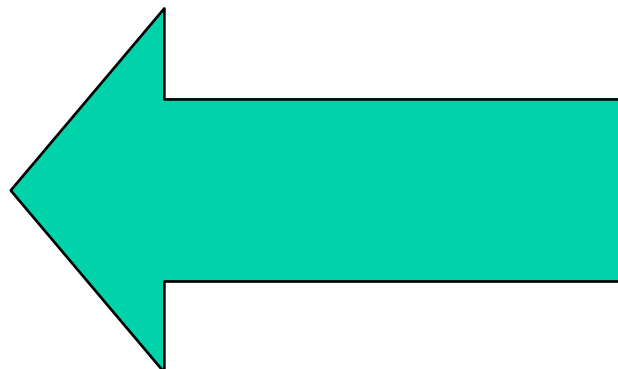
$$\neg c$$

$$e$$

$$h$$

$$\neg a$$

all of the above



“NYS 3” Revisited

Given the statements

$\neg\neg c$

$c \rightarrow a$

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$b \rightarrow d$

$\neg(d \vee e)$

Show in HyperSlate® that each of the first four options can be proved using the PC entailment oracle.

which one of the following statements are provable?

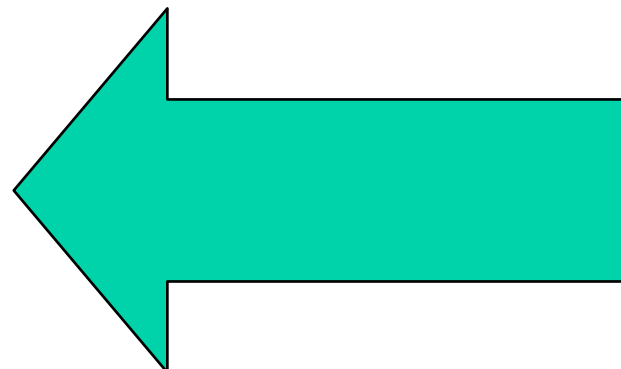
$\neg c$

e

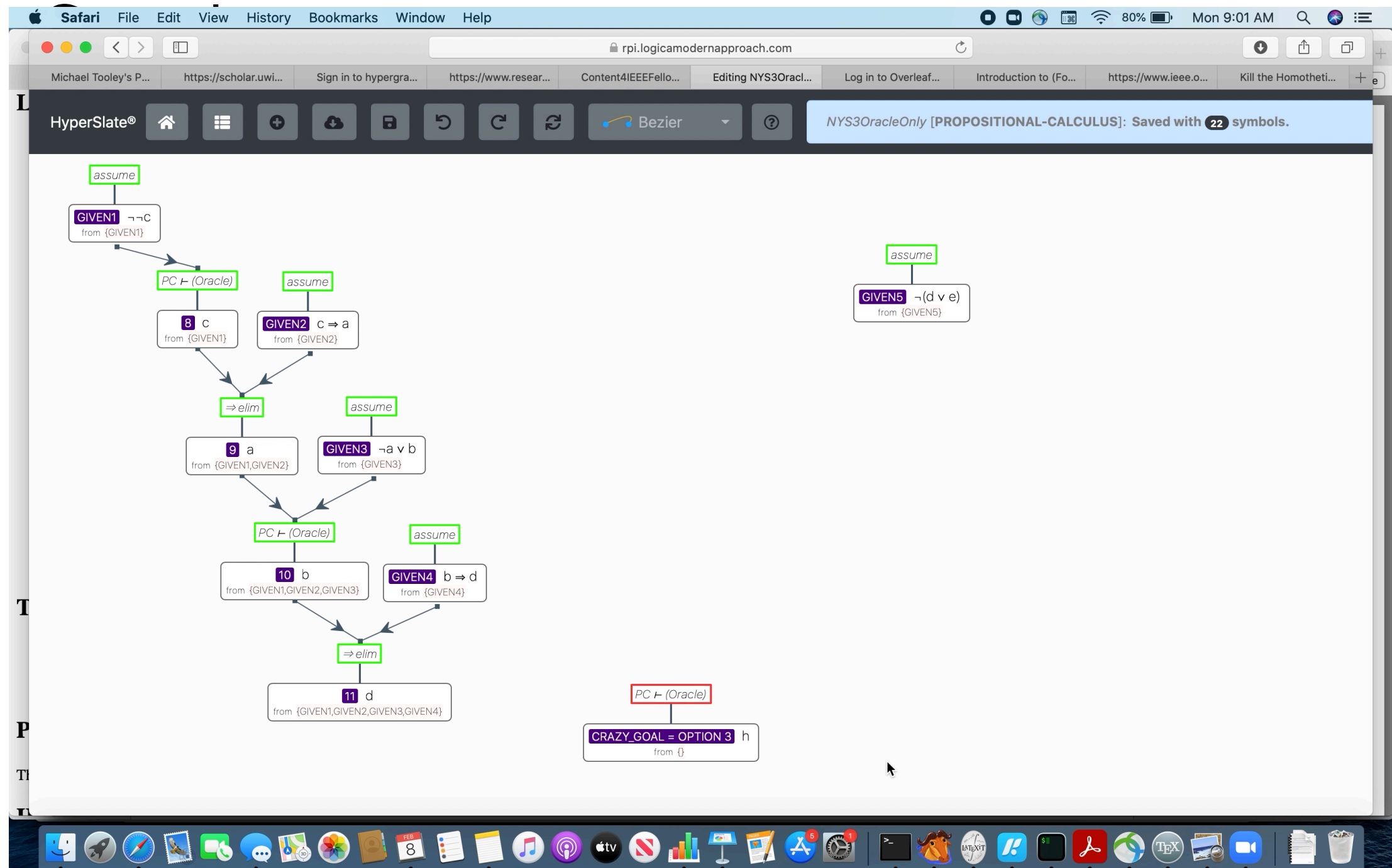
h

$\neg a$

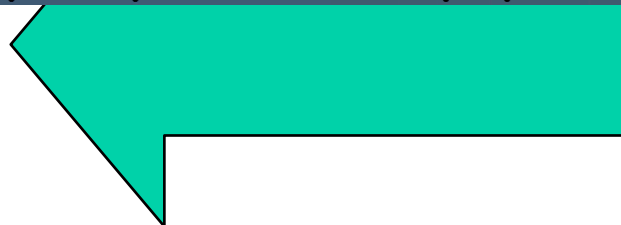
all of the above



“NYS 3” Revisited



all of the above



Det er en ære å lære formell logikk!

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Part II of Class