The Liar; Russell's Paradox; *Toward* Thoraf's Paradox

Selmer Bringsjord

Rensselaer AI & Reasoning (RAIR) Lab
Department of Cognitive Science
Department of Computer Science
Lally School of Management & Technology
Rensselaer Polytechnic Institute (RPI)
Troy, New York 12180 USA

IFLAI1
3/2/2023

R A I R
Rensselaer AI and Reasoning Lab

Logistics …

Test 1 grades now appear in your HG® account, in “My Progression” (but not Test 2 and Test 3).

Test 1 grades now appear in your HG® account, in "My Progression" (but not Test 2 and Test 3).

If you wish to attempt "resurrection" from a Test-1 grade of C (or nothing), pls email me bc now the underlying process for this has been implemented.

Also, last class I saw a proof or two with nodes having no assumption {•} sets visible. I need to get to the bottom of that, so please see me up front and/or come to office hours today (now 3:30–5:15).

Types of Paradoxes

- Deductive Paradoxes
- Inductive Paradoxes — coming (e.g. The Lottery Paradox & The St Petersburg Paradox)

Types of Paradoxes

First:

- Deductive Paradoxes
- Inductive Paradoxes — coming (e.g. The Lottery Paradox & The St Petersburg Paradox)

The Liar (Paradox) …

The (Economical) Liar

The (Economical) Liar

L: This sentence is false.

The (Economical) Liar

L: This sentence is false.

If T(**L)** then ¬T(**L**)

The (Economical) Liar

L: This sentence is false.

If T(**L)** then ¬T(**L**)

If ¬T(**L**) then T(**L)**

The (Economical) Liar

L: This sentence is false.

If T(**L)** then ¬T(**L**)

If ¬T(**L**) then T(**L)**

T(**L**) iff (i.e., if & only if) ¬T(L)

The (Economical) Liar

L: This sentence is false.

If T(**L)** then ¬T(**L**)

If ¬T(**L**) then T(**L)**

T(**L**) iff (i.e., if & only if) ¬T(L)

Contradiction!

The (Verbose) Liar — With a Twist

The (Verbose) Liar — With a Twist

Theorem: 2+2 = 5.

The (Verbose) Liar — With a Twist

Theorem: 2+2 = 5.

Proof: Set:

The (Verbose) Liar — With a Twist

Theorem: 2+2 = 5.

Proof: Set:

L: This sentence is false.

The (Verbose) Liar — With a Twist

Theorem: 2+2 = 5.

Proof: Set:

L: This sentence is false.

L is either true or false. Suppose that it's true. Then since what it says is that it's false, it *is* false; i.e., **L** is false, on this supposition. So we've proved that if **L** is true, L is false. Now suppose instead that **L** is false. Then since it says that it's false, it's true; i.e., **L** is true, on our current supposition. We have thus proved that if **L** is false, **L** is true. Combining the conditionals we've proved yields this: **L** is true if and only if **L** is false, which is a contradiction. (*P* if and only if ¬*P* is logically equivalent to *P* and ¬*P*.) By inference schema *explosion*, it follows that 2+2 = 5. **QED**

Outlawing Self-Referential Sentences Isn't the Answer!

Outlawing Self-Referential Sentences Isn't the Answer!

- For the following sentences, e.g., are perfectly innocuous, and obviously true or false (only) as the case may be, without complication.

Outlawing Self-Referential Sentences Isn't the Answer!

- For the following sentences, e.g., are perfectly innocuous, and obviously true or false (only) as the case may be, without complication.
  - `This sentence is a sentence.`

Outlawing Self-Referential Sentences Isn't the Answer!

- For the following sentences, e.g., are perfectly innocuous, and obviously true or false (only) as the case may be, without complication.
  - `This sentence is a sentence.`
  - `This sentence contains the letter ‘r’.`

Outlawing Self-Referential Sentences Isn't the Answer!

- For the following sentences, e.g., are perfectly innocuous, and obviously true or false (only) as the case may be, without complication.
  - `This sentence is a sentence.`
  - `This sentence contains the letter ‘r’.`
  - `This sentence has more than three letters in it.`

Outlawing Self-Referential Sentences Isn't the Answer!

- For the following sentences, e.g., are perfectly innocuous, and obviously true or false (only) as the case may be, without complication.
  - `This sentence is a sentence.`
  - `This sentence contains the letter ‘r’.`
  - `This sentence has more than three letters in it.`
  - `This sentence ends with a period, starts with a capital ’T’,  and has more than two words.`

Outlawing Self-Referential Sentences Isn't the Answer!

- For the following sentences, e.g., are perfectly innocuous, and obviously true or false (only) as the case may be, without complication.
  - `This sentence is a sentence.`
  - `This sentence contains the letter ‘r’.`
  - `This sentence has more than three letters in it.`
  - `This sentence ends with a period, starts with a capital ’T’,  and has more than two words.`
  - ...

Outlawing Self-Referential Sentences Isn't the Answer!

Outlawing Self-Referential Sentences Isn't the Answer!

Box 1

Outlawing Self-Referential Sentences Isn't the Answer!

Box 1

Box 2

Outlawing Self-Referential Sentences Isn't the Answer!

Box 1

The sentence in Box 2 is true.

Box 2

The sentence in Box 1 is false.

Outlawing Self-Referential Sentences Isn't the Answer!

Box 1

The sentence in Box 2 is true.

Neither sentence is self-referential.

Box 2

The sentence in Box 1 is false.

Outlawing Self-Referential Sentences Isn't the Answer!

Box 1		Box 2
The sentence in Box 2 is true.	Neither sentence is self-referential.	The sentence in Box 1 is false.

Outlawing Self-Referential Sentences Isn't the Answer!

Box 1		Box 2
The sentence in Box 2 is true.	Neither sentence is self-referential.	The sentence in Box 1 is false.

Suppose that the sentence in Box 1 is true. Then the sentence in Box 2 is true (because the sentence in Box 1 says that that sentence is true). But then the sentence in Box 1 is false (because the sentence in Box 2 says that that sentence is false). So, if the sentence in Box 1 is true, it's false. On the other hand, by parallel deduction, if the sentence in Box 1 is false, the sentence in Box 1 is true. (Make sure you work out and verify the reasoning that establishes the previous sentence.) We thus have again a contradiction: The sentence in Box 1 is true if and only if it's not true.

Well do you have a solution, Selmer?

Well do you have a solution, Selmer?

Of course :). But …

Well do you have a solution, Selmer?

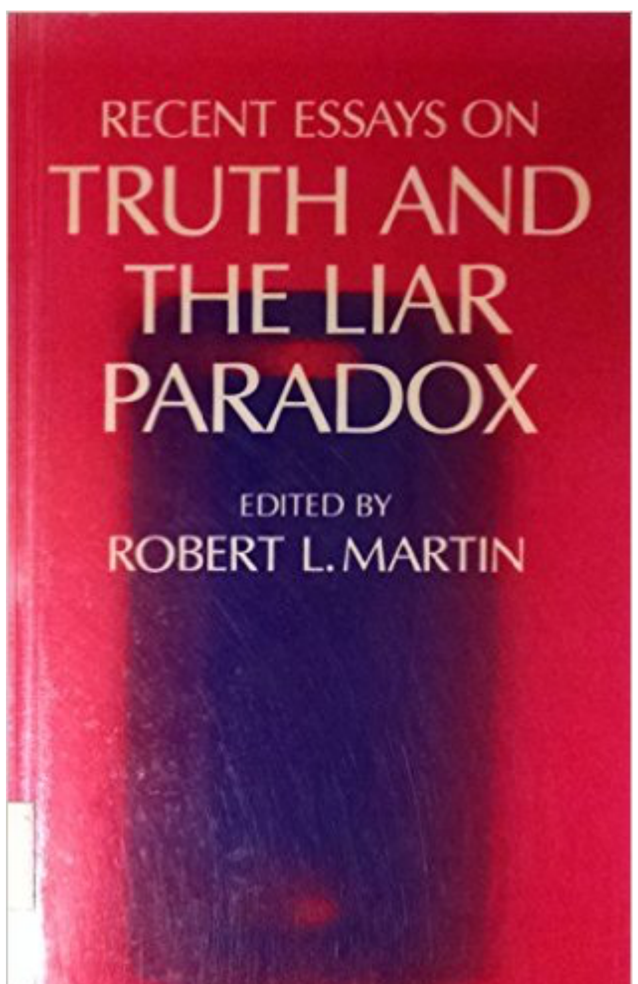
Of course :). But …

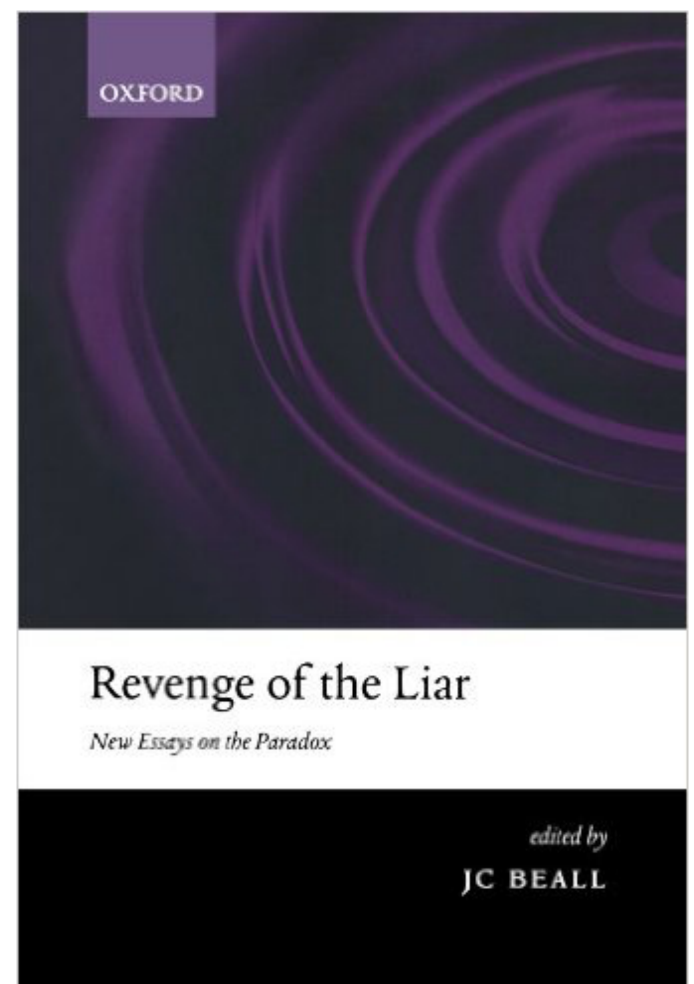
Further Reading …

Well do you have a solution, Selmer?

Of course :). But …

Further Reading …





Russell's Paradox …

Friday's Hill, Haslemere, 16 June 1902

Dear colleague,

For a year and a half I have been acquainted with your *Grundgesetze der Arithmetik*, but it is only now that I have been able to find the time for the thorough study I intended to make of your work. I find myself in complete agreement with you in all essentials, particularly when you reject any psychological element [[Moment]] in logic and when you place a high value upon an ideography [[Begriffsschrift]] for the foundations of mathematics and of formal logic, which, incidentally, can hardly be distinguished. With regard to many particular questions, I find in your work discussions, distinctions, and definitions that one seeks in vain in the works of other logicians. Especially so far as function is concerned (§ 9 of your *Begriffsschrift*), I have been led on my own to views that are the same even in the details. There is just one point where I have encountered a difficulty. You state (p. 17 [[p. 23 above]]) that a function, too, can act as the indeterminate element. This I formerly believed, but now this view seems doubtful to me because of the following contradiction. Let w be the predicate: to be a predicate that cannot be predicated of itself. Can w be predicated of itself? From each answer its opposite follows. Therefore we must conclude that w is not a predicate. Likewise there is no class (as a totality) of those classes which, each taken as a totality, do not belong to themselves. From this I conclude that under certain circumstances a definable collection [[Menge]] does not form a totality.

I am on the point of finishing a book on the principles of mathematics and in it I should like to discuss your work very thoroughly.[1] I already have your books or shall buy them soon, but I would be very grateful to you if you could send me reprints of your articles in various periodicals. In case this should be impossible, however, I will obtain them from a library.

The exact treatment of logic in fundamental questions, where symbols fail, has remained very much behind; in your works I find the best I know of our time, and therefore I have permitted myself to express my deep respect to you. It is very regrettable that you have not come to publish the second volume of your *Grundgesetze*; I hope that this will still be done.

Very respectfully yours,

BERTRAND RUSSELL

The above contradiction, when expressed in Peano's ideography, reads as follows:

$$w = \mathrm{cls} \cap x \ni (x \sim\varepsilon\, x) .\supset: w \,\varepsilon\, w \;.=.\; w \sim\varepsilon\, w.$$

Friday's Hill, Haslemere, 16 June 1902

Dear colleague,

For a year and a half I have been acquainted with your *Grundgesetze der Arithmetik*, but it is only now that I have been able to find the time for the thorough study I intended to make of your work. I find myself in complete agreement with you in all essentials, particularly when you reject any psychological element ⟦Moment⟧ in logic and when you place a high value upon an ideography ⟦Begriffsschrift⟧ for the foundations of mathematics and of formal logic, which, incidentally, can hardly be distinguished. With regard to many particular questions, I find in your work discussions, distinctions, and definitions that one seeks in vain in the works of other logicians. Especially so far as function is concerned (§ 9 of your *Begriffsschrift*), I have been led on my own to views that are the same even in the details. There is just one point where I have encountered a difficulty. You state (p. 17 ⟦p. 23 above⟧) that a function, too, can act as the indeterminate element. This I formerly believed, but now this view seems doubtful to me because of the following contradiction. Let w be the predicate: to be a predicate that cannot be predicated of itself. Can w be predicated of itself? From each answer its opposite follows. Therefore we must conclude that w is not a predicate. Likewise there is no class (as a totality) of those classes which, each taken as a totality, do not belong to themselves. From this I conclude that under certain circumstances a definable collection ⟦Menge⟧ does not form a totality.

I am on the point of finishing a book on the principles of mathematics and in it I should like to discuss your work very thoroughly.[1] I already have your books or shall buy them soon, but I would be very grateful to you if you could send me reprints of your articles in various periodicals. In case this should be impossible, however, I will obtain them from a library.

The exact treatment of logic in fundamental questions, where symbols fail, has remained very much behind; in your works I find the best I know of our time, and therefore I have permitted myself to express my deep respect to you. It is very regrettable that you have not come to publish the second volume of your *Grundgesetze*; I hope that this will still be done.

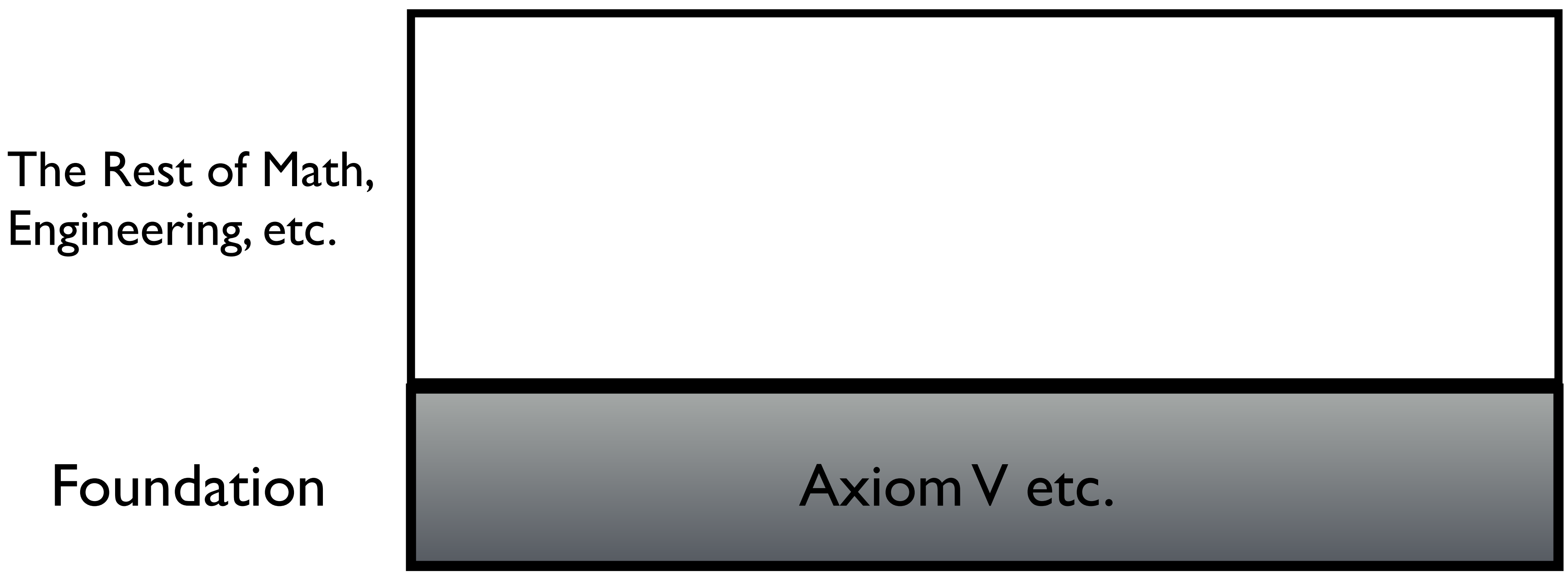
Very respectfully yours,

BERTRAND RUSSELL

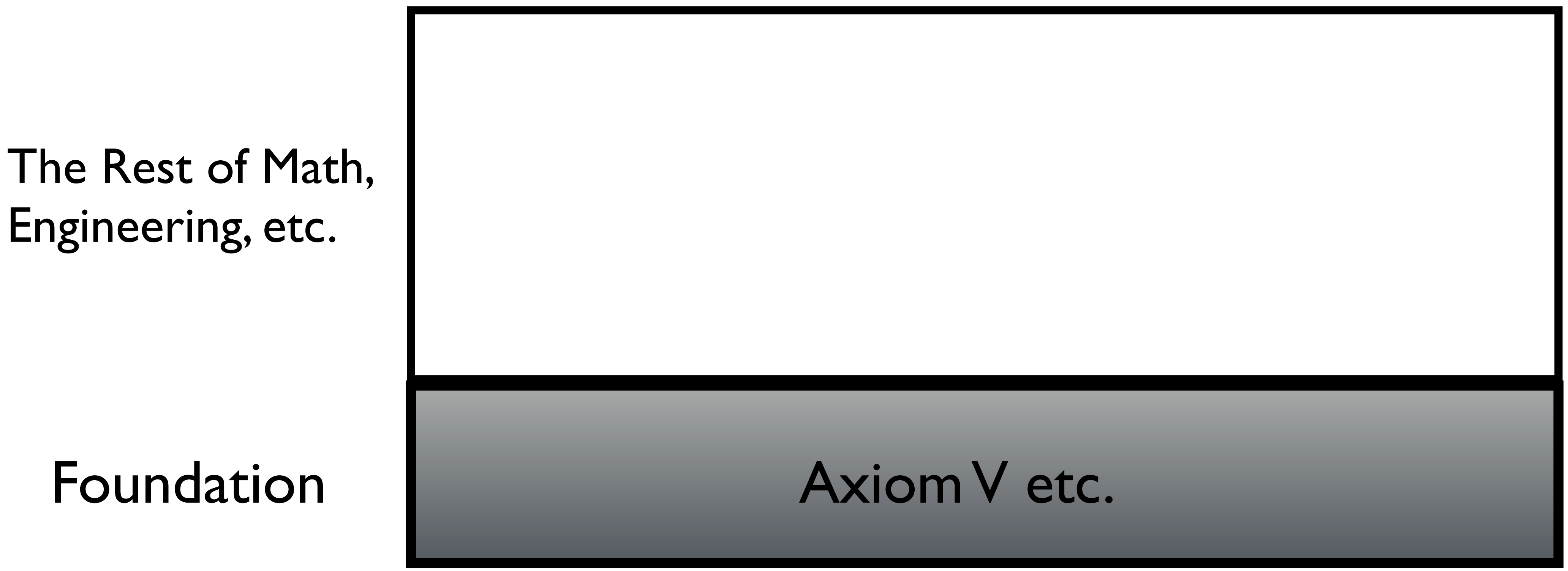
The above contradiction, when expressed in Peano's ideography, reads as follows:

$$w = \mathrm{cls} \cap x \ni (x \sim\varepsilon\, x) .\supset:\, w\,\varepsilon\, w .=.\, w \sim\varepsilon\, w.$$

The Foundation Crumbles

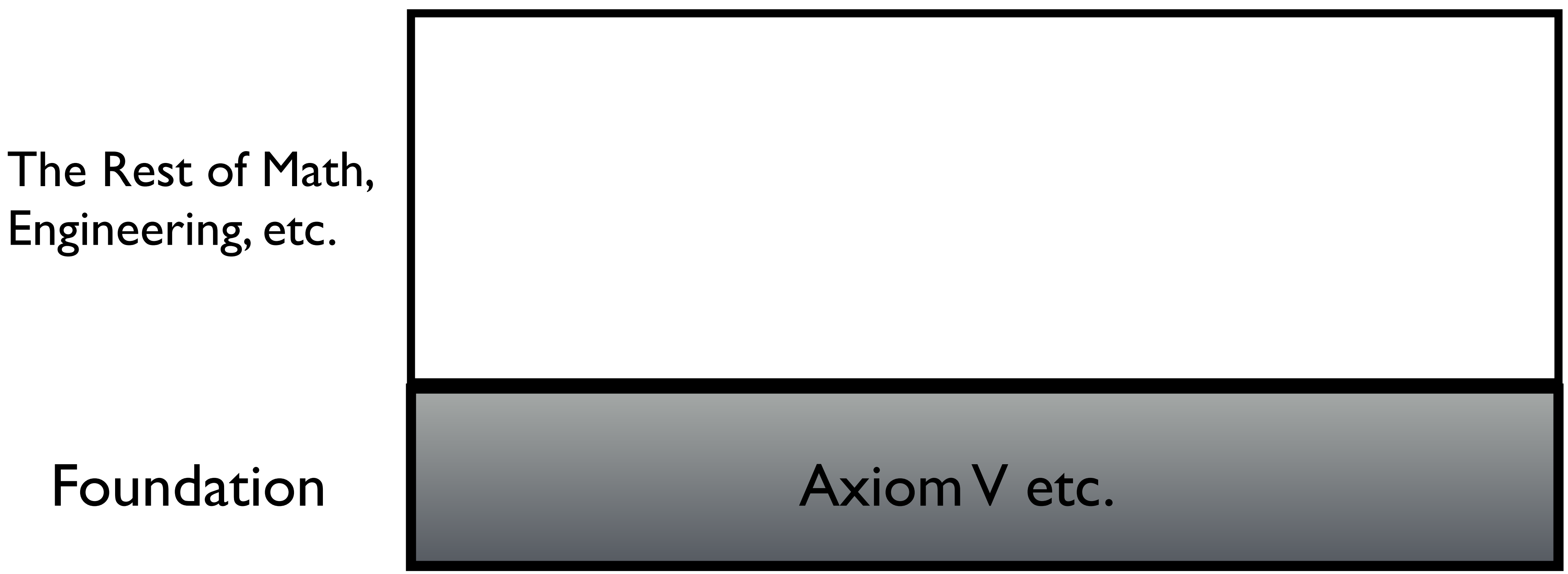


The Foundation Crumbles



$$\text{Axiom V} \qquad \exists x \forall y [y \in x \leftrightarrow \phi(y)]$$

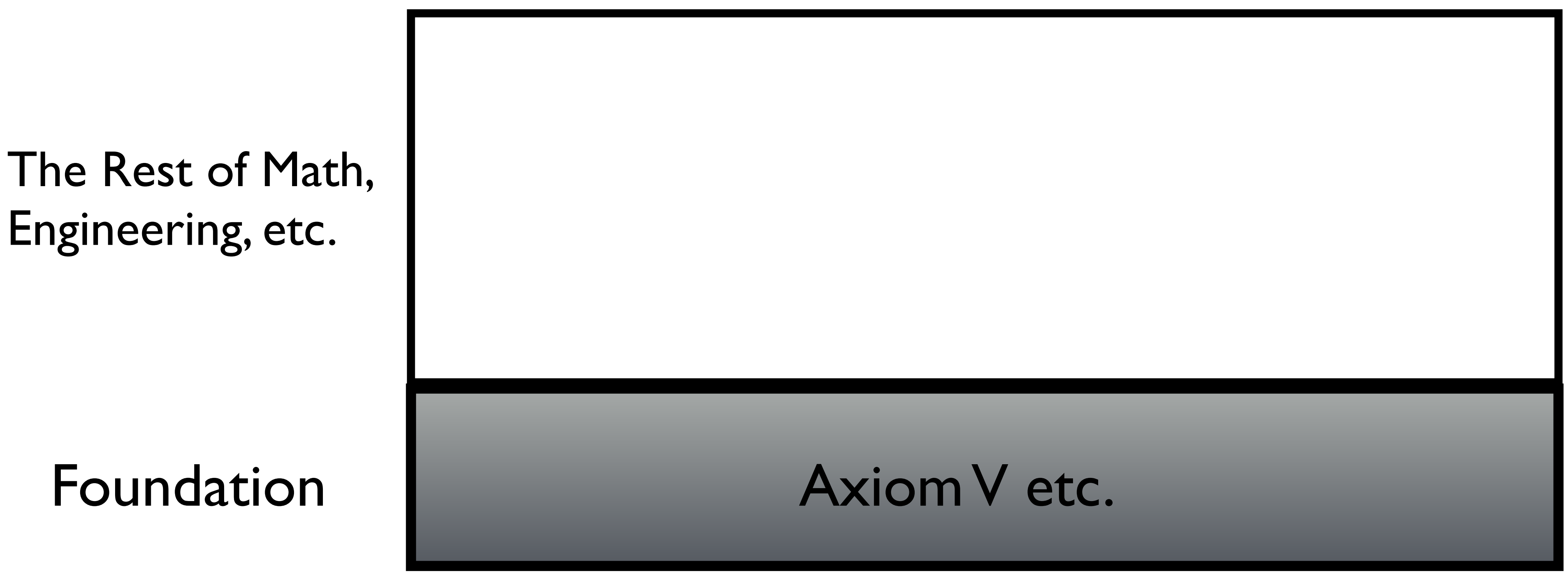
The Foundation Crumbles



$$\text{Axiom V} \qquad \exists x \forall y [y \in x \leftrightarrow \phi(y)]$$

a formula of arbitrary size in which the variable *y* is free; this formula ascribes a property to *y*

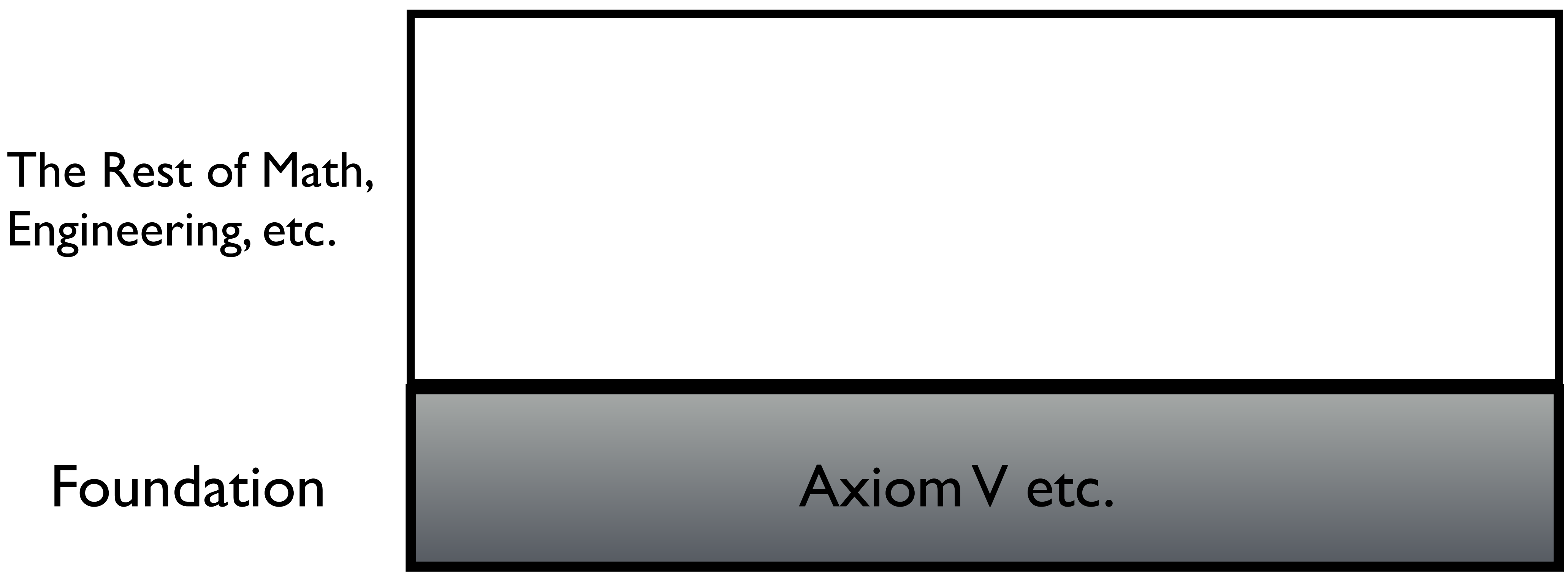
The Foundation Crumbles



$$\text{Axiom V} \qquad \exists x \forall y [y \in x \leftrightarrow \phi(y)]$$

a formula of arbitrary size in which the variable *y* is free; this formula ascribes a property to *y*

The Foundation Crumbles



The Foundation Crumbles

The Foundation Crumbles

The Rest of Math,
Engineering, etc.

Foundation

The Foundation Crumbles

The Rest of Math,
Engineering, etc.

Foundation

There was once a small town in Norway in which there was a barber who shaved all and only the men residing in the town who didn't shave themselves.

There was once a small town in Norway in which there was a barber who shaved all and only the men residing in the town who didn’t shave themselves.

There was once a small town in Norway in which there resided a barber who shaved all and only the men residing in the town who didn’t shave themselves.

There was once a small town in Norway in which there was a barber who shaved all and only the men residing in the town who didn't shave themselves.

There was once a small town in Norway in which there resided a barber who shaved all and only the men residing in the town who didn't shave themselves.

There was once a small town in Norway in which there resided a male barber who shaved all and only the men residing in the town who didn't shave themselves.

Such a situation is impossible!

Such a situation is impossible!

Proof: Let's assume for the sake of argument that such a situation can be. Without loss of generality, let the town be Lyngdal and the male Lyngdalian barber be Olaf. Either Olaf shaves himself or he doesn't. But either case leads straight to a contradiction. Therefore the situation is in fact impossible. Here we go …

Suppose Olaf shaves himself. Then it follows that he doesn't shave himself. Suppose on the other hand that Olaf doesn't shave himself. Then is follows that he does shave himself. Hence, Olaf shaves himself if and only if he doesn't shave himself, which is a contradiction. **QED**

Axiom V $\quad \exists x \forall y[y \in x \leftrightarrow \phi(y)]$

$$\text{Axiom V} \qquad \exists x \forall y[y \in x \leftrightarrow \phi(y)]$$

Russell's Theorem:

$$\text{Axiom V} \qquad \exists x \forall y[y \in x \leftrightarrow \phi(y)]$$

Russell's Theorem:

$$\vdash \neg \exists x \forall y(y \in x \leftrightarrow y \notin y)$$

$$\text{Axiom V} \qquad \exists x \forall y [y \in x \leftrightarrow \phi(y)]$$

Russell's Theorem:

$$\vdash \neg \exists x \forall y (y \in x \leftrightarrow y \notin y)$$

(Take that, Frege!)

Axiom V $\quad \exists x \forall y [y \in x \leftrightarrow \phi(y)]$

Russell's Theorem:

$$\vdash \neg \exists x \forall y (y \in x \leftrightarrow y \notin y)$$

(Take that, Frege!)

http://plato.stanford.edu/entries/russell-paradox/#HOTP

Friday's Hill, Haslemere, 16 June 1902

Dear colleague,

For a year and a half I have been acquainted with your *Grundgesetze der Arithmetik*, but it is only now that I have been able to find the time for the thorough study I intended to make of your work. I find myself in complete agreement with you in all essentials, particularly when you reject any psychological element [[Moment]] in logic and when you place a high value upon an ideography [[Begriffsschrift]] for the foundations of mathematics and of formal logic, which, incidentally, can hardly be distinguished. With regard to many particular questions, I find in your work discussions, distinctions, and definitions that one seeks in vain in the works of other logicians. Especially so far as function is concerned (§ 9 of your *Begriffsschrift*), I have been led on my own to views that are the same even in the details. There is just one point where I have encountered a difficulty. You state (p. 17 [[p. 23 above]]) that a function, too, can act as the indeterminate element. This I formerly believed, but now this view seems doubtful to me because of the following contradiction. Let w be the predicate: to be a predicate that cannot be predicated of itself. Can w be predicated of itself? From each answer its opposite follows. Therefore we must conclude that w is not a predicate. Likewise there is no class (as a totality) of those classes which, each taken as a totality, do not belong to themselves. From this I conclude that under certain circumstances a definable collection [[Menge]] does not form a totality.

I am on the point of finishing a book on the principles of mathematics and in it I should like to discuss your work very thoroughly.[1] I already have your books or shall buy them soon, but I would be very grateful to you if you could send me reprints of your articles in various periodicals. In case this should be impossible, however, I will obtain them from a library.

The exact treatment of logic in fundamental questions, where symbols fail, has remained very much behind; in your works I find the best I know of our time, and therefore I have permitted myself to express my deep respect to you. It is very regrettable that you have not come to publish the second volume of your *Grundgesetze*; I hope that this will still be done.

Very respectfully yours,

BERTRAND RUSSELL

The above contradiction, when expressed in Peano's ideography, reads as follows:

$$w = \mathrm{cls} \cap x \ni (x \sim\varepsilon\, x) . \supset : w \,\varepsilon\, w \,.=.\, w \sim\varepsilon\, w.$$

Friday's Hill, Haslemere, 16 June 1902

Dear colleague,

For a year and a half I have been acquainted with your *Grundgesetze der Arithmetik*, but it is only now that I have been able to find the time for the thorough study I intended to make of your work. I find myself in complete agreement with you in all essentials, particularly when you reject any psychological element [[Moment]] in logic and when you place a high value upon an ideography [[Begriffsschrift]] for the foundations of mathematics and of formal logic, which, incidentally, can hardly be distinguished. With regard to many particular questions, I find in your work discussions, distinctions, and definitions that one seeks in vain in the works of other logicians. Especially so far as function is concerned (§ 9 of your *Begriffsschrift*), I have been led on my own to views that are the same even in the details. There is just one point where I have encountered a difficulty. You state (p. 17 [[p. 23 above]]) that a function, too, can act as the indeterminate element. This I formerly believed, but now this view seems doubtful to me because of the following contradiction. Let w be the predicate: to be a predicate that cannot be predicated of itself. Can w be predicated of itself? From each answer its opposite follows. Therefore we must conclude that w is not a predicate. Likewise there is no class (as a totality) of those classes which, each taken as a totality, do not belong to themselves. From this I conclude that under certain circumstances a definable collection [[Menge]] does not form a totality.

I am on the point of finishing a book on the principles of mathematics and in it I should like to discuss your work very thoroughly.[1] I already have your books or shall buy them soon, but I would be very grateful to you if you could send me reprints of your articles in various periodicals. In case this should be impossible, however, I will obtain them from a library.

The exact treatment of logic in fundamental questions, where symbols fail, has remained very much behind; in your works I find the best I know of our time, and therefore I have permitted myself to express my deep respect to you. It is very regrettable that you have not come to publish the second volume of your *Grundgesetze*; I hope that this will still be done.

Very respectfully yours,

Bertrand Russell

The above contradiction, when expressed in Peano's ideography, reads as follows:

$$w = \mathrm{cls} \cap x \ni (x \sim\varepsilon\, x) .\supset:\, w\, \varepsilon\, w .=.\, w \sim\varepsilon\, w.$$

Friday's Hill, Haslemere, 16 June 1902

Dear colleague,

For a year and a half I have been acquainted with your *Grundgesetze der Arithmetik*, but it is only now that I have been able to find the time for the thorough study I intended to make of your work. I find myself in complete agreement with you in all essentials, particularly when you reject any psychological element [[Moment]] in logic and when you place a high value upon an ideography [[Begriffsschrift]] for the foundations of mathematics and of formal logic, which, incidentally, can hardly be distinguished. With regard to many particular questions, I find in your work discussions, distinctions, and definitions that one seeks in vain in the works of other logicians. Especially so far as function is concerned (§ 9 of your *Begriffsschrift*), I have been led on my own to views that are the same even in the details. There is just one point where I have encountered a difficulty. You state (p. 17 [[p. 23 above]]) that a function, too, can act as the indeterminate element. This I formerly believed, but now this view seems doubtful to me because of the following contradiction. Let w be the predicate: to be a predicate that cannot be predicated of itself. Can w be predicated of itself? From each answer its opposite follows. Therefore we must conclude that w is not a predicate. Likewise there is no class (as a totality) of those classes which, each taken as a totality, do not belong to themselves. From this I conclude that under certain circumstances a definable collection [[Menge]] does not form a totality.

I am on the point of finishing a book on the principles of mathematics and in it I should like to discuss your work very thoroughly.[1] I already have your books or shall buy them soon, but I would be very grateful to you if you could send me reprints of your articles in various periodicals. In case this should be impossible, however, I will obtain them from a library.

The exact treatment of logic in fundamental questions, where symbols fail, has remained very much behind; in your works I find the best I know of our time, and therefore I have permitted myself to express my deep respect to you. It is very regrettable that you have not come to publish the second volume of your *Grundgesetze*; I hope that this will still be done.

Very respectfully yours,

Bertrand Russell

The above contradiction, when expressed in Peano's ideography, reads as follows:

$$w = \mathrm{cls} \cap x \ni (x \sim\varepsilon\, x) .\supset\colon w \,\varepsilon\, w .=. w \sim\varepsilon\, w.$$

What about Thoraf’s paradox?

(Skolem’s Paradox)

(For a nice overview of Skolem’s Paradox, see https://plato.stanford.edu/entries/paradox-skolem.)

What about Thoraf's paradox?

(Skolem's Paradox)

3.8.4.1 Can First-Order Logic Capture Infinitude and Finitude?

Does the machinery introduced in the previous section enable us to show that the concepts of finitude and infinitude can be captured by suitable use of first-order logic? If so, how? We should first immediately sharpen this question, which as it stands is somewhat unclear. Let's first target the capturing of infinitude in FOL. Then our initial sharpening move is to stipulate that we are interested specifically in figuring out how we might use FOL to express that a set is countably infinite. (Recall that we defined what it is for a set to be countably infinite in §1.5.3.) In further sharpening of the intuitively expressed question that kicked off the present section, what shall be looking for is how to specify a set Φ that is such that a given interpretation

$$\mathfrak{I} \models \Phi \text{ iff domain } \mathscr{D} \text{ in } \mathfrak{I} \text{ is countably infinite}$$

where the set Φ contains only formulae in FOL. If we can somehow obtain such a set Φ, then we will have found a way to capture countable infinitude because the domain $\mathscr{D}$ here must be countably infinite. Can you meet this challenge, by drawing upon what was done in the previous section?

Now, what about finitude? Can it be captured by formulae in FOL? The question here can be taken to consist in the challenge to find a set Ψ such that a given interpretation

$$\mathfrak{I} \models \Psi \text{ iff domain } \mathscr{D} \text{ in } \mathfrak{I} \text{ is finite}$$

where, again, the set in question nce again contains only formulae in FOL.

(For a nice overview of Skolem's Paradox, see https://plato.stanford.edu/entries/paradox-skolem.)

3.8.4.1 Can First-Order Logic Capture Infinitude and Finitude?

Does the machinery introduced in the previous section enable us to show that the concepts of finitude and infinitude can be captured by suitable use of first-order logic? If so, how? We should first immediately sharpen this question, which as it stands is somewhat unclear. Let's first target the capturing of infinitude in FOL. Then our initial sharpening move is to stipulate that we are interested specifically in figuring out how we might use FOL to express that a set is countably infinite. (Recall that we defined what it is for a set to be countably infinite in §1.5.3.) In further sharpening of the intuitively expressed question that kicked off the present section, what shall be looking for is how to specify a set Φ that is such that a given interpretation

$$\mathfrak{I} \models \Phi \text{ iff domain } \mathscr{D} \text{ in } \mathfrak{I} \text{ is countably infinite}$$

where the set Φ contains only formulae in FOL. If we can somehow obtain such a set Φ, then we will have found a way to capture countable infinitude because the domain $\mathscr{D}$ here must be countably infinite. Can you meet this challenge, by drawing upon what was done in the previous section?

Now, what about finitude? Can it be captured by formulae in FOL? The question here can be taken to consist in the challenge to find a set Ψ such that a given interpretation

$$\mathfrak{I} \models \Psi \text{ iff domain } \mathscr{D} \text{ in } \mathfrak{I} \text{ is finite}$$

where, again, the set in question nce again contains only formulae in FOL.

3.8.4.1 Can First-Order Logic Capture Infinitude and Finitude?

Does the machinery introduced in the previous section enable us to show that the concepts of finitude and infinitude can be captured by suitable use of first-order logic? If so, how? We should first immediately sharpen this question, which as it stands is somewhat unclear. Let's first target the capturing of infinitude in FOL. Then our initial sharpening move is to stipulate that we are interested specifically in figuring out how we might use FOL to express that a set is countably infinite. (Recall that we defined what it is for a set to be countably infinite in §1.5.3.) In further sharpening of the intuitively expressed question that kicked off the present section, what shall be looking for is how to specify a set Φ that is such that a given interpretation

$$\boxed{\mathfrak{I} \models \Phi \text{ iff domain } \mathscr{D} \text{ in } \mathfrak{I} \text{ is countably infinite}}$$

where the set Φ contains only formulae in FOL. If we can somehow obtain such a set Φ, then we will have found a way to capture countable infinitude because the domain $\mathscr{D}$ here must be countably infinite. Can you meet this challenge, by drawing upon what was done in the previous section?

Now, what about finitude? Can it be captured by formulae in FOL? The question here can be taken to consist in the challenge to find a set Ψ such that a given interpretation

$$\mathfrak{I} \models \Psi \text{ iff domain } \mathscr{D} \text{ in } \mathfrak{I} \text{ is finite}$$

where, again, the set in question nce again contains only formulae in FOL.

3.8.4.1 Can First-Order Logic Capture Infinitude and Finitude?

Does the machinery introduced in the previous section enable us to show that the concepts of finitude and infinitude can be captured by suitable use of first-order logic? If so, how? We should first immediately sharpen this question, which as it stands is somewhat unclear. Let's first target the capturing of infinitude in FOL. Then our initial sharpening move is to stipulate that we are interested specifically in figuring out how we might use FOL to express that a set is countably infinite. (Recall that we defined what it is for a set to be countably infinite in §1.5.3.) In further sharpening of the intuitively expressed question that kicked off the present section, what shall be looking for is how to specify a set Φ that is such that a given interpretation

$$\boxed{\mathfrak{I} \models \Phi \text{ iff domain } \mathscr{D} \text{ in } \mathfrak{I} \text{ is countably infinite}}$$

$20!

where the set Φ contains only formulae in FOL. If we can somehow obtain such a set Φ, then we will have found a way to capture countable infinitude because the domain $\mathscr{D}$ here must be countably infinite. Can you meet this challenge, by drawing upon what was done in the previous section?

Now, what about finitude? Can it be captured by formulae in FOL? The question here can be taken to consist in the challenge to find a set Ψ such that a given interpretation

$$\mathfrak{I} \models \Psi \text{ iff domain } \mathscr{D} \text{ in } \mathfrak{I} \text{ is finite}$$

where, again, the set in question nce again contains only formulae in FOL.

$$\Phi := \{\underline{\phi_k} : k \in \mathbb{Z}^+\}$$

Remember from earlier today: This formula uses quantifiers to say there are at at least *k* objects.

$$\Phi := \{\underline{\phi_k} : k \in \mathbb{Z}^+\}$$

Remember from earlier today: This formula uses quantifiers to say there are at at least *k* objects.

$$\Phi := \{\underline{\phi_k} : k \in \mathbb{Z}^+\}$$

Remember from earlier today: This formula uses quantifiers to say there are at at least *k* objects.

$$\Phi := \{\underline{\phi_k} : k \in \mathbb{Z}^+\}$$

Any interpretation that renders all members of this set true must have a countably infinite domains, and *vice versa*!

Remember from earlier today: This formula uses quantifiers to say there are at at least *k* objects.

$$\Phi := \{\underline{\phi_k} : k \in \mathbb{Z}^+\}$$

Any interpretation that renders all members of this set true must have a countably infinite domains, and *vice versa*!

Now, can you find a set of formulae s.t. any interpretation that renders all members of it true must have a *finite* domain, and *vice versa*?

Remember from earlier today: This formula uses quantifiers to say there are at at least *k* objects.

$$\Phi := \{\underline{\phi_k} : k \in \mathbb{Z}^+\}$$

Any interpretation that renders all members of this set true must have a countably infinite domains, and *vice versa*!

Now, can you find a set of formulae s.t. any interpretation that renders all members of it true must have a *finite* domain, and *vice versa*?

$1000!

Remember from earlier today: This formula uses quantifiers to say there are at at least *k* objects.

$$\Phi := \{\underline{\phi_k} : k \in \mathbb{Z}^+\}$$

Any interpretation that renders all members of this set true must have a countably infinite domains, and *vice versa*!

Prove it!

Now, can you find a set of formulae s.t. any interpretation that renders all members of it true must have a *finite* domain, and *vice versa*?

$1000!

Remember from earlier today: This formula uses quantifiers to say there are at at least *k* objects.

$$\Phi := \{\underline{\phi_k} : k \in \mathbb{Z}^+\}$$

Any interpretation that renders all members of this set true must have a countably infinite domains, and *vice versa*!

Prove it!

Now, can you find a set of formulae s.t. any interpretation that renders all members of it true must have a *finite* domain, and *vice versa*?

$1000!

Selmer.Bringsjord@gmail.com

Hvis du forstår det, kan du bevise det.